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Khalid, Asma and Beg, Ismat

Lahore School of Economics

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Asma Khalid · Ismat Beg

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Abstract The aim of this paper is to introduce the notion of truthfulness in an influence based decision making model. An expert may submit his opinions truthfully or he may dismantle the original situation by undermining the actual opinion, such a decision maker is called an evasive decision maker or an almost truthful decision maker in this paper. It is assumed that experts in the panel are dignified members hence even though they are not habitual liars, they are either "almost truthful" or evasive. To measure their degree of truthfulness, we use the information provided by them in the form of preference relations. We use this information to state the foundation of influence model of evasive decision makers. Finally, a ranking method is proposed to find best possible solutions.

Keywords Truthfulness; group decision making ; social influence networks; additive reciprocity

1 Introduction

Influence models discussed in literature are yet to involve innate human behaviors. Truthfulness is an attribute that impacts the expressed opinions of the expert. It is a fact that in all decision modeling processes, the panel of experts comprise of the most dignified and reliable personals of the society. However, since they are human beings after all, there is possibility of deviating from the

Asma Khalid
Center for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore, 53200
Tel.: +92-42-35608000
Fax: +92 42 3572 2591
E-mail: asmakhalid4444@gmail.com

Ismat Beg
Center for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore, 53200

facts and portraying a picture different from reality. With this understanding, we refer to such decision makers as "almost truthful".
In the influence based process, experts present initial opinions that are revised because of social influence [3, 5] of other group members. In [11], Liang et al describes social influence as the changes incurred by an individual after interacting with one another. Social influence network (SIN) builds on the idea that there exists interdependence among actors and their actions [15, 19, 14, 8, 13]. Some interesting work on self management mechanism for non-cooperative behaviors in large-scale group consensus reaching processes is discussed in [7]. The similarities and differences between influence based decision making method and opinion dynamic based GDM has been discussed in [25]. The notion that plays a vital role in the influence model is an $n \times n$ adjacency matrix denoted by $W = (w_{ij})$. This matrix represents the interpersonal influence degree among the experts. In this matrix, w_{ij} represents the influence of expert j on the expert i [4]. Each row of the matrix satisfy the normalization property $\sum_{j=1}^{m} \omega_{ij} = 1$ for all $i \in \{1, 2, ..., m\}$. Some experts are prone to influence while others are less susceptible to change. From the matrix W, the diagonal matrix of susceptibility of each expert is deduced and named as A. It is assumed that W is such that the influence model reaches an equilibrium and hence in the long run, the influence model provides the final opinions of all the experts [4, 6, 14, 18, 24].

In this paper, we model an influence based problem for evasive or "almost truthful" decision makers. If all experts are speaking the truth then the matrix of truthfulness becomes an identity matrix. This is a particular case, which reduces this study to the basic influence model already discussed in literature. However, if any decision makers' opinions are farther from the reality and decision makers are "almost truthful" then the regular influence model will not adequately model the situation and this is the major difference between the proposed work and the work that is already existing in literature. All around the world, there are many cases of corruption and mishaps that are undermined and misrepresented by media and political parties in power. Specifically before elections, the political parties tend to undermine the problems faced by the country to reflect that their tenure has been successful while the reality may be different. Dishonest decision makers are studied recently by Dong et. al in [7].

We assert that the revised opinions will change completely based on how truthful an expert is. Hence, this innate behavior is important and must be studied. We also assume that the choice of the decision makers is such that they are reliable people who are mostly truthful. Also, even if they defy from the truth it is evasive lies. Because of this assumption that they are not habitual liars, our measure of truthfulness belongs to the interval $[0.5, 1]$ and not the unit interval. It is assumed in this paper that if a person is not truthful, it will be evident from the preference relation provided by him. Such a decision maker would show indifference when comparing most alternatives. The reason is that he does not want to express his original preferences over alternatives. Another contribution of this paper is that decision makers are not expected to provide crisp opinions. On the other hand, given a set of alternatives, the decision makers provide preference relations to express their liking for each alternative over others. In this study, we extract important information out of the preference relation in the form of priority vectors and use it in the influence model for evasive decision makers. Since the decision makers are "almost truthful" and not complete liars, this measure starts at 0.5. Similarly, the input to the influence model is not the entire preference relation but a column vector called the priority vector deduced from the preference relation. Once, final opinions are calculated, we define a ranking method to find best possible solution from the set of alternatives. We compare the truth based influence model with the present model to show that if truthfulness is not taken into account then this may lead to policy making that my not be realistic.

The paper is arranged as follows: Section 2 states some definitions that are used in the sequel. Section 3 proposes the influence model for evasive decision makers. For this purpose, we introduce a measure of truthfulness to find the degree of truthfulness of each expert. The degree of truthfulness of all experts helps us form a diagonal matrix which is used in the influence based model. In this section we find the final opinions on all alternatives and then convert them back into the final utility vectors provided by all experts. For this purpose, we calculate the degree of truthfulness from the given preference relations. Similarly, the initial opinion of the expert is calculated with the help of a ranking method that is proposed to rank the alternatives. A comparison of the existing method with the proposed method has also been established in this section. Section 4 concludes the paper and proposes some future directions.

2 Preliminaries

This section presents preliminary definitions that are required for the sequel sections. Given a non-empty finite set of alternatives $X = \{x_1, x_2, ...x_n\}$.

Definition 1 [23] A fuzzy preference relation R on X is defined by the membership function $\mu_R : X \times X \to [0,1]$. The membership function $\mu_R(x_i, x_j) = r_{ij}$ is interpreted as follows:
The alternative x_i is absolutely preferred over the alternative x_j if $r_{ij} = 1$.
The alternative x_j is absolutely preferred over the alternative x_i if $r_{ji} = 1$.
The alternative x_i is preferred over x_j if $r_{ij} \in (0.5, 1]$.
The alternative x_j is preferred over x_i if $r_{ji} \in (0.5, 1]$.
There exists indifference between the alternatives x_i and x_j if $r_{ij} = 0.5$.
Additive reciprocity property [2,12] in a fuzzy preference relation R is defined as $r_{ij} + r_{ji} = 1$ for all $i, j \in \{1, 2, ..., n\}$. Moreover, additive consistency in R [16,17] is defined as $r_{ij} = r_{ik} + r_{kj} - 0.5$ for all $i, j, k \in \{1, 2, ..., n\}, i \neq j$.

Now we define ordered weighted averaging operator (OWA) [20–22] in the following definition. In OWA operators, magnitude of the values to be aggregated determines the reordering step. These aggregation operators are used as we aggregated column matrices in the next section.

Definition 2 An OWA operator of dimension n is a mapping $\phi : \mathrm{R}^n \rightarrow \mathrm{R}$ with associated weights $W = (w_1, w_2, ..., w_n)^T$ with $w_i \in [0, 1], \sum_{i=1}^{n} w_i = 1$,

$$\phi(a) = \phi(p_1, p_2, .., p_n) = \sum_{i=1}^{n} w_i p_{\sigma(i)}$$

where $\sigma : \{1, 2, ..., n\} \rightarrow \{1, 2, ..., n\}$ is a permutation function such that $p_{\sigma(i)} \geq p_{\sigma(i+1)}$ for all $i = 1, 2, ..., n-1$. These weighting vectors can be obtained using the soft majority concept by using quantifier guided aggregation given by Yager [20–22].

3 Influence based decision making method

With the advent of social networking sites like Facebook and Twitter, human beings can interact not just with the people in the vicinity of their neighborhoods but with anyone and everyone belonging to any part of the globe. With these advancements, decision modelling also needs to cater for the fact that human beings may influence one another and the chance of this happening has increased by leaps and bounds over the past two decades.

In group settings, experts interact with one another and influence each other to some extent. The degree to which an expert is influenced by others depends on the susceptibility of the expert to interpersonal influence. Experts form their opinions in complicated interpersonal environment in which preferences are modified because of social influence. We can use directed graph to model the presence of influence in a social influence network (SIN) [4,9]. For this purpose, the adjacency matrix of interpersonal influences, $W = (w_{ij})$, can be formed and represented with the help of a directed graph.

If expert i has some influence on expert j, then there will be a directed arc between expert i and j starting from expert i and heading towards expert j. Similarly, if expert i has no influence on expert j then there will be no such arc. Each arc has a weight $w_{ij} \in [0, 1]$ representing the intensity with which the jth expert has influenced the ith expert. It is assumed that these weights satisfy the normalization property so that influence of all experts on one expert sums up to 1. Mathematically, this implies that the row sum is 1 for all rows.

Consider the following iterative scheme that depicts how opinions of an expert are revised over time because of the adjacency matrix of interpersonal influences. In the following model, $y^{(1)}$ represents the initial opinion and $y^{(t)}$ represents opinion of the expert at time t. Note that since W^t is positive, opinions will reach the state of equilibrium. This means that each expert will revise her opinion and that it will eventually converge in the long run. The same theory will hold for all of the experts in the decision making process. This phenomenon that opinions of all experts will converge over time is exhibited below.

$$y^{(t)} = \mathrm{W} y^{(t-1)}$$

For the convergence of this iterative scheme, one problem could be that of acyclicity. Consider the following matrix of interpersonal influences among two experts as $W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Suppose that opinion of the first expert is 0.2 whereas the second expert states it as 0.7. That is, $y^{(1)} = \begin{pmatrix} 0.2 \\ 0.7 \end{pmatrix}$. According to the model, we have,

$$y^{(2)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0.7 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.7 \end{pmatrix}$$

Whereas,

$$y^{(3)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.7 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.2 \end{pmatrix} = y^{(1)}$$

We can see that this results in a never ending cycle and because of such a choice of W the iterative scheme will not converge. This problem was taken care of by Groot [4] who suggested that W is such that there exists a positive integer t for which W^t is positive. Because of this condition, the opinions will converge after a finite number of iterations.

The diagonal matrix of susceptibility of experts represented as $A = diag(a_{11}, ..., a_{mm})$ is deduced form the matrix W. This matrix A represents the susceptibility of all experts to interpersonal influence. Friedkin et al [9] suggested to include susceptibility of each expert to interpersonal influence as $a_{ii} = 1 - w_{ii}$. The closer the degree of $a_{ii} \in [0, 1]$ is to 1, the more susceptible the expert is to interpersonal influence.

Consider the matrix W of interpersonal influences among three experts in the following. Let,

$$W = \begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Then the diagraph of interpersonal degrees of three experts is represented in figure 1. It can be seen that $w_{21} = 0$ which means that expert 1 has no influence on expert 2, hence in figure 1 there is no directed line from the first expert to the second expert.

With the help of the matrix of interpersonal influences W, matrix of susceptibility of each expert A, identity matrix I, and initial opinion $y^{(1)}$, the following iterative scheme is defined to find the revised and final opinion.

$$y^{(t)} = AWy^{(t-1)} + (I - A)y^{(1)} \tag{1}$$

Note that opinion of an expert at time t is stated as a convex combination of his initial opinion and the influenced opinion at the time $t - 1$. The basic assumption of this model is that $(I - AW)$ is non-singular. Because of this underlying assumption, it is assumed that this process reaches an equilibrium in the long run. That is, $y^{(\infty)} = \lim_{t\to\infty} y^{(t)}$ exists and it is equal to the following.

$$y^{(\infty)} = (I - AW)^{-1}(I - A)y^{(1)} \tag{2}$$

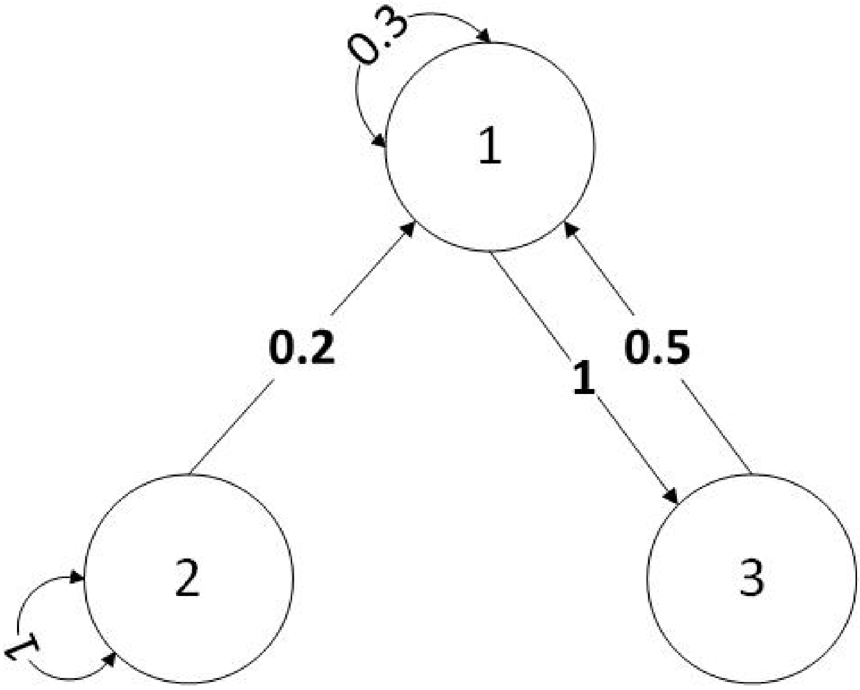

Fig. 1 Diagraph of interpersonal influence degrees

where $(I - A\mathrm{W})^{-1}$ is non-singular.
Note that, for the sake of clarity and better understanding, equation 1 can be expanded and re-written as follows:

$y^{(t)} = ((A\mathrm{W})^{(t-1)} + (I - A)\sum_{i=0}^{t-2}((A\mathrm{W})^{i}))y^{(1)} =$

$(A\mathrm{W})^{(t-1)}y^{(1)} + (I - A)(A\mathrm{W})^{(t-2)}y^{(1)} + ... + (I - A)(A\mathrm{W})y^{(1)} + (I - A)y^{(1)}$

It should be noted here that this is a bounded monnotonic geometric series and hence it is convergent. In the next section, we take into account the truthfulness of an expert and define a measure to calculate it. We propose a model that includes this attribute of each decision maker to produce more realistic results. [25]

4 Influence models for evasive decision makers

In this section, we are asserting that the influence model is not complete unless we cater for truthfulness. We know that truthfulness is a human attribute and it needs to be incorporated for in the influence model. We assume in this

study that the decision makers selected as members of the panel are dignified members of the society. By which we mean that they are not liars instead they are either evasive or "almost truthful" decision makers. In the following, we will define the measure of truthfulness and because of our assumption that the members are "almost truthful" ,or evasive, this measure will belong to the domain of 0.5 to 1. In this section, we define that if a person is not truthful, it will be visible in the preference relation provided by him. Such a decision maker will be indifferent between most alternatives because he is evasive and does not want to express his preferences truthfully.
As mentioned in the introduction section, all countries in the world are facing a handful of problems ranging from poverty to lack of education and absence of clean drinking water. However, the casualties that take place because of the poor health facilities are often understated by the people in power. We define the diagonal matrix of truthfulness $T = (t_{ii})_{m\times m}$, where t_{ii} represents truthfulness of the $i-th$ expert. In the following definition, we explain how to find truthfulness t of any expert.

Definition 3 Let $t : P^n \to [0.5, 1]$ be the measure of truthfulness defined as

$$t = \frac{Tr(PP^{\mathrm{T}})-0.25n}{n(n-1)/2}$$

where Tr is the trace of the preference relation PP^{T} and P^{T} stands for the transpose of the matrix P.

According to definition 3, a decision maker is truthful if his degree of truthfulness is 1. Whereas, the expert is "almost truthful" or evasive, if the degree of truthfulness of the expert is less than 1. The closer the value is to 0.5, the distant the opinions of the decision maker is from truth. Since it is assumed that the experts are not liars hence, the degree of truthfulness never goes below 0.5 and towards 0.

Consider the following preference relation provided by a decision maker e_1 over the set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$. It can be noted that the expert is indifferent between all the four alternatives presented to him. This is an extreme case where the expert does not want to express his preference of alternative at all.

$$P^1 = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

Clearly, the decision maker is indifferent about all alternatives and hence according to definition 3, truthfulness of this expert is 0.5 which is the lowest. This means that this expert is evasive and his degree of truthfulness is the least possible value. The other extreme is when a decision maker e_2 is very clear, expressive and decisive and not indifferent about presenting his preferences over any two choices presented to him. Consider the following preference relation,

$$P^2 = \begin{pmatrix} 0.5 & 1 & 1 & 0 \\ 0 & 0.5 & 0 & 1 \\ 0 & 1 & 0.5 & 0 \\ 1 & 0 & 1 & 0.5 \end{pmatrix}$$

According to definition 3, the truthfulness of decision maker e_2 is 1. As stated earlier, choice of the decision makers is such that they are either truthful and even if they are twisting facts, they are evasive decision makers and not habitual liars. We now introduce the "almost truthful" decision makers in the influence model for evasive decision makers as follows:

$$y^{(t)} = A\mathrm{W}y^{(t-1)} + T(I-A)y^{(1)} \tag{3}$$

The assumption is that if the expert is truthful then the initial opinion will be truthfully expressed and T will become the identity matrix. This is a particular case of our proposed model and hence in this case, the model will reduce to the original influence model. Otherwise, if the expert is evasive and "almost truthful", then he will ameliorate the original opinion by understating the intensity of his opinion.

For convergence of the iterative scheme presented in equation 3, note that T is a diagonal matrix hence the convergence condition for the influence iterative scheme holds true here as well. Note that opinion of an expert at time t is a convex combination of the decision maker's initial opinion, based on his truthfulness in expression, and the influenced opinion at time $t-1$.

For this iteration to reach an equilibrium, $(I - AW)$ must be non-singular. That is, $y^{(\infty)} = \lim_{t->\infty} y^{(t)}$ exists, and we have,

$$y^{(\infty)} = (I - AW)^{-1}T(I-A)y^{(1)} \tag{4}$$

Let us re-write equation 4 as

$$y^{(\infty)} = V_T y^{(1)}$$

where $V_T = (I - AW)^{-1}T(I - A)$. Note that equation 3 can be expanded as follows.

$$y^{(t)} = (AW)^{(t-1)}y^{(1)} + T(I-A)\sum_{i=0}^{t-2}((AW)^i y^{(1)} =$$

$$= (AW)^{(t-1)}y^{(1)}+T(AW)^{(t-2)}(I-A)y^{(1)}+....+T(AW)(I-A)y^{(1)}+(I-A)y^{(1)}$$

We note that from the second term onwards, it is a monotonic convergent geometric series. Note that in this study, we are provided with fuzzy preference relations by the decision makers and not just single values from the unit interval. Preference relations cannot become the input to the influence model for evasive decision makers. Therefore, we need to use the information given in the preference relations in order to state the corresponding priority vectors. This indicates that the model will be applied to each alternative separately. Because of this reason, we find the need to re-write equation 3 as follows:

$$y_{x_i}^{(t)} = AWy_{x_i}^{(t-1)} + T(I-A)y_{x_i}^{(1)} \tag{5}$$

where x_i is the alternative under consideration.
Note that $y_{x_1}^{(1)}$ is the column vector comprising of initial opinions of all the decision makers over the alternative x_1. As mentioned earlier, we need to convert the preference relation into priority vectors. The initial priority vectors will later be revised based on the degree of truthfulness of the evasive or "almost truthful" decision makers. Consider the fuzzy preference relation provided by the $k-th$ expert, P_k, over the set of alternatives $X = \{x_1, x_2, .., x_n\}$ as follows:

$$P_k = \begin{pmatrix} p_{11}^k & - & - & - & p_{1n}^k \\ - & - & -- & - & - \\ - & - & -p_{ij}^k & - & p_{in}^k \\ - & - & - & - & - \\ p_{n1}^k & - & - & - & p_{nn}^k \end{pmatrix}$$

Set of all preference relations have been defined as P^n and the set of alternatives as X. Let $\mathbb{U}$ represent set of $n \times 1$ priority vectors. Then we define the ranking rule $f : P^n \to \mathbb{U}$ which is defined as $f(P_k) = U^k$ where $U^k \in \mathbb{U}$. Note that, $U^k = (u_i^k)_{n\times 1}$ is the priority vector corresponding to the kth preference relation P_k. Now we will convert the preference relation into the priority vector. In the following, we define a method to calculate $(u_i^k)_{n\times 1}$ in two steps.
Step 1: The first step is to find the column sum of the $j-th$ column in P_k as $\sum_{l=1}^{n} p_{lj}$ and then dividing each value p_{ij}^k of the preference relation by its corresponding column sum. This is done as follows:

$$\begin{pmatrix} p_{11}^k & - & - & - & p_{1n}^k \\ - & - & -- & - & - \\ - & - & -p_{ij}^k & - & p_{in}^k \\ - & - & - & - & - \\ p_{n1}^k & - & - & - & p_{nn}^k \\ \hline \sum_{l=1}^{n} p_{l1}^k & - & \sum_{l=1}^{n} p_{lj} & - & \sum_{l=1}^{n} p_{ln} \end{pmatrix}$$

Now that we have the column sum of each column present in the preference relation, we divide each preference value by the column sum as shown in the following matrix. Note that this matrix $(u_{ij})_{n\times n}$ will have a column sum of 1.

$$(u_{ij})_{n\times n}^k = \begin{pmatrix} \frac{p_{11}^k}{\sum_{l=1}^{n} p_{l1}^k} & - & - & - & \frac{p_{1n}^k}{\sum_{l=1}^{n} p_{ln}} \\ - & - & - & - & - \\ - & - & \frac{p_{ij}^k}{\sum_{l=1}^{n} p_{lj}} & - & \frac{p_{in}^k}{\sum_{l=1}^{n} p_{ln}} \\ - & - & - & - & - \\ \frac{p_{n1}^k}{\sum_{l=1}^{n} p_{l1}^k} & - & - & - & \frac{p_{nn}^k}{\sum_{l=1}^{n} p_{ln}} \end{pmatrix}$$

This matrix is normalized by its column sum. We simplify this by re-writing the above matrix as follows. The last row shows that the column sum will be 1.

$$(u_{ij})^k_{n\times n} = \begin{pmatrix} u^k_{11} & - & - & - & u^k_{1n} \\ - & - & -- & - & - \\ - & - & -u^k_{ij} & - & u^k_{in} \\ - & - & - & - & - \\ u^k_{n1} & - & - & - & u^k_{nn} \\ \overline{1} & \overline{1} & \overline{1} & \overline{1} & \overline{1} \end{pmatrix}$$

Step 2: The second step is to calculate the column vector $U^k = (u^k_i)_{n\times 1}$ corresponding to the fuzzy preference relation P_k. We use normalized matrix $(u_{ij})^k_{n\times n}$ from step 1, calculate the row sum and divide it by the dimension of the matrix. Let the $i-j$th element of this matrix be represented as u^k_{ij}. Then,

$$u^k_i = \frac{\sum_{l=1}^{n} u^k_{il}}{n}$$

With this method we transform a preference relation into a priority vector without losing the information provided in the preference relation. Accordingly, the corresponding priority vector of P_k is as follows:

$$U^k = \begin{pmatrix} u^k_1 \\ ... \\ u^k_i \\ ... \\ u^k_n \end{pmatrix}$$

where u^k_i represents that alternative x_i is preferred over other alternatives by this value.

Consider $\{U_1, ..., U_m\}$ as the collection of column vectors where each U_i represents a vector of preference of an alternative over others. As already discussed, we need information pertaining to each alternative in separate column vectors because this is the requirement of our model in equation 5. Therefore, we separate information relevant to the first alternative x_1 in order to form $y^{(1)}_{x_1}$ which is explained as:

$$y^{(1)}_{x_1} = \begin{pmatrix} u^1_1 \\ ... \\ u^k_1 \\ ... \\ u^m_1 \end{pmatrix}$$

Similarly, we separate the information provided for each alternative $x_2, .., x_n$ to form corresponding column vectors. In the following we segregate information relevant to alternative x_n.

$$y^{(1)}_{x_n} = \begin{pmatrix} u^1_n \\ ... \\ u^k_n \\ ... \\ u^m_n \end{pmatrix}$$

This column matrix represents preferences given by all decision makers to alternative x_n over other alternatives. We use this in the truth based influence model to attain the final column matrix for each alternative. For instance, for alternative x_n, we have the following.

$$y_{x_n}^{(\infty)} = \begin{pmatrix} u_n^{1\infty} \\ ... \\ ... \\ ... \\ u_n^{m\infty} \end{pmatrix}$$

Now that we have the final opinions on each alternative, we use this information to restate the priority matrix that has been revised by the influence model of evasive decision makers. Here, the column vector represents the final preference of each alternative over the set of alternatives X by the evasive or "almost truthful" decision maker m.

$$U^{(m\infty)} = \begin{pmatrix} u_1^{m\infty} \\ ... \\ ... \\ ... \\ u_n^{m\infty} \end{pmatrix}$$

The final step is to rank the alternatives. We have $\phi^{'} : U^{(1\infty)} \times ... \times U^{(m\infty)} \to R^+$ where R^+ is the set of all positive real numbers. For alternative x_i for $i \in \{1, 2, .., n\}$, we define the following.

$$\phi^{'}(u_i^{1\infty}, ..., u_i^{m\infty}) = \textstyle\sum_{k=1}^{m} u_i^{k\infty}$$

Now we use the operation $\max\{\sum_{k=1}^{m} u_1^{k\infty}, ..., \sum_{k=1}^{m} u_n^{k\infty}\}$ in order to find the most preferred alternative. Then we construct another set excluding the most preferred alternative to find the second most preferred alternative and so on till all the alternatives have been ranked. We illustrate this with the help of the following example.

Example 1 Suppose that there are four issues $X = \{x_1 = education, x_2 = health, x_3 = security, x_4 = environment\}$ faced by the country of Greencity. Also suppose that there are three evasive or "almost truthful" policy makers who need to address these problems. Since they are partially truthful, instead of giving a fair opinion on the issues, they understate the problem to undermine the intensity of the issues.
Fuzzy preference relations provided by the experts are as follows.

$$P_1 = \begin{pmatrix} 0.5 & 0.2 & 0.4 & 0.6 \\ 0.5 & 0.5 & 0.7 & 0.3 \\ 0.6 & 0.3 & 0.5 & 0.7 \\ 0.1 & 0.4 & 0.2 & 0.5 \end{pmatrix}, P_2 = \begin{pmatrix} 0.5 & 0.9 & 1 & 0.5 \\ 0.1 & 0.5 & 0.9 & 0.2 \\ 0 & 0 & 0.5 & 0.6 \\ 0.8 & 0.4 & 0.3 & 0.5 \end{pmatrix}, P_3 = \begin{pmatrix} 0.5 & 0.3 & 0.2 & 0.5 \\ 0.7 & 0.5 & 0.6 & 0.3 \\ 0.5 & 0.4 & 0.5 & 0.7 \\ 0.6 & 0.7 & 0.4 & 0.5 \end{pmatrix},$$

$$\text{and } P_4 = \begin{pmatrix} 0.5 & 0.8 & 0.5 & 0.4 \\ 0.2 & 0.5 & 0.7 & 0.7 \\ 0.5 & 0.2 & 0.5 & 0.9 \\ 0.6 & 0 & 0.1 & 0.5 \end{pmatrix}$$

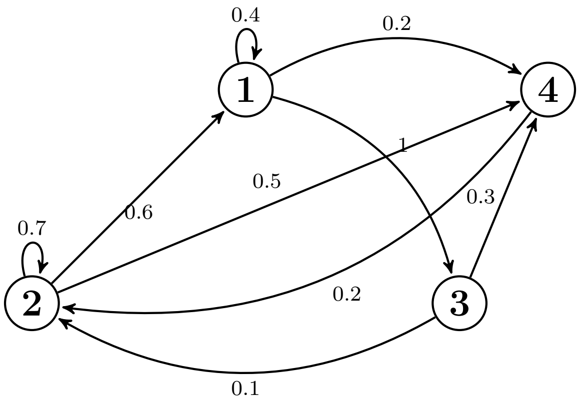

Diagram 1: Graphical representation of interpersonal influence matrix W

Let us first convert these preference relations into priority vectors. Otherwise, the truth based influence model cannot take input of a preference relation. It is known that the experts are evasive or "almost truthful", so before using the priority vectors, we w ill first find the degree of truthfulness of each expert. The corresponding priority vectors are calculated as follows:

$$U^1 = \begin{pmatrix} 0.23622782 \\ 0.295751634 \\ 0.2945845 \\ 0.173436041 \end{pmatrix}, U^2 = \begin{pmatrix} 0.376322751 \\ 0.198412698 \\ 0.12962963 \\ 0.295634921 \end{pmatrix},$$
$$U^3 = \begin{pmatrix} 0.185733275 \\ 0.267611724 \\ 0.268008817 \\ 0.278646184 \end{pmatrix}, U^4 = \begin{pmatrix} 0.31222 \\ 0.278333 \\ 0.26222 \\ 0.147222222 \end{pmatrix}$$

As stated earlier, we need to derive information from these priority vectors pertaining to each alternative. Information relevant to each alternative is to be used in the truth based influence model. The following column matrices are compiled such that each one represents the information available in the priority vectors according to each alternative.
Accordingly, we have,

$$y_{x_1}^{(1)} = \begin{pmatrix} 0.23622 \\ 0.3763 \\ 0.1857 \\ 0.3122 \end{pmatrix}, y_{x_2}^{(1)} = \begin{pmatrix} 0.295 \\ 0.198 \\ 0.267 \\ 0.278 \end{pmatrix} y_{x_3}^{(1)} = \begin{pmatrix} 0.295 \\ 0.129 \\ 0.268 \\ 0.262 \end{pmatrix}, y_{x_4}^{(1)} = \begin{pmatrix} 0.173 \\ 0.295 \\ 0.278 \\ 0.147 \end{pmatrix}$$

Consider the adjacency matrix of interpersonal influence among the four experts as given. As mentioned earlier, the row sum must be 1 and the entry w_{ij} represents the interpersonal influence of decision maker j on the expert i.

$$W = \begin{pmatrix} 0.4 & 0.6 & 0 & 0 \\ 0 & 0.7 & 0.1 & 0.2 \\ 1 & 0 & 0 & 0 \\ 0.2 & 0.5 & 0.3 & 0 \end{pmatrix}$$

Accordingly, the matrix A of susceptibilities of each expert is as follows.

$$A = \begin{pmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now, we find the truthfulness of the four experts using definition 3 and it is 0.59,0.861666,0.705,0.7566 respectively. Accordingly, the matrix of truthfulness of all experts is stated as follows.

$$T = \begin{pmatrix} 0.59 & 0 & 0 & 0 \\ 0 & 0.862 & 0 & 0 \\ 0 & 0 & 0.705 & 0 \\ 0 & 0 & 0 & 0.7566 \end{pmatrix}$$

In order to find the final opinions pertaining to each alternative, we use equation 5 and get the following.

$$y_{x_1}^{(\infty)} = \begin{pmatrix} 0.22322 \\ 0.31638 \\ 0.22322 \\ 0.2698 \end{pmatrix}, y_{x_2}^{(\infty)} = \begin{pmatrix} 0.17252 \\ 0.1708 \\ 0.17252 \\ 0.17167 \end{pmatrix}$$

$$y_{x_3}^{(\infty)} = \begin{pmatrix} 0.1455 \\ 0.1139 \\ 0.1456 \\ 0.1297 \end{pmatrix}, y_{x_4}^{(\infty)} = \begin{pmatrix} 0.1711 \\ 0.2477 \\ 0.1711 \\ 0.2093 \end{pmatrix}$$

Accordingly, the revised opinions of all four experts are as follows.

$$U^{(1\infty)} = \begin{pmatrix} 0.22322 \\ 0.17252 \\ 0.1455 \\ 0.1711 \end{pmatrix}, U^{(2\infty)} = \begin{pmatrix} 0.31638 \\ 0.1708 \\ 0.1139 \\ 0.2477 \end{pmatrix}$$

$$U^{(3\infty)} = \begin{pmatrix} 0.22322 \\ 0.17252 \\ 0.1456 \\ 0.1711 \end{pmatrix}, U^{(4\infty)} = \begin{pmatrix} 0.2698 \\ 0.17167 \\ 0.1297 \\ 0.2093 \end{pmatrix}$$

According to the ranking method defined above, we use $\phi^{'}$ to rank the four alternatives. According to the method we have, $\max\{1.03262, 0.68751, 0.5347, 0.7992\}$ which implies that $x_1 \succeq x_4 \succeq x_2 \succeq x_3$.

In order to make comparison of the influence model for almost truthful decision makers with the previous model, we do not consider the truthfulness of the decision makers at all. This implicity assumes that all decision makers are truthful and hence the matrix T is an identity matrix. The initial opinions of the judges stay the same, and accordingly equation 2 concludes the following.

$$y_{x_1}^{(\infty)} = \begin{pmatrix} 0.299709 \\ 0.370253 \\ 0.299709 \\ 0.334981 \end{pmatrix}, y_{x_2}^{(\infty)} = \begin{pmatrix} 0.251036 \\ 0.202187 \\ 0.251036 \\ 0.226612 \end{pmatrix}$$

$$y_{x_3}^{(\infty)} = \begin{pmatrix} 0.219763 \\ 0.136165 \\ 0.219763 \\ 0.177964 \end{pmatrix}, y_{x_4}^{(\infty)} = \begin{pmatrix} 0.228295 \\ 0.289734 \\ 0.228295 \\ 0.259014 \end{pmatrix}$$

It can be seen that expert whose degree of truthfulness is closer to 1 has less variation in his final outcome as compared to the experts whose truthfulness is closer to 0.5. If truthfulness of experts is not catered for then the ranking of alternatives would have been $x_1 = x_3 \succeq x_4 \succeq x_2$. This shows that the attention of the policy makers can be on different aspects if the truthfulness of the expert is not catered for.

5 Conclusion

In the classical models of decision modeling, role of influence is not incorporated. Moreover, personal attributes of human beings are not addressed. Which means that when studying interpersonal influence of experts over one another in a group setting, it is assumed that all experts are truthful.

It is obvious that in any matter, the panel of decision makers comprises of dignified personals. But since they are humans, there is still chance of them to deviate from the truth deliberately, or because they are lying evasively. This paper identifies this behavior of decision makers and calls them "almost truthful" decision makers. It is a generalization of the influence model provided in [14]. We generalize this model by also including the innate ability of experts to be truthful or to understate the original scenario. The attribute of truthfulness plays an important role in decision making and hence must be incorporated. We propose a method of calculating truthfulness of each expert based on the preference relation provided by the expert. Since the decision makers are "almost truthful", the values belong to $[0.5, 1]$ and not the unit interval. If all decision makers are truthful then their degree of truthfulness is 1, and hence, the matrix of truthfulness becomes the identity matrix and reduces this case is reduced to the regular influence model. However, if decision makers are "almost truthful" then this has an impact on their revised opinions. Which in return changes the choice of alternatives after ranking.

Another assumption in this study is that decision makers provided preference relations instead of mere opinions in the form of a number from unit interval. In this truth based influence model of "almost truthful" decision makers, the preference relations provided by experts are converted into priority vectors. From these priority vectors, information is extracted for each alternative and stated as column vectors. These column vectors are used in the truth based influence model. With the help of the model, we find the final utility vectors. These utility vectors are then ranked to find the best alternative.

In future, we would like to introduce more traits of human nature [1] by incorporating the theory of image processing. Moreover, similar influence model needs to be studied if interpersonal influences are not mere numbers from the unit interval but they are sub-intervals. Apart from this, consensus reaching is an area that has not been studied in influence based GDM models in our study, in our future work we would like to study how consensus reaching is affected by the degree of interpersonal influences among experts [24–26].

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