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**Abstract** Specialized production tasks are often performed by laborers equipped with the same technical capabilities to accomplish other tasks equally well. I demonstrate how production among laborers given identical technology with no comparative advantage leads to a complete specialization of different tasks. The switch cost between production tasks is formalized to show how the division of labor maximizes the output with the same technology among laborers. The degree of specialization is limited by the cost of switching among tasks. The improvement of technology in production that reduces switch cost makes workers or firms choose not to specialize in tasks.

**Keywords** Specialization · Switch cost · Division of Labor · Production

## 1 Introduction

A coffee store like Starbucks often trains the workers to be able to perform all tasks. However, each worker is assigned to specialize only on one task at a time. For example, one worker takes the orders and another one makes the drink. Could there be specialization if all laborers have no comparative advantage? This article contributes to the literature by showing that workers who have no comparative advantage choose to completely specialize on one task and the production becomes more efficient. This article formalizes the idea of switch cost, which is often recognized as a cost to adapt to a different task or as the degree of cognitive flexibility in industrial organization in the

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field of psychology. The idea of specialization is to minimize or avoid the switch cost. Smith (1776) proposes the idea of specialization within a pin factory that production would become more efficient when the laborers engaged in specialization. Often the knowledge and technical capabilities in a standardized production among the laborers are simple and often costless to acquire them. Even though those workers can perform all simple tasks within the production, they are all assigned to perform on a specific task at a given time. This paper studies the decision-theoretic foundation of tasks organization by establishing the condition for strategic interactions.

Some studies show that different comparative advantage among workers that leads to specialization like Stilger (1951), Roy (1951), Rosen (1978), and Kim (1989). Other studies like Becker and Murphy (1993), Garicano (2000), Hart and Moore (2005), and Alonso, Dessein and Matouschek (2008) show coordination cost is one of the determining factor and extent of specialization. This article complements the past literature because it provides a formal illustration on immediately intuitive findings of the cause of division of labor with identical technology. When there is an improvement in technology that reduces switch cost, the production in firms become less specialized.

## 2 A Simple Case

Suppose two workers need to perform two complementary tasks, $x_1$ and $x_2$, to complete a production of a good. The two tasks can be viewed as two types of goods that need to be completed to satisfy a production preference. A strictly convex production preference is assumed that there is no perfect substitution because both tasks are required to complete a good. The production incurs a switch cost, $c > 0$, when a worker switches between tasks and spent the resource to adapt for the new task. For illustration purposes, let's assume the time is the resource constraint faced by the workers.

Without loss of generality, the required time to perform each task is normalized to 1. Each worker faces with an identical resource constraint, $t = \min\{x_1, \alpha x_1 + (1-\alpha)x_2 - c, x_2\}$ and $0 \leq \alpha \leq 1$, which can be illustrated as the convex production possibilities frontier in solid red line shown in the Figure 1 and 2. Given a strictly convex production preference function, firms maximize the output $y(x_1, x_2)$ subject to the constraint $t(x_1, x_2)$.

If they choose to generalize and perform both tasks, they would incur a switch cost. Let $a = \alpha x_1^A + (1-\alpha)x_2^A - c$ and each worker produces $y(a)$ for a strictly convex production preference function.

If they choose to divide the labor and specialize only on one task, no switch cost is incurred. Then both workers specialize to produce at $x_1^A$and $x_2^A$ respectively. Let $a' = \alpha x_1^A + (1-\alpha)x_2^A$. The average output for division of labor is $y(a')$. Since $a' > a$, it implies that

$$y(a') > y(a) \tag{1}$$

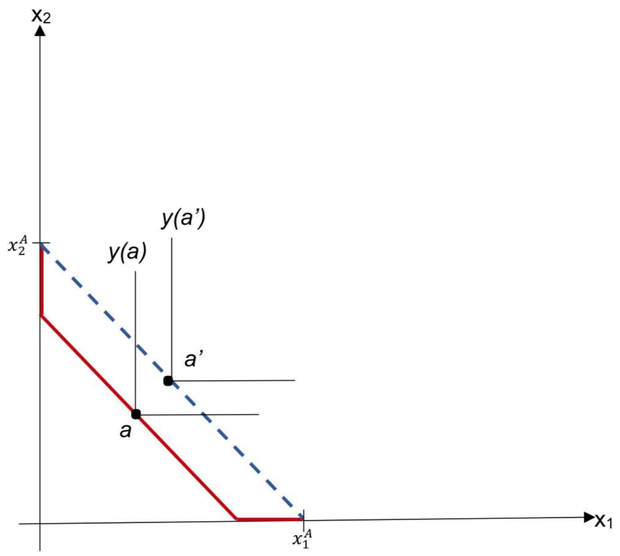


**Fig. 1** A convex production possibilities frontier shows an increase in output before (in red line) and after (in blue line) specialization.

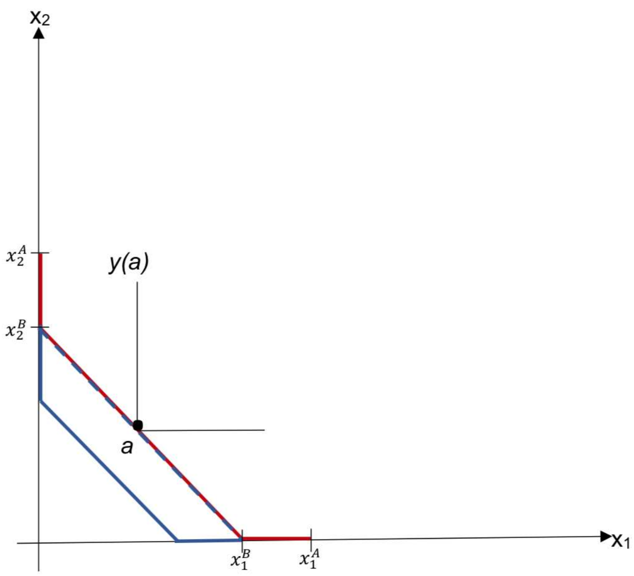


**Fig. 2** A convex production possibilities frontier shows a decrease in cost before (in red line) and after (in blue line) specialization.

**Proposition 1** *Specialization in the division of labor increases output if there exists a switch cost between tasks.*

When a firm produces at a fixed level of output, the objective of a firm is to minimize the cost $t(x_1, x_2)$ subject to the constraint $y(x_1, x_2)$. If they divide the labor and have the workers specialize on one task to produce the same output level, no switch cost is incurred. Suppose $x_1' = x_1 - c$ and $x_2' = x_2 - c$, then $t' = \min\{x_1', \alpha x_1' + (1-\alpha)x_2' - c, x_2'\}$. Then both workers specialize to produce at $x_1' = x_1^B$ and $x_2' = x_2^B$ respectively. With the division of labor, both workers produce an average output level such that $y(\alpha(x_1^A - c) + (1 - \alpha)(x_1^A - c)) = y\,(a)$. Since $c > 0$, it implies that

$$t' < t. \tag{2}$$

**Proposition 2** *Specialization in division of labor reduces cost if there exists a switch cost between tasks.*

## 3 A General Case

In this section, I illustrate a general case with $n$ complementary tasks and $n$ workers with identical technical capabilities. $y\,(x)$ represent a strictly convex production function with $x$ representing $n$-vector of tasks. The objective of a firm is to maximize the output $y\,(x)$ subject to the resource constraint $t\,(x)$. The resource constraint, $t = \min\{(n-1)c + \sum\limits_{i=1}^{n} \alpha_i x_i, x_1, x_2, ..., x_n\}$ for each worker is identical where $\sum\limits_{i=1}^{n} \alpha_i = 1$.

If the workers choose to generalize their production that each worker performs all tasks, each worker incurs a switch cost, $(n-1)c$. Let $a = (n-1)c + \sum\limits_{i=1}^{n} \alpha_i x_i$ and each worker produces $y\,(a)$.

If the workers choose to completely specialize so that each worker performs only one task, then every worker produces at $x^A$and no switch cost incurs. The average output per worker is $y\,(a')$ where $a' = \sum\limits_{i=1}^{n} \alpha_i x_i$. Since $a' > a$, this implies

$$y\,(a') > y\,(a)\,. \tag{3}$$

This proof above reaffirms Proposition 1 for the general case of division of labor.

Suppose only some $m$ tasks are generalized and the remaining $n - m$ tasks are specialized. Then, the average output is $y\,(a'')$ where $a'' = (n-m-1)c + \sum\limits_{i=1}^{n-m} \alpha_i x_i + \sum\limits_{i=n-m+1}^{n} \alpha_i x_i$. Since $a' > a''$, this implies that $y\,(a') > y\,(a'')$. Thus, some degree of specialization of tasks is not the most efficient way to organize a production, either maximizing output or minimizing cost.

Similarly, the proof for the general case for Proposition 2 is straightforward, thus is not shown here. A full complete specialization in a division of labor is the most efficient arrangement of production.

## 4 A Case with Increasing Opportunity Costs

Given the production technology with an increasing opportunity cost where the production possibilities frontier bows out, there is a combination of mixing tasks that yields a higher output than producing more of one task in favor of another task. In relation to a typical Ricardian or Heckscher-Ohlin model, some partial specialization is optimal and never a complete specialization. This section shows a complete specialization is optimal and preferred for a technology with an increasing opportunity cost when a large switch cost is present.

Let $y\,(a)$ be the most efficient production for a technology with an increasing opportunity cost. Each worker faces with an identical resource constraint, $t = \min\{x_1, f(x_1, x_2) - c, x_2\}$ and $0 \leq \alpha \leq 1$, where $f(x_1, x_2)$ is a concave function in $x_1$ and $x_2$ and is illustrated as the production possibilities frontier in solid red line shown in the Figure 3. The value for $f(x_1, x_2)$ at the axis is $x^0$. Suppose it is the most efficient production at $a$ when one worker produces two tasks. A tangent at $a$ is defined such that $f\,(a) + \partial a f'\,(a) \geq f(x_1, x_2)$. Let the intercept for the tangent at $a$ to be $x_1^c$ and $x_2^c$ for both axis respectively. So, $y\,(a) = y(\alpha x_1^c + (1-\alpha)x_2^c)$. Workers choose to specialize if $y\,(a') > y\,(a)$. Then $\alpha x_1^A + (1-\alpha)x_2^A > \alpha x_1^c + (1-\alpha)x_2^c$ becomes $\alpha(c + x_1^0) + (1-\alpha)(c + x_2^0) > \alpha(x_1^c + x_1^0 - x_1^0) + (1-\alpha)(x_2^c + x_1^0 - x_1^0)$ by substituting $x_0$ into the equation, and can be further simplified to $c > \alpha(x_1^c - x_1^0) + (1-\alpha)(x_2^c - x_2^0)$. The organization of workers are more efficient when they specialize at $x_1^A$ and $x_2^A$ if and only if

$$c > x^c - x^0. \tag{4}$$

Workers choose to specialize when the cost of switching between tasks outweighs the benefit of completing multi-task by a person due to increasing opportunity cost. Instead of choosing to mix the tasks when the technology is a concave function, workers are more efficient with division of labor if the switch cost is sufficiently large. Conventionally, workers always choose partial specialization when the technology is a concave function. This result presents a different insight that workers can be more efficient with complete specialization when the technology is a concave function.

Conversely, workers choose to not specialize in tasks when the switch cost becomes low such that $c < x^c - x^0$. We often observe that a workers tend to perform more tasks when there is an improvement of technology in a production that reduces switch costs among tasks for workers. So, the degree of specialization is also limited by the cost of switching among tasks. The finding is counterintuitive that the improvement in technology always results in more specialization. When most tasks have an increasing opportunity cost, the higher switch cost, the higher the degree of specialization. It shows that the

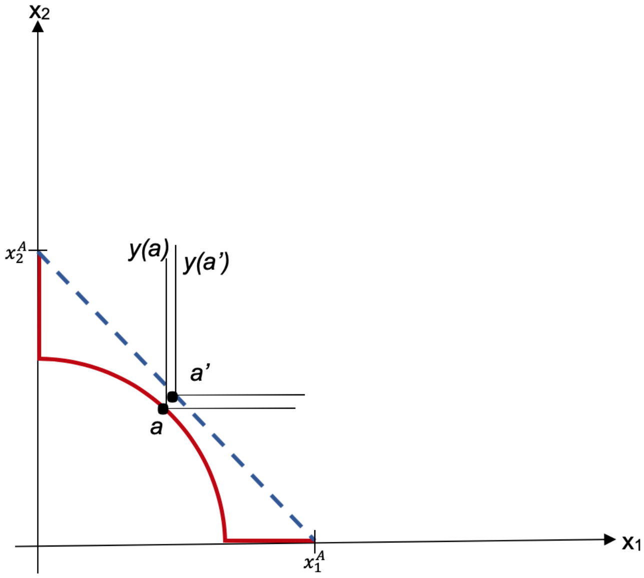


**Fig. 3** A production possibilities frontier with an increasing opportunity cost shows an increase in output before (in red line) and after (in blue line) specialization.

improvement in technology leads workers to become less specialized when the new technology reduces the switch cost to be sufficiently small.

## 5 Conclusion

The time spent to adapt for switching to a new task imposes a cost in the efficiency of a production. Even the production technology for all the workers is the same, specialization in the division of labor can avoid the switch cost between tasks to increase the efficiency in a production. The results can be extended to trade models where complete specialization among countries is possible even with increasing opportunity costs.

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