Innovation spillover, licensing, and ex-post privatization in international duopoly

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Innovation Spillover, Licensing, and Ex-post Privatization in International Duopoly∗

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Abstract

This paper studies the impact of innovation spillover and licensing on optimal ex-post privatization policies by involving an exogenous R&D activity in a partial-equilibrium international duopoly setting. By assuming a domestic public firm is relatively inefficient compared to its foreign private rival, we characterize and discuss optimal privatization policies under both foreign private and domestic public innovation. The theoretical results suggest that foreign private (domestic public) innovation, including both spillover and licensing, reduces (increases) the optimal degree of ex-post privatization. In addition, innovation spillover and licensing have the same impact direction on privatization policies. The numerical evidence supports these theoretical findings.

Keywords: Spillover, Licensing, Ex-post Privatization, International Duopoly

JEL Classification: L24, L33, L13, H44, F13

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1 Introduction

Privatization, the transfer of state-owned assets to the private sector, has prevailed globally for more than 50 years. Privatization is common for the telecommunications, finance, transportation, and energy sectors. As a result, private sector development and the privatization of state-owned enterprises (SOEs) has played an important role in many developing countries and emerging markets such as Brazil, China, Russia, and Vietnam (Cai and Li, 2011; Huang and Yang, 2016; Huang et al., 2017; Fridman, 2018). Privatization applies to China in a significant way accompanied by R&D spillover and licensing through foreign direct investment FDI. The Chinese government began nationwide mixed ownership reform in 2005 while nurturing a policy intended to make China an “innovation-oriented” country by 2020 and a “leading science power” by 2050 (Wang et al., 2014b).

As one of the key modes of technology diffusion under FDI, innovation spillover has an absorptive or imitative capacity with regard to advanced technology. It is common to observe spillovers in the same industry when new technology is introduced. However, the degree of spillover effect depends on the patent protection and technological sophistication of the new technology (Heywood and Ye, 2009). As one of the largest countries attracting FDI inflows, China has benefitted from the positive externality of innovation spillover through FDI inflows since the economic reforms of 1979 (Kuo and Yang, 2008; Abraham et al., 2010; Jiang et al., 2010; Lin et al., 2009; Liu, 2008). In addition, innovation licensing, another critical channel through which innovation is transferred, is widely observed in China’s domestic market. For example, in the early 1980s, Ring Round Company, which is associated with US Western Petroleum, obtained hybrid rice technology via China Seed Corporation through domestic public licensing. Zoomlion, a Chinese state-owned machin-

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1 For instance, China used approximately $1267.7 billion as FDI inflows in 2015 while the FDI average for BRIC nations – Brazil, Russia, India, and China – was $93.9 billion in 2015. This average was higher than the world average of $29.3 billion, the G7 average of $82.8 billion, and the European Union average of $19.1 billion.

2 In fact, according to the State Intellectual Property Office of China, from year 2000 to 2012, there were 27,412 license agreements covering 91,551 transferred patents in China.
ery enterprise, acquired Jost designs from Germany in June 2011 through foreign private licensing. Nevertheless, Arrow (1972) notes the negative impact of innovation spillover on the motivation to innovate, which provides evidence of the essential association between the two technology diffusion modes under FDI — R&D spillovers and licensing. In a mixed market, while numerous studies investigate optimal R&D policies in the presence of spillover effects, the literature on optimal privatization policies that align with R&D spillover is limited. Gil-Moltó et al. (2018) develop a mixed oligopoly model to examine the role of R&D subsidies and evaluate the welfare effects of privatization. Gil-Moltó et al (2011) analyze the use of R&D subsidies in a mixed and private duopoly market by involving R&D spillovers. Haruna and Goel (2015) examine international mixed duopoly behavior with research spillovers. Payogo-Theotoky (1995) provide an oligopoly model with information spillover. However, Heywood and Ye (2009) investigate the incentive for partial privatization in a mixed duopoly with R&D rivalry. However, licensing is not addressed in the article.

There is also a strand of literature examining the impact of licensing on privatization policies in mixed oligopolies. For instance, Mukherjee and Sinha (2014) find that it is unnecessary to privatize a public firm if the cost-asymmetry between the duopolists can be bridged by technology licensing. Wang and Zeng (2019) discuss the impact of domestic pri-

\[ \text{\textsuperscript{3}} \text{There are studies that investigate the association between R&D spillovers and licensing in different directions. Yan and Yang (2018) discuss optimal licensing schemes for a jointly owned enterprise when facing uncertain R&D outcomes and technology spillover. Bhattacharya and Guriev (2006) develop a two-stage cumulative R&D model in which a research unit engages in research to produce an interim innovative idea. Mukherjee (2006) shows the effect of patent protection on R&D investment in the presence of non-infringing imitation and technology licensing. Yun et al. (2000) examine several types of R&D organization, including non-cooperative and cooperative R&D, by taking spillover into consideration.} \]

\[ \text{\textsuperscript{4}} \text{There is substantial literature discussing the determination of optimal licensing schemes/strategies. For instance, Heywood et al. (2019) examine an optimal licensing strategy for a welfare-maximizing public firm to a more efficient competing foreign firm. Ye (2012) investigates and compares optimal public licensing by means of fixed fee royalties in an international mixed oligopoly. Wang (1998, 2002) studies and compares licensing through a fixed-fee and licensing using a royalty in a homogeneous and differentiated Cournot duopoly market, respectively. Chen et al. (2014) explore domestic private licensing strategies by three different means: a fixed fee, royalties, and a two-part tariff in a mixed oligopoly market. Kim et al. (2018) examine technology licensing by means of a fixed fee in a mixed duopoly where public and private firms purchase eco-technology from a foreign innovator.} \]
vate licensing on the optimal degree of *ex-ante* privatization in a domestic mixed oligopoly with entry. Wang *et al.* (2019) investigate the impact of both domestic public and foreign private licensing on *ex-ante* privatization policy. Niu (2015) analyzes the influence of licensing from an external institution privatization policy in a pure monopoly market. A remarkable assumption of the stated literature is that the innovator firm is more efficient compared to the licensee firm; that is, the cost asymmetry is reduced due to innovation licensing. However, the cost asymmetry between licensor and licensee firms can increase due to technology licensing if the licensor firm is less efficient compared to the licensee firm. As long as the efficiency gains for the licensee overwhelm the cost advantages of the licensor, the latter may achieve a larger total income from licensing irrespective of whether the licensor is more or less efficient than the licensee (Wang, 2002). None of the aforementioned articles address optimal privatization policy with both innovation spillover and licensing given the increased cost asymmetry between innovator and licensee firms caused by innovation spillover and licensing.

Hence, this paper follows Mukherjee and Sinha (2014) and Wang *et al.* (2019) and examines the impact of innovation spillover (with no licensing) and licensing (with no spillover) on the *ex-post* privatization policies of public firms in an international duopoly. This pa-

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5 *Ex-ante* privatization with technology licensing indicates that the local government optimally chooses the degree of privatization before determining the optimal licensing contract known as a licensing-then-privatization model. A few studies focus on privatization and technology licensing in a mixed oligopoly market and adopt the licensing-then-privatization model, see Wang *et al.* (2019) and Niu for more details.

6 Practically, CSR Corporation Limited and General Electric (GE) established a US-based joint venture to supply high-speed rail (HSR) passenger trains for two proposed dedicated US HSR corridors in Florida and California. The partnership represents an investment of approximately $50 million in a joint venture and public HSR technology licensing for GE in 2010. However, we find that the gross profit margins (GPM) of CSR and GE were 5.2% and 12.36%, respectively, in 2009, and the GPMs of CSR and GE were 5.6% and 9.35%, respectively, in 2010 (Source: CSR’s annual report and GE’s fact sheet in 2010). Thus, the practical evidence suggests that domestic public licensing to foreign partners does exist irrespective of whether the public licensor is more or less efficient than the foreign plant in the Chinese market. Therefore, the cost asymmetry between CSR and GE is effectively enlarged after public technology licensing.

7 Heywood *et al.* (2019) investigate optimal license contracts by assuming a public firm innovator licenses to a more efficient foreign rival. However, optimal privatization policy is not addressed in this study.

per includes an exogenous R&D activity under both foreign private and domestic public innovation, and assumes that the domestic public firm is less efficient than its foreign rival\(^9\). By adopting a licensing-then-privatization model, we assert that local government does not commit a certain degree of privatization after observing the licensing contract, which is realistic. The theoretical evidence suggests that foreign private (domestic public) innovation, including spillover and licensing, decreases (increases) the optimal degree of \textit{ex-post} privatization due to the reduction (augmentation) of cost asymmetry between the two firms. Additionally, innovation spillover and licensing have the same directional impact on privatization policies. However, the effect of innovation spillover is weaker than that of licensing on the degree of privatization if the innovation is not perfectly easily imitated \(^{10}\).

The remainder of this paper is arranged as follows. We discuss the model in section 2, and foreign private licensing is analyzed in section 3. We investigate domestic public licensing in section 4, a numerical analysis is presented in section 5, and we offer concluding remarks in the last section.

## 2 The Model

We consider an international duopoly market including a domestic public firm (firm 0) and a foreign private firm (firm 1) \(^{11}\). Firms produce homogeneous products for which the inverse demand function is linear; that is,

\[
p = a - Q,
\]

where \(p\) is the price, and \(Q\) is the total output. The constant marginal cost of firm \(i\), before R&D innovation, is \(c_i\) \((i = 0, 1)\). We assume that \(c_0 > c_1\). That is, initial cost asymmetry

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\(^9\)See also Chinese domestic public licensing of HSR toward the United States for examples where innovators are less efficient compared to foreign licensees mentioned in footnote 6.

\(^{10}\)We find that if the innovation spillover is perfect and complete, there is no demand for licensing.

between the domestic public firm and the foreign private firm exists. Firms $i$ exogenously invest $\varepsilon^2/2$ in R&D to develop a non-drastic cost-reducing innovation, which reduces the marginal cost by $\varepsilon$ where $0 < \varepsilon < c_0 - c_1$. We assume that the appropriability of research knowledge is imperfect, and there exists an innovation leakage that benefits the rival. Thus, by taking R&D activity and associated innovation spillovers into consideration, the profits of firm $i$ with cost-reducing innovation ($\pi_i$) and of firm $j$ without cost-reducing innovation ($\pi_j, i \neq j$) are, respectively,

$$
\pi_i = (p - c_i + \varepsilon) \cdot q_i - \frac{\varepsilon^2}{2}
$$

$$
\pi_j = (p - c_j + \phi \varepsilon) \cdot q_j,
$$

where $q_i$ ($q_j$) is firm $i$ ($j$)'s output, and $\phi \in [0, 1)$ is the degree of innovation spillover. Note that, $\phi = 0$ represents that innovation remains confidential and private. On the contrary, $\phi = 1$ indicates that innovation is easily imitated. Here, innovation licensing no longer exists if the innovation is perfectly easy to imitate (i.e., $\phi = 1$). Therefore, we merely consider $\phi \in [0, 1)$. Local welfare $SW$ is given by,

$$
SW = \frac{1}{2} (q_0 + q_1)^2 + \pi_0,
$$

given that private firm 1 is fully foreign-owned. Following Matsumura (1998), the public firm's objective function is a convex combination of social surplus and its own profit, $\theta \pi_0 + (1 - \theta) \cdot SW$. $\theta \in [0, 1]$ represents the degree of privatization. In the case of full nationalization (i.e., $\theta = 0$), firm 0 maximizes welfare. In the case of full privatization (i.e., $\theta = 1$), firm 0 maximizes its profit. The foreign private firm's objective is profit.

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12If $c_0 \leq c_1$, the public monopoly emerges in equilibrium, and there is no room to discuss mixed oligopolies. We assume that constant marginal costs with a cost disadvantage for a public firm is popular, as in the literature on mixed oligopolies. See Pal (1998), Mujumdar and Pal (1998), and Matsumura and Ogawa (2010), for instance. For a discussion on the endogenous cost disadvantage of public firms, see Matsumura and Matsushima (2004) and Ohnishi (2006). Many empirical studies illustrate that public firms in developing countries and emerging markets produce less efficiently than private firms. See also Vickers and Yarrow (1988), Megginson and Netter (2001), and La Porta et al. (2002) for example.


14See also Haruna and Goel (2015), Gil-Moltó et al. (2011), and Payogo-Theotoky (1995).

15See also Heywood and Ye (2009), Haruna and Goel (2015).
We attempt to examine the impact of innovation spillover (in the absence of licensing) and licensing (in the absence of spillover) on the *ex-post* privatization policy of the public firm. Note that unofficial innovation spillover automatically disappears in the presence of innovation licensing. A three-stage game is considered in this licensing-then-privatization model\textsuperscript{16}. In the first stage, a licensor firm offers the licensing contract by means of a fixed fee, \( f^* \), to a licensee firm for profit maximization. A licensing contract is signed if neither the licensor nor the licensee are worse off compared to a situation with no licensing. In the second stage, the government optimally chooses the degree of privatization \( \theta^* \) in the domestic public firm to maximize social welfare by observing the optimal level of licensing contract. In the third stage, these two duopolists engage in Cournot competition. The game is solved by backward induction. Note that the non-drastic innovation hypothesis; that is, \( c_0 - \varepsilon \geq c_1 \), holds for foreign private and domestic public innovation cases.

3 Foreign Private Innovation

In this section, the foreign firm is assumed to be the innovator and licensor firm of the cost-reducing innovation. Therefore, we intend to examine the impact of spillover (in the absence of licensing) and licensing (in the absence of spillover) under foreign private innovation on the public firm’s *ex-post* privatization policy.

3.1 Spillover & No Licensing

We first consider the game with spillover and with no licensing to examine the effect of innovation spillover and to compare the impact of innovation licensing on privatization policy under foreign private innovation. The timing of the game with no licensing is as

\textsuperscript{16}Except for Mukherjee and Sinha (2014), Haraguchi and Matsumura (2018) develop a mixed triopoly to investigate the optimal strategy of private licensing under the presence of *ex-post* privatization. Xu et al. (2017) investigate the impact of foreign penetration on both *ex-ante* and *ex-post* privatization and liberalization policies in a free-entry market. Lee et al. (2018) investigate the impact of the timing of privatization on the optimal degree of *ex-post* privatization in a free-entry market. Sato and Matsumura (2018) formulate a two-period model of mixed oligopoly assuming that the local government can change the optimal degree of privatization after observing the firm-level cost structure.
follows. In the first stage, the government optimally chooses the degree of privatization. In the second stage, the two duopolists; that is, the jointly owned public firm, competes with a foreign rival in Cournot fashion given the innovation spillover effect.

In the quantity competition, the public (foreign) firm chooses the optimal output level $q_0 (q_1)$ to maximize the weighted average utility $U_0$ (profit, $\pi_1$). The utility of firm 0 is as follows.

$$U_0^{N0} = (1 - \theta) \cdot SW^{N0} + \theta \cdot \pi_0^{N0}, \quad (1)$$

where superscript “N0” denotes no innovation transfers to public firm 0. Particularly, we assume foreign firm 1 is the innovator in this section, thus, the profits of both firm 0 and firm 1 are,

$$\pi_0^{N0} = (a - q_0^{N0} - q_1^{N0} - c_0 + \phi \varepsilon) \cdot q_0^{N0},$$

$$\pi_1^{N0} = (a - q_0^{N0} - q_1^{N0} - c_1 + \varepsilon) \cdot q_1^{N0} - \frac{\varepsilon^2}{2},$$

where $\phi \varepsilon$ indicates the spillover effect of cost-reducing innovation w.r.t. the domestic public firm. As a result, the first-order conditions are listed as,

$$q_0^{N0} = \frac{2(a - c_0) - \theta(a - c_1 + \varepsilon) + 2\phi \varepsilon}{\theta + 2}, \quad (2)$$

$$q_1^{N0} = \frac{(c_0 - c_1 + \varepsilon) + \theta(a - c_1 + \varepsilon) - \phi \varepsilon}{\theta + 2}. \quad (3)$$

In the first stage, the local government optimally chooses the privatization level of public firm 0 to maximize social welfare where

$$SW^{N0} = \frac{1}{2} \cdot (q_0^{N0} + q_1^{N0})^2 + \pi_0^{N0}.$$ 

The differentiation of $SW^{N0}$ w.r.t. the privatization degree $\theta$, generates

$$\frac{\partial SW^{N0}}{\partial \theta} = \frac{(2a - c_0 - c_1 + \varepsilon + \phi \varepsilon) \cdot [(c_0 - c_1 + \varepsilon) - \theta(3a - 2c_0 - c_1 + \varepsilon + 2\phi \varepsilon)]}{(\theta + 2)^3}$$

Given the satisfaction of the second-order condition, the optimal degree of privatization in public firm is

$$\theta^{N0} = \frac{c_0 - c_1 + \varepsilon - \phi \varepsilon}{3a - 2c_0 - c_1 + \varepsilon + 2\phi \varepsilon}. \quad (4)$$
Lemma 1 If there is foreign private spillover but no licensing, the optimal degree of privatization in the public firm is partial; that is, $\theta^N_0$ in (4) when $a > (4c_0 - c_1 + \varepsilon - 4\phi\varepsilon)/3$.

Proof See the Appendix.

We explain the intuition behind Lemma 1. The presence of cost asymmetry between the public firm and foreign firm creates a rationale for privatization (Mukherjee and Sinha, 2014; Matsumura, 1998). On the one hand, privatization leads to a less aggressive inefficient public firm in terms of production by moving its objective function more toward profit maximization. On the other hand, privatization encourages the efficient private firm to produce more, thus reducing the domestic welfare loss due to production inefficiency. However, in contrast to Mukherjee and Sinha (2014), the partial privatization policy must be optimal to reduce the profit transferred to the foreign country in an international duopoly market.

Next, we investigate the effect of exogenous spillover effect on the equilibrium outputs and on the optimal degree of privatization. Differentiating $q^N_0$, $q^N_1$, and $\theta^N_0$ in (2), (3), and (4) respectively, yields,

$$
\frac{\partial q^N_0}{\partial \phi} = \frac{2\varepsilon}{\theta + 2}, \quad \frac{\partial q^N_1}{\partial \phi} = -\frac{\varepsilon}{\theta + 2}, \quad \frac{\partial Q^N_0}{\partial \phi} = -\frac{\varepsilon}{\theta + 2}, \quad \frac{\partial \theta^N_0}{\partial \phi} = \frac{3\varepsilon \cdot (a - c_1 + \varepsilon)}{(3a - 2c_0 - c_1 + \varepsilon + 2\phi\varepsilon)^2}.
$$

From (5), we obtain the following lemma.

Lemma 2 $q^N_0$ and $Q^N_0$ are increasing in $\phi$, and $q^N_1$ and $\theta^N_0$ are decreasing in $\phi$.

We now explain the intuition. An increase in spillover effect, $\phi$, reduces the initial cost asymmetry between public firm 0 and foreign firm 1 given the assumption $c_0 - c_1 > \varepsilon$. Therefore, the increase in innovation spillover improves the production inefficiency of the public firm, induces the public firm to be more aggressive (decreases $\theta^N_0$), and produces more (increases $q^N_0$) when $\phi$ is larger (direct effect). Through the strategic interaction between the duopolists, firm 1 produces less when firm 0 produces more. Therefore, an increase in $\phi$ decreases $q^N_1$ (the indirect effect). Since the direct effect is stronger than the
indirect effect, $Q^{N_0}$ is increasing in $\phi$.

### 3.2 Licensing & No Spillover

In this subsection, we consider the presence of innovation transfers from foreign firm 1 to public firm 0 with *ex-post* privatization where both firms make licensing decisions before the government’s privatization decision. Note that innovation licensing only occurs if it renders both the innovator (foreign firm 1) and licensee (public firm 0) better off compared to the situation where no innovation is transferred. Accordingly, the innovation spillover effect disappears in this case due to the official foreign licensing toward the domestic public enterprise. The timing of the game is as stated previously.

In the quantity competition, with the presence of innovation transfers, the profits of public firm 0 and foreign firm 1 are

\[
\pi_{0}^{L_0} = (a - q_{0}^{L_0} - q_{1}^{L_0} - c_0 + \varepsilon) \cdot q_{0}^{L_0} - f^{L_0},
\]

\[
\pi_{1}^{L_0} = (a - q_{0}^{L_0} - q_{1}^{L_0} - c_1 + \varepsilon) \cdot q_{1}^{L_0} - \frac{\varepsilon^2}{2} + f^{L_0}.
\]

where the superscript “$L_0$” denotes licensing to public firm 0. Thus, the public firm 0 (foreign firm 1) chooses an output level to maximize its utility (profit) where

\[
U_{0}^{L_0} = (1 - \theta) \cdot SW^{L_0} + \theta \cdot \pi_{0}^{L_0},
\]

and the domestic welfare is

\[
SW^{L_0} = \frac{1}{2} (q_{0}^{L_0} + q_{1}^{L_0})^2 + \pi_{0}^{L_0}.
\]

As a result, the first-order conditions in the third stage are listed as.

\[
q_{0}^{L_0} = \frac{2(a - c_0 + \varepsilon) - \theta(a - c_1 + \varepsilon)}{\theta + 2},
\]

\[
q_{1}^{L_0} = \frac{c_0 - c_1 + \theta(a - c_1 + \varepsilon)}{\theta + 2}.
\]

In the second stage, the government optimally chooses the degree of privatization to maximize welfare given the presence of technology licensing from foreign firm 1 to public
firm 0, which yields,
\[
\frac{\partial SW^L_0}{\partial \theta} = \left(2a - c_0 - c_1 + 2\varepsilon\right) \cdot \left[c_0 - c_1 - \theta(3a - 2c_0 - c_1 + 3\varepsilon)\right] / (\theta + 2)^3
\]

By setting \(\partial SW^L_0 / \partial \theta = 0\), given the satisfaction of the second-order condition, we have
\[
\theta^L_0 = \frac{c_0 - c_1}{3a - 2c_0 - c_1 + 3\varepsilon}.
\]

Due to the prior literature on technology licensing (e.g., Wang and Zeng, 2019 and Wang, 1998), the innovator (foreign firm 1 in this section) is a dominant player in the licensing game and can extract the licensee’s (public firm 0) entire benefit from innovation transfers. Therefore, the maximum license fee foreign firm 1 can charge is solved by \(f^L_0 = U^L_0 - U^N_0\) given the equilibrium outcomes of \(U^L_0\) from (8) and \(U^N_0\) from (1). However, the fixed fee is a lump-sum transfer from public firm 0 to foreign firm 1 under foreign private licensing. Hence, the fixed fee cannot affect the output equilibrium level and the degree of privatization. The equilibrium \(f^L_0\) is presented in (19) in the appendix. By assuming the motivation of foreign private licensing, this discussion leads to the following lemma.

**Lemma 3** If there is foreign private licensing but no spillover, the optimal degree of ex-post privatization is partial; that is, \(\theta^L_0\) in (12) when \(a > (4c_0 - c_1 - 3\varepsilon)/3\).

**Proof** See the Appendix.

Under foreign private licensing, the cost asymmetry between the two duopolists decreases. However this cost-reducing innovation is non-drastic, which creates a rationale for privatization. In addition, by examining the degree of privatization with spillover & no licensing and the degree of privatization with licensing & no spillover, we find the spillover and licensing of foreign private innovation affects the degree of ex-post privatization as follows.

**Proposition 1** (i) Compared to no spillover (i.e., \(\phi = 0\)), unofficial innovation leakage from foreign private innovation reduces the optimal level of privatization in the domestic public firm; (ii) compared to no licensing, foreign private licensing to the domestic public
firm further reduces the optimal degree of ex-post privatization if \( a > (4c_0 - c_1 - 3\varepsilon)/3; \)

(iii) the impact of innovation spillover on the optimal degree of privatization can be exactly
the same as that of innovation licensing on the optimal degree of privatization under foreign
private innovation as long as the cost-reducing innovation is completely easy to imitate (i.e.,
\( \phi = 1 \)).

**Proof** See the Appendix and Figure 1.

The intuition is straightforward. Given the cost asymmetry between the licensee (firm 0)
and the innovator (firm 1); that is, \( c_0 - c_1 > \varepsilon \), the initial cost differential is reduced due to
either innovation spillover or foreign private licensing. First, when cost-reducing innovation
unofficially spills from the foreign firm to the public firm, aligned with the absence of
foreign private licensing, the public firm becomes more aggressive (decreasing in \( \theta^N0 \)) to
increase output in terms of production, and efficiency improvements reduce the marginal
cost to \( c_0 - \phi \varepsilon \). However, as long as the innovation is not completely easily imitated, (i.e.,
\( 0 < \phi < 1 \)), foreign private licensing must further reduce the cost asymmetry between the
two firms. In other words, compared to unofficial innovation leakage, foreign private licensing
could fully improve the production inefficiency of the domestic public firm by reducing its
marginal cost to \( c_0 - \varepsilon \). As a result, the profit margin of the public firm increases. Hence,
the benefit of privatization is diminished, indicating a further decrease in the degree of
privatization. That is, from \( \theta^N0|_{0<\phi<1} \) to \( \theta^L0 \) given the presence of cost-reductions *via* both
innovation spillover and licensing\(^\text{17}\). Figure 1 illustrates the fact discussed above. The figure
also shows that foreign private licensing is unnecessary if the innovation spillover is fully
complete (i.e., \( \phi = 1 \)). That is, as \( \phi \) increases, \( \theta^N0|_{0<\phi<1} \) closes to \( \theta^L0 \).

\(^{17}\text{Mukherjee and Sinha (2014) reveal full nationalization as the optimal policy with the presence of domestic private licensing in an *ex-post* privatization model due to the absence of cost asymmetry between firms. In contrast, Wang et al. (2019) show that foreign private licensing increases the optimal degree of privatization by adopting a privatization-then-licensing model.}
Figure 1: **Spillover, Licensing, and Degrees of Privatization**

Note: $\theta^N_1$ ($\theta^N_0$) denotes the optimal degree of *ex-post* privatization when there is spillover but no domestic public (foreign private) licensing toward the foreign private firm, firm 1 (domestic public firm, firm 0). $\theta^L_1$ ($\theta^L_0$) represents the optimal degree of *ex-post* privatization when there is domestic public (foreign private) licensing toward the foreign private firm, firm 1 (domestic public firm, firm 0). In addition, $\phi = 0$ indicates the absence of innovation spillover; $0 < \phi < 1$ signals imperfect/incomplete spillover effect; $\phi = 1$ illustrates perfect spillover effect.

4 **Domestic Public Innovation**

In this section, the domestic public firm is assumed to be the innovator and licensor firm of the cost-reducing innovation. Therefore, we intend to examine the impact of spillover (with the absence of licensing) and licensing (with the absence of spillover) under domestic public innovation on the *ex-post* privatization policy of this public firm.

4.1 **Spillover & No Licensing**

The benchmark game of domestic public innovation is examined in this subsection to investigate the effect of innovation spillover and to compare the impact of innovation licensing on privatization under domestic public innovation. The timing is the same as in section 3.1. Note that although the public firm is the innovator of the cost-reducing technology, the cost asymmetry between firm 0 and firm 1 still exists; that is, $c_0 - c_1 > \varepsilon$. In fact, the cost differential between the public and foreign firm is further increased by either innovation leakage or domestic public licensing under the assumption $c_0 - c_1 > \varepsilon$. However,
despite being relatively inefficient compared to the foreign private licensee, the domestic public innovator/licensor can extract some of the efficiency gain through fixed fee licensing. When the benefit extracted from licensing overwhelms the cost advantage of the licensor, the inefficient licensor is motivated to license to obtain a larger total income (Wang, 2002).

The profits of both licensor and licensee change as follows.

\[ \pi_0^{N1} = (a - q_0^{N1} - q_1^{N1} - c_0 + \varepsilon) \cdot q_0^{N1} - \frac{\varepsilon^2}{2} \]

\[ \pi_1^{N1} = (a - q_0^{N1} - q_1^{N1} - c_1 + \phi \varepsilon) \cdot q_1^{N1} \]

where the superscript “N1” denotes no technology licensing to the foreign firm 1. Therefore, the utility of public firm 0 is

\[ U_0^{N1} = (1 - \theta) \cdot SW^{N1} + \theta \cdot \pi_0^{N1} \]

and domestic welfare is thus,

\[ SW^{N1} = \frac{1}{2} (q_0^{N1} + q_1^{N1})^2 + \pi_0^{N1}, \] (13)

given private firm 1 is totally foreign. In the last stage, the foreign firm 1 (public firm 0) optimally chooses quantity to maximize its profit, \( \pi_1^{N1} \) (utility, \( U_0^{N1} \)), where the equilibrium production quantity from the last stage is

\[ q_0^{N1} = \frac{2(a - c_0 + \varepsilon) - \theta(a - c_1 + \phi \varepsilon)}{\theta + 2}, \] (14)

\[ q_1^{N1} = \frac{(c_0 - c_1 - \varepsilon) + \theta(a - c_1 + \phi \varepsilon) + \phi \varepsilon}{\theta + 2}. \] (15)

In the first stage, local government optimally chooses the privatization level of public firm 0 to maximize social welfare in (13). The differentiation of \( SW^{N1} \) w.r.t. the privatization degree, \( \theta \), generates,

\[ \frac{\partial SW^{N1}}{\partial \theta} = \frac{(2a - c_0 - c_1 + \varepsilon + \phi \varepsilon) \cdot [(c_0 - c_1 - \varepsilon + \phi \varepsilon) - \theta(3a - 2c_0 - c_1 + 2\varepsilon + \phi \varepsilon)]}{(\theta + 2)^3}. \]
Given the satisfaction of the second-order conditions, the optimal degree of privatization in public firm 0 with innovation spillover and no domestic public licensing is

$$\theta^{N1} = \frac{c_0 - c_1 - \varepsilon + \phi\varepsilon}{3a - 2c_0 - c_1 + 2\varepsilon + \phi\varepsilon}. \quad (16)$$

**Lemma 4** If there is domestic public spillover but no licensing in an international duopoly market, the optimal degree of privatization in public firm is partial; that is, $\theta^{N1}$ in (16) when $a > (4c_0 - c_1 - 4\varepsilon + \phi\varepsilon)/3$.

**Proof** See the Appendix.

We can explain the intuition as follows. When a foreign firm has an initial technology advantage (i.e., $c_0 - c_1 > \varepsilon$), the cost asymmetry between domestic public and foreign private firms is enlarged aligning with the unofficial innovation spillover (that is, $\phi\varepsilon$). In other words, the public innovator is even less efficient relative to the foreign licensee. Thus, a partial privatization policy would help to reduce the output of the public firm, thus reducing production inefficiency created by the sharp cost asymmetry between these two firms.

In addition, we now discuss the effect of exogenous spillover effect on the equilibrium outputs and the optimal degree of privatization under domestic public innovation. Differentiating $q_0^{N1}$, $q_1^{N1}$ and $\theta^{N1}$ in (14), (15), and (16), respectively, which yields,

$$\frac{\partial q_0^{N1}}{\partial \phi} = -\frac{\theta\varepsilon}{\theta + 2}, \quad \frac{\partial q_1^{N1}}{\partial \phi} = \frac{\varepsilon(\theta + 1)}{\theta + 2}, \quad \frac{\partial Q^{N1}}{\partial \phi} = \frac{\varepsilon}{\theta + 2}, \quad \frac{\partial \theta^{N1}}{\partial \phi} = \frac{3\varepsilon \cdot (a - c_0 + \varepsilon)}{(3a - 2c_0 - c_1 + 2\varepsilon + \phi\varepsilon)^2}. \quad (17)$$

From (17), we obtain the following lemma.

**Lemma 5** $q_1^{N1}$ and $\theta^{N1}$ are increasing in $\phi$, and $q_0^{N1}$ and $Q^{N1}$ are decreasing in $\phi$.

The intuition is straightforward. An increase in the spillover effect, $\phi$, enlarges the initial cost asymmetry between public firm 0 and foreign firm 1 given the assumption $c_0 - c_1 > \varepsilon$. Therefore, the increasing innovation spillover deteriorates the relative production inefficiency of the public firm to its foreign rival and induces the efficient foreign firm 1 to produce more when $\phi$ gets larger (direct effect)$^{18}$. Through the strategic interaction between these

---

$^{18}$The spillover effect directly impacts the foreign private firm.
two duopolists, the public firm 0 is less aggressive (increasing in $\theta^{N1}$) and producing less (decreasing in $q_0^{N1}$) as $\phi$ increases (indirect effect). Since the direct effect is stronger than the indirect effect, $Q^{N1}$ is increasing in $\phi$.

4.2 Licensing & No Spillover

In this subsection, we investigate the impact of domestic public licensing on the optimal degree of ex-post privatization under a licensing-then-privatization model. The three-stage game, which is exactly the same as the game in section 3.2, is solved again by backward induction. In the quantity competition, with domestic public licensing, the profits of both the innovator and licensee are as follows.

$$\pi^L_{L10} = (a - q^L_{L10} - q^L_{L11} - c_0 + \varepsilon) \cdot q^L_{L10} - \frac{\varepsilon^2}{2} + f^{L1},$$

$$\pi^L_{L11} = (a - q^L_{L01} - q^L_{L11} - c_1 + \varepsilon) \cdot q^L_{L11} - f^{L1}.$$

where $f^{L1}$ is the fixed fee charged by the public firm 0 due to the licensing contract. The superscript “L1” denotes technology licensing to foreign firm 1. In addition, the utility of public firm and domestic welfare are, respectively,

$$U^L_{L10} = (1 - \theta) \cdot SW^{L1} + \theta \cdot \pi^L_{L10},$$

$$SW^{L1} = \frac{1}{2} (q^L_{L01} + q^L_{L11})^2 + \pi^L_{L10}.$$

The equilibrium outputs from the third stage, chosen by maximizing profit/utility, are

$$q^L_{L10} = \frac{2(a - c_0 + \varepsilon) - \theta(a - c_1 + \varepsilon)}{\theta + 2},$$

$$q^L_{L11} = \frac{(c_0 - c_1) + \theta(a - c_1 + \varepsilon)}{\theta + 2}.$$

In the second stage, the government optimally chooses the degree of privatization to maximize welfare given the presence of technology licensing from public firm 0 to foreign firm 1, which generates

$$\frac{\partial SW}{\partial \theta} = \frac{(2a - c_0 - c_1 + 2\varepsilon) \cdot [(c_0 - c_1) - \theta(3a - 2c_0 - c_1 + 3\varepsilon)]}{(\theta + 2)^3}.$$
By letting $\partial SW/\partial \theta = 0$, due to the satisfaction of the second-order condition, the optimal degree of privatization in the second stage is,

$$\theta_{L1} = \frac{c_0 - c_1}{3a - 2c_0 - c_1 + 3\varepsilon} \quad (18)$$

Since public firm 0 is the innovator in this case, we can extract the foreign firm 1's entire benefit from technology licensing\(^\footref{19}\). Hence, in the first stage, the maximum fixed fee the public licensor can charge is,

$$f_{L1} = \pi_{L1}^L - \pi_{N1}^L = \frac{4\varepsilon(1 - \phi)(2c_0 - 2c_1 - \varepsilon + \phi\varepsilon)}{9}.$$ 

Note that the motivation for technology licensing from public firm 0 to foreign firm 1 exists; that is, $U_{L1}^L - U_{N1}^N > 0$ if $(4c_0 - c_1 - 3\varepsilon)/3 < a < (23c_0 - 20c_1 - 13\varepsilon)/3$ at the equilibrium level.

**Lemma 6** If there is domestic public licensing but no spillover, the optimal degree of ex-post privatization is partial; that is, $\theta_{L1}$ in (18) when $(4c_0 - c_1 - 3\varepsilon)/3 < a < (23c_0 - 20c_1 - 13\varepsilon)/3$.

**Proof** See the Appendix.

The intuition can be explained as follows. There may be limited room for the discussion of domestic public licensing given that the domestic public innovator is inefficient initially, and the cost asymmetry between the two duopolists cannot be bridged by technology innovation. However, note that, the domestic public licensor can extract some efficiency gain from the licensing payment. As long as the efficiency extracted from the foreign licensee overwhelms the cost advantages lost by the public innovator due to technology licensing, the public innovator is motivated to license the cost-reducing innovation to its foreign rivals to

\footnotetext[19]{We find that $\theta_{L0}^L$ in (12) is equal to $\theta_{L1}^L$ in (18). The main reason is that under both foreign private and domestic public licensing, the profit margin of the two firms is exactly the same. The difference between the model setups in foreign private licensing and domestic public licensing is the direction of fixed fee payments and the cost of R&D activity. Since fixed fee payments are a transfer from licensee to innovator, and the R&D cost is exogenous, they cannot influence the optimal level of ex-post privatization. Thus, we have $\theta_{L0}^L = \theta_{L1}^L$, which is reasonable.}
achieve more total profit (Wang, 2002). However, a rationale for privatization is created by domestic public licensing in terms of the increased cost asymmetry triggered by domestic public licensing.

Next, by examining the degree of privatization with spillover & no licensing and the degree of privatization with spillover with licensing & no spillover, we find that the spillover and licensing of domestic public innovation affects the degree of ex-post privatization as follows.

**Proposition 2**  
(i) Compared to no spillover (i.e., \( \phi = 0 \)), unofficial innovation leakage from domestic public innovation increases the optimal level of privatization in domestic public firms; (ii) compared to no licensing, domestic public licensing to a foreign private firm further increases the optimal degree of ex-post privatization if \( (4c_0 - c_1 - 3\varepsilon)/3 < a < (23c_0 - 20c_1 - 13\varepsilon)/3 \); (iii) the impact of spillover on the optimal degree of privatization can be exactly the same as the impact of spillover of licensing on the optimal degree of privatization under domestic public innovation as long as the cost-reducing innovation is completely easy to imitate (i.e., \( \phi = 1 \)).

**Proof**  
See the Appendix and Figure 1.

The intuition is straightforward. Given the cost asymmetry between the innovator (firm 0) and the licensee (firm 1); that is, \( c_0 - c_1 > \varepsilon \), the initial cost differential is increased due to innovation spillover or domestic public licensing. First, when cost-reducing innovation unofficially spills from a public firm to a foreign firm, aligned with the absence of licensing, the public firm becomes less aggressive (an increase in \( \theta^{N1} \)) to decrease the output in terms of a deterioration in relative production inefficiency compared to its foreign rival. Nevertheless, as long as the innovation is not completely easily imitated, (i.e., \( 0 < \phi < 1 \)), the domestic public licensing must further increase the cost asymmetry between the two firms. In other words, compared to unofficial innovation leakage, domestic public licensing could fully reduce

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20Wang et al. (2019) demonstrate the presence and validity of domestic public licensing in the ex-ante privatization model under the assumption that the domestic public firm is more efficient than the foreign private firm.
the marginal cost of the foreign private firm to $c_1 - \varepsilon$. As a result, the relative profit margin of the public firm compared to its foreign rival decreases. Hence, the benefit of privatization increases indicating a further rise in the degree of privatization; that is, from $\theta^N|_{0<\phi<1}$ to $\theta^L$ given the presence of cost reductions via both innovation spillover and licensing. Figure 1 illustrates the fact investigated above and shows that it is unnecessary for domestic public licensing if the innovation spillover is fully complete (i.e., $\phi = 1$). That is, as $\phi$ increases, $\theta^N|_{0<\phi<1}$ closes to $\theta^L$.

5 Numerical Analysis

In this section, we compute the optimal levels of \textit{ex-post} privatization with innovation spillovers and with innovation licensing by parameter specification. To be more specific, the numerical result of foreign private (domestic public) innovation is presented in Panel A (B). For instance, $\theta^N_0|_{\phi=1/3}$ ($\theta^N_1|_{\phi=1/3}$) in Table 1 represents the optimal degrees of privatization when a foreign private (domestic public) firm innovates this cost-reducing technology in (4) (in (16)) given that the effect of unofficial innovation leakage is one third of the official innovation licensing (i.e., $\phi = 1/3$). Particularly, $\theta^N_0|_{\phi=0}$ represents the optimal degree of \textit{ex-post} privatization under foreign private innovation in the absence of both innovation spillover and licensing. Additionally, $\theta^L_0$ ($\theta^L_1$) represents the optimal degree of \textit{ex-post} privatization in the presence of foreign private (domestic public) licensing in (12) (in (18)). Note that the domain of market size (a) is computed based on the parameter specification of $c_0$ and $c_1$ aligned with the restrictions from each lemma and proposition.

Under foreign private innovation (Panel A of Table 1), the cost asymmetry between the domestic public and foreign private firm is reduced when either innovation spillover or foreign private licensing occurs. Therefore, we find that the optimal degree of \textit{ex-post} privatization decreases due to the increase in the spillover effect of foreign private innovation. In addition, the privatization level further decreases in terms of improvements in the production inefficiency of the domestic public firm induced by foreign private licensing. On the
Table 1: Optimal Degrees of Ex-post Privatization

<table>
<thead>
<tr>
<th></th>
<th>Foreign Private Innovation</th>
<th>Domestic Public Innovation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A</td>
<td>Panel B</td>
</tr>
<tr>
<td></td>
<td>( \theta^{N0}</td>
<td>_{\phi=0} )</td>
</tr>
<tr>
<td>( a )</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>6.1</td>
<td>0.1214</td>
<td>0.1070</td>
</tr>
<tr>
<td>6.2</td>
<td>0.1189</td>
<td>0.1048</td>
</tr>
<tr>
<td>6.3</td>
<td>0.1164</td>
<td>0.1027</td>
</tr>
<tr>
<td>6.4</td>
<td>0.1141</td>
<td>0.1007</td>
</tr>
<tr>
<td>6.5</td>
<td>0.1118</td>
<td>0.0987</td>
</tr>
<tr>
<td>6.6</td>
<td>0.1097</td>
<td>0.0968</td>
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<tr>
<td>6.7</td>
<td>0.1076</td>
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</tr>
<tr>
<td>6.9</td>
<td>0.1037</td>
<td>0.0916</td>
</tr>
<tr>
<td>7.0</td>
<td>0.1018</td>
<td>0.0900</td>
</tr>
</tbody>
</table>

Note: Panel A (B) verifies lemma 2(5) and proposition 1 (2). Thus the domain of market size \( a \) follows the condition; that is, \( 4c_0 - 3c_1 - \varepsilon < a < (23c_0 - 20c_1 - 13\varepsilon)/3 \) where \( 4c_0 - 3c_1 - \varepsilon = 5.1 \) and \( (23c_0 - 20c_1 - 13\varepsilon)/3 = 7.8 \) under the specification of \( c_0 = 2, c_1 = 0.8, \) and \( \varepsilon = 0.5 \).

contrary, under domestic public innovation (Panel B in Table 1), the cost asymmetry between the domestic public and foreign private firm is enlarged by either innovation spillover or domestic public licensing, which further deteriorates the relative production inefficiency of the public firm compared to the foreign firm. Hence, the privatization level increases due to the worsening of the production inefficiency of the domestic public firm induced by either spillover or licensing under domestic public innovation.

6 Concluding Remarks

In this paper, we investigated the following research questions. Does innovation spillover and licensing under foreign private (domestic public) innovation play a role in determining the optimal degree of ex-post privatization of the domestic firm in an international duopoly market? How do they interact in determining privatization policies? We find the answers to be positive and interesting. We introduced a licensing-then-privatization model and
assumed that the local government does not commit to a certain degree of privatization. We demonstrated that the spillover and licensing due to foreign private (domestic public) innovation reduces (increases) the degree of ex-post privatization in terms of improvements (deterioration) in the relative production inefficiency of domestic public firms compared to their foreign rivals. In addition, innovation spillover and licensing have the same impact direction on privatization policies. However, the effect of innovation spillover is weaker than that of licensing on the degree of privatization if the innovation is not perfectly easily imitated.

This study provides insights into mixed ownership reform with respect to SOEs in non-strategic industries in China given that many sectors in the Chinese market face technology innovation decisions. However, our model and the related results are subject to a number of shortcomings. Constant returns to scale, linear preferences, and the consideration of a duopoly market without entry are some important limitations. However, we consider our results to be suggestive, and similar results would hold under broader contexts at the cost of a substantial increase in the analysis complexities.
Appendix

Proof of Lemma 1
To keep $\theta^{N0}$ in (4) valid, the equilibrium output from both firm 0 and firm 1 will be positive; that is,

$$q_{N0^*}^0 = \frac{3a - 4c_0 + c_1 - \varepsilon + 4\phi\varepsilon}{3}$$
$$q_{N0^*}^1 = \frac{2c_0 - 2c_1 + 2\varepsilon - 2\phi\varepsilon}{3} > 0$$

Obviously, $q_{N0^*}^1$ is greater than zero, and $q_{N0^*}^0 > 0$ if $a > (4c_0 - c_1 + \varepsilon - 4\phi\varepsilon)/3$.

Proof of Lemma 3
First, to keep $\theta^{L0}$ in (12) valid, the equilibrium output from both firm 0 and firm 1 should be positive; that is,

$$q_{L0^*}^0 = \frac{3a - 4c_0 + c_1 + 3\varepsilon}{3},$$
$$q_{L0^*}^1 = \frac{2(c_0 - c_1)}{3} > 0$$

Obviously, $q_{L0^*}^1$ is greater than zero, and $q_{L0^*}^0 > 0$ if $a > (4c_0 - c_1 - 3\varepsilon)/3$.

Second, the maximum license fee a foreign firm can charge is,

$$f^{L0} = 
\begin{bmatrix}
18(2a - c_0 - c_1 + 2\varepsilon)^2 \\
\varepsilon \cdot (1 - \phi)
\end{bmatrix}.
$$

Second, to keep the validity of the optimal degree of ex-post privatization in (12), the motivation of foreign private licensing will exist. That is, $\pi_1^{L0} - \pi_1^{N0}$ shall be greater than
zero, meaning that the foreign licensor is not worse off after fixed fee licensing; that is,
\[ \Delta \pi_1 = \pi_1^{L0} - \pi_1^{N0} = \frac{\varepsilon(1 - \phi)}{18(2a - c_0 - c_1 + 2\varepsilon)^2} \cdot \Omega > 0, \tag{20} \]
where
\[
\Omega = 192ac_0^2 - 244a^2c_0 - 80ac_1^2 + 28a^2c_1 + 8c_0c_1^2 - 60c_0c_1 + 62a\varepsilon^2 + 139a^2\varepsilon - 161c_0\varepsilon^2 + 170c_1^2\varepsilon + 99c_1\varepsilon^2 - 96c_1^2\varepsilon + 77\phi\varepsilon^3 + 72a^3 - 44c_0^3 + 24c_1^3 - 5\varepsilon^3 + 154a\phi\varepsilon^2 + 77a^2\phi\varepsilon - 83c_0\phi\varepsilon^2 + 22c_1\phi\varepsilon
- 71c_1\phi\varepsilon^2 + 16c_1^2\phi\varepsilon + 104ac_0c_1 - 405ac_0\varepsilon + 127ac_1\varepsilon + 65c_0c_1\varepsilon - 83ac_0\phi\varepsilon - 71ac_1\phi\varepsilon + 39c_0c_1\phi\varepsilon
\]
Accordingly, the occurrence of foreign private licensing requires \( \Omega > 0 \). With this assumption, the optimal degree of \textit{ex-post} privatization under foreign private licensing is valid.

\textbf{Proof of Propositions 1}

The optimal degrees of privatization with or without spillover effect from (4) are as follows,
\[
\theta^{N0}|_{\text{spillover}} = \frac{c_0 - c_1 + \varepsilon - \phi\varepsilon}{3a - 2c_0 - c_1 + \varepsilon + 2\phi\varepsilon}, \quad \text{if} \quad 0 \leq \phi < 1
\]
\[
\theta^{N0}|_{\text{no spillover}} = \frac{c_0 - c_1 + \varepsilon}{3a - 2c_0 - c_1 + \varepsilon}, \quad \text{if} \quad \phi = 0
\]
Obviously, \( \theta^{N0}|_{\text{spillover}} < \theta^{N0}|_{\text{no spillover}} \). Proposition 1 (i) is proved. In addition, the comparison between \( \theta^{N0} \) in (4) and \( \theta^{L0} \) in (12) generates
\[
\theta^{L0} - \theta^{N0} = \frac{3\varepsilon(\phi - 1)(a - c_1 + \varepsilon)}{(3a - 2c_0 - c_1 + 3\varepsilon)(3a - 2c_0 - c_1 + \varepsilon + 2\phi\varepsilon)} < 0, \quad \text{given} \quad 0 \leq \phi < 1,
\]
when \( a > (4c_0 - c_1 - 3\varepsilon)/3 \). Therefore, we obtain \( \theta^{L0} < \theta^{N0}|_{\text{spillover}} < \theta^{N0}|_{\text{no spillover}} \). Proposition 1 (ii) is proved. Third, \( \theta^{N0}|_{\text{spillover}} = \theta^{L0} \) if \( \phi = 1 \). Proposition 1 (iii) is proved.

\textbf{Proof of Lemma 4}

To keep \( \theta^{N1} \) in (16) valid, the equilibrium output from both firm 0 and firm 1 must be positive; that is,
\[
q_0^{N1*} = \frac{3a - 4c_0 + c_1 + 4\varepsilon - \phi\varepsilon}{3} > 0
\]
\[
q_1^{N1*} = \frac{2(c_0 - c_1 - \varepsilon + \phi\varepsilon)}{3} > 0
\]
Obviously, \( q_{1}^{N1^*} \) is greater than zero given \( c_0 - c_1 > \varepsilon \). Additionally, \( q_{0}^{N1^*} > 0 \) if \( a > (4c_0 - c_1 - 4\varepsilon + \phi\varepsilon)/3 \).

**Proof of Lemma 6**

First, to keep \( \theta^{L1} \) in (18) valid, the equilibrium output from both firm 0 and firm 1 must be positive; that is,

\[
q_{0}^{L1^*} = \frac{3a - (4c_0 - c_1 - 3\varepsilon)}{3}, \\
q_{1}^{L1^*} = \frac{2(c_0 - c_1)}{3} > 0.
\]

Obviously, \( q_{1}^{L1^*} \) is greater than zero. Additionally, \( q_{0}^{L1^*} > 0 \) if \( a > (4c_0 - c_1 - 3\varepsilon)/3 \).

Second, to keep the validity of the optimal degree of ex-post privatization in (18), the motivation for domestic public licensing must exist. That is, \( U_{0}^{L1} - U_{0}^{N1} \) must be greater than zero, meaning the domestic licensor is not worse off after fixed fee licensing. That is,

\[
\Delta U_0 = U_{0}^{L1} - U_{0}^{N1} = -\frac{\varepsilon(1 - \phi)(3a - 23c_0 + 20c_1 + 13\varepsilon - 10\phi\varepsilon)}{18} > 0. \tag{21}
\]

The positivity of \( \Delta U_0 \) in (21) requires \( a < (23c_0 - 20c_1 - 13\varepsilon + 10\phi\varepsilon)/3 \). Thus, \( \theta^{L1} \) in (18) is valid if \( (4c_0 - c_1 - 3\varepsilon)/3 < a < (23c_0 - 20c_1 - 13\varepsilon)/3 \).

**Proof of Proposition 2**

The optimal degrees of privatization with or without spillover effect from (16) are as follows,

\[
\theta^{N1}\bigg|_{\text{spillover}} = \frac{c_0 - c_1 - \varepsilon + \phi\varepsilon}{3a - 2c_0 - c_1 + 2\varepsilon + \phi\varepsilon}, \quad \text{if} \quad 0 \leq \phi < 1
\]

\[
\theta^{N1}\bigg|_{\text{no spillover}} = \frac{c_0 - c_1 - \varepsilon}{3a - 2c_0 - c_1 + 2\varepsilon}, \quad \text{if} \quad \phi = 0
\]

where

\[
\theta^{N1}\bigg|_{\text{spillover}} - \theta^{N1}\bigg|_{\text{no spillover}} = \frac{3\phi\varepsilon(a - c_0 + \varepsilon)}{(3a - 2c_0 - c_1 + 2\varepsilon) \cdot (3a - 2c_0 - c_1 + 2\varepsilon + \phi\varepsilon)} > 0
\]
Thus, we have $\theta_{N1}\big|_{\text{spillover}} > \theta_{N1}\big|_{\text{no spillover}}$. Proposition 2 (i) is proved. In addition, the comparison between $\theta_{N1}$ in (16) and $\theta_{L1}$ in (18) generates

$$\theta_{L1} - \theta_{N1} = \frac{3\varepsilon(1 - \phi)(a - c_0 + \varepsilon)}{(3a - 2c_0 - c_1 + 3\varepsilon) \cdot (3a - 2c_0 - c_1 + 2\varepsilon + \phi\varepsilon)} > 0, \quad \text{given } 0 \leq \phi < 1,$$

when $(4c_0 - c_1 - 3\varepsilon)/3 < a < (23c_0 - 20c_1 - 13\varepsilon)/3$. We also obtain $\theta_{L1} > \theta_{N1}\big|_{\text{spillover}} > \theta_{N1}\big|_{\text{no spillover}}$. Proposition 2 (ii) is proved. Third, $\theta_{N1}\big|_{\text{spillover}} = \theta_{L1}$ if $\phi = 1$. Proposition 2 (iii) is proved.
References


