Effects of Patents on the Transition from Stagnation to Growth

Chu, Angus C. and Kou, Zonglai and Wang, Xilin

University of Liverpool, Fudan University, Fudan University

January 2019

Effects of Patents on the Transition from Stagnation to Growth

Angus C. Chu Zonglai Kou Xilin Wang

August 9, 2019

Abstract

This study provides a growth-theoretic analysis of the effects of intellectual property rights on the take-off of an economy from an era of stagnation to a state of sustained economic growth. We incorporate patent protection into a Schumpeterian growth model in which take-off occurs when the population size crosses an endogenous threshold. We find that strengthening patent protection has contrasting effects on economic growth at different stages of development. Specifically, it leads to an earlier take-off but also reduces economic growth in the long run.

JEL classification: O31, O34
Keywords: intellectual property rights, endogenous take-off, innovation, economic growth

Chu: anguscсc@gmail.com. Management School, University of Liverpool, Liverpool, United Kingdom. Kou: zlkou@fudan.edu.cn. China Center for Economic Studies, School of Economics, Fudan University, Shanghai, China. Shanghai Institute of International Finance and Economics, Shanghai, China. Wang: 17110680014@fudan.edu.cn. China Center for Economic Studies, School of Economics, Fudan University, Shanghai, China. The authors would like to thank Oded Galor, Madeline Zavodny and three anonymous Referees for their helpful comments. Kou gratefully acknowledges financial support from the Ministry of Education of China for the Key Research Base Project (18JJD790003) in Humanities and Social Sciences. Wang gratefully acknowledges financial support (grant no. 71773020) from the National Natural Science Foundation of China. The usual disclaimer applies. The previous version of this manuscript was circulated under the title "Effects of Patents on the Industrial Revolution".

"England [...] by 1700 [...] had developed an efficient set of property rights embedded in the common law [and...] begun to protect private property in knowledge with its patent law. The stage was now set for the industrial revolution." North and Thomas (1973, p. 156)

1 Introduction

The differential timing of countries experiencing a transition from stagnation to growth has governed patterns of comparative economic development across the world and contributed significantly to the divergence in income across the world over the past two centuries.[1] Given the importance of intellectual property rights (IPR) to the pace of technological progress and therefore to the transition from stagnation to growth, this study explores the role that the patent system may have played in the pace of this transition and on economic growth in the long run.

The UK experienced this transition during the late 18th/early 19th century. Figure 1 plots real GDP per capita in the UK.[2] Figure 2 plots the log level of real GDP per capita, in which the slope shows the growth rate of income. In the 18th century, income in the UK grew very slowly. Specifically, the average annual growth rate of income in the UK from 1701 to 1800 was 0.4%. Then, the average growth rate from 1801 to 1900 increased to 1.0%. From the 20th century onwards, the average growth rate stabilized at about 1.7%.

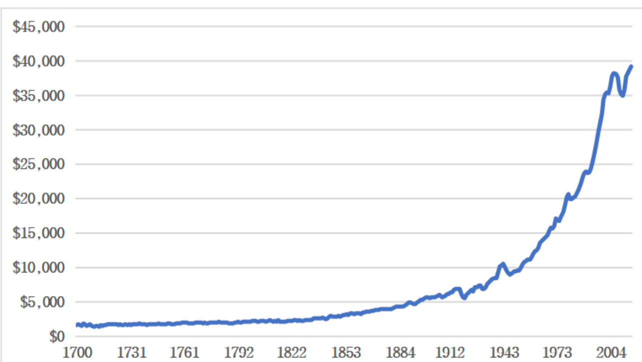

Figure 1: Real GDP per capita in the UK from 1700 to 2016

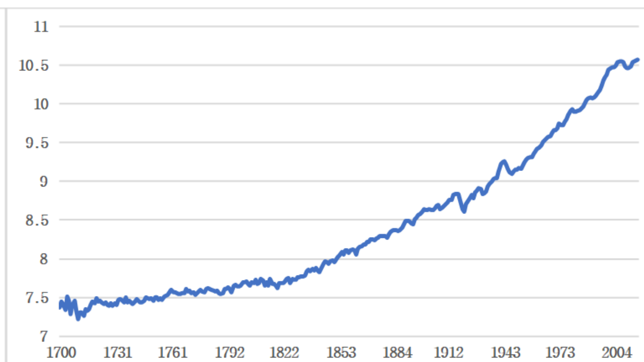

Figure 2: Log of real GDP per capita in the UK from 1700 to 2016

We incorporate patent protection into the Schumpeterian growth model of endogenous take-off in Peretto (2015). In this model, the economy first experiences stagnation with zero growth in output per capita when the market size is small. Here population size plays the crucial role of determining the market size, which in turn implies that population growth gives rise to an expansion of the market. As the market size becomes sufficiently large, innovation takes place and the economy gradually experiences growth. In the long run, the

[1]For a discussion of the great divergence, see Pomeranz (2001).
[2]Data source: Maddison Project Database.

economy converges to a balanced growth path (BGP) with steady-state growth. Within this growth-theoretic framework that is consistent with the growth pattern in Figure 1 and 2, we obtain the following results.

Strengthening patent protection leads to an earlier take-off. Incentives for innovation to take place depend on the market value of inventions, which in turn depends on the level of patent protection and the market size. Therefore, when stronger patent protection increases the market value of patents by reducing price competition and making firms more profitable, it also reduces the market size required for innovation to take place. As a result, the economy starts to experience innovation and growth at an earlier time (i.e., an earlier industrial revolution). Our finding that stronger IPR protection leads to an earlier (but not necessarily immediate) take-off is consistent with historical evidence on the effects of IPR on the industrial revolution.[3] However, stronger patent protection eventually reduces innovation and growth as recent studies tend to find.[4] Intuitively, although stronger patent protection encourages entry and increases the number of products in the economy, this larger number of products reduces the market size of each product and redirects resources away from the quality-improving innovation of each product, which determines long-run growth.[5]

This study relates to the literature on innovation and economic growth. Romer (1990) develops the seminal variety-expanding growth model in which innovation is driven by new products, whereas Aghion and Howitt (1992) develop the Schumpeterian quality-ladder growth model in which innovation is driven by higher-quality products. Peretto (1998, 1999) and Smulders and van de Klundert (1995) combine the two dimensions of innovation and develop a Schumpeterian growth model with endogenous market structure. This study explores the effects of IPR in this vintage of the Schumpeterian growth model.

In the literature on IPR and innovation, other studies also explore the effects of IPR in the innovation-driven growth model.[6] These studies mostly focus on either variety expansion or quality improvement. Only a few studies, such as Chu, Cozzi and Galli (2012) and Chu, Furukawa and Ji (2016), explore the effects of IPR in the Schumpeterian growth model with both dimensions of innovation. However, these studies do not consider the case in which the effects of IPR can change at different stages of the economy. Iwaisako (2013), Chu, Cozzi and Galli (2014) and Chu, Cozzi, Fan, Pan and Zhang (2019) show that the growth or welfare effects of IPR can depend on, respectively, the level of public services, the distance to the technology frontier and the level of financial development in the economy. However, none of these studies consider how IPR affects the endogenous take-off of an economy. The novel contributions of this study are to explore the effects of IPR in a Schumpeterian growth model of endogenous take-off and to highlight the contrasting effects of IPR on economic growth at different stages of the economy with different dimensions of innovation.

This study also relates to the literature on endogenous take-off and economic growth.

[3] See e.g., North and Thomas (1973), North (1981), Dutton (1984) and Khan (2005).

[4] See Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008) for evidence.

[5] See Peretto and Connolly (2007) for a theoretical explanation on quality-improving innovation being the only plausible engine of long-run growth.

[6] See e.g., Cozzi (2001), Li (2001), Goh and Olivier (2002), O'Donoghue and Zweimuller (2004), Furukawa (2007), Chu (2009), Acemoglu and Akcigit (2012), Iwaisako and Futagami (2013), Cozzi and Galli (2014), Huang *et al.* (2017) and Yang (2018).

Galor and Weil (2000) provide the seminal study and develop unified growth theory,[7] which explores how the quality-quantity tradeoff in childrearing and human capital accumulation allow a country to escape from the Malthusian trap and lead to the endogenous take-off of the economy.[8] Peretto (2015) develops a Schumpeterian growth model of endogenous take-off, which features exogenous population growth and does not capture the Malthusian trap; instead, it describes an economy in which take-off is driven by innovation, which also relates to the industrial revolution and is suitable for our analysis of patent policy. The Peretto model features both quality improvement and variety expansion, under which endogenous growth in the number of products provides a dilution effect that removes the scale effect of population size on long-run growth. Therefore, although the population size affects the timing of the take-off, it does not affect the steady-state growth rate. We incorporate patent protection into the Peretto model to explore its effects on endogenous take-off.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 explores the effects of patent policy at different stages of the economy. Section 4 concludes.

2 A Schumpeterian model of endogenous take-off

The theoretical framework is based on the Schumpeterian growth model with both variety-expanding innovation and quality-improving innovation in Peretto (2015). In this model, labor is used as a factor input for the production of final good. Final good is used for consumption and as a factor input for entry, in-house R&D, the production and operation of intermediate goods. We incorporate a patent policy parameter into the model and analyze its effects on the take-off, transitional dynamics and the BGP of the economy.

2.1 Household

The representative household has a utility function given by

$$U = \int_0^\infty e^{-(\rho-\lambda)t} \ln c_t dt, \tag{1}$$

where $c_t \equiv C_t/L_t$ denotes per capita consumption of final good (numeraire) at time t, and $\rho > 0$ is the subjective discount rate. Population grows at an exogenous rate $\lambda \in (0, \rho)$ with initial population normalized to unity (i.e., $L_t = e^{\lambda t}$). The household maximizes (1) subject to

$$\dot{a}_t = (r_t - \lambda)\, a_t + w_t - c_t, \tag{2}$$

where $a_t \equiv A_t/L_t$ is the real value of assets owned by each member of the household, and r_t is the real interest rate. Each member supplies one unit of labor to earn w_t. Standard dynamic optimization yields

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

[7] See also Galor and Moav (2002), Weisdorf (2004), Galor and Mountford (2008), Ashraf and Galor (2011), Galor (2011) and Desmet and Parente (2012).

[8] Other early studies on endogenous takeoff include Jones (2001) and Hansen and Prescott (2002).

2.2 Final good

Final output Y_t is produced by competitive firms using the following production function:

$$Y_t = \int_0^{N_t} X_t^{\theta}(i) \left[Z_t^{\alpha}(i) Z_t^{1-\alpha} L_t / N_t^{1-\sigma} \right]^{1-\theta} di, \tag{4}$$

where $\{\theta, \alpha, \sigma\} \in (0,1)$. $X_t(i)$ is the quantity of non-durable intermediate goods $i \in [0, N_t]$. The productivity of $X_t(i)$ depends on its quality $Z_t(i)$ and the average quality of all intermediate goods $Z_t \equiv \int_0^{N_t} Z_t(j)\, dj / N_t$ capturing technology spillovers. The private return to quality is determined by α, and the degree of technology spillovers is determined by $1-\alpha$. The parameter $1-\sigma$ captures a congestion effect of variety, and hence, the social return to variety is measured by σ.

Profit maximization yields the following conditional demand functions for L_t and $X_t(i)$:

$$L_t = (1-\theta) Y_t / w_t, \tag{5}$$

$$X_t(i) = \left(\frac{\theta}{p_t(i)} \right)^{1/(1-\theta)} Z_t^{\alpha}(i) Z_t^{1-\alpha} L_t / N_t^{1-\sigma}, \tag{6}$$

where $p_t(i)$ is the price of $X_t(i)$. Perfect competition implies that firms pay $\theta Y_t = \int_0^{N_t} p_t(i) X_t(i)\, di$ for intermediate goods.

2.3 Intermediate goods and in-house R&D

Monopolistic firms produce differentiated intermediate goods with a linear technology that requires $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good $i \in [0, N_t]$. Therefore, the marginal cost for the firm in industry i to produce $X_t(i)$ with quality $Z_t(i)$ is one. The firm also incurs $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of final good as a fixed operating cost. To improve the quality of its products, the firm devotes $I_t(i)$ units of final good to in-house R&D. The innovation process is

$$\dot{Z}_t(i) = I_t(i), \tag{7}$$

and the firm's (before-R&D) profit flow at time t is

$$\Pi_t(i) = [p_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{8}$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^{\infty} \exp\left(- \int_t^s r_u du \right) [\Pi_s(i) - I_s(i)]\, ds. \tag{9}$$

The monopolistic firm maximizes (9) subject to (7) and (8). We solve this dynamic optimization problem in the proof of Lemma 1 and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. To analyze the effects of patent breadth, we introduce a policy parameter $\mu > 1$, which determines the unit cost for imitative firms to produce $X_t(i)$ with the same quality $Z_t(i)$[9] as the monopolistic firm in industry i.[10] Intuitively, a larger patent breadth μ increases the cost of imitation and allows the monopolistic producer of $X_t(i)$, who owns the patents, to charge a higher markup without losing her market share to potential imitators;[11] see also Li (2001), Goh and Olivier (2002) and Iwaisako and Futagami (2013). The equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{10}$$

We assume that $\mu < 1/\theta$. In this case, increasing patent breadth raises the markup.

We follow previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$ and the size of each intermediate-good firm is identical across all industries $X_t(i) = X_t$.[12] From (6) and $p_t(i) = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \tag{11}$$

We define the following transformed variable:

$$x_t \equiv \mu^{1/(1-\theta)} \frac{X_t}{Z_t} = \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}, \tag{12}$$

which is a state variable determined by the quality-adjusted firm size and not directly affected by μ (but indirectly via N_t). In Lemma 1, we derive the rate of return on quality-improving R&D, which is increasing in x_t and μ.

Lemma 1 *The rate of return on quality-improving in-house R&D is*[13]

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi\right]. \tag{13}$$

Proof. See the Appendix. ■

2.4 Entrants

Following previous studies, we assume that entrants have access to aggregate technology Z_t to ensure symmetric equilibrium at any time t. A new firm pays βX_t units of final good to

[9] Alternatively, one can assume that the imitative firms have the same unit cost of production as the incumbent monopolist but can only offer a lower-quality version of $X_t(i)$ due to the monopolist's patents.

[10] In other words, this setup implicitly assumes a knowledge diffusion of quality improvement, perhaps via the patents filed by the monopolistic firms.

[11] This setup is consistent with Gilbert and Shapiro's (1990) insight on patent "breadth as the ability of the patentee to raise price" and originates from the patent-design literature; e.g., Gallini (1992) also assumes that a larger patent breadth increases the imitation cost of imitators.

[12] Symmetry also implies $\Pi_t(i) = \Pi_t$, $I_t(i) = I_t$ and $V_t(i) = V_t$.

[13] Note that $(\mu - 1)/\mu^{1/(1-\theta)}$ is increasing in μ for $\mu < 1/\theta$.

enter the market with a new variety of intermediate goods and set up its operation. $\beta > 0$ is an entry-cost parameter. The asset-pricing equation implies that the return on assets is

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{14}$$

When entry is positive, free entry implies

$$V_t = \beta X_t. \tag{15}$$

Substituting (7), (8), (12), (15) and $p_t = \mu$ into (14) yields the return on entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t}\right] + \frac{\dot{x}_t}{x_t} + z_t, \tag{16}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of aggregate quality.

2.5 Equilibrium

The equilibrium is a time path of allocations $\{A_t, Y_t, C_t, X_t, I_t\}$ and prices $\{r_t, w_t, p_t, V_t\}$ such that

- the household maximizes utility taking $\{r_t, w_t\}$ as given;
- competitive firms produce Y_t and maximize profits taking $\{w_t, p_t\}$ as given;
- incumbents for intermediate goods choose $\{p_t, I_t\}$ to maximize V_t taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $A_t = N_t V_t$; and
- the following market-clearing condition of final good holds:

$$Y_t = C_t + N_t\left(X_t + \phi Z_t + I_t\right) + \dot{N}_t \beta X_t. \tag{17}$$

2.6 Aggregation

Substituting (6) and $p_t = \mu$ into (4) and imposing symmetry yield aggregate output as

$$Y_t = \left(\theta/\mu\right)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_t. \tag{18}$$

The growth rate of output per capita is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t, \tag{19}$$

where $y_t \equiv Y_t / L_t$ denotes output per capita. Its growth rate g_t is determined by both the variety growth rate $n_t \equiv \dot{N}_t / N_t$ and the quality growth rate z_t.

3 Dynamics of the economy

The dynamics of the economy is determined by the dynamics of $x_t = \theta^{1/(1-\theta)} L_t / N_t^{1-\sigma}$. Its initial value is $x_0 = \theta^{1/(1-\theta)} / N_0^{1-\sigma}$. In the first stage of the economy, there is neither variety expansion nor quality improvement. At this stage, x_t increases solely due to population growth. When x_t becomes sufficiently large, innovation begins to happen. The following inequality ensures the realistic case in which the creation of products (i.e., variety-expanding innovation) happens before the improvement of products (i.e., quality-improving innovation).

$$\alpha < \frac{\mu - 1 - (\rho - \lambda)\beta}{(\rho - \lambda)\beta\phi} \left\{ \rho + \frac{(\theta/\mu)\left[\mu - 1 - (\rho - \lambda)\beta\right]}{1 - (\theta/\mu)\left[\mu - (\rho - \lambda)\beta\right]}\lambda \right\}. \tag{20}$$

Variety-expanding innovation happens when x_t crosses the first threshold x_N defined as

$$x_N \equiv \frac{\mu^{1/(1-\theta)}\phi}{\mu - 1 - (\rho - \lambda)\beta}, \tag{21}$$

which is the value of x_t that yields $n_t = 0$ when $z_t = 0$. Then, quality-improving innovation also happens when x_t crosses the second threshold x_Z defined as

$$x_Z \equiv \underset{x}{\arg\operatorname{solve}} \left\{ \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x - \phi \right] \left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\beta x} \right] = \rho - \sigma(\rho - \lambda) \right\}, \tag{22}$$

which is the value of x_t that yields $z_t = 0$ when $n_t > 0$. The inequality in (20) implies $x_N < x_Z$. In the long run, x_t converges to its steady-state value x^*. The following inequalities ensure that when the economy is on the BGP, the variables $\{x^*, z^*, g^*\}$ are positive:

$$\beta\phi > \frac{1}{\alpha}\left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] > \mu - 1. \tag{23}$$

The following proposition adapted from Peretto (2015) summarizes the dynamics of x_t.

Proposition 1 *When the initial condition of the economy satisfies*[14]

$$\mu^{1/(1-\theta)}\phi/(\mu - 1) < x_0 < x_N, \tag{24}$$

the dynamics of x_t is given by[15]

$$\dot{x}_t = \begin{cases} \lambda x_t > 0 & x_0 \le x_t \le x_N \\ \bar{v}(\bar{x}^* - x_t) > 0 & x_N < x_t \le x_Z \\ v(x^* - x_t) \ge 0 & x_Z < x_t \le x^* \end{cases}, \tag{25}$$

where

$$\bar{v} \equiv \frac{1-\sigma}{\beta}\left[\mu - 1 - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right],$$

[14] The inequality $x_0 > \mu^{1/(1-\theta)}\phi/(\mu - 1)$ implies that $\Pi_0 > 0$.

[15] It can be shown that (20) and (23) imply $x_N < x_Z < \bar{x}^* < x^*$.

$$\bar{x}^* \equiv \frac{\mu^{1/(1-\theta)}\phi}{\mu - 1 - \beta\left[\rho + \lambda\sigma/(1-\sigma)\right]},$$

$$v \equiv \frac{1-\sigma}{\beta}\left[(1-\alpha)(\mu - 1) - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right],$$

$$x^* \equiv \mu^{1/(1-\theta)}\frac{(1-\alpha)\phi - \left[\rho + \lambda\sigma/(1-\sigma)\right]}{(1-\alpha)(\mu-1) - \beta\left[\rho + \lambda\sigma/(1-\sigma)\right]}.$$

Proof. See the Appendix. ■

3.1 Stage 1: Stagnation

When the market size is not large enough (i.e., $x_t \leq x_N$), there are insufficient incentives for firms to develop new products or improve the quality of existing products. In this case, output per capita is

$$y_t = (\theta/\mu)^{\theta/(1-\theta)} N_0^\sigma Z_0, \tag{26}$$

and the growth rate of y_t is $g_t = 0$. In this regime, strengthening patent protection μ decreases y_t due to monopolistic distortion that reduces intermediate production X_t. However, stronger patent protection also leads to an earlier (but not necessarily immediate) take-off by decreasing x_N in (21). Intuitively, stronger patent protection increases the profitability of firms and provides more incentives for firms to develop new products. As a result, the economy starts to experience innovation at an earlier time.

Proposition 2 *When $x_t \leq x_N$, stronger patent protection reduces the level of output per capita but leads to an earlier take-off.*

Proof. Use (21) and (26) to show that x_N and y_t are decreasing in μ. Given that x_t increases at the exogenous rate λ when $x_t \leq x_N$, a smaller x_N implies an earlier take-off. ■

3.2 Stage 2: Variety expansion

When the market size is sufficiently large (i.e., $x_t > x_N$), firms have incentives to develop new products. In this case, output per capita is

$$y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^\sigma Z_0, \tag{27}$$

and the growth rate of y_t is $g_t = \sigma n_t$. In the Appendix, we show that whenever $n_t > 0$, c_t/y_t always jumps to a steady state. Therefore, we can substitute r_t^e in (16) into the Euler equation $r_t = \rho + g_t = \rho + \sigma n_t$ in (3) and also use (12) to derive the variety growth rate as[16]

$$n_t = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t}\right] - \rho + \lambda. \tag{28}$$

[16] Note from (21) and (28) that $n_t > 0$ if and only if $x_t > x_N$.

For a given level of x_t, a larger patent breadth μ raises the rate of return on variety-expanding innovation and increases the equilibrium growth rate $g_t = \sigma n_t$ as in previous studies, such as Li (2001) and O'Donoghue and Zweimuller (2004).

Proposition 3 *For a given $x_t \in (x_N, x_Z)$, stronger patent protection increases the equilibrium growth rate.*

Proof. Use (28) to show that $g_t = \sigma n_t$ is increasing in μ for a given x_t. ■

3.3 Stage 3: Quality improvement and variety expansion

When the market size becomes even larger (i.e., $x_t > x_Z$), firms have incentives to improve the quality of products in addition to inventing new products. Then, output per capita is

$$y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t, \tag{29}$$

and the growth rate of y_t is $g_t = \sigma n_t + z_t$. We can then substitute r_t^q in (13) into the Euler equation $r_t = \rho + g_t = \rho + \sigma n_t + z_t$ in (3) to derive the quality growth rate as[17]

$$z_t = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right] - \rho - \sigma n_t. \tag{30}$$

For a given level of x_t, a larger patent breadth μ raises the rate of return on quality-improving innovation and continues to increase the equilibrium growth rate $g_t = \sigma n_t + z_t = r_t^q - \rho$, where $r_t^q = \alpha \left[\frac{\mu-1}{\mu^{1/(1-\theta)}} x_t - \phi \right]$.

Proposition 4 *For a given $x_t \in (x_Z, x^*)$, stronger patent protection increases the equilibrium growth rate.*

Proof. Use (30) to show that $g_t = \sigma n_t + z_t$ is increasing in μ for a given x_t. ■

3.4 Stage 4: Balanced growth path

In the long run, x_t converges to x^*. Then, the steady-state quality growth rate is

$$z^* = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x^* - \phi \right] - \rho - \sigma n^*, \tag{31}$$

where $n^* = \lambda/(1-\sigma) > 0$ and

$$x^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \lambda\sigma/(1-\sigma)]}{(1-\alpha)(\mu - 1) - \beta[\rho + \lambda\sigma/(1-\sigma)]}, \tag{32}$$

[17] One can use (16) to derive n_t when $z_t \geq 0$ and then substitute n_t into (30) to show that $z_t > 0$ if and only if $x_t > x_Z$.

which is decreasing in μ. Intuitively, stronger patent protection increases the number of products, which leads to a smaller market size for each product. This smaller firm size x^* in turn reduces the incentives for quality-improving innovation and the steady-state equilibrium growth rate $g^* = \sigma n^* + z^*$. This result generalizes the one in Chu *et al.* (2016), who assume zero social return to variety (i.e., $\sigma = 0$).

Proposition 5 *On the BGP (i.e., $x_t = x^*$), stronger patent protection decreases the steady-state equilibrium growth rate.*

Proof. Use (31) and (32) to show that $g^* = \sigma n^* + z^*$ is decreasing in μ. ■

3.5 Summary

We summarize the dynamics of the economy in the following figures. Figure 3 plots the relationship between the quality-adjusted firm size x_t and the equilibrium growth rate g_t. It shows that when x_t is below the first threshold x_N, the economy does not grow due to the absence of variety-expanding innovation (and also quality-improving innovation). When x_t crosses the first threshold x_N, variety-expanding innovation begins to happen. When x_t crosses the second threshold x_Z, quality-improving innovation also happens. A larger patent breadth μ shifts the curve to the left giving rise to a higher growth rate for any given x_t.

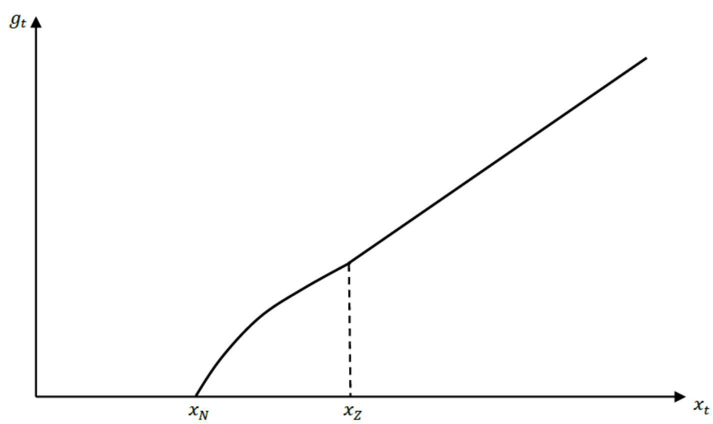

Figure 3: Relationship between firm size and growth

Figure 4 plots the transition path of the quality-adjusted firm size x_t.[18] It shows how x_t evolves from an initial state x_0 to the steady state x^*, which is decreasing in the level of patent breadth μ. Finally, Figure 5 summarizes the transition path of the equilibrium growth rate g_t and shows that strengthening patent protection leads to an earlier take-off (by decreasing x_N) but also lower long-run growth (by decreasing x^*).

[18] T_N (T_Z) is the time when variety-expanding (quality-improving) innovation is activated. In this example, we plot the case in which T_Z increases and x_Z decreases, but other cases are also possible.

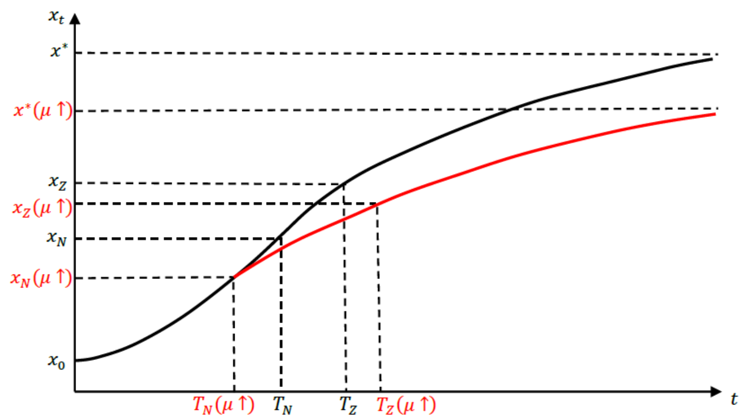

Figure 4: Transition path of the firm size

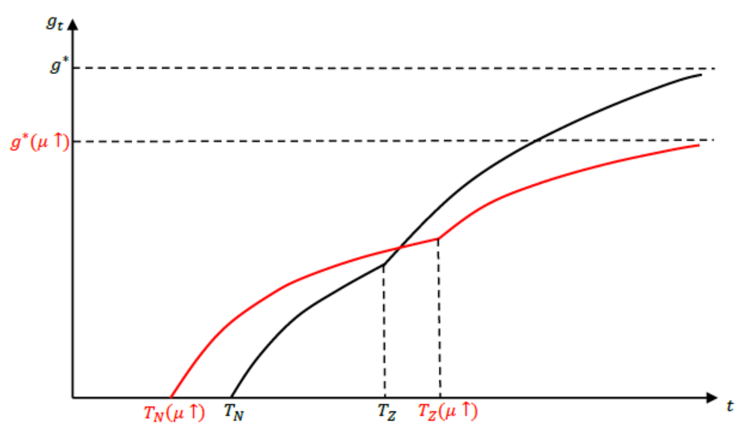

Figure 5: Transition path of the growth rate

4 Conclusion

In this study, we analyze the effects of IPR in a Schumpeterian growth model with endogenous take-off and find that strengthening patent protection causes an earlier take-off by increasing the profitability of firms and providing more incentives for firms to innovate. However, stronger patent protection eventually slows down economic growth by increasing the number of products that reduces the market size of each product and the incentives for quality-improving innovation. These contrasting effects of IPR at different stages of the economy are consistent with historical evidence on the industrial revolution and recent evidence on the effects of the patent system.

These results are also consistent with the fact that the UK implemented a patent system before the US and experienced an earlier industrial revolution but eventually lower economic growth than the US. Our analysis also addresses some critiques on the hypothesis that IPR contributed to the occurrence of the industrial revolution; see for example, Mokyr (2009).

These critiques can be summarized as follows. First, the emergence of the patent system occurred much earlier than the industrial revolution. Second, many inventions at that time were not patented. Our analysis shows that strengthening IPR does not necessarily lead to an immediate take-off but only an earlier take-off. Furthermore, although our analysis does not feature unpatented inventions, the no-arbitrage condition in a model with both patented and unpatented inventions should imply that when the rate of return on patented inventions increases, the rate of return on unpatented inventions also increases.

Finally, this study considers a closed economy for simplicity. In an open economy, the strengthening of patent protection and the endogenous take-off of one country may have the following effects on other countries. On the one hand, it may lead to technology spillovers to other countries. On the other hand, it may cause the industrializing country to specialize in industrial production and other countries to specialize in agricultural production, resulting into a delay of their take-off.[19] We leave this interesting extension to future research.

5 Compliance with ethical standards

The authors declare that they have no conflict of interest.

[19] See Galor and Mountford (2008).

References

[1] Acemoglu, D., and Akcigit, U., 2012. Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, 2012, 10, 1-42.

[2] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica*, 60, 323-351.

[3] Ashraf, Q., and Galor, O., 2011. Dynamics and stagnation in the Malthusian epoch. *American Economic Review*, 101, 2003-41.

[4] Bessen, J., and Meurer, M., 2008. *Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk*. Princeton University Press.

[5] Boldrin, M., and Levine, D., 2008. *Against Intellectual Monopoly*. Cambridge University Press.

[6] Chu, A., 2009. Effects of blocking patents on R&D: A quantitative DGE analysis. *Journal of Economic Growth*, 14, 55-78.

[7] Chu, A., Cozzi, G., Fan, H., Pan, S., and Zhang, M., 2019. Do stronger patents stimulate or stifle innovation? The crucial role of financial development. *Journal of Money, Credit and Banking*, forthcoming.

[8] Chu, A., Cozzi, G., and Galli, S., 2012. Does intellectual monopoly stimulate or stifle innovation? *European Economic Review*, 56, 727-746.

[9] Chu, A., Cozzi, G., and Galli, S., 2014. Stage-dependent intellectual property rights. *Journal of Development Economics*, 106, 239-249.

[10] Chu, A., Furukawa, Y., and Ji, L., 2016. Patents, R&D subsidies and endogenous market structure in a Schumpeterian Economy. *Southern Economic Journal*, 82, 809-825.

[11] Cozzi, G., 2001. Inventing or spying? Implications for growth. *Journal of Economic Growth*, 6, 55-77.

[12] Cozzi, G., and Galli, S., 2014. Sequential R&D and blocking patents in the dynamics of growth. *Journal of Economic Growth*, 19, 183-219.

[13] Desmet, K., and Parente, S., 2012. The evolution of markets and the revolution of industry: A unified theory of growth. *Journal of Economic Growth*, 17, 205-234.

[14] Dutton, H., 1984. *The Patent System and Inventive Activity During the Industrial Revolution, 1750-1852*. Manchester University Press.

[15] Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control*, 31, 3644-3670.

[16] Gallini, N., 1992. Patent policy and costly imitation. *RAND Journal of Economics*, 23, 52-63.

[17] Galor, O., 2011. *Unified Growth Theory.* Princeton University Press.

[18] Galor, O., and Moav, O., 2002. Natural selection and the origin of economic growth. *Quarterly Journal of Economics*, 117, 1133-1192.

[19] Galor, O., and Mountford, A., 2008. Trading population for productivity: Theory and evidence. *Review of Economic Studies*, 75, 1143-1179.

[20] Galor, O., and Weil, D., 2000. Population, technology and growth: From the Malthusian regime to the demographic transition. *American Economic Review*, 110, 806-828.

[21] Gilbert, R., and Shapiro, C., 1990. Optimal patent length and breadth. *RAND Journal of Economics*, 21, 106-112.

[22] Goh, A.-T., and Olivier, J., 2002. Optimal patent protection in a two-sector economy. *International Economic Review*, 43, 1191-1214.

[23] Hansen, G., and Prescott, E., 2002. Malthus to Solow. *American Economic Review*, 2002, 92, 1205-1217.

[24] Huang, C., Yang, Y., and Cheng, C., 2017. The growth and welfare analysis of patent and monetary policies in a Schumpeterian economy. *International Review of Economics & Finance*, 52, 409-426.

[25] Iwaisako, T., 2013. Welfare effects of patent protection and productive public services: Why do developing countries prefer weaker patent protection?. *Economics Letters*, 118, 478-481.

[26] Iwaisako, T., and Futagami, K., 2013. Patent protection, capital accumulation, and economic growth. *Economic Theory*, 52, 631-668.

[27] Jaffe, A., and Lerner, J., 2004. *Innovation and Its Discontents: How Our Broken System Is Endangering Innovation and Progress, and What to Do About It.* Princeton University Press.

[28] Jones, C., 2001. Was an industrial revolution inevitable? Economic growth over the very long run. *B.E. Journal of Macroeconomics (Advances)*, 1, 1-45.

[29] Khan, Z., 2005. *The Democratization of Invention: Patents and Copyrights in American Economic Development, 1790-1920.* Cambridge University Press.

[30] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal*, 111, C164-C179.

[31] Mokyr, J., 2009. Intellectual property rights, the industrial revolution, and the beginnings of modern economic growth. *American Economic Review*, 99, 349-355.

[32] North, D., 1981. *Structure and Change in Economic History.* New York and London: W. W. Norton.

[33] North, D., and Thomas, R., 1973. *The Rise of the Western World: A New Economic History.* Cambridge University Press.

[34] O'Donoghue, T., and Zweimuller, J., 2004. Patents in a model of endogenous growth. *Journal of Economic Growth*, 9, 81-123.

[35] Peretto, P., 1998. Technological change and population growth. *Journal of Economic Growth*, 3, 283-311.

[36] Peretto, P., 1999. Cost reduction, entry, and the interdependence of market structure and economic growth. *Journal of Monetary Economics*, 43, 173-195.

[37] Peretto, P., 2015. From Smith to Schumpeter: A theory of take-off and convergence to sustained growth. *European Economic Review*, 78, 1-26.

[38] Peretto, P., and Connolly, M., 2007. The Manhattan metaphor. *Journal of Economic Growth*, 12, 329-50.

[39] Pomeranz, K., 2001. *The Great Divergence: China, Europe, and the Making of the Modern World Economy.* Princeton University Press.

[40] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.

[41] Smulders, S. and van de Klundert T., 1995. Imperfect competition, concentration and growth with firm-specific R&D. *European Economic Review*, 39, 139-160.

[42] Weisdorf, J., 2004. From stagnation to growth: Revisiting three historical regimes. *Journal of Population Economics*, 17, 455-472.

[43] Yang, Y., 2018. On the optimality of IPR protection with blocking patents. *Review of Economic Dynamics*, 27, 205-230.

Appendix

Proof of Lemma 1. The current-value Hamiltonian for monopolistic firm i is

$$H_t(i) = \Pi_t(i) - I_t(i) + \eta_t(i)\dot{Z}_t(i) + \omega_t(i)[\mu - p_t(i)], \tag{A1}$$

where $\omega_t(i)$ is the multiplier on $p_t(i) \leq \mu$. Substituting (6)-(8) into (A1), we can derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial p_t(i)} = \omega_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \eta_t(i) = 1, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \alpha \left\{ [p_t(i) - 1] \left[\frac{\theta}{p_t(i)} \right]^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} - \phi \right\} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} = r_t\eta_t(i) - \dot{\eta}_t(i). \tag{A4}$$

If $p_t(i) < \mu$, then $\omega_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial p_t(i) = 0$ yields $p_t(i) = 1/\theta$. If the constraint on $p_t(i)$ is binding, then $\omega_t(i) > 0$. In this case, we have $p_t(i) = \mu$. Therefore,

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{A5}$$

Given that we assume $\mu < 1/\theta$, the monopolistic firm sets its price at $p_t(i) = \mu$. Substituting (A3), (12) and $p_t(i) = \mu$ into (A4) and imposing symmetry yield

$$r_t^q = \alpha\frac{\Pi_t}{Z_t} = \alpha\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}}x_t - \phi\right], \tag{A6}$$

which is the rate of return on quality-improving in-house R&D. ■

Before we prove Proposition 1, we first derive the dynamics of the consumption-output ratio C_t/Y_t when $n_t > 0$.

Lemma 2 *When $n_t > 0$, the consumption-output ratio always jumps to*

$$C_t/Y_t = \beta(\theta/\mu)(\rho - \lambda) + 1 - \theta. \tag{A7}$$

Proof. The total value of assets owned by the household is

$$A_t = N_tV_t. \tag{A8}$$

When $n_t > 0$, the no-arbitrage condition for entry in (15) holds. Then, substituting (15) and $\mu X_t N_t = \theta Y_t$ into (A8) yields

$$A_t = N_t\beta X_t = (\theta/\mu)\beta Y_t, \tag{A9}$$

which implies that the asset-output ratio A_t/Y_t is constant. Substituting (A9), (3) and (5) into $\dot{A}_t = r_t A_t + w_t L_t - C_t$ yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} = r_t + \frac{w_t L_t}{A_t} - \frac{C_t}{A_t} = \rho + \frac{\dot{C}_t}{C_t} - \lambda + \frac{(1-\theta)\,\mu}{\beta\theta} - \frac{\mu}{\beta\theta}\frac{C_t}{Y_t}, \tag{A10}$$

which can be rearranged as

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\mu}{\beta\theta}\frac{C_t}{Y_t} - \frac{(1-\theta)\,\mu}{\beta\theta} - (\rho - \lambda)\,. \tag{A11}$$

Therefore, the dynamics of C_t/Y_t is characterized by saddle-point stability, such that C_t/Y_t jumps to its steady-state value in (A7). ■

Proof of Proposition 1. Using (12), we can derive the growth rate of x_t as

$$\frac{\dot{x}_t}{x_t} = \lambda - (1-\sigma)\,n_t. \tag{A12}$$

When $x_0 \le x_t \le x_N$, we have $n_t = 0$ and $z_t = 0$. In this case, the dynamics of x_t is given by

$$\dot{x}_t = \lambda x_t. \tag{A13}$$

When $x_N < x_t \le x_Z$, we have $n_t > 0$ and $z_t = 0$. In this case, Lemma 2 implies that C_t/Y_t is constant and $\dot{c}_t/c_t = \dot{y}_t/y_t$. Therefore, we can substitute r_t^e in (16) and (A12) into $r_t = \rho + \sigma n_t$ in (3) to obtain (28). Substituting (28) into (A12) yields the dynamics of x_t as

$$\dot{x}_t = \frac{1-\sigma}{\beta}\left\{\phi\mu^{1/(1-\theta)} - \left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] x_t\right\}. \tag{A14}$$

Defining $\overline{v} \equiv \frac{1-\sigma}{\beta}\left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right]$ and $\overline{x}^* \equiv \frac{\phi\mu^{1/(1-\theta)}}{\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)}$, we can express (A14) as

$$\dot{x}_t = \overline{v}(\overline{x}^* - x_t). \tag{A15}$$

When $x_t > x_Z$, we have $n_t > 0$ and $z_t > 0$. In this case, Lemma 2 implies that C_t/Y_t is also constant, and $\dot{c}_t/c_t = \dot{y}_t/y_t$. Then, we use (3), (19) and $\dot{c}_t/c_t = \dot{y}_t/y$ to obtain

$$r_t = \rho + \sigma n_t + z_t. \tag{A16}$$

Substituting r_t^e in (16) and (A12) into (A16) yields

$$n_t = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t}\right] - \rho + \lambda. \tag{A17}$$

We substitute (30) into (A17) to derive

$$n_t = \frac{\left[(1-\alpha)(\mu - 1) - (\rho - \lambda)\beta\right]\left[x_t/\mu^{1/(1-\theta)}\right] - (1-\alpha)\phi + \rho}{(\beta x_t)/\mu^{1/(1-\theta)} - \sigma}. \tag{A18}$$

Substituting (A18) into (A12) yields the dynamics of x_t as

$$\dot{x}_t = \frac{1-\sigma}{\beta - \sigma\mu^{1/(1-\theta)}/x_t} \left\{ \left[(1-\alpha)\,\phi - \left(\rho + \frac{\lambda\sigma}{1-\sigma} \right) \right] \mu^{1/(1-\theta)} - \left[(1-\alpha)\,(\mu - 1) - \beta \left(\rho + \frac{\lambda\sigma}{1-\sigma} \right) \right] x_t \right\}. \tag{A19}$$

Using $v \equiv \frac{1-\sigma}{\beta - \sigma\mu^{1/(1-\theta)}/x_t} \left[(1-\alpha)\,(\mu-1) - \beta \left(\rho + \frac{\lambda\sigma}{1-\sigma} \right) \right]$ and x^* in (32), we express (A19) as

$$\dot{x}_t = v\,(x^* - x_t)\,, \tag{A20}$$

where we approximate $\sigma\mu^{1/(1-\theta)}/x_t \cong 0$ for $x_t > x_Z$, so $v \cong \frac{1-\sigma}{\beta} \left[(1-\alpha)\,(\mu-1) - \beta \left(\rho + \frac{\lambda\sigma}{1-\sigma} \right) \right]$ becomes a constant. ■