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ABSTRACT

The monetary policy framework of many countries has been developed under an Inflation Targeting Framework, which is a fixed central bank interest rate. The well-known Taylor's Rule is the rule of monetary policy applied in empirical evidence for the mode of transmission mechanisms of the Central Bank. Microfoundations in Log-linear terms are consistent in line with Kranz (2015), however countries such as: China, Nigeria, Bolivia, Yemen, Suriname, among others, are in a different framework, control of the money supply (the IMF defines as Monetary Objective Aggregate). The MacCallum's Rule proposed in the 1980s would be more appropriate to describe the transmission mechanisms of monetary policy in this type of policy. But in the present investigation it is based on a monetary policy rule different from the conventional ones. Thanks to the contribution of William Poole in 1970, our Policy Rule explains that the money supply reacts to the behavior of five (5) variables: product gap, interest rate gap, observed interest rate, product expectations and inflation; for what we call this instrument the Poole's Rule. Through a Dynamic Stochastic General Equilibrium Model (DSGE) we check if said rule is appropriate for economies under a different Inflation Targeting Framework.

JEL Classification: E51, E60, E61

Palabras Clave: Poole's Rule, Taylor's Rule, MacCallum's Rule, Dynamic Stochastic General Equilibrium Model (DSGE), Bayesian Estimation.

[1] This document expresses the exclusive point of view of the authors and not of the institutions to which they belong.
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Introduction

The implementation of famous Taylor's Rule for modeling monetary policy is a current consensus in many Central Banks. The monetary authority that fixes interest rate scheme (Inflation Targeting Framework) to generate price level stability and control fluctuations in the product gap. However, there are countries that are classified in different ways, Monetary Aggregate Target, according to the International Monetary Fund, the modeling of this scheme in many investigations the execution through the MacCallum´s Rule, but such instrument does not result in the feasibility of characterizing stylized facts in the transmission mechanisms.

In this paper the foundations of an unconventional monetary policy rule are developed, Poole´s Rule. This proposal was designed by William Poole in 1970, later, many investigations until the late 80´s checking the position of the author, Turnovsky (1975), Woglom (1979), Yoshikawa (1981), Cazoneri et al. (1983), Daniel (1986) and Fair (1987) test the effectiveness of this rule, at that time they call it "A combination, between control of the stock of money and fixing of rates". The predominant role of estimating the parameters of that rule determines its validness. The equation found postulates that the monetary authority must fix the money stock (money supply) based on five key variables: product gap, interest rate gap, observed interest rate, expectations of product and inflation. To validate its effectiveness, a Dynamic Stochastic General Equilibrium Model (DSGE) was built for a small and closed economy. The results are promising, because the exercise performed captures stylized facts of an economy under a money supply control scheme and the parameters estimation were relevant to confirm the evolution of the Poole´s Rule; an expansive monetary policy (money supply shocks) has positive effects on the real sector, in addition to controlling inflationary pressures, through an indirect effect (interest rate).

On the other hand, the weighting of the loss function of a Central Bank prevails in the construction of the model and a higher value of parameter allow to monetary authority can further stimulate economic growth, control inflationary pressures from idiosyncratic shocks of the New Phillips Keynesian curve and stabilize household expectations.

The paper is organized as follows: I) Literature Review, II) Microfundaments of a Monetary Policy Rule, Control of the Money Supply, III) A simple exercise and IV) Conclusions.

I) Literature Review

Between the 60's and the late 80's, there was a debate in the academy about the use of the optimal instrument of the monetary authority, the setting of the interest rate or the control of the money supply. The mainstream research at that time was by William Poole (1970), who developed a model from the perspective of the well-known IS-LM model in a stochastic context. The investigation covers the "target problem[2]", if the monetary authority can operate through changes in the interest rate or changes in the money supply (the author defines it as a stock of money), therefore, the monetary authority must choose only one policy instrument. Depending on value of the model parameters, Poole indicates that one instrument is superior to another or vice versa, in the section IV of his investigation the proposal of a combination of both instruments (interest rate setting and control of the stock of money), in this context, the evaluation of the parameters would not be worthwhile.

The objective function that assumes for the minimum loss of the desired level of the product is quadratic, that is, the variation of the product with respect to the natural level[3]. The empirical evidence of Poole's position is done by Stephen Turnovsky (1975), confirming the position in relation to the parameters, the value of the same helps the monetary authority to choose one instrument over the other, stating that under uncertainty, the offer Optimal monetary is pro-cyclical to the money stock. When the money supply affects real expenses indirectly through the interest rate, the dominance of the instrument in rates is appropriate.

In 1981, similarly Hiroshi Yoshikawa studies the decision of the monetary authority to choose an optimal instrument, control of the money supply, a primary result refers to elasticity of the money demand and the influence on stability of the dynamic stochastic equilibrium model, the value that assume with respect to the interest rate. Yoshikawa points out that under uncertainty the objective of monetary policy is to adapt to shocks, changing the growth rate of money and to make the variance of the interest rate independent of the elasticity of money demand. Under this premise, the instrumental instability of money supply variance is possible, while its average must converge to some constant rate.

From another point of view, Ray Fair (1987) asks the following question in relation to the Poole´s model: "Are the variances, covariances, and parameters in the model such as to favor one instrument over the other, in particular the interest rate over the money supply? The answer (results), reveals that both instruments are optimal in terms of reducing the variance of the Gross National Product, although the Federal Reserve prefers the use of the interest rate as an instrument.

Then Bennett MacCallum in 1984, proposes a monetary policy rule under the scoop that if there is a constant growth of the stock of money, good macroeconomic performance is expected, being able to improve the results with the extension of a rule that adjusts the intervals of the stock monetary according to the fluctuations of GDP to reach a desired path of this variable (this target is non-inflationary), this instrument (rule) is active and not discretionary. MacCallum in this investigation and subsequent lately in 1987, 1988, 1993, 1999, among others, uses the monetary

[2] The author makes a discussion about the terms "target" or "goal." In other words, economic policy must make adjustments to the instruments to influence the "target" or "goal" variables. It also considers intermediate or upcoming instruments such as the discount rate, open market operations, reserve requirements, among others. In his paper points out that the money stock can be set exactly at the desired level, so the money stock can also be called a monetary policy instrument instead of a near target.

[3] In Poole´s paper indicates that this function is set out in the book "Optimal Decision Rules for Government and Industry" by Henry Theil.

aggregates M1 or M2 as a proxy for the money stock. Specifically in 1993, the application of this rule is carried out for the Japanese economy, through a model of Autoregressive Vectors (VAR) with Keynesian characteristics, showed that using this non-discretionary instrument, GDP can be kept close to its target.

Betty Daniel (1986) recalling Poole (1970), analyzes whether the monetary authority should use a specific instrument, interest rate or money supply; in other words, argues whether the Central Bank when making use of any instrument; the interest rate, some monetary aggregate or a combination of both is appropriate to stabilize the product in relation to natural level, she proposes that the monetary policy rules should allow temporary deviations from the long-term money supply path to compensate prognosis errors of interest rate. Regardless of the combination of the money supply and the objective of the interest rate in "t", the money supply is expected to return to its pre-established growth trajectory for the next period "$t + 1$". Confirming the Poole theory in presence of shocks by the LM curve, the stabilization of the real interest rate is the best instrument for the inflation forecast, however if the shocks come from the aggregate supply, set an interest rate to stabilize the product around the target is not optimal. Finally, concludes by demonstrating that if the monetary authority is not aware of the source of the shocks, a rate rule will not be a product stabilizing instrument.

Thanks to John Taylor (1993) and his mainstream article in relation to the discretion or use of a monetary policy rule, the transmission mechanism that many investigations use nowadays is the interest rate as a variable that stabilizes the product with respect to its target and reacts to the market inflationary pressures.

II) Microfundaments of a Monetary Policy Rule, Control of the Money Supply

In the previous section, we demonstrate the duality about the use of a monetary policy instrument, the setting of the interest rate (Taylor´s Rule) or the control of the money supply (MacCallum´s Rule). Under the current DSGE precept, we intend to provide microfundaments with a slightly different monetary policy rule than those known, in line with Kranz (2015) the variables will be expressed in Log-linear version.

The typical way to find an optimal Taylor Rule is the minimization of a quadratic loss function, this function proposed by Henri Theil in 1964. Poole (1970) adopts this function as: $L = E\left(Y - Y_f\right)^2$ where "$Y - Y_f$", are the deviations of the product from the desired (natural). This formulates that this type of function is not necessarily exclusive to determine an optimal monetary policy rule in rates.

From the New Keynesian perspective with rigidities of prices à la Calvo, the rule of a Central Bank will be derived from minimizing the function of discounted loss in all periods.

$$\underset{\tilde{\pi},\ \tilde{x}}{\text{Min}}\ E_t\left\{\sum_{t=0}^{\infty} \Omega^t[(\tilde{\pi}_t - \tilde{\pi}^*)^2 + \Theta\tilde{x}_t^2]\right\} \tag{1}$$

Where, $\tilde{x}_t$ is the product gap, $\tilde{\pi}_t$ is the observed inflation and $\tilde{\pi}^*$is the target inflation. We assume that $\tilde{\pi}^* = 0$, because the essence of obtaining the monetary policy rule does not change. On the other hand, the parameter Θ is weighting factor and Ω^t is the subjective discount rate of the monetary authority. The restrictions to this minimization problem will be the New Keynesian Phillips Curve (NKPC), the IS equation and the Microfounded Money Demand, all expressed around their steady state (Log-linear).

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\widetilde{mc}_t \tag{2}$$

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \frac{1}{\sigma}(\tilde{\imath}_t - E_t\tilde{\pi}_{t+1}) \tag{3}$$

$$\tilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{C}_t - \frac{\beta}{\sigma^M}\tilde{\imath}_t \tag{4}$$

It should be noted that the marginal cost "$\widetilde{mc}_t$" is an approximation of the product gap, in a model with two (2) factors of capital production (K_t) and labor (N_t), this variable is determined by:

$$\widetilde{mc}_t = \frac{(\sigma + \eta)(1 - \alpha)}{1 + \eta\alpha}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{\alpha(1 + \eta)}{1 + \eta\alpha}\left[\tilde{Z}_t - \tilde{Z}_t^f\right] \tag{5}$$

Where "$\tilde{Y}_t - \tilde{Y}_t^f$" is the product gap ($\tilde{x}_t$), in line with Poole (1970) this expresses the deviations of the product from the desired (natural). The expression "$\tilde{Z}_t - \tilde{Z}_t^f$", is the gap in the marginal productivity of capital in relation to the natural one, if this is so, there is doubt about the relationship of the interest rate with this variable. Metzler (1950) indicates that the marginal productivity of capital "Z_t" will not necessarily be equal to the interest rate due to:

$$PMgK \equiv Z_t = \frac{Value\ of\ increase\ in\ lumber\ output}{Increase\ in\ capital}$$

$$i_t = \frac{Value\ of\ increase\ in\ lumber\ output}{Capital\ use\ in\ extending\ period\ of\ production}$$

This means that the price of capital will be higher than interest rate $Z_t > i_t$[4], in the Real Bussines Cycle Models (RBC) which assume price flexibility in steady state the price of capital is: $Z_{ss} = \frac{1}{\beta} - (1-\delta)$, similarly the steady-state interest rate from the point of view of a DSGE with new Keynesian characteristics is: $i_{ss} = \frac{1}{\beta} - 1$, so we can express that "$Z_{ss} = i_{ss} + \delta$".

So, we get $Z_t = i_t - \delta$, log-linearizing version:

$$Z_{ss}\left(1 + \tilde{Z}_t\right) = i_{ss}(1 + \tilde{\imath}_t) - \delta$$

$$Z_{ss} + Z_{ss}\tilde{Z}_t = i_{ss} + i_{ss}\tilde{\imath}_t - \delta$$

$$Z_{ss}\tilde{Z}_t = i_{ss}\tilde{\imath}_t$$

$$\left[\frac{1}{\beta} - (1-\delta)\right]\tilde{Z}_t = \left[\frac{1}{\beta} - 1\right]\tilde{\imath}_t$$

$$\tilde{Z}_t = \frac{\left[\frac{1}{\beta} - 1\right]\tilde{\imath}_t}{\left[\frac{1}{\beta} - (1-\delta)\right]} = \frac{\left[\frac{1-\beta}{\beta}\right]\tilde{\imath}_t}{\left[\frac{1-\beta+\beta\delta}{\beta}\right]} = \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t$$

$$\tilde{Z}_t = \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t \tag{6}$$

Similarly, the price or marginal productivity of natural capital is defined as:

$$\tilde{Z}_t^f = \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t^f \tag{7}$$

Where "$\tilde{\imath}_t^f$" is the natural interest rate, concept introduced by Knut Wicksell (1898) in his seminal work "Interest and Prices", Michael Woodford indicates that this variable "natural interest rate" guarantees equilibrium when wages and prices are flexible, given the current production factors. In this sense, Woodford points out ... "In Wicksell's view, price stability depended on keeping the interest rate controlled by the central bank in line with the natural rate determined by real factors (such as the marginal product of capital)". In other words, nominal rates must be controlled so that they fluctuate around the natural to maintain stable inflation and a product gap very low volatile[5].

Replacing the expressions (6) and (7) in (5):

$$\widetilde{mc}_t = \frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{\alpha(1+\eta)}{1+\eta\alpha}\left\{\left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t - \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t^f\right\} \tag{8}$$

(8) in (2)

[4] In both cases the numerator is the same, however the denominator of the price of capital is slightly lower than the interest rate.

[5] In chapter 4 (A Neo-Wicksellian Framework) Michael Woodford´s book "Interest & Prices", the expression (1.15) corresponding to the percentage deviation of the natural interest rate with respect to its steady state is observed.

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\left\{\frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{\alpha(1+\eta)}{1+\eta\alpha}\left\{\left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t - \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t^f\right\}\right\}$$

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\left\{\frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha}[\tilde{x}_t] + \frac{\alpha(1+\eta)}{1+\eta\alpha}\left\{\left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t - \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t^f\right\}\right\}$$

For simplicity the following expressions will be defined as:

$$\frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha} = \varphi$$

$$\frac{\alpha(1+\eta)}{1+\eta\alpha} = \gamma$$

$$\frac{1-\beta}{1-\beta(1-\delta)} = \varpi$$

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\left\{\varphi\tilde{x}_t + \gamma\varpi\tilde{\imath}_t - \gamma\varpi\tilde{\imath}_t^f\right\} \tag{9}$$

Redefining the microfounded IS curve[6] and changing the expression of the Money Demand microfounded by $\tilde{C}_t \cong \tilde{Y}_t$:

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \frac{1}{\sigma}(\tilde{\imath}_t - E_t\tilde{\pi}_{t+1})$$

$$\tilde{x}_t + \tilde{Y}_t^f = E_t\tilde{x}_{t+1} + E_t\tilde{Y}_{t+1}^f - \frac{1}{\sigma}\tilde{\imath}_t + \frac{1}{\sigma}E_t\tilde{\pi}_{t+1} \tag{10}$$

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{Y}_t - \frac{\beta}{\sigma^M}\tilde{\imath}_t$$

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\left(\tilde{x}_t + \tilde{Y}_t^f\right) - \frac{\beta}{\sigma^M}\tilde{\imath}_t \tag{11}$$

The restrictions of Central Bank are (9), (10) and (11). The Lagrangian problem for the monetary authority will be:

$$\mathcal{L} = E_t\sum_{t=0}^{\infty}\Omega^t\left\{\begin{matrix}\tilde{\pi}_t^2 + \Theta\tilde{x}_t^2 - \chi_t\left[\tilde{x}_t + \tilde{Y}_t^f - E_t\tilde{x}_{t+1} - E_t\tilde{Y}_{t+1}^f + \frac{1}{\sigma}\tilde{\imath}_t - \frac{1}{\sigma}E_t\tilde{\pi}_{t+1}\right] \\ -\Phi_t\left[\tilde{\pi}_t - \beta E_t\tilde{\pi}_{t+1} - \kappa\varphi\tilde{x}_t - \kappa\gamma\varpi\tilde{\imath}_t + \kappa\gamma\varpi\tilde{\imath}_t^f\right] - \psi_t\left[\widetilde{m}_t - \frac{\sigma}{\sigma^M}\left(\tilde{x}_t + \tilde{Y}_t^f\right) + \frac{\beta}{\sigma^M}\tilde{\imath}_t\right]\end{matrix}\right\}$$

The first order conditions:

$$\frac{\partial\mathcal{L}}{\partial\tilde{\pi}_t} \qquad 2\tilde{\pi}_t - \Phi_t = 0 \tag{i}$$

$$\frac{\partial\mathcal{L}}{\partial\tilde{x}_t} \qquad 2\Theta\tilde{x}_t - \chi_t + \Phi_t\kappa\varphi + \psi_t\frac{\sigma}{\sigma^M} = 0 \tag{ii}$$

$$\frac{\partial\mathcal{L}}{\partial\tilde{\imath}_t} \qquad -\frac{\chi_t}{\sigma} + \Phi_t\kappa\gamma\varpi - \psi_t\frac{\beta}{\sigma^M} = 0 \tag{iii}$$

$$\frac{\partial\mathcal{L}}{\partial\widetilde{m}_t} \qquad -\psi_t = 0 \tag{iv}$$

Condition (iv) is equal to zero because the minimized loss will not change if the microfounded IS curve shifts. As the mechanism of the Central Bank can counteract this movement by restoring the interest rate through changes of $\widetilde{m}_t$, if we combine (i), (ii) and (iii) we obtain:

[6] The output gap is $\tilde{x}_t = \tilde{Y}_t - \tilde{Y}_t^f$

$$\Phi_t \kappa\gamma\varpi\sigma = \chi_t$$

$$2\Theta\tilde{x}_t - \Phi_t\kappa\gamma\varpi\sigma + \Phi_t\kappa\varphi = 0 \quad \rightarrow \quad 2\Theta\tilde{x}_t + \Phi_t\kappa(\varphi - \gamma\varpi\sigma) = 0$$

$$-\frac{2\Theta}{\kappa(\varphi - \gamma\varpi\sigma)}\tilde{x}_t = \Phi_t \tag{v}$$

(v) in (i):

$$2\tilde{\pi}_t = -\frac{2\Theta}{\kappa(\varphi - \gamma\varpi\sigma)}\tilde{x}_t$$

$$\tilde{\pi}_t = -\frac{\Theta}{\kappa(\varphi - \gamma\varpi\sigma)}\tilde{x}_t \quad or \quad \tilde{x}_t = -\frac{\tilde{\pi}_t[\kappa(\varphi - \gamma\varpi\sigma)]}{\Theta} \tag{vi}$$

We redefine the expression $\kappa(\varphi - \gamma\varpi\sigma) = \varrho$, obtain in the Phillips curve:

$$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\{\varphi\tilde{x}_t + \gamma\varpi\tilde{\imath}_t - \gamma\varpi\tilde{\imath}_t^f\}$$

$$-\frac{\Theta\tilde{x}_t}{\varrho} = \beta E_t\tilde{\pi}_{t+1} + \kappa\varphi\tilde{x}_t + \kappa\gamma\varpi\tilde{\imath}_t - \kappa\gamma\varpi\tilde{\imath}_t^f$$

$$0 = \tilde{x}_t\left[\frac{\varrho\kappa\varphi + \Theta}{\varrho}\right] + \beta E_t\tilde{\pi}_{t+1} + \kappa\gamma\varpi\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right)$$

$$\tilde{x}_t = -\left[\frac{\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} - \frac{\varrho\kappa\gamma\varpi}{\varrho\kappa\varphi + \Theta}\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right) \tag{vii}$$

Rewriting the money demand equation based on the natural product.

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\left(\tilde{x}_t + \tilde{Y}_t^f\right) - \frac{\beta}{\sigma^M}\tilde{\imath}_t$$

$$\widetilde{m}_t - \frac{\sigma}{\sigma^M}\left(\tilde{x}_t + \tilde{Y}_t^f\right) + \frac{\beta}{\sigma^M}\tilde{\imath}_t = 0$$

$$\widetilde{m}_t - \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\beta}{\sigma^M}\tilde{\imath}_t = \frac{\sigma}{\sigma^M}\tilde{Y}_t^f$$

$$\tilde{Y}_t^f = \frac{\sigma^M}{\sigma}\left[\widetilde{m}_t + \frac{\beta}{\sigma^M}\tilde{\imath}_t\right] - \tilde{x}_t \tag{viii}$$

The expressions (vii) and (viii) by inserting in the microfounded IS curve we are able to obtain a monetary policy rule[7].

$$\tilde{x}_t + \tilde{Y}_t^f = E_t\tilde{x}_{t+1} + E_t\tilde{Y}_{t+1}^f - \frac{1}{\sigma}\tilde{\imath}_t + \frac{1}{\sigma}E_t\tilde{\pi}_{t+1}$$

$$-\left[\frac{\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} - \frac{\varrho\kappa\gamma\varpi}{\varrho\kappa\varphi + \Theta}\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right) + \frac{\sigma^M}{\sigma}\widetilde{m}_t + \frac{\beta}{\sigma}\tilde{\imath}_t - \tilde{x}_t = E_t\tilde{Y}_{t+1} - \frac{1}{\sigma}\tilde{\imath}_t + \frac{1}{\sigma}E_t\tilde{\pi}_{t+1}$$

$$\frac{\sigma^M}{\sigma}\widetilde{m}_t - \tilde{x}_t = E_t\tilde{Y}_{t+1} - \frac{1}{\sigma}\tilde{\imath}_t + \frac{1}{\sigma}E_t\tilde{\pi}_{t+1} - \frac{\beta}{\sigma}\tilde{\imath}_t + \left[\frac{\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\varrho\kappa\gamma\varpi}{\varrho\kappa\varphi + \Theta}\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right)$$

$$\sigma^M\widetilde{m}_t - \sigma\tilde{x}_t = \sigma E_t\tilde{Y}_{t+1} - \tilde{\imath}_t + E_t\tilde{\pi}_{t+1} - \beta\tilde{\imath}_t + \left[\frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\varrho\kappa\varphi + \Theta}\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right)$$

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}\tilde{\imath}_t(1+\beta) + \frac{\sigma}{\sigma^M}\tilde{x}_t + E_t\tilde{\pi}_{t+1}\frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right] + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}\left(\tilde{\imath}_t - \tilde{\imath}_t^f\right)$$

Defining the interest rate gap as $\tilde{x}_t^i = \tilde{\imath}_t - \tilde{\imath}_t^f$

[7] Keeping the expression of $\tilde{Y}_{t+1} = \tilde{x}_{t+1} + \tilde{Y}_{t+1}^f$.

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}\left(\tilde{x}_t^i\right) \qquad (12)$$

The expression (12) constitutes our monetary policy rule, similar to that proposed by McCallum; however, the offer for money in this case responds not only to the expectations of the GDP activity ($\tilde{Y}_t$) and inflation ($\tilde{\pi}_t$), in addition to this it reacts to the output gap ($\tilde{x}_t$), the interest rate ($\tilde{\imath}_t$) and the interest rate gap, that is, the monetary authority observes the deviations of the interest rate from the natural level ($\tilde{x}_t^i$). As mentioned earlier, Woodford points out… “In Wicksell’s view, price stability depended on **keeping the interest rate controlled by the central bank in line with the natural rate** determined by real factors (such as the marginal product of capital)”.

So, in line with Woodford for maintaining the interest rate around its natural level and based on the findings of Poole (1970)[8], this rule beyond having similarities with the McCallum´s Rule in aggregates can be defined as a Poole´s Rule in honor of Willam Poole, for his work in May 1970.

[8] In his section IV "The Combination Policy" the expression (16), shows a combination of what he defines as the interest rate of pure policy and a stock of pure policy money, assuming values of certain parameters indicates that the **combination of policies are superior** to individual instruments, interest rate fixing and money stock control. The approach is defined as: $c_0M = c_1^* + c_2^*r$. Where c_1^* and c_2^* depend at the same time on the elasticity of money demand, the natural product and other parameters of interest.

III) A simple exercise

To verify the viability of this monetary policy rule, it will be evaluated in a DSGE model with rigidity price à la Calvo for a small and closed economy. As Poole (1970), Turnovsky (1975), Yoshikawa (1981), Daniel (1986) and Fair (1987) point out, the value of the parameters determines the viability of the instrument, for this reason a Bayesian estimation of some parameters will be made.

Households

There is a continuum of households indexed by j in an economy, each one maximizes a utility function, choosing an optimal path of real consumption (C_t), labor supply (N_t) and money demand in real balances (M_t/P_t)[9].

$$\max_{C_t, N_t, B_{t+1}, M_t} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \zeta \frac{N_t^{1+\eta}}{1+\eta} + \gamma^m \frac{\left(\frac{M}{P}\right)_t^{1-\sigma^M}}{1-\sigma^M} \right] \right\}$$

Where $\beta \epsilon (0,1)$ is the subjective discount rate, σ is the risk aversion coefficient of households or the inverse of the elasticity of intertemporal substitution of consumption, η is the inverse of the elasticity of the labor supply of Frish (elasticity of work respect to real wages) and σ^M is the inverse of the elasticity of money demand respect to the interest rate. The insertion of real balances in the instant utility function is due to Sidrauski (1967), known as Money in The Utility Function (MIU).

For there to be an optimal condition in the behavior of the representative agent, $\forall t$ the constraint facing is described as:

$$P_t C_t + B_{t+1} + M_t - M_{t-1} = W_t N_t + \Pi_t + (1 + i_{t-1}) B_t$$

The Lagrangian problem to solve the representative agent is:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \zeta \frac{N_t^{1+\eta}}{1+\eta} + \gamma^m \frac{\left(\frac{M}{P}\right)_t^{1-\sigma^M}}{1-\sigma^M} \right] + \\ \lambda_t [W_t N_t + \Pi_t + (1 + i_{t-1}) B_t - P_t C_t - B_{t+1} - M_t + M_{t-1}] \end{array} \right\}$$

The first order conditions:

$$\frac{\partial \mathcal{L}}{\partial C_t} \qquad C_t^{-\sigma} = \lambda_t P_t \implies \lambda_t = \frac{C_t^{-\sigma}}{P_t};\ \forall t$$

$$\frac{\partial \mathcal{L}}{\partial N_t} \qquad \zeta N_t^{\eta} = \lambda_t W_t$$

[9] The aggregation of consumption, labor supply and demand for money in real balances, inserted in the utility function of households indexed in this economy is: $C_t = \left(\int_0^1 C_{t,j}^{\frac{\varepsilon^C - 1}{\varepsilon^C}} dj \right)^{\frac{\varepsilon^C}{\varepsilon^C - 1}}$; $N_t = \left(\int_0^1 N_{t,j}^{\frac{\varepsilon^N - 1}{\varepsilon^N}} dj \right)^{\frac{\varepsilon^N}{\varepsilon^N - 1}}$ and $(M/P)_t = \left(\int_0^1 (M/P)_{t,j}^{\frac{\varepsilon^{(M/P)} - 1}{\varepsilon^{(M/P)}}} dj \right)^{\frac{\varepsilon^{(M/P)}}{\varepsilon^{(M/P)} - 1}}$, respectively; ε^C,ε^Ny $\varepsilon^{(M/P)}$ they are elasticities of substitution: of the set of the household consumption basket, among all the different jobs in the labor market and of the preference of the real balances.

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} \qquad -\lambda_t + \beta E_t \lambda_{t+1}(1+i_t) = 0 \quad \Longrightarrow \quad \lambda_t = \beta E_t \lambda_{t+1}(1+i_t)$$

$$\frac{\partial \mathcal{L}}{\partial M_t} \qquad \gamma^m M_t^{-\sigma^M} \left(\frac{1}{P}\right)_t^{1-\sigma^M} - \lambda_t + \beta E_t \lambda_{t+1} = 0 \qquad \Longrightarrow \gamma^m \left(\frac{M}{P}\right)_t^{-\sigma^M} \frac{1}{P_t} = \lambda_t - \beta E_t \lambda_{t+1}$$

Reducing the previous expressions we get:

$$\zeta N_t^\eta = \frac{W_t}{P_t} C_t^{-\sigma}$$

$$\zeta N_t^\eta = w_t C_t^{-\sigma} \tag{13}$$

We define $w_t = {}^{W_t}/_{P_t}$, as the real salary. To obtain the Euler equation we substitute λ_t in the derivative with respect to financial assets.

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} (1+i_t) \frac{P_t}{P_{t+1}}$$

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{(1+i_t)}{(1+\pi_{t+1})} \tag{14}$$

The expression $\frac{(1+i_t)}{(1+\pi_{t+1})}$, converges to "$(1+R_t)$", known as the Fisher equation, where R_t is the real interest rate; on the other hand, the Money Demand with microfundaments is obtained by the substitution of λ_t and equality $\frac{C_t^{-\sigma}}{(1+i_t)} = \beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}$.

$$\gamma^m \left(\frac{M}{P}\right)_t^{-\sigma^M} \frac{1}{P_t} = \frac{C_t^{-\sigma}}{P_t} - \beta E_t \frac{C_{t+1}^{-\sigma}}{P_{t+1}}$$

$$\gamma^m \left(\frac{M}{P}\right)_t^{-\sigma^M} = C_t^{-\sigma} - \beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}$$

$$\gamma^m \left(\frac{M}{P}\right)_t^{-\sigma^M} = C_t^{-\sigma} - \frac{C_t^{-\sigma}}{(1+i_t)}$$

$$\gamma^m \left(\frac{M}{P}\right)_t^{-\sigma^M} = C_t^{-\sigma} \frac{i_t}{(1+i_t)}$$

$$\gamma^m C_t^{\sigma} \frac{(1+i_t)}{i_t} = \left(\frac{M}{P}\right)_t^{\sigma^M}$$

$$m_t^{\sigma^M} = \gamma^m C_t^{\sigma} \frac{(1+i_t)}{i_t} \tag{15}$$

Where $m_t = \left(\frac{M}{P}\right)_t$ is real money balances. The sequence of budget constraints $\Sigma_{t=0}^{\infty}$ satisfies the transversality condition $\lim_{t\to\infty} \beta^t \lambda_t B_{t+1} = 0$ when $B_{t+1} > 0$.

Intermediate Producers

An intermediate producing firm of goods with certain market power is assumed to set prices[10]. This firm takes as prices the factors of production and from this determines the optimal capital and labor for the minimization of costs.

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[10] Monopolistic Competition is a market with many firms that produce in a similar way, but the products are heterogeneous and when new firms signal the entrance to the market, this causes a variety in differentiation both in intrinsic quality of the products, the location of the signatures and the provision of Services to other industries.

$$\underset{\{N_t(j),K_t(j)\}}{\text{Min}} \; W_t N_{t,j} + Z_t K_{t,j}$$

The restriction for each period is described by a Cobb-Douglas production function $Y_{t,j} = A_t K_{t,j}^{\alpha} N_{t,j}^{1-\alpha}$. $Y_{t,j}$, is GDP, $K_{t,j}^{\alpha}$, stock of capital, $N_{t,j}^{1-\alpha}$ labor demand and $A_t$is the Total Factor Productivity (TFP). The problem of minimizing costs to be solved by these firms is[11]:

$$\mathcal{L} = W_t N_{t,j} + Z_t K_{t,j} + \Xi_{t,j}\left(Y_{t,j} - A_t K_{t,j}^{\alpha} N_{t,j}^{1-\alpha}\right)$$

Where, $\frac{\Xi_{j,t}}{P_t} = mc_{j,t}$, is the Real Marginal Cost. The first order conditions:

$$\frac{\partial \mathcal{L}}{\partial N_{t,j}} \qquad W_t - (1-\alpha)\, \Xi_{t,j}\, A_t K_{t,j}^{\alpha} N_{t,j}^{-\alpha} = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t,j}} \qquad Z_t - \alpha\, \Xi_{t,j}\, A_t K_{t,j}^{\alpha-1} N_{t,j}^{1-\alpha} = 0$$

In real wages terms (marginal productivity of labor) and the price of capital (marginal productivity of capital), operating we obtain:

$$w_t = (1-\alpha)\, mc_{j,t}\, \frac{A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}}{N_{j,t}}$$

$$N_t = (1-\alpha)\, mc_t\, \frac{Y_t}{w_t} \tag{16}$$

$$Z_t = \alpha\, mc_{j,t}\, \frac{A_t K_{j,t}^{\alpha} N_{j,t}^{1-\alpha}}{K_{j,t}}$$

$$K_t = \alpha\, mc_t\, \frac{Y_t}{Z_t} \tag{17}$$

(15) and (16) in the production function.

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

$$Y_t = A_t \left[\alpha\, mc_{j,t}\, \frac{Y_t}{Z_t}\right]^{\alpha} \left[(1-\alpha)\, mc_t\, \frac{Y_t}{w_t}\right]^{1-\alpha}$$

$$Y_t = A_t \alpha^{\alpha} mc_t^{\alpha}\, \frac{\mathrm{Y}_t^{\alpha}}{\mathrm{Z}_t^{\alpha}} (1-\alpha)^{\alpha} mc_t^{1-\alpha}\, \frac{\mathrm{Y}_t^{1-\alpha}}{w_t^{1-\alpha}}$$

$$1 = A_t \alpha^{\alpha} mc_t\, \frac{1}{\mathrm{Z}_t^{\alpha}} (1-\alpha)^{\alpha}\, \frac{1}{w_t^{1-\alpha}}$$

$$\frac{1}{mc_t} = A_t \alpha^{\alpha}\, \frac{1}{\mathrm{Z}_t^{\alpha}} (1-\alpha)^{\alpha}\, \frac{1}{w_t^{1-\alpha}}$$

[11] The variety of existing firms in the economic one implies an indexation, therefore the aggregate form is: $Y_t = \left(\int_0^1 Y_{t,j}^{\frac{\varepsilon^Y - 1}{\varepsilon^Y}} dj\right)^{\frac{\varepsilon^Y}{\varepsilon^Y - 1}}$ and $(K)_t = \left(\int_0^1 (K)_{t,j}^{\frac{\varepsilon^K - 1}{\varepsilon^K}} dj\right)^{\frac{\varepsilon^K}{\varepsilon^K - 1}}$. Where ε^Y is the elasticity of substitution of the production of the firms under monopolistic competition and ε^K is the elasticity of substitution of the capital stock used in the production process. On the other hand, in terms of labor demand $(N_{t,j}^{1-\alpha})$ the labor market is always in equilibrium $N_{t,j} = N_t = \left(\int_0^1 N_{t,j}^{\frac{\varepsilon^N - 1}{\varepsilon^N}} dj\right)^{\frac{\varepsilon^N}{\varepsilon^N - 1}}$

$$mc_t = \frac{1}{A_t}\left[\frac{Z_t}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{1-\alpha} \tag{18}$$

The expression (18) is converted to its steady state (log-linearization).

$$mc_{ss}(1+\widetilde{mc}_t) = \frac{1}{A_{ss}}\left[\frac{Z_{ss}}{\alpha}\right]^{\alpha}\left[\frac{w_{ss}}{(1-\alpha)}\right]^{1-\alpha}\left\{1+\alpha\tilde{Z}_t + (1-\alpha)\tilde{w}_t - \tilde{A}_t\right\}$$

$$\widetilde{mc}_{j,t} = \alpha\tilde{Z}_t + (1-\alpha)\tilde{w}_t - \tilde{A}_t \tag{19}$$

Under monopolistic competition and the New Keynesian framework with price rigidities such as Calvo (1983) there is a fraction of firms that set prices with probability (θ).When this parameter is $\theta = 0$, then we can visualize that $P_{j,t}^{*} = \mu mc_{t+i}^{f}$, $\frac{1}{\mu} = mc_{t+i}^{f}$, this would denote perfect competition, under this assumption and full flexibility prices exist:

$$w_t = (1-\alpha)\,\frac{1}{\mu}\,\frac{A_t K_t^{\alpha} N_t^{1-\alpha}}{N_t}$$

$$w_t^{f} = (1-\alpha)\,\frac{1}{\mu}\,\frac{Y_t^{f}}{N_t^{f}}$$

$$K_t = \alpha\,\frac{1}{\mu}\,\frac{A_t K_t^{\alpha} N_t^{1-\alpha}}{Z_t}$$

$$K_t^{f} = \alpha\,\frac{1}{\mu}\,\frac{Y_t^{f}}{Z_j^{f}}$$

Where the variables X_t^{f} with superscript "f"denote the same variable in its natural state. Returning to the expression (13) and remembering that $C_t \cong Y_t$, the log-linearization version with flexible prices we obtain:

$$\zeta N_t^{\eta} C_t^{\sigma} = w_t^{f}$$

$$\zeta N_t^{\eta} Y_t^{\sigma} = (1-\alpha)\,\frac{1}{\mu}\,\frac{Y_t^{f}}{N_t^{f}}$$

$$\zeta N_{ss}^{\eta} Y_{ss}^{\sigma}\left(1+\eta\tilde{N}_t^{f}+\sigma\tilde{Y}_t^{f}\right) = (1-\alpha)\,\frac{1}{\mu}\,\frac{Y_{ss}^{f}}{N_{ss}^{f}}\left(1+\tilde{Y}_t^{f}-\tilde{N}_t^{f}\right)$$

$$\eta\tilde{N}_t^{f} + \sigma\tilde{Y}_t^{f} = \tilde{Y}_t^{f} - \tilde{N}_t^{f}$$

$$\eta\tilde{N}_t^{f} + \tilde{N}_t^{f} = \tilde{Y}_t^{f} - \sigma\tilde{Y}_t^{f}$$

$$\tilde{N}_t^{f}(\eta+1) = \tilde{Y}_t^{f}(1-\sigma)$$

$$\tilde{N}_t^{f} = \tilde{Y}_t^{f}\,\frac{(1-\sigma)}{(1+\eta)} \tag{20}$$

In deviations around its steady state of the Cobb Douglas production function.

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

$$\tilde{Y}_t = \tilde{A}_t + \alpha\tilde{K}_t + (1-\alpha)\tilde{N}_t \tag{21}$$

Alternatively we get $(1-\alpha)\tilde{N}_t = \left(\tilde{Y}_t - \tilde{A}_t - \alpha\tilde{K}_t\right)$[12].The capital with flexible prices is $K_t^f = \alpha \frac{1}{\mu}\frac{Y_t^f}{Z_t^f}$, around its steady state you have:

$$K_{ss}^f\left(1+\tilde{K}_t^f\right) = \alpha \frac{1}{\mu}\frac{Y_{ss}^f}{Z_{ss}^f}\left(1+\tilde{Y}_t^f - \tilde{Z}_t^f\right)$$

$$\tilde{K}_t^f = \tilde{Y}_t^f - \tilde{Z}_t^f \tag{22}$$

The expression (21) inserting it into the production function with flexible prices.

$$(1-\alpha)\tilde{N}_t^f = \tilde{Y}_t^f - \tilde{A}_t - \alpha\tilde{K}_t^f$$

$$(1-\alpha)\tilde{N}_t^f = \tilde{Y}_t^f - \tilde{A}_t - \alpha\left(\tilde{Y}_t^f - \tilde{Z}_t^f\right)$$

$$(1-\alpha)\tilde{N}_t^f = \tilde{Y}_t^f - \tilde{A}_t - \alpha\tilde{Y}_t^f + \alpha\tilde{Z}_t^f$$

$$(1-\alpha)\tilde{N}_t^f = \tilde{Y}_t^f(1-\alpha) - \tilde{A}_t + \alpha\tilde{Z}_t^f$$

$$\tilde{N}_t^f = \tilde{Y}_t^f - \frac{1}{(1-\alpha)}\tilde{A}_t + \frac{\alpha}{(1-\alpha)}\tilde{Z}_t^f \tag{22}$$

For Friedman (1968) we have a natural unemployment rate, under this precept the economy is in full employment $(\tilde{N}_t^f)$, Walrasian equilibrium concept. Through equation (20), we can define:

$$\tilde{Y}_t^f\frac{(1-\sigma)}{(1+\eta)} = \tilde{Y}_t^f - \frac{1}{(1-\alpha)}\tilde{A}_t + \frac{\alpha}{(1-\alpha)}\tilde{Z}_t^f$$

$$0 = \tilde{Y}_t^f - \tilde{Y}_t^f\frac{(1-\sigma)}{(1+\eta)} - \frac{1}{(1-\alpha)}\tilde{A}_t + \frac{\alpha}{(1-\alpha)}\tilde{Z}_t^f$$

$$0 = \tilde{Y}_t^f\left[1 - \frac{(1-\sigma)}{(1+\eta)}\right] - \frac{1}{(1-\alpha)}\tilde{A}_t + \frac{\alpha}{(1-\alpha)}\tilde{Z}_t^f$$

$$0 = \tilde{Y}_t^f\left[\frac{1+\eta-1+\sigma}{(1+\eta)}\right] - \frac{1}{(1-\alpha)}\tilde{A}_t + \frac{\alpha}{(1-\alpha)}\tilde{Z}_t^f$$

$$\frac{1}{(1-\alpha)}\left[\tilde{A}_t - \alpha\tilde{Z}_t^f\right] = \tilde{Y}_t^f\left[\frac{\eta+\sigma}{1+\eta}\right]$$

$$\tilde{Y}_t^f = \left[\frac{1+\eta}{\sigma+\eta}\right]\left[\frac{1}{(1-\alpha)}\right]\left[\tilde{A}_t - \alpha\tilde{Z}_t^f\right] \quad or \quad \tilde{A}_t = \tilde{Y}_t^f\left[\frac{(\sigma+\eta)(1-\alpha)}{1+\eta}\right] + \alpha\tilde{Z}_t^f \tag{23}$$

From the expression "$\zeta N_t^\eta C_t^\sigma = w_t$" (13), we obtain $N_{ss}^\eta Y_{ss}^\sigma\left(1+\eta\tilde{N}_t + \sigma\tilde{Y}_t\right) = w_{ss}(1+\tilde{w}_t) \Longrightarrow \eta\tilde{N}_t + \sigma\tilde{Y}_t = \tilde{w}_t$, combining them with (19) and (21).

$$\widetilde{mc}_{j,t} = \alpha\tilde{Z}_t + (1-\alpha)\tilde{w}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = (1-\alpha)\left(\eta\tilde{N}_t + \sigma\tilde{Y}_t\right) + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = (1-\alpha)\left\{\eta\left[\frac{1}{(1-\alpha)}\left(\tilde{Y}_t - \tilde{A}_t - \alpha\tilde{K}_t\right)\right] + \sigma\tilde{Y}_t\right\} + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = \eta\tilde{Y}_t - \eta\tilde{A}_t - \eta\alpha\tilde{K}_t + (1-\alpha)\sigma\tilde{Y}_t + \alpha\tilde{Z}_t - \tilde{A}_t$$

[12] The production function in natural state will be: $\tilde{Y}_t^f = \tilde{A}_t + \alpha\tilde{K}_t^f + (1-\alpha)\tilde{N}_t^f$

From (17) "$K_t = \alpha\, mc_t\, \frac{Y_t}{Z_t}$", the log-linear expression is $K_{ss}(1+\tilde{K}_t) = \alpha\, mc_{ss}\, \frac{Y_{ss}}{Z_{ss}}(\widetilde{mc}_t + \tilde{Y}_t - \tilde{Z}_t), \Longrightarrow \tilde{K}_t = \widetilde{mc}_t + \tilde{Y}_t - \tilde{Z}_t$. And finally combining it with (23).

$$\widetilde{mc}_t = \eta\tilde{Y}_t - \eta\tilde{A}_t - \eta\alpha(\widetilde{mc}_t + \tilde{Y}_t - \tilde{Z}_t) + (1-\alpha)\sigma\tilde{Y}_t + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = \eta\tilde{Y}_t - \eta\tilde{A}_t - \eta\alpha\,\widetilde{mc}_t - \eta\alpha\tilde{Y}_t + \eta\alpha\tilde{Z}_t + (1-\alpha)\sigma\tilde{Y}_t + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = \eta\tilde{Y}_t - \eta\tilde{A}_t - \eta\alpha\,\widetilde{mc}_t - \eta\alpha\tilde{Y}_t + \eta\alpha\tilde{Z}_t + (1-\alpha)\sigma\tilde{Y}_t + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = \eta\tilde{Y}_t - \eta\tilde{A}_t - \eta\alpha\,\widetilde{mc}_t - \eta\alpha\tilde{Y}_t + \eta\alpha\tilde{Z}_t + \sigma\tilde{Y}_t - \alpha\sigma\tilde{Y}_t + \alpha\tilde{Z}_t - \tilde{A}_t$$

$$\widetilde{mc}_t = \tilde{Y}_t(\sigma+\eta) - \alpha\tilde{Y}_t(\sigma+\eta) - \eta\alpha\,\widetilde{mc}_t + \alpha\tilde{Z}_t(1+\eta) - \tilde{A}_t(1+\eta)$$

$$\widetilde{mc}_t + \eta\alpha\,\widetilde{mc}_t = \tilde{Y}_t(\sigma+\eta)(1-\alpha) + \alpha\tilde{Z}_t(1+\eta) - \tilde{A}_t(1+\eta)$$

$$\widetilde{mc}_t(1+\eta\alpha) = \tilde{Y}_t(\sigma+\eta)(1-\alpha) + \alpha\tilde{Z}_t(1+\eta) - \left\{\tilde{Y}_t^f\left[\frac{(\sigma+\eta)(1-\alpha)}{1+\eta}\right] + \alpha\tilde{Z}_t^f\right\}(1+\eta)$$

$$\widetilde{mc}_t(1+\eta\alpha) = \tilde{Y}_t(\sigma+\eta)(1-\alpha) + \alpha\tilde{Z}_t(1+\eta) - \left\{\tilde{Y}_t^f\left[\frac{(\sigma+\eta)(1-\alpha)}{1+\eta}\right] + \alpha\tilde{Z}_t^f\right\}(1+\eta)$$

$$\widetilde{mc}_t(1+\eta\alpha) = \tilde{Y}_t(\sigma+\eta)(1-\alpha) + \alpha\tilde{Z}_t(1+\eta) - \tilde{Y}_t^f(\sigma+\eta)(1-\alpha) - (1+\eta)\alpha\tilde{Z}_t^f$$

$$\widetilde{mc}_t(1+\eta\alpha) = (\sigma+\eta)(1-\alpha)\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \alpha(1+\eta)\left[\tilde{Z}_t - \tilde{Z}_t^f\right]$$

$$\widetilde{mc}_t = \frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{\alpha(1+\eta)}{1+\eta\alpha}\left[\tilde{Z}_t - \tilde{Z}_t^f\right] \qquad (24)$$

Equation (24) shows that the real marginal cost is an approximation of the output gap ($\tilde{Y}_t - \tilde{Y}_t^f$), as marginal cost is the inverse of the markup (profit margin), then $\frac{1}{\mu} = mc_{t+i}^f$, where, $\frac{1}{\mu} = \frac{\varepsilon}{\varepsilon-1}$, and ε, is the elasticity of substitution between the wholesale products of the firms that produce the final good. If output gap is positive, then the real marginal cost is above its desirable state, so the margins are lower (equivalent to a less distorted economy), the opposite happens when gap is negative.

Final Good Producer

The aggregation and monopolistic competition the modeling of final production is expressed from a representative firm of goods that adds intermediate inputs according to a technology of Constant Substitution Elasticity (CES). Due to the large number of intermediary firms, the final good producing firm is also an aggregation using capital and labor, assuming that the firms are identical to each other, the maximization of benefits is obtained:

$$\underset{\{Y_t(j)\}}{\text{Max}}\ P_tY_t - \int_0^1 P_t(j)Y_t(j)\ dj$$

Technology aggregation (Dixit-Stiglitz, 1977) is represented by the restriction:

$$Y_t = \left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\ dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

Replacing the Dixit-Stiglitz aggregator:

$$\underset{\{Y_t(j)\}}{\text{Max}}\ P_t\left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\ dj\right\}^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(j)Y_t(j)\ dj$$

The first order conditions:

$$\frac{\varepsilon}{\varepsilon-1}P_t\left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}-1}\frac{\varepsilon-1}{\varepsilon}[Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}-1}-\;P_t(j)=0$$

$$\left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{1}{\varepsilon-1}}\;[Y_t(j)]^{-\frac{1}{\varepsilon}}=\frac{P_t(j)}{P_t}$$

$$\underbrace{\left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{-\frac{\varepsilon}{\varepsilon-1}}}_{Y_t^{-1}}\quad Y_t(j)=\left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon}$$

$$Y_t^{-1}\quad Y_t(j)=\left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon}$$

$$Y_t(j)=\left[\frac{P_t}{P_t(j)}\right]^{\varepsilon}Y_t$$

This equation expresses the demand relative for intermediate goods produced (j), which is directly proportional to aggregate demand (Y_t) and inversely proportional to the relative price $\left[{}^{1}/{}_{P_t(j)/\,P_t}\right]$. Derivation of the price index is:

$$Y_t=\left\{\int_0^1 [Y_t(j)]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_t=\left\{\int_0^1 \left[\left(\frac{P_t}{P_t(j)}\right)^{\varepsilon}Y_t\right]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_t=Y_t\,P_t^{\varepsilon}\left\{\int_0^1 \left[\left(\frac{1}{P_t(j)}\right)^{\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$P_t^{\varepsilon}=\frac{1}{\left\{\int_0^1\left[\left(P_t(j)\right)^{-\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}}$$

$$P_t^{\varepsilon}=\left\{\int_0^1 \left[\left(P_t(j)\right)^{\varepsilon}\right]^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

Price aggregation level will be:

$$P_t=\left\{\int_0^1 \left[\left(P_t(j)\right)\right]^{\varepsilon-1}\,dj\right\}^{\frac{1}{\varepsilon-1}}$$

Sticky Prices

We assume that prices do not adjust instantaneously in each period, "$1-\theta$" is the probability of defining the prices of goods for all periods "t". However, exists a fraction of firms that are not willing to change prices with probability θ. Then, the dynamic problem for the firm in maximizing benefits to readjust the price will be:

$$\max_{P_{j,t}^*} E_t\left\{\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}\left[\frac{P_{j,t}^*}{P_{t+i}}\ C_{j,t+i}-mc_{t+i}\ C_{j,t+i}\right]\right\}$$

Where $\Delta_{i,t+i}=\beta^i\left(\frac{C_{t+i}}{C_t}\right)^{-\sigma}$ is the stochastic discount factor and the restriction in all periods that define price is:

$$C_{j,t}=\left[\frac{P_{j,t}^*}{P_t}\right]^{-\varepsilon}\ C_t$$

Replaced

$$\max_{P_{j,t}^*} E_t\left\{\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}\left[\frac{P_{j,t}^*}{P_{t+i}}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\ C_{t+i}-mc_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\ C_{t+i}\right]\right\}$$

$$\max_{P_{j,t}^*} E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}\left[\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{1-\varepsilon}-mc_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\right]$$

The first order conditions:

$$E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}\left[(1-\varepsilon)\left(\frac{1}{P_{t+i}}\right)^{1-\varepsilon}P_{j,t}^{*\ -\varepsilon}+\varepsilon\ mc_{t+i}\left(\frac{1}{P_{t+i}}\right)^{-\varepsilon}P_{j,t}^{*\ -\varepsilon-1}\right]=0$$

$$E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}P_{j,t}^{*\ -\varepsilon}\left[(1-\varepsilon)\left(\frac{1}{P_{t+i}}\right)^{1-\varepsilon}+\varepsilon\ mc_{t+i}\left(\frac{1}{P_{t+i}}\right)^{-\varepsilon}\frac{1}{P_{j,t}^*}\right]=0$$

$$E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}P_{j,t}^{*\ -\varepsilon}\left(\frac{1}{P_{t+i}}\right)^{-\varepsilon}\left[(1-\varepsilon)\ \frac{1}{P_{t+i}}+\varepsilon\ mc_{t+i}\frac{1}{P_{j,t}^*}\right]=0$$

$$E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\left[(1-\varepsilon)\ \frac{1}{P_{t+i}}+\varepsilon\ mc_{t+i}\frac{1}{P_{j,t}^*}\right]=0$$

$$E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}(1-\varepsilon)\ \frac{1}{P_{t+i}}+E_t\sum_{i=0}^{\infty}\theta^i\Delta_{i,t+i}C_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\varepsilon\ mc_{t+i}\frac{1}{P_{j,t}^*}=0$$

Replacing $\Delta_{i,t+i}=\beta^i\left(\frac{C_{t+i}}{C_t}\right)^{-\sigma}$

$$E_t\sum_{i=0}^{\infty}\theta^i\beta^i\left(\frac{C_{t+i}}{C_t}\right)^{-\sigma}C_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}(1-\varepsilon)\ \frac{1}{P_{t+i}}=-E_t\sum_{i=0}^{\infty}\theta^i\beta^i\left(\frac{C_{t+i}}{C_t}\right)^{-\sigma}C_{t+i}\left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\varepsilon}\varepsilon\ mc_{t+i}\frac{1}{P_{j,t}^*}$$

$$(1-\varepsilon)\frac{P_{j,t}^{*\ -\varepsilon}}{C_t^{-\sigma}}E_t\sum_{i=0}^{\infty}(\theta\beta)^i\frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{1-\varepsilon}}=-\ \varepsilon\frac{P_{j,t}^{*\ -\varepsilon}}{C_t^{-\sigma}}\frac{1}{P_{j,t}^*}E_t\sum_{i=0}^{\infty}(\theta\beta)^i\frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{-\varepsilon}}mc_{t+i}$$

$$\varepsilon\frac{1}{P_{j,t}^*}E_t\sum_{i=0}^{\infty}(\theta\beta)^i\frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{-\varepsilon}}mc_{t+i}=-(1-\varepsilon)E_t\sum_{i=0}^{\infty}(\theta\beta)^i\frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{1-\varepsilon}}$$

$$\varepsilon \frac{1}{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon}\, mc_{t+i} = (\varepsilon - 1) E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma}\, P_{t+i}^{\varepsilon-1}$$

$$\frac{\varepsilon}{(\varepsilon - 1)} \frac{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon}\, mc_{t+i}}{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma}\, P_{t+i}^{\varepsilon-1}} = P_{j,t}^*$$

Firms set their prices at the same level of the mark up and marginal cost. Therefore, in all periods firms set a price level. Updated in each $t + i$, it can be re-expressed in a compact way:

$$P_{j,t}^* = \mu \frac{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon}\, mc_{t+i}}{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma}\, P_{t+i}^{\varepsilon-1}} \qquad or \qquad P_{j,t}^* = \mu \frac{\mathrm{A}_t}{\mathrm{B}_t} \tag{25}$$

This expression is called New Keynesian Phillips Curve (NKPC).

On the other hand, the aggregated prices (P_t) is determined by:

$$P_t^{1-\varepsilon} = (1-\theta) {P_{j,t}^*}^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}$$

$$\left[\frac{P_t}{P_{t-1}}\right]^{1-\varepsilon} = (1-\theta) \frac{{P_{j,t}^*}^{1-\varepsilon}}{P_{t-1}^{1-\varepsilon}} + \theta$$

$$\pi_t^{1-\varepsilon} = \theta + (1-\theta) \left[\frac{P_{j,t}^*}{P_{t-1}}\right]^{1-\varepsilon}$$

The steady state inflation is one, $\pi_{ss} = \frac{P_{ss}}{P_{ss}}$ =1. The price dynamics with frictions in log-linear expression is:

$$\pi_{ss}^{1-\varepsilon}[1 + (1-\varepsilon)\tilde{\pi}_t] = \theta + (1-\theta) \left[\frac{P_{ss}}{P_{ss}}\right]^{1-\varepsilon} \left[1 + (1-\varepsilon)\widetilde{P_{j,t}^*} - (1-\varepsilon)\tilde{P}_{t-1}\right]$$

$$1 + (1-\varepsilon)\tilde{\pi}_t = \theta + \left[1 + (1-\varepsilon)\tilde{P}_{j,t}^* - (1-\varepsilon)\tilde{P}_{t-1}\right] - \theta\left[1 + (1-\varepsilon)\tilde{P}_{j,t}^* - (1-\varepsilon)\tilde{P}_{t-1}\right]$$

$$1 + (1-\varepsilon)\tilde{\pi}_t = \theta + 1 + (1-\varepsilon)\tilde{P}_{j,t}^* - (1-\varepsilon)\tilde{P}_{t-1} - \theta - \theta(1-\varepsilon)\tilde{P}_{j,t}^* + \theta(1-\varepsilon)\tilde{P}_{t-1}$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\tilde{P}_{j,t}^* - (1-\varepsilon)\tilde{P}_{t-1} - \theta(1-\varepsilon)\tilde{P}_{j,t}^* + \theta(1-\varepsilon)\tilde{P}_{t-1}$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\left(\tilde{P}_{j,t}^* - \tilde{P}_{t-1}\right) - \theta(1-\varepsilon)\left(\widetilde{P_{j,t}^*} - \tilde{P}_{t-1}\right)$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\left(\tilde{P}_{j,t}^* - \tilde{P}_{t-1}\right)(1-\theta)$$

$$\tilde{\pi}_t = \left(\tilde{P}_{j,t}^* - \tilde{P}_{t-1}\right)(1-\theta)$$

$$\frac{\tilde{\pi}_t}{1-\theta} + \tilde{P}_{t-1} = \tilde{P}_{j,t}^* \tag{26}$$

From (25) we obtain the following expressions in Log-linear version[13].

$$\widetilde{\mathrm{A}}_t - \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} - (1-\theta\beta)\widetilde{mc}_t = \left[(1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t\right](1-\theta\beta) \tag{27}$$

$$\widetilde{\mathrm{B}}_t - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} + \tilde{P}_t(1-\theta\beta) = \left[(1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t\right](1-\theta\beta) \tag{28}$$

Operating (26), (27) and (28) we get the New Keynesian Phillips Curve (NKPC) Log-linear:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \kappa\, \widetilde{mc}_t \tag{29}$$

[13] In appendixes we obtained the log-linear Phillips New Keynesian Curve (NKPC) in detail.

Fisher Equation

Strictly from Fisher's equation and remembering that $\pi_{ss} = 1$:

$$\frac{(1 + i_t)}{(1 + E_t\pi_{t+1})} = 1 + R_t$$

Steady state:

$$1 + i_{ss} = 1 + R_{ss} + \pi_{ss} + \pi_{ss}R_{ss}$$

$$i_{ss} = R_{ss} + 1 + R_{ss}$$

$$i_{ss} - 1 = 2R_{ss}; \frac{\frac{1-\beta}{\beta} - 1}{2} = R_{ss} = \frac{\frac{1-\beta-\beta}{\beta}}{2} = \frac{\frac{1-2\beta}{\beta}}{2} = \frac{1-2\beta}{2\beta}$$

$$\frac{(1 + i_t)}{(1 + E_t\pi_{t+1})} = 1 + R_t$$

$$1 + i_t = 1 + R_t + E_t\pi_{t+1} + R_tE_t\pi_{t+1}$$

$$1 + i_{ss}(1 + \tilde{\imath}_t) = 1 + R_{ss}(1 + \tilde{R}_t) + \pi_{ss}(1 + E_t\tilde{\pi}_{t+1}) + R_{ss}\pi_{ss}(1 + \tilde{R}_t + E_t\tilde{\pi}_{t+1})$$

$$i_{ss}(1 + \tilde{\imath}_t) = R_{ss}(1 + \tilde{R}_t) + 1 + E_t\tilde{\pi}_{t+1} + R_{ss}(1 + \tilde{R}_t + E_t\tilde{\pi}_{t+1})$$

$$i_{ss} + i_{ss}\tilde{\imath}_t = R_{ss} + R_{ss}\tilde{R}_t + R_{ss} + R_{ss}E_t\tilde{\pi}_{t+1} + R_{ss}\tilde{R}_t + 1 + E_t\tilde{\pi}_{t+1}$$

$$i_{ss} - 1 + i_{ss}\tilde{\imath}_t = 2R_{ss} + 2R_{ss}\tilde{R}_t + R_{ss}E_t\tilde{\pi}_{t+1} + E_t\tilde{\pi}_{t+1}$$

$$2R_{ss} + i_{ss}\tilde{\imath}_t = 2R_{ss} + 2R_{ss}\tilde{R}_t + R_{ss}E_t\tilde{\pi}_{t+1} + E_t\tilde{\pi}_{t+1}$$

$$i_{ss}\tilde{\imath}_t = 2R_{ss}\tilde{R}_t + E_t\tilde{\pi}_{t+1}(1 + R_{ss})$$

$$i_{ss}\tilde{\imath}_t = 2R_{ss}\tilde{R}_t + E_t\tilde{\pi}_{t+1}\left(1 + \frac{1-2\beta}{2\beta}\right)$$

$$\frac{1-\beta}{\beta}\tilde{\imath}_t = 2\left(\frac{1-2\beta}{2\beta}\right)\tilde{R}_t + E_t\tilde{\pi}_{t+1}\left(\frac{1-2\beta+2\beta}{2\beta}\right)$$

$$\tilde{\imath}_t = \frac{\beta}{1-\beta}\frac{1-2\beta}{\beta}\tilde{R}_t + \frac{\beta}{1-\beta}E_t\tilde{\pi}_{t+1}\left(\frac{1}{2\beta}\right)$$

$$\tilde{\imath}_t = \frac{1-2\beta}{1-\beta}\tilde{R}_t + \frac{1}{2(1-\beta)}E_t\tilde{\pi}_{t+1}$$

$$\tilde{\imath}_t - \beta\tilde{\imath}_t = \tilde{R}_t - 2\beta\tilde{R}_t + \frac{1}{2}E_t\tilde{\pi}_{t+1}$$

$$2(\tilde{\imath}_t - \beta\tilde{\imath}_t - \tilde{R}_t + 2\beta\tilde{R}_t) = E_t\tilde{\pi}_{t+1} \tag{30}$$

Monetary Policy

Investigation´s goal was to find a monetary policy rule out of the conventional, in the previous section the Poole´s Rule was obtained, in log-linear version. The monetary authority is aware of the behavior of the Aggregate Demand, Money Demand and Sticky Prices in the market (New Keynesian Phillips Curve, NKPC), described by (14), (15) and (29) respectively[14].

$$\tilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}(\tilde{x}_t^i)$$

[14] The log-linearization of IS curve microfounded and Money Demand is detailed in Appendix.

Equilibrium Condition, Capital Accumulation Equation and Stochastic Processes

The evaluation of Poole´s Rule within the proposed model in a closed economy without government, would assume the following expressions

$$Y_t = C_t + I_t$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

Expressed around its steady state we get

$$\tilde{Y}_t = \frac{C_{ss}}{Y_{ss}}\tilde{C}_t + \frac{I_{ss}}{Y_{ss}}\tilde{I}_t \tag{31}$$

$$\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + \delta\tilde{I}_t \tag{32}$$

Also, in the Walrasian system the equilibrium condition of production factors are reached through (16) and (17), around its steady state is:

$$N_t = (1-\alpha)\, mc_t\, \frac{Y_t}{w_t}$$

$$\tilde{N}_t = \widetilde{mc}_t + \tilde{Y}_t - \tilde{w}_t$$

$$K_t = \alpha\, mc_t\, \frac{Y_t}{Z_t}$$

$$\tilde{K}_t = \widetilde{mc}_t + \tilde{Y}_t - \tilde{Z}_t$$

By $\widetilde{mc}_t$ the condition converges:

$$\tilde{K}_t + \tilde{Z}_t = \tilde{N}_t + \tilde{w}_t \tag{33}$$

In the proposed exercise, some variables follow an autoregressive process AR (1) such as TPF ($\tilde{A}_t$) and the natural interest rate ($\tilde{\imath}_t^f$). Additionally, shocks were introduced in Poole´s Rule ($\tilde{\phi}_t^{\widetilde{m}}$), in the New Keynesian Phillips Curve (NKPC, $\tilde{\phi}_t^{\tilde{\pi}}$) and aggregated demand ($\tilde{\phi}_t^{\widetilde{AD}}$), which similarly follow an AR (1) process, the log-linear form are[15]:

$$\tilde{A}_t = \rho^{\tilde{A}}\tilde{A}_{t-1} + \varepsilon_t^{\tilde{A}} \tag{34}$$

$$\tilde{\imath}_t^f = \rho^{\tilde{\imath}^f}\tilde{\imath}_{t-1}^f + \varepsilon_t^{\tilde{\imath}^f} \tag{35}$$

$$\tilde{\phi}_t^m = \rho^{\tilde{\phi}^m}\tilde{\phi}_{t-1}^m + \varepsilon_t^{\tilde{\phi}^m} \tag{36}$$

$$\tilde{\phi}_t^{\tilde{\pi}} = \rho^{\tilde{\pi}}\tilde{\phi}_{t-1}^{\tilde{\pi}} + \varepsilon_t^{\tilde{\pi}} \tag{37}$$

$$\tilde{\phi}_t^{\widetilde{AD}} = \rho^{\tilde{\phi}^{\widetilde{AD}}}\tilde{\phi}_{t-1}^{\widetilde{AD}} + \varepsilon_t^{\tilde{\phi}^{\widetilde{AD}}} \tag{38}$$

$\varepsilon_t^{\tilde{A}}, \varepsilon_t^{\widetilde{m}}, \varepsilon_t^{\tilde{\imath}^f}, \varepsilon_t^{\widetilde{AD}}, \varepsilon_t^{\tilde{\pi}}$ are the stochastic processes $N(0, \vartheta^2)$.

[15] The nonlinear form of the five (5) variables is $X_t = X_{t-1}^{\rho^{\tilde{X}}}\varepsilon_t^{\tilde{X}}$.

Competitive equilibrium definition

Log-linear equations Walrasian competitive equilibrium under monopolistic competition with Sticky Prices follow a stochastic process:

$$\left\{\tilde{Y}_t, \tilde{C}_t, \tilde{I}_t, \widetilde{K}_t, \widetilde{m}_t, \tilde{x}_t, \tilde{x}_t^i, \tilde{\pi}_t, \tilde{\imath}_t, \tilde{R}_t, \widetilde{mc}_t, \tilde{x}_t^{\tilde{Z}}, \tilde{Z}_t, \tilde{Z}_t^f, \tilde{Y}_t^f, \tilde{\imath}_t^f, \widetilde{w}_t, \widetilde{N}_t, \tilde{A}_t, \tilde{\phi}_t^{\widetilde{m}}, \tilde{\phi}_t^{\widetilde{AD}}, \tilde{\phi}_t^{\tilde{\pi}}\right\}_t^\infty$$

Stochastic processes are:

$$\left\{\varepsilon_t^{\tilde{A}}, \varepsilon_t^{\widetilde{m}}, \varepsilon_t^{\tilde{\imath}^f}, \varepsilon_t^{\widetilde{AD}}, \varepsilon_t^{\tilde{\pi}}\right\}_t^\infty$$

Model Structure:

Equation	Definition
$\tilde{C}_t = E_t\tilde{C}_{t+1} - \frac{1}{\sigma}(\tilde{\imath}_t - E_t\tilde{\pi}_{t+1}) + \tilde{\phi}_t^{\widetilde{AD}}$	Euler´s Equation
$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{C}_t - \frac{\beta}{\sigma^M}\tilde{\imath}_t$	Money Demand
$\widetilde{mc}_t = \frac{(\sigma+\eta)(1-\alpha)}{1+\eta\alpha}\left[\tilde{Y}_t - \tilde{Y}_t^f\right] + \frac{\alpha(1+\eta)}{1+\eta\alpha}\left[\tilde{Z}_t - \tilde{Z}_t^f\right]$	Marginal Cost
$\tilde{\pi}_t = \beta E_t\tilde{\pi}_{t+1} + \kappa\, \widetilde{mc}_t + \tilde{\phi}_t^{\tilde{\pi}}$	New Keynesian Phillips Curve
$\tilde{Y}_t^f = \left[\frac{1+\eta}{\sigma+\eta}\right]\left[\frac{1}{(1-\alpha)}\right]\left[\tilde{A}_t - \alpha\tilde{Z}_t^f\right]$	Natural Output
$\tilde{Z}_t = \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t$	Capital Price
$\tilde{Z}_t^f = \left[\frac{1-\beta}{1-\beta(1-\delta)}\right]\tilde{\imath}_t^f - \frac{\beta}{1-\beta(1-\delta)}$	Natural Capital Price
$\widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \delta\tilde{I}_t$	Capital Accumulation Equation
$\widetilde{w}_t = \eta\widetilde{N}_t + \sigma\tilde{Y}_t$	Labor Supply
$2\left(\tilde{\imath}_t - \beta\tilde{\imath}_t - \tilde{R}_t + 2\beta\tilde{R}_t\right) = E_t\tilde{\pi}_{t+1}$	Fisher´s Equation
$\widetilde{K}_t + \tilde{Z}_t = \widetilde{N}_t + \widetilde{w}_t$	Equilibrium Condition of Production Factors
$\tilde{Y}_t = \tilde{A}_t + \alpha\widetilde{K}_t + (1-\alpha)\widetilde{N}_t$	Cobb-Douglas, Production Function
$\tilde{Y}_t = \frac{C_{ss}}{Y_{ss}}\tilde{C}_t + \frac{I_{ss}}{Y_{ss}}\tilde{I}_t$	Equilibrium Condition
$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi+\Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi+\Theta]}\left(\tilde{x}_t^i\right) + \tilde{\phi}_t^m$	Poole´s Rule
$\tilde{x}_t^i = \tilde{\imath}_t - \tilde{\imath}_t^f$	Interest Rate Gap

$\tilde{x}_t = \tilde{Y}_t - \tilde{Y}_t^f$	Output Gap
$\tilde{x}_t^{\tilde{Z}} = \tilde{Z}_t - \tilde{Z}_t^f$	Capital Price Gap
$\tilde{A}_t = \rho^{\tilde{A}} \tilde{A}_{t-1} + \varepsilon_t^{\tilde{A}}$	*Shock* Productive (TPF)
$\tilde{\phi}_t^m = \rho^{\tilde{\phi}^m} \tilde{\phi}_{t-1}^m + \varepsilon_t^{\widetilde{m}}$	*Shock* in Poole´s Rule
$\tilde{\imath}_t^f = \rho^{\tilde{\imath}^f} \tilde{\imath}_{t-1}^f + \varepsilon_t^{\tilde{\imath}^f}$	Natural Interest Rate
$\tilde{\phi}_t^{\widetilde{AD}} = \rho^{\tilde{\phi}^{\widetilde{AD}}} \tilde{\phi}_{t-1}^{\widetilde{AD}} + \varepsilon_t^{\widetilde{AD}}$	*Shock* Aggregate Demand
$\tilde{\phi}_t^{\tilde{\pi}} = \rho^{\tilde{\pi}} \tilde{\phi}_{t-1}^{\tilde{\pi}} + \varepsilon_t^{\tilde{\pi}}$	*Shock* (Cost-Push Inflation)

Modeling of monetary policy for economies with Inflation Targeting Framework is carried out through the well-known Taylor´s Rule; however, for economies under a money supply control scheme such as China, Nigeria, Bolivia, Malawi, Tanzania, Ethiopia, Yemen, Suriname, among others, there is no clarity about the application of a monetary policy rule. Therefore, the application of the Taylor´s Rule is not appropriate[16].

Li and Liu (2017) design a DSGE model for the Chinese economy, they evaluate monetary policy rules. The authors use Bayesian techniques estimate the parameters of three types of rules: i) Taylor's Rule, ii) MacCallum's Rule and iii) the latter they define as a combination of both "combination policy" they call in the spirit of Poole´s, 1970. All rules they incorporate in the model are ad-hoc, there is no microeconomic fundamentals for expressions, although a relevant conclusion is about the third rule (they call Expanded Taylor Rule[17]); the interest rate reacts to the money growth rate gap, they indicate it is the most appropriate to capture the characteristics of China's economy over the Taylor and MacCallum Rules.

In Bolivian case, two documents incorporate the MacCallum Rule for modeling monetary policy: Valdivia J. (2017)[18] and Zeballos, Heredia and Yujra (2018)[19]. In both investigations the results are counterintuitive and against the economic theory, positive shocks in the Aggregates Rule instead of encouraging real variables: product, consumption and investment, the effect is contractive. Likewise, the result on the variables of interest of the monetary authority, inflation and interest rate, is

[16] The International Monetary Fund (IMF) defines these countries under a Monetary Aggregate Target scheme.

[17] The equation is: $\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho^R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\gamma_\pi^R} \left(\frac{y_t}{y_t^*}\right)^{\gamma_y^R} \left(\frac{\omega_t}{\bar{\omega}}\right)^{\gamma_\omega^R}\right]^{1-\rho^R}$ exp (ε_t^R).Where ω_t is the money growth rate.

[18] Research presented at the XXII Meeting of the Central Bank Researchers Network (CEMLA).

[19] Winning research of the technical cooperation program "Strengthening Research in Economic Development in Bolivia" of the development bank of Latin America (CAF) together with the Bolivian Academy of Economic Sciences (ABCE); under the technical and operational management of the INESAD Foundation.

not plausible; Increases in the money supply generate downward pressures in the price level and upward interest rate[20].

The empirical evidence for application of monetary policy rules in economies under a different scheme of inflation targeting is ambiguous. The objective of this investigation was the finding of the foundations of the Poole´s Rule, so an exhaustive evaluation of the parameters of the model was carried out in line with Poole (1970,) Turnovsky (1975), Yoshikawa (1981), Daniel (1986) and Fair (1987), because the values they assume play an important role in the empirical (stylized facts) and theoretical validity of the Policy Rule. The model proposed for the Bolivian economy (DSGE) was estimated with Bayesian econometrics[21].

Results

Shocks in the Poole´s Rule have positive effects on real variables: GDP, Consumption and Investment, in the first and third cases the effects are immediate as in the Impulse Response Functions (IRF); 0.44 percentage points (pp, Figure 1) of product growth reacts to shocks ($\varepsilon_t^{\widetilde{m}}$) in the Monetary Policy Rule (the posterior standard deviation of $\varepsilon_t^{\widetilde{m}}$, under the Bayesian methodology is 0.69). The result is consistent with paper of Li and Liu (2017), although in "expanded Taylor´s Rule" result is not specifically visualized[22], if we make a simple clearance of the money growth rate gap $(\omega_t - \omega^*)$ we can obtain the similar form of Poole´s Rule.

$$\omega_t - \omega^* = \frac{1}{(1-\rho^R)\gamma_\omega^R}(R_t - \rho^R R_{t-1}) - \frac{\gamma_\pi^R}{\gamma_\omega^R}(\pi_t - \pi^*) - \frac{\gamma_y^R}{\gamma_\omega^R}(y_t - y^*) - \varepsilon_t^R$$

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}\left(\tilde{x}_t^i\right) + \tilde{\phi}_t^m$$

Li and Liu´s version as a result of the clearing the shock is negative (ε_t^R), the effect of IRF is contractive in economic growth, approximately 0.02pp (in DSGE models shocks are symmetric, so that if (ε_t^R) were positive the result of "expanded Taylor´s Rule" would be in line with results found in this investigation). The response of consumption to these types of shocks in our model is positive in the second period (0.58pp, Figure 1). As a result of this shock the fall in the interest rate is congruent in the same periodicity, such an effect can be expected because the transmission mechanism (Poole's Rule) is not contemporary to real variables. Finally, the nature of this shock is not immediately inflationary, from the third period the expansionary

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[20] The expression used for the Bolivian case is:

$$m_t = (m_{t-1})^{\rho^m}\left[\left(\frac{\pi_t}{\pi^*}\right)^{\varphi_m^\pi}\left(\frac{y_t}{y^*}\right)^{\varphi_m^y}\right]^{1-\rho^m}\phi_t^m \qquad or \qquad M_t^d = \left(M_{t-1}^d\right)^{\rho_M}\left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi}\left(\frac{y_t}{y^*}\right)^{\gamma_y}\right]^{1-\rho_M}\exp\left(\epsilon_t^{MP}\right)$$

[21] See appendix for model results. The observed variables are GDP, Consumption, Inflation, and M2 Monetary Aggregate. This information can be obtained from the National Statistics Institute (INE) and Central Bank of Bolivia (BCB), data of access to the general public.

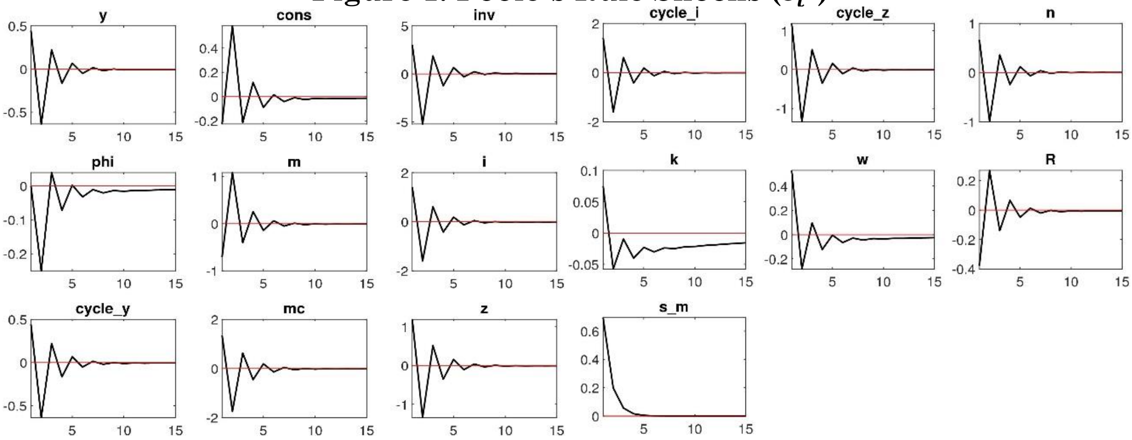
[22] The Log-linear version of "Expanded Taylor Rule" is: $R_t = \rho^R R_{t-1} + (1-\rho^R)\left[\gamma_\pi^R(\pi_t - \pi^*) + \gamma_y^R(y_t - y^*) + \gamma_\omega^R(\omega_t - \omega^*)\right] + \varepsilon_t^R$. From ad-hoc expression we clear the money growth rate gap $(\omega_t - \omega^*)$ we get:

$$\frac{1}{(1-\rho^R)\gamma_\omega^R}R_t - \frac{\rho^R}{(1-\rho^R)\gamma_\omega^R}R_{t-1} - \frac{1}{\gamma_\omega^R}\gamma_\pi^R(\pi_t - \pi^*) - \frac{1}{\gamma_\omega^R}\gamma_y^R(y_t - y^*) - \varepsilon_t^R = \omega_t - \omega^*$$

$$\omega_t - \omega^* = \frac{1}{(1-\rho^R)\gamma_\omega^R}(R_t - \rho^R R_{t-1}) - \frac{\gamma_\pi^R}{\gamma_\omega^R}(\pi_t - \pi^*) - \frac{\gamma_y^R}{\gamma_\omega^R}(y_t - y^*) - \varepsilon_t^R$$

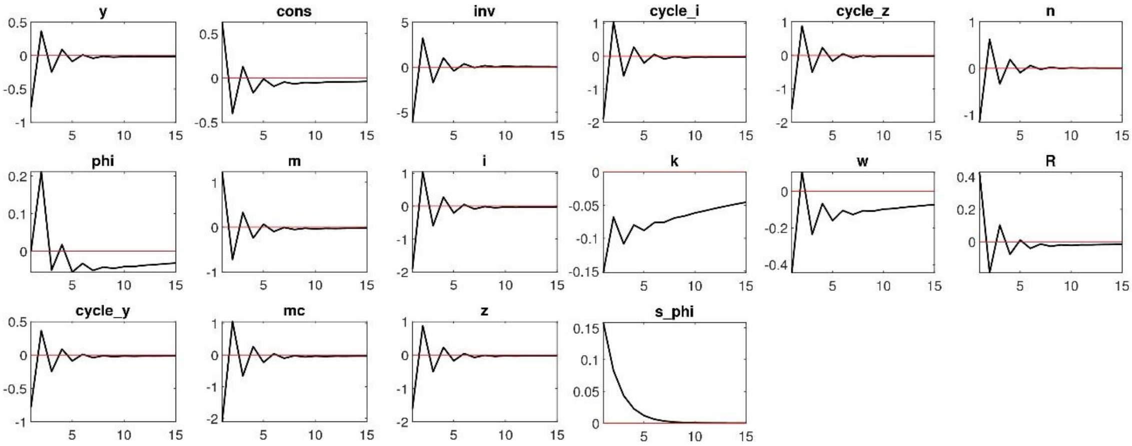
effect of the monetary policy is visualized (the stability price and the positive response of the interest rate in the first period support this finding).

Figure 1: Poole´s Rule Shocks ($\varepsilon_t^{\widetilde{m}}$)



Moreover, we evaluate the behavior of the monetary authority against cost push inflation in the NKPC, the variables of interest by the monetary authority ($\tilde{m}_t$ and $\tilde{\imath}_t$) react with a lag period, the money is withdrawn from the economy and the interest rate rises to contain higher inflationary pressures. The disquisition is due to the behavior of consumption determined by the Euler´s Equation, the positive relationship of this variable with inflationary expectations (households are more adverse to the future behavior of the economy, therefore they consume in "t" to protect the purchasing loss that the money may suffer in "$t+1$" against to inflationary expectations). In the literature, in front of such shocks ($\varepsilon_t^{\tilde{\pi}}$), GDP, investment, salary and employment decrease because increasing price translates into the firms' costs, then a negative gap the product is feasible (Figurate 2) [23].

Figure 2: Cost Push Inflation ($\varepsilon_t^{\tilde{\pi}}$)



The behavior of the variables in front other types of shocks, the natural interest rate ($\varepsilon_t^{\tilde{\imath}^f}$), aggregate demand ($\varepsilon_t^{\widetilde{AD}}$) or in the technological process ($\varepsilon_t^{\tilde{A}}$) are intuitively coherent (data relationship, stylized facts) and backed by economic theory.

[23] The posterior standard deviation of cost push inflation ($\varepsilon_t^{\tilde{\pi}}$), is 0.16.

Parameter Evaluation

As indicated by Poole (1970,) Turnovsky (1975), Yoshikawa (1981), Cazoneri et al. (1983), Daniel (1986) and Fair (1987) the parameters of the rule that Poole raised in his investigation are relevant to establish if this instrument is effective to control fluctuations for output gap. Some parameters have a predominant role under the core Poole model, elasticity of money demand with respect to the interest rate and the elasticity of income effect of money demand. In our version of the Poole´s Rule and the DGSE model, monetary policy is influenced by the parameters of the IS microfounded, the NKPC and the money demand.

$$\widetilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}\left(\tilde{x}_t^i\right) + \tilde{\phi}_t^m$$

Where:

σ = *Coefficient of Relative Risk Aversion*
σ^M = *Inverse of the elasticity of money holdings with respect to the interest rate*
β = *Discount Factor*
θ = *Probability that this price remains fixed*
η = *Inverse of the Frisch elasticity of labor supply*
α = *Elasticity of the level of production with respect to capital*
δ = *Capital Depreciation Rate*
Θ = *Relative Weight Attached to Cyclical Movements in Output*

The last parameter (Θ) was never estimated for the Bolivian economy and the value can fluctuate between 0.05 and 0.33 according to the estimation of some authors according to Tobias Kranz (2015). In the bayesian estimation for two exercises were performed, consequently, two different Priors of Θ were conjectured, to validate the Poole´s Rule in the DSGE model. The first was 0.5 and the subsequent estimate resulted in 0.2657 (Table 1) value in line with Kranz (the author calibrates the value of this parameter at 0.25). For the second Prior, the value was 0.01, which implied a Posterior of 0.0297, in this case the model had counterintuitive and unlikely results[24].

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[24] Although the Priors of the other parameters did not change in both exercises, the influence of the relative weight of the monetary authority that it adopts in relation to the product gap (Θ) determines the validity of the Poole´s Rule in the entire system of equations, the results are inadmissible from the second Prior, see appendix.

Table 1: Prior and posterior distribution

Parámetro	Prior Mean	Post Mean	10%	90%	Distribución	S.D.
σ	2	2.0595	2.0595	2.0674	norm	0.1
σ^M	2	2.4225	2.3979	2.4511	norm	0.1
Θ	0.5	0.2657	0.2359	0.2885	beta	0.1
ρ^{π}	0.5	0.5229	0.5174	0.5270	beta	0.1
ρ^m	0.5	0.2853	0.2315	0.3183	beta	0.1
ρ^d	0.5	0.9522	0.9512	0.9529	beta	0.1
ρ^A	0.5	0.2985	0.2555	0.3220	beta	0.1
ρ^{i^n}	0.5	0.4702	0.4633	0.4752	beta	0.1
ε^A	0.01	0.6835	0.6396	0.7218	invg	Inf
ε^{π}	0.01	0.1583	0.1551	0.1618	invg	Inf
ε^{i^n}	0.01	0.0085	0.0031	0.0153	invg	Inf
ε^m	0.01	0.6947	0.6645	0.7292	invg	Inf
ε^d	0.01	0.0688	0.0612	0.0760	invg	Inf

Note: The Prior value of σ and σ^M were reviewed by Benchimol (2013), the initial value (Prior) of the persistence parameters of the AR processes (1) was extracted from Smets and Wouters (2007) but their standard deviation is from Benchimol. Finally, the standard deviations and the distribution function are from Julliard M. et al. (2006) and Valdivia J. (2017).

The other parameters, we decided to calibrate based on previous research and national accounts**.**

Table 2: Calibration

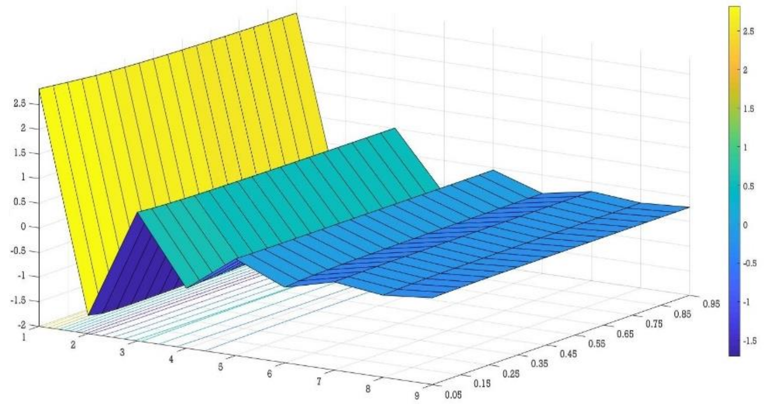
Parameter	Source	Value
β	Valdivia D. (2008)	0.88
θ	Costa Junior (2016)	0.7
η	Costa Junior (2016)	1.5
α	Valdivia J. (2017)	0.33
δ	Kliem y Kriwoluzky (2016)	0.025
$\frac{C_{ss}}{Y_{ss}}$	Consumption/ GDP ratio (2018). National Accounts	0.7
$\frac{I_{ss}}{Y_{ss}}$	Gross Fixed Capital Formation/GDP ratio (2018). National Accounts	0.2

Simulation

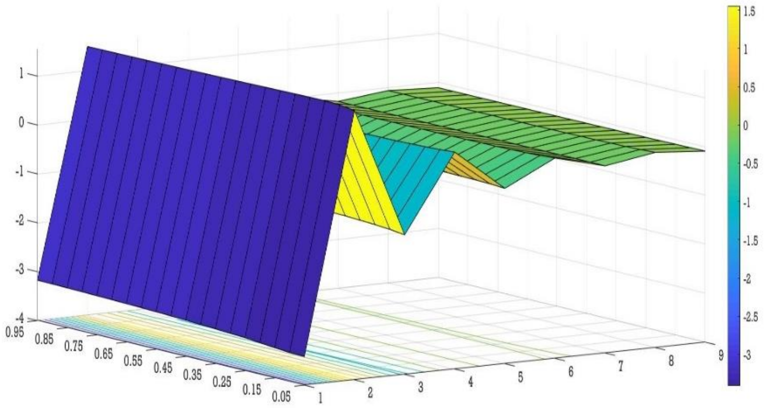
Depending on the value assigned by the monetary authority to Zeta (Θ), the IRF response may change substantially. A simple simulation was performed with respect to the value of Zeta, the results indicate that the effects of the shocks may change when the monetary authority weights in greater proportion the fluctuations of the product observed with respect to the natural one (the exercise was carried out by shocks in the NKPC and in the Poole´s Rule).

Figure 3: Numerical Simulation (different values of Θ)

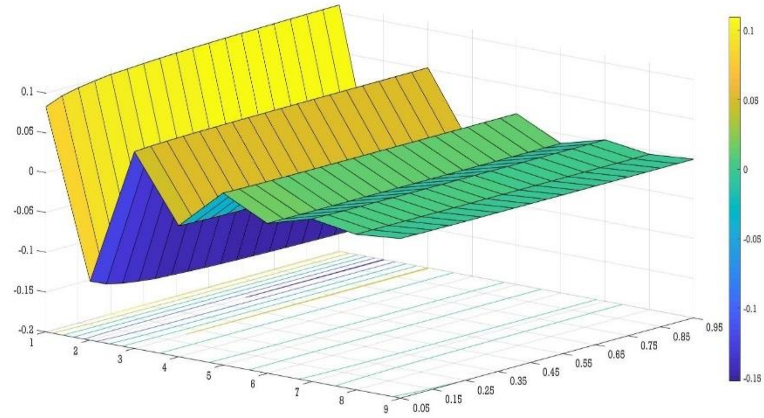
Consumption response to shocks (Cost-Push Inflation)



GDP response to shocks in the Phillips Curve (Cost-Push Inflation)



GDP response to shocks in Poole´s Rule



When monetary authority is more concerned with the deviation of the observed product with respect to its natural state (Θ = 0.95), consumption reacts positively against shocks in the NKPC but to a lesser extent than a minimum Zeta weighting (Θ = 0.05)[25]. Likewise, GDP`s response in front to the nature of this shock is still contractive, however, the value of Zeta influences the magnitude of the IRF, the contraction of the product reaches 3.4pp to smaller values Zeta (Θ) but when Zeta converges to 0.95 the decrease in GDP is reduced to 3.1pp. Finally, the expansive

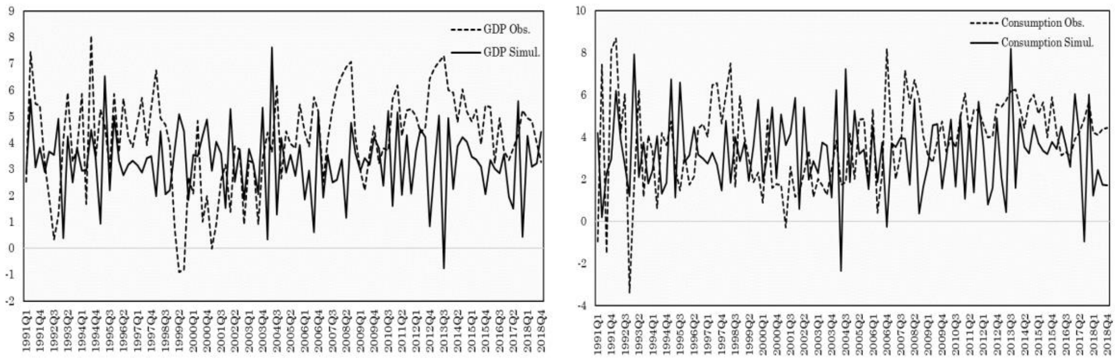
[25] The IRF of consumption increases in 2.5pp when Zeta (Θ = 0.05), but when Zeta (Θ = 0.95), consumption only increases by 2pp.

behavior of monetary policy (shocks in Poole´s Rule, $\varepsilon_t^{\widetilde{m}}$) is more effective in economic growth when the same authority "worries" more about the product gap.

Finally, a complementary exercise for the validity of proposed model is obtaining simulated data from DSGE. The simulation of 120 observations, of GDP and Consumption reveal that the model partially replicates the behavior of the observed variables and certain stylized facts of the Bolivian economy.

Figure 4: Data simulation

GDP Consumption



IV) Conclusions

In this paper, a monetary policy rule with microfundaments, Poole`s Rule is elucidated. In the current literature in the field of macroeconomics there is no such rule based on a loss function that a Central Bank has as its objective. A first approximation is made by Li and Liu (2017) for the Chinese economy, applying a rule that they call the "Taylor`s Rule expanded", but the ad-hoc equation has a similarity to the Poole Rule we find. The debate on the application of this rule was generated between the 70's and the late 80's, authors such as Turnovsky (1975), Woglom (1979), Yoshikawa (1981), Cazoneri et al. (1983), Daniel (1986) and Fair (1987) confirm the findings of the mainstream publication of Poole (1970). All authors converge on a common point of view on the rule called as a "combination" of control of the stock of money and setting the interest rate, this instrument is appropriate to control the volatility of the product with respect to its natural state. However, as Poole points out the monetary policy rule and its effectiveness depends on values of certain parameters can assume, essentially the elasticity of money demand with respect to changes in the interest rate, the income effect elasticity of money demand and the standard deviation of shocks (stochastic variables) raised in their model; from an econometric evaluation by Turnovsky, Yoshikawa and Fair, they ratify Poole's arguments. Turnovsky indicates that pro-cyclical adjustments of the money supply are an optimal instrument under uncertainty of the parameters of the IS-LM model. Yoshikawa points out that the monetary authority must adapt to shocks, and depending on their nature, monetary policy changes its instrument, controlling the money supply to interest rates or vice versa. Finally, Fair's conclusions are that both instruments are optimal for reducing the variance of the Gross National Product.

In the preliminary exercise for the Bolivian economy, some parameters were estimated and calibrated, in the loss minimization function of the Central Bank the Prior of Zeta (Θ) has a relevant influence on the validity of Poole`s Rule, the results indicate that the Central Bank of Bolivia (BCB) weighs 0.2657 of aversion in relation to fluctuations in the product gap, this corollary is in line with Kranz (2017). Thanks to the estimation of parameters, the Impulse Response Functions in analogy with shocks from the Poole Rule have positive effects on economic growth (0.44pp) confirming the BCB's expansive position. The BCB's response to shocks in the NKPC is right in the proposed model (decrease in the money supply and increases in interest rate).

Zeta values simulation (Θ) intuitively approximates the orientation of the monetary policy of any Central Bank. When monetary authority ponders even more the deviation of the observed product with respect to its natural state (product gap), the effects in real sector are greater (GDP). Shocks in the NKPC although they contract economic growth, the result is lower when Zeta ($\Theta \cong 0.95$); and in the same way consumption reacts positively but to a lesser extent thanks to the monetary authority stabilizing agents' expectations. In the exercise carried out, the Poole`s Rule responds to structure Bolivia economy based on the characteristics of households and firms.

$$\tilde{m}_t = \frac{\sigma}{\sigma^M}\tilde{x}_t + \frac{\sigma}{\sigma^M}E_t\tilde{Y}_{t+1} - \frac{1}{\sigma^M}(1+\beta)\,\tilde{\imath}_t + \frac{1}{\sigma^M}\left[1 + \frac{\sigma\beta\varrho}{\varrho\kappa\varphi + \Theta}\right]E_t\tilde{\pi}_{t+1} + \frac{\sigma\varrho\kappa\gamma\varpi}{\sigma^M[\varrho\kappa\varphi + \Theta]}\left(\tilde{x}_t^i\right)$$

$$\tilde{m}_t = \Upsilon\tilde{x}_t + \Upsilon E_t\tilde{Y}_{t+1} - \Phi\,\tilde{\imath}_t + \Gamma E_t\tilde{\pi}_{t+1} + \xi\tilde{x}_t^i$$

Where:

$$\Upsilon = 0.85015480$$

$$\Phi = 0.77605779$$

$$\Gamma = 0.67942925$$

$$\xi = 0.02325362$$

The control of supply or demand money reacts in 0.85pp to the output gap and to the expectations of the economic growth, inversely proportional to the interest rate that is determined by market (0.77pp), with respect to the inflationary expectations in 0.67 pp and finally with 0.02pp in relation to the deviations of the interest rate with respect to its natural state.

Poole mentioned that the parameters will not necessarily remain fixed when there is interaction with fiscal policy (fiscal result). This indicates that there is the challenge of evaluating the Poole`s Rule with the introduction of other agents in the economy: Fiscal Policy, Financial Sector, Household Heterogeneity, External Sector, Informality, Insertion of Costs of Adjustment to Capital and Investment, between others. The most appropriate for estimation of parameters the would be by time varying parameters or a model with regime switching to more conveniently extract the characteristics of an economy that is not defined in a targeting inflation scheme.

In conclusion, the objective of the investigation was to provide a theoretical contribution of an unconventional rule for the management of monetary policy. Under the preliminary exercise, Poole´s Rule for the Bolivian economy was validated, capturing certain characteristics.

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Appendix

Obtaining the New Keynesian Phillips Curve (NKPC) Log-Linear version:

$$\max_{P_{j,t}^*} E_t \left\{ \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} \left[\frac{P_{j,t}^*}{P_{t+i}} \, C_{j,t+i} - mc_{t+i} \, C_{j,t+i} \right] \right\}$$

The constraint is:

$$C_{j,t} = \left[\frac{P_{j,t}^*}{P_t} \right]^{-\varepsilon} \, C_t$$

$$\max_{P_{j,t}^*} E_t \left\{ \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} \left[\frac{P_{j,t}^*}{P_{t+i}} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} - mc_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \right\}$$

$$\max_{P_{j,t}^*} E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} \left[\left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{1-\varepsilon} - mc_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} \right]$$

FOC:

$$E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} \left[(1-\varepsilon) \left(\frac{1}{P_{t+i}} \right)^{1-\varepsilon} P_{j,t}^{*\,-\varepsilon} + \varepsilon \, mc_{t+i} \left(\frac{1}{P_{t+i}} \right)^{-\varepsilon} P_{j,t}^{*\,-\varepsilon-1} \right] = 0$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} P_{j,t}^{*\,-\varepsilon} \left[(1-\varepsilon) \left(\frac{1}{P_{t+i}} \right)^{1-\varepsilon} + \varepsilon \, mc_{t+i} \left(\frac{1}{P_{t+i}} \right)^{-\varepsilon} \frac{1}{P_{j,t}^*} \right] = 0$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} P_{j,t}^{*\,-\varepsilon} \left(\frac{1}{P_{t+i}} \right)^{-\varepsilon} \left[(1-\varepsilon) \, \frac{1}{P_{t+i}} + \varepsilon \, mc_{t+i} \frac{1}{P_{j,t}^*} \right] = 0$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} \left[(1-\varepsilon) \, \frac{1}{P_{t+i}} + \varepsilon \, mc_{t+i} \frac{1}{P_{j,t}^*} \right] = 0$$

$$E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} (1-\varepsilon) \, \frac{1}{P_{t+i}} + E_t \sum_{i=0}^{\infty} \theta^i \Delta_{i,t+i} C_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} \varepsilon \, mc_{t+i} \frac{1}{P_{j,t}^*} = 0$$

$$\Delta_{i,t+i} = \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma}$$

$$E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} C_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} (1-\varepsilon) \, \frac{1}{P_{t+i}} = -E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} C_{t+i} \left(\frac{P_{j,t}^*}{P_{t+i}} \right)^{-\varepsilon} \varepsilon \, mc_{t+i} \frac{1}{P_{j,t}^*}$$

$$(1-\varepsilon) \frac{P_{j,t}^{*\,-\varepsilon}}{C_t^{-\sigma}} E_t \sum_{i=0}^{\infty} (\theta\beta)^i \frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{1-\varepsilon}} = -\,\varepsilon \frac{P_{j,t}^{*\,-\varepsilon}}{C_t^{-\sigma}} \frac{1}{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\theta\beta)^i \frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{-\varepsilon}} mc_{t+i}$$

$$\varepsilon \frac{1}{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\theta\beta)^i \frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{-\varepsilon}} mc_{t+i} = -(1-\varepsilon) E_t \sum_{i=0}^{\infty} (\theta\beta)^i \frac{C_{t+i}^{1-\sigma}}{P_{t+i}^{1-\varepsilon}}$$

$$\varepsilon \frac{1}{P_{j,t}^*} E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon} \, mc_{t+i} = (\varepsilon-1) E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} \, P_{t+i}^{\varepsilon-1}$$

$$\frac{\varepsilon}{(\varepsilon-1)} \frac{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon} \, mc_{t+i}}{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} \, P_{t+i}^{\varepsilon-1}} = P_{j,t}^*$$

$$P_{j,t}^{*} = \mu \frac{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon}\, mc_{t+i}}{E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma}\, P_{t+i}^{\varepsilon-1}}$$

$$P_{j,t}^{*} = \mu \frac{\mathrm{A}_t}{\mathrm{B}_t}$$

$$P_{ss}^{*}\left(1 + \tilde{P}_{j,t}^{*}\right) = \mu \frac{\mathrm{A}_{ss}}{\mathrm{B}_{ss}}\left(1 + \widetilde{\mathrm{A}}_t - \widetilde{\mathrm{B}}_t\right)$$

$$\tilde{P}_{j,t}^{*} = \widetilde{\mathrm{A}}_t - \widetilde{\mathrm{B}}_t$$

Price dynamics

$$P_t^{1-\varepsilon} = (1-\theta) {P_{j,t}^{*}}^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}$$

$$\left[\frac{P_t}{P_{t-1}}\right]^{1-\varepsilon} = (1-\theta)\frac{{P_{j,t}^{*}}^{1-\varepsilon}}{P_{t-1}^{1-\varepsilon}} + \theta$$

$$\pi_t^{1-\varepsilon} = \theta + (1-\theta)\left[\frac{P_{j,t}^{*}}{P_{t-1}}\right]^{1-\varepsilon}$$

$$\pi_{ss}^{1-\varepsilon}[1 + (1-\varepsilon)\tilde{\pi}_t] = \theta + (1-\theta)\left[\frac{P_{ss}}{P_{ss}}\right]^{1-\varepsilon}\left[1 + (1-\varepsilon)\widetilde{P_{j,t}^{*}} - (1-\varepsilon)\tilde{P}_{t-1}\right]$$

$$1 + (1-\varepsilon)\tilde{\pi}_t = \theta + \left[1 + (1-\varepsilon)\tilde{P}_{j,t}^{*} - (1-\varepsilon)\tilde{P}_{t-1}\right] - \theta\left[1 + (1-\varepsilon)\tilde{P}_{j,t}^{*} - (1-\varepsilon)\tilde{P}_{t-1}\right]$$

$$1 + (1-\varepsilon)\tilde{\pi}_t = \theta + 1 + (1-\varepsilon)\tilde{P}_{j,t}^{*} - (1-\varepsilon)\tilde{P}_{t-1} - \theta - \theta(1-\varepsilon)\tilde{P}_{j,t}^{*} + \theta(1-\varepsilon)\tilde{P}_{t-1}$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\tilde{P}_{j,t}^{*} - (1-\varepsilon)\tilde{P}_{t-1} - \theta(1-\varepsilon)\tilde{P}_{j,t}^{*} + \theta(1-\varepsilon)\tilde{P}_{t-1}$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\left(\tilde{P}_{j,t}^{*} - \tilde{P}_{t-1}\right) - \theta(1-\varepsilon)\left(\widetilde{P_{j,t}^{*}} - \tilde{P}_{t-1}\right)$$

$$(1-\varepsilon)\tilde{\pi}_t = (1-\varepsilon)\left(\tilde{P}_{j,t}^{*} - \tilde{P}_{t-1}\right)(1-\theta)$$

$$\tilde{\pi}_t = \left(\tilde{P}_{j,t}^{*} - \tilde{P}_{t-1}\right)(1-\theta)$$

$$\frac{\tilde{\pi}_t}{1-\theta} + \tilde{P}_{t-1} = \tilde{P}_{j,t}^{*} \tag{a}$$

Rewriting the previous expression and replacing $\tilde{P}_{j,t}^{*}$, in $\frac{\tilde{\pi}_t}{1-\theta} + \tilde{P}_{t-1} = \widetilde{\mathrm{A}}_t - \tilde{B}_t$

$$\mathrm{A}_t = E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon}\, mc_{t+i}$$

$$\mathrm{A}_t = C_t^{1-\sigma} P_t^{\varepsilon} mc_t + E_t \sum_{i=1}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon} mc_{t+i}$$

$$\mathrm{A}_t = C_t^{1-\sigma} P_t^{\varepsilon} mc_t + \theta\beta\, C_{t+1}^{1-\sigma} P_{t+1}^{\varepsilon}\, mc_{t+1} + E_t \sum_{i=2}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon} mc_{t+i}$$

$$\mathrm{A}_t = C_t^{1-\sigma} P_t^{\varepsilon} mc_t + \theta\beta\, C_{t+1}^{1-\sigma} P_{t+1}^{\varepsilon}\, mc_{t+1} + (\theta\beta)^2\, C_{t+2}^{1-\sigma} P_{t+2}^{\varepsilon}\, mc_{t+2} + E_t \sum_{i=3}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon} mc_{t+i}$$

$$\mathrm{A}_t = C_t^{1-\sigma} P_t^{\varepsilon} mc_t + \theta\beta E_t \mathrm{A}_{t+1}$$

$$\mathrm{B}_t = E_t \sum_{i=0}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon-1}$$

$$\mathrm{B}_t = C_t^{1-\sigma} P_t^{\varepsilon-1} + E_t \sum_{i=1}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon-1}$$

$$\mathrm{B}_t = C_t^{1-\sigma} P_t^{\varepsilon-1} + \theta\beta\, C_{t+1}^{1-\sigma} P_{t+1}^{\varepsilon-1} + E_t \sum_{i=2}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon-1}$$

$$\mathrm{B}_t = C_t^{1-\sigma} P_t^{\varepsilon} + \theta\beta\, C_{t+1}^{1-\sigma} P_{t+1}^{\varepsilon-1} + (\theta\beta)^2\, C_{t+2}^{1-\sigma} P_{t+2}^{\varepsilon-1} + E_t \sum_{i=3}^{\infty} (\theta\beta)^i C_{t+i}^{1-\sigma} P_{t+i}^{\varepsilon-1}$$

$$\mathrm{B}_t = C_t^{1-\sigma} P_t^{\varepsilon-1} + \theta\beta E_t \mathrm{B}_{t+1}$$

$$\mathrm{A}_{ss} = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon} mc_{ss} + \theta\beta \mathrm{A}_{ss} \qquad \mathrm{B}_{ss} = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon-1} + \theta\beta \mathrm{B}_{ss}$$

$$\mathrm{A}_{ss}(1-\theta\beta) = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon} mc_{ss} \qquad \mathrm{B}_{ss}(1-\theta\beta) = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon-1}$$

$$\mathrm{A}_t = C_t^{1-\sigma} P_t^{\varepsilon} mc_t + \theta\beta E_t \mathrm{A}_{t+1}$$

$$\mathrm{A}_{ss}\left(1+\widetilde{\mathrm{A}}_t\right) = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon} mc_{ss}\left[1 + (1-\sigma)\tilde{C}_t + \varepsilon \tilde{P}_t + \widetilde{mc}_t\right] + \theta\beta \mathrm{A}_{ss}\left(1 + E_t \widetilde{\mathrm{A}}_{t+1}\right)$$

$$\mathrm{A}_{ss}\left(1+\widetilde{\mathrm{A}}_t\right) = \mathrm{A}_{ss}(1-\theta\beta)\left[1 + (1-\sigma)\tilde{C}_t + \varepsilon \tilde{P}_t + \widetilde{mc}_t\right] + \theta\beta \mathrm{A}_{ss}\left(1 + E_t \widetilde{\mathrm{A}}_{t+1}\right)$$

$$\begin{aligned}\mathrm{A}_{ss}\left(1+\widetilde{\mathrm{A}}_t\right) = \mathrm{A}_{ss}\{&(1-\theta\beta)\left[1 + (1-\sigma)\tilde{C}_t + \varepsilon \tilde{P}_t + \widetilde{mc}_t - \theta\beta - \theta\beta(1-\sigma)\tilde{C}_t - \theta\beta\varepsilon\tilde{P}_t\right.\\ &\left.- \theta\beta\widetilde{mc}_t\right] + \theta\beta\left(1 + E_t\widetilde{\mathrm{A}}_{t+1}\right)\}\end{aligned}$$

$$1 + \widetilde{\mathrm{A}}_t - \theta\beta - \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} = 1 + (1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t + \widetilde{mc}_t - \theta\beta - \theta\beta(1-\sigma)\tilde{C}_t - \theta\beta\varepsilon\tilde{P}_t - \theta\beta\widetilde{mc}_t$$

$$\widetilde{\mathrm{A}}_t - \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} = (1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t + \widetilde{mc}_t - \theta\beta(1-\sigma)\tilde{C}_t - \theta\beta\varepsilon\tilde{P}_t - \theta\beta\widetilde{mc}_t$$

$$\widetilde{\mathrm{A}}_t - \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} - (1-\theta\beta)\widetilde{mc}_t = \left[(1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t\right](1-\theta\beta) \qquad \text{(b)}$$

$$\mathrm{B}_t = C_t^{1-\sigma} P_t^{\varepsilon-1} + \theta\beta E_t \mathrm{B}_{t+1}$$

$$\mathrm{B}_{ss}\left(1+\tilde{B}_t\right) = C_{ss}^{1-\sigma} P_{ss}^{\varepsilon-1}\left[1 + (1-\sigma)\tilde{C}_t + (\varepsilon-1)\tilde{P}_t\right] + \theta\beta \mathrm{B}_{ss}\left(1 + E_t \widetilde{\mathrm{B}}_{t+1}\right)$$

$$\mathrm{B}_{ss}\left(1+\tilde{B}_t\right) = \mathrm{B}_{ss}(1-\theta\beta)\left[1 + (1-\sigma)\tilde{C}_t + (\varepsilon-1)\tilde{P}_t\right] + \theta\beta \mathrm{B}_{ss}\left(1 + E_t \widetilde{\mathrm{B}}_{t+1}\right)$$

$$1 + \widetilde{\mathrm{B}}_t - \theta\beta - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} = 1 + (1-\sigma)\tilde{C}_t + (\varepsilon-1)\tilde{P}_t - \theta\beta - \theta\beta(1-\sigma)\tilde{C}_t - \theta\beta(\varepsilon-1)\tilde{P}_t$$

$$\widetilde{\mathrm{B}}_t - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} = (1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t - \tilde{P}_t - \theta\beta(1-\sigma)\tilde{C}_t - \varepsilon\theta\beta\tilde{P}_t + \theta\beta\tilde{P}_t$$

$$\widetilde{\mathrm{B}}_t - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} = \left[(1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t\right](1-\theta\beta) - \tilde{P}_t(1-\theta\beta)$$

$$\widetilde{\mathrm{B}}_t - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} + \tilde{P}_t(1-\theta\beta) = \left[(1-\sigma)\tilde{C}_t + \varepsilon\tilde{P}_t\right](1-\theta\beta) \qquad \text{(c)}$$

Matching (b) and (c):

$$\widetilde{\mathrm{B}}_t - \theta\beta E_t \widetilde{\mathrm{B}}_{t+1} + \tilde{P}_t(1-\theta\beta) = \widetilde{\mathrm{A}}_t - \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} - (1-\theta\beta)\widetilde{mc}_t$$

$$-\theta\beta E_t \widetilde{\mathrm{B}}_{t+1} + \tilde{P}_t(1-\theta\beta) + \theta\beta E_t \widetilde{\mathrm{A}}_{t+1} + (1-\theta\beta)\widetilde{mc}_t = \widetilde{\mathrm{A}}_t - \widetilde{\mathrm{B}}_t$$

$$\theta\beta E_t\left(\widetilde{\mathrm{A}}_{t+1} - \widetilde{\mathrm{B}}_{t+1}\right) + (1-\theta\beta)\left(\widetilde{mc}_t + \tilde{P}_t\right) = \widetilde{\mathrm{A}}_t - \widetilde{\mathrm{B}}_t$$

Rewriting in price dynamics $\frac{\tilde{\pi}_t}{1-\theta} + \tilde{P}_{t-1} = \widetilde{\mathrm{A}}_t - \widetilde{\mathrm{B}}_t$, we obtain:

$$\theta\beta\left(\frac{E_t\tilde{\pi}_{t+1}}{1-\theta}+\tilde{P}_t\right)+(1-\theta\beta)\left(\widetilde{mc}_t+\tilde{P}_t\right)=\frac{\tilde{\pi}_t}{1-\theta}+\tilde{P}_{t-1}$$

$$\theta\beta\frac{E_t\tilde{\pi}_{t+1}}{1-\theta}+\theta\beta\tilde{P}_t+\widetilde{mc}_t+\tilde{P}_t-\theta\beta\,\widetilde{mc}_t-\theta\beta\tilde{P}_t-\tilde{P}_{t-1}=\frac{\tilde{\pi}_t}{1-\theta}$$

$$\frac{\tilde{\pi}_t}{1-\theta}=\theta\beta\frac{E_t\tilde{\pi}_{t+1}}{1-\theta}+\widetilde{mc}_t(1-\theta\beta)\ +\tilde{\pi}_t$$

$$\tilde{\pi}_t=\theta\beta E_t\tilde{\pi}_{t+1}+(1-\theta)(1-\theta\beta)\widetilde{mc}_t+\tilde{\pi}_t-\theta\tilde{\pi}_t$$

$$\tilde{\pi}_t=\beta E_t\tilde{\pi}_{t+1}+\frac{(1-\theta)(1-\theta\beta)}{\theta}\widetilde{mc}_t$$

$$\boldsymbol{\tilde{\pi}_t=\beta E_t\tilde{\pi}_{t+1}+\kappa\,\widetilde{mc}_t}$$

Where $\kappa=\frac{(1-\theta)(1-\theta\beta)}{\theta}$

Log-linearization of Euler`s equation

The transformation around steady state we take into account this version of the Fisher`s equation $\tilde{R}_t=\tilde{\imath}_t-E_t\tilde{\pi}_{t+1}$in Log-linear version.

$$C_t^{-\sigma}=\beta E_tC_{t+1}^{-\sigma}\frac{(1+i_t)}{(1+\pi_{t+1})}$$

$$C_t^{-\sigma}=\beta E_t\ C_{t+1}^{-\sigma}(1+R_t)$$

$$C_t^{-\sigma}=\beta E_t\ C_{t+1}^{-\sigma}\ R_t$$

$$C_{ss}^{-\sigma}\left(1-\sigma\tilde{C}_t\right)=\beta C_{ss}^{-\sigma}\ R_{ss}\left(1-\sigma E_t\tilde{C}_{t+1}+\tilde{R}_t\right)$$

$$\left(1-\sigma\tilde{C}_t\right)=\left(1-\sigma E_t\tilde{C}_{t+1}+\tilde{R}_t\right)$$

$$\boldsymbol{\tilde{C}_t=E_t\tilde{C}_{t+1}-\frac{1}{\sigma}(\tilde{\imath}_t-E_t\tilde{\pi}_{t+1})}$$

Log-linearization of Money Demand

$$m_t^{\sigma^M}=C_t^{\sigma}\frac{(1+i_t)}{i_t}$$

$$m_{ss}^{\sigma^M}=\frac{C_{ss}^{\sigma}}{i_{ss}}+C_{ss}^{\sigma}=C_{ss}^{\sigma}\left[\frac{1}{i_{ss}}+1\right];\ \frac{1}{i_{ss}}+1=\frac{m_{ss}^{\sigma^M}}{C_{ss}^{\sigma}}$$

$$m_{ss}^{\sigma^M}(1+\sigma^M\tilde{m}_t)=\frac{C_{ss}^{\sigma}}{i_{ss}}\left(1+\sigma\tilde{C}_t-\tilde{\imath}_t\right)+C_{ss}^{\sigma}\left(1+\sigma\tilde{C}_t\right)$$

$$\frac{m_{ss}^{\sigma^M}}{C_{ss}^{\sigma}}(1+\sigma^M\tilde{m}_t)=\frac{1}{i_{ss}}\left(1+\sigma\tilde{C}_t-\tilde{\imath}_t\right)+\left(1+\sigma\tilde{C}_t\right)$$

$$\left[\frac{1}{i_{ss}}+1\right](1+\sigma^M\tilde{m}_t)=\frac{1}{i_{ss}}\left(1+\sigma\tilde{C}_t-\tilde{\imath}_t\right)+\left(1+\sigma\tilde{C}_t\right)$$

$$\frac{1}{1-\beta}(1+\sigma^M\tilde{m}_t)=\frac{\beta}{1-\beta}\left(1+\sigma\tilde{C}_t-\tilde{\imath}_t\right)+\left(1+\sigma\tilde{C}_t\right)$$

$$1+\sigma^M\tilde{m}_t=\beta+\beta\sigma\tilde{C}_t-\beta\tilde{\imath}_t+1+\sigma\tilde{C}_t-\beta-\beta\sigma\tilde{C}_t$$

$$\boldsymbol{\tilde{m}_t=\frac{\sigma}{\sigma^M}\tilde{C}_t-\frac{\beta}{\sigma^M}\tilde{\imath}_t}$$

Log-linearization of Labor Supply

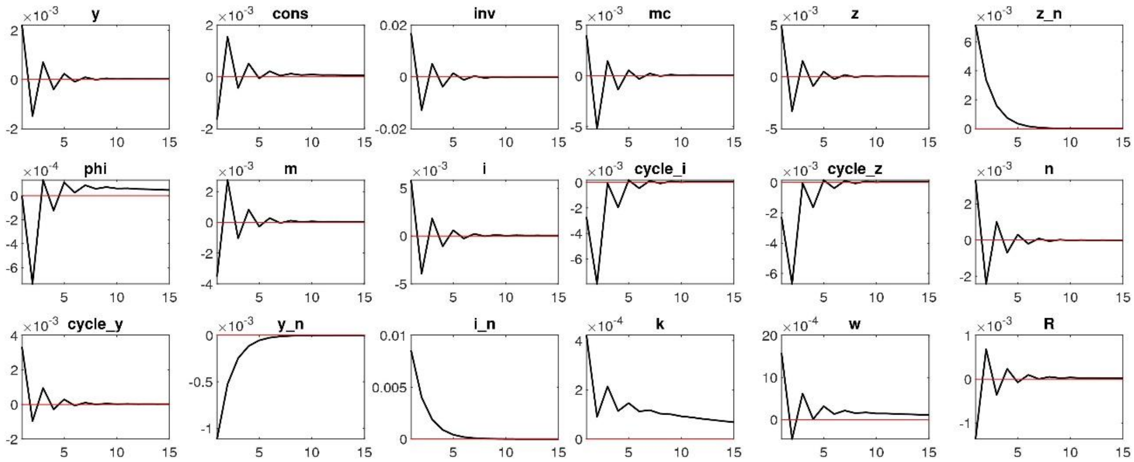
$$\zeta N_t^{\eta} = \frac{W_t}{P_t} C_t^{-\sigma}$$

$$\zeta N_{ss}^{\eta}\left(1 + \eta \tilde{N}_t\right) = w_{ss} C_{ss}^{-\sigma}\left(1 + \tilde{w}_t - \sigma \tilde{C}_t\right)$$

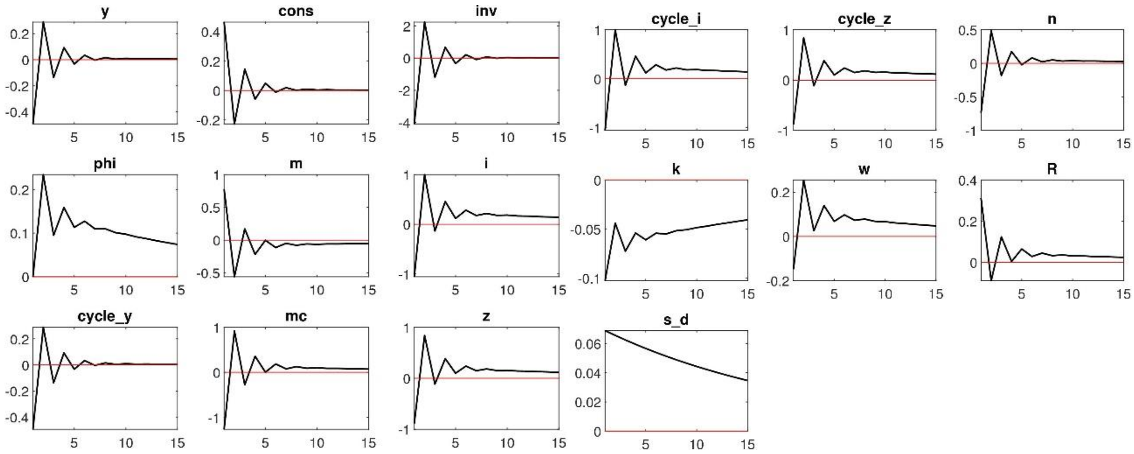
$$\boldsymbol{\eta \tilde{N}_t + \sigma \tilde{C}_t = \tilde{w}_t}$$

Impulse Response Function (with Prior of Θ = 0.5)

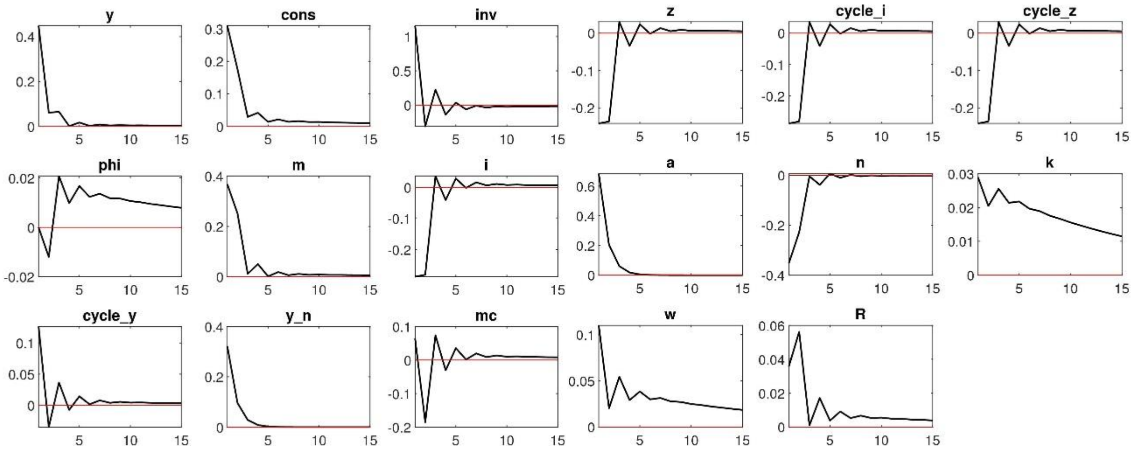
Shocks in the Natural Interest Rate ($\varepsilon_t^{\tilde{i}^f}$)



Aggregate Demand Shocks ($\varepsilon_t^{\widetilde{AD}}$)

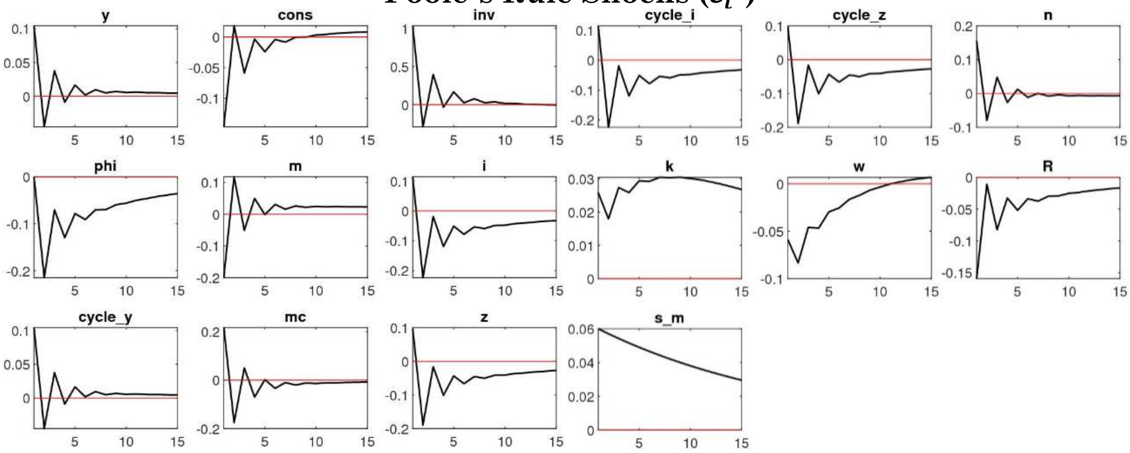


Shocks in Productivity Total Factors ($\varepsilon_t^{\tilde{A}}$)

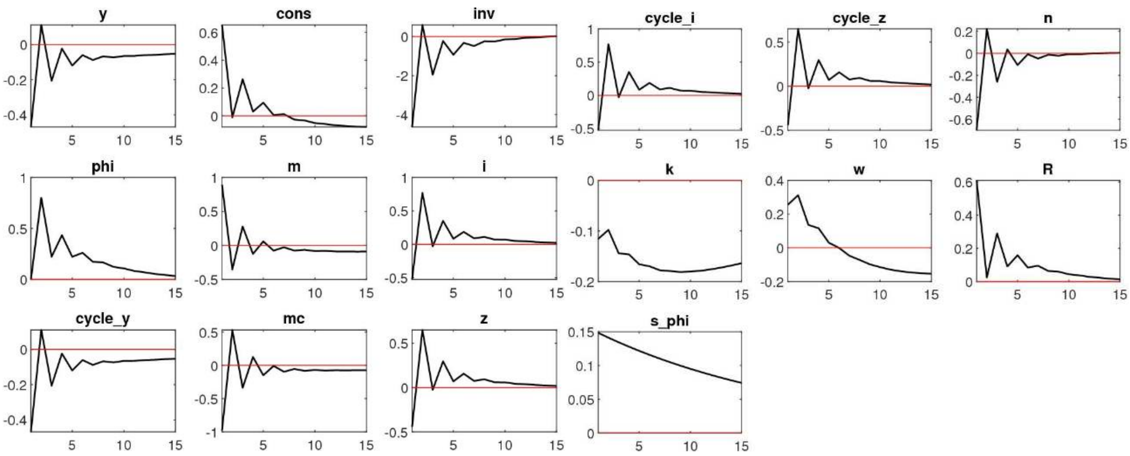


Impulse Response Function (with Prior of Θ = 0.01)

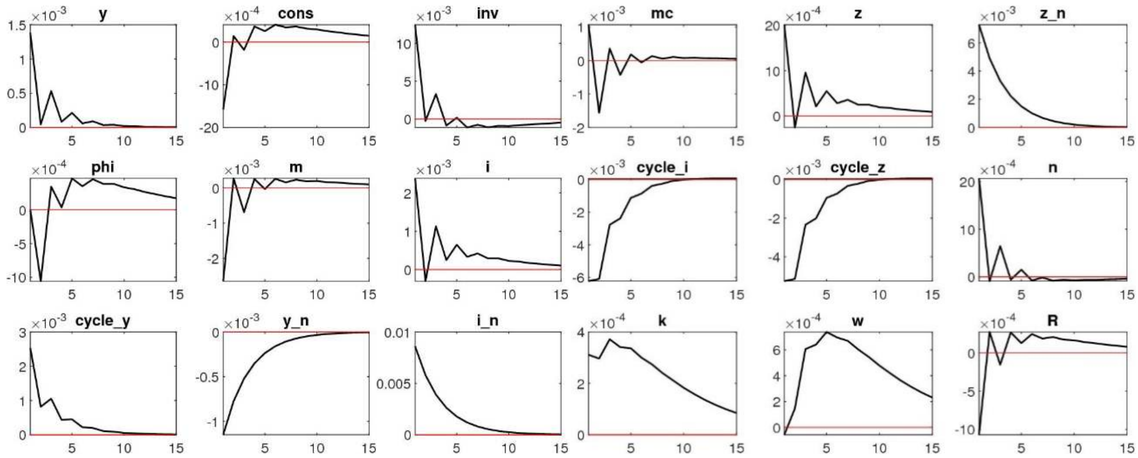
Poole´s Rule Shocks ($\varepsilon_t^{\tilde{m}}$)



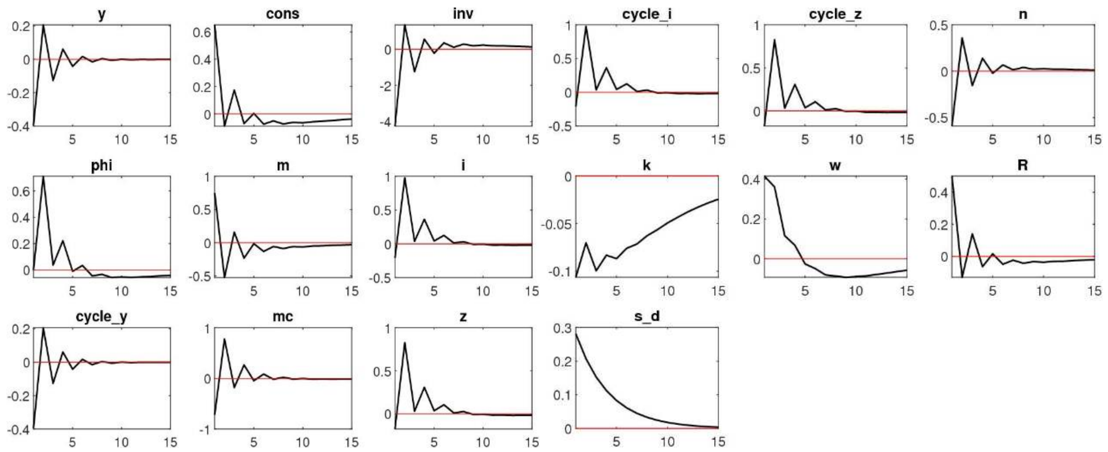
Cost Push Inflation ($\varepsilon_t^{\tilde{\pi}}$)



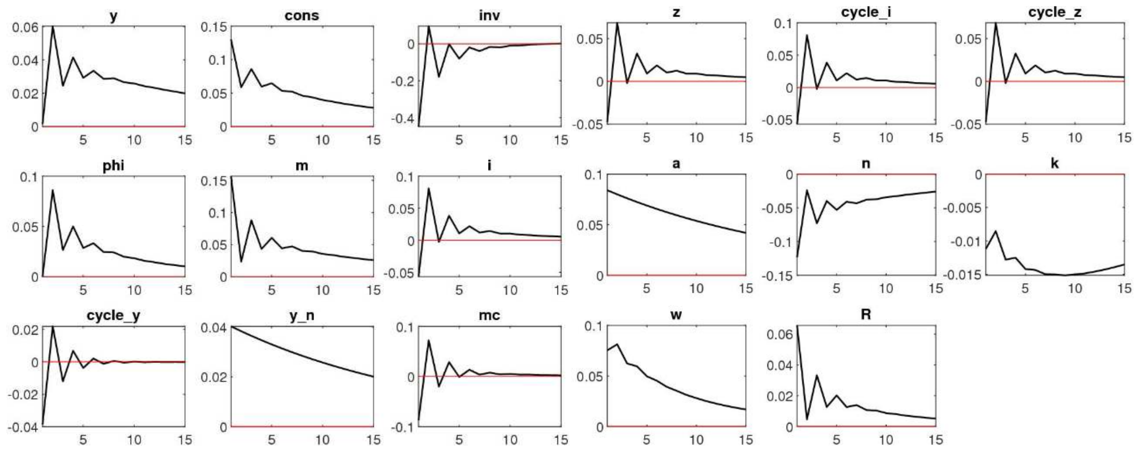
Shocks in the Natural Interest Rate ($\varepsilon_t^{\tilde{i}^f}$)



Aggregate Demand Shocks ($\varepsilon_t^{\widetilde{AD}}$)



Shocks in Productivity Total Factors ($\varepsilon_t^{\tilde{A}}$)



Estimation Methodology

The parameters of model were evaluated with an econometric methodology from the bayesian point of view to measure the effect of the shocks raised previously in the observed variables. The bayesian econometric approach provides much more information to the decisions under uncertainty, unlike the classic "frequentist" econometrics, this approach considers different types of information often subjective, which may have on the parameters to estimate before taking into account the data. Bayesian estimation can be seen as a bridge between calibration and maximum likelihood estimation (MV).

The estimated model is based on Fernández-Villaverde and Rubio-Ramírez (2004) and Smets and Wouter (2007). The estimation is based on a likelihood function generated by the solution of the log-linearized version of the model. Prior distributions of the parameters of interest are used to provide additional information in the estimate. The whole set of linearized equations form a system of linear equations of rational expectations, which can be written as follows:

$$\Gamma_0(\vartheta)\ \mathrm{z}_t = \Gamma_1(\vartheta)\ \mathrm{z}_{t-1} + \Gamma_2(\vartheta)\ \varepsilon_t + \Gamma_3(\vartheta)\ \Theta_t$$

Where z_t is a vector that contains the variables of the model expressed as logarithmic deviations of its stationary states, ε_t is a vector that contains white noise from the exogenous shocks of the model and Θ_t is a vector that contains the rational expectations of prediction errors. The matrices Γ_1 are non-linear functions of the structural parameters contained in the vector ϑ. The vector z_t contains the endogenous variables of the model and the exogenous shocks: $\varepsilon_t^{\tilde{A}}, \varepsilon_t^{\widetilde{m}}, \varepsilon_t^{\tilde{\imath}^f}, \varepsilon_t^{\widetilde{AD}}, \varepsilon_t^{\widetilde{\pi}}$. The solution to this system can be expressed as follows:

$$\mathrm{z}_t = \Omega_z(\vartheta)\ \mathrm{z}_{t-1} + \Omega_\varepsilon(\vartheta)\ \varepsilon_t + \Gamma_3(\vartheta)\ \Theta_t$$

Ω_z and Ω_ε are functions of the structural parameters. In addition, let y_t be a vector of the observed variables, which is related to the variables in the model through a measurement equation:

$$\mathrm{y}_t = H\mathrm{z}_t$$

Where, H is a matrix that selects elements of z_t, and y_t that contain observed variables (the sample is from 1991Q1 - 2018Q4), the number of observed variables must be equal to or less than the number of shocks in the model to avoid stochastic singularity problem:

$$\mathrm{y}_t = [\tilde{Y}_t, \tilde{C}_t, \tilde{\pi}_t, \widetilde{m}_t]$$

These equations correspond to the state-space form that represent y_t. If we assume the white noise, ε_t is normally distributed, and using the Kalman filter we can calculate the conditional likelihood function for the structural parameters. $p(\vartheta)$ the prior density function of the structural parameters and $L\ (\vartheta/Y^T)$, where $Y^T = \{y_1, y_T\}$ contains the observed variables. The subsequent density function of the parameters is calculated using Bayes' theorem.

The conditional likelihood function has no solution with an analytical expression, the use of numerical methods based on the Metropolis-Hastings algorithm was made. The estimates were obtained with the Dynare 4.5.7 program.

Prios and Results

The following tables present the prior values of parameters and shocks, which are in line with international literature that incorporates beliefs about possible traits of the prior density and behavior of the variables (Juillard M et al., 2006; Smets and Wouters, 2007; Benchimol 2013 and Valdivia J., 2017).

Prior and posterior distribution (Prior $\Theta = 0.5$)

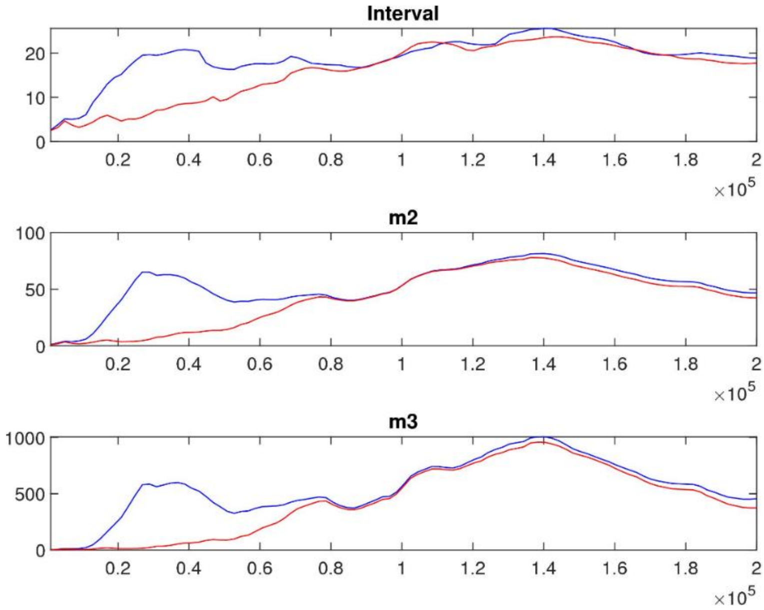
Parámetro	Prior Mean	Post Mean	10%	90%	Distribución	S.D.
σ	2	2.0595	2.0595	2.0674	norm	0.1
σ^M	2	2.4225	2.3979	2.4511	norm	0.1
Θ	0.5	0.2657	0.2359	0.2885	beta	0.1
ρ^π	0.5	0.5229	0.5174	0.5270	beta	0.1
ρ^m	0.5	0.2853	0.2315	0.3183	beta	0.1
ρ^d	0.5	0.9522	0.9512	0.9529	beta	0.1
ρ^A	0.5	0.2985	0.2555	0.3220	beta	0.1
ρ^{i^n}	0.5	0.4702	0.4633	0.4752	beta	0.1
ε^A	0.01	0.6835	0.6396	0.7218	invg	Inf
ε^π	0.01	0.1583	0.1551	0.1618	invg	Inf
ε^{i^n}	0.01	0.0085	0.0031	0.0153	invg	Inf
ε^m	0.01	0.6947	0.6645	0.7292	invg	Inf
ε^d	0.01	0.0688	0.0612	0.0760	invg	Inf

Prior and posterior distribution (Prior $\Theta = 0.01$)

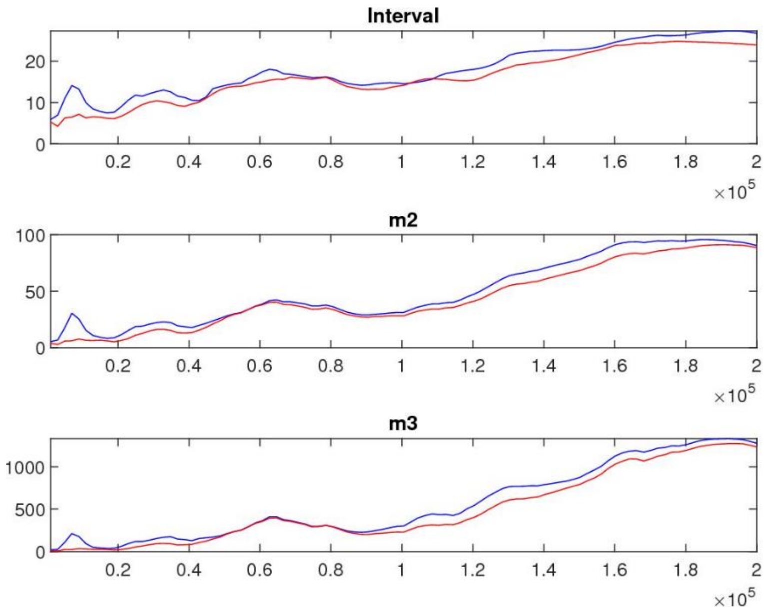
Parámetro	Prior Mean	Post Mean	10%	90%	Distribución	S.D.
σ	2	1.9978	1.9768	2.0193	norm	0.1
σ^M	2	1.9794	1.9542	2.0037	norm	0.1
Θ	0.01	0.0297	0.0270	0.2885	beta	0.01
ρ^π	0.5	0.9517	0.9502	0.9529	beta	0.1
ρ^m	0.5	0.9506	0.9478	0.9529	beta	0.1
ρ^d	0.5	0.9522	0.6851	0.7920	beta	0.1
ρ^A	0.5	0.9515	0.9498	0.9529	beta	0.1
ρ^{i^n}	0.5	0.6734	0.6531	0.6902	beta	0.1
ε^A	0.01	0.0841	0.0747	0.0932	invg	Inf
ε^π	0.01	0.1483	0.1318	0.1652	invg	Inf
ε^{i^n}	0.01	0.0086	0.0025	0.0193	invg	Inf
ε^m	0.01	0.0601	0.0522	0.0677	invg	Inf
ε^d	0.01	0.2809	0.2165	0.3555	invg	Inf

On the other hand, the convergence of the Markov-Monte Carlo Chain (MCMC) is satisfactory, implying that the multivariate analysis of the model parameters converges towards its steady state given the different iterations of the requested Metropolis Hastings (MH) algorithm (100,000 draws). There are three measures: "interval" that represents a confidence interval of 80% around the average, "m2" measures the variance and "m3" the third moment. The blue and red lines converge in a satisfactory manner (The blue lines represent measurements of the parameter vectors within the requested chains).

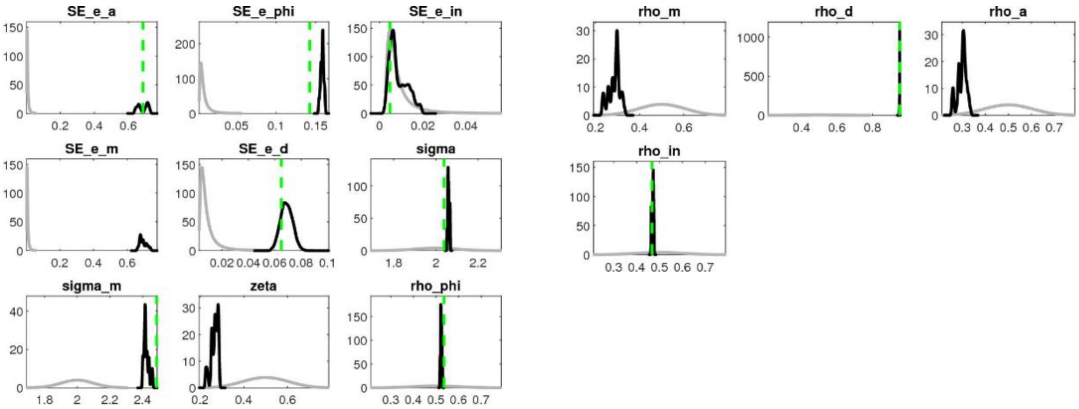
Convergence of the Markov-Monte Chain (Prior $\Theta = 0.5$)



Convergence of the Markov-Monte Chain (Prior $\Theta = 0.01$)



Priors y Posteriors density (Prior $\Theta = 0.5$)



Priors y Posteriors density (Prior $\Theta = 0.01$)