Investigating double counting terms in the value-added decomposition of gross exports

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Several papers using inter-country input-output tables have developed frameworks to decompose value-added in gross exports and to remove potential double counting in intermediate inputs. We point out that while domestic value-added can indeed be 'double counted' in the domestic content of exports, the concept of foreign double counting is more complex to define and does not always imply value-added counted twice from the point of view of the exporting economy. When talking about foreign double counting in exports, it is important to distinguish the global level (world) from the country level and to be precise about the perspective from which value-added is decomposed. We provide a general framework and introduce different decompositions based on a global consistency or country consistency in order to shed light on the distribution of value-added in aggregate and bilateral export flows. We thus clarify the meaning of double counting terms and explain differences in the decompositions that have been proposed in the literature.

1. Introduction

To better understand the fragmentation of production and trade in the context of global value chains (Gereffi and Fernandez-Stark, 2016), a series of papers have introduced frameworks for decomposing gross exports in inter-country input-output tables. These papers aim at measuring the value-added contribution of all countries involved in the production process (Daudin et al., 2011; Johnson and Noguera, 2012; Koopman et al., 2014; Foster-McGregor and Stehrer, 2013; Los et al., 2016; Miroudot and Ye, 2017; Borin and Mancini, 2017; Johnson, 2018; Arto et al., 2019). One motivation for developing value-added measures of trade is to remove the ‘double counting’ in gross exports. In the input-output framework, the concept of ‘double counting’ comes from the measurement of intermediate inputs. Output is equal to (domestic) value-added plus intermediate inputs. But intermediate inputs are also produced with (domestic or foreign) value-added and other intermediate inputs. Double counting can be regarded as a subset of intermediate inputs in output decomposition.

Since gross exports correspond to the share of output sold to foreign consumers, there is also a ‘double counting’ involved. This double counting in intermediate inputs can be removed by looking at net trade (Trefler and Zhu, 2010) or by working with measures of value-added trade derived from final demand (Johnson and Noguera, 2012). But when authors start to introduce double counting terms in the decomposition of gross exports, things become more complicated since intermediate inputs are both part of exported goods and foreign inputs used in their production. Moreover, the concept of ‘foreign value-added’ (FVA) in trade, which is the variable of interest to understand global production, leads to further questions on what is double counted. When looking at exports of all countries in the world, any foreign value-added is by definition double counted, since it is ‘domestic value-added’ (DVA) in other countries. What authors try to define as double counting is therefore no longer the intermediate inputs double counted in output but some share of value-added that would be counted several times from the point of view of the exporting economy, including in the FVA term (something sometimes referred to as ‘pure double counting’).

In the domestic content of gross exports, there is some consensus in the way a 'pure' domestic double counting (DDC) can be separated out from DVA.[1] This double counting corresponds to domestic intermediate inputs that come back to the exporting economy embodied in foreign inputs and are exported again after further processing. An example would be steel produced by China and exported to Thailand to be incorporated in parts and components from the motor vehicles industry that are then exported back to China and used in exports of Chinese cars. Within Chinese gross exports, the value-added related to this production of steel will be counted twice (in exports of steel and in exports of cars).

This double counting can also happen with foreign inputs. The same Chinese steel, for example, could be used in Thailand to produce cylinders for an engine manufactured in Malaysia. This engine could then be exported back and incorporated in cars assembled in Thailand and exported to Japan. This kind of 'circular trade' is what creates double counting. When decomposing gross exports of Thailand, the value-added associated to Chinese steel (which is foreign) will be counted twice: first in exports of engines to Malaysia and then in exports of cars to Japan.

There is no consensus yet on the definition and calculation of this foreign double counting (FDC) in gross exports decompositions. As a consequence, there is no

[1] There are however still differences among papers when considering the bilateral level due to different assumptions on the way bilateral measures sum (or not) to aggregate measures.

consensus either on the correct measure for FVA since the two terms should sum to a foreign content already defined as the difference between the domestic content (DVA+DDC) and gross exports.

Some authors, such as Koopman et al. (2014), Nagengast and Stehrer (2016) or Borin and Mancini (2017) propose to base the definition of double counting (domestic or foreign) on the number of international border crossings with the objective of identifying value-added that crosses the same border several times (at least twice). Alternatively, Miroudot and Ye (2017) rely on a supply-side input-output model. In their framework, double counting terms can be measured by the second and later input rounds in the generation of value-added in exported goods (using the Ghosh decomposition). Finally, Los and Timmer (2018) point out that double counting also depends on whether aggregate or bilateral exports are decomposed. In particular, they identify a 'double count of domestic value added in summing bilateral measures' which is the difference between domestic value-added in aggregate exports (i.e with partner world) and the sum of bilateral domestic value-added across all partners.

In this paper, we investigate more closely the concept of 'double counting' in the decomposition of gross exports. First, we show that while DVA can indeed be 'double counted' in the domestic content of exports, the concept of FDC is more complicated and does not always imply value-added counted twice from the point of view of the exporting economy. We review the existing literature and introduce a new decomposition framework (consistent with Los et al., 2016) to show that there are several possible answers to the definition of double counting in gross exports. Using numerical examples and calculations with the World Input-Output Database (WIOD), we suggest that these decompositions lead to a different economic interpretation and can answer different types of questions in relation to global production.

Section 2 discusses the concept of double counting in gross exports decompositions and how it was dealt with in previous papers. Section 3 introduces a new input-output framework (consistent with Los et al., 2016) that allows us to clarify the definition of double counting terms and to distinguish a country consistency and global consistency approach, while also dealing with bilateral measures. Section 4 develops numerical examples and use WIOD data to illustrate how the different decompositions compare to each other and what we can learn through the double counting terms. Section 5 concludes.

2. Defining double counting terms in the decomposition of gross exports: main issues

In the framework developed by Koopman, Wang and Wei (2014), KWW hereafter, double counting is defined as the value-added that crosses international borders more than once. Therefore, all the FVA is already double counted. It makes sense since the authors are interested in removing double counting from aggregate world trade statistics. In this case, FVA in exports is by definition DVA in the exports of another country and double counted. In order to decompose gross exports of a specific country and to introduce a FVA term, the authors then refer to a 'pure' double counting, which is the difference between gross exports and the sum of DVA and FVA. This 'pure double counting' is then split between a domestic and foreign component so that at the end gross trade is decomposed into four terms: DVA, DDC, FVA and FDC.[2] Defined as a residual, this pure double counting can be calculated but there is no clear interpretation of what it exactly measures.[3]

Pointing out the issue with KWW, Borin and Mancini (2017) propose a different definition for double-counted terms. From the point of view of a specific exporting economy, double counting corresponds to the value-added that has crossed the country's border more than once. It is a better starting point but the issue with a definition of double counting based on the number of border crossings is that the input-output framework cannot tell us how many times value-added has crossed borders. The input-output matrix identifies international and domestic transactions but there are many paths through which value-added can reach final consumers and these paths are not known. They are summarized in a single input-output matrix that has collapsed the different production stages (Los and Timmer, 2018). As we will see, some assumptions

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[2] There are actually 9 terms in the KWW decomposition but the additional terms further decompose DVA and FVA on the basis of where value-added is absorbed.

[3] For another paper defining double counting as the difference between gross exports and value-added embodied in trade, see Arto et al. (2019). This paper has different results than the KWW paper, highlighting that this definition is compatible with several types of decompositions and therefore unclear about what FVA and FDC measure.

have to be made to allocate FVA and to decide whether or not it is double counted. This subtlety explains why there is no simple formula to calculate FVA in exports (net of double counting) and why there is no consensus yet in the literature on how it can be done.

The definition that Borin and Mancini (2017) propose for double counting in the sense of value-added coming twice to the same economy is conceptually sound and it is the definition we also suggest. But its implementation in the input-output framework is problematic. As we will formally show in the next Section, value-added ratios multiplied by the Leontief inverse can be used to measure value-added when it enters a specific country "for the first time". But before entering a specific country, this value-added has already crossed all possible borders according to the input-output table. Therefore, there is no clarity in terms of how many times borders are crossed. Moreover, the concept of 'border' is not the same when dealing with global exports (exports to the world) and bilateral exports. This further complicates the reference to border crossings in the definition of double counting. What is the 'border' already depends on the initial assumptions and the setting of the decomposition.

In Miroudot and Ye (2017), this issue is avoided by relying on the supply-side input-output model to define double counting. The Ghosh insight already refers to different rounds in the process of value generation. There is, embedded in the model, the concept of an initial round and value-added measured in all later rounds is by definition double counted. This framework provides a clear definition of double counting and is straightforward when it comes to its implementation in the context of an inter-country input-output table. However, there are debates on the foundations and assumptions behind the supply-side input-output model (Oosterhaven, 1988; Dietzenbacher, 1997). Value-added in this case is regarded as exogenous, which helps when it comes to its allocation to different countries, but one can question this assumption.

Something common to Borin and Mancini (2017) and Miroudot and Ye (2017) is a definition of double counting that assumes that there is a first country where value-added is generated (and exported) and that any time this value-added is measured somewhere else in the exports of another country, it has to be regarded as part of double counting terms. We will see that such approach defines double counting on the basis of global (wold) exports (global consistency). It is explicit in Miroudot and Ye (2017) but maybe less clear in the context of Borin and Mancini (2017) since they refer to value-added crossing twice the border of the same country. We show in Section 4 that the decompositions by Miroudot and Ye (2017) and Borin and Mancini (2017) provide the same results. Referring to the work of Nagengast and Stehrer (2016), we could also call this approach 'source-based', i.e. from the point of view of the (first) exporting economy.[4]

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[4] Nagengast and Stehrer (2016) distinguish a 'source-based' approach from a 'sink-based' approach in the value-added analysis of bilateral trade balances. The source-based approach takes the perspective of the exporting economy and the sink-based approach the

Lastly, the paper by Los et al. (2016) is the only one that does not introduce double counting terms. It also has no explicit formula for FVA. Nevertheless, the methodology it applies to derive DVA in gross exports (an hypothetical extraction) can also be used to estimate FVA. The difference between the sum of DVA and FVA in such framework also creates a residual that can be interpreted as double counting. Even more interesting is the fact that this double counting is different from the one calculated by KWW and by Borin and Mancini (2017) or Miroudot and Ye (2017). We believe that this residual corresponds to the value-added coming actually twice to the exporting economy (domestic or foreign). The framework of Los et al. (2016), further developed in Los and Timmer (2019), is also the one that explains why the sum of bilateral measures of value-added is different from the decomposition of aggregate measures (i.e. exports to the world). This is why we use it as a starting point in our analysis.

3. Decomposition framework for gross exports: country consistency versus global consistency

Looking at the literature it seems that there are two main approaches in the way authors allocate value-added across countries when decomposing gross exports. These

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perspective of the country of final absorption. The distinction we make in this paper between the 'country consistency' and 'global consistency' is different. It asks whether the decomposition takes the perspective of world exports or exports of a given country (based on the type of Lentief inverse used in the decomposition).

approaches are not different when it comes to DVA but lead to significant differences in the calculation of FVA due to the definition of double counting (FDC).

To distinguish these two approaches, we refer to the 'global consistency' and 'country consistency'. The country consistency is the approach where double counting can be defined as value-added (domestic or foreign) that crosses the border of the exporting economy more than once. When allocating value-added found in exports across countries, this approach disregards what is measured in other countries and only takes the perspective of the exporting economy.

The 'global consistency' approach is different. Even when authors start from the exports of a given country or bilateral exports, the Leontief inverse they use implies that the way they allocate value-added in exports takes the perspective of global exports. We would describe the results they obtain as a mapping across countries of the decomposition of world exports. As previously pointed out, world exports can be decomposed into two terms: value-added (in trade) and intermediate inputs (in trade). There is no concept of domestic or foreign at the world level. All intermediate inputs double count (or count multiple times) the value-added in exports and this value-added in exports is consistent with world GDP (the share of GDP going to exports). World value-added is DVA and can be allocated to the different countries where it was generated. Then, the value of intermediate inputs (already counted in DVA) can be split into FVA and double counting terms. FVA in this case is the allocation across countries of DVA in exports that is embodied in exports of other countries. Measured globally, FVA and double counting (which is the difference between DVA, FVA and gross exports) are different.

To show this, we start with the standard Leontief (1936) input-output framework extended to G countries and N sectors in an inter-country input-output (ICIO) table, as it is usually done in the trade in value-added literature. The basic input-output relationship states that all gross output must be used either as an intermediate good or as a final good:

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Yu} \tag{1}$$

where, $\mathbf{x}$ is the $NG \times 1$ gross output vector, $\mathbf{Y}$ is the $NG \times G$ final demand matrix, in which $\mathbf{u}$ is a unit column $G \times 1$ vector, and $\mathbf{A}$ is the $NG \times NG$ I-O coefficients matrix.

As previously emphasised, gross exports is a subset of gross output. Our objective is to recreate the basic equations of the Leontief model but separating out exports from the rest of gross output. As such, our methodology is close to the hypothetical extraction proposed by Los et al. (2016). But the value-added decomposition (and definition of double counting) depends on whether we separate out exports of all countries (global consistency) or exports of a single country (country consistency). We can also extract exports to a single partner (which is the right way to get bilateral measures of value-added in trade) but we will come back later to the question of bilateral exports.[5] The framework is 'universal' in the sense that we can keep the same equations but with different 'extraction' matrices when separating out exports.

When focusing on the exports of a given country i (country consistency), we split the output vector into an exports vector $\mathbf{e}$ that has the length G times N with the exports for all industries in country i corresponding to elements $\mathbf{e}_i$ and zeros elsewhere ($\mathbf{e}=[\mathbf{0},\ldots,\mathbf{e}_i,\ldots,\mathbf{0}]$).[6] In the global consistency approach, the exports vector becomes the array of global exports ($\mathbf{e}=[\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_i, \ldots, \mathbf{e}_G]$). In both cases, the remaining term is $\mathbf{h}$ with $\mathbf{x}=\mathbf{e}+\mathbf{h}$.

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[5] Authors following the global consistency approach also have bilateral measures but in this case what they provide is a mapping of the country's exports across different partners rather than a decomposition of bilateral gross exports. With such an approach, the sum of bilateral measures is equal to the aggregate value-added decomposition. It is also the case for decompositions following the country consistency approach but assuming that the sum of bilateral measures is equal to the aggregate decomposition (e.g. Arto et al., 2019).

[6] We omit subscripts related to industries to simplify the presentation of the framework but all matrices and vectors refer to an inter-country input-output table with G countries and N industries as previously indicated. Note that extracting exports of a single industry in the exporting country also leads to different results and can be used to obtain a decomposition of gross exports at the industry level.

Then, the following accounting equations can be obtained: $\mathbf{e} = \mathbf{A^I(e+h)} + \mathbf{Y^I u}$ and $\mathbf{h} = \mathbf{A^*(e+h)} + \mathbf{Y^* u}$, where $\mathbf{A^I}$ is the given export measurement matrix including the IO coefficients for the use of intermediate inputs from one country into another country and $\mathbf{A^*}$ is the corresponding extraction matrix, so that we have $\mathbf{A} = \mathbf{A^I} + \mathbf{A^*}$. $\mathbf{Y^I}$ is the foreign final demand for the given exports and $\mathbf{Y^*}$ is the extraction final demand matrix, so that $\mathbf{Y} = \mathbf{Y^I} + \mathbf{Y^*}$.

For example, assuming that we have three countries *i*, *j* and *k*, the intermediate inputs coefficients matrix is:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

With the country consistency approach, gross exports from country *i* to other countries are extracted ($\mathbf{e=[e}_i\mathbf{, 0,0]}$) and the **A** matrix is split into:

$$\mathbf{A^I} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}^* = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

With the global consistency approach where exports of all countries are extracted ($\mathbf{e=[e}_i\mathbf{,e}_j\mathbf{,e}_k\mathbf{]}$), the corresponding matrices are:

$$\mathbf{A^I} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{0} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}^* = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{jj} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{kk} \end{pmatrix}$$

Finally, if we measure value-added in bilateral exports between country *i* and *j* ($\mathbf{e=[e}_{ij}\mathbf{,0,0]}$), the matrices become:

$$\mathbf{A^I} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}^* = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

Using these different extraction matrices for the decomposition of value-added in gross exports has implications on the 'boundaries' of exports and what we will identify as double counted. It is also intuitive that the Leontief inverse based on these different matrices of inputs coefficients cannot lead to the same value-added decomposition. Domestic input coefficients (the diagonal of the A matrix) are the same across these matrices (explaining that results for DVA and DDC are not affected) but off-diagonal elements (impacting FVA and FDC) depend on the type of extraction.

After re-arrangement, the accounting relationship between exports and final demand in destination countries in the ICIO model can be expressed as:

$$\mathbf{e} = \tilde{\mathbf{A}}\mathbf{e} + \tilde{\mathbf{Y}}\mathbf{u} \tag{2}$$

with $\tilde{\mathbf{Y}} = \mathbf{Y^I} + \tilde{\mathbf{A}}\mathbf{Y}^*$ and $\tilde{\mathbf{A}} = \mathbf{A^I}(\mathbf{I} - \mathbf{A}^*)^{-1}$.

Each element of the $\tilde{\mathbf{A}}$ matrix describes how domestic intermediate goods are sent abroad (or sold domestically) to produce one unit of exports in foreign countries (or sales in the domestic economy). For example, the element $\tilde{\mathbf{A}}_{ji}$ ($N \times N$ matrix) means that in order to produce one unit of exports in country *i*, country *j* needs to produce $\tilde{\mathbf{A}}_{ji}$ units of intermediate inputs that are then embodied in exports in country *j*. $\tilde{\mathbf{A}}_{ji}\mathbf{e}_i$ ($N \times 1$ vector) means that country *j* needs to produce $\tilde{\mathbf{A}}_{ji}\mathbf{e}_i$ intermediate inputs for exports $\mathbf{e}_i$ ($N \times 1$ vector) in country *i*. We can regard $\tilde{\mathbf{A}}$ as the 'direct exports requirements matrix'. Re-arranging equation (2) above, we obtain $\mathbf{e} = \tilde{\mathbf{B}}\tilde{\mathbf{Y}}\mathbf{u}$, and $\tilde{\mathbf{B}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$, similar to $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ in the IO model. We can define matrix $\tilde{\mathbf{B}}$ as the 'total exports requirements matrix'. Still we have:

$$\begin{aligned}\tilde{\mathbf{B}} &= (\mathbf{I} - \tilde{\mathbf{A}})^{-1} = [\mathbf{I} - \mathbf{A}^{\mathbf{I}}(\mathbf{I} - \mathbf{A}^{*})^{-1}]^{-1} = [(\mathbf{I} - \mathbf{A}^{*})(\mathbf{I} - \mathbf{A}^{*})^{-1} - \mathbf{A}^{\mathbf{I}}(\mathbf{I} - \mathbf{A}^{*})^{-1}]^{-1} \\ &= [(\mathbf{I} - \mathbf{A}^{*} - \mathbf{A}^{\mathbf{I}})(\mathbf{I} - \mathbf{A}^{*})^{-1}]^{-1} = (\mathbf{I} - \mathbf{A}^{*})\mathbf{B} = (\mathbf{I} - \mathbf{A} + \mathbf{A}^{\mathbf{I}})\mathbf{B} = \mathbf{I} + \mathbf{A}^{\mathbf{I}}\mathbf{B}\end{aligned} \tag{3}$$

If we define $\mathbf{B}^{*} = (\mathbf{I} - \mathbf{A}^{*})^{-1}$, we can also show that:

$$\begin{aligned}\mathbf{B}^{*}\tilde{\mathbf{B}} &= (\mathbf{I} - \mathbf{A}^{*})^{-1}(\mathbf{I} - \tilde{\mathbf{A}})^{-1} = [(\mathbf{I} - \tilde{\mathbf{A}})(\mathbf{I} - \mathbf{A}^{*})]^{-1} = \{[\mathbf{I} - \mathbf{A}^{\mathbf{I}}(\mathbf{I} - \mathbf{A}^{*})^{-1}](\mathbf{I} - \mathbf{A}^{*})\}^{-1} \\ &= \{[(\mathbf{I} - \mathbf{A}^{*})(\mathbf{I} - \mathbf{A}^{*})^{-1} - \mathbf{A}^{\mathbf{I}}(\mathbf{I} - \mathbf{A}^{*})^{-1}](\mathbf{I} - \mathbf{A}^{*})\}^{-1} \\ &= [(\mathbf{I} - \mathbf{A}^{*} - \mathbf{A}^{\mathbf{I}})(\mathbf{I} - \mathbf{A}^{*})^{-1}(\mathbf{I} - \mathbf{A}^{*})]^{-1} \\ &= (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{B}\end{aligned} \tag{4}$$

For $\mathbf{e}_i$ ($N \times 1$ vector) that are exports in country *i*, all the intermediate inputs needed are $\sum_j^G \tilde{\mathbf{A}}_{ji}\mathbf{e}_i$. We can thus calculate the value-added in exports of country *i* as $\mathbf{w}(i)^{\mathbf{T}} = \mathbf{e}_i - \sum_j^G \tilde{\mathbf{A}}_{ji}\mathbf{e}_i$ (where $\mathbf{w}(i)$ is a $1 \times N$ vector). This value-added does not only include country *i*'s value-added (DVA) but also other countries' value-added (FVA). We can then express the value-added multiplier coefficients in exports in the form of a $1 \times NG$ vector $\tilde{\mathbf{v}}$, defined as:

$$\tilde{\mathbf{v}} = \mathbf{t}(\mathbf{I} - \tilde{\mathbf{A}}) = \mathbf{t}(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A}^{*})^{-1} = \mathbf{v}(\mathbf{I} - \mathbf{A}^{*})^{-1} \tag{5}$$

where $\mathbf{v}$ is a $1 \times NG$ vector of direct value-added coefficients. Each element of $\mathbf{v}_i$ ($1 \times N$ vector) gives the share of direct domestic value-added in total output. It is equal to one minus the intermediate input share from all countries (including domestically produced intermediates): $\mathbf{v}_i = \mathbf{t}[\mathbf{I} - \sum_{j}^{G} \mathbf{A}_{ji}]$, where $\mathbf{t}$ is a 1×N unit vector. Using $\mathbf{B}^* = (\mathbf{I} - \mathbf{A}^*)^{-1}$, we can obtain the expression for value-added coefficients in exports for country i: $\tilde{\mathbf{v}}_i = \mathbf{v}_i \mathbf{B}^*_{ii} + \sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}^*_{ji}$. We can see in this equation the value-added from country i (DVA) $\mathbf{v}_i \mathbf{B}^*_{ii}$ and the value-added from other countries (FVA) $\sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}^*_{ji}$ [7].

These two terms ($\mathbf{v}_i \mathbf{B}^*_{ii}$ and $\sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}^*_{ji}$) explicitly measure value-added when it enters a specific 'border' and is embodied in exports for the 'first time'. But the definition of the 'border' depends on the initial extraction matrix.

To measure double counting as value-added coming twice to the exporting economy (or having crossed a specific border more than once), the total value-added coefficients matrix (**vB**) (or total value-added multiplier) can be re-written by merging equations (3) and (4):

$$\mathbf{e}^T = \mathbf{vB\hat{e}} = \tilde{\mathbf{v}}\tilde{\mathbf{B}}\hat{\mathbf{e}} = \mathbf{vB}^*\tilde{\mathbf{B}}\hat{\mathbf{e}} = \mathbf{vB}^*(\mathbf{I} + \mathbf{A}^{\mathbf{I}}\mathbf{B})\hat{\mathbf{e}} \quad (6)$$

This equation explains how value-added is decomposed in our framework: we have the value-added coefficients $\mathbf{vB}^*$ corresponding to the first time value-added (domestic or foreign) was embodied in exports and then a residual term $\mathbf{vB}^*(\mathbf{A}^{\mathbf{I}}\mathbf{B})$.

[7] These terms are consistent with Los et al. (2016). They also show that when measuring value-added in world exports there is no longer a distinction between DVA and FVA.

$\mathbf{A^I}$ is the matrix that was used to identify exports and separate them from the rest of gross output (exports of all countries with the global consistency approach, exports of a single country with the country consistency approach or bilateral exports). This matrix has the ICIO coefficients for the use of intermediate inputs from one country into another country. It defines the concept of 'border' when decomposing exports. With the country consistency approach, it is the border between the specific exporting economy and other countries, while for bilateral exports it corresponds to the border between the exporting economy and its partner country. With the global consistency approach, the concept of 'border' encompasses all exporting economies so that once value-added leaves the country of origin it is recorded as DVA in exports and then as FVA when it leaves the foreign country where it is embodied for the first time, crossing the border twice.

Coefficients $\mathbf{A^I B}$ point at flows of value-added crossing the 'border' twice and we can interpret $\mathbf{vB}^*(\mathbf{A^I B})$ as the expression for double counting. It measures value-added (domestic or foreign) that has crossed the given border (as defined by $\mathbf{A^I}$) more than once and which is already accounted for in the $\mathbf{vB}^*$ expression. As we can see, this double counting depends on the definition of $\mathbf{A^I}$ and this is why we have different measures of double counting and different values for FVA.

To put it in a nutshell, our framework starts with the definition of an identification matrix $\mathbf{A^I}$ and corresponding extraction matrix $\mathbf{A}^*$ and decomposes gross exports of country *i* as:

$$\text{DVA} = \mathbf{v}_i \mathbf{B}^*_{ii} \mathbf{e}_i \qquad (7)$$

$$\text{FVA} = \sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}^*_{ji} \mathbf{e}_i \qquad (8)$$

$$\text{DDC} = \mathbf{v}_i [\mathbf{B}^* \mathbf{A}^{\mathbf{I}} \mathbf{B}]_{ii} \mathbf{e}_i \qquad (9)$$

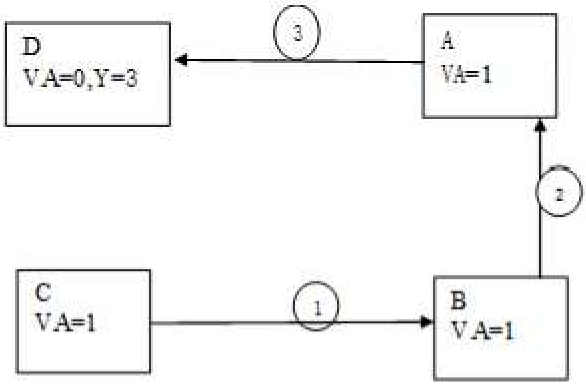
$$\text{FDC} = \sum_{j \neq i}^{G} \mathbf{v}_j [\mathbf{B}^* \mathbf{A}^{\mathbf{I}} \mathbf{B}]_{ji} \mathbf{e}_i \qquad (10)$$

4. Numerical examples and empirical analysis

To further understand the difference between the global consistency and country consistency approach, as well as between aggregate and bilateral measures, we first develop in this Section three simple numerical examples that illustrate differences in double counting and what they mean. For each example, we show both the 'global value chain' (GVC) and the corresponding ICIO as already the information in the ICIO has collapsed the different stages of production. This is where we can see the assumptions made to go from the ICIO (which is the only empirical information we have) to an allocation of value-added that could recreate the GVC (something we do not observe in aggregate statistics but that authors try to recreate on the basis of the ICIO when decomposing gross exports).

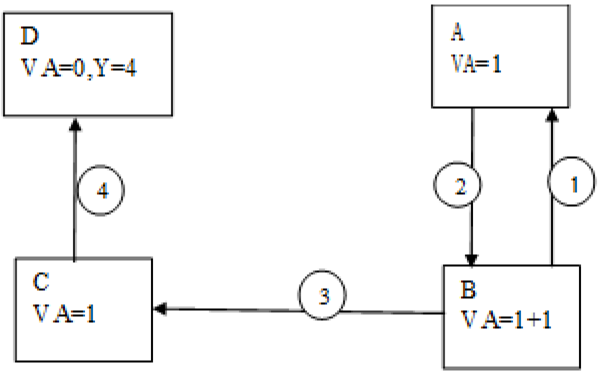
4.1 Numerical examples illustrating the difference between the global consistency and country consistency approach

Case 1: country C exports 1 unit of intermediate inputs to country B, then B exports 2 units to country A (using as input the production of country C), then A exports 3 units to country D (using as input the production of country B) that are finally absorbed by D. The value chain and the corresponding ICIO table can be represented as below[8]:



	A	B	C	D	A	B	C	D
A	0	0	0	0	0	0	0	3
B	2	0	0	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	1	1	1	0				

Case 2: country B exports 1 unit to country A at the beginning, then A exports 2 units back to country B, then B re-exports 3 units to country C, then C exports 4 units to country D, finally absorbed by D.



	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	0
B	1	0	3	0	0	0	0	0
C	0	0	0	0	0	0	0	4
D	0	0	0	0	0	0	0	0
VA	1	2	1	0				

Case 3: this case is similar to the previous one but with a simple modification. For the

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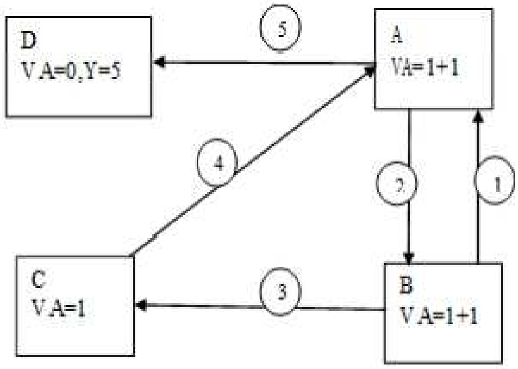
[8] These examples omit industries for simplicity. The first block in the ICIO is the intermediate consumption matrix and the second block the final demand. VA is at the bottom of the intermediate consumption matrix. VA = value-added, Y = final demand.

fourth step in the value chain, country C now exports 4 units back to country A again, then A exports 5 units to country D, finally absorbed by D.



	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	5
B	1	0	3	0	0	0	0	0
C	4	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	2	2	1	0				

Next, we show results for the decomposition of gross exports into four terms: DVA, DDC, FVA, FDC. We first report results using the KWW approach as it was the first one proposed in the literature. This approach belongs to the category of the 'global consistency' as it uses a global multiplier to calculate FVA. But the way it separates FVA from FDC is influenced by the country of final absorption so that it could also be described as sink-based.[9] Results were obtained using the formulas provided by KWW. We then present results based on the framework we have introduced in the previous Section, distinguishing the global consistency approach (extraction of exports of all countries) from the country consistency approach (extraction of exports of a given country). The equations provided by Borin and Mancini (2017) and Miroudot and Ye

[9] See Nagengast and Stehrer (2016), Borin and Mancini (2017) and Arto et al. (2019) for a discussion of specific issues with some of the terms of the KWW decomposition and whether it is a sink-based approach.

(2017) give results that are identical to the global consistency approach. Although not including the decomposition for G countries, the framework proposed by Johnson (2018) for two countries also falls under the global consistency approach. For the country consistency approach, results are consistent with Los et al. (2016)[10] and the equations proposed by Arto et al. (2019) lead to similar results.[11] We can therefore classify existing decompositions of gross exports into three groups with a different split between FVA and FDC.

Table 1: Decomposition of Case 1

		KWW				Global consistency				Country consistency			
	Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
A	3	1	0	2	0	1	0	1	1	1	0	2	0
B	2	1	0	0	1	1	0	1	0	1	0	1	0
C	1	1	0	0	0	1	0	0	0	1	0	0	0
D	0	0	0	0	0	0	0	0	0	0	0	0	0

[10] We thank Bart Los for having provided information on the calculation of FVA according to this framework, something that was not included in the published paper but developed by the authors.

[11] Identical at the country level but not at the bilateral level as Arto et al. (2019) assume that the sum of bilateral measures is equal to aggregate measures of value-added for a given country.

Table 2: Decomposition of Case 2

		KWW				Global consistency				Country consistency			
	Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
A	2	1	0.33	0	0.67	1	0.33	0.5	0.17	1	0.33	0.5	0.17
B	4	2	0.67	0	1.33	2	0.67	1	0.33	2	0.67	1	0.33
C	4	1	0	3	0	1	0	1.5	1.5	1	0	3	0
D	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 3: Decomposition of Case3

		KWW				Global consistency				Country consistency			
	Gross exports	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
A	7	2	0.8	3	1.2	2	0.8	1.5	2.7	2	0.8	3	1.2
B	4	2	0.8	0	1.2	2	0.8	0.57	0.63	2	0.8	0.86	0.34
C	4	1	0.3	0	2.7	1	0.3	1.5	1.2	1	0.3	2.08	0.62
D	0	0	0	0	0	0	0	0	0	0	0	0	0

The results illustrate the advantages and disadvantages of the various approaches. First, it should be noted that all decompositions are the same for DVA and DDC. As previously noted, there is no difference in the way domestic double counting is measured with the three approaches. However, we see important differences in the way value-added is allocated to FVA and FDC.

As the KWW equations are close to a sink-based approach, the measurement of FVA and FDC depends more on the country of absorption. When value-added crosses more than one country and is still not finally absorbed, KWW counts this value-added as double counting. This is reflected in the gross exports decomposition of country B in all cases and of country C in case 3. Because the export flow is not absorbed by the direct importer, the value of FVA is 0 in the KWW framework. As such, this approach leads to counter-intuitive results with high values for FDC. But it can be understood if one accepts the idea that foreign value-added that ‘continues to travel’ is passed to the FDC term. It is also consistent with the definition of double counting in the KWW paper as value-added crossing borders twice. But it should be clarified that this border is not the border of the exporting economy. Crossing the border of the exporting economy plus the border of another country downstream before final absorption also qualifies to become part of FDC.

The global consistency approach also leads to high values for FDC but for a different reason. This time it is based on the source country and the fact that when value-added has already crossed a border and is measured a second time in the exports of another country upstream, this value-added contributes to FDC. This is illustrated with the decomposition of gross exports of country A in case 1, country C in case 2, or countries B and C in case 3. If we look at country A in case 1, it exports 3 units of value-added: one unit is domestic, one unit is from country B and one from country C. In the global consistency approach, the 1 unit of value-added from country C is measured as DVA in C’s exports and measured as FVA in B’s exports because it is the first time it leaves its originating country and the foreign country where it was embodied. Since this value-added was already recorded by country B as FVA, it goes to FDC when it leaves country A, even if it has not crossed twice the border of country

A and leaves it for the first time. This is different with the country consistency approach where from the point of view of A, the same unit of value-added from country C becomes part of FVA.

From these examples, we can see that the definition of double counting is not about value-added crossing twice the same border but more about value-added being measured twice in the value-added generation with a different perspective when deciding where to measure it for ‘the first time’: in the country of origin, in the exporting economy or in the country before final absorption. Only the country consistency approach provides a definition of double counting consistent with the concept of value-added crossing twice the border of the same (exporting) economy.

We can therefore suggest to use a country consistency approach when trying to disentangle DVA from FVA in exports of a specific country and trying to remove only double counting related to inputs coming back to the same exporting economy. But the global consistency approach has other properties that may be needed when looking at global issues (for example CO2 emissions embodied in world trade). With the global consistency approach, all FVA is the double counted form of DVA and there is consistency with global GDP (Miroudot and Ye, 2018). FVA already accounted for in exports of other countries in this case may be legitimately regarded as FDC when adding values across countries in order to not overestimate FVA terms.

4.2 Bilateral results

Using case 3 as a numerical example, Table 4 provides the decomposition of bilateral exports for country A. It illustrates how in the bilateral analysis the concept of border becomes the bilateral border. For example, in bilateral trade between country A and D, since there is no value-added crossing the border between A and D more than once, there is no double counting.

Table 4: Decomposition of bilateral exports of country A in case 3

	Exports	DVA	DDC	FVA	FDC
A_B	2	0.57	0.23	0.86	0.34
A_C	0	0	0	0	0
A_D	5	2	0	3	0
Sum	7	2.57	0.23	3.86	0.34
Aggregate	7	2	0.8	3	1.2

An important point previously made by Los and Timmer (2019) is that the sum of bilateral measures is not equal to the aggregate value-added decomposition. It can be seen in Table 4 where the sum of DVA, DDC, FVA and FDC across partners is not the same as the aggregate measure (that was reported in Table 3 and is reproduced in the last row of Table 4). Since the border in bilateral exports has been changed, we can easily explain within our framework why the decomposition of bilateral exports is not the mapping of the aggregate decomposition across partners. For some analysis, it might be useful to have bilateral values that sum to the aggregate ones. But in this case authors should be clear about the fact that there is a mismeasurement involved.

From the point of view of our framework, it is also problematic to create bilateral measures based on the global consistency approach as the extraction matrix further departs from the one that is needed to isolate bilateral exports. But authors relying on decompositions consistent with this approach sometimes propose bilateral measures. Again, it should be regarded as a mapping of the aggregate measure across partners with some approximation in the identification of DVA and FVA terms.

4.3 Empirical results using the WIOD database

Numerical examples are useful to understand differences across decompositions, but one could argue that actual GVCs are more complex and that maybe differences are exaggerated using these simple examples. In Table 5, we provide results based on the World Input-Output Database (WIOD) tables (Timmer et al., 2015) for 44 countries for the year 2014 (aggregating across industries).

Table 5: Decomposition of gross exports, % (WIOD, 2014)

		KWW				Global consistency				Country consistency			
	Gross (million	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC	DVA	DDC	FVA	FDC
AUS	287,162	85.83	0.14	10.08	3.95	85.83	0.14	10.47	3.56	85.83	0.14	14.01	0.02
AUT	210,995	63.86	0.29	23.24	12.61	63.86	0.29	24.7	11.15	63.86	0.29	35.65	0.21
BEL	383,014	53.96	0.39	30.81	14.84	53.96	0.39	32.71	12.94	53.96	0.39	45.21	0.44
BGR	31,698	61.81	0.03	25.51	12.65	61.81	0.03	28.02	10.14	61.81	0.03	38.13	0.03
BRA	270,262	87.16	0.06	9.69	3.09	87.16	0.06	9.69	3.09	87.16	0.06	12.77	0.01
CAN	563,511	75.77	0.42	20.29	3.52	75.77	0.42	19.03	4.77	75.77	0.42	23.68	0.12
CHE	352,570	74.48	0.2	19.96	5.37	74.48	0.2	18.29	7.03	74.48	0.2	25.23	0.09
CHN	2,425,464	83.15	0.94	12.69	3.22	83.15	0.94	11.68	4.23	83.15	0.94	15.69	0.23
CYP	9,347	71.94	0.04	17.14	10.87	71.94	0.04	20.12	7.9	71.94	0.04	28	0.02
CZE	161,570	54.02	0.33	30.34	15.31	54.02	0.33	30.73	14.92	54.02	0.33	45.36	0.29
DEU	1,682,253	71.85	1.39	19.22	7.53	71.85	1.39	18.77	7.98	71.85	1.39	26.12	0.63

DNK	170,293	62.47	0.17	28.99	8.37	62.47	0.17	27.31	10.05	62.47	0.17	37.26	0.1
ESP	389,005	68.87	0.26	23.02	7.84	68.87	0.26	22.56	8.3	68.87	0.26	30.71	0.16
EST	18,266	56.55	0.09	30.77	12.59	56.55	0.09	28.83	14.53	56.55	0.09	43.28	0.08
FIN	100,453	64.97	0.12	24.01	10.9	64.97	0.12	25.83	9.07	64.97	0.12	34.82	0.09
FRA	759,654	72.28	0.46	19.96	7.3	72.28	0.46	19.44	7.82	72.28	0.46	27.06	0.2
GBR	751,599	80.74	0.29	13.7	5.27	80.74	0.29	13.84	5.13	80.74	0.29	18.89	0.08
GRC	56,261	69.58	0.04	22.61	7.77	69.58	0.04	23.19	7.19	69.58	0.04	30.35	0.02
HRV	23,269	72.68	0.05	19.36	7.91	72.68	0.05	19.37	7.9	72.68	0.05	27.25	0.02
HUN	116,445	48.13	0.16	35.84	15.87	48.13	0.16	35.46	16.25	48.13	0.16	51.51	0.2
IDN	210,599	82.74	0.11	13.15	3.99	82.74	0.11	12.61	4.54	82.74	0.11	17.13	0.02
IND	369,456	79.28	0.11	15.78	4.82	79.28	0.11	16.13	4.47	79.28	0.11	20.57	0.04
IRL	262,751	50.65	0.13	39.39	9.83	50.65	0.13	41.7	7.53	50.65	0.13	49.12	0.1
ITA	588,585	73.63	0.32	18.94	7.11	73.63	0.32	18.5	7.56	73.63	0.32	25.91	0.14
JPN	817,514	76.41	0.32	17.19	6.09	76.41	0.32	17.89	5.38	76.41	0.32	23.15	0.12
KOR	697,935	64.79	0.35	26.03	8.84	64.79	0.35	26.74	8.13	64.79	0.35	34.65	0.22
LTU	32,722	64.29	0.05	24.9	10.76	64.29	0.05	27.42	8.24	64.29	0.05	35.61	0.05
LUX	118,439	33.96	0.08	49.29	16.67	33.96	0.08	57.23	8.72	33.96	0.08	65.79	0.16
LVA	14,719	68.98	0.1	21.87	9.04	68.98	0.1	20.78	10.14	68.98	0.1	30.87	0.05
MEX	368,185	66.44	0.26	29.7	3.59	66.44	0.26	25.43	7.86	66.44	0.26	33.17	0.12
MLT	13,420	34.51	0.03	51.53	13.93	34.51	0.03	44.67	20.79	34.51	0.03	65.39	0.07
NLD	575,068	63.15	0.8	23.84	12.2	63.15	0.8	26.22	9.83	63.15	0.8	35.6	0.45
NOR	188,131	82.96	0.25	10.88	5.91	82.96	0.25	12.16	4.64	82.96	0.25	16.75	0.04
POL	251,642	69.04	0.27	20.82	9.87	69.04	0.27	21.52	9.18	69.04	0.27	30.56	0.13
PRT	76,633	68.84	0.09	22.42	8.65	68.84	0.09	21.47	9.6	68.84	0.09	31.01	0.06
ROU	77,648	73.31	0.07	18.17	8.46	73.31	0.07	18.35	8.28	73.31	0.07	26.59	0.03
RUS	493,789	92.36	0.14	4.86	2.64	92.36	0.14	5.27	2.22	92.36	0.14	7.49	0.01
SVK	82,119	51.86	0.2	33.75	14.18	51.86	0.2	30.87	17.06	51.86	0.2	47.72	0.22
SVN	30,812	62.63	0.08	25.29	12	62.63	0.08	25.15	12.15	62.63	0.08	37.24	0.05
SWE	235,354	71.2	0.28	19.81	8.71	71.2	0.28	20.75	7.77	71.2	0.28	28.38	0.14
TUR	249,783	71.47	0.13	22.02	6.39	71.47	0.13	19.31	9.1	71.47	0.13	28.35	0.06
TWN	369,923	58.17	0.4	28.08	13.35	58.17	0.4	29.87	11.56	58.17	0.4	41.15	0.29
USA	1,927,091	87.15	0.7	8.84	3.32	87.15	0.7	9.45	2.71	87.15	0.7	12.04	0.12
ROW	3,833,149	73.53	1.68	17.88	6.91	73.53	1.68	20.83	3.96	73.53	1.68	24.24	0.55

Table 5 confirms that there is a consensus for the calculation of DVA and that all the frameworks provide the same DDC, which is generally a small percentage of gross exports (most of the time below 1%). When it comes to FVA, we find important differences across the three approaches, as it was the case with the simple numerical examples. For example, KWW and the global consistency approach have a FDC equal to about 15% for the Czech Republic. FDC is only 0.29% with the country consistency approach. We can see from these results that choosing one approach or the other has important consequences for the analysis. FDC with the country consistency approach is small and can be seen as the symmetric of DDC for foreign inputs. FDC is generally smaller than DDC, indicating that FVA coming back to the same exporting economy is something even less common than DVA.

Someone interested in understanding how important are foreign inputs for the production of exports in the Czech Republic would have a more accurate answer with the value 45% (country consistency) rather than 30% (global consistency). Someone interested in measuring circular trade in foreign inputs in the Czech Republic should also look at the country consistency column (0.29%).

But the global consistency approach is interesting to identify some components of FVA that have been part of more complex value chains than the direct import of foreign inputs. The high FDC (15%) in exports of the Czech Republic highlights that a high share of the foreign content comes from vertical trade upstream in the value chain. From the above table, we can see that two-thirds of the foreign inputs in Czech exports are embodied for the first time in exports in the Czech Republic while one third was already FVA in exports of other countries.

When measuring jobs embodied in trade or CO2 emissions embodied in trade (based on value-added), the two types of double counting may be useful. One could look specifically at one country or aggregate measures in a region or across the world and in this latter case the global consistency (and consistency with world GDP) could be useful.

Finally, in Table 6, we can see the decomposition of bilateral exports of China with all WIOD partners in 2014 using our framework. DDC and FDC become very small at the bilateral level, suggesting that it might not even be worth trying to identify these double-counted terms when working with bilateral data.[12] While also small, the difference between the sum of bilateral measures and the aggregate measure confirms that decompositions assuming their equivalence introduce some approximation in the analysis. They tend to overestimate DDC and FDC and as such could be less accurate than decompositions simply omitting DDC and FDC and just using a foreign content and domestic content.

Table 6: Decomposition of Chinese bilateral exports, % (WIOD, 2014)

	Exports (million USD)	DVA	DDC	FVA	FDC
CHN_AUS	48459	83.22	0.011	16.77	0.002
CHN_AUT	4242	83.99	0.000	16.01	0.000

[12] China was picked as one country where circular trade is more pronounced. When going at the industry level, double counting becomes even more marginal. The same logic applies as when going from the extraction of exports with world to bilateral exports. Extracting a single industry allocates to this industry the FVA that would otherwise be regarded as double counted across different industries.

CHN_BEL	11804	84.93	0.002	15.07	0.000
CHN_BGR	1029	83.77	0.000	16.23	0.000
CHN_BRA	38988	82.92	0.004	17.08	0.001
CHN_CAN	49636	83.65	0.003	16.35	0.001
CHN_CHE	7293	82.67	0.001	17.33	0.000
CHN_CYP	583	85.03	0.000	14.97	0.000
CHN_CZE	8898	79.04	0.002	20.95	0.001
CHN_DEU	88465	83.31	0.013	16.67	0.003
CHN_DNK	6199	85.45	0.001	14.55	0.000
CHN_ESP	21496	84.48	0.001	15.52	0.000
CHN_EST	1073	81.97	0.000	18.03	0.000
CHN_FIN	6870	83.64	0.001	16.36	0.000
CHN_FRA	41291	83.97	0.004	16.03	0.001
CHN_GBR	51850	83.11	0.003	16.89	0.001
CHN_GRC	4190	83.53	0.000	16.47	0.000
CHN_HRV	714	83.63	0.000	16.37	0.000
CHN_HUN	5396	78.66	0.002	21.34	0.000
CHN_IDN	34969	83.46	0.005	16.54	0.001
CHN_IND	44869	82.48	0.004	17.51	0.001
CHN_IRL	3471	82.75	0.001	17.25	0.000
CHN_ITA	28865	84.17	0.002	15.83	0.000
CHN_JPN	172861	82.77	0.051	17.17	0.013
CHN_KOR	101924	81.78	0.145	18.04	0.039
CHN_LTU	947	83.87	0.000	16.13	0.000
CHN_LUX	911	79.94	0.000	20.06	0.000
CHN_LVA	654	84.60	0.000	15.40	0.000
CHN_MEX	38330	81.47	0.002	18.53	0.001
CHN_MLT	455	84.06	0.000	15.94	0.000
CHN_NLD	42640	81.94	0.007	18.06	0.001
CHN_NOR	4563	84.41	0.000	15.59	0.000
CHN_POL	14316	82.28	0.001	17.72	0.000
CHN_PRT	2251	83.47	0.000	16.53	0.000
CHN_ROU	2614	81.64	0.000	18.35	0.000
CHN_RUS	65198	87.73	0.003	12.26	0.001
CHN_SVK	2002	81.47	0.000	18.53	0.000
CHN_SVN	1369	84.46	0.000	15.54	0.000
CHN_SWE	11173	85.80	0.001	14.20	0.000
CHN_TUR	23149	81.88	0.002	18.12	0.000
CHN_TWN	43622	80.29	0.167	19.50	0.051
CHN_USA	347311	82.36	0.013	17.62	0.003
CHN_ROW	1038525	85.14	0.501	14.25	0.111
Sum	**2425464**	**83.86**	**0.230**	**15.86**	**0.052**
Aggregate	**2425464**	**83.15**	**0.938**	**15.69**	**0.225**

5. Concluding remarks

This paper has further investigated the concept of double counting in the decomposition of gross exports and found that differences in definitions and approaches to the measurement of double counting can explain why several decompositions are proposed in the literature with results that are the same for DVA and DDC but quite different when it comes to FVA and FDC.

When looking at world exports, the concept of 'border' between countries does not exist and world gross exports can be decomposed into value-added (equal to world GDP in exports) and intermediate inputs (that are the double counting part). When looking at exports of specific countries, one can start to disentangle DVA from FVA (which is DVA in other countries) and two approaches are available. One can assume that FVA is the strict symmetric of DVA and ensures that globally FVA is measured only once. This is the global consistency approach where double counting is the value-added measured twice (or more) in exports (including for FVA, i.e. FVA measured twice or more in exports, acknowledging that it was already measured as DVA somewhere else). Exports in the global consistency approach mean exports of any country.

In the country consistency approach, double counting is defined as the value-added coming twice (or more) to the same exporting economy. For FVA, it implies that some value-added could have already been measured as FVA somewhere else (with some potential global FVA double counting) but the perspective is the exporting economy. This is conceptually closer to what seems to be the objective of the trade in value-added literature in the analysis of GVCs, i.e the identification of the foreign contribution in exports removing double counting related to inputs coming back to the exporting economy. Such an approach was proposed by Los et al. (2016) but without deriving the formulas for FVA and FDC. As such, our paper offers a useful complement to calculate these additional terms and to fully decompose gross exports with the country consistency approach.

In addition, our framework allows the decomposition of bilateral exports in a consistent way and confirms that double counting in bilateral exports should not be regarded as a bilateral mapping of double counting in exports with world. When introducing bilateral borders, one needs to redefine what is double counted as crossing the same border twice (or more) no longer has the same meaning.

Empirically, we find small values for double-counted terms with the country consistency approach and even smaller in bilateral exports. For some analysis and for countries not too much involved in 'circular trade', we could argue that papers that have used a simple approach in decomposing value-added in trade with just a domestic content and foreign content (such as the decomposition provided in the OECD Trade in Value-Added database) have used rather good estimates. Analysis based on the KWW framework or the global consistency approach with high shares of FDC and lower shares of FVA are not wrong but it is important for users to understand how double counting is defined in these approaches and the difference with the country consistency approach. As we have highlighted, a global consistency approach may be needed when consistency across countries or with world GDP is important.

As illustrated by the different frameworks found in the literature, there is no simple answer to the decomposition of value-added in gross exports. The area is also highly technical. The challenge in the future will be to provide measures of trade in value-added based on ICIOs that do not overlook this complexity but are more accessible and clearer in terms of what is measured for researchers and policymakers.

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