Barter, Money and Direct Search Equilibrium

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Abstract

Given the adoption of crypto-bartering during the 2018 financial crisis in Venezuela, direct Internet search for bartering opportunities, or communities in Greece and Russia exploring barter systems, why does the use of direct search technology not undermine the importance of traditional forms of money? In this paper I demonstrate how a market with more types of goods leads to greater dependence on money for exchange transactions. Ultimately, agents self-impose a constraint on their search for goods only if they hold money when the number of good types is infinite or sufficiently large. This result serves to reconcile competing mainstream views by showing that the cash-in-advance constraint emerges as a subset of the money search model. Bartering exists if and only if the number of good types is sufficiently small that a double coincidence of wants is more likely to occur. The condition for existence of a monetary equilibrium depends on a large number of different specialized goods. The reversion to a monetary economy from sporadic bartering becomes inevitable when the economy expands and agents demand more types of specialized goods.

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1 Introduction

Random-matching models, spawned from two seminal papers by Kiyotaki and Wright (1989, 1993), have been common in the money search theory literature. These two papers are the successful attempts to brilliantly model money explicitly as a medium of exchange in economics models. Technical restrictions in those models have drawn criticism. One criticism is the randomness in matching. As noted by Howitt (2003), most people do not conduct their daily economic transactions in a random-matching setting. Randomness in matching is unrealistic but has been maintained in the models for tractability. Further developments in money search models have moved away from random-matching towards more directed-matching. For example, Corbae, Temzelides and Wright (2003) present a directed search model with a cooperative mechanism in an exchange market, while Goldberg (2007) introduces partial directed money search in a fully decentralized market.

For the analysis of monetary equilibrium, non-cooperative direct search is a more realistic feature than partial directed or cooperative directed search. People normally visit stores to get the goods they want without a third party coordinator. The entire search process is self-direct and each agent can generally reach the sellers of their desired good. The non-cooperative direct search model of money introduced in this paper is similar to the random money search model of Burdett, Coles, Kiyotaki and Wright (1995), henceforth BCKW, except that people directly search and know which available goods other agents sell. Ex-ante, there are many identical sellers of a good, so the searcher can either randomize the choice of seller or revisit a known seller to obtain the desired good. Unlike in partial directed search, a seller in non-cooperative direct search is always able to provide the good whenever approached by a buyer.

The random-matching models generated from Kiyotaki and Wright's (1993), henceforth KW, framework pose a technical challenge. An increasing number of good types, denoted k, will reduce the expected utility with or without money in a random search model[1]. When k is sufficiently large in a random search model, it will deter people from trade because the expected utility diminishes. Even with the presence of money, people will not choose to trade when there is a sufficiently large variety of goods because the expected utility diminishes as k increases. In the random meeting money search model, the role of trade and money would disappear and the expected utility approaches zero as $k \to \infty$. In other random meeting papers (e.g. BCKW), a higher k increases the likelihood of an autarkic equilibrium, even when money is introduced, given that search cost is imposed. This means the variety of goods is not always positively correlated to trade or to the role of money in the random-matching setting. However, it conflicts with the general perception that people are not less willing to hold money to trade when the variety of goods increases in the market. This paper shows that this is not the case in a direct money search model: when $k \to \infty$, the role of money becomes more apparent. This occurs because k is not in the matching probability function with money holdings in the present direct search model.

When the information of the seller's desire is absent, and the probability of a single coincidence of want matching is low because k is sufficiently large, money plays a role to facilitate exchange. This would be sufficient for money to appear as a medium of exchange.

When $k \to \infty$, the cash-in-advance constraint framework is shown to be a very special case of the money search framework. When there is a sufficiently large number of good types, trading without money is not profitable because the possibility of barter is near zero, so agents will only search for a good when they have money. The variety of goods is positively correlated with trade with money, but negatively correlated with trade without money. No agent will find it profitable to barter unless they have the cash in advance before searching for

[1]In Kiyotaki and Wright (1993), $\lim_{k\to\infty} rV_1 = \dfrac{\beta(1-M)}{k(k-1)}(U-\epsilon+V_n-V_1) + \frac{\beta M}{k}\pi(V_m-V_n-1) = 0$ and $\lim_{k\to\infty} rV_m = \dfrac{\beta(1-M)}{k}\Pi(U-\epsilon+V_n-V_m) = 0$ where V_1, V_m and V_n are the value functions for not holding money, holding money, and drawing a new preference shock. In the probability measure of k, the number of desired good types being searched has to be infinitely many. Otherwise, if k is finite, this gives a zero measure. In KW, $\frac{\partial V_1}{\partial k} < 0$ and $\frac{\partial V_m}{\partial k} < 0$.

a good. The cash-in-advance framework also proves to be a special case of the direct money search framework, where k only needs to be sufficiently large that there exists a transaction cost or search cost.

The model setup is described in the next section. In Section 3, I describe the decision problem faced by an agent. The autarkic equilibrium is also described in Section 3. In Section 4, a monetary equilibrium with direct search is presented. A comparison between the random-matching model and direct search, in terms of the effect of k, is shown in Section 5. A welfare comparison between random-matching and direct search is shown in Section 6. In Section 7, I conclude and discuss further lines of research.

2 The Model

The environment used in this paper is a non-cooperative direct search framework. The framework used in this paper draws its ideas mainly from BCKW, and KW.

2.1 The Environment

A continuum of agents, $i \in [0, 1]$, live in discrete time with an infinite horizon and a finite number of good types, $k \geq 3$, where k is a finite integer. Each good is indivisible, uniform in size and perishable at the end of each period. Each agent continues to specialize in producing one unit of an agent-specific commodity good at the beginning of each period[2] with a disutility (cost) of production, x. There are no production shocks, and each agent can use his own production for consumption or possible bilateral exchange.

Each agent is a generalist in consumption and receives positive utility, u, by consuming the type of good desired in that period, where $u > x$. Each agent is randomly assigned a preference for the type of good he initially desires. If he happens to desire his own production type, the agent will immediately consume it and draw a new preference. A new random

[2] This feature of producing a good in every period is a closer fit to the non-monetary general equilibrium model.

preference will be drawn at the end of the period only after the agent acquires the desired good. Otherwise the agent will continue to desire the same type of good. If the agent consumes a good not desired, his utility equals the disutility (cost) of producing a unit of the good and it gives zero net utility[3]. An agent always consumes his own production when there is no trade by the end of each period. The preference shocks are assumed to be identically and independently drawn from a known uniform distribution of good types. The preference drawn is private information known only to the agent, but his production type is public information.

Money is indivisible, not perishable, cannot be produced and is randomly assigned initially to a fraction of agents, m. Each agent can only hold one unit of money balances. A "non-moneyholder" possesses only one unit of a self-produced commodity good; a "moneyholder" possesses one unit of money in addition to one unit of a commodity good[4]. A non-moneyholder can only bring one unit of a commodity into the marketplace to trade; whereas a moneyholder can bring one unit of money or one unit of a commodity, or both. For each successful trade, both agents incur a transaction cost, ϵ.

There are two decisions to make in every period: (i) search or stay, and (ii) reject or accept trade. Both moneyholder and non-moneyholder can choose to search or stay. Each agent chooses to search and trade only once and meets only one agent in a given period. Otherwise, an agent has to wait for the next period to decide to search and trade again. The trading decision is straightforward in the sense that an agent will only trade when he finds the good he desires or when he is willing to exchange his production good for money. Besides that, no agent would want to trade for a good he does not desire because each good is perishable within the period and all agents exhibit self-interest; he would end up with negative net utility from the trade due to the transaction cost. So, we focus more on the decision to search or stay.

[3]This simplifies the model by ensuring that an agent will not receive any utility from consuming his own production when he does not desire it.

[4]The setup in the conventional money search framework differs from this paper, where an agent expects to produce in every period and is able to possess a unit of a good and money at the same time.

2.2 Search Process / Mechanisms

A moneyholder becomes a *buyer* when he exchanges his money for a good. A non-moneyholder becomes a *seller* when he trades his good for money. Anyone becomes a *barterer* whenever he swaps his good for another agent's good. If an agent gets his desired good, he consumes it immediately. All agents who search will return to their home at the end of the period.

All sellers of the same type of production are located in a specific marketplace known to all agents. The matching is endogenous such that the searcher self-directs exactly whom to meet. However, the searcher does not know the seller's desire. The information of the seller's desired good is revealed only after meeting. If an agent decides to search, he is assumed to approach only a person who holds and owns the good he desires. So he know with certainty who holds which goods, but he doesn't know which good(s) those persons desire. When a searcher arrives at a marketplace, the assumption of a continuum of agents means there are infinitely many identical sellers of the desired good type for potential trade[5]. A searcher is indifferent about the identical sellers at the marketplace. Hence, a searcher is always matched with a seller of the searcher's desired good. The searcher knows his desired good is *available*, but he does not know if one of the sellers will *accept* his offer[6]. This feature is more realistic than the partial directed search: we normally have prior knowledge about a seller's available stock of the desired good when we want to make a purchase, but we don't always know what the seller desires in return.

Every meeting between agents in this exchange economy will produce either a single coincidence of wants or double coincidence of wants. No coincidence of wants does not happen because an agent always directs himself to the seller of his desired good in this non-cooperative direct search[7]. Revisitation is permissible in this model, but has no measurable

[5]This is a mathematical convenience to assume away the number of agents in matching as long as not all agents are searching. The assumption of infinite agents is less realistic, but makes the model more tractable.

[6]Goldberg (2007) motivates the frictions in partially directed search with a probabilistic measure that a searcher will acquire a good from a seller of his desired good. Unlike in Goldberg's (2007) model, searchers here also know which seller's shop is open and has the capacity to supply the desired good.

[7]The absence of no coincidence of wants distinguishes this model from random-search or partially directed-search models.

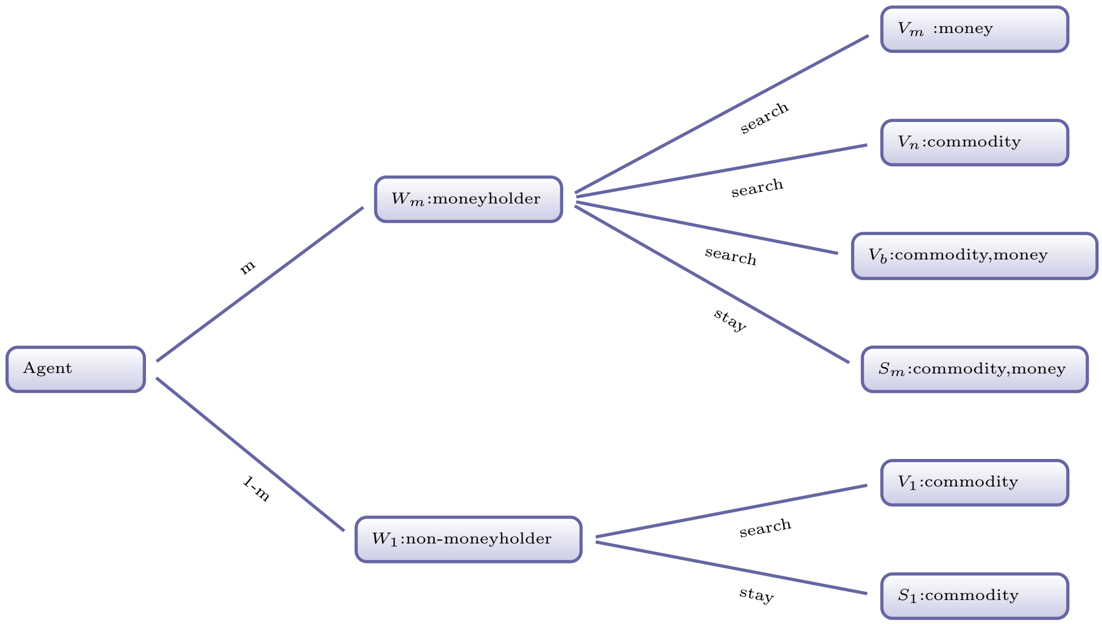

Figure 1: The roles and strategies of agents.

effect on payoffs. A searcher is indifferent between randomly picking or revisiting a seller because there is no advantage to knowing the seller's past desire from a bartering exchange in the previous period[8]. A searcher without money can only barter and a searcher with money can either barter or exchange with money if the seller is willing to accept.

On the other hand, the stayer cannot dictate who will visit him for an exchange. As a stayer, you remain at your store to welcome traders at no cost. A stayer who does not hold money can either barter or be a seller, and a stayer who holds money can only barter because of the assumed degenerate distribution of money holdings. The stayer has the opportunity to meet with those who are searching only for that specific good type. This is symmetric for all other agents.

[8]Let a searcher who draws the same preference choose to revisit the same seller who previously bartered with him. He may not be able to barter with the seller because the seller may desire a good other than the searcher's production good due to the seller's preference shock. When the searcher draws the same preference again and intends to barter with the same trader, the probability of a successful barter by revisiting a previously traded seller is the same as a random pick for any other identical seller.

2.3 Value Functions and Equilibrium

The probability you will meet an agent who wants your good is $\frac{1}{k-1}$, and the probability that you would want his good is always 1 because you would only look for the good you desire in this self-direct mechanism. The probability that a searcher would have the good you desire is $\frac{1}{k-1}$. Let m be the probability that you would meet with someone with money and $1-m$ without money.

Let c_1 and c_m be the cost of searching and transport with commodity and with money, where $c_1 > c_m > 0$. Assume that the cost of transport with money is zero, so c_m is simply the search cost[9]. The rate of time preference is $r > 0$, and π and Π denote the best response of an agent to accept money and the likelihood of acceptance of money by the other agent. Let n_1, n_m, n_b and n_s be the proportions of non-moneyholders who search with a good, moneyholders who prefer to search with money, moneyholders who search with both money and commodity, and moneyholders who prefer to search with a good.

Let W_j denote the value function for an agent, where $j \in \{1, m\}$; subscript 1 denotes a non-moneyholder and subscript m denotes a moneyholder who holds a unit of money and a unit of commodity. V_j and S_j denote the sub-value functions of a searcher and a stayer, respectively. V_c and V_b are special cases for a moneyholder who prefers to search with commodity only and with both money and commodity.

If the agent is a non-moneyholder, he chooses to search with commodity or stay at steady-state, so as to maximize:

[9]Because the transport cost with money is zero, c_m is the search cost, and so $c_1 - c_m$ is the transport cost of a commodity.

$$
\begin{aligned}
W_1 = max\{ & \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_1 \text{ :search with commodity}}, \\
& \underbrace{\frac{n_1(1-m)}{r(k-1)}U + \frac{mn_c}{r(k-1)}U + \frac{mn_b}{r(k-1)}U + \max_{\pi}[\pi(\frac{mn_m}{r} + \frac{mn_b}{r}\frac{k-2}{k-1})(-x-\epsilon+W_m-W_1)]\}}_{S_1 \text{ :stay with commodity}}
\end{aligned}
\tag{1}
$$

where[10] $U = \frac{(u-x)k}{k-1} - \epsilon$.

If the agent is a moneyholder, he chooses to search with money at steady-state, with commodity, with money and commodity, or stay with money and commodity, so as to maximize:

$$
\begin{aligned}
W_m = max\{ & \underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m] - \frac{c_m}{r}}_{V_m \text{ :search with money}}, \quad \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_c \text{ :search with commodity}}, \\
& \underbrace{\frac{\Pi(1-m)}{r}(1-\frac{1}{k-1})[U+x+W_1-W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r} - \frac{c_m}{r}}_{V_b \text{ :search with money and commodity}}, \\
& \underbrace{\frac{n_1(1-m)+m(n_c+n_b)}{r(k-1)}U}_{S_m \text{ :stay with money and commodity}} \}
\end{aligned}
\tag{2}
$$

An equilibrium is defined to be a Nash equilibrium such that each agent chooses the pure and stationary strategy which maximizes his expected utility, contingent upon other agents' strategies and the distributions of n_1, n_m, n_b, n_s resulting from the strategies chosen by other

[10]Similar to BCKW, if you draw a preference desiring your own good, you would consume it immediately and then draw a new preference. The value function of drawing a preference desiring your own production type, V_0, is:

$$V_0 = (u-x) + \frac{1}{k}V_0 + \frac{k-1}{k}max\{V_1, S_1\} = \frac{k}{k-1}(u-x) + max\{V_1, S_1\}$$

u denotes the utlity in consuming one unit of desired good, and, U denotes the expected utility gained from acquiring and consuming his desired good, including a draw to desire his own production type from a taste shock.

agents.

Assumption 1. *A continuum of agents means there are infinitely many identical sellers of the desired good type for potential trade as long as the fraction of agents who search is not equal to 1.*

Assumption 2. $u \geq x > c_1 > c_m > \epsilon > 0$.

Lemma 1. $V_i, S_i \geq 0$ *for* $i \in \{1, m, c, b\}$.

Any strategy used must have a value greater than or equal to zero because an agent could choose the strategy to consume his own production forever to have a zero expected utility.

3 Non-monetary Equilibrium

In this model, the friction of matching is due to incomplete information: not knowing the desire of the seller. If knowledge of the seller's desire can be easily acquired, money is not needed for the agents to perfectly coordinate. Hence, a non-monetary Walrasian equilibrium could occur when everyone knows each seller's desire.

Lacking information of the seller's desire, there can be two possible non-monetary equilibria in a direct search money model. First, a non-monetary autarkic equilibrium could exist when the searching strategies imply that no one is willing to search. Everyone stays, and no trade occurs (similar to the results of Goldberg (2007) and BCKW).

Proposition 1. *There exists a non-monetary autarkic equilibrium iff* $c_1 > \frac{U}{(k-1)}$ *and* $c_m > \Pi(1-m)[U+x]$.

This result appears to be consistent with the standard money search models: (i) a non-monetary autarkic equilibrium exists when the cost of searching with a commodity and money is higher than the possible gain from trade, (*ii*) no agent would want to search with commodity when the variety of goods is sufficiently large which diminishes the utility gained from trade, and (*iii*) no agent wanting to search with money depends on the probability of money being accepted and the fraction of money supplied to agents in the market. This makes the probability of meeting another agent willing to accept money low, making it unprofitable to search with money and a commodity. When each agent believes that all agents follow these strategies, his optimal strategy is to stay. Since no agent chooses to search, no trade occurs. This is an interesting result, in that the threshold of searching with money is independent of the number of good types.

Proposition 2. *There exists a non-monetary barter equilibrium iff* $c_1 < \frac{U}{k-1}$, *with the resulting distribution of* $n_1 = 1 - \frac{c_1(k-1)}{u}$ *and* $c_m > \Pi(1-m)[U+x]$.

When the number of good types is very low and the value of having money as a medium of exchange is very small, a barter economy can emerge. This is consistent with a number of observable cases. In ancient or primitive economies, where the variety of goods is limited and the number of traded goods is low, bartering tends to prevail. Similarly, even in more modern war-torn economies, survival depends on fewer types of goods, primarily food and shelter, increasing the reliance on barter transactions. Indeed, bouts of hyperinflation during such periods might be viewed as efforts to flee from the use of money in favor of the direct exchange of goods.

4 Monetary Equilibrium

A monetary equilibrium is a Nash equilibrium with a list (W_1, W_m) that satisfies the incentive condition to hold money, $-x - \epsilon + W_m - W_1 > 0$, such that the gain in accepting money exceeds zero, given a stationary distribution of n_1, n_m, n_s. Only the equilibria where people accept money with probability $\Pi = \pi = 1$ are considered.

There would exist a monetary equilibrium when agents get higher value by choosing the strategy to accept money when the cost of searching is lower than the benefit of being matched with the desired good in exchanging with money. The cost of searching with a commodity must exceed the benefit of being matched in a bartering exchange, given the utility in successfully acquiring the desired good and the probability of getting matched $\frac{1}{k-1}$.

Proposition 3. *There exists a monetary equilibrium if a moneyholder would not carry only a commodity to search.*

A moneyholder would not carry only a commodity to the market to search because this implies a moneyholder chooses a strategy for which the payoff is the same as for a non-moneyholder. No one would accept money if a non-moneyholder can be equally as well-off as a moneyholder. If that is the case, monetary equilibrium does not exist. A moneyholder who carries only a commodity to search implies that search with money is not profitable; this strategy is not possible in monetary equilibrium. It must be the case that searching with money is profitable for a moneyholder; so this strategy of carrying only a commodity to the marketplace is Pareto dominated in a monetary equilibrium.

Lemma 2. *When an agent searches with money and a commodity, he will choose the strategy to barter if possible; if not, then he would offer money in exchange for a good.*

Proposition 4. *In monetary equilibrium, a moneyholder would carry both money and a commodity to search iff* $\frac{U}{k-1} + c_m < c_1 < \frac{1}{k-1}[mU + (1-m)x] + c_m$*, or there exists a* k *such that* $\frac{U}{c_1 - c_m} + 1 < k \leq \frac{mU+(1-m)x}{c_1 - c_m} + 1$.

It would not be profitable for the moneyholder to search with money and a commodity if the cost of carrying the commodity, $c_1 - c_m$, exceeds the gain of engaging in barter with

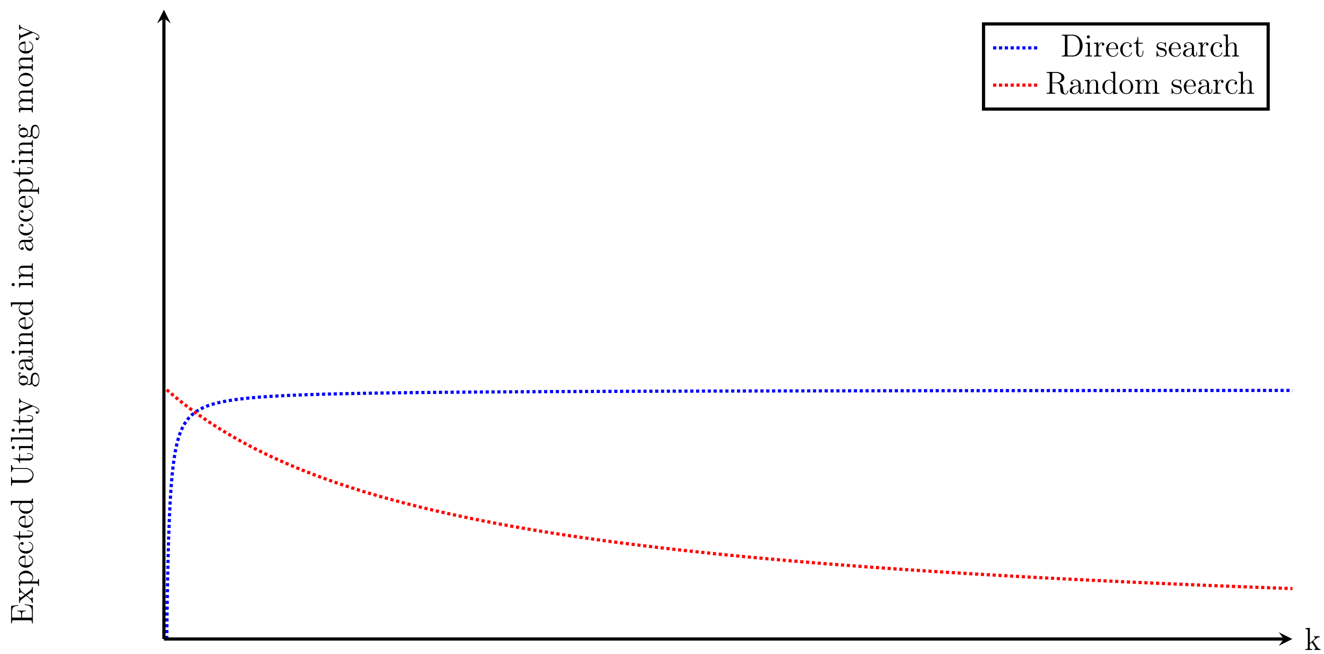

Figure 2: A comparison of utility gained in accepting money between random and direct money search frameworks.

a probability of $\frac{1}{k-1}$. Searching with both will save on the search cost, since search cost is incurred only once for each search, regardless if the agent carries money, a good, or both.

Proposition 5. *In monetary equilibrium, a moneyholder would carry only money to search iff* $c_1 > \frac{1}{k-1}[mU + (1-m)x] + c_m$ *or there exists a sufficiently large* k *such that* $k > \frac{mU+(1-m)x}{c_1-c_m} + 1$.

Figures 2, 3, and 4 show some numerical results for this model. For all numerical results, we set $r = 0.001, m = 0.5, c = 0.01, x = 0.1$ and $U = 1$. Molico (2006), who provides a good example of numerical analysis of a money search model, finds that $r = 0.001$ yields an approximately normal distribution for money holdings and other key variables. In my model, welfare is maximized when m is set to approximately 0.5, and the decision to set $U = 1$ follows BCKW, mostly for reasons of tractability. The value $c = 0.01$ and $x = 0.1$ are used to introduce a simple form of transaction and production costs to the model.

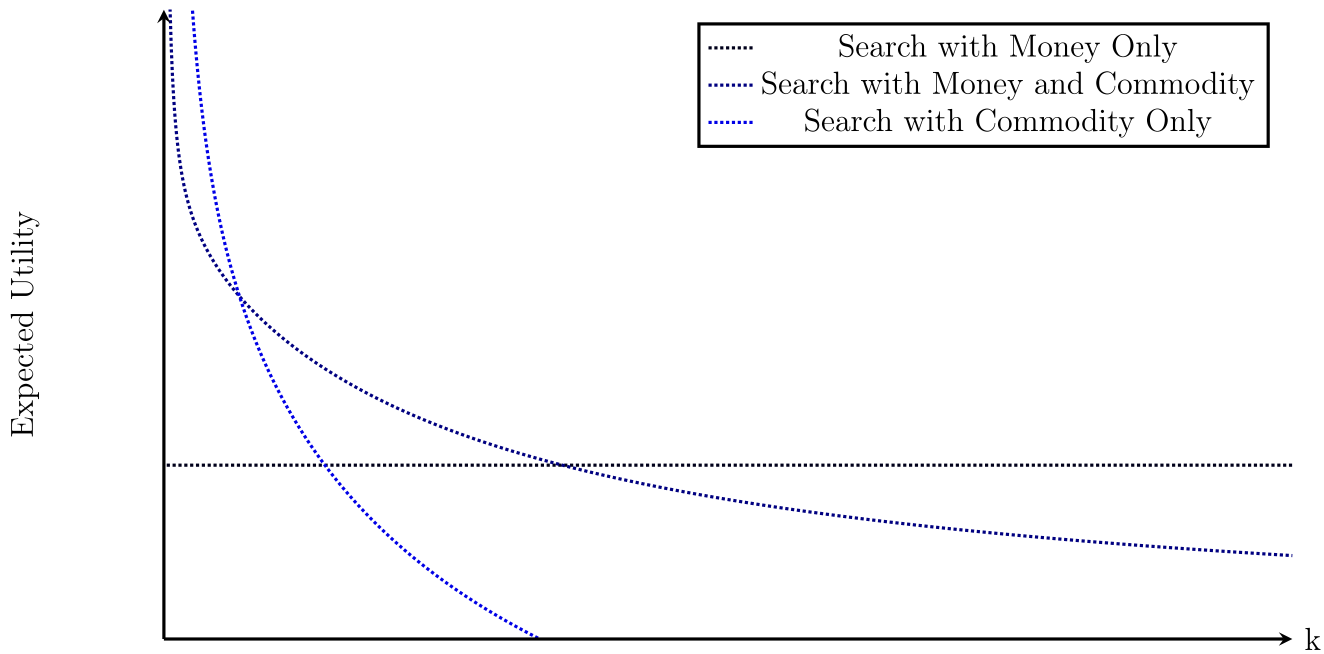

Figure 3: Expected utility for different searching strategies of a moneyholder at steady state in non-cooperative direct money search.

Figure 2 shows that the gain for accepting money in the random money search model diminishes in k, as in BCKW. Note, however, that direct money search shows that the gain for accepting money monotonically increases in k. This is particularly important, in that it conforms with economic intuition about the gain in accepting money being positively related to the number of good types, k. The gain from searching with a commodity diminishes when there are more type of goods in the market.

In a random search framework, the expected utility of a moneyholder and a barterer in monetary and non-monetary equilibria approaches 0 when k approaches infinity. With random money search, the likelihood of searching with money decreases because the expected utility of accepting money decreases with an increasing number of good types, k. However, in reality, the opposite seems to be more plausible: having more types of good to choose from should not decrease the value of holding money. In the random search framework, whether money is introduced or absent, the expected utility converges to zero as the number of good types increases, implying that no one would trade and the system would converge

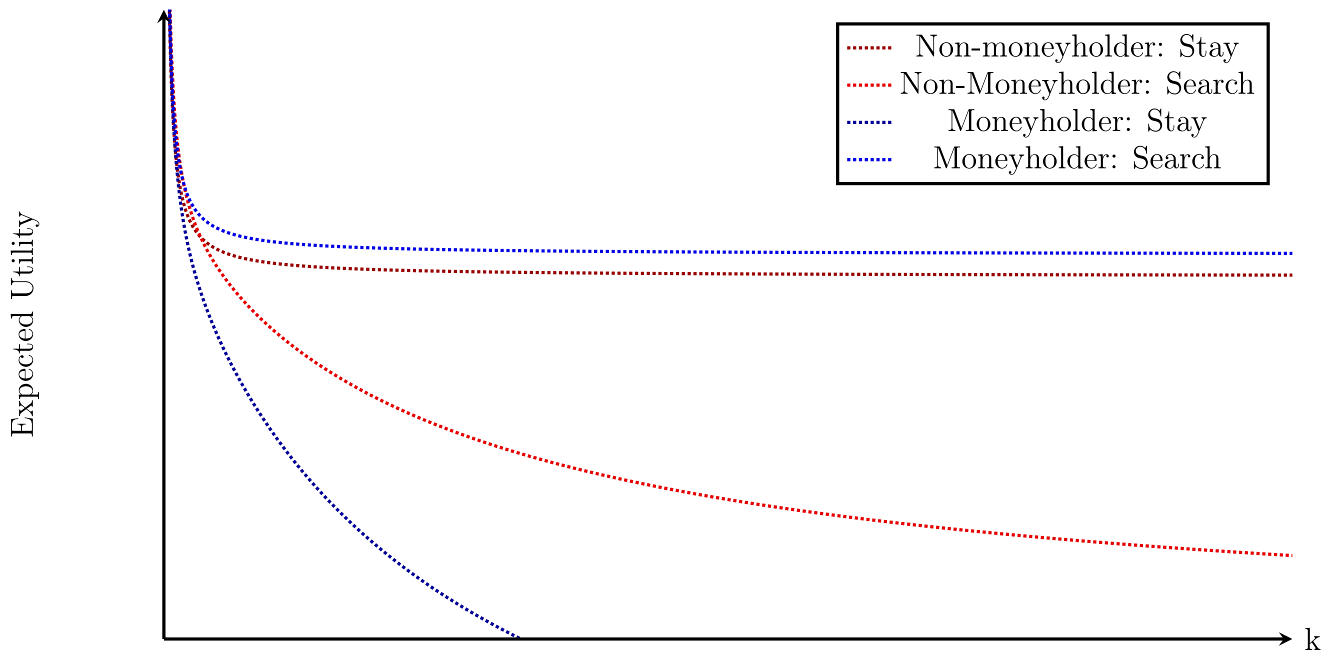

Figure 4: Expected utility at steady state for value functions in non-cooperative direct money search.

to an autarkic equilibrium.

Even given some search cost in random search[11], there exists a sufficiently large k such that the expected utility of a moneyholder and barterer will be zero. Hence, it would result in autarkic equilibrium for any given value of m, c_1, c_m, r and x. It seems unrealistic that, when the number of goods is large enough, no one will trade with or without money. For example, we have a healthy monetary exchange economy with millions of types of good in the world today.

In comparison, non-cooperative direct search presents a result closer to the general economic intuition that the benefit of accepting money increases or at least does not diminish with k.

Proposition 6. *When $k \to \infty$, monetary equilibrium still exists and a moneyholder would*

[11] The traditional KW random search framework contains storage costs rather than search costs, while BCKW use search cost in their money search model.

carry only money to search for any $c_1, c_m, r, m > 0$ *and* $m, r \in (0, 1)$*.*

Figure 3 shows different value functions for the direct money search model. The value function for holding money and a commodity to search converges to a positive value as k approaches infinity. The value function of searching with a commodity only quickly goes to zero as k increases. This result has an interesting implication: agents will be unlikely to search without money unless there are few good types in the market. The more good types in the market the more likely the agents will search with money and commodities or money alone. These intuitive results follow from the present direct search framework, but contradict the earlier random search models.

Corollary 1. *The cash-in-advance constraint framework is a (strict) special case of the money search model with conditions that satisfy Proposition 5 (Proposition 6).*

People will only search and buy when they have money in hand. This phenomenon is equivalent to the cash-in-advance constraint framework. That being said, the cash-in-advance framework can be regarded as a special case of the direct money search model, where the agent would search only when they have the money (or cash) in hand when the search cost is positive and k is large enough to deter search with a commodity, which appears to be unprofitable. This implies that for any given parameter values, there exists a unique solution to the equation for monetary equilibrium.

5 Conclusion

People have different desires and needs at different times. Without perfect knowledge of the agent's desire, there exists a role for money in exchange, given a sufficiently large number of good types. Compared to the the random search framework, direct money search conforms better with basic economic intuition. The idea that the monetary equilibrium using random search becomes infeasible with a sufficiently large number of goods is particularly troubling.

In the direct money search model with a less rigid form of the Wicksellian triangle presented here, monetary equilibrium can be satisfied with increasingly large k. The inefficiency of bartering due to low probability of matching depends on the large number of good types. In addition, the value of accepting money is at least non-decreasing when the number good types increases. Generally, the variety of goods is positively correlated with trade with money, however, random matching in money search models leads to a *negative* correlation. Given these technical challenges, there must exist other reasons to proceed with the equivalent class of random money search models. The use of random money search may be applicable to specific cases, but non-cooperative direct money search appears to be a useful alternative.

By focusing on the number of types of goods in a direct money search model, this paper offers a novel connection between two well-known frameworks, the cash-in-advance framework and the money search framework. This connection may serve as a starting point for intuitively understanding both frameworks and combining their features to improve the tools of monetary policy analysis. In this regard, future research should fully extend the integration of cash-in-advance models with money search models.

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A Mathematical Appendix

Proof of Lemma 1. An agent can always choose to stay at home or at his market, which gives him a lower bound of zero utility. It costs him nothing to stay at home or his market. If other strategies give value lower than zero, he will always choose to stay home. So, he can always choose the strategy to stay home or at his market. Hence the value function cannot be lower than zero. This lemma is also established by Goldberg (2007). □

Proof of Lemma 2. If an agent carries both money and a good into the marketplace for trade, he would barter to get his desired good if he finds a seller who likes his good. Otherwise he would exchange with money, unless the seller is already a moneyholder.

Suppose that exchanging with money is preferred to bartering when a searcher brings both money and a good to the market.

$$\begin{aligned}&\underbrace{\frac{\Pi(1-m)}{r}[U+x+W_1-W_m]+\frac{mU}{r(k-1)}-\frac{c_1}{r}}_{\text{where the offer of money is preferred for trade}}\\&>\underbrace{\frac{\Pi(1-m)}{r}(1-\frac{1}{k-1})[U+x+W_1-W_m]+\frac{U}{r(k-1)}-\frac{c_1}{r}}_{\text{where the bartering is preferred for trade}}\end{aligned} \tag{A.1}$$

The RHS denotes the value function for a searcher searching with a good and money who prefers to barter, whereas the LHS denotes the value function for a searcher seaching with a good and money who prefers to offer money in exchange for a good. The inequality becomes:

$$\frac{mU}{r(k-1)}>\frac{-\Pi(1-m)}{r(k-1)}[U+x+W_1-W_m]+\frac{U}{r(k-1)} \tag{A.2}$$

Rearranging this inequality gives

$$U(1-\frac{1}{\Pi})+x>W_m-W_1 \tag{A.3}$$

The incentive condition for holding money is

$$-x-\epsilon+W_m-W_1>0 \text{ and } \Pi=1, \tag{A.4}$$

hence, $W_m - W_1 > x + \epsilon > x$ for $\epsilon > 0$.

But the inequality from the incentive condition for holding money and $\Pi = 1$ imply:

$$x > W_m - W_1, \tag{A.5}$$

which contradicts.

□

Proof of Proposition 1. Assume that an agent will choose not to search if the alternative strategies give the same utility. We have to show that no one would accept money in an exchange and bringing money to market are redundant, such that,

$$max\{V_1, S_1\} \geq -x - \epsilon + max\{V_m, V_b\} \tag{A.6}$$

and no one would choose to search. To show that no one would choose to search, it also means that all agents would prefer to stay, such that $V_1 \leq S_1 = 0, max\{V_m, V_b\} \leq S_m = 0$. This implies that $n_1, n_c, n_m = 0$. This causes the Equations [1] and [2] to become:

$$W_1 = max\{ \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_1 \text{ :search with commodity}}, \underbrace{0}_{S_1 \text{ :stay with commodity}} \} \tag{A.7}$$

$$W_m = max\{ \underbrace{\frac{\Pi(1-m)}{r}[U + x] - \frac{c_m}{r}}_{V_m \text{ :search with money}}, \underbrace{\frac{U}{r(k-1)} - \frac{c_1}{r}}_{V_c \text{ :search with commodity}}, \underbrace{0}_{S_m \text{ :stay with money and commodity}} \} \tag{A.8}$$

In a non-monetary autarkic equilibrium, $S_1 > V_1$, therefore:

$$c_1 \geq \frac{u}{(k-1)} \tag{A.9}$$

and, $S_m \geq V_m$ gives:

$$c_m \geq (1-m)[U + x] \tag{A.10}$$

Since no one chooses the strategy to search, no matching will occur and no exchange can take place. This is an equilibrium because the cost of searching exceeds the expected gain in utility from matching.

□

Proof of Proposition 2. For a non-monetary bartering equilibrium, agents are indifferent between searching with a commodity and staying, such that $V_1 = S_1$, $V_c = S_m$, and $max\{V_c, S_m\} > V_m$. This implies that $n_m = n_b = 0$, so the Equations [1] and [2] become:

$$W_1 = max\{\frac{U}{r(k-1)} - \frac{c_1}{r}, \frac{n_1 U(1-m)}{r(k-1)} + \frac{mn_c U}{r(k-1)}\} \tag{A.11}$$

$$W_m = max\{\underbrace{\frac{\Pi(1-m)}{r}[U+x] - \frac{c_m}{r}}_{V_m\text{ :search with money}}, \frac{U}{r(k-1)} - \frac{c_1}{r}, \frac{n_1 U(1-m)}{r(k-1)} + \frac{mn_c U}{r(k-1)}\} \tag{A.12}$$

When a moneyholder abandons searching with money because $max\{V_c, S_m\} > V_m$, his payoff for W_1 is the same as for a non-moneyholder. This means their strategic behavior is the same, so the distribution of agents with or without money will be the same, such that $n_1 = n_c = \theta$.

Letting $n_c = n_1 = \theta$ gives:

$$\frac{u}{r(k-1)} - \frac{c_1}{r} = \frac{\theta u(1-m)}{r(k-1)} + \frac{\theta um}{r(k-1)} \tag{A.13}$$

and solving for θ gives:

$$\theta = 1 - \frac{c_1(k-1)}{u} \tag{A.14}$$

and, $0 < \theta < 1$ since $0 < n_1, n_c < 1$, then

$$\frac{u}{c_1} + 1 > k > 1 \tag{A.15}$$

It must be the case that the value of searching with a commodity is greater than 0, satisfying Lemma 1.

$$c_1 < \frac{u}{k-1} \tag{A.16}$$

$$k < \frac{u}{c_1} + 1. \tag{A.17}$$

$\square$

Proof of Proposition 3. To search with money, the condition for a moneyholder is:

$$max\{V_m, V_b\} > max\{V_c, S_m\} \tag{A.18}$$

To accept money in exchange for a good, the condition for the non-moneyholder is

$$-x - \epsilon + max\{V_m, V_b, V_c, S_m\} > max\{V_1, S_1\} \tag{A.19}$$

From the condition to search with money, the equation above can be simplified to

$$-x - \epsilon + max\{V_m, V_b\} > max\{V_1, S_1\} \tag{A.20}$$

It would be sufficient to show that $W_m > W_1$ to ensure the monetary equilibrium condition is satisfied. Thus, V_c is redundant to check in the sense that $W_m > W_1$ equals $W_m > max\{V_1, S_1\}$ where $W_1 = max\{V_1, S_1\}$. Given that the value function of a moneyholder searching with commodity only is V_c, $V_c = V_1$ means $W_m > max\{V_1, S_1\} \geq V_c$. As long as the condition for monetary equilibrium is satisfied, search with a commodity is not profitable for a moneyholder.

□

Proof of Proposition 4. From Proposition 2, we know that an agent will only search when $c_1 > \frac{u}{k-1}$, hence the LHS of the inequality is satisfied such that $k > \frac{u}{c_1} + 1$.

As for the RHS of the inequality, from Proposition 3, we know that the inequality must satisfy:

$$\begin{aligned} &\underbrace{\frac{\Pi(1-m)}{r}[U + x + W_1 - W_m] - \frac{c_m}{r}}_{\text{searching only with money}} \\ &\le \underbrace{\frac{\Pi(1-m)}{r}(1 - \frac{1}{k-1})[U + x + W_1 - W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r}}_{\text{searching with money and a commodity}} \end{aligned} \tag{A.21}$$

$$c_1 \le \frac{1}{k-1}[mU + (1-m)[x + W_1 - W_m]] + c_m \tag{A.22}$$

Hence, there exists a k such that $k \le \frac{mU+(1-m)[x+W_1-W_m]}{c_1-c_m} + 1$.

□

Proof of Proposition 5. For an agent to choose a pure strategy of searching only with money, the value function of searching with only money must exceed the value of holding both money and a commodity, as follows:

$$\begin{aligned} &\underbrace{\frac{\Pi(1-m)}{r}[U + x + W_1 - W_m] - \frac{c_m}{r}}_{\text{searching only with money}} \\ &> \underbrace{\frac{\Pi(1-m)}{r}(1 - \frac{1}{k-1})[U + x + W_1 - W_m] + \frac{U}{r(k-1)} - \frac{c_1}{r}}_{\text{searching with money and a commodity}} \end{aligned} \tag{A.23}$$

$$c_1 - c_m > \frac{-\Pi(1-m)}{k-1}[U + x + W_1 - W_m] + \frac{U}{(k-1)} \tag{A.24}$$

$$c_1 - c_m > \frac{1}{k-1}[U(1-\Pi(1-m)) + \Pi(1-m)[x + W_1 - W_m]] \tag{A.25}$$

For a monetary equilibrium, Π is assumed to be 1. Hence,

$$c_1 > \frac{1}{k-1}[mU + (1-m)[x + W_1 - W_m]] + c_m \tag{A.26}$$

The above inequality is satisfied when k is sufficiently large. □

Proof of Proposition 6. For any r, m, c_1, c_m :

$$\lim_{k\to\infty} V_1 = \lim_{k\to\infty} \frac{U}{r(k-1)} - \frac{c_1}{r} = -\frac{c_1}{r} < 0 \tag{A.27}$$

$$\lim_{k\to\infty} S_m \leq 0 \tag{A.28}$$

$$\lim_{k\to\infty} S_1 \geq 0 \tag{A.29}$$

$$\lim_{k\to\infty} (V_m - V_b) \geq 0 \tag{A.30}$$

$$\lim_{k\to\infty} (V_m - S_1) \geq 0 \tag{A.31}$$

This satisfies the incentive constraint for holding money such that $V_m = max\{V_m, V_b\} > S_1 \geq 0$ and Lemma 1 for the exchange participating constraint. □

Table 2.1: A table showing the sensitivity analysis for the expected utility of holding money.