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**Abstract**

I study mergers where each firm owns multiple shops across a country. Presently, the European Commission views every shop, together with the shops from its catchment area, as an isolated market. Such an approach is internally inconsistent. I show how to extend the European Commission's approach to consistently take overlaps in catchment areas into account. My model is a specialization of the existing network theory that is aimed to be feasible in real merger cases. As a demonstration, I study a past merger case and I find that neglecting overlaps in catchment areas can result in substantial biases.

**Key Words:** mergers, networks, spatial competition, consumer demand.

**JEL Classification:** D43, D85, L13, L14, L40.

## 1 Introduction

In the recent past the European Commission (EC) investigated a number of merger cases where the merging parties owned multiple shops across a country.[1] A typical case is as follows. The shops can be supermarkets,

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[1]See cases M.1221, M.1684, M.4686, M.6506, M.6822.

pharmacies, gas stations, swimming pools, cinemas, etc. Due to transportation costs, the competition between the shops is local in scope. The number of shops can go into hundreds or thousands and it is infeasible for the merging parties or the EC to analyze them all on a case by case basis. Initial screening for potentially problematic areas is necessary. In many cases the EC has adopted the following approach.

Given the type of shops under investigation, the maximum travel time is defined so that it is sensible to assume that consumers are willing to travel that amount of time to reach a competing shop. Then, for each shop of the merging parties a local market is defined with the center at that shop and capturing all competing shops that fall within the maximum travel time. For each of the so-defined local markets a concentration analysis is performed. Normally, the EC follows the general guidelines with some industry-specific modifications, e.g. local markets with the HHI below 2 000 and the HHI increase below 250 are considered non-problematic.[2] All local markets that are not automatically cleared will require more thorough investigation.

Effectively, the EC uses partial equilibrium analysis of horizontally differentiated markets to study what impact a merger can have on prices. In the rest of the paper I refer to this approach as *the local markets approach*. However, local markets overlap and if the corresponding connections are taken into account, i.e. if we look at the general equilibrium—as applied to these horizontally differentiated markets—then the effect on prices can potentially be different. I use the network theory to develop this new approach, and so I refer to it as *the network approach*. Just how important then is this possible difference between the outcomes of the local markets and the network approaches?

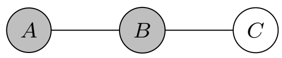
Firstly, some mergers are cleared subject to divestments. In such cases the initial screening procedure might also be used to decide on which divestments are necessary. If the network approach delivers a different set of problematic areas in comparison with the local markets approach, then that means the firms might be divesting wrong shops (from the society's point of view). Secondly, the outcome of the initial screening procedure might decide the outcome of the merger case. We can speculate that in the worst case

[2] See paras. 17–21 in "Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings," Official Journal C 31, 05.02.2004, pp. 5–18.

scenario a merger might be cleared unconditionally using the local markets approach whereas it should be blocked based on the networks approach, or vice versa.

In this paper I combine the insights from the literature on differentiated Bertrand competition and the literature on network games to setup a model that, on one hand, is in line with the EC practice of defining local markets and, on the other hand, allows for the general equilibrium analysis of horizontally differentiated markets. I then present an example where the local markets and the network approaches deliver opposite results. To assess whether such differences occur in practice, as well as to demonstrate that my network approach is feasible, I revisit a past merger case of two Dutch supermarket chains. I find that for a number of shops the local markets approach of the EC is strongly biased. In light of these results, there is scope for improvement within the current antitrust practices when it comes to mergers of firms with multiple points of sale. From the academic perspective, this paper demonstrates a novel practical application of the network theory.

In general, competition economists are aware that network effects can be important. See, e.g. the *chain of substitution* argument in Bishop and Walker (2010, p. 145). The argument goes as follows. Suppose the geography is such that shops $A$ and $B$ compete with each other, shops $B$ and $C$ compete with each other, but not shops $A$ and $C$ directly. The chain of substitution argument says that even if shops $A$ and $B$ belong to the merging parties, there might not be a substantial increase in prices after the merger, because the prices in shop $B$ will be constrained by shop $C$, and the prices in shop $A$ will be constrained by shop $B$. Whereas the local markets approach identifies a monopoly market centered around shop $A$.[3] In this paper I undertake a more formal analysis of this critique and show with a specific example what the chain of substitution argument might imply for merger analysis.




I define consumer preferences following Dixit (1979), Singh and Vives (1984), and Häckner (2000). Conceptually, Dixit's specification implies that consumers diversify their purchases across all neighbouring shops. Given a suitable time horizon, this feature is realistic for some of the examples given

[3]The EC has accepted the chain of substitution argument in its most general form—by considering a global market instead of a set of local markets when local markets' overlaps are large enough—in cases M.1221 and M.1684.

earlier: supermarkets, gas stations. Another possible modelling approach is to use discreet choice models. Arguably, discreet choice models are more suitable for, e.g., swimming pools. However, these models—of which the logit model is the simplest example—are likely to deliver multiple equilibria when complex networks are considered. And multiplicity of equilibria is a hurdle for normative applications of the model.

When the EC defines local markets, they define which shops exerts competitive pressure on the current shop. I will assume that consumers are concentrated at shop locations. Then saying that shop $A$ exerts competitive pressure on shop $B$ is equivalent to saying that consumers that are located at $B$ can shop not only at $B$ but also at $A$. In this way, I can take the local markets as defined by the EC as a starting point and from there draw the corresponding network of shops and consumers.

Placing consumers at network nodes results in a model that is easily tractable even for thousands of shops. Following Hotelling, one can also place consumers at the edges, see Heijnen and Soetevent (2014). These models can give new theoretical insights but they are hardly tractable for empirical work with many locations.

Generally speaking, I analyze a linear-quadratic network game, with firms competing in prices. This type of games has been studied in the literature, see e.g., Ballester et al. (2006), Ballester and Calvó-Armengol (2010). These authors, as well as Bramoullé et al. (2014), also remark that a linear-quadratic network game can be rationalized with linear-quadratic utility functions. I follow up on their remarks with a formal specification suitable for studying price competition. Bloch and Quérou (2013) also explore linear-quadratic games where several locations can be owned by the same firm, but they give a behavioural foundation to their model due to a different focus of their paper.

My research goal is to develop a model which can be used for preliminary assessments of mergers of firms with hundreds or thousands of sale locations. Such a model needs to be robust, e.g. a model with potentially multiple equilibria is not, and it needs to work with minimal data requirements as otherwise it can never be used in practice by the parties or the EC. The model that I develop is a specialization of the more general models considered in the literature. This specialization is not arbitrary but developed so as to adhere to my research goal.

To demonstrate that my network approach is indeed practically feasible, I analyse a past merger case of two Dutch supermarket chains, Jumbo and C1000. The parties and their competitors had between themselves 4,149 shops at the time of the merger. I show how to use my approach to assess post-merger price increases using no more information that is typically available on such a merger case. Additionally, I compare the results of the network approach with the results of the local markets approach currently used by the EC. While for many supermarkets both approaches give comparable results, there is a number of supermarkets where the local markets approach is strongly biased.

Using a local markets approach effectively results in a geographic market definition that is too narrow, because it does not account for the chain-of-substitution effects. Whenever this narrow definition excludes the shops of the competitors, the EC can overstate expected price increases. Similarly, but more consequential from the practical perspective, whenever this narrow definition excludes the shops of the parties, the EC can substantially understate the expected price increases. These biases are not simply theoretical but are found in practice, as my analysis of the Jumbo and C1000 merger reveals.

The rest of the paper is organized as follows. In Sections 2 and 3, I setup and solve a general model of mergers on networks. In Section 4, I work through an example with a merger over a graph of 6 shops. In Section 5, I apply my methodology to a past merger case of two Dutch supermarket chains, and I compare the results with those obtained when following the EC approach. Section 6 concludes.

## 2 Model Setup

There are $M$ firms and $N$ shops. The ownership structure is given by matrix $f$ such that $f_{ij} = 1$ whenever firm $i$ owns shop $j$ and $f_{ij} = 0$ otherwise. I impose $\sum_i f_{ij} = 1$, i.e. only one firm can own a given shop. Then

$$s_i = \{j | f_{ij} = 1\} \tag{1}$$

gives the set of all the shops owned by firm $i$ and

$$v(j) = \sum_i i f_{ij} \tag{2}$$

gives the firm that owns shop $j$.

Every shop $i$ sets its own price $p_i$. In practice, firms can sometimes have uniform price policies so that each shop belonging to the same firm sets the same price. However, product quality (e.g., product variety in grocery stores) and service quality (e.g., opening hours, queue waiting times) may still vary between the shops. The quality effectively constitute the negative of a price: higher quality is more costly for the firms to produce and it is beneficial to consumers. Therefore the assumption that prices can vary across shops owned by the same firm is generally preferable to the alternative assumption of a single price. The fact that the EC analyzes local markets in such merger cases instead of a single national market further corroborates this logic.

There is a number $w_i$ of consumers located around each shop $i$. Consumers can shop at their own location as well as at the neighbouring shops. Formally, let $g$ be an adjacency matrix such that $g_{ij} = 1$ if consumers from location $j$ can shop at location $i$ and $g_{ij} = 0$ otherwise. By construction, $g_{ii} = 1$. An equivalent interpretation is that whenever $g_{ij} = 1$ shop $i$ puts competitive pressure on shop $j$.

If shop 1 is a specialized grocery store and shop 2 is a supermarket, then 2 puts competitive pressure on 1 whereas 1 likely puts no competitive pressure on 2. To be able to model such situations I allow $g$ to be asymmetric, i.e. a network.

Let

$$N_i^+ = \{j | g_{ji} = 1\} \tag{3}$$

denote the in-neighbourhood of $i$. $N_i^+$ is a set of all the shops that put competitive pressure on shop $i$, plus shop $i$ itself.

Following Häckner (2000),[4] a representative consumer located at $i$ has

[4] There is a typo in that paper on p. 234. In the utility definition it should be either $\gamma \sum_{i \neq j} q_i q_j$ or $2\gamma \sum_{i>j} q_i q_j$ as otherwise the formula is not consistent with Singh and Vives (1984).

utility $U_i$ defined as follows:

$$U_i = \alpha \sum_{j \in N_i^+} q_{ij} - \frac{1}{2} \left( \sum_{j \in N_i^+} q_{ij}^2 + \gamma \sum_{\substack{k,j \in N_i^+ \\ k \neq j}} q_{ij} q_{ik} \right) + q_0$$

$$= \alpha \sum_j g_{ji} q_{ij} - \frac{1}{2} \left( \sum_j g_{ji} q_{ij}^2 + \gamma \sum_{k,j : k \neq j} g_{ji} g_{ki} q_{ij} q_{ik} \right) + q_0, \quad (4)$$

where $q_{ij}$ is the amount of goods that consumer located at $i$ buys at location $j$, $q_0$ is the outside good, $\alpha > 0$, and $0 \leq \gamma < 1$. With this $\gamma$ range the goods offered at different shops are gross substitutes. The associated budget constraint is given by

$$\sum_{j \in N_i^+} p_j q_{ij} + q_0 \leq I. \quad (5)$$

Define

$$q_j = \sum_i w_i q_{ij} \quad (6)$$

to be the total amount of goods purchased at shop $j$. Then the profits of firm $i$, which equal the sum of the profits of its shops, are given by

$$\pi_i = \sum_{j \in s_i} q_j (p_j - c) = \sum_j f_{ij} q_j (p_j - c), \quad (7)$$

where $c$ denotes marginal costs.

The timing of the model is as follows. In period one the firms simultaneously set the prices in their shops. In period two consumers shop so as to maximize their utility given the prices. I focus on the Nash equilibrium of this game.

Games where players have incomplete information about the network have received attention in the literature. For example, Galeotti et al. (2010) assume that each player knows his node's degree but not the degree of his neighbours. Where does the current model stand when we consider the information requirements for the players? Do we need to assume that the firms know their competitors, the competitors of those competitors, etc.? It is worth noting that this strong assumption is not required.

As we will see in the next section, the model implies linear demand and linear best response functions. Furthermore, prices in a differentiated Bertrand game are strategic complements, and are limited from above due to the linear demand. Under these conditions the equilibrium is unique and can be reached iteratively with localized contraction, see Belhaj et al. (2014). Hence, for the considered Nash equilibrium to be a practical solution concept it suffices to assume that the firms know the residual demand on their products given the prices of their competitors, and that they gradually adjust their prices towards their optimum level. This is a milder assumption that is satisfied if, for example, firms regularly conduct marketing research to reveal the demand they face.

# 3 General Equilibrium

Denote with $d_i = |N_i^+| = \sum_j g_{ji}$ the in-degree of shop $i$. From (4) and (5) it follows that the consumer utility is maximized when

$$q_{ij} = g_{ji}\left(\beta_i + \delta_i \sum_k g_{ki} p_k - \eta p_j\right), \tag{8}$$

where

$$\beta_i = \frac{\alpha}{1 + \gamma(d_i - 1)}, \tag{9}$$

$$\delta_i = \frac{\gamma}{(1-\gamma)(1 + \gamma(d_i - 1))}, \tag{10}$$

$$\eta = \frac{1}{1-\gamma}. \tag{11}$$

Summing up over $i$ we obtain

$$q_j = \sum_i g_{ji} w_i \beta_i + \sum_{i,k} g_{ji} g_{ki} w_i \delta_i p_k - \eta p_j \sum_i g_{ji} w_i. \tag{12}$$

Substituting this expression for $q_j$ into (7) and differentiating $\pi_r$ with respect to $p_t$ (whenever $f_{rt} = 1$) gives

$$\frac{\partial \pi_r}{\partial p_t} = a_{rt} + \sum_j b_{rtj} p_j - h_t p_t, \tag{13}$$

where

$$a_{rt} = -c\sum_{i,j} f_{rj} g_{ji} g_{ti} w_i \delta_i + \eta c \sum_i g_{ti} w_i + \sum_i g_{ti} w_i \beta_i, \tag{14}$$

$$b_{rtj} = (1 + f_{rj}) \sum_i g_{ji} g_{ti} w_i \delta_i, \tag{15}$$

$$h_t = 2\eta \sum_i g_{ti} w_i. \tag{16}$$

Putting all the first order conditions together and solving gives the prices in the general equilibrium:

$$p = X^{-1} y, \tag{17}$$

where $X_{ij} = b_{v(i)ij} - \mathbf{1}_{i=j} h_i$, $y_i = -a_{v(i)i}$, and $\mathbf{1}_A = 1$ if $A$ is true and $\mathbf{1}_A = 0$ otherwise.

It is straightforward to derive sufficient second order conditions for the profit maximization problem. Whenever shops $t$ and $k$ belong to firm $r$ ($f_{rt} = 1$, $f_{rk} = 1$) we have

$$\frac{\partial \pi_r}{\partial p_t \partial p_k} = b_{rtk} - \mathbf{1}_{t=k} h_t. \tag{18}$$

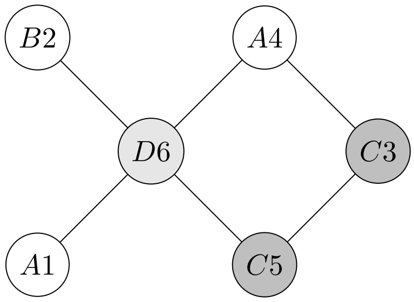
For each firm $r$ I require the Hessian matrix $\left\{ \frac{\partial \pi_r}{\partial p_t \partial p_k} \right\}$ to be negative semidefinite. Then $\pi_r$ is concave for each $r$ and (17) gives profit maximizing equilibrium prices.

A block-diagonal matrix is negative semidefinite if and only if all the block submatrices are negative semidefinite. Further, renumbering both rows and columns in the same way does not change whether a matrix is negative semidefinite or not. Therefore the Hessian matrices for all firms are negative semidefinite if and only if matrix $\Omega$ is negative semidefinite, where

$$\Omega_{ij} = b_{v(i)ij} f_{v(i)j} - \mathbf{1}_{i=j} h_i. \tag{19}$$

Hence, to check whether the second-order conditions for profit maximization are satisfied we simply need to check whether matrix $\Omega$ is negative semidefinite. In the example and the merger study that follow, the second-order conditions have been checked and they hold.

Figure 1: A Merger Example on a Graph




## 4 Example

Consider the example depicted in Fig. 1. There are 4 firms denoted $A$ through $D$. Between them they have 6 shops denoted 1 through 6. Firms $C$ and $D$ are planning a merger. What would be the expected price increase across all the shops?

In this example we have

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}. \tag{20}$$

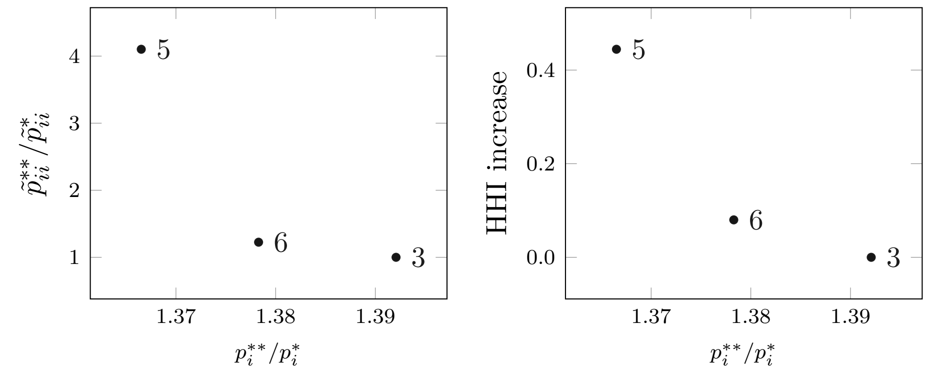
Let $f^*$ denote the ownership matrix before the merger and $f^{**}$—after the merger. Then

$$f^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{21}$$

$$f^{**} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}. \tag{22}$$

Finally, for this example take $\alpha = 1.0$, $\gamma = 0.9$, $c = 0$, $w_i = 1$.

Figure 2: Example Analysis




Then from (17) for each shop $i$ we can compute prices before and after the merger, denote them respectively $p^*_i$ and $p^{**}_i$. These are the prices from the network approach. However, the EC uses the local markets approach. We can emulate the local markets approach by considering each shop together with its closest neighbours as a separate graph. For example, the graph centered around shop 5 includes shops 5, 6, and 3. For each such subgraph $j$ let $\tilde{p}^*_{ji}$ and $\tilde{p}^{**}_{ji}$ denote the prices of shop $i$ before and after the merger.

Fig. 2, left side, plots $\tilde{p}^{**}_{ii}/\tilde{p}^*_{ii}$ against $p^{**}_i/p^*_i$ for $i \in \{3, 5, 6\}$, i.e. for the shops of the merging parties. We can see that the expected increases in prices are completely opposite depending on the chosen approach. If we use the local markets approach, then the market around shop 5 becomes a monopoly and a four-fold price increase is expected. Whereas if we use the network approach, the price rise at 5 is the smallest among the shops of the merging parties because the price at 5 is limited by the price at 6, which in turn continues to be under competitive pressure from the three neighbouring shops. And the situation is just the opposite when we look at 3. According to the local markets approach there is no expected price increase, because the local market around 3 does not change after the merger. However, according to the networks approach this area is most problematic, because the only remaining competitor in the area, shop 4, is now surrounded by the shops of the merging parties that are all raising prices.

In practice, the EC often will not compute price increases as such but rather approximate them with HHI increases. Fig. 2, right side, plots HHI increases computed on the respective neighbourhood subgraphs against $p^{**}_i/p^*_i$. The outcome is the same: the local markets approach yields results that are completely at odds with the results from the network approach.

## 5 Merger of Jumbo and C1000

In Section 4 we have seen that it is possible to construct an example, where HHI increments computed for local catchment areas give wrong information about the expected price increases. This problem arises, because analysing local catchment areas does not take chain of substitution effects into account. In other words, there are examples where the local markets approach with its partial equilibrium analysis of horizontally differentiated markets is a very poor approximation to the network approach, which employs general equilibrium analysis of these markets.

Does the same problem manifest itself in practice? In principle, this need not be the case. The example was specifically chosen to demonstrate the problem and it is conceivable that in real cases such a problem does not arise. However, if it does, that means that wrong problematic areas are given attention in the type of mergers we consider, that wrong divestments are chosen, or even that wrong clearing or blocking decisions are made.

To asses the significance of chain of substitution effects in practice, and to demonstrate how my model can be applied to an actual case, I revisit a Dutch merger of two supermarket chains, Jumbo and C1000, that took place in the beginning of 2012.[5] The data on floor sizes and addresses of all Dutch supermarkets prior to the merger is available from a third party (Ondernemers Pers Nederland). I have geocoded the addresses using Google's geocoding service. In total, there were 4,149 supermarkets with complete information about them in December 2011, out of which 461 belonged to C1000 and 271 belonged to Jumbo. I will skip over the further details of this merger as they are irrelevant for the following exercise. An interested reader is referred to Argentesi et al. (2016), which is a post merger study commissioned by the Dutch Competition Authority (ACM).

In this merger case, the ACM has considered each locality as a separate market, and there has been no further analysis of catchment areas of individual shops. Such geographic definition is problematic, because big localities such as Amsterdam or Rotterdam will never raise competition concerns, even if the market power is substantially increased after the merger

[5] See the ACM decision 7323/81 (in Dutch).

Table 1: Catchment Area Definition

| Shop Size | Drive Time | Distance |
|---|---|---|
| Below 280 $m^2$ | 5 min. | 0.5 miles |
| 280–1000 $m^2$ | 5–10 min. | 0.75 miles* |
| Above 1000 $m^2$ | 10–15 min. | 1.25 miles* |

Source: Freeman et al. (2008), see paras. 12, 15, 4.102, 4.103. *The distances for mid-sized and large shops are derived to be proportional to drive times (not present in Freeman et al.).

in certain areas in those localities. The recent practice of the EC of drawing catchment areas around each shop is aimed to addresses precisely this problem. However, as I argue in this paper, the EC practice does not take chain of substitution effects into account, which might be problematic in its own way.

To study the EC practice, I substitute the ACM geographic market definition with catchment areas. I adopt the definition for a catchment area from Freeman et al. (2008), which is a study conducted by the UK Competition and Market Authority. This study is one of the most comprehensive studies to date on catchment areas of supermarkets, and their definition is based on the analysis of consumer purchase data.

Freeman et al. (2008) define a catchment area in terms of drive times, and further differentiate between shops of different sizes. Their definition is reproduced in Table 1. I adopt their definition but replace drive times with corresponding distances. While the definition in terms of drive times is more accurate, for the purpose of my comparative analysis it suffices to use the simpler definition in terms of distances.

Freeman et al. (2008) further stipulate that a bigger shop puts competitive pressure on a smaller shop but that a smaller shop does not put competitive pressure on a bigger shop (asymmetric competition in the product market). Consequently, I set $g_{ij} = 1$ if 1) shop $j$ falls within the catchment area of shop $i$, and 2) shop $i$ belongs to the same size class as shop $j$, or to a bigger size class. Otherwise I set $g_{ij} = 0$. So, using a network allows us to capture the essential elements of the product and geographic market definitions that are typically used in spatial mergers.

The resulting network consists of 1,253 weakly connected components, with the largest component having 172 shops. For any given component, the merger can impact the corresponding prices only when the shops of both merging parties are present in that component. I obtain that there are 91 affected components with a total of 1,658 shops, which are shown in Fig. 3 (the shops from the non-affected components are shown as dots in the figure). The largest affected components correspond to the main Dutch cities, albeit there are many smaller affected components scattered around the country.

So far, we have constructed the network showing which shops put competitive pressure on which shops, and that gives us matrix $g$. The ownership matrices before and after the merger, namely $f^*$ and $f^{**}$, follow directly from the data. What remains is to calibrate the following parameters of the model: the number of consumers at each location ($w_i$), the overall strength of the demand ($\alpha$), the extend to which the products from different shops are substitutes ($\gamma$), and the marginal costs ($c$).

First, consider the vector of weights $w$. From eqs. (14)–(17) it immediately follows that the equilibrium prices are homogeneous of degree zero with respect to the weights, i.e. $p_i(kw) = p_i(w)$. Hence, we only need to define the weights up to a scalar.

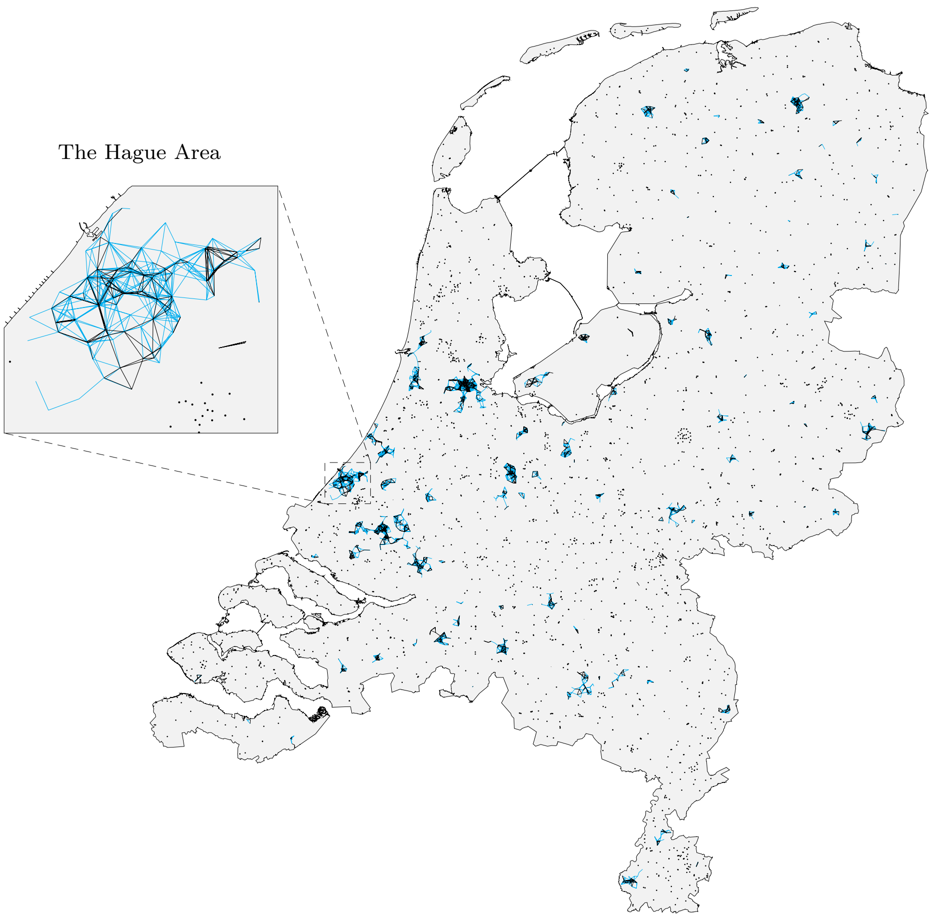
Preferably, we would use the data on population density at each location $i$ to define $w_i$. However, even in the absence of such data, a reasonable approximation can be made if we are ready to assume that the industry is in a state of long-run equilibrium. Let $v_i$ be the size of shop $i$ (I am using floor size, which is available in my dataset). Then, assuming a long-run equilibrium, $w_i$ can be chosen so that $q_i = v_i$. In practice, this procedure can be numerically demanding. For example, in our case we need to numerically solve 172 equations in 172 unknowns when calibrating the weights for the largest component of the network. My goal in this paper is to recommend a practically feasible alternative to the current approach of the EC. I therefore suggest to simply use $w_i = v_i$ as an approximate calibration.

Next, let us turn to $\alpha$ and $c$. We will need the following proposition (the proof is in the appendix).

**Proposition 1.** *In the equilibrium, the absolute markup, $p_i - c$, is proportional to $\alpha - c$. That is, for any $\alpha_1$, $c_1$ and $\alpha_2$, $c_2$ we have*

$$\frac{p_i(\alpha_1, c_1) - c_1}{\alpha_1 - c_1} = \frac{p_i(\alpha_2, c_2) - c_2}{\alpha_2 - c_2}.$$

Figure 3: Dutch Supermarkets




The map shows the locations of Dutch supermarkets in December 2011. If supermarket $i$ puts competitive pressure on supermarket $j$, but not the other way around ($g_{ij} = 1$, $g_{ji} = 0$), then these supermarkets are connected with a *cyan* line. If the competitive pressure is mutual, then the supermarkets are connected with a *black* line. The lines are drawn only between those supermarkets that might be affected by the merger (the corresponding network has shops from both merging parties). Supermarkets that cannot be affected by the merger are shown as standalone dots. The Hague area is magnified as an example.

An immediate corollary of the proposition is that the model is homogeneous of degree 1 in $\alpha$ and $c$. That is,

$$p_i(k\alpha, kc) = kp_i(\alpha, c). \tag{23}$$

Hence, for the purpose of the analysis of relative prices increases we can normalize the model by imposing

$$\alpha + c = 1. \tag{24}$$

Let $L_i = (p_i - c)/p_i$ be a Lerner index for shop $i$. Let $L = \sum_i L_i/N$ be the average Lerner index across all shops. Proposition 1, together with $\alpha + c = 1$, allows us to compute $\alpha$ and $c$ if $L$ is given. First, set $\alpha_1 = 1$ and $c_1 = 0$ (any arbitrary $\alpha_1$ and $c_1$ can be used in this first step) and compute the corresponding equilibrium prices, which we denote with $p_i^1$. We want to find $\alpha$ and $c$ such that

$$\frac{1}{N}\sum_i \frac{p_i(\alpha, c) - c}{p_i(\alpha, c)} = L. \tag{25}$$

Using Proposition 1 and using $\alpha + c = 1$, we obtain

$$\frac{1}{N}\sum_i \frac{c}{(1 - 2p_i^1)c + p_i^1} = 1 - L. \tag{26}$$

Eq. (26) can be solved numerically to obtain $c$ given $L$. I use $L = 0.05$, which, according to Freeman et al. (2008), is the average Lerner index in the supermarket industry.[6]

It remains to choose the degree of substitution $\gamma$. Preferably, $\gamma$ should be estimated based on the actual consumer behaviour. However, just like with the weights, we can make a reasonable choice of $\gamma$ even in the absence of additional data if we assume a long-run equilibrium. Specifically, in the long-run equilibrium the profits of all shops should be equal as otherwise it would be profitable to move shops from less profitable locations to more profitable locations.

Following this logic, I propose to choose $\gamma$ so that the coefficient of variation computed over the equilibrium profits is minimized. I use coefficient

[6] See para. 5.46.

of variation instead of variance, because $\gamma$ also influences the average profits and coefficient of variation is scale-invariant while variance is not. While I do not have data on fixed costs, I can simply compute coefficient of variation for each size class (assuming similar fixed costs within a size class). However, from my experiments it turns out that considering different size classes has little impact on the optimal gamma, therefore I simply group all shops together. Formally,

$$\hat{\gamma} = \arg\min_{\gamma} \frac{\sum_i (\pi_i(\gamma) - \mu(\gamma))^2}{\mu(\gamma)}, \quad \mu(\gamma) = \frac{\sum_i \pi_i(\gamma)}{N}, \tag{27}$$

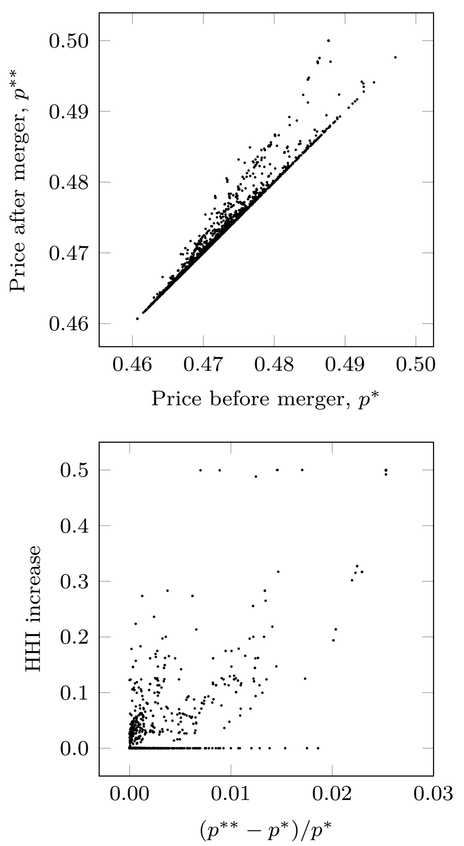
where the summation is done across all shops.

Following the procedure outlined above I obtain $\alpha = 0.55$, $c = 0.45$, and $\gamma = 0.39$. Having the model calibrated, we can proceed with calculating the prices before and after the merger. The result is depicted in Fig. 4, top left panel; only the shops from the affected components are shown in the figure. The prices before and after the merger both vary between 0.46 and 0.5. The prices at most locations are hardly affected the merger. Only at 71 out of 1,658 potentially affected locations is there a price increase of more than 1%, with a maximum increase of 2.5%.

We can use the same network $g$ and weights $w$ to compute HHI before and after the merger in accordance with the local markets approach of the EC. Specifically, for each shop $i$ I compute HHI using all shops $j$ such that $g_{ij} = 1$ (by construction, this set always includes shop $i$ itself), with weights $w_j$. A picture similar to earlier obtains, see Fig. 4, top right panel. Only at 314 locations is there an HHI increase of more than 250, with the largest increase being 5000 (the numbers in the figure are scaled down by 10,000).

We can compare now the approximate local markets approach to the theoretically consistent network approach that I propose in this paper. This comparison is done in Fig. 4, bottom left panel, which plots predicted prices increases against HHI changes. At many locations where a substantial price increase is predicted, both approaches yield consistent results. Crucially, however, there is a number of locations where HHI is a poor approximation of the predicted price increase. At some locations the change in HHI is high, whilst no substantial price increase is expected. This bias is less consequential and can be corrected as long as further detailed analysis is performed for these locations. However, the presence of this bias highlights the

Figure 4: Merger Analysis



necessity of such further analysis. No conclusions should be drawn based on HHI alone. A more consequential bias can be seen at those locations, where there is no change in HHI but there is a high expected price increase. Such a bias occurs precisely, because analysing individual catchment areas, e.g. when computing HHI, misses on the chain-of-substitution effects. This bias means that the EC might be missing on potentially problematic areas in their initial screenings in this type of merger cases.

The network approach I propose in this paper is flexible. When data are available, store and consumer locations can be decoupled, demand parameters as well as catchment area delineations can be estimated from the consumer purchase data, additional product varieties can be introduced. In other words, the proposed methodology can be used for a complete merger simulation. Having said that, my primary goal is to advocate this approach for preliminary screening, as in its simplest form the only additional data required is the average Lerner index in the industry. The rest of the data are the same data as used when computing HHI increases, and so have to be collected in either case. Importantly, this approach is feasible within the time constraints imposed by first stage investigations, because the complete algorithm has been outlined and can be preprogrammed in advance of the data analysis. For instance, the analysis done in this paper is fully automated.[7]

# 6 Concluding Remarks

The EC plays a fundamental role in ensuring a fair, competitive, and united European market. The decisions of the EC have direct impact on business profits and on consumer welfare. In this paper, I show that the local markets approach that the EC uses for mergers of firms with multiple points of sale can yield incorrect predictions regarding post-merger price increases. I propose an alternative network approach, which has strict theoretical foundations and which, in first approximation, requires no more data than is normally available on such merger cases. Given the feasibility of the proposed network approach, there is no reason not to advice a change of practice to the EC.

[7] The relevant code (python) will soon be posted online. In principle, the code can be readily used with the data from any other spatial merger.

Every model is an approximation. And while some parts of a model can be viewed as approximations that are too rough to be practically useful in their own right, the model itself might allow enough flexibility to compensate for those rough approximations. One clear example is binomial trees in derivatives pricing. The price does not just go up or down, but the big number of nodes compensates for this extreme approximation. And so it is, in my opinion, with network models. The network theory has yielded some interesting theoretical intuition, but its largest value added might lie in practical applications. I use catchment areas as a basis for my network construction. A catchment area is essentially a rough approximation of transportation costs: zero till a certain threshold, and infinite after that. However, when hundreds of such approximations are knit together within a network, a richer structure emerges, which might well be a good approximation of the real world.

# Appendix

**Proposition 1.** *In the equilibrium, the absolute markup, $p_i - c$, is proportional to $\alpha - c$. That is, for any $\alpha_1$, $c_1$ and $\alpha_2$, $c_2$ we have*

$$\frac{p_i(\alpha_1, c_1) - c_1}{\alpha_1 - c_1} = \frac{p_i(\alpha_2, c_2) - c_2}{\alpha_2 - c_2}.$$

*Proof.* In vector notation we have

$$\frac{p(\alpha_1, c_1) - \iota c_1}{\alpha_1 - c_1} = \frac{p(\alpha_2, c_2) - \iota c_2}{\alpha_2 - c_2}, \tag{28}$$

where $\iota$ denotes an all-ones vector.

It follows from (10), (11) and (15), (16) that $X$ is independent of $\alpha$ and $c$, hence

$$\frac{X^{-1}y(\alpha_1, c_1) - \iota c_1}{\alpha_1 - c_1} = \frac{X^{-1}y(\alpha_2, c_2) - \iota c_2}{\alpha_2 - c_2}, \tag{29}$$

or, equivalently,

$$\frac{y(\alpha_1, c_1) - X\iota c_1}{\alpha_1 - c_1} = \frac{y(\alpha_2, c_2) - X\iota c_2}{\alpha_2 - c_2}. \tag{30}$$

If we prove that (30) holds, then we also prove the proposition. Let us consider $y(\alpha, c) - X\iota c$. By definition, we have

$$y_i(\alpha, c) - (X\iota)_i c = -a_{v(i)i} - \sum_j (b_{v(i)ij} - \mathbf{1}_{i=j} h_i)c. \tag{31}$$

Dropping index restrictions, we can show that in general

$$\begin{aligned} - a_{rt} - \sum_j (b_{rtj} - \mathbf{1}_{t=j} h_t)c \\ = c \sum_{i,j} f_{rj} g_{ji} g_{ti} w_i \delta_i - \eta c \sum_i g_{ti} w_i - \sum_i g_{ti} w_i \beta_i \\ - c \sum_{i,j} g_{ji} g_{ti} w_i \delta_i - c \sum_{i,j} f_{rj} g_{ji} g_{ti} w_i \delta_i + 2\eta c \sum_i g_{ti} w_i \\ = \sum_i g_{ti} w_i (\eta c - \beta_i - c \delta_i d_i) = \frac{c - \alpha}{\gamma \eta} \sum_i g_{ti} w_i \delta_i, \end{aligned} \tag{32}$$

where we have used the following

$$\sum_{i,j} g_{ji} g_{ti} w_i \delta_i = \sum_i g_{ti} w_i \delta_i \sum_j g_{ji} = \sum_i g_{ti} w_i \delta_i d_i. \tag{33}$$

Given that $1/(\gamma\eta) \sum_i g_{ti} w_i \delta_i$ is independent of $\alpha$ and $c$, from (31) and (32) it immediately follows that (30) holds. $\square$

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