Economic-Based Synchronization and Control of New Fractional-Order Chaotic System Based on Lyapunov Theorem

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***Abstract*— The chaotic systems play a critical role in a wide range of communication systems, form both economical and technical. Recently, a new fractional-order chaotic system was proposed and dynamical analysis was investigated. Here, we focus on synchronization of the *drive and response systems, which can have significant impact on the economic side of the problem*. Furthermore, stability of mentioned system is discussed based on Lyapunov theorem. It is shown that Lyapunov theorem can be extended to the systems which have terms with orders higher than 2. Numerical simulations prove our claims from the objective evaluation.**

Keywords—Chaotic dynmics; fractional-order; synchronization; Lyapanov theorem; Economic

I. Introduction

Fractional calculus has a large history in mathematics over 300-years-old. However, its applications to engineering are a recent focus of interest [1-7]. Many systems are known to display fractional order dynamics, such as viscoelastic systems [8], dielectric polarization [9], and electrode-electrolyte polarization [10]. Also, secure communications and information encryption based on fractional-order systems are a field of research [11-14]. There are three approaches to solve fractional-order chaotic systems: frequency-domain method [15], Adomian decomposition method (ADM) [16], and Adams-Bashforth-Moulton algorithm [17, 18]. However, Tavazoei et al. [19] reported that the frequency-domain method is not always reliable in detecting chaos behavior in nonlinear systems. In this paper, discretization is performed by Adams-Bashforth-Moulton decomposition method.

Chaos control and synchronization are two important ways to utilize chaos in practice. The synchronization of chaotic fractional-order system has attracted great attention due to its potential application in secure communication. Synchronization of fractional order chaotic system can be divided into two principle groups: Lyapunov-based method [20] and Laplacian-based method [21-23]. The Lypaunov-based methods uses Lyapunov functions and related lemmas to achieve the synchronization goal. In this paper we focus on Lyapunov-based method to achieve the synchronization goal.

Stability is a minimum requirement for control systems, certainly including fractional-order chaotic systems [24]. In nonlinear systems, Lyapunov direct method (also called the second method of Lyapunov) provides a way to analyze the stability of a system without explicitly solving the differential equations. The method generalizes the idea that the system is stable if there are some Lyapunov function satisfies for the system.

II. Mathematical Aspects of the Chaotic System

A. New chaotic system and its dynamical analysis

Based on [25-30], the new fractional-order chaotic system is as follows:

$$\begin{aligned} \frac{d^{q_1}x}{dt^{q_1}} &= z \\ \frac{d^{q_2}y}{dt^{q_2}} &= z^3 + z^2 + 3xz \\ \frac{d^{q_3}z}{dt^{q_3}} &= x^2 + y^2 - r^2 - 4yz^2 \end{aligned} \tag{1}$$

where $r = 0.992$ and x, y, z are the state variables and q_1, q_2 and q_3 are the order of fractional derivatives.

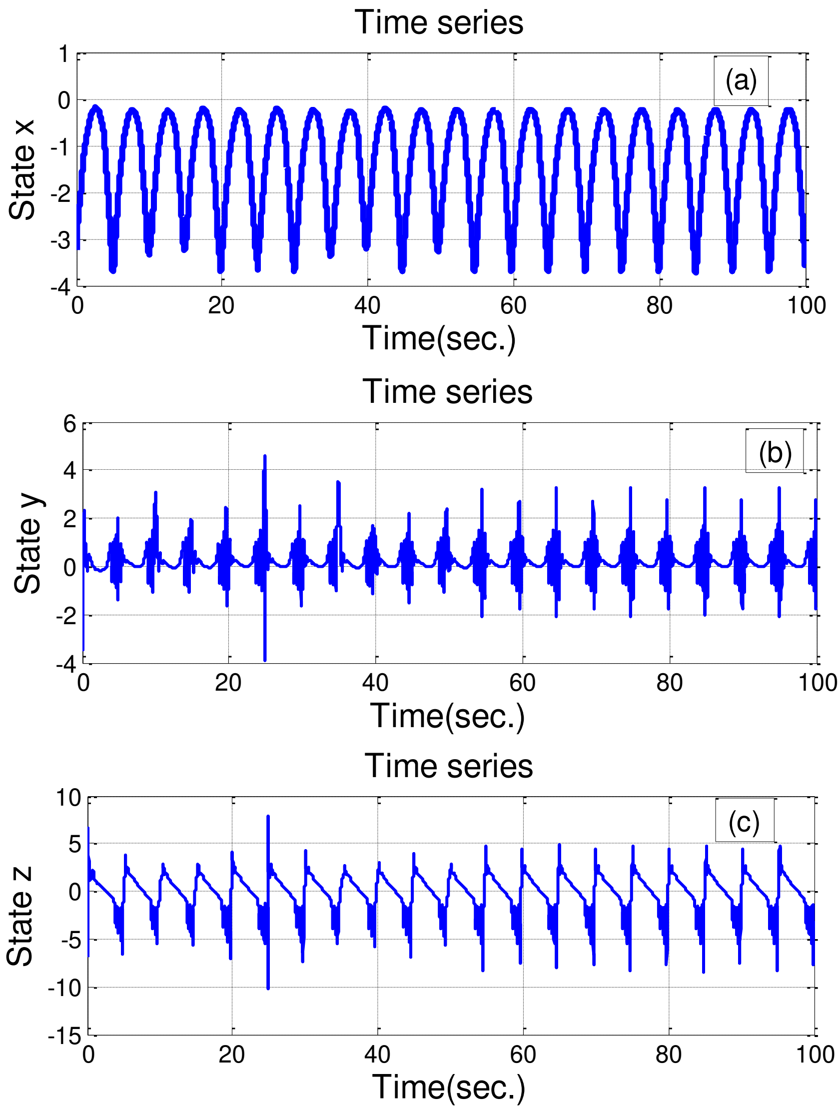

Fig. 1.Time series of state variables versus time; (a) State x(t), (b) State y(t), (c) State z(t)

It should be noted that this technique is practical in other sciences such as electrical, economics, mechanical, and oceanography [31-50]. Time series of (1) are shown in Fig.1 for the commensurate condition with $q1=q2=q3=0.97$. It should be noted that chaos exist in (1) for the order less than 3. There are different method to solve (1). In this paper we use Adams Bashforth–Moulton predictor–corrector [17, 18]. This method is based on the Caputo definition of the fractional-order derivative. Furthermore, the phase portraits of system (1) are shown in Fig. 2.

B. Synchronization based on Lyapunov theory

In this section, we exert the Lyapunov theory to our system in order to achieve synchronization goal [20]. In this corresponding, consider the derive system as below:

$$\begin{cases} \dfrac{d^{q_1} x_m}{dt^{q_1}} = z_m \\ \dfrac{d^{q_2} y_m}{dt^{q_2}} = z_m{}^3 + z_m{}^2 + 3x_m z_m \\ \dfrac{d^{q_3} z_m}{dt^{q_3}} = x_m{}^2 + y_m{}^2 - r^2 - 4y_m z_m \end{cases} \tag{2}$$

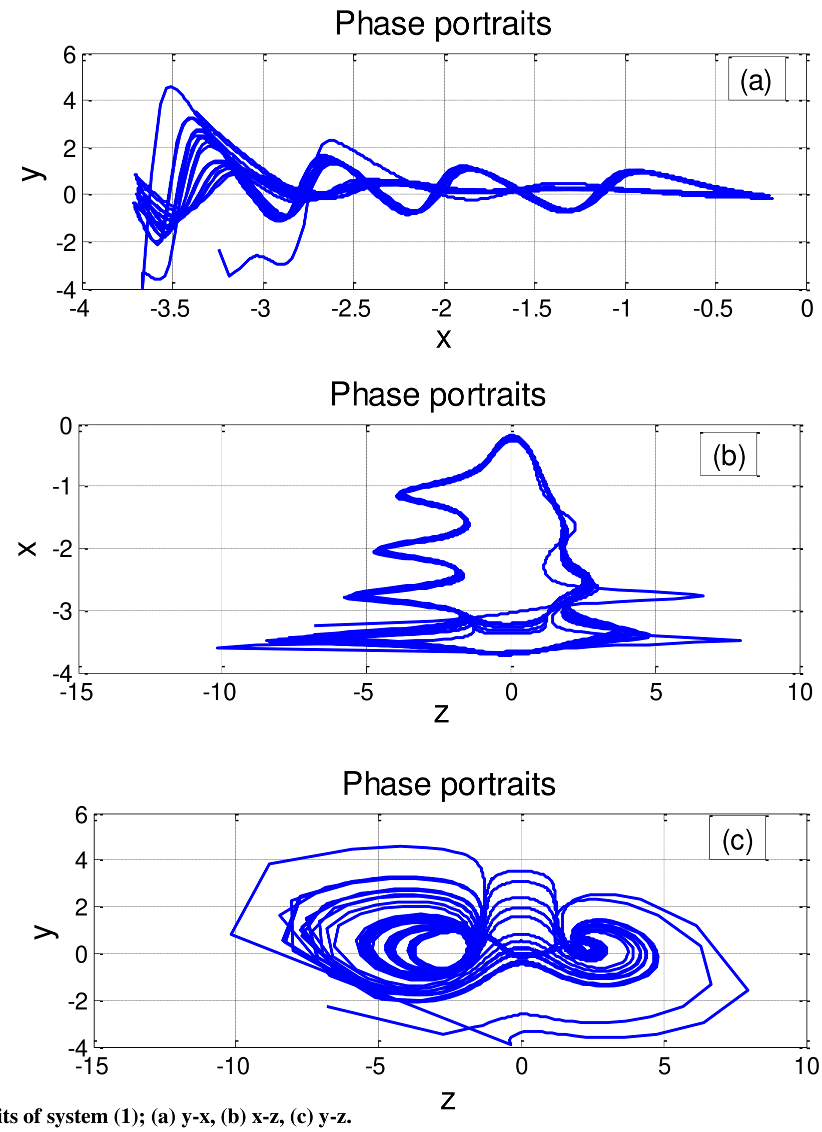

Fig. 2. Phase portraits of system (1); (a) y-x, (b) x-z, (c) y-z.

And also the controlled system named response system:

$$\begin{cases} \dfrac{d^{q_1} x_s}{dt^{q_1}} = z_s + u_1(x_s, x_m) \\ \dfrac{d^{q_2} y_s}{dt^{q_2}} = z_s{}^3 + z_s{}^2 + 3x_s z_s + u_2(y_s, y_m) \\ \dfrac{d^{q_3} z_s}{dt^{q_3}} = x_s{}^2 + y_s{}^2 - r^2 - 4y_s z_s + u_3(z_s, z_m) \end{cases} \tag{3}$$

where u1, u2 and u3 are the controllers.

To study the chaos synchronization, we define the error signals as $e_1(t) = x_s(t) - x_m(t)$, $e_2(t) = y_s(t) - y_m(t)$ and $e_3(t) = z_s(t) - z_m(t)$. Then the error system can be obtained as follow:

$$\begin{cases} \frac{d^{q_1} e_1}{dt^{q_1}} == e_3 + u_1 \\ \frac{d^{q_2} e_2}{dt^{q_2}} = z_s^{\ 3} - z_m^{\ 2} + z_s^{\ 2} - z_m^{\ 2} + 3x_s z_s - 3x_m z_m + u_2 \\ \frac{d^{q_3} e_3}{dt^{q_3}} = y_x^{\ 2} - y_m^{\ 2} + x_s^{\ 2} - x_m^{\ 2} - 4y_s z_s + 4y_m z_m + u_3 \end{cases} \tag{4}$$

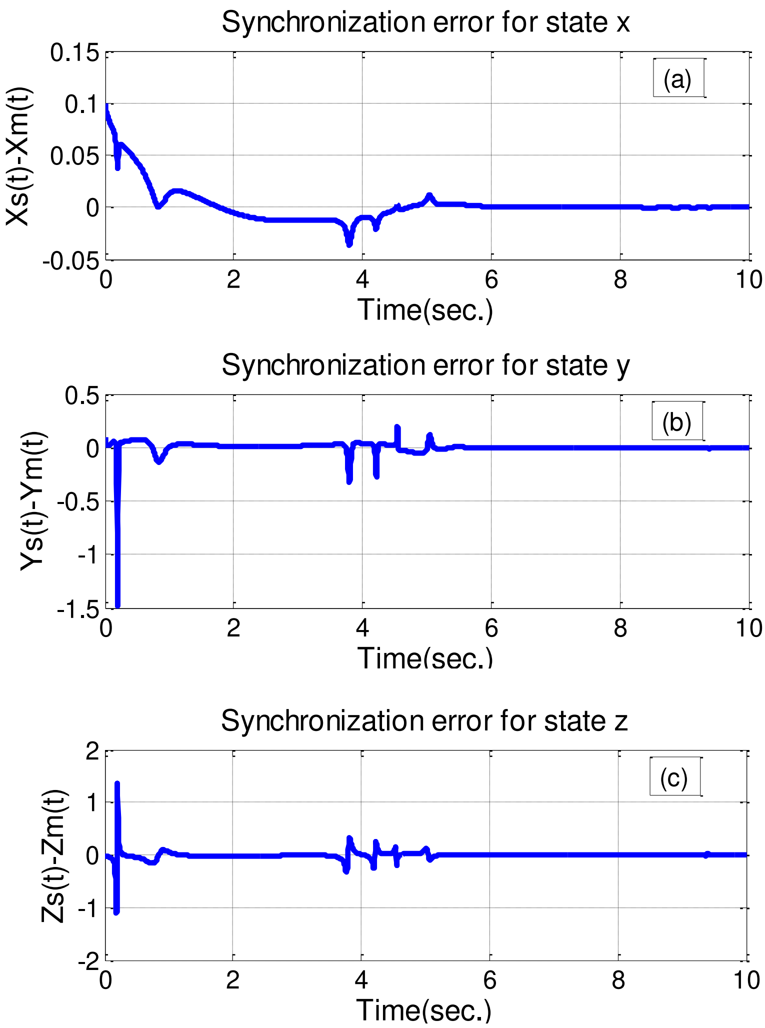

Fig. 3. Synchronization errors versus time; (a) Error x(t), (b) Error y(t), (c) Error z(t).

If the controller is given by:

$$\begin{cases} u_1(t) = k_1 e_1(t) = k_1(x_s - x_m) \\ u_2(t) = k_2 e_2(t) = k_2(y_x - y_m) \\ u_3(t) = k_3 e_3(t) = k_3(z_s - z_m) \end{cases} \tag{5}$$

And adaptive control parameters update according to the following laws:

$$\begin{cases} \dfrac{d^{q_1}k_1}{dt^{q_1}} = -\gamma_1 e_1{}^2(t) = -\gamma_1(x_s(t) - x_m(t))^2 \\ \dfrac{d^{q_2}k_2}{dt^{q_2}} = -\gamma_2 e_2{}^2(t) = -\gamma_2(y_x(t) - y_m(t))^2 \\ \dfrac{d^{q_3}k_3}{dt^{q_3}} = -\gamma_3 e_3{}^2(t) = -\gamma_3(z_s(t) - z_m(t))^2 \end{cases} \quad (6)$$

where $\gamma_i > 0 (i = 1,2,3)$ are arbitrary constants. Then synchronization of two commensurate and incommensurate fractional-order chaotic systems can be achieved. All the theory and assumptions behind this method have been proved in [20] in general conditions, so here we only use the consequences of laws. In this paper, simulations are done with $\gamma_i = 1\ for\ i = 1,2,3$ and initial conditions $(x_s(0), y_s(0), z_s(0)) = (-3.24, -2.3, -6.83)$ and $(x_m(0), y_m(0), z_m(0)) = (-3.14, -2.2, -6.91)$. The results for this subsection are shown in Fig. 3.

C. Controlling systems based on Lyapunov theory

In this section, we investigate the stability and control of the above mentioned fractional-order chaotic system by using the Lyapunov stability theorem [24]. To prove the mathematically calculation simulation are brought at the end of this subsection. Now, we can consider a fractional-order chaotic system with its control as follows:

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = z - k_1 x \\ \dfrac{d^{q_2}y}{dt^{q_2}} = z^3 + z^2 + 3xz - k_2 y \\ \dfrac{d^{q_3}z}{dt^{q_3}} = x^2 + y^2 - r^2 - 4yz - k_3 z \end{cases} \quad (7)$$

It is obvious that the system (8) has chaotic attractors when the parameters $k_1, k_2, k_3 = 0$.

Referring to the Lyapunov stability theorem [24], we can write a special function as:

$$\begin{aligned} & x\frac{d^{q_1}x}{dt^{q_1}} + y\frac{d^{q_2}y}{dt^{q_2}} + z\frac{d^{q_3}z}{dt^{q_3}} = xz - k_1x^2 + yz^3 + yz^2 + 3xyz - k_2y^2 + x^2zy^2z - 4yz^3 - k_3x^2 \\ & - r^2z \le (-k_3 + 0.5)x^2 + (-k_2 + y^2) + (-k_3 + 0.5)z^2 + yz^3 + yz^2 + 3xyz + x^2z + y^2z - 4yz^3 - r^2z \\ & \le (-k_1 + 0.5 + 1.5\max|z|)x^2 + (-k_2 + 1.5\max|z|)y^2 + (-k_3 + 0.5 + 2.5\max|z|)x^2 - 3yz^3 + yz^2 + x^2z \\ & + y^2z - r^2z \le (-k_1 + 0.5 + 2.5\max|z|)x^2 + (-k_2 + 2.5\max|z|)y^2 + (-k_3 + 0.5 + \max|y|)y^2 \\ & + (-k_3 + 0.5 + \max|y|)z^2 + 3yz^2\max|z| - r^2z \le (-k_1 + 0.5 + 2.5\max|z|)x^2 + (k_2 + 2.5\max|z|)y^2 \\ & + (-k_3 + 0.5 + 4\max|y|)z^2 + r^2\max|z| \le 0. \end{aligned}$$

(8)

Therefore, system (8) is stable to zero when:

$$\begin{aligned} & (-k_1 + 0.5 + 2.5\max|z|) < 0, (-k_2 + 2.5\max|z|) < 0, \\ & (-k_3 + 0.5 + 4\max|y|) < 0. \end{aligned} \quad (9)$$

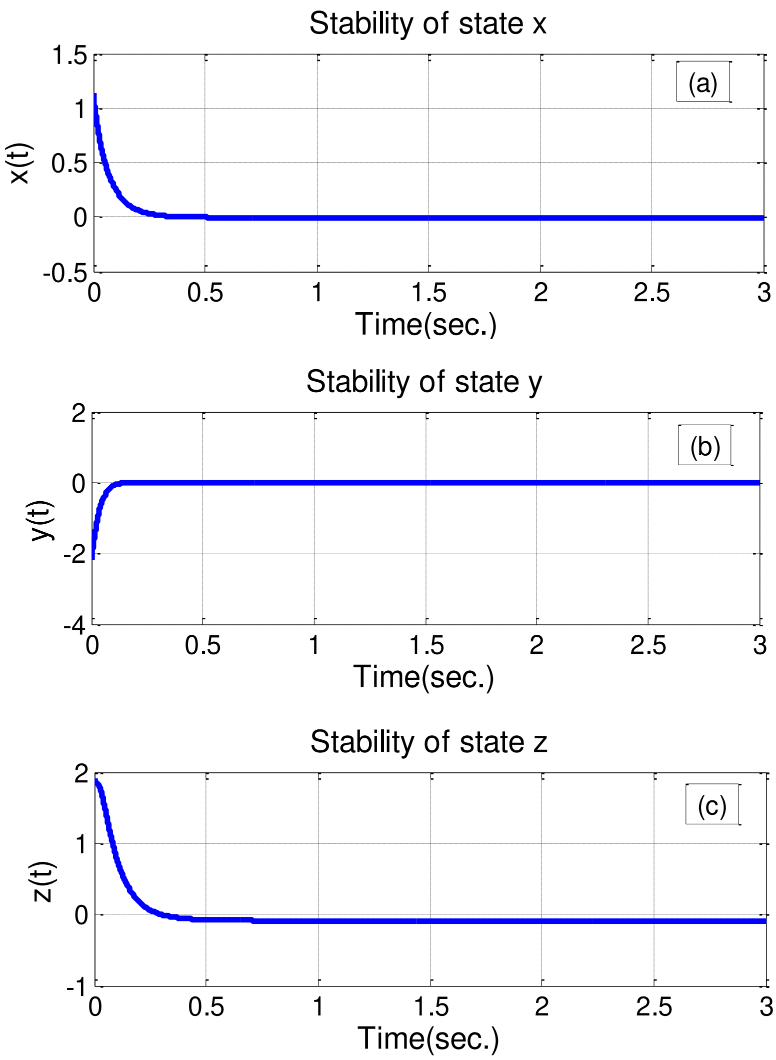

Fig. 4. Stability of states based on Lyapunov theory; (a) State x(t), (b) State y(t), (c) State z(t)

We choose the parameters $(k_1, k_2, k_3) = (16.6, 16.1, 11.2)$ in order to satisfy the above condition. Also, taking initial conditions $x(0) = 1.14$, $y(0) = -2.2$ $z(0) = 1.91$, we have done numerical simulations and results are shown in Fig. 4. It is worth to say that, we have extended the Lyapunov theory to the systems which have DC terms and also have terms with the orders greater than 2.

III. CONCLUSION

In this paper, the Lyapunov theorem was applied to synchronize and control a novel proposed chaotic system. It has been shown than the Lyapunov theorem can be generalized to the chaotic systems which have the order greater than 2. For the objective evaluation of the designed controller, the simulation results have been shown.

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