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Abstract

We study incentives for innovations that enable firms to enter backward into the input market. Such innovations are disruptive in that they lead to structural changes and even reversal of supply-customer relationships. We first show that Arrow's *replacement effect* is also present in our vertical setting which gives rise to two general results: (1) vertical integration lowers the R&D incentive of the integrated downstream firm; and (2) vertical integration raises the R&D incentive of the non-integrated downstream firm. We then identify, respectively, situations for strategic integration, which is driven by the motive to preempt R&D of the target firm, and for strategic separation, which occurs as a means to not trigger R&D by the downstream rival. An otherwise profitable raising rival's cost strategy may not be chosen for fear of counterattack by the rival in the form of disruptive R&D.

Keywords: innovation, structural change, replacement effect.
JEL Code: L13, L42, O31

1 Introduction

There are many real-life situations where downstream producers in vertically related industries enter backward into the upstream market as a result of internal R&D or by acquiring independent innovating firms. For example, Apple Inc. had once discussed acquiring Imagination, a major supplier of graphics processors to the iPhone, but finally decided to take development of graphics design in-house in order to reduce its future reliance on Imagination's technology.[1][2] On the software side, Apple had recently launched the mobile payment system Apple Pay, which is viewed by many as posing a direct competition threat to the incumbent Paypal, the dominant leader in on-line payment services.[3]Another case in point is that of Dell, which in 2012 created its software division,

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[1] "The Apple discount", *Financial Times*, April 8, 2017; https://www.ft.com/content/3d49b76a-1b76-11e7-a266-12672483791a

[2] Similarly, Apple had relied on its main competitor Samsung for the production of chips used in the iPhone and iPad, and was actively developing its own chips in alliance with Taiwan Semiconductor Manufacturing Company.

[3] Apple Pay, linked with existing credit cards such as Visa, Mastercard and UnionPay, enables customers to make payments on their mobile phones at the point of sales in physical stores. See, e.g., "Apple Pay Takes on Paypal with

Dell Software Group, based on a series of acquisitions that enabled Dell to enter into software and services businesses.[4][5] What affects firm incentive for innovation in a related (upstream) market? What are the effects of such potentially disruptive R&D based entry on existing firms in the related market? What competitive strategies might be available for affected firms to defend themselves against such threats?

In this paper, we study the R&D incentives of downstream firms for innovations that enable them to enter backward into the upstream market (a type of cross-market R&D). In our model, two downstream firms produce differentiated products initially using an input supplied by an upstream monopolist. We consider mostly the case where only the downstream firms can conduct R&D which when successful results in a new input of a higher quality, or equivalently at a lower cost, and enables the innovating firm to enter the upstream market. A central feature of our model is the structural changes associated with post-R&D entry into the upstream sector and exit of the incumbent supplier if the innovation is drastic (which some would argue is the case for Apply Pay and PayPal). Incentives are measured by the willingness to pay for the new input.[6]

We first consider the case where the market structure is exogenously given as one of two types: vertical separation, whereby all three firms are independent entities, and vertical integration, whereby the upstream firm is merged with one of the two downstream firms. Our first set of results show that the downstream firm's R&D incentive is stronger if it is a separate firm than if it is integrated with the upstream incumbent supplier. Hence, vertical integration reduces the target downstream firm's R&D incentive for developing new input. The results also show that vertical integration increases the R&D incentive of the competitor of the acquired downstream firm. These results hold for general demand functions, regardless of whether the innovation is drastic, and under both Cournot and Bertrand competition downstream.

The driving force for these results is the *replacement effect* of innovation identified by Arrow (1962) and the extensive studies in the literature on horizontal innovations, which says that a firm's R&D incentive is inversely related to its pre-innovation profit (see, e.g., Tirole 1998; Reinganum 1989; Gilbert 2006a and 2006b). If integrated with the current upstream supplier, the downstream's innovation on new input simply replaces its upstream unit's current business, whereas the innovation firm is not concerned with replacing the upstream input supply under vertical separation. Similarly, integration between the upstream incumbent and a downstream firm hurts its downstream rival because the latter faces a cost disadvantage with the integrated firm in the downstream market. This decline in the pre-R&D profit for the non-integrating downstream firm induces it to invest more in R&D that leads to not only self-sufficiency in the firm's input supply, but also enables

Long-awaited Functions on Websites," *Financial Review*, 13 June, 2016; http://www.afr.com/technology/technology-companies/apple/apple-pay-takes-on-paypal-with-longawaited-function-for-websites-20160613-gpia4f.

[4] "Dell changes focus from hardware to software, services", Dallas Business Journal, Aug 7, 2013. Since its creation, Dell Software Group has introduced many new products, including its Operating System 10 (OS10) which is based on a native, unmodified Linux kernel that can support a broad range of applications and services from the Linux ecosystem. "S10 represents an interesting new direction for Dell as it continues to extend and enhance its networking portfolio with innovations in software and hardware," said Brad Casemore, Research Director, Datacenter Networks, IDC. Press Release, Dell, January 20, 2016. http://www/dell/com/learn/us/en/vn/press-releases/2016-01-20-dell-raises-the-bar-for-open-networking.

[5] Other examples include that of China South Rail Corporation, which in 2015 successfully developed its own technology for producing the Insulated Gate Bipolar Transistor, a core component in high-speed train system that used to be imported from more advanced countries. Toyota has developed a way to make hybrid and electric vehicles without the use of expensive rare earth metals that had to be imported from China.

[6] This covers the cases where a downstream firm enters backward into the upstream market by acquiring an R&D firm, as is the case of Dell Software Group.

the firm to enter into the upstream market and drive the integrated firm out of the input business (which we call the relationship-reversal effect).

These general results enable us to further consider strategic incentives on the part of the upstream incumbent for vertical integration. We show that for certain R&D projects (to be conducted by a downstream target firm or its competitor) (1) the incumbent input supplier can use vertical integration to preempt disruptive input R&D by its target firm, which welfare is likely to decline; and (2) under some other circumstances it pays the incumbent upstream firm to not vertically merge with its target firm, so as to prevent input R&D (and the aftermentioned relationship reversal effect) by the other downstream firm; Put differently, an otherwise profitable raising rival's cost strategy (namely, vertical integration) is not chosen in our model for fear that so doing would invite counterattack by the rival in the form of disruptive innovation. The policy implication of result (1) is that authorities should be aware of the R&D related preemptive motive for vertical mergers. Regarding result (2), the traditional removal of double marginalization incentive for vertical integration, which is very strong in our model as in most vertical settings, is outweighed by the fear of the otherwise disruptive input innovation by a downstream rival.

We also consider the case of R&D competition where both the downstream firms in our model can conduct R&D. A firm's success rate increases with its R&D investment; if both firms succeed, each firm receives the patent with equal probability. Here, each firm has two incentives to innovate: a standalone incentive (as considered in the early parts of the paper) and a competitive incentive (given that its rival succeeds). We find that the conclusion remains the same: for both drastic and non-drastic innovations, vertical integration reduces the equilibrium R&D investment of the integrated firm and raises that of the non-integrated firm. In addition, the strategic incentives for vertical integration and separation, as identified previously, also exist under R&D competition.[7]

Stimulated by the arguments of Schumpeter (1942) and Arrow (1962) on the relationship between market structure and firms' R&D incentives, economists have devoted much attention to innovation in the past several decades. The central question is whether market power enhances or hurts innovation. However, most of the vast literature focuses on horizontal settings. Innovations that enable firms to enter another tier of the vertical industry, e.g., backward to the input producing sector, have received little attention. Our paper is among the first attempts to study incentives for potentially disruptive R&D that lead to entry into the upstream market, and to show that *the replacement effect* identified by Arrow (1962) also exits along vertical businesses of an integrated firm.

This may be viewed as supplementing the Arrownian counter-argument to the traditional Schumpeterian view that larger firms are more innovative. Most of the studies mentioned so far are conducted in a horizontal setting. Among the few studies of R&D in vertical settings, Chen and Sappington (2010) examine the effect of vertical structure on process innovation conducted by upstream suppliers. They show that vertical integration generally enhances innovation when the downstream competition is Cournot, but can dampen it if the competition is Bertrand. The major force is that the upstream firm internalises the positive externality of its R&D investment under VI. Our paper considers R&D by downstream firms that lead to entry to the other level

[7] In the Appendix B, we consider the pure innovation incentive by the upstream firm to develop the new input. We show that, for a linear demand, the upstream firm has a greater incentive to innovate under vertical integration than under vertical separation. Unlike the case of downstream innovation, upstream R&D is not disruptive in that the market structure remains unchanged following innovation. Thus, for upstream R&D, there is no structural benefit to the innovator. As the upstream firm innovates, the input price goes down which benefits both downstream firms. This positive externality is partially internalised under vertical integration but not under vertical separation.

of the industry and thus structural change, and we also study strategic incentives for vertical integration/separation, both of which are absent in Chen and Sappington's study.

In a recent paper, Loertscher and Riordan (2014) consider a procurement model with competing upstream firms that invest in process R&D and a monopolist downstream firm that can source internally by vertical integration. The authors show that vertical integration discourages cost-reducing investments of independent upstream suppliers by creating a favoured, internal source of supply. As a result, vertical integration in their model, while increasing the R&D incentive of the integrated supplier, decreases such incentive of the non-integrated suppliers, which is detrimental to the monopoly customer. Loertscher and Riordan (2014) show that it may pay the monopoly customer to remain vertically separate so as to encourage innovation by upstream suppliers. Symmetrically, we consider innovation by downstream producers and show that vertical integration reduces R&D incentive of the integrated firm but increases that of the non-integrated firm. In our model, the monopoly upstream supplier may strategically choose not to integrate with a downstream firm in order to deter R&D investment by the non-integrating downstream firm.

Allain, Chambolle, and Rey (2015) study the effects of vertical integration on downstream firms' incentives to increase the quality of the final products. In their model, downstream R&D requires information exchanges with an upstream supplier, which may hinder downstream innovation if sensitive information is leaked to downstream rivals. Vertical integration reduces the integrated supplier's ability to interact with non-integrated competitors. In our paper, the reward for innovation is the (endogenous) extra profit that an innovator can earn, and the driving forces for our results are the structural changes brought about by downstream R&D.[8]

The rest of this paper is organised as follows: Section 2 sets up the model and derives some basic expressions. Sections 3 and 4 examine standalone incentives for R&D of the integrated and non-integrated downstream firms, respectively. Section 5 discusses strategic vertical integration and separation. Extension of R&D competition is considered in Sections 6. Section 7 concludes the paper. Proofs are regulated in Appendix A and the analysis of upstream innovation is provided in Appendix B.

2 Model

Consider a model of two vertically related industries: a downstream industry and an upstream industry. In the downstream industry, two firms, D_1 and D_2, compete with horizontally differentiated products. The demand function for D_i's product is $p_i(q_i, q_j)$, which satisfies the following properties:

$$\frac{\partial p_i}{\partial q_j} < 0 \text{ for } i, j = 1, 2$$

and

$$\left|\frac{\partial p_i}{\partial q_i}\right| > \left|\frac{\partial p_i}{\partial q_j}\right| \text{ for } j \neq i.$$

[8]Some recent papers investigate the incentive to innovate in a vertical market structure when a downstream firm can integrate backward through innovation. Inderst, Jakubovic and Jovanovic (2015) examine the shift of innovation activity away from manufacturers and towards large retailers and show that there is a hold-up effect when upstream firms innovate and a rent appropriation effect when innovations come from the retailers. Chamboll, Christin and Meunier (2015) study a situation where a retailer may either choose to integrate backward with a small firm or rely on a national brand manufacturer to product its private label.

The downstream competition can be in either Cournot or Bertrand fashion. Our main results are derived based on the general demand system.[9]

The production of the final products requires a single input that is supplied by an upstream firm U_0 with marginal cost of production $c > 0$. Initially, U_0 is the only supplier of the input. One unit of each final product requires exactly one unit of the input. The costs of transforming the input into a final product are normalised to zero.

There are two alternative market structures: vertical separation (VS), in which all three firms are independent entities, and vertical integration (VI), in which U_0 and D_1 are vertically integrated. We refer to D_1 as the integrating firm, and D_2 as the non-integrating firm. Under VS, U_0 sells the input to both D_1 and D_2. Under VI, the integrating firm supplies the input to D_1 internally at cost and to the downstream rival D_2 at some chosen price.

Both downstream firms have the capability to invent a new input that can be used in place of the one currently produced by U_0. We refer to the new input producer as U_n and assume that the new input has a higher quality than the old one. This quality premium can be transformed to a cost premium,[10] i.e., the new input is identical to the existing input but can be produced at a lower effective marginal cost $c - d$, where the cost differential $d \in [0, c]$ measures the significance of the innovation. After an innovation, there may or may not be competition between U_n and U_0 in the upstream industry depending on the initial market structure. The upstream competition, if any, is assumed to be in price.

Let t_i^j denote firm D_i's equilibrium profit under market structure t when firm j is the innovator, where $t = S$ (VS) or V (VI) and $i, j = 1, 2$. Firm D_i's pre-innovation profit (i.e., when neither firm innovates) is denoted as S_i^0 and V_i^0 for VS and VI, respectively.

2.1 Downstream competition

In the downstream industry, suppose that firm D_i obtains the input at price w_i and its rival obtains the input at price w_j, and denote the equilibrium output and profit of D_i as $q(w_i, w_j)$ and $\pi(w_i, w_j)$, respectively. We assume the usual properties (they all hold for both Cournot and Bertrand profits):

$$\frac{\partial \pi(w_i, w_j)}{\partial w_i} < 0, \quad \frac{\partial \pi(w_i, w_j)}{\partial w_j} > 0, \quad \text{and} \quad \frac{d\pi(w, w)}{dw} < 0.$$

Note that w_i and w_j are endogenous, to be determined by market competition under either the VS or VI market structure.

2.2 Pre-innovation equilibrium under vertical separation_

Under VS, the two downstream firms are symmetric, so U_0 charges the same price, w, to D_1 and D_2 for supplying the input: $w_1 = w_2 = w$. The derived demand for U_0's input is thus $2q(w, w)$. U_0 solves the following optimisation problem:

$$\max_{w} \; 2(w - c)q(w, w). \tag{1}$$

[9] For results on R&D competition in section 6, we will consider the following linear demand system:

$$p_i = a - q_i - \beta q_j, \quad i, j = 1, 2, \quad i \neq j,$$

where $\beta \in (0, 1)$ represents the degree of product substitution.

[10] This assumption enables us to avoid the complication of modelling how differentiated inputs are transformed into differentiated final products.

Let $w^S(c)$ denote the solution, and note that $w^S(c) > c$. The two downstream firms' profits are

$$S_1^0 = S_2^0 = \pi(w^S(c), w^S(c)).$$

2.3 Pre-innovation equilibrium under vertical integration_

Under VI, D_1 and U_0 are integrated, so U_0 supplies the input to D_1 at cost c. Suppose that U_0 sells the input to D_2 at price w_2.[11] The derived demand from D_2 for the input is thus $q(w_2, c)$. Receiving profits from both upstream and downstream businesses, the integrating firm solves the following problem:

$$\max_{w_2} \quad (w_2 - c)q(w_2, c) + \pi(c, w_2). \tag{2}$$

Let $w^I(c)$ denote the solution, which represents the optimal input price that a vertically integrated firm charges to its downstream rival when the input production cost is c. Depending on whether the downstream competition is Cournot or Bertrand, the expression of $w^I(c)$ may differ (see the Appendix), but $w^I(c)$ always increases in c, which is to be expected. Once $w^I(c)$ is derived, the resulting equilibrium profit of the integrating firm is then

$$V_1^0 = [w^I(c) - c]q(w^I(c), c) + \pi(c, w^I(c)) \equiv V(c),$$

and the profit of D_2 is $V_2^0 = \pi(w^I(c), c)$.

Remark 1 *(Raising rival's cost effect of VI): $w^I(c) > c$ and $V_2^0 < S_2^0$.*

Remark 1 is a form of raising rival's cost effect of vertical integraton well studied in the literature (see, e.g., Ordover, Saloner, and Salop, 1990). Vertical integration between D_1 and the upstream incumbent firm raises the input price facing D_2 relative to that of D_1, thereby lowering the pre-innovation profit of D_2. This effect is beyond the result of Proposition 2 as well that on strategic separation (Proposition 3b) to be derived below.

3 R&D incentive by the integrating firm

To study how VI alters a downstream firm's incentive to innovate, we first investigate what Tirole (1988) calls "the pure incentive to innovate". That is, we assume that only one downstream firm has the capacity to innovate: either D_1 (this section) or D_2 (next section). Section 6 considers the case where both firms compete in R&D investments.

3.1 Post-innovation equilibrium under vertical integration_

Suppose that D_1 is the only firm capable of R&D. If D_1 and $U_0$are initially integrated, D_1's invention does not change the market structure. D_1 and the upstream supplier (now U_n instead of U_0) remain integrated and continue to supply their downstream rival D_2. D_1's optimisation problem now becomes

$$\max_{w_2} \quad [w_2 - (c - d)]q(w_2, c - d) + \pi(c - d, w_2). \tag{3}$$

[11] Foreclosure is allowed, as w_2 is under D_1's control and can therefore be set at such a high level that D_2 buys no input from D_1. However, in our model with product differentiation, it can be shown that downstream foreclosure never occurs unless the final products are homogeneous.

This problem is identical to (2) except that the input production cost is $c - d$ instead of c. Consequently, the solution must be $w_2 = w^I(c-d)$, and the two downstream firms' profits are $V_1^1 = V(c-d)$ and $V_2^1 = \pi(w^I(c-d), c-d)$, respectively, using the equilibrium expressions defined earlier.

3.2 Post-innovation equilibrium under vertical separation_

Suppose that D_1 and U_0 are initially separated. After D_1invents the new input, it no longer needs to purchase the input from U_0; it produces its own input in-house. Under VS, therefore, input innovation enables the innovating downstream firm to become vertically integrated. In addition, D_1 is now capable of entering backward into the upstream market and competing with the incumbent supplier U_0 in supplying the input to its downstream competitor, D_2.[12] Given the cost advantage of the new input and Bertrand competition in the upstream market, D_1 is able to grab the entire input market by undercutting the incumbent supplier, U_0.

Specifically, D_1 faces the following optimisation problem after its innovation: Choose the input price charged to D_2, w_2, so as to maximize its total profit from both downstream and upstream businesses, subject to the constraint that w_2 cannot exceed its upstream rival U_0's unit cost, c:

$$\max_{w_2} \ [w_2 - (c-d)]q(w_2, c-d) + \pi(c-d, w_2), \ \text{ s.t. } w_2 \leq c. \tag{4}$$

This optimisation problem is identical to (3) except for the constraint of $w_2 \leq c$. There are therefore two possibilities. If the solution to the unconstrained optimization problem (3), $w_2 = w^I(c-d)$, is no greater that c, then U_0 does not impose any competitive pressure on U_n, and D_1's optimal input price in (4) equals $w^I(c-d)$. Such a case is referred to as *drastic innovation.*[13] Recall that $w^I(c)$ increases with c, which implies that $w^I(c-d)$ decreases with d. Then an innovation is drastic if and only if d is sufficiently large. If, however, $w^I(c-d) > c$, then D_1 is constrained by the competition from U_0 and has to charge $w_2 = c$. This case is referred to as *non-drastic innovation.*

To summarise, if D_1 innovates under VS, then the equilibrium input price charged to D_2 is $w_2^* = \min\{w^I(c-d), c\}$. The resulting profit of D_1 equals

$$S_1^1 = [w_2^* - (c-d)]q(w_2^*, c-d) + \pi(c-d, w_2^*),$$

whereas D_2's equilibrium profit is $S_2^1 = \pi(w_2^*, c-d)$.

Note that under VS, the incumbent upstream supplier, U_0, is always driven out of business regardless of the significance of the innovation. The innovating downstream firm not only obtains its own input but also supplies the input to its downstream rival. Therefore, innovation brings structural change under VS, but preserves market structure under VI as seen earlier. Also note that when the innovation is drastic, $w_2^* = w^I(c-d)$ and therefore $S_1^1 = V_1^1$ and $S_2^1 = V_2^1$, i.e.,

[12] While obtaining extra business from selling its input, U_n's participation in the upstream competition reduces the supply price that D_2 pays for its input, thus hurting D_1's downstream business. D_1 may therefore wish (and claim) to refrain from entering the input market so as to keep its rival's cost high. However, such a claim may not be credible. Given the nature of Bertrand competition in the upstream industry, for any price that U_0 charges D_2, U_n can undercut and grab the whole business. Such commitment issue (by a vertically integrated firm to supply its downstream competitor) also arises in Ordover et. al (1990), as pointed out by Hart and Tirole (1990) and Reiffen (1992).

[13] Arrow (1962) has used the term drastic to refer to an innovation for which existing products or processes do not constrain the inventor's profit-maximizing price. See also Gilbert (2006b).

the post-innovation equilibria under VS and VI are the same. These results are highlighted in the following lemma.

Lemma 1: *Consider an innovation by the integrating firm.*
(i) The innovation changes the market structure under VS *but not* under VI.
(ii) If the innovation is drastic, the post-innovation equilibrium is the same whether the original market structure was VS *or* VI.

3.3 Effect of vertical integration on D_1's R&D incentive

We are now ready to compare D_1's R&D incentives between the two market structures, i.e., to compare $\Delta_1^V \equiv V_1^1 - V_1^0$ under VI with $\Delta_1^S \equiv S_1^1 - S_1^0$ under VS. We consider drastic and non-drastic innovations in turn. Note that $V_1^1 - V_1^0 = V(c-d) - V(c)$, that is, R&D by D_1 under VI does not lead to any structural change and the innovation simply entails the replacement of the old input by the new input.

If the innovation is drastic, we have shown that the post-innovation equilibrium is exactly the same regardless of whether the original market structure was VI or VS: $V_1^1 = S_1^1$. Before the innovation, it is obvious that D_1 earns greater profit under VI than it does under VS: $V_1^0 > S_1^0$. It follows immediately that $V_1^1 - V_1^0 < S_1^1 - S_1^0$. For drastic innovation, therefore, D_1's R&D incentive is greater under VS than under VI.

One way to understand the proceeding result is to decompose D_1's R&D incentive under VS into two parts. For drastic innovation,

$$\Delta_1^S \equiv S_1^1 - S_1^0 = V_1^1 - S_1^0 = \left(V_1^0 - S_1^0\right) + (V_1^1 - V_1^0).$$

The first part of the last expression, $V_1^0 - S_1^0$, can be called *the integration effect:* input R&D transforms D_1 from a vertically separated firm to an integrated firm that can earn a profit of V_1^0 even if it uses the old technology. The second part, $V_1^1 - V_1^0$, captures an efficiency effect whereby an already integrated D_1 can further increase its profit by using the new technology. The integration effect highlights the structural change brought about by input R&D under VS; no such structural change exists if the market structure was VI initially. The above decomposition shows clearly the extra incentive for R&D under VS, as D_1's R&D incentive under VI contains only the efficiency effect ($\Delta_1^V = V_1^1 - V_1^0$).

If the innovation is non-drastic, the comparison between Δ_1^V and Δ_1^S is no longer so straightforward. After the innovation, the market structure under VS is similar but not identical to that under VI. In both cases, D_1 is the only upstream supplier and its own input is acquired at cost $c-d$. The only difference is that the input price charged to D_2 is constrained under VS ($w_2 = c$) but not under VI ($w_2 = w^I(c-d) > c$). Therefore, D_1's post-innovation profit is greater under VI than under VS ($V_1^1 > S_1^1$). Before the innovation, D_1's pre-innovation profit is also higher under VI than under VS ($V_1^0 > S_1^0$), as pointed out earlier. To compare $V_1^1 - V_1^0$ with $S_1^1 - S_1^0$, it helps to first look at the extreme case of $d = 0$. Under VS, both downstream firms obtain their inputs at $w^S(c)$ before the innovation, and at c after the innovation (for D_1, it it the new supplier U_n's cost $c - d = c$; for D_2, it is the old supplier U_0's cost c). As the input price is lower, both firms benefit. In particular, $S_1^1 > S_1^0$. Notice an interesting observation here: D_1's innovation raises not only its own profit, but also D_2's profit. In a vertical setting, therefore, a firm may benefit from its competitor's innovation, which never happens in horizontal settings.

When $d = 0$, therefore, $S_1^1 - S_1^0 > 0$. Meanwhile, $V_1^1 - V_1^0 = V(c-d) - V(c) = 0$. It follows that $V_1^1 - V_1^0 < S_1^1 - S_1^0$. That is, if the innovation does not reduce the production cost, it has no value under VI, as it does not change the market structure. However, such innovation is still valuable under VS because it enables the innovator to become vertically integrated with its supplier and thus obtain its input at a lower price even though the input production cost has not changed. For this reason, D_1's R&D incentive is stronger under VS than under VI when $d = 0$.

What about $d > 0$? As mentioned earlier, in the post-innovation equilibrium under VS, D_1 obtains its own input at $c - d$ and supplies the input to D_2 at the constrained price, c. The cost differential d measures how severe the constraint is: a larger d means a more relaxed constraint. (In fact, when d is sufficiently large so that the innovation becomes drastic, the constraint disappears completely.) As D_1 faces the constraint under VS but not under VI, a more relaxed constraint favors D_1's performance under VS over VI. In the previous paragraph we have established that even under the most severe constraint of $d = 0$, D_1's R&D incentive under VS is still greater than that under VI. If $d > 0$ so that D_1's performance under VS is further enhanced, the conclusion must continue to hold.

To summarise, for both drastic and non-drastic innovations, we have the following result (the proof can be found in the Appendix):

Proposition 1: *Vertical integration reduces the integrating downstream firm's innovation incentive:* $\Delta_1^V < \Delta_1^S$.

For drastic innovation, the result can be explained by the replacement effect, as the post-innovation outcome is independent of the pre-innovation market structure. For non-drastic innovation, the driving force is that D_1's innovation brings structural changes only under VS, and is therefore more valuable there than under VI.[14]

4 R&D incentive by the non-integrating firm

We now turn to the effect of VI on the non-integrating downstream firm's R&D incentive. Suppose that D_2 is the only firm that can innovate. Before the innovation, D_2 relies on U_0 for the supply of the input under both VI (where U_0 and D_1 are integrated) and VS, and its pre-innovation profits under the two market structures have already been derived.

4.1 Post-innovation equilibrium under vertical separation_

Under VS, D_1 and D_2 are symmetric, so D_2's post-innovation profit as the innovator is the same as D_1's when D_1 is the innovator: $S_2^2 = S_1^1$. Specifically, the input innovation by D_2 transforms it into an integrated firm that supplies the new input to its own downstream business at cost $c - d$, and to its downstream competitor D_1 at either the non-constrained price $w^I(c-d)$ if the innovation is drastic, or the constrained price c if the innovation is non-drastic. Note that the non-innovator's profit is also symmetric: $S_1^2 = S_2^1$.

[14] Our results are robust to other forms of contracting. For example, if the upstream supplier uses a two-part tariff to capture the entire downstream profit, the downstream firm's innovation incentive is still higher under VS than VI. To see this, note that the innovating firm earns the same post-innovation profit under both VS and VI. However, there is a positive pre-innovation profit for the integrating firm under VI but none for the downstream firm under VS. Similarly, if secret contracts would be offered, the downstream firm would earn less pre-innovation profit under VS and thus its innovation incentive would still be higher under VS than V

4.2 Post-innovation equilibrium under vertical integration_

Under VI after D_2 has innovated, there are two vertically integrated entities: one is between D_1 and U_0, which produces the input at c; the other is between D_2 and U_n, which produces the input at $c-d$. If U_n supplies the input to D_1 at price $c-\varepsilon$, where ε is a very small positive value, $U_0 \bigcup D_1$ certainly accepts the deal. The sale also increases D_2's total profit because its downstream profit is not hurt ($w_1 = c$ and $w_2 = c - d$ in either case), and it now has some extra upstream business (as the supply price to D_1, c, is greater than the marginal production cost of the new input, $c-d$). As a result, it is a dominant strategy for U_n to supply the input to D_1 (at a price no greater than c), and D_1 accepts the offer.[15]

In supplying D_1, U_n chooses the input price w_1 to maximise the sum of its profit from supplying its downstream competitor D_1 and its own downstream unit D_2's business, subject to the constraint that this price does not exceed U_0's own cost, c:

$$\max_{w_1} \quad [w_1 - (c-d)]q(w_1, c-d) + \pi(c-d, w_1) \;\; \text{s.t. } w_1 \leq c. \tag{5}$$

This optimisation problem faced by D_2 under VI is exactly the same as that faced by D_1 under VS, equation (4). As D_1 and D_2 are symmetric under VS, this implies that when D_2 is the innovator, the post-innovation equilibrium is exactly the same whether D_1 and U_0 were originally integrated (the case of VI) or separated (the case of VS). After D_2's innovation, the original upstream supplier U_0, which is an in-house supplier for D_1 under VI and an independent supplier under VS, always foreclosed due to its cost disadvantage, but the mechanism of the foreclosure is slightly different between the two vertical structures. If U_0 and D_1 were integrated, their input production cost and hence the internal supply price are fixed at c, which is undercut by the new supplier U_n. If U_0 and D_1 were separated, U_n and U_0 engage in Bertrand competition in the upstream market, which drives the price down to a level equal to or below U_0's unit cost, c.

To summarise:

Lemma 2. *Consider an innovation by the non-integrating firm.*

(i) Innovation changes the market structure under both VS and VI.

(ii) Innovation leads to the same market structure and equilibrium outcome regardless of the initial market structure (whether VS or VI) or the significance of the innovation (whether drastic or non-drastic).

4.3 Effect of vertical integration on D_2's R&D incentive

Given that D_2 is the only innovator, the net gain of a successful R&D to D_2 is $\Delta_2^V \equiv V_2^2 - V_2^0$ under VI, and is $\Delta_2^S \equiv S_2^2 - S_2^0$ under VS. According to Lemma 2, $V_2^2 = S_2^2$, i.e., D_2's post-innovation profit is the same regardless whether the initial vertical structure was VS or VI. D_2's pre-innovation profit, however, is smaller if U_0 and D_1 were integrated $V_2^0 < S_2^0$ (Remark 1). As a result, D_2's R&D incentive is stronger under VI than under VS: $\Delta_2^V = V_2^2 - V_2^0 > S_2^2 - S_2^0 = \Delta_2^S$. We therefore reach the following conclusion.

Proposition 2: *Vertical integration between a monopoly input supplier and a downstream firm raises the non-integrating downstream firm's innovation incentive: $\Delta_2^V > \Delta_2^S$.*

[15] Notice the interesting reversal of customer-supplier relationship. Before the innovation, the upstream supplier (U_0) is vertically integrated with D_1 and supplies D_2. After the innovation, the upstream supplier (U_n) is vertically integrated with D_2 and supplies D_1.

As established by Lemma 2, innovation by D_2 changes the market structure under both VI or VS. Such a structural change always gives D_2 a competitive advantage in the post-innovation market. Prior to the innovation, D_2 was on an equal footing with D_1 under VS, but faced competitive disadvantage in relation to D_1 under VI. Successful R&D enables D_2 to get rid of the disadvantage under VI, thereby conferring a greater value than under VS.

Proposition 1 and Proposition 2 deliver a consistent message. Proposition 1 says that an advantaged firm (i.e., D_1 under VI) is less eager to innovate than a non-advantaged firm (i.e., D_1 under VS), whereas Proposition 2 says that a disadvantaged firm (D_2 under VI) is more eager to innovate than a non-disadvantaged firm (D_2 under VS). Both can be understood in terms of *the replacement effect* identified by Arrow (1962). Since innovation by D_1 replaces the input business of the upstream incumbent, vertical integration between the two firms internalises such negative externality, leading to the result in Proposition 1. Likewise, integration between D_1 and the upstream incumbent reduces the pre-innovation profit of D_2, thereby increasing D_2's incentive for R&D (Proposition 2).

5 Strategic Integration/Separation

In this section, we extend our main model to consider strategic incentives of the upstream monopoly supplier in using vertical organisations to influence downstream innovation. In particular, we intend to make two points. First, under VS where only one of the downstream firms has access to R&D, potential innovation may be deterred by a strategic vertical integration, which is mutually beneficial to the upstream firm and the innovating downstream firm. Second, the upstream incumbent may choose to restrain itself from acquiring a downstream non-innovating firm to reduce the innovation incentive of the downstream rival that has access to R&D. These results have important policy implications.

5.1 Strategic vertical integration

Consider first the possibility of vertical integration as a tool of deterring innovation. The market is initially vertically separate and D_1 is the only firm with access to R&D. There is a fixed cost F_1 for D_1 to come up with the innovation as considered in the previous sections.

The game sequence is as follows. At stage 1, U_0 first makes a take-it-or-leave-it offer (with transfer payment) to fully acquire D_1; D_1 then decides on whether accept or reject the offer. At stage 2, D_1 decides whether to innovate by incurring the fixed cost. Innovation by D_1, if undertaken, leads to the new input with unit cost of product $c-d$. At stage 3, the firms compete in the product markets, each receiving their equilibrium profits as derived in the previous sections.

There are four possible market/technology structures: (i) D_1 accepts VI and innovates; (ii) D_1 accepts VI and does not innovate; (iii) D_1 rejects VI and innovates; and (iv) D_1 rejects VI and does not innovate. We consider the set of R&D projects such that

$$V_1^1 - V_1^0 \leq F_1 < S_1^1 - S_1^0 \tag{6}$$

Based on Proposition 1, such F_1 exists. For such values of R&D costs, innovation is profitable for D_1 if it has rejected U_0's VI offer in stage 1 of the game by not profitable if it has accepted the offer. Given this, the joint profits of U_0 and D_1 are equal to V_1^0 if D_1 accepts the offer in stage 1, in which case U_0 and D_1 integrate with each other and choose not to conduct R&D afterwards, leaving the industry as integrated with the old input technology d. The joint profits of U_0 and D_1 equal $S_1^1 - F_1$ if D_1 rejects U_0's offer in stage 1, in which case the independent D_1 innovates and subsequently enters backward to the input market and drives the incumbent firm U_0 out of the market. Note that $V_1^1 \geq S_1^1$, i.e., D_1 earns greater profit if it is the sole upstream firm with the new input than if it innovates as a separated firm and then enter the upstream market to compete with incumbent U_0.

This, together with the above assumptions, implies that $V_1^0 > V_1^1 - F_1 \geq S_1^1 - F_1$. Thus, vertical integration is mutually profitable for U_0 and D_1 and hence they are able to find a mutually beneficial offer with properly chosen transfer payment for D_1 to accept the VI offer in stage 1 of the previous game. Hence, we have the following result.

Proposition 3(a). *Under vertical separation, where only one of the downstream firms has access to R&D, innovation may be deterred by a strategic vertical integration between the upstream firm and the downstream firm, if the R&D cost is such that* $V_1^1 - V_1^0 \leq F_1 < S_1^1 - S_1^0$.

When it occurs, strategic vertical integration deters innovation by a downstream firm that would enable the firm to enter backward into the upstream sector and compete with the incumbent supplier with a better input. The benefit of such integration to the incumbent monopoly supplier U_0 is that it prevents it from being driven out by the R&D product and entry of D_1. The benefit to the acquired downstream firm that would otherwise conduct R&D is that it saves on R&D cost and is able to obtain the input (albeit old) at marginal cost. The independent downstream firm D_2 is actually worse off. Absent of such integration between U_0 and D, D_1 would conduct R&D and be able to enter the upstream market with the new input, which would result in a lower input price that D_2 has to pay.

With such strategic vertical integration, the industry becomes $(U_0 - D_1, D_2)$ with the old input and monopoly pricing on input by the vertically integrated $U_0 - D_1$. Absent such integration, the industry would also become integrated as D_1 enters the upstream market with its new innovation, either replacing the incumbent firm or competing with it depending on whether the innovation is drastic. In sum, strategic vertical integration in our model deters downstream innovation and entry to the upstream sector, thereby necessarily reducing welfare (gross of R&D cost). Therefore, in contrast with the relatively lenient treatment of vertical mergers in the EU and the Unite States, our analysis suggests that antitrust authorities should consider blocking such vertical mergers.

5.2 Strategic vertical separation

We now consider an alternative scenario where the target firm for vertical integration does not have access to R&D, but the non-target firm does. In particular, we assume that D_2 is the firm with access to R&D, with fixed cost F_2. As before, U_0 and D_1 are initially separate firms and U_0 makes a "take-it-or-leave-it" vertical integration offer to D_1. After D_1's decision on whether to accept U_0 offer, D_2 decides on whether to innovate. Here, we consider the set of R&D projects such that

$$S_2^2 - S_2^0 \leq F_2 < V_2^2 - V_2^0 \tag{7}$$

From Proposition 2, which tells us that the R&D incentive of D_2 is greater if U_0 and D_1 merge than if they do not, such F_2 exists.

If D_1 accepts the VI offer by U_0, then D_2 chooses to innovate because $V_2^2 - F_2 > V_2^0$. If, instead, D_1 rejects the VI offer, D_2 does not find innovation profitable because $S_2^2 - F_2 \leq S_2^0$. Given D_2's choice, the joint profit of U_0 and D_1 under VS is higher than their profit under VI if

$$S_1^0 + U_0^0 > V_1^2. \tag{8}$$

Condition (8) is likely to hold when d is small. To illustrate, take an extreme case, $d = 0$. In this case, the innovation by D_2 is merely to take over the upstream production without any efficiency improvement and thus, $V_1^2 < S_1^0$.[16] Hence, (8) holds when d is sufficiently small. Note that under (7) and (8), the firms contemplating vertical mergers choose not to merge, so as to deter D_2's innovation that would disrupt the industry by driving U_0 out of the market and making D_2 a more competitive downstream firm in competing with D_1. We summarise the discussions in the following proposition.

Proposition 3(b). *For an R&D cost such that $S_2^2 - S_2^0 \leq F_2 < V_2^2 - V_2^0$ and a certain range of $d > 0$, the upstream incumbent firm chooses to restrain itself from acquiring a downstream firm and the other downstream firm chooses to not conduct R&D in the subgame perfect equilibrium of the game.*

It is worth noting again that absent of R&D, the incentive for integration is particularly strong in our model as it eliminates double-marginalization between the merged firms (U_0 and D_1), and the resulting raising rival's cost effect as mentioned in Remark 1, which give them a cost-advantage in downstream competition over rival firm D_2. The downside of such vertical integration, as shown above, is that it forces the downstream rival firm D_2 into a corner which increases its incentive for input R&D that disrupts the industry by driving the input division of the integrated firm out of business. The above proposition shows that such a negative impact to the upstream firm and the downstream target firm can be so huge that they rather choose to stay separate so as to not trigger such a disruptive innovation by the downstream competitor. R&D as a counterattack measure on the part of D_2 severs as a credible threat that can deter vertical integration by the incumbent supplier and its target firm.

As mentioned in the Introduction, the above result is similar to a finding in Loertscher and Riordan (2014) who consider a procurement model with competing upstream firms that invest in process R&D and a monopolist downstream that firm can source internally by vertical integration. Loertscher and Riordan show that, among other things, it may pay the monopoly customer to remain vertically separate so as to encourage innovation by upstream suppliers. They obtained this result because vertical integration, which will result in an internal supplier which the downstream customer can procure from, discourages R&D incentive by independent suppliers who anticipate their disadvantageous post-R&D procurement position vis-a-vis the internal supplier. Similarly, our model also identifies a strategic incentive for firms to maintain vertical separation, in an attempt to influence subsequent R&D activity by other firms.

Another way of looking at the issue is through the value of commitment created by vertical separation. As Loertscher and Riordan (2014) point out, in their model, "vertical divestiture is a commitment to a level playing field that encourages independent suppliers to invest in cost reduction." In our model, vertical separation is a commitment by the upstream monopoly supplier to not discriminate against the independent downstream firm (D_2) in its input price decision (by engaging in price squeeze). This commitment confers a level playing field in downstream competition, thereby reducing (eliminating in the model above) the incentive of the otherwise disadvantegeous independent firm to conduct input R&D.[17]

[16] Formally, if $d = 0$, then $V_1^2 = V_2^1 = V_2^0 < S_2^0 = S_1^0$.

[17] In a model without R&D, Lin (2006) shows that strategic separation enables the once-integrated firm to credibly increase its supply to downstream rivals, thereby reducing the market shares of upstream rivals.

Our result provides a new explanation for vertical disintegration as a way to deter innovation from downstream rivals. Such a strategic separation, similar to strategic vertical integration, may have a negative effect on welfare because it may hinder innovation that would otherwise occur in the absence of such spin-off and may also lead to double-marginalisations on consumers of the products of both downstream firms, because without the spin-off, the innovating downstream firm would choose to innovate and hence become the integrated firm, eliminating the double markups faced by its consumers.

6 R&D competition

So far we have discussed how vertical integration affects the two downstream firms' R&D incentives, assuming that only one of them can conduct R&D. In this section we extend our analysis to the case of R&D competition, where both downstream firms can conduct R&D that may lead to invention of a new input. Specifically, in the first stage of the game, for a given market structure (either VS or VI), D_1 and D_2 simultaneously and independently choose their investments in R&D. As in Allain, Chambolle and Rey (2015), the cost of R&D is assumed to be $C(\rho_i)$ for firm D_i, where $\rho_i \in [0,1]$ is D_i's probability of R&D success and is referred to as its R&D investment, with $C' > 0$ and $C'' > 0$. The outcomes of R&D projects are independent between the two firms. If only one firm succeeds in innovation, the firm is granted the patent for the new input. If both succeed, then each obtains the patent with a probability of one half.[18] In the second stage of the game, production and competition take place.

Recall the notation we have used so far: t_i^j denotes firm D_i's payoff under market structure t when firm j is the innovator, where $t = S$ (vertical separation) or V (vertical integration) and $i, j = 1, 2$. D_i's pre-innovation payoff is denoted as t_i^0. If both firms succeed in R&D, D_i's expected payoff is denoted as t_i^b. By assumption, $t_i^b = \frac{1}{2}(t_i^i + t_i^j)$ for $i, j \in \{1, 2\}$ and $i \neq j$. As firms D_1 and D_2 are symmetric under VS, we have $S_1^2 = S_2^1$, $S_1^1 = S_2^2$, $S_1^0 = S_2^0$, and $S_1^b = S_2^b$.

6.1 Vertical separation

Under VS and given the R&D investments of the two firms, ρ_1 and ρ_2, the payoff matrix (gross of R&D costs) for D_1 and D_2 is

$D_1 \ddots D_2$	Successful	Not Successful
Successful	$S_1^b,\ S_2^b$	$S_1^1,\ S_2^1$
Not Successful	$S_1^2,\ S_2^2$	$S_1^0,\ S_2^0$

Given ρ_j, D_i chooses ρ_i to maximise its expected profit under VS:

$$\max_{\rho_i} \Pi_i^S \equiv \rho_i[\rho_j S_i^b + (1-\rho_j)S_i^i] + (1-\rho_i)[\rho_j S_i^j + (1-\rho_j)S_i^0] - C(\rho_i).$$

The first-order condition is

$$(1-\rho_j)(S_i^i - S_i^0) + \rho_j(S_i^b - S_i^j) = C'(\rho_i). \tag{9}$$

[18] We are grateful to Patrick Rey for his useful comments and suggestions.

The first term on the left-hand side, $S_i^i - S_i^0 \equiv \Delta_i^S$, measures the value of D_i's successful R&D given that D_j does not succeed, and is referred to hereafter as D_i's *standalone incentive* of R&D. Apparently, a firm's standalone incentive is the same as its R&D incentive when it is the only firm capable of R&D, which we analysed earlier in Sections 3 and 4. The second term, $S_i^b - S_i^j$, measures the value of D_i's successful R&D given that D_j succeeds in R&D. It is therefore referred to as D_i's *competitive incentive* of R&D. More specifically, $S_i^b - S_i^j = \frac{1}{2}(S_i^i + S_i^j) - S_i^j = \frac{1}{2}(S_i^i - S_i^j)$. Given D_j's success, if D_i also succeeds, it has a 50% chance of winning the patent, in which case its payoff increases by $S_i^i - S_i^j$. With the remaining 50% of the chance, D_i's successful R&D fails to win the patent, so there is no change in its payoff. Thus, the competitive incentive for a firm equals half of the difference between its payoff as the winner and its payoff as the loser. The above first-order condition says that firm D_i's R&D investment is optimal if the marginal cost of its R&D investment equals the marginal benefit, which is a weighted average of its standalone and competitive incentives.

The two firms' equilibrium R&D investments depend also on the strategic interaction between their R&D choices. Note that

$$\frac{\partial^2 \Pi_i^S}{\partial \rho_i \partial \rho_j} = S_i^0 - S_i^b.$$

Thus, the two firms' R&D investments are strategic substitutes (under VS) if and only if $S_i^0 < S_i^b$, i.e., each firm earns greater profit if both firms succeed in R&D than if none succeeds. As the two firms are symmetric under VS, this condition is equivalent to $S_i^0 + S_j^0 < S_i^i + S_j^i$, i.e., the innovation under VS raises the two downstream firms' joint profits, which apparently holds. Therefore, under VS the two firms' R&D investments are strategic substitutes, meaning that a firm reduces its investment in response to its rival's greater investment.

Let (ρ_1^S, ρ_2^S) denote the Nash equilibrium of R&D competition under vertical separation. We assume that the equilibrium is unique and stable. As D_1 and D_2 are symmetric under VS, we have $\rho_1^S = \rho_2^S = \rho^S$.

6.2 Vertical integration

Now suppose that U_0 and D_1 were integrated initially. The payoff matrix of the R&D game (gross of R&D cost) under VI is similar to that under VS:

$D_1 \cdot\cdot\cdot D_2$	Successful	Not Successful
Successful	$V_1^b,\ V_2^b$	$V_1^1,\ V_2^1$
Not Successful	$V_1^2,\ V_2^2$	$V_1^0,\ V_2^0$

Given ρ_j, D_i chooses ρ_i to maximise its expected profit under VI:

$$\max_{\rho_i} \Pi_i^V \equiv \rho_i[\rho_j V_i^b + (1-\rho_j)V_i^i] + (1-\rho_i)[\rho_j V_i^j + (1-\rho_j)V_i^0] - C(\rho_i).$$

The first-order condition is

$$(1-\rho_j)(V_i^i - V_i^0) + \rho_j(V_i^b - V_i^j) = C'(\rho_i).$$

As before, $V_i^i - V_i^0 \equiv \Delta_i^V$ is D_i's standalone incentive under VI, whereas $V_i^b - V_i^j = \frac{1}{2}(V_i^i - V_i^j)$ is D_i's competitive incentive under VI.

Note that D_1 and D_2 are no longer symmetric under VI. For D_1,

$$\frac{\partial^2 \Pi_1^V}{\partial \rho_1 \partial \rho_2} = V_1^0 - V_1^b.$$

Recall that $V_1^b = \frac{1}{2}(V_1^1 + V_1^2)$, $V_1^0 = V(c)$ and $V_1^1 = V(c-d)$. If $d = 0$, we have $V_1^0 = V_1^1$ and therefore $\frac{\partial^2 \Pi_1^V}{\partial \rho_1 \partial \rho_2} = \frac{1}{2}(V_1^1 - V_1^2) > 0$. Intuitively, if d is small, winning the patent is not very rewarding for D_1, while losing it to D_2 is still damaging. Then, D_1's payoff is higher when no firm succeeds than when both succeed. In that case, D_1's R&D investment is a strategic complement of D_2's investment, meaning that D_1 increases its investment in response to D_2's greater investment. Of course, this is true only when the innovation is not significant (i.e., d is small). If the innovation is significant (i.e., d is sufficiently large), the sign of $\frac{\partial^2 \Pi_1^V}{\partial \rho_1 \partial \rho_2}$ is reversed, and D_1's R&D investment becomes a strategic substitute of D_2's investment under VI, as in the case of VS.

For D_2,

$$\frac{\partial^2 \Pi_2^V}{\partial \rho_1 \partial \rho_2} = V_2^0 - V_2^b.$$

Thus, D_2's R&D investment is a strategic substitute of D_1's investment (i.e., $\frac{\partial^2 \Pi_2}{\partial \rho_1 \partial \rho_2} < 0$) if and only if D_2's payoff is greater when both firms succeed in R&D than when neither succeeds, which is likely to be true in general (it holds for both Cournot and Bertrand profits under linear demand).

Assume there exists a unique Nash equilibrium in R&D competition under VI and denote the equilibrium by (ρ_1^I, ρ_2^I). We have the following result:

Proposition 4. *When the two firms compete in R&D, vertical integration lowers the R&D investment of the integrating firm and raises that of the non-integrating firm ($\rho_1^V < \rho_1^S = \rho_2^S < \rho_2^V$) if the innovation is drastic. For non-drastic innovation, the same is true if the demand system is linear and the R&D cost is quadratic.*

The proposition says that R&D competition leads to the same conclusion as in the case of stand-alone R&D: vertical integration reduces D_1's R&D investment and raises D_2's. The intuition for drastic innovation is relatively easy to understand. R&D competition differs from standalone R&D in two aspects: each firm has an extra competitive incentive, and the two firms' R&D investments are interdependent. If the innovation is drastic, the extra competitive incentive is the same for both firms in both VS and VI (Lemma 3). Driven by the difference in stand-alone incentives, then, vertical integration tends to reduce D_1's investment and raise D_2's. Furthermore, the two firms' R&D investments being strategic substitutes, D_1's reduced investment further raises D_2's investment and vice versa. (For non-drastic innovation, the derivations are tedious. Nevertheless, we are able to show that the conclusion still holds when the demand for the final product is linear and the R&D cost is quadratic.)

In Section 5, we have shown that the upstream incumbent chooses to restrain itself from acquiring a downstream non-innovating firm in order to reduce the innovation incentive of the downstream rival firm. Under R&D competition, we obtain a similar result by numerical analysis.

Remark 2. *When the two downstream firms compete in R&D, there exist parameter values under which the joint profit of U_0 and D_1 is greater under VS than under VI.*

One immediate implication of the result is that U_0 *and* D_1 may choose to restrain themselves from vertical integration for fear of raising D_2's R&D incentive. In fact, with R&D competition, vertical integration between U_0 and D_1leads to two opposing effects on the integrating firm's profit.

On the one hand, the merge of U_0 and D_1 eliminates double-marginalisation which would give them a cost-advantage in downstream competition with rival firm D_2. This increases their joint profit. On the other hand, integration raises D_2's R&D incentive, which negatively affects the merged firm's profit because the probability of D_2 becoming the supplier in the upstream is higher (Proposition 4). We are able to show that, under some parameter values, the latter effect dominates and consequently, U_0 and D_1 strategically choose not to merge even if they have such an option. Put differently, while having the option to raise rival's cost in the current product market through vertical integration, the upstream firm chooses not to do so in anticipation of the counter measure (R&D) by its downstream rival.

6.3 Industry rate of innovation

We have established that vertical integration reduces D_1's equilibrium investment and raises D_2's investment. What about the aggregate industrial R&D activity? To answer this question, we use the probability of at least one firm succeeding in R&D as a measure of the aggregate rate of R&D, or social rate of innovation, calculated as $\theta^t \equiv 1-(1-\rho_1^t)(1-\rho_2^t)$ under market structure $t = V, S$.

To simplify the calculation, we assume that the R&D cost function is quadratic: $C(\rho_i) \equiv \frac{\gamma}{2}(\rho_i)^2$, where $\gamma > 0$ measures the cost or complexity of R&D projects. We have the following result:

Proposition 5: *Assume that innovation is drastic and the demand system for final products is linear. Then, for the above R&D cost fuction, there exists* $\bar{\gamma} > 0$ *such that vertical integration lowers the industry rate of innovation (*$\theta^V < \theta^S$*) if and only if* $\gamma > \bar{\gamma}$ *(i.e., R&D is sufficiently costly).*

The proposition states that vertical integration increases social innovation when the R&D project is not very costly, but reduces social innovation when R&D is costly. The intuition can be understood as follows. We have established that vertical integration reduces the R&D investment of the integrated downstream firm and raises that of the non-integrated downstream firm (Proposition 3). When R&D is not very costly (i.e., γ is small), D_2's investment in R&D under vertical integration can be so large that its success rate is close to 1, so R&D is almost certain to succeed under VI, which is not the case under VS. When R&D is costly (i.e., γ is large), the intuition can be best understood by considering drastic innovation, although the conclusion still holds for non-drastic innovation. Roughly, the two firms' total investment is determined by the extra joint profit the innovation can bring. Since the innovation is drastic, the post-innovation outcome is the same regardless of the pre-innovation market structure or who wins the patent, so the total investment must be inversely related to the two firms' pre-innovation joint profit. Because vertical integration raises the two firms' joint profit before the innovation, their total R&D investment is smaller under VI than under VS. Again, part of the benefit of innovation is vertical integration; if firms are already integrated vertically before the innovation, the joint reward of innovation is reduced, leading to a lower rate of innovation on the aggregate.

The result that vertical integration may enhance the social rate of innovation is particularly interesting. When investigating how market structure affects innovation (mostly in a horizontal setting), existing researches usually consider the R&D incentive of a single firm.[19] In this paper, we consider not only the integrating firm, but also the non-integrating firm. Since the former's incentive

[19] There are some papers looking at how the aggregate innovation changes with the number of symmetric firms (e.g., Lee and Wilde 1980; Delbono and Denicolo 1991). Here, we compare the aggregate innovation between a symmetric structure (i.e., VS) and an asymmetric structure (i.e., VI) with a fixed number of firms.

is weakened while the latter's is enhanced, we further consider the joint effect, and find that market concentration can be conducive to innovation. Market concentration puts a non-integrated firm at a disadvantage, and thereby gives it greater incentive to conduct R&D.

7 Conclusions

Cross-market R&D, which enables the innovator to enter related markets, has not received much attention in the literature. We study downstream firms' incentive to invent a new input in a two-tier vertical industry. Successful input innovation brings about structural changes to the vertically related industry as the innovative downstream firm enters backward into the upstream market and competes with, or even drives out, the existing input supplier. We show that integration of the incumbent input supplier with a downstream producer reduces the R&D incentive of the integrated downstream firm and raises the incentive of the non-integrated downstream firm. The results hold for general demands and R&D cost functions, whether one or both downstream firms conduct R&D, regardless of the degree of product differentiation or the mode of competition at either level of the industry,[20] and for both drastic and non-drastic innovations.

Our model identifies the structural changes that can be brought about by input innovation. In a vertically separated industry, downstream input R&D transforms the industry into an vertically integrated one as the innovating downstream firm enters backward into the upstream industry. In a vertically integrated industry, input R&D by the non-integrated firm leads to relationship reversal, whereby the innovating non-integrated firm supplies the new input to its former supplier after R&D. When such structural changes take place, a firm's innovation brings the benefits of both vertical integration and cost reduction. By contrast, R&D by an already integrated firm generates only the benefit of cost reduction, and is thereby not as attractive to the firm.

The fear of structural changes caused by disruptive downstream R&D can produce strategic incentives for vertical integration as well as for vertical separation. We show that situations exist where an upstream supplier may takeover a downstream firm so as to preempt its otherwise disruptive R&D that would overthrow the supplier. Likewise, there are cases where the incumbent supplier foregoes vertical integration with its downstream target firm and instead commits and to remaining separated for fear of increasing the R&D incentive of the downstream competitor, despite such integration would eliminate the double-marginalisation between the integrating firms and raise the cost of the rival. These incentives for strategic integration/separation have not been identified in the literature, to the best of our knowledge.

One can relate our findings to the long-time debate between the Schumpeterian view of R&D, which states that larger firms are more innovative, and the Arrownian argument, which stipulates that smaller firms have stronger incentives for R&D because of their lower pre-innovation profits. Economists have come to understand that both arguments have their merits and neither dominates the other in theory. Unlike the case of horizontal mergers, our model predicts that vertical mergers unambiguously reduce the R&D incentive of the merging firm but raise that of the non-merging firm.

[20] In our model, competition in the downstream market can be either Bertrand or Cournot, but competition in the upstream market is in price. This assumption simplifies the analysis because in the post-innovation equilibrium there is always only one supplier in the upstream market. If, instead, upstream competition is in Cournot fashion, more scenarios may arise. In particular, for non-drastic innovation, the innovating downstream firm may enter the upstream input market and compete with the incumbent supplier when the extent of innovation (the value of d) is moderate, or choose to not enter the upstream market at all when d is sufficiently small. It can be shown that our main results still hold, although the analyses are much more tedious.

In addition to being readily testable, these results push us to think deeper about the similarities and differences between vertical and horizontal settings when investigating innovation incentives.

Our findings can help antitrust authorities in deciding whether to challenge a vertical merger. The model predicts that vertical integration reduces or even preempts innovation. When both firms can conduct R&D, vertical integration may raise or reduce industry innovation depending on how costly the R&D is. Thus, such possible dynamic effects of vertical mergers should not be ignored in antitrust enforcement. Also, the elimination of double marginalisation has been recognized as having a major welfare-enhancing effect and hence used as a major defense of vertical integration. However, in innovative industries such a positive effect can also be achieved through innovation rather than by vertical integration, as is the case in our model. Therefore, in assessing the competition effects of vertical mergers, no integration may not be the proper counterfactual; one may need to consider the likelihood of vertical integration forced by disruptive innovation.

Appendix A

Proof of Proposition 1.

If the innovation is drastic, then $S_1^1 = V_1^1$. However,

$$\begin{aligned} V_1^0 &\equiv \max_w \pi(c,w) + (w-c)q(w,c) \\ &\geq \pi(c,w^S) + (w^S - c)q(w^S,c) \\ &\geq \pi(c,w^S) \\ &> \pi(w^S,w^S) \equiv S_1^0. \end{aligned}$$

As a result, $V_1^1 - V_1^0 < S_1^1 - S_1^0$.

For non-drastic innovation, first consider the case of $d = 0$. Then, $\Delta_1^V = V(c-d) - V(c) = V(c) - V(c) = 0$. Before the innovation, D_1 earns $S_1^0 = \pi(w^S(c), w^S(c))$. After the innovation, D_1 obtains its own input at cost c. Meanwhile, upstream competition between U_n and U_0 (both producing the input at cost c) means D_2 obtains its input also at cost c. Therefore, D_1's post-innovation profit is $S_1^1 = \pi(c,c)$. Because $w^S(c) > c$ and given our assumption that $\frac{d\pi(w,w)}{dw} < 0$, we conclude that $S_1^1 = \pi(c,c) > \pi(w^S(c), w^S(c)) = S_1^0$. In sum, when $d = 0$, $\Delta_1^V = V_1^1 - V_1^0 = 0$ while $\Delta_1^S = S_1^1 - S_1^0 > 0$, so $V_1^1 - V_1^0 < S_1^1 - S_1^0$, or equivalently $V_1^1 - V_1^0 < S_1^1 - S_1^0$.

We next show that $V_1^1 - S_1^1$ (which is positive) decreases in d. Note that V_1^1 is the value of the unconstrained optimisation problem (3), while S_1^1 is the value of the same optimisation problem with a constraint that the choice variable w_2 cannot exceed c (equation (4)). Let $f(w,d)$ denote D_1's objective function, where w is its choice variable (this is w_2 in the text) and d is the parameter. Then, $V_1^1 = \max_w f(w,d)$ and $S_1^1 = \max_{w \text{ , s.t. } w \leq c} f(w,d)$. Let $w(d)$ denote the solution to the unconstrained optimisation problem. Then, it must satisfy the first-order condition (FOC): $f_w(w(d),d) \equiv 0$, where f_w means $\frac{\partial f}{\partial w}$. Take the total differentiation of the FOC on d: $\frac{\partial f_w}{\partial w} \cdot w'(d) + \frac{\partial f_w}{\partial d} = 0$. However, $\frac{\partial f_w}{\partial w} = \frac{\partial^2 f}{\partial w^2} < 0$ by the second-order condition, while we know that $w'(d) < 0$. Then, it must be the case that $\frac{\partial f_w}{\partial d} < 0$. Note that $\frac{\partial f_w}{\partial d} = \frac{\partial^2 f(w,d)}{\partial w \partial d} = \frac{\partial f_d(w,d)}{\partial w}$, where f_d means $\frac{\partial f}{\partial d}$. Now, V_1^1 is the objective function evaluated at the optimal choice $w = w(d)$,so$V_1^1 = f(w(d),d)$, while S_1^1 is the same objective function evaluated at c because the constraint $w \leq c$ is binding for non-drastic innovation: $S_1^1 = f(c,d)$. Then, $\frac{\partial}{\partial d}(V_1^1 - S_1^1) = \frac{\partial f(w(d),d)}{\partial d} - \frac{\partial f(c,d)}{\partial d}$. By the envelope theorem, $\frac{\partial f(w(d),d)}{\partial d} = \frac{\partial f(\bar{w},d)}{\partial d} = f_d|_{w=\bar{w}}$, while $\frac{\partial f(c,d)}{\partial d} = f_d|_{w=c}$. Since $\bar{w} = w(d) > c$ and $\frac{\partial f_d(w,d)}{\partial w} < 0$, it must be that $f_d|_{w=\bar{w}} < f_d|_{w=c}$, i.e., $\frac{\partial}{\partial d}(V_1^1 - S_1^1) < 0$.

Finally, note that $V_1^1 - V_1^0 < S_1^1 - S_1^0$ if and only if $V_1^1 - S_1^1 < V_1^0 - S_1^0$. The right-hand side is independent of d, while we conclud that (1) the left-hand side decreases in d; and (2) the inequality $V_1^1 - S_1^1 < V_1^0 - S_1^0$ holds for $d = 0$. Then, the inequality must hold for any $d > 0$ (such that the innovation is non-drastic).

Payoffs when downstream competition is Cournot.

Suppose that the downstream competition is Cournot and the demand for firm D_i's final product is:

$$p_i = a - q_i - \beta q_j,$$

where $\beta \in [0,1]$ represents the degree of product substitution. Given the input prices w_i and w_j, it can be shown that firm D_i's Cournot output is $q(w_i, w_j) = \frac{(2-\beta)a - 2w_i + \beta w_j}{4-\beta^2}$, and its downstream Cournot profit is

$$\pi(w_i, w_j) = \left[\frac{(2-\beta)a - 2w_i + \beta w_j}{4-\beta^2}\right]^2.$$

The equilibrium input price under VI (solution to (2)) is then solved as $w^I(c) = a - \frac{a-c}{2}\frac{8-2\beta^2-\beta^3}{8-3\beta^2}$, and that under VS (solution to (1)) is $w^S(c) = \frac{a+c}{2}$. The innovation is drastic (i.e., $w^I(c-d) \le c$) if and only if $\delta \equiv \frac{d}{a-c} \ge \frac{8-4\beta^2+\beta^3}{8-2\beta^2-\beta^3} \equiv \lambda_J$, where δ is the normalised significance of the innovation. Let $A = (a-c)^2$. We have the following profit expressions:

$$V_1^0 = A\frac{(12-8\beta+\beta^2)}{4(8-3\beta^2)}, \quad V_2^0 = A\frac{4(1-\beta)^2}{(8-3\beta^2)^2}.$$

$$S_1^0 = S_2^0 = \frac{A}{4(2+\beta)^2}.$$

$$V_1^1 = V_1^0(1+\delta)^2, \quad V_2^1 = V_2^0(1+\delta)^2.$$

For non-drastic innovation,

$$S_1^1 = A\left\{\frac{[(2-\beta)-\beta\delta]\delta}{4-\beta^2} + \frac{[(2-\beta)+2\delta]^2}{(4-\beta^2)^2}\right\}, \quad S_2^1 = A\frac{[(2-\beta)-\beta\delta]^2}{(4-\beta^2)^2}.$$

Payoffs when downstream competition is Bertrand.

Now consider downstream Bertrand competition. The demand is given by[21]

$$q_i = h(1-b) - p_i + bp_j,$$

[21] A general linear demand for Bertrand competition can be represented by $q_i = h' - kp_i + bp_j$. Because h', k and b are homogeneous of degree one, we can normalize $k = 1$ without any loss of generality. It turns out that we may have another normalization: $\delta = \frac{d}{\frac{h'}{1-b} - c}$, which can be achieved by assuming $h' = (1-b)h$ in the demand function. Then all profits depend only on b and δ ($(h-c)^2$ is a common factor for all profits), just like in the case of Cournot competition. Note that $h' = (1-b)h$ is just a transformation of variable. It does not place any constraint on the demand system, which still contains two independent parameters. Since our major results are not about comparative statics concerning b, it does not matter whether the vertical intercept is h' or $h(1-b)$.

where $b \in [0, 1]$ represents the degree of product substitution. Given the input prices w_i and w_j, it can be shown that firm D_i's equilibrium price for its final product is $p(w_i, w_j) = \frac{(2-b-b^2)h+2w_i+bw_j}{4-b^2}$, and its equilibrium downstream profit is

$$\pi(w_i, w_j) = \left[\frac{(2-b-b^2)h - (2-b^2)w_i + bw_j}{4-b^2}\right]^2.$$

The equilibrium input price is $w^I(c) = h - \frac{h-c}{2}\frac{8-6b^2+b^3+b^4}{8-7b^2+b^4}$ under VI, and is $w^S(c) = \frac{h+c}{2}$ under VS. The innovation is drastic (i.e., $w^I(c-d) \le c$) if and only if $\delta \equiv \frac{d}{h-c} \ge \frac{8-8b^2-b^3+b^4}{8-6b^2+b^3+b^4} \equiv \lambda_J$.

Let $\delta \equiv \frac{d}{h-c}$ and $A = (h-c)^2$. Then, the equilibrium profits under different market structures are:[22]

$$V_1^0 = \frac{A(1-b)^2}{4}\frac{12+16b+5b^2}{8-7b^2+b^4}, \quad V_2^0 = \frac{A(1-b)^2}{4}\left(\frac{4-4b^2-b^3}{8-7b^2+b^4}\right)^2.$$

$$S_1^0 = S_2^0 = \frac{A(1-b)^2}{4}\frac{1}{(2-b)^2}.$$

$$V_1^1 = V_1^0(1+\delta)^2, \quad V_2^1 = V_2^0(1+\delta)^2.$$

For non-drastic innovation, we have

$$S_1^1 = A\left\{\frac{[(2-b-b^2)-b\delta]\delta}{4-b^2} + \frac{[(2-b-b^2)+(2-b^2)\delta]^2}{(4-b^2)^2}\right\}, \quad S_2^1 = A\frac{[(2-b-b^2)-b\delta]^2}{(4-b^2)^2}.$$

Proof of Proposition 4.

The proof proceeds as follows. We first characterise the equilibrium investments in R&D under VS and VI. Then, we provide a result (Lemma 3) on the competitive incentives. Finally, we show that vertical integration lowers the R&D investment of the integrating firm and raises that of the non-integrating firm.

Step 1. We first compare equilibrium investments in R&D under the two market structures VS and VI. The equilibrium under VS is solved from:

$$\begin{cases} (1-\rho_2)(S_1^1 - S_1^0) + \rho_2(S_1^b - S_1^2) = C'(\rho_1) \\ (1-\rho_1)(S_2^2 - S_2^0) + \rho_1(S_2^b - S_2^1) = C'(\rho_2) \end{cases}$$

Similarly, the equilibrium under VI is solved from:

$$\begin{cases} (1-\rho_2)(V_1^1 - V_1^0) + \rho_2(V_1^b - V_1^2) = C'(\rho_1) \\ (1-\rho_1)(V_2^2 - V_2^0) + \rho_1(V_2^b - V_2^1) = C'(\rho_2) \end{cases}$$

Since the marginal cost of R&D is the same across the two market structures, any difference in equilibrium R&D investment must be driven by the difference in the marginal benefit of R&D, which, as mentioned earlier, is a weighted average of a firm's standalone and competitive incentives.

[22] We assume $b < 0.9$ so that the final products are not too closely substitutable, so as to guarantee that the non-integrating firm is still active in the downstream market. No such condition about foreclosure is needed under downstream Cournot competition.

Propositions 1 and 2 have established that vertical integration reduces D_1's stand-alone incentive ($V_1^1 - V_1^0 < S_1^1 - S_1^0$) and raises D_2's standalone incentive ($V_2^2 - V_2^0 > S_2^2 - S_2^0$).

Step 2. Regarding the competitive incentives, we have the following result.

Lemma 3:

(i) If the innovation is drastic, a firm's competitive incentive of R&D is independent of the market structure or the firm's identity (i.e., $V_1^b - V_1^2 = S_1^b - S_1^2 = V_2^b - V_2^1 = S_2^b - S_2^1$*).*

(ii) If the innovation is non-drastic, both firms' competitive incentives of R&D are greater under VI *than under* VS *(i.e.,* $V_1^b - V_1^2 > S_1^b - S_1^2$*, and* $V_2^b - V_2^1 > S_2^b - S_2^1$*).*

The proof is given as follows. By Lemmas 1 and 2, when the innovation is drastic, the post-innovation equilibrium under VI is the same as that under VS regardless of who the innovator is: $V_i^i = S_i^i$ and $V_i^j = S_i^j$ for $i = 1, 2$ and $j \neq i$. As a result, $V_i^b = \frac{1}{2}(V_i^i + V_i^j) = \frac{1}{2}(S_i^i + S_i^j) = S_i^b$ and consequently $V_i^b - V_i^j = S_i^b - S_i^j$. That is, if the innovation is drastic, a firm's competitive incentive is the same whether the market structure is VI or VS.

For non-drastic innovation, D_1's competitive incentive under VI is $V_1^b - V_1^2 = \frac{1}{2}(V_1^1 + V_1^2) - V_1^2 = \frac{1}{2}(V_1^1 - V_1^2)$, and that under VS is similarly $S_1^b - S_1^2 = \frac{1}{2}(S_1^1 - S_1^2)$. Because $V_1^1 > S_1^1$ (D_1 achieves unconstrained maximum for V_1^1, but is constrained by the independent U_0 for S_1^1), and $V_1^2 = S_1^2$ by Lemma 2 (when D_2 is the innovator, the post-innovation equilibrium is the same whether it's VI or VS), we conclude that $V_1^b - V_1^2 > S_1^b - S_1^2$, i.e., vertical integration raises D_1's competitive incentive.

D_2's competitive incentive under VI is $V_2^b - V_2^1 = \frac{1}{2}(V_2^2 - V_2^1)$, and that under VS is $S_2^b - S_2^1 = \frac{1}{2}(S_2^2 - S_2^1)$. Because $V_2^2 = S_2^2$ by Lemma 2, while $V_2^1 > S_2^1$ (between S_2^1 and V_2^1, $w_1 = c - d$ is the same, but $w_2 = c$ for S_2^1, while $w_2 = w^I(c - d) > c$ for V_2^1), we conclude that $V_2^b - V_2^1 > S_2^b - S_2^1$, i.e., vertical integration raises D_2's competitive incentive.

To understand the intuition of Lemma 3, recall that a firm's competitive incentive is related to the difference between its payoff as the winner and that as the los in R&D competition. If the innovation is drastic, the equilibrium is the same between VI and VS whether the innovator is D_1 (Lemma 1) or D_2 (Lemma 2). In particular, the vertical structure does not affect a given firm's payoff either as the winner or loser, and hence does not affect the difference between the two payoffs. This means that for drastic innovation, a firm's competitive incentive is the same whether the vertical structure is VI or VS. Now consider non-drastic innovation. For D_1, its payoff as the loser is independent of the vertical structure (Lemma 2, as D_2 is the winner)), but its payoff as the winner is larger under VI than under VS, where it has to compete with the incumbent supplier U_0 in supplying D_2. As a result, D_1's competitive incentive is greater under VI. For D_2, its payoff as the winner is independent of the vertical structure (Lemma 2), but its payoff as the loser is smaller under VI than under VS, so its competitive incentive is also greater under VI.

Step 3.

For drastic innovation, Lemma 3 together with Propositions 1 and 2 indicate that vertical integration lowers the marginal benefit of R&D for D_1 and raises that for D_2. Refer to Figure 1 for the two firms' best response curves for drastic innovation, where the solid lines are those under VS and the dashed lines are those under VI. Consider firm D_1's best responses under the two market structures. If $\rho_2 = 1$, D_1's marginal benefit of R&D equals its competitive incentive, which is the same whether it is VS or VI, so D_1's two best response curves intersect at this point. When $\rho_2 < 1$, D_1's marginal benefit is a weighted average of its stanalone and competitive incentives. Its competitive incentive is the same, but its standalone incentive is smaller under VI than under VS, so D_1's best response under VI is everywhere below its best response under VS. Moving from VS

to VI, therefore, D_1's best response rotates downward around the upper intercept (where $\rho_2 = 1$). By a similar argument, moving from VS to VI, D_2's best response rotates upward around the right intercept (where $\rho_1 = 1$). Now consider the equilibrium R&D investment under the two vertical structures. Under VS, the two firms' best response curves are symmetric and downward sloping. Moving from VS to VI, then, the two firms' best responses intersect at a point to the northwest of the VS intersection, which means that D_1's R&D investment is smaller, whereas D_2's R&D investment is larger.

Figure 1: Best response curves for drastic innovation

For non-drastic innovation, refer to Figure 2 for the two firms' best response curves. The two symmetric best response curves under VS (the solid lines) are similar to those in Figure 1. For D_2, vertical integration raises its standalone incentive (Proposition 2) and competitive incentive (Lemma 3), so moving from VS to VI, D_2's best response shifts up for any value of ρ_1. For D_1, vertical integration lowers its standalone incentive (Proposition 1) and raises its competitive incentive (Lemma 3), so D_1's best response curve under VI may be upward or downward sloping, but what matters for the comparison between (ρ_1^S, ρ_2^S) and (ρ_1^V, ρ_2^V) is where $\rho_1^V(\rho_2^V)$ intersects $\rho_1^S(\rho_2^S)$, not the slope of D_1's best response curve.

Figure 2: Best response curves for non-drastic innovation

Given that the R&D cost function is $C(\rho) = \frac{\gamma}{2}\rho^2$, the two firms' best response functions under VS (which are symmetric) are determined by

$$(1-\rho_2)(S_1^1 - S_1^0) + \rho_2(S_1^b - S_1^2) = \gamma\rho_1 \tag{10}$$
$$(1-\rho_1)(S_2^2 - S_2^0) + \rho_1(S_2^b - S_2^1) = \gamma\rho_2 \tag{11}$$

and their best response functions under VI are determined by

$$(1-\rho_2)(V_1^1 - V_1^0) + \rho_2(V_1^b - V_1^2) = \gamma\rho_1 \tag{12}$$
$$(1-\rho_1)(V_2^2 - V_2^0) + \rho_1(V_2^b - V_2^1) = \gamma\rho_2 \tag{13}$$

The left-hand side of (10) can be rewritten as $(S_1^1 - S_1^0) - \rho_2(S_1^b - S_1^0)$, and the left-hand side of (12) is $(V_1^1 - V_1^0) - \rho_2(V_1^b - V_1^0)$. Then the intersection of these two functions is characterised by $(S_1^1 - S_1^0) - \rho_2(S_1^b - S_1^0) = (V_1^1 - V_1^0) - \rho_2(V_1^b - V_1^0)$, or $\rho_2^* = \frac{(S_1^1 - S_1^0) - (V_1^1 - V_1^0)}{(S_1^b - S_1^0) - (V_1^b - V_1^0)}$. The VS equilibrium (i.e., the intersection between (10) and (11)) leads to $\rho_2^s = \frac{S_1^1 - S_1^0}{\gamma + S_1^b - S_1^0}$. Then, $\rho_2^* > \rho_2^s$ if and only if $\gamma > \frac{(S_1^b - S_1^0)(V_1^b - V_1^0) - (S_1^1 - S_1^0)(V_1^b - V_1^0)}{(S_1^1 - S_1^0) - (V_1^1 - V_1^0)} \equiv \gamma_1$. However, for (12) and (13) to intersect within the range of $\rho_1 \in (0,1)$ and $\rho_2 \in (0,1)$, we must have $\gamma > V_2^2 - V_2^0$. It can be shown from both the Cournot profits and the Bertrand profits that $V_2^2 - V_2^0 > \gamma_1$. Therefore, $\rho_2^* > \rho_2^s$. When the demand is linear (either Cournot or Bertrand in downstream competition) and R&D cost is quadratic, the intersection between $\rho_1^V(\rho_2^V)$ and $\rho_1^S(\rho_2^S)$ leads to a ρ_2 that is above the VS equilibrium ρ_2^S. It is then clear from Figure 2 that the VI equilibrium has a smaller ρ_1 and a greater ρ_2 than the VS equilibrium. **Q.E.D.**

Proof of Proposition 5.

Note that $C'(\rho_i) = \gamma\rho_i$ and hence the first-order conditions (FOCs) can be rewritten as

$$\gamma\rho_i^t + M_i^t\rho_j^t = A_i^t,$$

where $A_i^t = t_i^i - t_i^0$ is firm D_i's standalone incentive under market structure $t = V, S$, while M_i^S is the difference between D_i's standalone incentive and competitive incentive: $M_i^t = (t_i^i - t_i^0) - (t_i^b - t_i^j) = t_i^b - t_i^0$. The R&D competition equilibrium under vertical structure t is consequently solved from the following two FOCs:

$$\gamma\rho_1^t + M_1^t\rho_2^t = A_1^t$$
$$M_2^t\rho_1^t + \gamma\rho_2^t = A_2^t$$

Closed-form solutions can be easily calculated as $\rho_1^t = \frac{A_1^t\gamma - A_2^t M_1^t}{\gamma^2 - M_1^t M_2^t}$ and $\rho_2^t = \frac{A_2^t\gamma - A_1^t M_2^t}{\gamma^2 - M_1^t M_2^t}$.

If the innovation is drastic, the competitive incentive is the same regardless of the innovator or market structure: $t_i^b - t_i^j = P_i^t = P$ for $i = 1, 2$ and $t = V, S$. Then the equilibrium investment can be rewritten as $\rho_i^t = 1 - (\gamma - P)\frac{(\gamma - M_i^t)}{\gamma^2 - M_i^t M_j^t}$. As a result, $\theta^t \equiv 1 - (1 - \rho_1^t)(1 - \rho_2^t) = 1 - (\gamma - P)^2 h^t$, where $h^t = \frac{(\gamma - M_1^t)(\gamma - M_2^t)}{(\gamma^2 - M_1^t M_2^t)^2}$. Then, $\theta^V < \theta^S$ if and only if $h^V > h^S$. Note that $\rho_2^V \leq 1$ requires $\gamma \geq M_2^V$. When $\gamma = M_2^V$, $h^V = 0$ whereas $h^S > 0$, so $h^V < h^S$, meaning that $\theta^V > \theta^S$.

Note that the equilibrium under VS is symmetric: $M_1^S = M_2^S = M^S$, so $h^S = \frac{(\gamma - M_1^S)(\gamma - M_2^S)}{\left(\gamma^2 - M_1^S M_2^S\right)^2} = \frac{1}{(\gamma + M^S)^2}$. It can be shown that $h^V - h^S$ has the same sign as a cubical function in γ, where the coefficient of γ^3 is $2M^S - M_1^V - M_2^V$. Because the innovation is drastic, the post-innovation outcome is the same under both VI and VS, so $2M^S - M_1^V - M_2^V = (V_1^0 + V_2^0) - (S_1^0 + S_2^0)$. It can be shown for both Bertrand and Cournot competition that $V_1^0 + V_2^0 > S_1^0 + S_2^0$. Therefore, $h^V - h^S > 0$ when γ is sufficiently large, and as a result $\theta^V < \theta^S$.

For non-drastic innovation, analytical comparison cannot be obtained. We resort to numerical simulation and confirm the conclusion: $\theta^V > \theta^S$ when γ is small, and $\theta^V < \theta^S$ when γ is large. Therefore, there exists a critical γ, below which vertical integration increases the industry rate of innovation.

Appendix B

We consider the innovation incentive by the upstream firm under two different market structures, vertical separation and vertical integration. We address the following question: under which vertical market structure does the upstream firm have higher innovation incentive? Following the framework in the previous sections, we consider the pure incentive to innovate. Specifically, we compare the incentives to innovate under vertical separation and vertical integration when only the upstream firm conducts innovation.[23] For simplicity and tractability, we assume the following linear demand system in downstream competition: $p_i = a - q_i - \beta q_j$ where $\beta \in (0, 1)$ represents the degree of product substitution.

Consider first the case of vertical separation in which U_0 supplies the input both downstream firms at price w. Thus, U_0 solves the following optimisation problem:

[23] We have also considered a setting as in Gilbert and Newsbery (1982) and found that a downstream firm has higher incentive to innovate than the upstream firm.

$$\max_{w} \; 2\,(w-c)\,q\,(w,w)\,.$$

To find the optimal solution of w to the above problem, we first solve the subgame of downstream competition. In particular, given that U_0 supplies the input at price w, downstream firm D_i maximises $(a - q_i - \beta q_j - w)\,q_i$. We can show that the quantity of input each downstream firm demands in the symmetric equilibrium is

$$q\,(w,w) = \frac{a-w}{2+\beta}.$$

Thus, given $q\,(w,w)$, U_0's optimisation problem becomes

$$\max_{w} \; 2\,(w-c)\left(\frac{a-w}{2+\beta}\right).$$

The optimal solution to the above problem is

$$w^S\,(c) = \frac{a+c}{2}.$$

and U_0 earns profit

$$\Pi^S\,(c) = \frac{(a-c)^2}{2\,(2+\beta)}.$$

We next turn to the case of vertical integration in which U_0 and D_1 are vertically integrated. In this case, U_0 supplies D_1 (internally) at cost c but charges price w_2 to D_2. Thus, U_0 solves the following optimisation problem:

$$\max_{w_2} \;\; (w_2 - c)\,q\,(w_2, c) + \pi\,(c, w_2)\,.$$

We can show that the profit for integrating firm is

$$\Pi^I\,(c) = \frac{(2-\beta)\,(6-\beta)}{4\left(8-3\beta^2\right)}\,(a-c)^2\,.$$

Now, we are in a position to examine the innovation incentive under two different vertical market structures. Note that the effects of a reduction of input cost (from c to $c-d$) on innovation incentive under vertical separation and vertical integration are, respectively,

$$S_0\,(c-d) - S_0\,(c) = \frac{1}{2\,(2+\beta)}\left[(a-c+d)^2 - (a-c)^2\right]$$

and

$$V_0\,(c-d) - V_0\,(c) = \frac{(2-\beta)\,(6-\beta)}{4\left(8-3\beta^2\right)}\left[(a-c+d)^2 - (a-c)^2\right].$$

Since

$$\frac{(2-\beta)\,(6-\beta)}{2\left(8-3\beta^2\right)} - \frac{1}{2+\beta} = \frac{1}{2}\,\frac{-4\beta+\beta^3+8}{(\beta+2)\left(8-3\beta^2\right)} > 0,$$

we have the following result, similar to that of Chen and Sappington (2010).

Proposition 6. *Under linear demand system and quadratic R&D* cost function, *the upstream firm has a higher incentive to innovate under vertical integration than under vertical separation:* $V_0(c-d) - V_0(c) > S_0(c-d) - S_0(c)$.

To understand the result, it is worth noting the following points. First, note that in this upstream R&D setting, there is no market structure change following innovation: after the innovation by the upstream firm, the market structure remains VS (VI) if it was VS (VI) before R&D. This contrasts with the case of downstream R&D under VS, where the innovation transforms the industry to VI. In other words, for upstream R&D, there is no structural benefit to the innovator. Second, in this upstream R&D setting, the replacement effect of R&D exists under both VI and VS: as the upstream firm innovates, it replaces the original profit regardless of the market structure. Third, there are positive externalities associated with R&D here: as the upstream firm innovates, both downstream firms benefit as the input price goes down. This positive externality is partially internalised under VI, but not under VS. Hence, the R&D incentive of the upstream firm is greater under VI than under VS.

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