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Abstract

This paper studies how optimal corporate tax rates differ when firm productivities are drawn from a lognormal distribution instead of a Pareto, the literature standard, in a model of monopolistic competition. Recent literature has demonstrated that lognormal distributions are a better fit for firm productivities; I not only find that this result holds in developing economies, but that the distributional choice has significant implications for the properties of the optimal corporate tax rates. I show this using an enhanced Melitz model with heterogeneous sectors subject to a framework of corporate taxation. This tax framework consists of a single economy-wide statutory tax that is augmented by a set of sector-specific depreciation allowance rates which distort the effective tax rate by sector. I find that using the Pareto distribution mutes a transmission channel between the corporate tax instruments and the equilibrium variables which leads to qualitative different policy implications compared to those obtained under the lognormal distribution. Additionally, my model can reconcile recent empirical studies that come to seemingly conflicting conclusions about the effects of statutory tax rates on export dynamics. I do this by showing that the level of the sector-specific tax rate determines whether or not changing the statutory tax rate will increase the probability of firms engaging in exporting.

Keywords: Corporate tax policy, Melitz-Pareto, asymmetric sectors.

JEL Classification Numbers: F12, F68, H25.

1 Introduction

The trade literature with heterogeneous firms has in its great majority assumed Pareto distributions of productivities.[1] Recent studies have started a debate on how this "standard" assumption affects the outcomes of the models in question, with particular attention to the most widely used model of this

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[1] The justification for this assumption has roots in empirical evidence from Axtell (2001), Del Gatto et al. (2006). However, the real advantage of using the Pareto distribution lyes in the analytical tractability that it provides to the models.

type: the Melitz model. For example, Head et al. (2014) finds that using a lognormal distribution, instead of Pareto, allows them to fit their model significantly better using sales data from French and Spanish firms. Additionally, Bee and Schiavo (2015) provide a thorough comparison between the gains of trade obtained under both distributions to highlight that the standard assumption might be overstating the gains of trade in a significant way. I follow in these steps, but on a parallel path, by investigating the implications to optimal corporate taxation in a Melitz model when one departs from the standard assumption of Pareto productivity distribution in favor of a lognormal distribution. I also provide evidence that the latter distribution is consistently a better fit for productivities in over 100 countries that are part of the World Bank Entrepreneurial Survey.

This paper studies a multi-sector trade model à la Melitz in which I include governments that must provide a fixed amount of public goods, which they finance through the taxation of firms' profits. The tax framework used is modeled after the corporate taxation systems observed in most countries, which usually contain a single statutory corporate profit tax rate (τ), which is imposed on all firms producing in the country; and a set of sector-specific depreciation allowance rates for capital (δ_s), which in the case of my model is assessed in the fixed cost of production. What is special about this corporate tax framework is that the *effective tax rate* is not only different from the statutory tax rate but it can vary significantly across sectors.[2]

The question of what are the optimal corporate tax rates is answered substantially differently depending on which productivity distribution is assumed. For example, the optimal statutory tax rate under lognormal is always lower than the rate derived under Pareto assumption. This property is complementary to the finding that depreciation allowance rates (δ), under the assumption of Pareto distributions, do not explicitly include sector specific fixed costs of production and/or entry cost. On the other hand, the optimal policy for the government in the lognormal model is to exploit these asymmetries in cost across the sectors by using a targeted approach through δ instead of τ which has an economy wide scope.

The difference in the optimal formulas for fiscal instruments is traced to a channel of transmission that is shut down when Pareto distributions are assumed. The channel is based on the ratio of productivities from the average firm and the marginal firm; this ratio is fixed under Pareto but

[2] Effective tax rates are usually defined as the ratio of taxes paid over net profits. For a recent study in the variability of this measure across sector see Barrios et al. (2014)

variable under lognormal distributions. This modification in the market landscape is obviated if we assume Pareto distributions, which eliminates one channel through which governments can influence the equilibrium outcomes via the fiscal instruments.

There are non-trivial welfare losses associated with using the simpler policy functions derived under the Pareto assumption in a country which has lognormal distributions. In the closed economy the welfare losses are enhanced with the degree of asymmetry across the sectors, with one of our numerical examples showing a 3 % loss of welfare relative to using the "correct" policy functions. When the open economy is considered, not only does the degree of asymmetry across sectors in one country plays role but a more important driver is the heterogeneity between countries. In this setting the same scenarios considered in the closed economy yield welfare losses 5 to 10 times as high. The significant welfare losses warrant the use of the more complicated policy functions (obtained under lognormal) when such corresponds to the appropriate distributional assumption of the country being studied.

Adding the proposed tax framework to a Melitz model also provides a basis to reconcile two contradictory findings about the relationship between corporate taxes and export dynamics. Using French firm level data Bernini and Treibich (2013) find that small and medium sized firms are less likely to export their products when they face higher corporate tax rates. On the other hand, Federici and Parisi (2014) use longitudinal data from Italian firms and find the opposite relation. My model is able to produce both relationships and it shows that the export cutoffs are not solely functions of domestic taxes but also depend on taxes from the target country.

The tax collected by the government is used to purchase an exogenous amount of a public good q_0^G, which is produced under perfect competition. Thus, my model uses the Ramsey approach in which governments choose tax rates to maximize the welfare of their citizens while raising enough tax revenue to cover an exogenous level of expenditure. This simple framework can be used to replace the decentralization scheme proposed by Nocco et al. (2014) – to achieve the efficient outcome in a multi-sector Melitz type model – which is based on subsidies and lump sum transfers.[3]

[3] Recent papers have shown that market outcomes are inefficient when the economy is composed of a perfect competitive sector and a monopolistic competitive one. In particular, Dhingra and Morrow (2012) show that resources are mis-allocated between such sectors in a Melitz type model with Variable Elasticity of Substitution preferences (see Zhelobodko et al. (2012) for VES preferences exposition) leading to inefficient outcomes that could be improved. Additionally, Nocco et al. (2014) propose a decentralization scheme to achieve the efficient outcome via subsidies and lump sum taxes on consumers and firms. While this scheme provides us with useful insights into the mechanics at play

If the amount q_0^G is set to the optimal amount found by Nocco et al., then my model provides a framework to compute the optimal tax rates that could be implemented in current tax codes to achieve such outcome.

2 Closed Model

This section presents an extended Melitz (2003) with asymmetric sectors and the addition of a set of fiscal instruments: a statutory corporate tax rate and depreciation allowance rates specific to each sector.[4] The model is first developed in a closed environment as it facilitates the discussion of the relations between the fiscal instruments and the equilibrium outcomes, specially: sector productivity and the number of firms producing in each sector. Special focus is put on the consequences that assuming Pareto distributions exert on the response of these variables to changes in the fiscal instruments. The following paragraphs define the model and its equilibrium.

Households

The country is home to L households who inelastically supply one unit of labor to fulfill demand from firms. The household receives a wage "w" per unit of labor and spends her income on a continuum of differentiated goods $q(\omega)$. Households also derive utility from consuming a public good q_0^G which is provided by the government. The functional form of utility is quasilinear thus the household maximization problem is:

$$\max_{Q_s} \quad q_0^G + \prod_{s=1}^{S} Q_s^{\alpha_s}$$

where Q_s is the aggregate consumption of sector "s" goods.

Let Ω_s represent the collection of available goods in sector "s"; the consumer problem can be

it is hard to imagine its applicability in the real world given the amount of information that the central authority would need but most importantly, the tax codes of most countries would have to be scratched entirely. This seems like an impossible task from a practical perspective and thus I decide to frame the corporate taxes in my model in a way that is closely related to what we observe in most countries.

[4] Bauer et al. (2014) provides a similar taxation framework but their model considers only one sector with heterogeneous firms with no fixed production and entry costs.

broken into S separated maximization problems given by:

$$Q_s = \max_{q(\omega)} \left[\int_{\omega \in \Omega_s} q(\omega)^{\rho_s} \right]^{1/\rho_s} \tag{2.1}$$

such that

$$\int_{\omega \in \Omega_s} p_s(\omega) q(\omega) \leq Y_s$$

where $Y_s = \alpha_s Y$ due to the Cobb-Douglas preferences over sectors. Equation (2.1) is a standard C.E.S utility with elasticity of substitution $\sigma_s = 1/(1-\rho_s)$. As shown in Dixit and Stiglitz (1977), the price index $P_s = \left[\int_{\omega \in \Omega_s} p_s(\omega)^{1-\sigma_s}\right]^{1/1-\sigma_s}$ is used to express quantities demanded as:

$$q_s(\omega) = \frac{Y_s p_i(\omega)^{-\sigma_s}}{P_s^{1-\sigma_s}} = Q_s \left[\frac{p_s(\omega)}{P_s} \right]^{-\sigma_s} \tag{2.2}$$

Firms

Firms operate in one of the S sectors of the economy which are characterize by monopolistic competition and costly entry. After paying the sector-specific entry cost of $F_{e,s}$, a firm randomly draws its productivity (φ) from the distribution $Z_s(\varphi)$. A firm in sector "s" with productivity φ requires $l = q/\varphi + f_s$ units of labor to produce q units of output. The fixed cost of production f_s is homogeneous across firms operating in sector s.

The government sets a statutory corporate profit tax rate (τ), that is common for firms regardless of sector; and a set of sector-specific depreciation allowance rates (δ_s), which allows firms to deduct $\delta_s f_s$ from their taxable income. The value of these "fiscal rates" is known by firms before they make any decision inclusive of entry into a market.

With the above notation, the formulas for taxes paid (t_s), after tax profits (π_s) and, the profit maximizing price for a firm with productivity φ in sector s are:

$$t_s(\varphi) = \tau\left(p_s q_s - w\frac{q_s}{\varphi} - \delta_s w f_s\right) \tag{2.3}$$

$$\pi_s(\varphi) = (1-\tau)\left(p_s q_s - w\frac{q_s}{\varphi} - u_s w f_s\right) \tag{2.4}$$

$$u_s = \frac{1-\delta_s\tau}{1-\tau} \tag{2.5}$$

$$p_s(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\frac{w}{\varphi}\,. \tag{2.6}$$

The variable u_s is the user cost of capital, in the spirit of Hall and Jorgensen (1967), when fixed costs of production f_s are interpreted as capital that firms spend in order to produce.[5] This capital (in a broad sense) could be any variable costs such as licenses, training, machinery costs, etc. However, the type of model that I use doesn't distinguish between labor and capital (in the neoclassical way), which makes the interpretation of δ_s less straightforward than a depreciation allowance on capital. Here, δ_s is a policy instrument that shifts the effective tax rate of firms sector "s" only . Holding τ fixed, increasing δ_s implies that the taxable income for firms in sector "s" is reduced and consequently their effective tax rates decrease; decreases in δ_s have the opposite effect.

2.1 Equilibrium

As is well known, in this type of model, the aggregate variables are functions of the average productivity of firms' that find it profitable to produce:

$$\tilde{\varphi}_s(\varphi_s^*) = \left[\frac{1}{1-Z_s(\varphi_s^*)}\int_{\varphi_s^*}^{\infty} \varphi^{\sigma_s - 1} z(\varphi_s)\, d\varphi\right]^{1/\sigma_s - 1} \tag{2.7}$$

[5] An implicit assumption in the above equations is a physical depreciation rate of capital of 100 %. However, if the real depreciation rate of capital for sector "s" is d_s, the model solution is exactly the same with modified user cost of capital:

$$u_s = \frac{d_s - \delta_s\tau}{1-\tau}$$

Furthermore, the solutions for the optimal tax problem remain valid by scaling the depreciation allowance rate and the fixed cost of production by the physical depreciation rate of capital.

$$\hat{\delta}_s = \frac{\delta_s}{d_s}$$

$$\hat{f}_s = d_s f_s$$

where φ_s^* is the productivity of the marginal firm in sector "s" i.e, the firm that makes zero after tax profit. Let M_s represent equilibrium number of firms producing in sector "s" then aggregation across firms in sector "s" yields the following sector-level economic variable

$$
\begin{aligned}
P_s &= M_s^{1/1-\sigma_s} p_s(\tilde{\varphi}_s) \\
Q_s &= M_s^{1/\rho_s} q_s(\tilde{\varphi}_s) \\
R_s &= M_s r_s(\tilde{\varphi}_s)
\end{aligned}
\qquad
\begin{aligned}
\Pi_s &= M_s \pi_s(\tilde{\varphi}_s) \\
T_s &= M_s t_s(\tilde{\varphi}_s)
\end{aligned}
$$

where $z_s(\tilde{\varphi}_s)$ is the average value of z_s whereas Z_s is the sector aggregate value.

The productivity cutoff φ_s^* is found by equating two conditions on average *after tax* profits. The first condition is derived from the marginal firm which makes zero after tax profit:

$$\bar{\pi}_s = (1 - \delta_s \tau) w f_s \left\{ \left[\frac{\tilde{\varphi}_s(\varphi_s^*)}{\varphi_s^*} \right]^{\sigma_s - 1} - 1 \right\}. \tag{ZPC}$$

Since the number of potential entrants into the market is unbounded, the average expected value of a firm equates the cost of entry $F_{e,s}$. Let ψ be the probability that a firm goes out of business, then the free entry condition is:

$$\bar{\pi}_s = \frac{\psi}{1 - Z(\varphi_s^*)} w F_{e,s} \quad . \tag{FEC}$$

In equilibrium, the (ZPC) and (FEC) conditions hold in every sector determining the equilibrium cutoff productivities. Figure I shows the graphical representation of the equilibrium φ_s^*.[6]

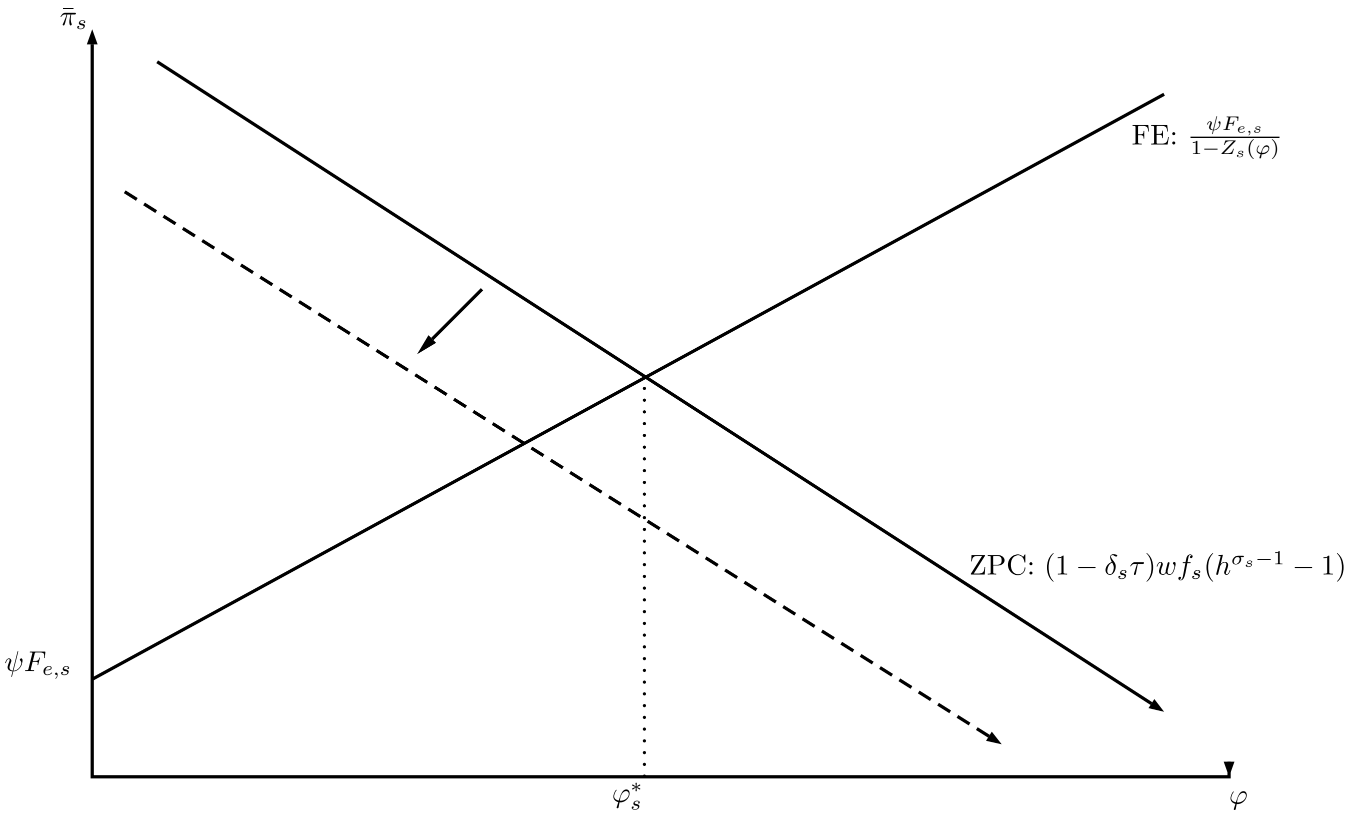
The last step is to solve for the number of firms in equilibrium which is obtained by clearing the labor market. The economy-wide labor supply L is allocated among firms in the monopolistic competition sectors and, a firm that produces the public good for the government and sells it at marginal cost. A firm with productivity φ has labor costs equal to $r(\varphi) - \pi(\varphi) - t(\varphi)$. Aggregating the expression across all firms in sector "s" results in total labor used for production in such sector

$$w L_{p,s} = R_s - \Pi_s - T_s \qquad \forall s \in S \; . \tag{2.8}$$

In equilibrium the number of successful new entrants equates the number of exiting firms, thus:

[6] An equilibrium in which all sectors are producing only exists if $\delta_s \tau \leq 1$ for all sectors.

Figure I: Equilibrium productivity cutoff using the FEC and ZPC curves



$(1-Z_s(\varphi_s^*))M_{e,s} = \psi M_s$. Using this equality and the FEC condition we find that labor costs spent in entry ($wL_{e,s}$) is equal to sector aggregate profit Π_s. Thus, total labor cost for sector "s" is:

$$wL_s \quad = \quad wL_{p,s} + wL_{e,s} = R_s - T_s \tag{2.9}$$

Summing the above across sectors gives the total labor expenditure by firms in the monopolistic competition sectors. Finally, the firm that produces public goods uses one unit of labor to produce one unit of q_0^G. Adding the labor used for the production of private consumption goods (eq. 2.8) plus that of the public good results in total labor income:

$$wL = \sum_{s=1}^{S} R_s - \sum_{s=1}^{S} T_s + wq_0^G \tag{2.10}$$

Using the aggregate variable identities defined earlier, the above is transformed into the equations for the equilibrium number of firms:

$$M_s = \frac{\alpha_s \left(wL + \sum_{i=1}^{S} T_i - p_0^G q_0^G \right)}{\sigma_s u_s f_s h_s^{\sigma-1}} \qquad \forall s \in S \tag{2.11}$$

where $p_0^G = w$ is the price of q_0^G. For the closed economy I will use the public good as the nummeraire which implies $w = 1$.

2.2 Fiscal Instruments and their effects on Equilibrium

In the following paragraphs I describe the relation between equilibrium variables and the "fiscal instruments": statutory tax rate (τ) and depreciation allowance rates (δ_s). The main results are a set of propositions that show the differences between the equilibrium responses under Pareto and lognormal distributional assumptions for firms' productivities, and trace such difference to a transmission channel that is erased when assuming a Pareto distribution.

Before proceeding, I define the following variables to facilitate notation and discussion:

$$h_s = \frac{\tilde{\varphi}_s(\varphi_s^*)}{\varphi_s^*} \qquad \xi_{x,y} = \frac{\partial X}{\partial Y}\frac{Y}{X}$$

where h_s is a measure of firm dispersion and $\xi_{x,y}$ is the elasticity of variable x with respect to variable y.[7]

I start by describing the negative relationship between the depreciation allowance rate and the equilibrium cutoff productivity for the relevant sector. To illustrate, consider an increase in $\delta_{s'}$ which translates into a reduction in the user cost $u_{s'}$ and therefore decreasing the after-tax fixed costs of production ($u_{s'} f_{s'}$). These changes imply that the revenue required to make a zero after tax profit has decreased; consequently, the productivity cutoff for sector s' falls. In terms of the equilibrium conditions, the increase in $\delta_{s'}$ shifts the ZPC curve downward for sector s' since τ is greater than zero as long as there is a positive supply of the public good. In Figure I, this shift is represented by the dash line which results in a smaller value of $\varphi_{s'}^*$.

Next, I explain the ambiguous relationship between τ and the productivity cutoffs which depends

[7] $h^{\sigma-1}$ is the ratio given by the revenue of the average firm with respect to the marginal firm. An h_s closer to one implies less heterogeneity in sector s, in terms of productivities, being $h_s = 1$ the model with one representative firm in sector s.

on the sign of the depreciation allowance rate for the sector. An important consequence is that changing τ affects all sectors simultaneously, but the direction of change of φ^* can be different across sectors. Instead of explaining each direction of the relationship, I find that is more useful to use the table below to show the sign of the changes after an increase in τ

$$\tau \uparrow \begin{cases} \varphi_s^* \downarrow & \text{if } \delta_s > 0 \\ \varphi_s^* \uparrow & \text{if } \delta_s < 0 \\ \varphi_s^* = & \text{if } \delta_s = 0 \end{cases}$$

The above relationships are a direct implication of the $(1 - \delta\tau)$ factor in the ZPC equation. To understand this relationship it is useful to note that net operating profit changes by $(\triangle\tau)\delta w f_s$. When $\delta > 0$, an increase in τ raises net profit, ceteris paribus, which reduces the threshold productivity for the marginal firm since making a zero profit is now "easier"; the case in which $\delta < 0$ has the exact opposite implication as net profits decrease for any level of productivity.

Now that the links between the tax instruments and the cutoff productivities have been determined I show that the change in average productivity has a special property under the Pareto assumption. Clearly, an increase in φ_s^* is raises $\tilde{\varphi}_s$, regardless of distribution, but the relation is stronger under Pareto:

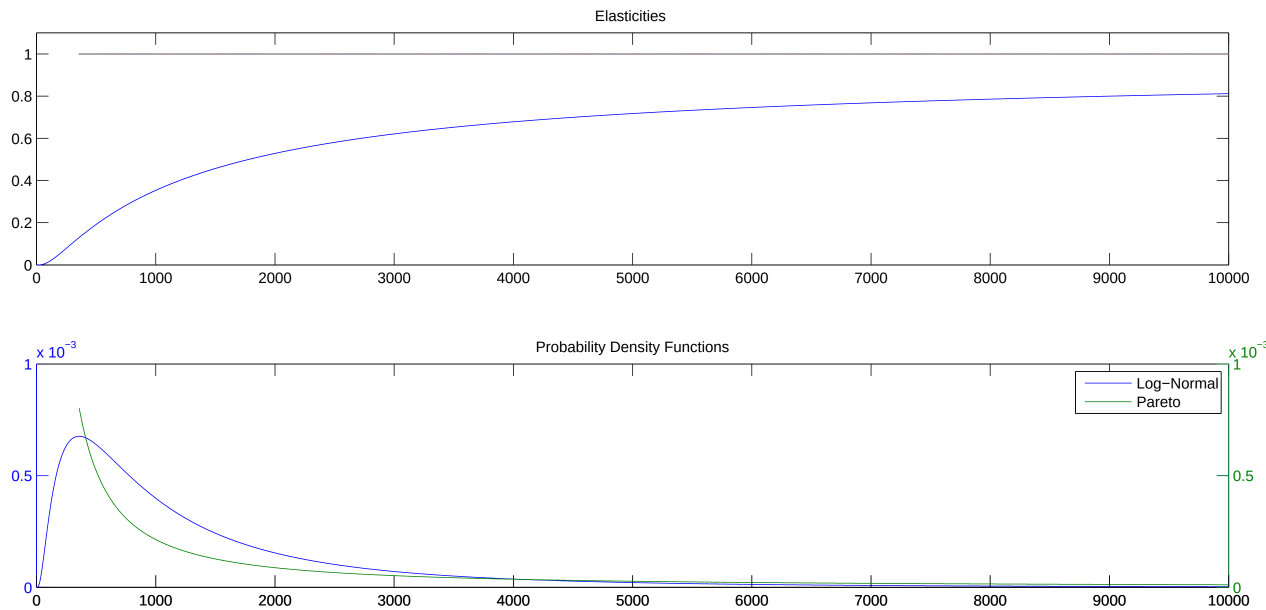
Proposition 2.1. *For any random distribution $Z(\varphi)$ the value of $\xi_{\tilde{\varphi},\varphi^*}$ is strictly positive. If $Z \sim log\mathcal{N}$ then $\xi_{\tilde{\varphi},\varphi^*} < 1$. If the random distribution is Pareto this elasticity is constant along the support of φ and $\xi_{\tilde{\varphi},\varphi^*} \equiv 1$*

Proof. Appendix C.1 □

The property in proposition 2.1 is key since changes in τ, δ lead to alterations in h when the distribution is lognormal, while a Pareto distribution implies a constant value of h. Simply put, the assumption of a Pareto distribution of productivity precludes a sector recomposition that results in a wider/narrower disparity between the marginal and average firm. Furthermore, the constant versus variable h has consequences for equilibrium since it appears in the ZPC equation.

The value $\xi_{\tilde{\varphi},\varphi^*}$ is determinant to the response of the number of firms to tax rate changes. To illustrate, the elasticities of number of firms with respect to statutory tax rate and depreciation

Figure II: Log-normal distributions with parameters $m = 6.88$ and $v = 1$. Pareto distribution parameters selected to match the mode and mean of the lognormal distribution



allowance rate are:

$$\xi_{M_s,\delta_{s'}} = \frac{\sum_{i=1}^{S} \dfrac{\partial T_i}{\partial \delta_{s'}} \delta_{s'}}{wL + \sum_{i=1}^{S} T_i - p^g q_0^G} - \left[\frac{-\tau\delta_s}{(1-\delta_s\tau)} + (\sigma_s - 1)\left(\xi_{\varphi_s^*,\delta_{s'}} \left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1 \right] \right) \right] \qquad \text{if s=s'}$$

$$\xi^s_{M_s,\tau} = \frac{\sum_{i=1}^{S} \dfrac{\partial T_i}{\partial \tau} \tau}{wL + \sum_{i=1}^{S} T_i - p^g q_0^G} - \left[\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)} + (\sigma_s - 1)\left(\xi_{\varphi^*,\tau} \left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1 \right] \right) \right]$$

Using proposition 2.1, we can clearly see that the Pareto distributions annihilate the last term inside the square bracket of the above elasticities. This erased term captures the change in the dispersion of the firms, which is a measure of the new competition landscape in the sector.

Building upon the previous results I provide ordinal statements regarding ξ_M under the two distributional assumptions of productivity.

Proposition 2.2. *Assume that the government runs a balanced budget. Let ξ^P be the elasticities implied from assuming a Pareto distribution and ξ^{log} be the elasticities obtained under a lognormal distribution of productivity.*

- *Let $s \neq s'$, then $\xi^{log}_{M_s,\delta_{s'}} = \xi^{P}_{M_s,\delta_{s'}} = 0$*

- *Let* $s = s'$ *then* $\xi^{log}_{M_s,\delta_{s'}} < \xi^{P}_{M_s,\delta_{s'}}$. *Furthermore if* $\delta > (\leq) 0$ *then* $\xi^{P}_{M_s,\delta_{s'}} > (\leq) 0$

Proof. See Appendix C.2 □

The above proposition says that $\xi^{log}_{M_{s'},\delta_{s'}}$ is always lower than its Pareto counterpart, but its sign is not always determined. When $\delta_{s'} \leq 0$ the magnitude of change in the number of firms under lognormal distribution is greater; however, it is not possible to sign $\xi^{log}_{M_{s'},\delta_{\prime}}$ when $\delta_{s'} > 0$. The last case is intriguing since it opens the possibility that the direction of change for $M_{s'}$, following changes to $\delta_{s'}$, will have different signs for each distributional assumption of productivities.

Turning to the statutory corporate tax rate:

Proposition 2.3. *Assume* $\sum_{i=1}^{S} T_i = p^g q_0^G$. *Let* ξ^P *be the elasticities implied from assuming a Pareto distribution and* ξ^{log} *be the elasticities obtained under a lognormal distribution of productivity.*

- *If* $\delta_s \leq 1$ *then* $\xi^{log}_{M_s,\tau} < \xi^{P}_{M_s,\tau} \leq 0$.

- *If* $\delta_s > 1$ *then* $\xi^{log}_{M_s,\tau} < \xi^{P}_{M_s,\tau}$ *Furthermore,* $\xi^{P}_{M_s,\tau}$ *is positive but* $\xi^{log}_{M_s,\tau}$ *can't be signed.*

Proof. See Appendix C.3 □

Interpretation and consequences of proposition 2.3 are similar to those of proposition 2.2 so they are skipped.

3 Optimal Fiscal Policy in the Closed Economy

This section describes and solves the optimal corporate tax rate under a fiscal framework designed to capture the important features of the corporate tax codes observed in the real world.

The government problem is to choose the optimal effective corporate tax rates that raise sufficient tax revenue to finance government expenditure $p^G q_0^G$, while maximizing aggregate welfare. Let $E(\tau, \{\delta_s\}_1^S)$ be the set of optimal consumption and price vectors for given τ and $\{\delta_s\}_1^S$. The government problem is:

$$\max_{\tau,\{\delta_s\}_1^S} \quad Lq_0^G + L\prod_{s=1}^{S} Q_s^{\alpha_s} \tag{3.1}$$

such that

$$\sum_{s=1}^{S} T_s \quad \geq \quad p^G q_0^G \tag{3.2}$$

$$(q^*, p^*) \quad \in \quad E(\tau, \{\delta_s\}_1^S) \tag{3.3}$$

$$0 < \tau \leq 1 \qquad \delta_s \quad < \quad 1/\tau \qquad \forall s \in S$$

Note that the fiscal authority must raise tax revenue using two instruments: a statutory corporate tax rate and depreciation allowance rates. In one hand, changing τ affects the equilibrium productivity in all sectors and, consequently, the price indexes which determine welfare. On the other hand, it can affect a specific sector by modifying the relevant depreciation allowance rate, thereby enhancing or mitigating the effects of τ in the sector equilibrium productivity and number producing firms. Thus, the government can use cross sector heterogeneity to impose "differentiated" effective tax rates between the sectors.

The F.O.Cs of the government optimization problem can be written in terms of elasticities:

$$\sum_{i=1}^{S} \alpha_i \left(\frac{1}{1-\sigma_i} \xi_{M_i,\delta_{s'}} - \mathcal{I}_{i=s'} \left(\xi_{\tilde{\varphi}_i,\varphi_i^*} \xi_{\varphi_i^*,\delta_{s'}} \right) \right) \quad \leq \quad \delta_{s'} \tilde{\lambda} \sum_{i=1}^{S} \frac{\partial T_i}{\partial \delta_{s'}} \qquad \forall s' \in S \tag{3.4}$$

$$\sum_{i=1}^{S} \alpha_i \left(\frac{1}{1-\sigma_i} \xi_{M_i,\tau} - \xi_{\tilde{\varphi}_i,\varphi_i^*} \xi_{\varphi_i^*,\tau} \right) \quad = \quad \tau \tilde{\lambda} \sum_{i=1}^{S} \frac{\partial Ti}{\partial \tau} \tag{3.5}$$

$$\lambda \left(q_0^G - \sum_{i=1}^{S} T_i \right) \quad = \quad 0 \tag{3.6}$$

$$\tilde{\lambda} \quad = \quad \frac{\mathbb{P}\lambda + 1}{Y} \tag{3.7}$$

where λ is the Lagrange multiplier associated with the government budget constraint, $\mathcal{I}$ is the indicator function and, $\mathbb{P}$ is the wide economy price index.[8] The second equation holds with equality since it is assumed that $q_0^G > 0$ and tax revenue can't be positive unless $\tau > 0$.

The modified FOCs shows in a clear way that the productivity distribution assumption will play a central role in the solutions to the optimal tax problem. As shown in section 2.2, the elasticities

8

$$\mathbb{P} = \Pi_{i=1}^{S} \left(\frac{\mathbb{P}_s}{\alpha_s} \right)^{\alpha_s}$$

appearing in the above equations are significantly different across the two distributional assumptions, particularly $\xi_{\tilde{\varphi}_i,\varphi_i^*}$ which is fixed to unity under Pareto and variable under lognormal.

I proceed to show the optimal tax/depreciation rates for the two different distributional assumptions of productivities for the case with a binding government budget constraint.[9] The Lagrange multiplier associated with the government budget constraint is defined in the following proposition:

Proposition 3.1. *Assuming that the government budget constraint is binding, the Lagrange multiplier (λ) is given by:*

$$\tilde{\lambda} = \frac{\sum_{i=1}^{S} \dfrac{\alpha_i}{\sigma_i - 1}}{wL \sum_{i=1}^{S} \dfrac{\alpha_i}{\sigma_i} - p^G q_0^G}$$

Proof. See Appendix C.4 □

3.1 Optimal tax policy under Pareto

Assume productivities follow a Pareto distribution with CDF $Z_s(x) = 1 - \left(\frac{\varphi_{min,s}}{x}\right)^{k_s}$. The optimal statutory tax rate and depreciation allowance rates are:

$$\xi_{\varphi_i^*,\delta_i} = \xi_{\varphi_i^*,\tau} = \frac{-\tau\delta_i}{k_i(1-\delta_i\tau)} \tag{3.8}$$

$$1-\tau = \left[\sum_{i=1}^{S} \frac{\alpha_i}{k_i}\right] \left[\tilde{\lambda} wL \sum_{i=1}^{S} \frac{\alpha_i \rho_i}{k_i}\right]^{-1} \tag{3.9}$$

$$1-\delta_{s'}\tau = \left(\sum_{i=1}^{S} \frac{\alpha_i}{k_i} \Big/ \sum_{i=1}^{S} \frac{\alpha_i \rho_i}{k_i}\right) \rho_{s'} \tag{3.10}$$

Proposition 3.2. *The differences between sector depreciation rates are proportional to the elasticities of substitutions between their sectors. Furthermore, the ratio of usercosts is solely a function of such elasticities:* $\dfrac{u_{s'}}{u_s} = \dfrac{\rho_{s'}}{\rho_s}$.

The above proposition simply says that in an economy with Pareto distributions, firms in sectors with higher elasticities of substitutions get smaller depreciation allowance rates relative to sectors with lower elasticities of substitution. Going a step further, the elasticity of substitution within each sector is the sole driver for the targeted depreciation allowance rates.

[9] Derivation of the optimal rates and the solution strategies are found in Appendix A.1.

Understanding the mechanics behind this result is useful since there are similar forces acting in the case of lognormal distributions. Consider two different sectors s', s with the same shape parameter k but different elasticities of substitution and without loss of generality assume that $\sigma_{s'} > \sigma_s$. The key variable that drives the equilibrium results is $h^{\sigma-1}$, which appears in the ZPC condition and the formulas for M_s (equation 2.11). By proposition 2.1 we know that under a Pareto distribution, $h^{\sigma-1}$ is constant regardless of the equilibrium value of φ^*; moreover, this variable is increasing in σ since in equilibrium $h_s^{\sigma_s-1} = \dfrac{k_s}{k_s-(\sigma_s-1)}$.

First, the result that h is constant under Pareto implies that changes in the tax instruments only modify the ZPC equation via the factor $(1-\delta\tau)$. Since this factor is multiplied by $(h^{\sigma-1}-1)$, changes in the tax instruments will have a greater effect in the productivity cutoff in sector s' relative to s. In subsection 2.2 we saw that decreasing δ_s increases the productivity cutoff φ_s^*; therefore, the government gives the smaller depreciation allowance rate to sector s' since it gains the most in terms of equilibrium productivities. The increase in productivities translates to higher welfare as the price index decreases.

Second, there is a trade off from having a high σ as it's negatively related to the number of equilibrium firms, which itself lowers the price indexes.[10] The denominator in equation 2.11 shows that the government could improve the number of firms by decreasing the usercost, i.e increasing the depreciation allowance rate. The government does this for sector s as it has a higher impact on M relative to sector s'. Hence, the government aims to decrease the price index for sector s by increasing M_s.

The next proposition contains a surprising and strong result regarding the relation of depreciation allowance rates across all sectors.

Proposition 3.3. *Let the economy consist of S sectors with equal expenditure shares i.e, $\alpha_i = \bar{\alpha} = 1/S$. When productivities are Pareto distributed with homogeneous shape parameter $\bar{k}$, then $\sum_{i=1}^{S} \delta_i^P = 0$.*

Proof. See Appendix □

The above result says that regardless of the degree of heterogeneity in fixed costs across sectors, if market shares and Pareto shape parameters are the same, then the depreciation allowance rates

[10] This is a common feature of monopolistic competition models with CES preferences.

will add up to zero. Notice that there isn't a condition on the distribution parameter φ_{min} only on the shape parameter k since h is only a function of the latter.

3.2 Optimal tax policy under lognormal

Now, assume productivities follow a distribution $Z_i \sim log\,\mathcal{N}(m_i, v_i)$. In this economy, the average productivity in equilibrium can be expressed as:

$$
\begin{aligned}
\tilde{\varphi}_i^{\sigma-1} &= \exp\left(m_i(\sigma_i - 1) + \frac{((\sigma_i-1)v_i)^2}{2}\right)\frac{\Phi((\sigma_i-1)v_i - d_i)}{\Phi(-d_i)} \\
&= A_i g_i(\varphi_i^*)
\end{aligned}
$$

where Φ is the standard normal distribution CDF and $d_i = \dfrac{log(\varphi_i^*) - m_i}{v_i}$. The marginal productivity cutoff has to be solved numerically using:

$$
\frac{A_i g_i(\varphi_i^*)}{(\varphi_i^*)^{\sigma-1}} = \frac{\psi F_{e,i}}{(1-\delta_i\tau)\Phi(-d_i)f_i} + 1
$$

While the optimal tax rates for this economy don't have closed form solutions, it is possible to make some analytical comparisons of these optimal tax rates with those obtained under the Pareto distribution. First, consider the elasticity of productivity cutoff with respect to τ, δ:

$$
\xi_{\varphi_i^*,\delta_i} = \xi_{\varphi_i^*,\tau} = \frac{\psi F_{e,i}}{X_i(1-\sigma_i)}\left(\frac{\tau\delta_i}{1-\tau\delta_i}\right) \tag{3.11}
$$

$$
X_i = \psi F_{e,i} + (1-\delta_i\tau)\Phi(-d_i)f_i \tag{3.12}
$$

Unlike the case of Pareto distributions, these elasticities are dependent on the fixed cost of production and entry.

The conditions to obtain optimal depreciations allowances equal to zero differ significantly across the two productivity distribution assumptions. The following proposition specifies such conditions:

Proposition 3.4. *Let $q_0^G > 0$ and $\lambda > 0$. The conditions for $\delta_i = 0 \quad \forall i$ are:*

1. *Pareto distribution: The shape parameter and elasticity of substitution must be equal across sectors ($k_i = \bar{k} \quad \forall i \in S$, $\sigma_i = \sigma \quad \forall i \in S$).*

2. Log-normal distribution: *The sectors in the economy must be symmetric in all respects.*

Proof. See Appendix C.5. □

The condition placed on the Pareto model is significantly weaker from that of lognormal model. Part of the condition imposes homogeneous shape parameters across sectors but not necessarily on the productivity cutoff parameter. Once again, this is a result of h being fully determined by σ, k and fixed to a constant value under Pareto. As mentioned previously, the optimal rates in the Pareto setting don't depend on the fixed cost of production, hence there is no need to impose symmetry on them. In contrast, the optimal rates in the lognormal environment are affected by such costs and thus a stringent condition is needed to obtain all depreciation allowances set optimally to zero.

A key difference between the optimal tax policies of government in the lognormal environment is given in the following proposition:

Proposition 3.5. *The optimal statutory corporate tax rate under Pareto productivities is greater than or equal to its counterpart found under lognormal distributions. The inequality is strict if there is at least one sector that is asymmetric to the rest.*

Proof. Appendix C.6 □

The result of this proposition highlights that the government in the lognormal scenario has another transmission channel of their policies via alterations of h, which is muted in the Pareto case. These additional channels allows the government to take full advantage of sector asymmetries by using δ more heavily than τ as the latter affects all sectors simultaneously.

3.3 Optimal fiscal tools as functions of selected parameters

I continue by exploring the difference in responses of optimal depreciation and tax rates to changes in the elasticity of substitution, country size, government spending and fixed costs. To ease the exposition the economy is restricted to two almost identical sectors whose only difference lie in their elasticity of substitution σ_i. The parameters for the model are found in table I, values are standard except for the productivity parameters which are explain in the footnote.[11]

[11] The lognormal distribution parameters (m_i, v_i) are set to the average of empirical estimates of the Latin American region (Section 6.2) while the Pareto distribution parameters $(k_i, \varphi_{min,i})$ are set to match the mean and variance of

The take away from all these response functions is twofold. First, the productivity distribution assumption is not important when sectors are identical but becomes critical when the economy is composed of asymmetric sectors. Moreover, the divergence between the optimal rates implied by each distributional assumption increases with the degree of asymmetry between sectors, especially when the asymmetry involves the elasticity of substitution. Second, if an sector experiences changes in fixed cost (production or entry) then each distributional assumption will result in completely different responses for the depreciation allowances and the corporate tax rates.

Although a full symmetric case is not used as a baseline, the response functions in Figure IV contain a point ($\sigma_2 = 2.5$) for which both sectors are completely symmetric. As stated in proposition 3.4, this special case generates depreciation rates equal to zero for both sectors regardless of distributional assumption. Intuitively, when both sectors are completely symmetrical they can be aggregated into a single sector with the same properties. In this case, the government can't improve upon the free market ("first best") outcome by shifting resources across the sectors. The free market equilibrium productivity is that of Melitz (2003), which is attained in my model by setting δ or τ to zero. Since $q_0^G > 0$, the statutory corporate tax rate (τ) is strictly positive which implies depreciation rates are optimally zero.

I now describe the sensitivity of optimal tax instruments rates and equilibrium responses as the elasticity of substitution in sector 2 varies along the interval $[2, 3.5]$, while sector 1 is fixed at 2.5. Figure IV contains the response functions, where solid lines are values under the lognormal assumption and dash lines represent values from assuming a Pareto distribution. Optimal depreciation rates produced under lognormal productivities exhibit a larger degree of responsiveness to changes in σ_2 when compared to their Pareto counterparts; the divergence between such rates increases as the distance between σ_1 and σ_2 grows larger. This divergence occurs even though the Pareto and

both distributions. I do not use the empirical values for k_i as they are in the neighborhood of 1 implying values of σ significantly lower than those used in the literature. By matching the variances we implicitly impose a finite variance for the Pareto distribution, which implies that k is strictly greater than 2. Solving for the Pareto distribution parameters leads to a quadratic polynomial for k; choosing the non-negative root gives the following formulas:

$$\begin{aligned} k_i &= 1 + \sqrt{\frac{\exp(v_i^2)}{\exp(v_i^2) - 1}} \\ \varphi_{min,i} &= \exp(m_i + \frac{v^2}{2})\frac{k_i}{k_i - 1} \end{aligned}$$

lognormal productivity distributions have the same unconditional mean and variance. Thus, the divergence is mainly a result of the extra channel of effect (through $\xi_{\tilde{\varphi},\varphi^*}$) that the lognormal setting posses.

In contrast to the optimal depreciation allowance rates, the response functions for τ are more responsive when Pareto distributions are assumed and, for all numerical experiments considered, $\tau^{log} \leq \tau^P$. The take away of this analysis is that a policymaker in an environment with Pareto distributed productivity will optimally distribute the burden of taxation more evenly across the sectors than the lognormal case. Importantly, the relative small differences in observed tax and depreciation allowance rates have significant implications for the number of firms in each sector and the efficiency of the marginal firm.

A common property of the optimal depreciation rates across both productivity distribution is that the sector with the smallest elasticity of substitution is given the lesser of the depreciation allowances. In proposition 3.2 I explained the mechanics for this property for the Pareto case. The same applies for the lognormal environment with the addition that the term $h^{\sigma-1}$ is variable for this setting, hence depreciation rates change more drastically in the lognormal environment.

Next, figure V shows the response functions for changes in government spending, country size, entry cost and fixed costs of production. As government expenditure increases, the budget constraint becomes tighter, which limits the ability of governments to exploit the variability of productivity distributions; hence, we observe a convergence in the values of δ and τ for the two distributional assumptions. When L increases, the corporate tax rate decreases as firms in both sectors earn higher revenues. Since changes in q_0^G, L affect both sectors equally via $\tilde{\lambda}$ and the income available to spend, response functions of τ, δ are approximately the same under both productivity distribution assumptions.

The last two rows show the responses to changes in fixed cost of production and entry in sector 2. The optimal δs response functions in a Pareto environment are invariant to changes in fixed costs while the optimal δs under lognormal present some response; the optimal response of τ exhibits the same property.

3.4 Inefficient outcomes from assuming a Pareto distribution

To finalize this section, I study the welfare implications of a government mis-specifying the productivity distribution when deciding the optimal depreciation and corporate tax rates. Based on recent theoretical and empirical research, as well as the empirical evidence in section 6.2, I posit that countries contain firms that draw their productivities from a lognormal distribution and conduct the following experiment. First, I compute the optimal δ and τ using the formulas implied by the Pareto setting. I call these the "null" optimal rates and use them used to compute the equilibrium for the economy.[12] Next, the process is repeated but using the "alternative" formulas for the optimal rates, i.e the formulas under the lognormal assumption. I then compare the outcomes of the model as well as the ratio of welfare of the "null" model and the "alternative" model. Welfare under both models is comparable since the amount of public good q_0^G is the same for the "alternative" and "null" model and, any difference between government expenditures and revenues is transferred/taken from households through a lump sum tax. Experiments are conducted under 5 different scenarios and the results are reported in Table I, where the "null" model outcomes are displayed on the top lines and "alternative" model values are directly underneath.[13]

The almost symmetric scenario shows that using the simpler Pareto formulas for the optimal δs and τ carries a 0.14% loss in welfare relative to using the "alternative" formulas. The "alternative" and "null" models have equilibrium outcomes that are almost identical, except for the depreciation allowances which are non-symmetric across sectors for the lognormal case.

The next two scenarios have sector asymmetries in the fixed cost of production or entry costs. For these scenarios the penalties in welfare are larger than that of the almost symmetric case; albeit, the equilibrium variables for both models are almost equal to each other. The optimal δ, τ under Pareto are the same as those of the almost symmetric scenario but, in the lognormal case, these rates differ across scenarios. The adaptation of fiscal rates to changes in fixed cost drives the improvement

[12] These rates are not the solution to the government problem and therefore the budget constraint may not hold with equality, i.e $\Sigma T_i \neq p^G q_0^G$. Hence, the number of firms for this equilibrium is found as the solution to the system of equations:

$$M_s = \frac{\alpha_s(wL + \Sigma_{i=1}^{S} T_i - p^g q_0^G)}{\bar{r}_s} \qquad s = 1, 2$$

[13] We continue to set the Pareto distribution parameters by matching the unconditional mean and variance to that of the lognormal distribution.

in welfare benefits from using the “alternative” rates.

The next scenario increases the difference between the elasticities of goods substitution between the sectors. This scenario generates the most significant losses in welfare from using the “null” rates in the economy whose firms have lognormal distributed productivity. The loss in welfare is over 2%, which is significantly higher than any of the other losses in the previous scenarios. Moreover, the equilibrium outcomes of the two models are considerably different particularly for the number of firms and optimal tax rates. The policies obtained from a lognormal rely on targeting specific sectors at different rates instead of heavily readjusting τ, as is the case with the Pareto assumption. These results, coupled with the high variability of empirical estimate for σ across sectors, illustrates the importance of computing the optimal depreciation and tax rates using the proper distributional assumption.

In conclusion, the analytically convenient assumption that productivities follow a Pareto distribution is not innocuous in the context of corporate tax policy.

4 Open Economy

This section extends the model into the open economy to study the linkage between export status and corporate taxation. I find that my model provides a basis for explaining conflicting empirical results regarding this linkage. In my model, modifications to the statutory corporate tax rate alone generates an ambiguous change in the probability of becoming an exporter, with the sign of the change being determined by the value of the depreciation allowance rate. Expanding on this point, in the next section I show that in a symmetric country setting, the probability of exporting is invariant to changes in tax rates when Pareto distribution are assumed. This property fails to hold in the lognormal case, reinforcing the argument that Pareto distributions eliminate important channels of economic change induced by modifications in effective corporate tax rates.

Additionally, including corporate taxes can solve an important issue of the multi-sector Melitz model regarding unilateral liberalization of some sectors.[14] The evidence tells us that following unilateral liberalization there is a stronger rise in productivity in the liberalized sectors, relative to

[14] Unilateral liberalization refers to a single country reducing their trade barriers/cost to imports

those that are not liberalized.[15] In theory, Demidova and Rodríguez-Clare (2013) find that a one sector Melitz model generates such implication; however, Segerstrom and Sugita (2015) find that such implication doesn't hold when a multi-sector Melitz model is considered. In fact, they find that such model generates the reverse implication under very general conditions. My model can reconcile the theory and empirical evidence by accounting for changes in effective corporate tax rates faced by specific sectors, which offsets/enhance the productivity gains from a unilateral tariff reduction.

The next paragraphs contain only the key elements and results of the model when countries open to trade and under the assumption that utilities are identical across countries. A general model derivation with N countries and asymmetric parameters of the utility (α, σ) is provided in Appendix B.

4.1 Setup, Aggregation and Equilibrium

I assume that household preferences in both nations have the same functional form and parameters as in section 2, with the exception of sector markets shares α, and no labor migration across borders is allowed. Since consumers can now buy products from another countries I use x_{jis} to represent a variable from country j with final market in country i, for sector s.

The timing of decisions by the firm is the same as in the closed economy, but firms serving the domestic market can choose to serve the foreign country via exports. Thus, after a firm (from sector s) in country j draws its productivity from the distribution $Z_s^j(\varphi)$ they decide whether to serve country i via exports or remain solely a domestic supplier. Shipping goods across countries involves an iceberg trade cost $\theta_{jis} \geq 1$; and exporting firms pay a fixed investment cost of f_{jis} every period which is also subject to the depreciation allowance rate δ_{js}. Hence, the after tax profit formula for a representative firm in country j is:

$$\pi_{js}(\varphi) = (1 - \tau_j)\left(\frac{r_{jjs}(\varphi)}{\sigma_s} - u_{js}w_j f_{jj} + \mathcal{I}_{export}\left(\frac{r_{jis}(\varphi)}{\sigma_s} - u_{js}w_j f_{jis}\right)\right) \quad (4.1)$$

$$r_{jis}(\varphi) = \left(\frac{p_{jis}(\varphi)}{\mathbb{P}_{is}}\right)^{(1-\sigma_s)} Y_{is} \quad (4.2)$$

[15] See for example Trefler (2004)

Define φ^*_{jj}, φ^*_{ji} as the cutoff productivity levels for the marginal firm that decides to serve the domestic market and the productivity level of the marginal firm that chooses to export to country i. Using $\tilde{\varphi}(\ )$ (equation 2.7) define the average productivity of all firms producing in j ($\tilde{\varphi}_{jj}$) and the average productivity of firms that export their goods to i ($\tilde{\varphi}_{ji}$):

$$\tilde{\varphi}_{jj} = \tilde{\varphi}^j(\varphi^*_{jj}) \qquad\qquad \tilde{\varphi}_{ji} = \tilde{\varphi}^j(\varphi^*_{ji})$$

The number of producing firms in sector "s", based in country j, is M_{js} with a subset $M_{jis} = \kappa^x_{jis} M_{js}$ serving country i via exports; where κ_{ji} is the conditional probability of becoming an exporter.[16] Hence, the total amount of products available to consumers in country j is $M^j_{tot,s} = M_{js} + M_{ijs}$.

With the above, the price index for sector s as well as the average productivity of firms *selling* in country j sector "s":

$$\tilde{\varphi}^j_{tot,s} = \left[\frac{1}{M^j_{tot,s}}\left(M_{js}\left(\tilde{\varphi}_{jj}\right)^{\sigma_s-1} + M_{ijs}\left(\hat{\theta}^{-1}_{ijs}\tilde{\varphi}_{ijs}\right)^{\sigma_s-1}\right)\right]^{\frac{1}{\sigma_s-1}} \tag{4.3}$$

$$\mathbb{P}_{js} = \left(M^j_{tot,s}\right)^{\frac{1}{1-\sigma_s}} p_{jjs}(\tilde{\varphi}^j_{tot,s}) \tag{4.4}$$

where $\hat{\theta}_{ijs} = \dfrac{w_i\theta_{ijs}}{w_j}$ measures a combination of shipping costs and wages (input costs in this model). The total average productivity ($\tilde{\varphi}_{tot,s}$) is the weighted average of mean productivities of all domestic firms and foreign firms selling products in country j.

The sector price index formulas are needed to solve for the equilibrium since the new zero profit condition (ZCP) contains domestic and export productivity cutoffs that have to be linked through the sector price index. To be more clear, the new ZCP condition is:

$$\bar{\pi}_{js} = (1-\delta_{js}\tau_j)\left[w_j f_{jjs}\left(\left(\frac{\tilde{\varphi}_{jjs}}{\varphi^*_{jjs}}\right)^{\sigma_s-1} - 1\right) + \kappa^x_{jis} w_{js} f_{jis}\left(\left(\frac{\tilde{\varphi}_{jis}}{\varphi^*_{jis}}\right)^{\sigma_s-1} - 1\right)\right] \tag{4.5}$$

and to solve φ^*_{jis} it must be expressed as a function of φ^*_{jjs}:

[16] $\kappa^x_{jis} = \dfrac{1 - Z_{js}(\varphi^*_{jis})}{1 - Z_{js}(\varphi^*_{jjs})}$

$$\varphi^*_{jis} = \left[\frac{M^i_{tot,s}}{M^j_{tot,s}}\right]^{\frac{1}{\sigma_s-1}} \frac{\tilde{\varphi}^i_{tot,s}}{\tilde{\varphi}^j_{tot,s}} \left[\frac{Y_{js}}{Y_{is}}\frac{f_{jis}}{f_{jjs}}\right]^{\frac{1}{\sigma_s-1}} \hat{\theta}_{jis}\varphi^*_{jjs} \tag{4.6}$$

Notice that the above equation expresses the export productivity cutoff for country j as a function of other productivity cutoffs, including those of country i. Many papers at this point invoke a symmetry assumption across the countries making the above sufficient to pin down the equilibrium productivities. However, in my model even if countries were completely symmetric in all their parameters but one of their corporate tax rates, it would generate different domestic cutoffs which translate into heterogeneous equilibrium outcomes between the countries. Borrowing from Segerstrom and Sugita (2015), I use the relationship between the domestic and import productivity cutoffs:

$$\varphi^*_{jis} = \left(\frac{u_{js}w_j f_{jis}}{u_{is}w_i f_{ii}}\right)^{\frac{1}{\sigma_s-1}} \hat{\theta}_{ji}\varphi^*_{ii} \tag{4.7}$$

to convert equation 4.6 into a function of φ^*_{jj} only.

Lastly, the number of firms is solved to complete the description of the equilibrium. This is simple as labor used for production is still given by $r(\varphi) - \pi(\varphi) - t(\varphi)$ and we can use the same procedure as in section 3 to obtain aggregate revenue $R = wL + \sum T - p^g q_0^G$. Therefore, the equilibrium is found by solving a $S \times 2 \times 2$ simultaneous system of equations consisting of the following 2 equations for each sector, for each country:

$$ZCP_s = FE_s \tag{4.8}$$

$$M_{js} = \frac{\alpha_{js}(w_j L_j + \Sigma^S_{s'=1} T_{js'} - p^g_j q^G_0)}{\sigma_{js} u_{js} w_j \left(f_{jjs} h^{\sigma_s-1}_{jjs} + \kappa^x_{jis} f_{jis} h^{\sigma_s-1}_{jis}\right)} \tag{4.9}$$

where $h_{jj} = \tilde{\varphi}_{jj}/\varphi^*_{jj}, \quad h_{ji} = \tilde{\varphi}_{ji}/\varphi^*_{ji}$

4.2 Tax rates and the decision to export

This subsection provides a detailed account of the relationship between the export productivity cutoffs and corporate tax rates. I find that the conditional probability of exporting κ is negatively related to the depreciation rate (in the source country), but the relationship with the statutory corporate tax rate is ambiguous. The first part of the result is not surprising as increasing δ decreases the cost of f_{ji} which incentives more firms to enter the export markets, all else equal. However, the direction of change for modification in τ is ambiguous as it depends on the level of δ. These properties help explain the mixed evidence regarding the effects of corporate tax rates on export dynamics.

The effects of changes in δ, τ on the probability of exporting (κ^x) are expressed in terms of the elasticities of φ^*. Let Z_{js} be the productivity distribution in country j sector s, then:

$$\Upsilon_{js}(x) = \frac{z_{js}(x)}{1 - Z_{js}(x)} x \tag{4.10}$$

$$\frac{\partial \kappa^x_{jis}}{\partial y} y = \kappa^x_{jis} \left(\Upsilon(\varphi^*_{jjs}) \xi_{\varphi^*_{jjs}, y} - \Upsilon(\varphi^*_{jis}) \xi_{\varphi^*_{jis}, y} \right) \qquad \text{for y} = \tau, \delta_s \tag{4.11}$$

the function $\Upsilon(x)$ has the following properties:

- If $Z_{js} \sim Pareto(k_{js}, \varphi_{min})$ then $\Upsilon(\varphi) = k_{js}$ for any φ in the support of Z_{js}.
- If $Z_{js} \sim log\mathcal{N}(m_{js}, v_{js})$ then $\Upsilon(\varphi)$ is an increasing function.

The above shows, once again, that distributional assumptions about productivity are important for the comparative statics of the model. A constant versus increasing Υ has implications for the effects of tax changes on the probability of becoming an exporter. For the special case of symmetric countries, it will be shown that, under the Pareto distribution, changing taxes have no effect in the probability of exporting (κ); this invariability property iis not present when assuming lognormal distributions. For the general case (assymetric countries), the effects on κ, following changes to tax rates, are determined by the difference between the domestic and export productivity cutoff elasticities. However, the subtraction's terms will be equally weighted for the Pareto case but, under the lognormal assumption, a higher weight is assigned to the export cutoff elasticity.

Figure III (below) illustrates the relation between tax rates and the probability of export. The panel presents heat maps for κ_{ji1}: the probability of export for firms in sector 1, country j; as a function of τ_j and δ_{j1}. The export probabilities come from solving the equilibrium for two countries (Home and Foreign) whose parameters are equal to those of the almost symmetric scenario. A surface plot of κ_{ji1} is generated by evaluating the model at grid points spawn by τ_j, δ_{j1}. The left graphs in the panel show that increasing the depreciation allowance rate (δ_{1j}) results in a decrease in the propensity to export by firms in country "j", but the relationship between the statutory tax rate (τ_j) and the probability of export is ambiguous. In the graphs we observe that increasing τ_j results in an increase in the probability of exporting but only when the value of δ_{j1} is below a certain threshold. In contrast, if δ_{j1} is above such threshold, the probability of export decreases with the statutory corporate tax rate. The reason behind the ambiguous effect goes back to the movement of the ZPC condition in closed economy, which was positive for $\delta > 0$ but negative for $\delta < 0$. In the open economy the new ZPC condition also contains the term φ^*_{ji} which is determine by ratio of user costs across countries; thereby, the threshold value for δ at which the relation between τ and productivity cutoffs change is different than zero.

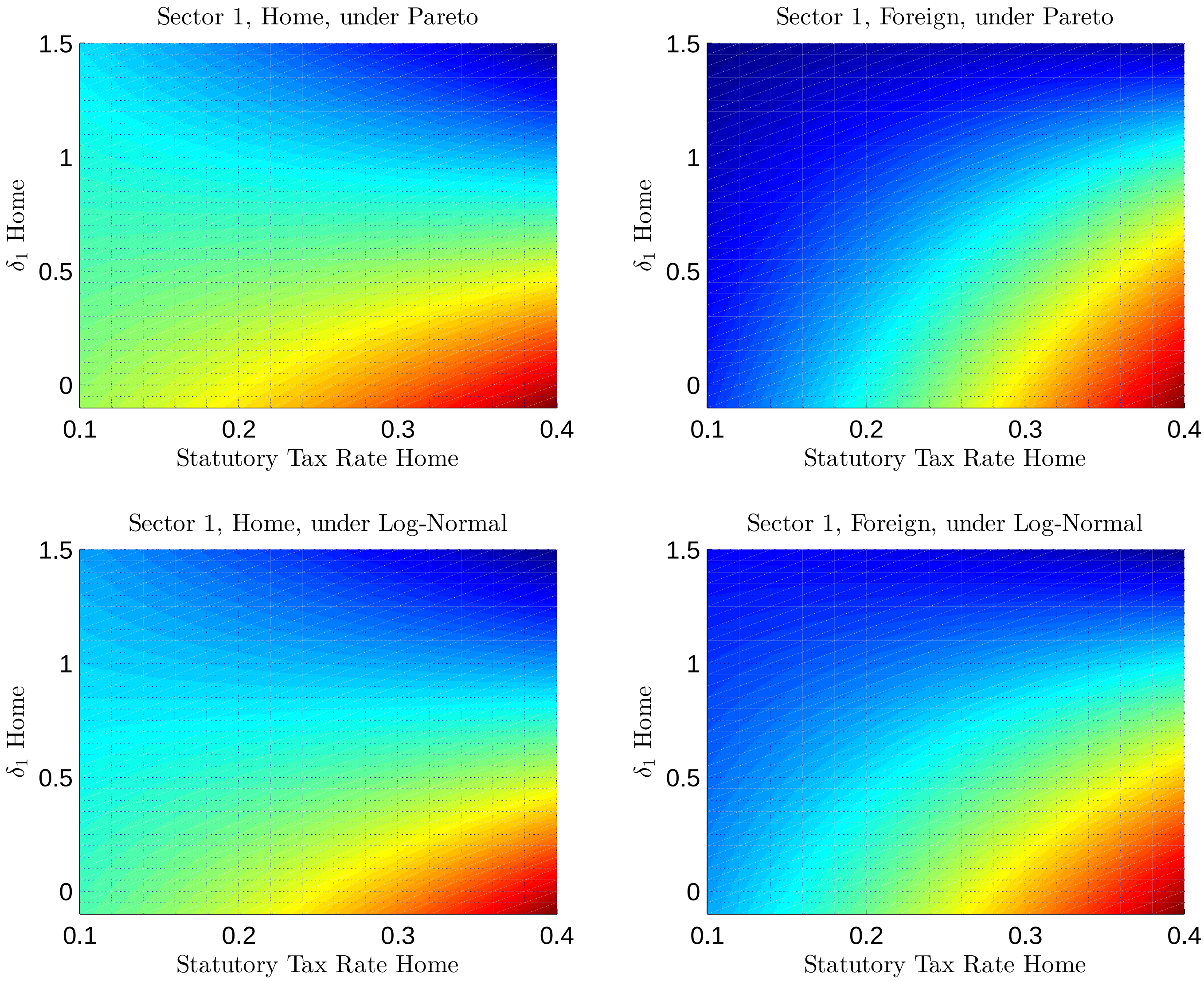
The relation shown in Figure III bridges two conflicting empirical findings regarding corporate tax effects on export dynamics. First, Bernini and Treibich (2013) find that corporate tax rates are negatively correlated with the probability that firms will engage in export activities.[17] Their results are obtained by exploiting an exogenous variation in the statutory tax rate charged to small-medium firms in France, which was reduced from 33.33% to 15% for the years 2001 to 2003, and compare the export outcomes of such firms relative to large firms as their statutory tax rate was unchanged. As we have seen in Figure III, my model predicts such relationship but only when the depreciation allowance rate is above a threshold. On the other hand, Federici and Parisi (2014) use data from Italian firms, for the years 2004 to 2006, to show that export propensity is positively associated with corporate taxation, which in their study is a measure of firms' specific effective tax rate. In my model, this would translate to a negative relationship between the sector depreciation allowance rate and the probability of exporting, which is what we observe in Figure III.[18]

Adding corporate taxation to a multi-sector Melitz model ameliorates the critique of Segerstrom and Sugita (2015) who find that such model is inconsistent with the data. In the data, sector productivity increases more strongly in liberalized sectors than in non-liberalized sectors; however, the multi-sector Melitz model generates the opposite relationship under fairly general conditions. Using equation 4.7, we can observe that the effects of a unilateral decrease in trade costs (θ) can be

[17] Alessandria and Choi (2014) also finds a negative relation between corporate taxation and export *growth*

[18] Increasing δ_s allows firms in sector "s" to increase their reduction in taxable income and thereby reduce their tax liability. Thus, all else equal, the ratio of taxes paid to profits will decrease i.e their effective tax rate will decrease.

Figure III: Heat Map for the probability of exporting obtained by simultaneously varying the values of the depreciation allowance rate of sector 1 and the statutory tax rate at Home.



directly offset via corporate tax changes in either country. Hence, the critique of Segerstrom and Sugita (2015) regarding the implication of a multi-sector Melitz model can be attenuated.

While the question of interest was on the relationship between exports and the corporate tax rates I also show that the model is consistent with other standard results. Using equation 4.6, we see that liberalization (reduction of θ) reduces the productivity cutoff to serve country i via exports. The same equation also provides a relationship between market competition and the export productivity required to "carve" a space in such market. For example, if there are many firms operating in country i and/or the productivity of such firms is high ($\tilde{\varphi}_{tot,i}$), then the required export productivity cutoff will be higher relative to other less competitive markets.

5 Optimal Corporate Tax Rates in the Open Economy

This section will provide the characterization of the optimal corporate tax rates in the open economy, for a general case; and its solutions, for the special case of symmetric countries.

Without loss of generality assume $j \neq i$. The following conditions are for country j but they are analogous for country i.

$$\max_{\tau_j, \{\delta_{js}\}_1^S} \quad L_j q_{j0}^G + L_j \prod_{s=1}^{S} Q_{js}^{\alpha_{js}} \tag{5.1}$$

$$\tag{5.2}$$

such that

$$\sum_{s=1}^{S} T_{js} \geq p_j^g q_0^G \tag{5.3}$$

$$(q^*, p^*) \in E(\tau_j, \{\delta_{js}\}_1^S) \tag{5.4}$$

$$0 < \tau_j \leq 1 \qquad \delta_{js} < 1/\tau_j \qquad \forall s \in S$$

Analysis is restricted for the case of a binding constraints leading to the following FOCs:

$$\left(\frac{\alpha_{js} a_{js}^{-1}}{\sigma_{js} - 1}\right) \left(\frac{\xi_{M_{js},\delta_{js}}}{\tilde{\varphi}_{jj}^{1-\sigma}} + \frac{\partial \tilde{\varphi}_{jj}^{\sigma-1}}{\partial \delta_{js}} \delta_{js} + \frac{M_{is}}{M_{js}} \hat{\theta}_{ijs}^{1-\sigma} \left(\frac{\partial \kappa_{ijs}^x}{\partial \delta_{js}} \delta_{js} \tilde{\varphi}_{ijs}^{\sigma-1} + \kappa_{ijs}^x \left(\frac{\xi_{M_{is},\delta_{js}}}{\tilde{\varphi}_{ijs}^{1-\sigma_s}} + \frac{\partial \tilde{\varphi}_{ij}^{\sigma-1}}{\partial \delta_{js}} \delta_{js}\right)\right)\right) = -\tilde{\lambda} M_{js} \left(\xi_{M_{js},\delta_{js}} \bar{t}_{js} + \frac{\partial \bar{t}_{js}}{\partial \delta_{js}} \delta_{js}\right) \qquad \forall s \in S$$

$$\sum_{s=1}^{S} \left(\frac{\alpha_{js} a_{js}^{-1}}{\sigma_{js} - 1}\right) \left(\frac{\xi_{M_{js},\tau_j}}{\tilde{\varphi}_{jj}^{1-\sigma}} + \frac{\partial \tilde{\varphi}_{jj}^{\sigma-1}}{\partial \tau_j} \tau_j + \frac{M_{is}}{M_{js}} \hat{\theta}_{ijs}^{1-\sigma} \left(\frac{\partial \kappa_{ijs}^x}{\partial \tau_j} \tau_j \tilde{\varphi}_{ijs}^{\sigma-1} + \kappa_{ijs}^x \left(\frac{\xi_{M_{is},\tau_j}}{\tilde{\varphi}_{ijs}^{1-\sigma_s}} + \frac{\partial \tilde{\varphi}_{ij}^{\sigma-1}}{\partial \tau_j} \tau_j\right)\right)\right) = -\tilde{\lambda} \sum_{s=1}^{S} M_{js} \left(\xi_{M_{js},\tau_j} \bar{t}_{js} + \frac{\partial \bar{t}_{js}}{\partial \tau_j} \tau_j\right)$$

with

$$
\begin{aligned}
a_{js} &= \tilde{\varphi}_{jjs}^{\sigma_s-1} + \kappa_{ijs}^{x} \frac{M_{is}}{M_{js}} \left(\hat{\theta}_{ijs}^{-1} \tilde{\varphi}_{ijs} \right)^{\sigma_s-1} \\
\bar{t}_{js} &= \tau_j \left(w_j f_{jj} (u_{js} h_{jjs}^{\sigma_s-1} - \delta_{js}) + w_j f_{ji} \kappa_{ji}^{x} (u_{js} h_{jis}^{\sigma_s-1} - \delta_{js}) \right)
\end{aligned}
$$

where $\bar{t}_{js}$ is the average tax revenue from sector s.

The FOCs tell us that the government faces a similar problem as in the closed economy section: the left hand side is the benefit/cost to the average productivity of firms and the right hand side is the benefit/cost to tax revenue. However, the left hand side now includes a term for the productivity of importers which is affected by tax policy in j as stated in equations 4.6 and 4.7. The right hand also includes an additional revenue factor from exporting products into i, which can be influenced by the fiscal instruments.

The elasticity of the number of firms with respect to the different tax rates is presented below:

it is useful to present the elasticity of the number of firms with respect to the different tax rates to aid in the understanding of the effects assuming Pareto distributions on the determination of the fiscal instruments. The elasticities are provided below:

$$
\begin{aligned}
\xi_{M_{js},\delta_{js}} &= -\left[\frac{-\tau_j \delta_{js}}{1-\tau_j \delta_{js}} + \frac{f_{jjs} \frac{\partial h_{jjs}^{\sigma_s-1}}{\partial \delta_{js}} \delta_{js} + f_{jis} \left(\frac{\partial h_{jis}^{\sigma_s-1}}{\partial \delta_{js}} \delta_{js} \kappa_{jis}^{x} + \frac{\partial \kappa_{jis}^{x}}{\partial \delta_{js}} h_{jis}^{\sigma_s-1} \delta_{js} \right)}{f_{jjs} h_{jjs}^{\sigma_s-1} + \kappa_{jis}^{x} f_{jis} h_{jis}^{\sigma_s-1}} \right] \\
\xi_{M_{js},\tau_j} &= -\left[\frac{(1-\delta_{js})\tau_j}{(1-\tau_j \delta_{js})(1-\tau_j)} + \frac{f_{jjs} \frac{\partial h_{jjs}^{\sigma_s-1}}{\partial \tau_j} \tau_j + f_{jis} \left(\frac{\partial h_{jis}^{\sigma_s-1}}{\partial \tau_j} \tau_j \kappa_{jis}^{x} + \frac{\partial \kappa_{jis}^{x}}{\partial \tau_j} h_{jis}^{\sigma_s-1} \tau_j \right)}{f_{jjs} h_{jjs}^{\sigma_s-1} + \kappa_{jis}^{x} f_{jis} h_{jis}^{\sigma_s-1}} \right]
\end{aligned}
$$

Just like in the closed economy, the response of the equilibrium number of firms with respect to τ, δ depend upon the distributional assumptions being made. This is clear from the terms $\partial h^{\sigma-1}/\partial x$ which are identical to zero when productivities are assumed to be distributed as Pareto. For the general distribution, the above elasticities contain an additional term that captures the changes in the export market. These alterations are a combination of effects on the productive term or the "intensive" margin; and the change in the ex-ante probability of entering the export market, the "extensive" margin.

5.1 Symmetric countries

The main result of this subsection shows that under the Pareto distribution assumption, optimal tax rates for the open economy are identical to those of the closed economy. This odd result is unique to the Pareto environment since it generates ex-ante probabilities of exporting that are invariant to changes in tax rates. In contrast, the optimal tax rates in the open economy under lognormal distribution are different since governments' power to affect M, φ^* via tax policy is diminish when the country opens to trade.

In this setting I impose the additional restriction that both countries are completely symmetric and both governments set their optimal fiscal policies together. In this case, we can think of countries having a "harmonization" scheme with respect to their statutory tax rates and depreciation allowance rates.[19] To avoid the nuisances of first-player advantages or incentives to deviate from the commonly agreed tax rates, I assume that there is a global planner that sets the tax rates.

The full symmetric assumption allows for a straightforward relationship between the export cutoff and the domestic productivity cutoff.

$$\varphi^*_{ji} = \left(\frac{f_{jis}}{f_{jjs}}\right)^{\frac{1}{\sigma_s-1}} \theta_{jis}\varphi^*_{jj} \tag{5.5}$$

$$M^j_{tot,s} = M_{js}\left(1+p^x_{jis}\right) \tag{5.6}$$

The particular relation of φ^*_{ji} with the domestic productivity cutoff has powerful implications for the optimal tax rates; in particular for the case of Pareto as highlighted in the following lemma:

Lemma 5.1. *Let $x_s = \tau, \delta_s$, under the symmetric assumption the following holds:*

$$\frac{\partial \kappa^x_{jis}}{\partial x}x = \kappa^x_{jis}\xi_{\varphi^*_{jj},x}\left(\Upsilon_{js}(\varphi^*_{jjs}) - \Upsilon_{js}(\varphi^*_{jis})\right) \qquad x = \tau, \delta_s \tag{5.7}$$

Furthermore,

- *If $Z \sim Pareto$ then $\dfrac{\partial \kappa^x_{jis}}{\partial x}x = 0$.*
- *If $Z \sim log\mathcal{N}$ then $\dfrac{\partial \kappa^x_{jis}}{\partial x}x > (<)\ 0$ if $\xi_{\varphi^*_{jjs},x_s} < 0\ (>0)$. This derivative is only equal to zero when*

[19] This "harmonization" scheme has been argued as optimal for the case of the Europe Union with Devereux as one of the main voices supporting this type of framework.

$\xi_{\varphi^*_{jjs},x_s} = 0$ *or as* $\varphi^*_{jj} \rightarrow \infty$

Lemma 5.1 says that under the country symmetry assumption and Pareto productivities there is no change in the *ex-ante* probability, of a successful firm , of entering the export market following changes to corporate tax rates. Thus, a symmetric country model with Pareto productivities can't explain the results found by either Bernini and Treibich (2013), Alessandria and Choi (2014) or Federici and Parisi (2014).

In contrast, when lognormal productivities are assumed the modifications to tax rates have an effect on the export probabilities and hence on the number of exporters in equilibrium. The intuition for the direction of the change is simple. First, assume that τ, δ have a negative effect on the domestic productivity cutoff. Since φ^*_{jis} is a fixed multiple of the domestic cutoff, the probability of obtaining a productivity above it – conditional on successful entry to domestic market – increases since the right tail of the lognormal distribution is monotonically decreasing. A more intuitive explanation: under the symmetry assumption, the foreign market has become less competitive due to the reduction in average productivities and making it easier for domestic firms to serve the foreign market via exports.

The invariability of the number of exporters to modifications in the tax rate, under the Pareto assumption, has the following implication:

Proposition 5.2. *Assume productivities are Pareto distributed. The optimal tax rates for the open economy under the symmetry assumption are exactly equal to those obtained in the closed economy.*

Proof. See Appendix □

While proposition 5.2 states that the optimal formula for τ, δ have not changed in this setting, it doesn't imply that equilibrium outcomes haven't changed. The model still generates gains from trade spawn from the increased productivity of the firms following the opening to trade that enhances competition.

Nonetheless, the implication that optimal taxes remain the same in the opening economy is striking, and might be judge as an undesirable property generated by the Pareto distribution. The explanation behind this odd outcome is quite simple. It was shown that the Pareto distribution muted a channel of transmission by precluding the rearrangement of the sector via h, which in this open economy setting is extended to the export market via h_{ji}. Moreover, the Pareto distribution also erases a channel of effect through the invariability of the number of exporting firms in equilibrium. Hence, the closed and open economy optimal rates are the same since the export channels of transmission are also annihilated under the Pareto distribution assumption.

In contrast, export market channels play a significant role in the determination of the optimal tax rates in the lognormal scenario. The transition from autarky to trade cuts the power of the government to influence equilibrium outcomes as stated in the proposition below:

Proposition 5.3. *Let $\varepsilon^{C}_{\varphi^*_{jjs},x_{js}}$, $\varepsilon^{O}_{\varphi^*_{jjs},x_{js}}$ be the elasticity of the domestic cutoff productivity in the closed and open economy respectively. If firms draw productivity from a lognormal distribution then the following holds:*

$$|\varepsilon^{O}_{\varphi^*_{jjs},x_{js}}| < |\varepsilon^{C}_{\varphi^*_{jjs},x_{js}}| \qquad \forall s \in S \;\; \textit{and} \;\; x_{js} = \tau_j, \delta_{js}$$

Proof. See Appendix □

From the discussion of 3.2, we saw that governments make a trade off between raising productivity in some sectors while increasing the number of firms in others. In the open economy the degree by which governments can influence the equilibrium productivities diminishes relative to the closed economy setting. In one hand, this is "bad" for sectors with high σ as the government loses power to raise equilibrium productivity. On the other hand, sectors in which government policies were reducing equilibrium productivity are affected to a lesser degree, a"good" outcome.

The effects of proposition 5.3 are passed into the equilibrium number of firms and therefore into the aggregate variables. If governments – in an economy with lognormal distributed productivities – didn't adapt their corporate tax rates when opening to trade, the policy recommendation under Pareto distributions, they will experience increases/decreases in their tax revenue thereby missing their target spending. Table II contains the results of an economy that opens to trade; assuming that governments keep using the optimal tax instrument rates of the closed economy. Consistent with Head et al. (2014) I find that gains from trade (GFT) under Pareto are significantly higher than those obtained by assuming lognormal distribution of productivities. Moreover, the tax revenue in the lognormal environment decreases for all scenarios which forces the government to tax households in order to meet their expenditure. This reduction in disposable income has a negative effect in the number of firms; therefore, this fiscal issue also plays a factor in the GFT differences.

To further illustrate the effects in tax revenue from moving into the open economy without changing the corporate tax rates, I present its response function in terms of several parameters in Figure VI. In these graphs the dash lines correspond to the Pareto distribution assumption while the solid lines are for the economy with lognormal distribution of productivities. In the first panel we see that the wedge between the public spending ($q_0^G = 0.5$) and tax revenue increases with the degree of asymmetry in the elasticity of substitution across sectors. Just as in the closed economy, when the sectors are completely symmetric there is no difference in the optimal tax rates between the Pareto and lognormal distribution assumptions. In term of the fixed cost of production we observe that the tax revenue increase with f_1 but decreases with f_2. This happens because the increase in fixed production cost reduces the number of firms and in the case of sector 1, which gets a positive depreciation allowance rate, it reduces the total amount of "subsidy" given to this sector. For sector 2 the explanation is analogous, but for this sector the depreciation allowance rate is negative.

Lastly, I provide some examples of the welfare loses that government can incur by using the incorrect policy recommendation for the corporate tax instrument rates. For the open economy case, the policy recommendation under Pareto is to keep taxes unchanged when switching from autarky to trade. Thus, the "null" model will use the optimal tax rates found in the closed economy, for the lognormal assumption, and compute the open economy equilibrium. These outcomes are compared to the "alternative" model in which the optimal tax rates have been updated to their new values. The welfare gains from using the correct taxes are found in the last row of Table II. Governments can gain an additional 0.12% to 0.32% in welfare by adjusting their corporate tax rates and, once more, the gains from using the correct tax rates increase with the degree of asymmetry across the sectors.

6 Empirical Evidence for using lognormal distributions

To finalize this paper I present some basic empirical findings that suggest lognormal distributions are a better fit for the empirical distribution of productivities for developing countries. This adds to the evidence first found by Sun et al. (2011) for Chinese firms, and Head et al. (2014) for French and Spanish firms.

I test the fitness of the Pareto distribution using multiple estimation methods on two different measure of productivity. The first measure is direct estimation of productivities under the assumption that the productive technology of firms is Cobb-Douglass. Under this approach I follow Del Gatto et al. (2006) as this paper has been cited multiple times to justify the validity of the Pareto assumption for European firms. Thus, I replicate their studies using data for developing countries. Nonetheless, there are many issues involving the direct estimation of productivity which can be reduced if I were to use Olley-Pakes method; however, the data available isn't a proper panel which precludes me from using such method. Therefore, the second approach I use involves using direct sales data for the firms. In this case the assumption isn't on the firms' technology but on the characteristic of the sector which is assumed to be monopolistic competitive with firms pricing their products at a markup.

Regardless of which measure of firm productivity is used, the results strongly point in the direction of a lognormal distribution over a Pareto distribution for firm level productivity. Moreover, for most empirical distributions the estimated parameters for the Pareto distribution violate the equilibrium conditions for the Melitz model, rendering it inapplicable.

6.1 Data

The necessary firm level data comes from the Enterprise Surveys database, which is provided by the World Bank. The survey is given to firms with 5 or more full time employees in 136 countries and contains a rich set of variables that provide a detailed picture of the firms' performance as well as the environment in which they operate. To ensure that data is comparable across countries, we make use of the standardized surveys for the period 2006 to 2013. These surveys were designed to be representative of the economy of each country, including its sector composition, with sample sizes chosen to ensure robust statistical inferences.

I restrict the database to manufacturing firms that have completed the *manufacturing questionnaire*.[20] Observations are dropped if they are missing any of the following variables: total sales, net book value of machinery and equipment and, number of full time employees. Monetary variables in the survey are reported in local currency units (LCU) in nominal terms which are transformed

[20]There are 3 types of questionnaires in the survey: core, manufacturing and service. The last two questionnaires contain the same questions as the core plus a set of extra questions related to manufacturing or service sectors. The manufacturing questionnaire is the only one that asks for the net book value of current machinery and equipment, which is our fixed capital variable.

into real values expressed in international 2010 dollars. The transformation is accomplished using GDP deflators and PPP exchanges obtained from the World Bank financial database. Labour input is measured by the number of full time *permanent* workers that the firm employed during the fiscal year. A permanent full time employee is a full time paid worker that has been in the firm for a year or more and/or full time workers that have been there for less than a year but have a renewal offer.[21]

The ISIC codes of the firms are used to classify them into 18 sectors. Table III shows the distribution of observations across these sectors and geographical regions. The Middle East region (MNA) is substantially underrepresented compared to other regions and it's dropped due to an insufficient number of observations. The "Petroleum and Coal" sector is omitted for the same reason.

6.2 Testing the fitness of distributions: productivity as the residual of the production function

Assuming a Cobb-Douglas production, the productivity of a firm j in sector i is estimated by $\exp(c_i + \epsilon_i)$:

$$log(sales_j) = c_i + a_i log(K_j) + b_i log(N_j) + \epsilon_{j,i} \tag{6.1}$$

This regression is computed separately for each sector/region pair and summary statistics are presented in table V. Eastern Europe and Central Asia region comes atop with an average (across sectors) of 222.62 while Africa stands last among all regions studied, with an alarming low 4.78. A minor surprise is Latin America ranking second, right above the Asia Pacific region.

Sectors inside each region are remarkably different reinforcing that such cross-sector heterogeneity should be explicitly consider in my corporate taxation model. "Electric machinery" and "professional and scientific equipment" are the two sectors that exhibit some of the best performance in all regions; however, no common worst performing sectors across regions were found. Nonethe-

[21] A second measure that takes into account the *temporary* full time workers was also considered. The importance of including the temporary workers stems from the vast differences in labor markets of the countries in the sample. Regulations, unions, internship requirement, etc are quite different across countries/regions and thus the firms' composition of permanent and temporary full time workers will differ greatly depending on location. We calculate the modified labor measure by computing the median (across firms in a particular country)of the average months a temporary worker is employed; the median is then divided by 12 and the resulting number is multiplied by the number of temporary full time workers the firm employed. This last number is added to the full time permanent workers to generate the modified labor measure.

less, the worst performing sector in ECA (wearing apparel) is 4 to 6 times better than the worst performer in the other regions, excluding Africa. If the paper product sector is not included then the top performer of ECA is less than twice as productive as the top performers of other regions, including Africa.

Pareto

Now that productivities have been estimated I test if their distribution can be properly fitted by a Pareto distribution. The functional form of the Pareto distribution implies that for a region r and sector s, the shape parameter k_s can be estimated by:

$$log(1 - F(x_{s,r})) = cons - k_s log(x_{s,r}) + \epsilon_{s,r,} \tag{6.2}$$

This estimation approach is used in Del Gatto et al. (2006) with the difference that I include fixed year effect in the OLS regression. Estimation results are found in Tables VI-X under the OLS headings.

It will be shown below that estimates for k_s using OLS are unreliable but they are reported for the sake of comparison with the values for Western Europe in Del Gatto et al. (2006). Most of the estimated k_s are below one which could present a problem, since the shape parameter (k_s) has to be greater than the elasticity of substitution minus one, for the existence of an equilibrium in the Melitz model. Even though there is no consensus among economist about the exact value of the Armington elasticity of substitution, the range is usually between 1 to 4.6; though there are estimates as high as 12 and as low as 0.51.[22] The estimated k_s under OLS are consistent with the model if the elasticities are in the lower range of what is commonly assumed in trade models. Thus, the elasticities bounds imply by the estimated k_s are plausible but not likely.

An alternative estimator for k_s has to be employed since the OLS estimator is biased, which is clear once 6.2 is re-written into:

$$log(1 - F(x_{s,r})) = k_{s,r} log(x_{min,s,r}) - k_s log(x_{s,r}) + \epsilon_{s,r,}$$

the constant term in the previous regression is a function of the shape parameter and the lower

[22]The most recent estimation of Armington elasticities can be found in Feenstra et al. (2014)

bound of the support of $F(x)$. Due to the unreliability of the estimators of k_s using simple regression I use a maximimum likelihood estimator instead; where I assume $x_{min,s,r}$ is equal to the minimum productivity observed in sector s in region r. [23]

Estimation using MLE generates a very different picture from what was obtained under OLS. First, the estimated shape parameters are smaller for all cases, which highlights the bias of the OLS estimator. A detail description of results under this estimation is not provided since the estimated distributions are not good approximations of the empirical distributions. These goodness of fit conclusions are derived using the Kolmogorov-Smirnov test with the associated $p - values$ reported in the same tables.[24] Using a threshold of $p > 0.05$, there is no case but one in which the estimated Pareto distributions fit the data well. The "Professional and Scientific equipment" in the SAR region is the only case that passes the KS test; however, the number of observations is 19, which is below the $n = 50$ sample size requirement to ensure the asymptotic properties.[25]

I continue by testing if the Pareto distributions fit only a part of the empirical distributions for productivity. Income distribution was believed to follow a Pareto distribution until Clementi and Gallegati (2005) , Brzezinski (2014) showed that such was not the case when the considering distribution of *all* incomes. The latter paper goes further and applies methodology developed in Clauset et al. (2009) to show that the right tails of the distribution are nicely fitted by a Pareto distribution. Following this insight, I employ the same methods to test the Pareto distributions one last time. The estimation procedure is simple. First, MLE estimation is perform in all observations and the KS statistics is computed, then the smallest observation is dropped and the estimation is re-run. This process continues until one of these happens: the KS statistic is below the threshold to pass or the next iteration would generate a bias that is greater than 0.10.

Surprisingly, no dramatic improvement was found with regards to the goodness of fit criteria as only two more cases passed the p-value threshold. Nonetheless, these cases are now a good fit

[23] As a robustness check, the same estimation is carried assuming that $x_{min,s,r}$ is equal across all sectors in the same region, and its value is given by the smallest productivity observed in such region. Results of both estimations are almost the same. Furthermore, it can be shown that the MLE estimator for x_{min} is the minimum observed value from the sample.

[24] Kolmogorov-Smirnov tests the null hypothesis that the estimated distribution and the empirical distribution are statistically no different.

[25] This case was re-estimated using a finite sample bias correction, which produced estimators not significantly different from the one reported in table X

without discarding a significant amount of the empirical data.[26] What is clear, is that the shape parameters under these estimations are consistenlty greater than those obtained by setting x_{min} equal to the lowest value observed in the full sample of the sector-region pair. The values for k_s are closer to those found in Del Gatto et al. (2006) and other studies conducted in developed countries. Furthermore, if the upper bound for $\hat{x}_{min}$ is removed then Pareto distributions are a decent approximation for the reduced data. This is a similar result to Head et al. (2014), which finds that only the right tails of productivity distributions can be approximated by a Pareto distribution.

Alternative Distribution: Log-Normal

I continue by testing if lognormal distributions perform better at describing the empirical data than the Pareto distributions. The pdf of the lognormal distribution is given by:

$$f(x) = \left(\frac{1}{x\sqrt{2\pi v}}\right) exp\left(-\frac{(ln(x)-m)^2}{2v^2}\right)$$

in which m, v are the scale and variance parameters. MLE is used to estimate the parameters and the results are reported in Tables VI-X.

The goodness of fit are a dramatic improvement over the Pareto distribution as attested by the Kolmogorov-Smirnov tests. Using the same $p - value$ threshold of 0.05, the estimated lognormal distributions are a good fit for 72 out of 85 possible cases. Africa is the region with the least sectors (9) that are satisfactory fitted while the rest of regions exhibit empirical productivity distributions that are well approximated for most, if not all, sectors.

The Kolmogorov-Smirnov tests strongly suggest that the data is well described by the lognormal distribution, but I perform an additional robustness check to confirm/reject these initial conclusions. Ross (2013) gives a thorough exposition of the advantages of using Monte Carlo simulations to obtain reliable $p - values$ that take into account the possibility that initial results were the product of chance. Synthetic data is generated for each sector/region pair by drawing values from the estimated distribution that best fitted it, where the number of draws is equal to the amount of observations used in the initial estimation. Then, the parameters to best fit the synthetic data are estimated and

[26] Paper product in EAP region discard 16% of observation while Electric Machinery in LAC discards only 7%

the Kolmogorov-Smirnov statistic computed. The whole procedure is repeated 10000 times (for each sector-region pair) to obtain a precision of $\epsilon = 0.005$.[27] The $p - value$ based on the Monte Carlo simulation is the fraction of KS statistics larger than the value obtained for the empirical data. In this case, higher $p - values$ are "good" in the sense that they imply a lower probability that the results from the KS test was just an outcome of chance.

Using a $p - value$ threshold of $p > 0.05$ ($p > 0.10$) only 44 (38) sector-region pairs pass the Monte-Carlo simulation confirmation. This number of successful fits is lower than the amount obtained by using the KS test criteria (72 cases) for which the estimated and empirical distribution were not statistically significantly different from each other. Nonetheless, the rejections/acceptance of fits based on the Monte Carlo simulations are in line with observations of the quantile-on-quantile plots.

6.3 Testing the fitness of distribution: sales data

The previous estimation using estimated values of firms' productivities is prone to many critics, specially regarding endogeneity issues between revenues and the amount of labor employed. Methods to solve this problem (such as Olley-Packes and its derivatives) require a proper panel data which is not available in these surveys.

Therefore, I perform an alternative analysis that uses revenues for firms to infer the productivity parameter consistent with the model presented in this paper. The Melitz model implies that a firm with productivity φ has revenue:

$$\begin{aligned} r(\varphi) &= p(\varphi)^{1-\sigma} \frac{Income}{\mathbb{P}^{1-\sigma}} \\ p(\varphi) &= \frac{w}{\rho}\varphi^{-1} \end{aligned}$$

Thus, revenues under this model have the same distributional form as φ since the transformation $Y = \varphi^{\sigma-1}$ preserves the shape of the distribution of φ. Specifically:

- If φ came from a Pareto distribution with shape parameter k, then $\varphi^{\sigma-1} \sim Pareto(\tilde{k})$, where

[27] For computational considerations, the procedure is only carried for sector-region pairs that have passed the initial K.S test ($p > 0.05$).

$$\tilde{k} = \frac{k}{\sigma - 1}$$

- If $\varphi \sim log\mathcal{N}(m, v)$ then $\varphi^{\sigma-1} \sim log\mathcal{N}((\sigma-1)m, (\sigma-1)v)$

The analysis using firms' revenues has additional advantages: it expands the number of non-missing observations significantly, and it can be used to test if the estimated parameters for the Pareto distribution satisfy the equilibrium conditions of the model. Previously, observations missing input for capital equipment had to be deleted since it was a necessary input to estimate the residual from the production function; however, for the current estimation method this is not necessary and thus valid observations are increased by approximately 8000. The distribution of valid observations across the sector and regions is found in Table IV.[28]

Pareto or lognormal?

Before proceeding to the more rigorous testing, using the Kolmogorov-Smirnov statistics, it is useful to analyze the histograms for the distribution of the logarithm of revenues. The distribution of the log of sales is expected to be: exponential if sales were Pareto distributed; and normal if the sales follow a lognormal distribution. Figures VII to XI contain the histograms for log sales and several of them favor the lognormal as the underlying distribution for sales. In particular, Latin America region and Eastern Europe have the most consistent patterns supporting the hypothesis of lognormal distributions.

Next, I conduct the same analysis as in section 6.2 and obtain similar findings for the fit of the Pareto distribution. Estimation results are found in Tables XI to XV with the first columns containing the estimated parameters for a Pareto distribution. Similarly to results using estimated productivities, the KS statistics for most sectors in each region are unfavorable to the hypothesis that revenues are Pareto distributed. Only 2 cases, out of a possible 85, pass the KS test with a threshold $p - value$ of 0.05. The modified MLE, in which the cutoff parameter is free to move, doesn't provide

[28] The analysis presented in the main body uses the full sample of firms. Nonetheless, concerns may arise since the sample has a mix of firms that sell only domestically with others that also engage in export. Therefore, separate analysis using: (i) firms whose revenues are fully realized from the domestic market, (ii) firms whose national sales account for 90 % or more of their revenue. The results are not significantly different from using the full sample. In fact, when the sample consist of firms that only sell on the domestic market the conclusion in favor of using lognormal distributions to approximate the empirical distribution of productivity is stronger.

significant improvements except for "Electric Machinery" in LAC region which now passes the KS test by dropping only 7% of the lower observations.

Furthermore, the MLE results in values of $\tilde{k}$ that are below unity for all cases which is problematic. The condition for the existence of an equilibrium in the Melitz model is $k > \sigma - 1 \implies \tilde{k} > 1$, therefore the estimated parameters using the Pareto distribution are inconsistent with this model. The modified MLE estimation barely improves the problem as it results in estimates of $\tilde{k}$ that are above one in most case but not by a significant amount. In fact, for Africa the average $\tilde{k}$ still remains below one and the averages for the other regions are at most 1.66.

Finally, the estimated lognormal distributions perform remarkably well (and strongly outperform the Pareto distribution) in fitting the sales data, corroborating the first impressions from looking at the histograms of the logarithm of revenues. The lognormal distributions pass the Kolmogorov-Smirnov test for 70 sector-region pairs, out of a possible 85 cases, a dramatic improvement over the performance of the Pareto distribution. Once again, Monte Carlo simulations were performed (10 000 repetitions) to confirm the initial conclusions of the KS test. Using a p-value of 0.10 (0.05) the KS test is confirmed for 35 (42) cases, which is half of the cases that passed the KS test.

7 Conclusion

The question of the implication of assuming productivities that are Pareto distributed in a Melitz model has largely been neglected until recently when Head et al. (2014) showed their effects in equilibrium outcomes and how this assumption enhances the gains from trade relative to using a model with lognormal distributed productivities. However, the implications for policy of this de facto assumption have not been explored; specifically, the question of the difference between optimal corporate tax rates derived under the Pareto distribution and the lognormal distribution.

Using an enhanced Melitz model with heterogeneous sectors and corporate taxation under a framework that resembles those observed in the real world, I have demonstrated that using the Pareto distribution assumption mutes a transmission channel between the corporate tax rates and the equilibrium outcomes. Thus, I find not only quantitative differences between the optimal tax rates derived under the Pareto and lognormal distribution assumptions, but also qualitative implications for the optimal corporate tax rates. Optimal rates derived under both distributional assumptions share many properties, especially the attribute that firms in sectors with higher elasticities of substitution get smaller depreciation allowance rates on their fixed cost of productions. Quantitatively, the differences between the optimal rates derived under both distributions become more prominent with the degree of cross sector heterogeneity. There are also many qualitative differences with one of the most important regarding the explicit inclusion of fixed production and entry costs in the determination of the statutory corporate tax rate and the sector specific depreciation allowance rate. Under the Pareto distribution assumption the optimal rates are not functions of these fixed costs; hence, the optimal rates formulas derived under the lognormal assumption exploit sector heterogeneity along all dimensions. This issue is particularly important given that changes in fixed cost of sectors occur, and such changes can be quite significant as in the case of entry costs following regulations targeting the competitiveness of the sector. Another example is the evolution of fixed production costs that sectors experience through their life cycle, from infancy to maturity.

Additionally, incorporating the corporate tax framework into the Melitz model allows me to provide the theoretical basis to explain conflicting empirical results regarding the relationship between corporate taxes and export dynamics. My model shows that decreasing the statutory corporate tax rate can increase or decrease the probability of becoming an exporter, the sign of this relationship depends on the level of the depreciation allowance rate on fixed costs. Nonetheless, increasing the depreciation allowance rate decreases the probability of exporting for all levels of the statutory corporate tax rate since this increase reduces the equilibrium productivity cutoff of domestic firms which makes them less competitive relative to firms in the other country.

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Table I: Parameters and results for the different scenarios used to compute the inefficiencies from using the incorrect distribution for productivities. For outcomes with two values, the top comes from the "null" model while the value for the "alternative" model is directly underneath

Scenario	Almost Symmetric		Different Entry Cost		Different Cost of Production		More asymmetric Elasticities		Different Variance	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
Parameters										
Wage	1		1		1		1		1	
Labor Size	5		5		5		5		5	
q_0^G	0.5		0.5		0.5		0.5		0.5	
ψ	0.02		0.02		0.02		0.02		0.02	
Elasticity of Subs.	2.5	3	2.5	3	2.5	3	**1.5**	3	2.5	3
Share (α)	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Fixed cost Production	0.1	0.1	0.1	0.1	0.1	**0.2**	0.1	0.1	0.1	0.1
Entry cost	0.5	0.5	0.5	**0.1**	0.5	0.5	0.5	0.5	0.5	0.5
m_i	2	2	2	2	2	2	2	2	2	**6**
v_i	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.7
k_i	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	3.12	2.61
φ_{min}	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.69	317.69
Results										
Number Firms	4.72 4.76	2.19 2.16	4.73 4.76	2.55 2.53	4.72 4.76	1.18 1.17	12.56 12.82	1.57 1.48	4.72 4.76	1.61 1.59
Sector Price Index	3.69 3.68	5.20 5.21	3.69 3.67	3.82 3.82	3.69 3.67	5.97 5.97	0.18 0.17	5.73 5.75	3.70 3.68	0.06 0.06
Depreciation Rate (%)	28.32 29.70	-28.32 -35.50	28.32 28.17	-28.32 -36.11	28.32 29.02	-28.32 -35.62	90.57 94.56	-90.57 -120.54	30.11 32.45	-25.11 -31.51
Corporate Tax (%)	30.71 30.31		30.71 29.91		30.71 30.13		40.15 35.85		31.25 31.06	
$\sum T_{alternative}$	0.5049		0.5083		0.5065		0.5696		0.5044	
$\mathbb{W}_{null}/\mathbb{W}_{alt}$	0.9986		0.9977		0.9982		0.9766		0.9987	

Table II: Results for the open economy equilibrium with symmetric countries using the Pareto distribution recommend policy: the "optimal" corporate tax rates are the same as in the closed economy. The welfare gain from changing the corporate tax rates to their optimal value is given by $\mathbb{W}_{alternative}/\mathbb{W}_{null}$

Scenario	Almost Symmetric		Different Entry Cost		More asymmetric Elasticities		Different Variance	
	Pareto	Log-Normal	Pareto	Log-Normal	Pareto	Log-Normal	Pareto	Log-Normal
Sector 1								
$\%\Delta\varphi_{jj}$	16.436	9.567	16.436	9.550	8.349	10.283	16.436	9.599
$\tilde{\varphi}$	20.267	17.162	20.267	17.181	9.297	11.265	20.216	17.127
M	2.453	3.245	2.453	3.245	10.248	9.314	2.453	3.243
φ_{ex}	16.283	15.883	16.283	15.906	10.392	11.683	16.243	15.840
$\tilde{\varphi}_{ex}$	25.175	20.374	25.175	20.398	14.726	15.533	25.112	20.329
M_{ex}	1.245	1.498	1.245	1.496	2.433	3.339	1.245	1.500
GFT($\%\Delta\tilde{\varphi}_{tot}$)	21.607	9.801	21.607	9.794	16.824	12.671	21.607	9.815
% decrease in Prices	16.436	9.555	16.436	9.531	8.349	9.674	16.436	9.579
Sector 2								
$\%\Delta\varphi_{jj}$	18.703	8.510	18.704	6.379	18.704	7.987	24.595	12.585
$\tilde{\varphi}$	42.955	21.059	71.879	26.645	46.187	22.148	217.370	38.734
M	0.534	1.467	0.534	1.766	0.367	1.006	0.105	1.043
φ_{ex}	26.716	18.931	44.704	25.240	28.726	20.173	71.630	31.110
$\tilde{\varphi}_{ex}$	50.825	24.115	85.048	30.792	54.649	25.422	257.196	44.296
M_{ex}	0.315	0.718	0.315	0.729	0.217	0.475	0.068	0.598
GFT($\%\Delta\tilde{\varphi}_{tot}$)	22.155	8.003	22.155	6.934	22.155	7.789	28.415	11.193
% decrease in Prices	18.703	8.503	18.704	6.368	18.704	7.868	24.595	12.573
Country								
Tax Collected	0.500	0.499	0.500	0.499	0.500	0.486	0.500	0.499
Welfare	77.430	64.804	99.428	74.543	305.773	275.690	125.039	82.248
Gains from Trade	16.901	8.632	17.048	7.624	13.284	8.383	19.956	10.665
% ($\mathbb{W}_{alt}/\mathbb{W}_{null} - 1$)		0.12		0.163		0.327		0.154

Table III: Distribution of observations across sectors and regions

	Region						
	AFR	EAP	ECA	LAC	MNA	SAR	Total
Food beverages and tobacco	1,532	402	1,130	2,195	211	549	6,019
Textiles	185	326	287	872	7	484	2,161
Wearing apparel except footwear	971	345	611	1,260	33	452	3,672
Leather products and footwear	111	42	59	263	3	357	835
Wood products except furniture	232	61	244	145	15	66	763
Paper products	70	38	68	62	6	40	284
Printing and Publishing	226	56	214	194	10	68	768
Petroleum and Coal	5	7	6	8	6	2	34
Chemicals	336	276	286	1,323	40	283	2,544
Rubber and plastic	177	314	195	546	40	109	1,381
Other non-metallic products	207	374	324	391	172	94	1,562
Metallic products	89	101	55	126	6	85	462
Fabricated metal products	499	248	604	895	47	75	2,368
Machinery except electrical	112	173	431	622	9	78	1,425
Electric machinery	61	159	165	144	6	70	605
Professional and scientific equipment	19	82	107	73	2	15	298
Transport equipment	48	128	64	134	2	33	409
other manufacturing	717	106	327	453	39	143	1,785
Total	5,597	3,238	5,177	9,706	654	3,003	27,375

Table IV: Distribution of non-missing observations, across sectors and regions, for the analysis using firms' revenues.

	Region					
	AFR	EAP	ECA	LAC	SAR	Total
Food beverages and tobacco	1,936	553	1,684	2,793	706	7,672
Textiles	272	418	387	1,120	639	2,836
Wearing apparel except footwear	1,199	458	899	1,645	506	4,707
Leather products and footwear	143	55	81	306	386	971
Wood products except furniture	324	97	360	186	103	1,070
Paper products	88	56	95	96	70	405
Printing and Publishing	318	71	347	261	77	1,074
Chemicals	418	380	413	1,582	333	3,126
Rubber and plastic	213	418	326	651	141	1,749
Other non-metallic products	284	522	591	540	133	2,070
Metallic products	125	125	90	156	159	655
Fabricated metal products	654	287	850	1,083	88	2,962
Machinery except electrical	142	188	698	748	112	1,888
Electric machinery	73	215	257	175	71	791
Professional and scientific equipment	21	109	180	81	15	406
Transport equipment	64	158	93	167	55	537
other manufacturing	1,032	148	504	575	195	2,454
Total	7,306	4,258	7,855	12,165	3,789	35,373

Table V: Summary Statistics for the estimate productivities. The means are in hundreds of 2010 International Dollars

	AFR			EAP			ECA			LAC			SAR		
	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.	Mean	σ	Obs.
Food beverages and tobacco	1.55	5.23	1623	5.95	9.42	403	122.5	203.11	1112	23.4	28.56	2172	15.3	38.48	542
Textiles	3.71	10.93	187	5.76	6.62	329	55.32	87.72	284	43.57	42.81	872	17.53	23.24	481
Wearing apparel except footwear	0.66	1.29	1008	20.79	29.24	343	31.96	51.02	601	37.13	41.8	1251	15.29	18.54	448
Leather products and footwear	3.87	7.57	112	21.89	22.83	42	129.24	859.62	59	7.61	6.68	268	12.36	15.48	352
Wood products except furniture	4.23	19.74	240	7.75	11.54	63	105.21	156.87	240	26.93	47.4	143	26.35	43.91	66
Paper products	16.04	39.23	72	8.93	6.74	38	8803.15	47086.7	68	56.6	54.26	62	24.75	33.99	40
Printing and Publishing	0.45	0.8	234	28.07	46.6	56	36.41	75.76	210	184.31	251.72	192	6.89	9.76	68
Chemicals	9.82	31.81	343	18.6	37.73	272	202.89	325.77	284	73.7	86.32	1306	11.97	18.44	279
Rubber and plastic	4.38	13.21	187	66.52	97.62	311	49.36	57.97	193	54.71	39.86	537	5.68	8.1	108
Other non-metallic products	4.94	27.06	215	11.61	19.31	372	74.92	97.92	320	17.42	34.29	388	213.41	362.35	95
Metallic products	16.53	40.68	91	17.62	25.01	99	161.27	534.39	55	17.52	35.56	125	55.78	72.53	85
Fabricated metal products	0.95	2.61	530	32.44	87.44	246	50.73	66.67	594	53.18	55.13	885	14.24	23.62	76
Machinery except electrical	5.83	24.95	124	23.39	38.69	171	267.15	489.27	423	45.49	47.66	620	24.93	38.71	78
Electric machinery	22.76	115.08	63	110.05	165.49	157	115.01	149.2	163	23.05	20.09	142	21.81	80	70
Professional and scientific equip	195.21	277.3	19	55.36	77	82	305.64	412.9	106	144.52	135.2	73	194.1	179.31	15
Transport equip	26.87	33.11	48	58.56	52.58	126	86.75	127.47	64	13.07	31.25	138	21.09	84.54	34
other manufacturing	9.16	45.94	765	12.19	14.42	104	78.03	142.34	324	7.19	8.73	463	56.93	57.57	142
Total	4.78	31.28	5861	27.68	64.97	3214	222.62	5494.41	5100	42.43	67.06	9637	25.63	82.84	2979

Table VI: Africa: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1623	0.63	0.9	0.26	0.89	0.00	0.96	98.75	0.00	0.74	3.66	1.49	0.00	
Textiles	187	0.76	0.93	0.37	8.31	0.00	1.04	145.54	0.00	0.57	4.83	1.24	0.39	0.052
Wearing apparel except footwear	1008	0.71	0.88	0.34	1.32	0.00	0.96	31.85	0.00	0.58	3.25	1.31	0.04	
Leather products and footwear	112	0.6	0.84	0.27	3.50	0.00	0.79	97.97	0.00	0.42	4.89	1.47	0.81	0.439
Wood products except furniture	240	0.58	0.91	0.25	1.25	0.00	0.77	174.50	0.00	0.78	4.16	1.62	0.05	0.000
Paper products	72	0.67	0.9	0.30	17.78	0.00	0.88	646.76	0.00	0.65	6.23	1.35	0.27	0.017
Printing and Publishing	234	0.73	0.9	0.36	1.18	0.00	0.95	18.73	0.00	0.52	2.92	1.26	0.36	0.044
Chemicals	343	0.65	0.95	0.31	8.91	0.00	0.72	128.87	0.00	0.40	5.38	1.48	0.01	
Rubber and plastic	187	0.65	0.95	0.33	4.63	0.00	0.72	54.36	0.00	0.37	4.61	1.47	0.00	
Other non-metallic products	215	0.62	0.94	0.29	1.90	0.00	0.70	25.96	0.00	0.25	4.14	1.54	0.03	
Metallic products	91	0.55	0.75	0.15	0.63	0.00	0.89	352.68	0.00	0.33	6.29	1.49	0.29	0.024
Fabricated metal products	530	0.66	0.9	0.30	1.00	0.00	0.86	37.15	0.00	0.62	3.33	1.43	0.14	0.003
Machinery except electrical	124	0.55	0.93	0.28	2.24	0.00	0.61	28.09	0.00	0.24	4.39	1.70	0.02	
Electric machinery	63	0.5	0.86	0.17	0.44	0.00	0.64	48.07	0.01	0.17	4.96	1.85	0.22	0.012
Professional and scientific equip.	19	0.58	0.78	0.35	534.23	0.04	1.17	10564.04	0.00	0.47	9.12	1.31	0.99	0.955
Transport equip.	48	0.86	0.89	0.47	186.02	0.00	0.89	695.75	0.01	0.19	7.36	1.02	0.87	0.553
Other manufacturing	765	0.58	0.91	0.20	0.78	0.00	0.69	82.49	0.00	0.42	4.86	1.64	0.00	
Average		0.64	0.89	0.29	45.59		0.84	778.33			4.96	1.45		

Table VII: East Asia Pacific: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	403	0.87	0.88	0.27	7.84	0.00	1.29	412.73	0.00	0.61	5.76	1.06	0.46	0.092
Textiles	329	0.96	0.82	0.37	25.57	0.00	1.44	353.29	0.00	0.46	5.93	0.92	0.26	0.017
Wearing apparel except footwear	343	0.97	0.86	0.36	79.90	0.00	1.47	1360.27	0.00	0.53	7.15	0.93	0.11	0.002
Leather products and footwear	42	1.16	0.87	0.60	301.09	0.00	1.37	1036.87	0.00	0.26	7.37	0.76	0.80	0.426
Wood products except furniture	63	0.77	0.79	0.35	25.47	0.00	1.33	367.45	0.00	0.37	6.06	1.08	0.33	0.029
Paper products	38	1.29	0.86	0.67	161.08	0.00	1.47	430.78	0.13	0.16	6.57	0.66	0.77	0.377
Printing and Publishing	56	0.9	0.91	0.47	181.12	0.00	1.35	1621.48	0.00	0.52	7.33	1.01	0.82	0.461
Chemicals	272	0.91	0.87	0.42	89.79	0.00	1.60	2001.69	0.00	0.76	6.90	1.02	0.55	0.148
Rubber and plastic	311	0.92	0.87	0.38	286.16	0.00	1.39	5351.66	0.00	0.66	8.26	0.99	0.45	0.076
Other non-metallic products	372	0.96	0.87	0.35	39.35	0.00	1.22	550.88	0.00	0.42	6.52	0.96	0.22	0.010
Metallic products	99	1.04	0.92	0.62	219.95	0.00	1.52	1513.42	0.00	0.66	7.00	0.89	0.81	0.451
Fabricated metal products	246	0.96	0.9	0.44	173.30	0.00	1.24	1480.11	0.00	0.47	7.41	0.98	0.19	0.007
Machinery except electrical	171	0.96	0.92	0.50	176.33	0.00	1.21	1164.73	0.00	0.48	7.17	0.98	0.45	0.081
Electric machinery	157	0.87	0.87	0.32	276.31	0.00	1.31	5890.42	0.00	0.47	8.71	1.03	0.14	0.003
Professional and scientific equipment	82	0.86	0.88	0.44	313.63	0.00	1.21	2802.60	0.00	0.44	8.04	1.04	0.49	0.100
Transport equipment	126	1.12	0.85	0.52	630.69	0.00	1.49	3914.81	0.00	0.46	8.36	0.79	0.65	0.228
other manufacturing	104	0.92	0.84	0.48	97.41	0.00	1.44	818.60	0.00	0.50	6.65	0.95	0.94	0.738
Average		0.97	0.87	0.45	181.47		1.37	1827.75			7.13	0.94		

Table VIII: East Europe & Central Asia: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1112	0.78	0.85	0.33	278.38	0.00	1.17	8683.76	0.00	0.65	8.69	1.17	0.19	0.007
Textiles	284	0.79	0.82	0.35	170.75	0.00	1.38	3966.73	0.00	0.58	7.99	1.12	0.78	0.403
Wearing apparel except footwear	601	0.95	0.9	0.42	169.76	0.00	1.31	2574.45	0.00	0.68	7.49	0.99	0.09	0.001
Leather products and footwear	59	0.88	0.95	0.45	110.01	0.00	0.94	460.45	0.01	0.19	6.91	1.30	0.02	
Wood products except furniture	240	0.87	0.9	0.44	563.66	0.00	0.99	3510.19	0.00	0.34	8.62	1.07	0.16	0.004
Paper products	68	0.67	0.8	0.26	3438.73	0.00	1.00	116026.90	0.00	0.38	11.94	1.38	0.26	0.016
Printing and Publishing	210	0.88	0.88	0.40	154.65	0.00	1.36	2775.91	0.00	0.67	7.54	1.04	0.62	0.194
Chemicals	284	0.79	0.81	0.29	330.44	0.00	1.44	14939.70	0.00	0.60	9.29	1.11	0.64	0.213
Rubber and plastic	193	0.9	0.78	0.34	161.90	0.00	1.43	3045.54	0.00	0.45	8.07	0.95	0.63	0.208
Other non-metallic products	320	0.76	0.81	0.31	155.42	0.00	1.33	5973.27	0.00	0.62	8.29	1.15	0.99	0.929
Metallic products	55	0.65	0.92	0.31	144.95	0.00	0.78	2364.67	0.00	0.38	8.23	1.44	0.19	0.006
Fabricated metal products	594	0.94	0.85	0.32	139.47	0.00	1.36	3627.29	0.00	0.59	8.03	0.97	0.12	0.002
Machinery except electrical	423	0.89	0.86	0.38	1070.20	0.00	1.49	33762.05	0.00	0.81	9.58	1.03	0.35	0.044
Electric machinery	163	0.88	0.84	0.36	426.96	0.00	1.39	8455.83	0.00	0.60	8.83	1.00	0.81	0.452
Professional and scientific equipment	106	0.81	0.86	0.41	1435.32	0.00	1.41	25665.57	0.00	0.63	9.71	1.10	0.66	0.236
Transport equipment	64	0.83	0.74	0.36	317.55	0.00	1.57	5243.98	0.00	0.42	8.57	0.98	0.74	0.336
other manufacturing	324	0.82	0.89	0.35	220.82	0.00	1.05	3104.72	0.00	0.44	8.22	1.13	0.14	0.002
Average		0.83	0.85	0.36	546.41		1.26	14363.59			8.59	1.11		

Table IX: Latin America and the Caribbean: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	2172	0.98	0.83	0.32	63.62	0.00	1.81	3743.14	0.00	0.85	7.32	0.92	0.05	
Textiles	872	0.91	0.74	0.27	74.84	0.00	1.84	4770.79	0.00	0.70	7.99	0.94	0.04	
Wearing apparel except footwear	1251	0.95	0.81	0.33	121.37	0.00	1.69	4004.55	0.00	0.72	7.79	0.93	0.43	0.071
Leather products and footwear	268	0.92	0.7	0.25	10.31	0.00	1.91	704.29	0.00	0.59	6.30	0.88	0.15	0.005
Wood products except furniture	143	1.03	0.87	0.46	186.39	0.00	1.66	2017.19	0.00	0.57	7.41	0.89	0.73	0.332
Paper products	62	1	0.71	0.43	405.45	0.00	1.22	2326.14	0.02	0.16	8.34	0.80	0.74	0.329
Printing and Publishing	192	1.29	0.8	0.38	988.92	0.00	2.06	12703.50	0.00	0.41	9.53	0.70	0.06	0.002
Chemicals	1306	1.01	0.82	0.31	194.52	0.00	2.01	11126.73	0.00	0.82	8.50	0.89	0.27	0.021
Rubber and plastic	537	1.24	0.79	0.46	481.08	0.00	2.35	5880.26	0.00	0.65	8.37	0.70	0.80	0.434
Other non-metallic products	388	0.97	0.88	0.37	65.87	0.00	1.34	884.71	0.00	0.45	6.89	0.95	0.14	0.003
Metallic products	125	0.89	0.82	0.37	63.97	0.00	1.27	931.45	0.00	0.46	6.88	1.00	0.98	0.898
Fabricated metal products	885	1.11	0.84	0.37	260.83	0.00	1.65	4216.79	0.00	0.56	8.24	0.81	0.37	0.045
Machinery except electrical	620	0.97	0.78	0.32	138.30	0.00	2.02	5270.53	0.00	0.72	8.04	0.89	0.54	0.132
Electric machinery	142	1.12	0.85	0.46	188.51	0.00	0.99	680.91	0.28	0.07	7.43	0.79	0.18	0.006
Professional and scientific equipment	73	1.08	0.85	0.53	1543.51	0.00	1.27	5755.48	0.03	0.15	9.25	0.80	0.09	0.001
Transport equipment	138	0.67	0.64	0.21	6.04	0.00	1.26	525.59	0.00	0.33	6.51	1.17	0.03	
other manufacturing	463	0.83	0.76	0.24	6.90	0.00	1.84	988.21	0.00	0.79	6.09	1.03	0.31	0.032
Average		0.99	0.79	0.36	282.38		1.66	3913.54			7.70	0.89		

Table X: South Asia: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities based on estimation of the residual from a Cobb-Douglas production technology

		OLS		MLE			MLE mod				Log-normal			
	Obs.	k_s	R^2	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	542	0.77	0.91	0.38	43.79	0.00	1.24	1569.81	0.00	0.78	6.42	1.21	0.10	0.001
Textiles	481	0.94	0.87	0.45	113.92	0.00	1.67	2221.36	0.00	0.77	6.97	0.97	0.37	0.045
Wearing apparel except footwear	448	1.04	0.88	0.49	128.15	0.00	1.80	2190.74	0.00	0.81	6.90	0.89	0.55	0.142
Leather products and footwear	352	0.86	0.83	0.37	50.71	0.00	1.71	1529.58	0.00	0.75	6.59	1.03	0.98	0.871
Wood products except furniture	66	0.97	0.83	0.55	259.30	0.00	1.18	1248.29	0.00	0.41	7.37	0.92	0.65	0.224
Paper products	40	0.76	0.91	0.43	121.32	0.00	0.91	728.25	0.00	0.33	7.13	1.13	0.47	0.082
Printing and Publishing	68	1.22	0.89	0.48	59.13	0.00	1.59	394.49	0.00	0.40	6.16	0.77	0.62	0.200
Chemicals	279	0.96	0.85	0.47	85.67	0.00	1.67	1212.45	0.00	0.70	6.59	0.95	0.93	0.709
Rubber and plastic	108	0.74	0.85	0.39	21.90	0.00	0.69	93.68	0.00	0.17	5.65	1.18	0.86	0.543
Other non-metallic products	95	0.72	0.74	0.19	53.87	0.00	1.38	16824.12	0.00	0.63	9.33	1.15	0.88	0.589
Metallic products	85	0.86	0.87	0.46	361.42	0.00	1.12	2496.12	0.00	0.39	8.07	1.03	0.93	0.730
Fabricated metal products	76	0.89	0.83	0.45	86.49	0.00	1.51	959.18	0.00	0.58	6.70	0.98	0.49	0.102
Machinery except electrical	78	1.08	0.83	0.46	183.44	0.00	1.21	988.14	0.00	0.23	7.41	0.83	0.95	0.775
Electric machinery	70	0.9	0.92	0.52	113.83	0.00	0.84	313.88	0.01	0.19	6.65	1.10	0.20	0.008
Professional and scientific equipment	15	1.08	0.9	1.06	5494.80	0.83	1.06	5494.80	0.83	0.00	9.56	0.77	0.83	0.473
Transport equipment	34	0.7	0.9	0.46	41.97	0.07	1.11	485.10	0.00	0.53	5.92	1.41	0.78	0.399
other manufacturing	142	0.95	0.82	0.41	323.44	0.00	1.06	1977.84	0.00	0.18	8.24	0.91	0.81	0.435
Average		0.91	0.86	0.47	443.71		1.28	2395.76			7.16	1.01		

Table XI: Africa: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1623	0.26	0.89	0.00	0.96	98.75	0.00	0.74	3.66	1.49	0.00	
Textiles	187	0.37	8.31	0.00	1.04	145.54	0.00	0.57	4.83	1.24	0.39	0.541
Wearing apparel except footwear	1008	0.34	1.32	0.00	0.96	31.85	0.00	0.58	3.25	1.31	0.04	
Leather products and footwear	112	0.27	3.50	0.00	0.79	97.97	0.00	0.42	4.89	1.47	0.81	0.081
Wood products except furniture	240	0.25	1.25	0.00	0.77	174.50	0.00	0.78	4.16	1.62	0.05	0.003
Paper products	72	0.30	17.78	0.00	0.88	646.76	0.00	0.65	6.23	1.35	0.27	0.097
Printing and Publishing	234	0.36	1.18	0.00	0.95	18.73	0.00	0.52	2.92	1.26	0.36	0.005
Chemicals	343	0.31	8.91	0.00	0.72	128.87	0.00	0.40	5.38	1.48	0.01	0.038
Rubber and plastic	187	0.33	4.63	0.00	0.72	54.36	0.00	0.37	4.61	1.47	0.00	0.167
Other non-metallic products	215	0.29	1.90	0.00	0.70	25.96	0.00	0.25	4.14	1.54	0.03	0.021
Metallic products	91	0.15	0.63	0.00	0.89	352.68	0.00	0.33	6.29	1.49	0.29	0.017
Fabricated metal products	530	0.30	1.00	0.00	0.86	37.15	0.00	0.62	3.33	1.43	0.14	0.002
Machinery except electrical	124	0.28	2.24	0.00	0.61	28.09	0.00	0.24	4.39	1.70	0.02	0.114
Electric machinery	63	0.17	0.44	0.00	0.64	48.07	0.01	0.17	4.96	1.85	0.22	0.775
Professional and scientific equip.	19	0.35	534.23	0.04	1.17	10564.04	0.00	0.47	9.12	1.31	0.99	0.323
Transport equip.	48	0.47	186.02	0.00	0.89	695.75	0.01	0.19	7.36	1.02	0.87	0.852
Other manufacturing	765	0.20	0.78	0.00	0.69	82.49	0.00	0.42	4.86	1.64	0.00	
Average		0.29	45.59		0.84	778.33		0.45	4.96	1.45		

Notes. The values for x_{min} have been divided by 1000

Table XII: East Asia Pacific: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	403	0.27	7.84	0.00	1.29	412.73	0.00	0.61	5.76	1.06	0.46	0.029
Textiles	329	0.37	25.57	0.00	1.44	353.29	0.00	0.46	5.93	0.92	0.26	
Wearing apparel except footwear	343	0.36	79.90	0.00	1.47	1360.27	0.00	0.53	7.15	0.93	0.11	0.001
Leather products and footwear	42	0.60	301.09	0.00	1.37	1036.87	0.00	0.26	7.37	0.76	0.80	0.078
Wood products except furniture	63	0.35	25.47	0.00	1.33	367.45	0.00	0.37	6.06	1.08	0.33	0.035
Paper products	38	0.67	161.08	0.00	1.47	430.78	0.13	0.16	6.57	0.66	0.77	0.239
Printing and Publishing	56	0.47	181.12	0.00	1.35	1621.48	0.00	0.52	7.33	1.01	0.82	0.003
Chemicals	272	0.42	89.79	0.00	1.60	2001.69	0.00	0.76	6.90	1.02	0.55	
Rubber and plastic	311	0.38	286.16	0.00	1.39	5351.66	0.00	0.66	8.26	0.99	0.45	
Other non-metallic products	372	0.35	39.35	0.00	1.22	550.88	0.00	0.42	6.52	0.96	0.22	0.001
Metallic products	99	0.62	219.95	0.00	1.52	1513.42	0.00	0.66	7.00	0.89	0.81	0.507
Fabricated metal products	246	0.44	173.30	0.00	1.24	1480.11	0.00	0.47	7.41	0.98	0.19	0.002
Machinery except electrical	171	0.50	176.33	0.00	1.21	1164.73	0.00	0.48	7.17	0.98	0.45	0.323
Electric machinery	157	0.32	276.31	0.00	1.31	5890.42	0.00	0.47	8.71	1.03	0.14	0.381
Professional and scientific equipment	82	0.44	313.63	0.00	1.21	2802.60	0.00	0.44	8.04	1.04	0.49	0.160
Transport equipment	126	0.52	630.69	0.00	1.49	3914.81	0.00	0.46	8.36	0.79	0.65	0.034
other manufacturing	104	0.48	97.41	0.00	1.44	818.60	0.00	0.50	6.65	0.95	0.94	0.135
Average		0.45	181.47		1.37	1827.75			7.13	0.94		

Notes. The values for x_{min} have been divided by 1000

Table XIII: Eastern Europe & Central Asia region: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	1112	0.33	278.38	0.00	1.17	8683.76	0.00	0.65	8.69	1.17	0.19	
Textiles	284	0.35	170.75	0.00	1.38	3966.73	0.00	0.58	7.99	1.12	0.78	
Wearing apparel except footwear	601	0.42	169.76	0.00	1.31	2574.45	0.00	0.68	7.49	0.99	0.09	0.414
Leather products and footwear	59	0.45	110.01	0.00	0.94	460.45	0.01	0.19	6.91	1.30	0.02	0.574
Wood products except furniture	240	0.44	563.66	0.00	0.99	3510.19	0.00	0.34	8.62	1.07	0.16	0.460
Paper products	68	0.26	3438.73	0.00	1.00	116026.90	0.00	0.38	11.94	1.38	0.26	0.895
Printing and Publishing	210	0.40	154.65	0.00	1.36	2775.91	0.00	0.67	7.54	1.04	0.62	0.239
Chemicals	284	0.29	330.44	0.00	1.44	14939.70	0.00	0.60	9.29	1.11	0.64	0.048
Rubber and plastic	193	0.34	161.90	0.00	1.43	3045.54	0.00	0.45	8.07	0.95	0.63	0.011
Other non-metallic products	320	0.31	155.42	0.00	1.33	5973.27	0.00	0.62	8.29	1.15	0.99	0.310
Metallic products	55	0.31	144.95	0.00	0.78	2364.67	0.00	0.38	8.23	1.44	0.19	0.096
Fabricated metal products	594	0.32	139.47	0.00	1.36	3627.29	0.00	0.59	8.03	0.97	0.12	0.180
Machinery except electrical	423	0.38	1070.20	0.00	1.49	33762.05	0.00	0.81	9.58	1.03	0.35	0.041
Electric machinery	163	0.36	426.96	0.00	1.39	8455.83	0.00	0.60	8.83	1.00	0.81	0.001
Professional and scientific equipment	106	0.41	1435.32	0.00	1.41	25665.57	0.00	0.63	9.71	1.10	0.66	0.302
Transport equipment	64	0.36	317.55	0.00	1.57	5243.98	0.00	0.42	8.57	0.98	0.74	0.596
other manufacturing	324	0.35	220.82	0.00	1.05	3104.72	0.00	0.44	8.22	1.13	0.14	0.065
Average		0.36	546.41		1.26	14363.59			8.59	1.11		

Notes. The values for x_{min} have been divided by 1000

Table XIV: Latin America and the Caribbean: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	2172	0.32	63.62	0.00	1.81	3743.14	0.00	0.85	7.32	0.92	0.05	
Textiles	872	0.27	74.84	0.00	1.84	4770.79	0.00	0.70	7.99	0.94	0.04	0.057
Wearing apparel except footwear	1251	0.33	121.37	0.00	1.69	4004.55	0.00	0.72	7.79	0.93	0.43	0.046
Leather products and footwear	268	0.25	10.31	0.00	1.91	704.29	0.00	0.59	6.30	0.88	0.15	0.342
Wood products except furniture	143	0.46	186.39	0.00	1.66	2017.19	0.00	0.57	7.41	0.89	0.73	0.473
Paper products	62	0.43	405.45	0.00	1.22	2326.14	0.02	0.16	8.34	0.80	0.74	0.006
Printing and Publishing	192	0.38	988.92	0.00	2.06	12703.50	0.00	0.41	9.53	0.70	0.06	0.025
Chemicals	1306	0.31	194.52	0.00	2.01	11126.73	0.00	0.82	8.50	0.89	0.27	
Rubber and plastic	537	0.46	481.08	0.00	2.35	5880.26	0.00	0.65	8.37	0.70	0.80	0.000
Other non-metallic products	388	0.37	65.87	0.00	1.34	884.71	0.00	0.45	6.89	0.95	0.14	0.000
Metallic products	125	0.37	63.97	0.00	1.27	931.45	0.00	0.46	6.88	1.00	0.98	0.149
Fabricated metal products	885	0.37	260.83	0.00	1.65	4216.79	0.00	0.56	8.24	0.81	0.37	0.004
Machinery except electrical	620	0.32	138.30	0.00	2.02	5270.53	0.00	0.72	8.04	0.89	0.54	0.151
Electric machinery	142	0.46	188.51	0.00	0.99	680.91	0.28	0.07	7.43	0.79	0.18	0.125
Professional and scientific equipment	73	0.53	1543.51	0.00	1.27	5755.48	0.03	0.15	9.25	0.80	0.09	0.009
Transport equipment	138	0.21	6.04	0.00	1.26	525.59	0.00	0.33	6.51	1.17	0.03	0.555
other manufacturing	463	0.24	6.90	0.00	1.84	988.21	0.00	0.79	6.09	1.03	0.31	0.151
Average		0.36	282.38		1.66	3913.54			7.70	0.89		

Notes. The values for x_{min} have been divided by 1000

Table XV: South Asia Region: Parameter estimation and goodness of fit for the Pareto and lognormal distributions. Empirical distribution of productivities using a transformation on firms' revenues.

		MLE			MLE mod				Log-normal			
	Obs.	k_s	x_{min}	K.S p-value	k_s	x_{min}	K.S p-value	ratio of $x < x_{min}$	m	v	K.S p-value	Monte Carlo p-value
Food beverages and tobacco	542	0.38	43.79	0.00	1.24	1569.81	0.00	0.78	6.42	1.21	0.10	
Textiles	481	0.45	113.92	0.00	1.67	2221.36	0.00	0.77	6.97	0.97	0.37	
Wearing apparel except footwear	448	0.49	128.15	0.00	1.80	2190.74	0.00	0.81	6.90	0.89	0.55	
Leather products and footwear	352	0.37	50.71	0.00	1.71	1529.58	0.00	0.75	6.59	1.03	0.98	
Wood products except furniture	66	0.55	259.30	0.00	1.18	1248.29	0.00	0.41	7.37	0.92	0.65	0.014
Paper products	40	0.43	121.32	0.00	0.91	728.25	0.00	0.33	7.13	1.13	0.47	0.424
Printing and Publishing	68	0.48	59.13	0.00	1.59	394.49	0.00	0.40	6.16	0.77	0.62	0.106
Chemicals	279	0.47	85.67	0.00	1.67	1212.45	0.00	0.70	6.59	0.95	0.93	0.027
Rubber and plastic	108	0.39	21.90	0.00	0.69	93.68	0.00	0.17	5.65	1.18	0.86	0.034
Other non-metallic products	95	0.19	53.87	0.00	1.38	16824.12	0.00	0.63	9.33	1.15	0.88	0.041
Metallic products	85	0.46	361.42	0.00	1.12	2496.12	0.00	0.39	8.07	1.03	0.93	0.622
Fabricated metal products	76	0.45	86.49	0.00	1.51	959.18	0.00	0.58	6.70	0.98	0.49	0.894
Machinery except electrical	78	0.46	183.44	0.00	1.21	988.14	0.00	0.23	7.41	0.83	0.95	0.112
Electric machinery	70	0.52	113.83	0.00	0.84	313.88	0.01	0.19	6.65	1.10	0.20	0.684
Professional and scientific equipment	15	1.06	5494.80	0.83	1.06	5494.80	0.83	0.00	9.56	0.77	0.83	0.954
Transport equipment	34	0.46	41.97	0.07	1.11	485.10	0.00	0.53	5.92	1.41	0.78	
other manufacturing	142	0.41	323.44	0.00	1.06	1977.84	0.00	0.18	8.24	0.91	0.81	0.081
Average		0.47	443.71		1.28	2395.75			7.16	1.01		

Notes. The values for x_{min} have been divided by 1000

Figure IV: Effects of Changes in the Elasticity of Substitution for sector 2

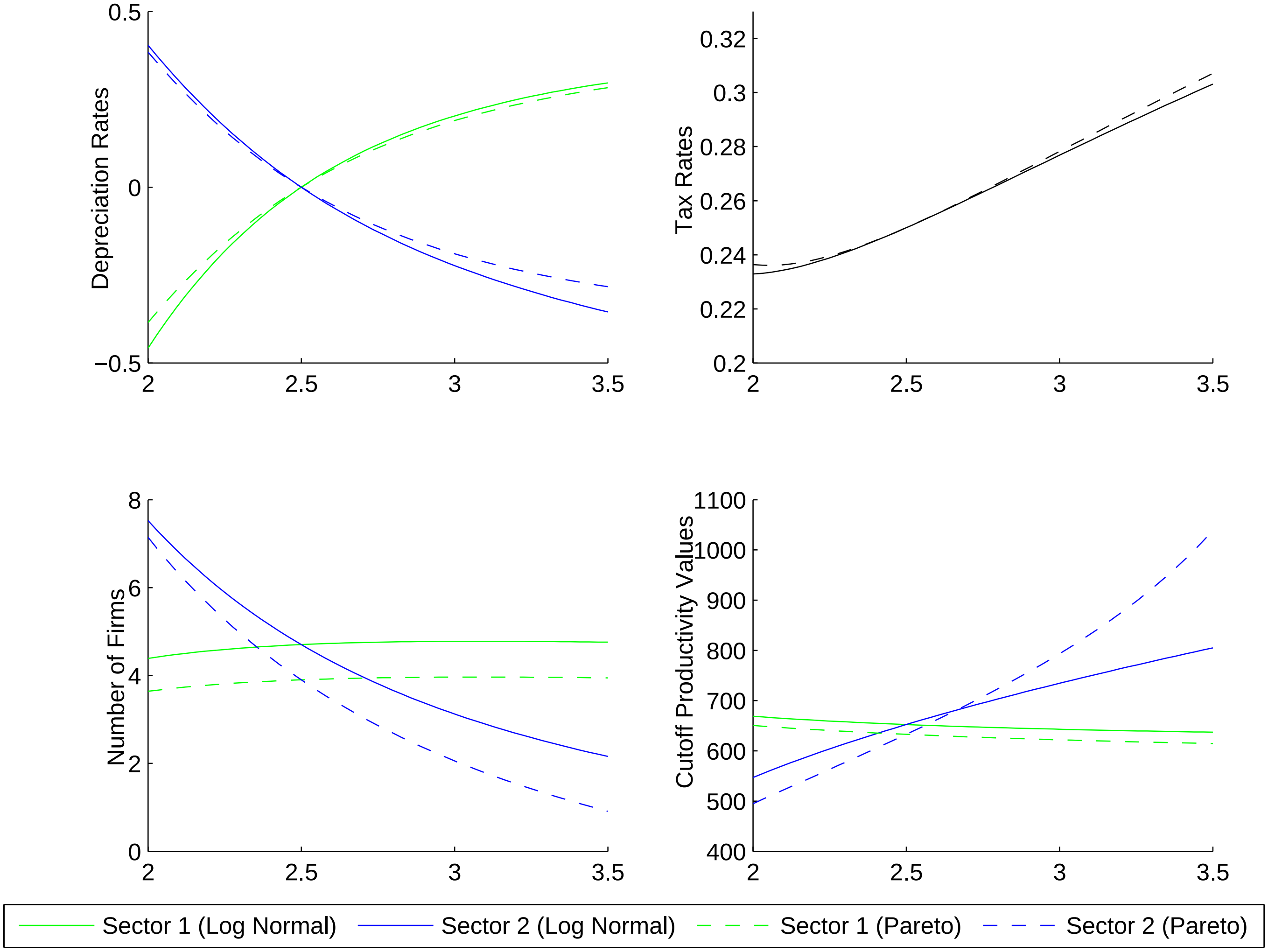


Figure V: Depreciation and tax rates as functions of different variables

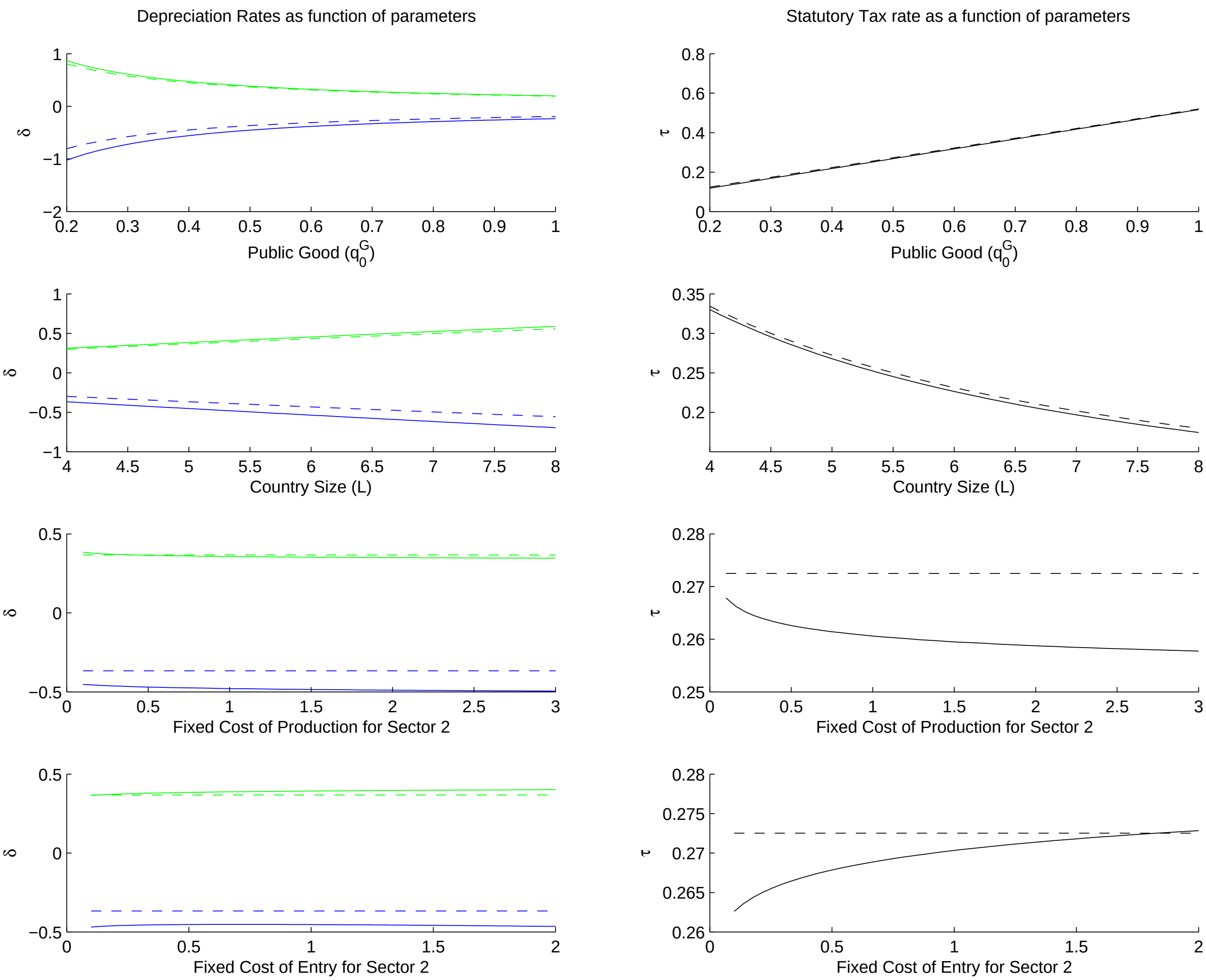


Figure VI: Tax revenue and gains from trade using the optimal corporate tax rates based in the closed economy formulas

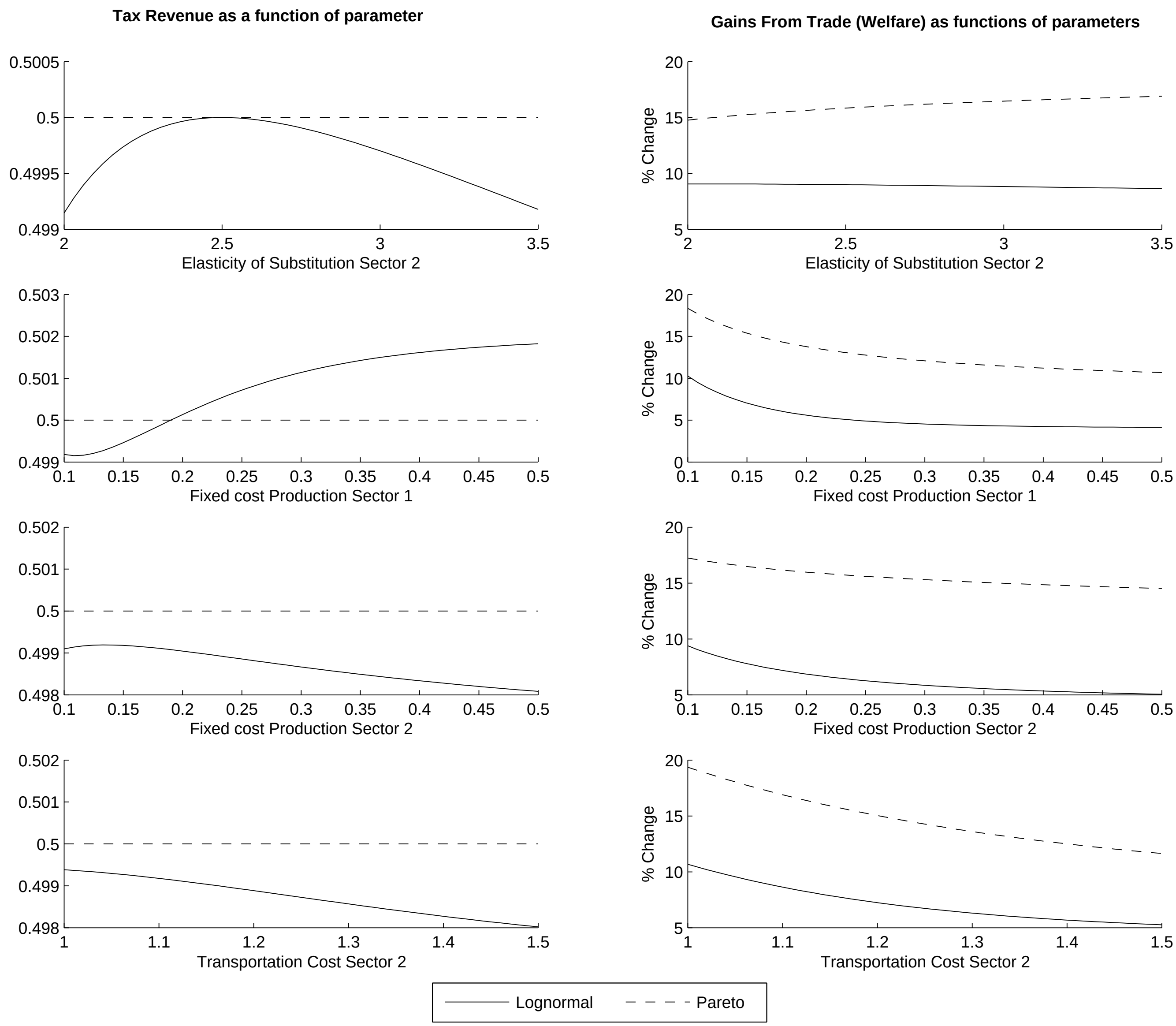


Figure VII: Distribution of log sales of firms for 17 sectors in the Africa region

Figure VIII: Distribution of log sales of firms for 17 sectors in the East and Pacific Asia region

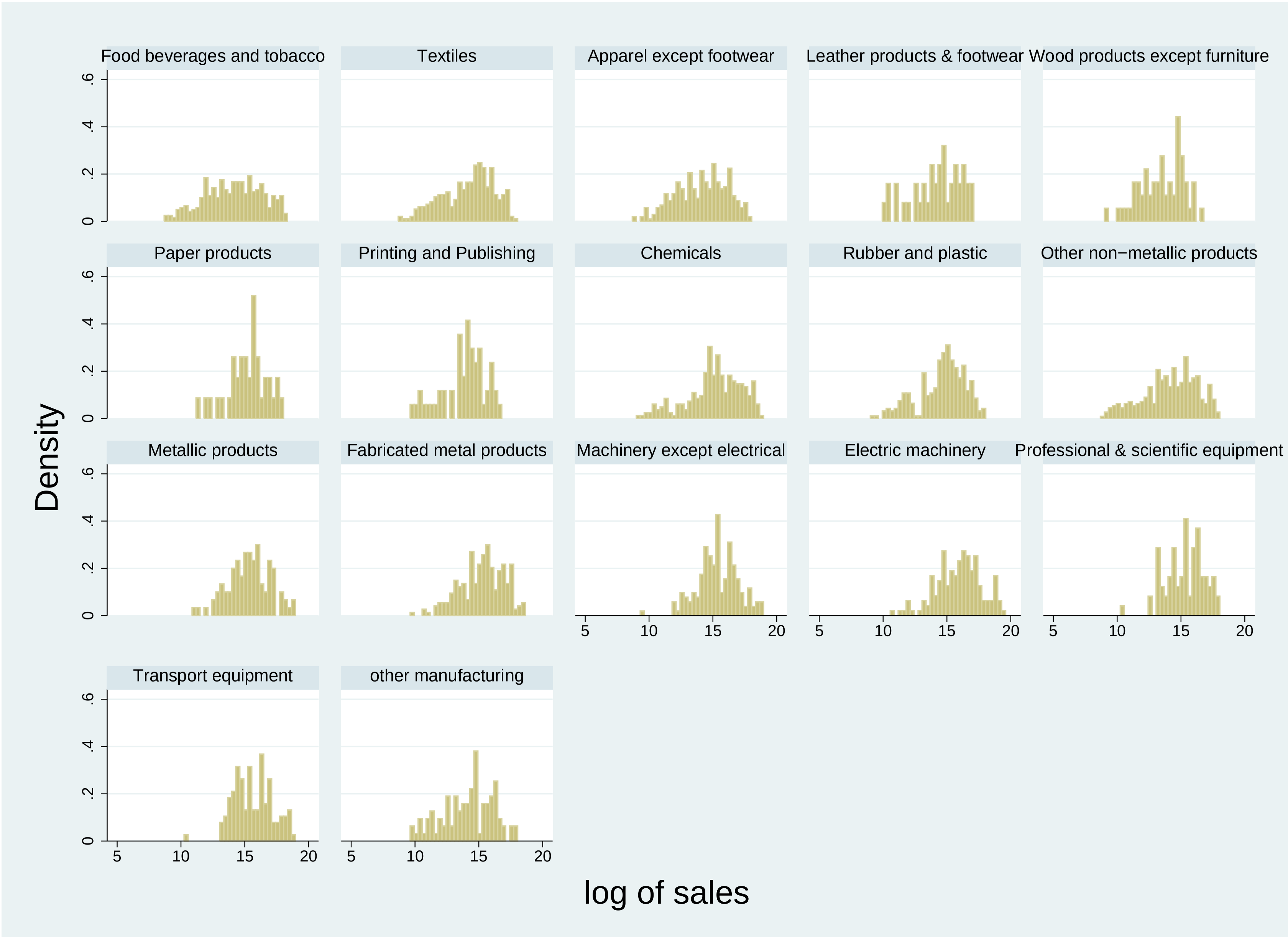


Figure IX: Distribution of log sales of firms for 17 sectors in the Eastern Europe and Central Asia region

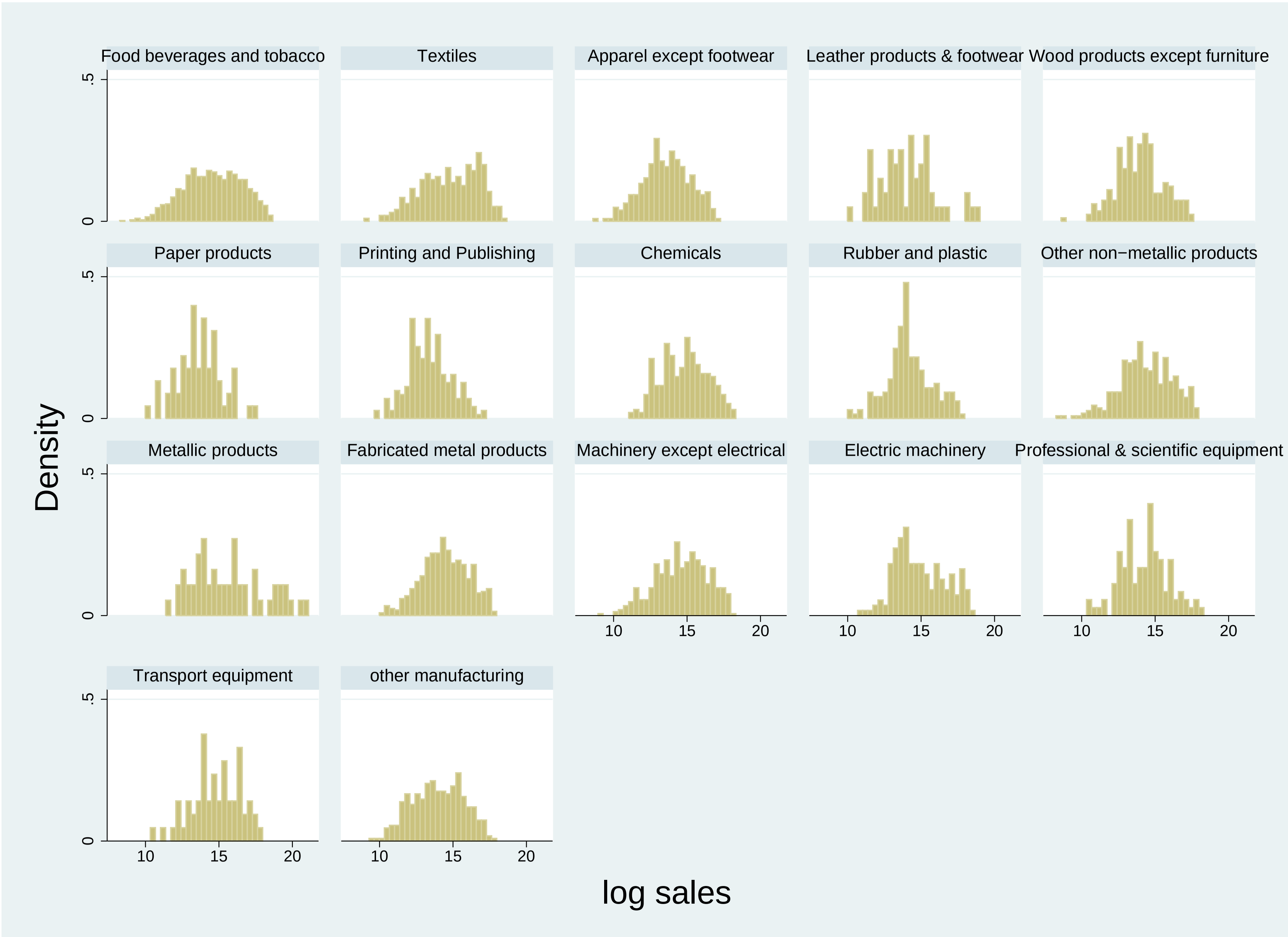


Figure X: Distribution of log sales of firms for 17 sectors in the Latin America region

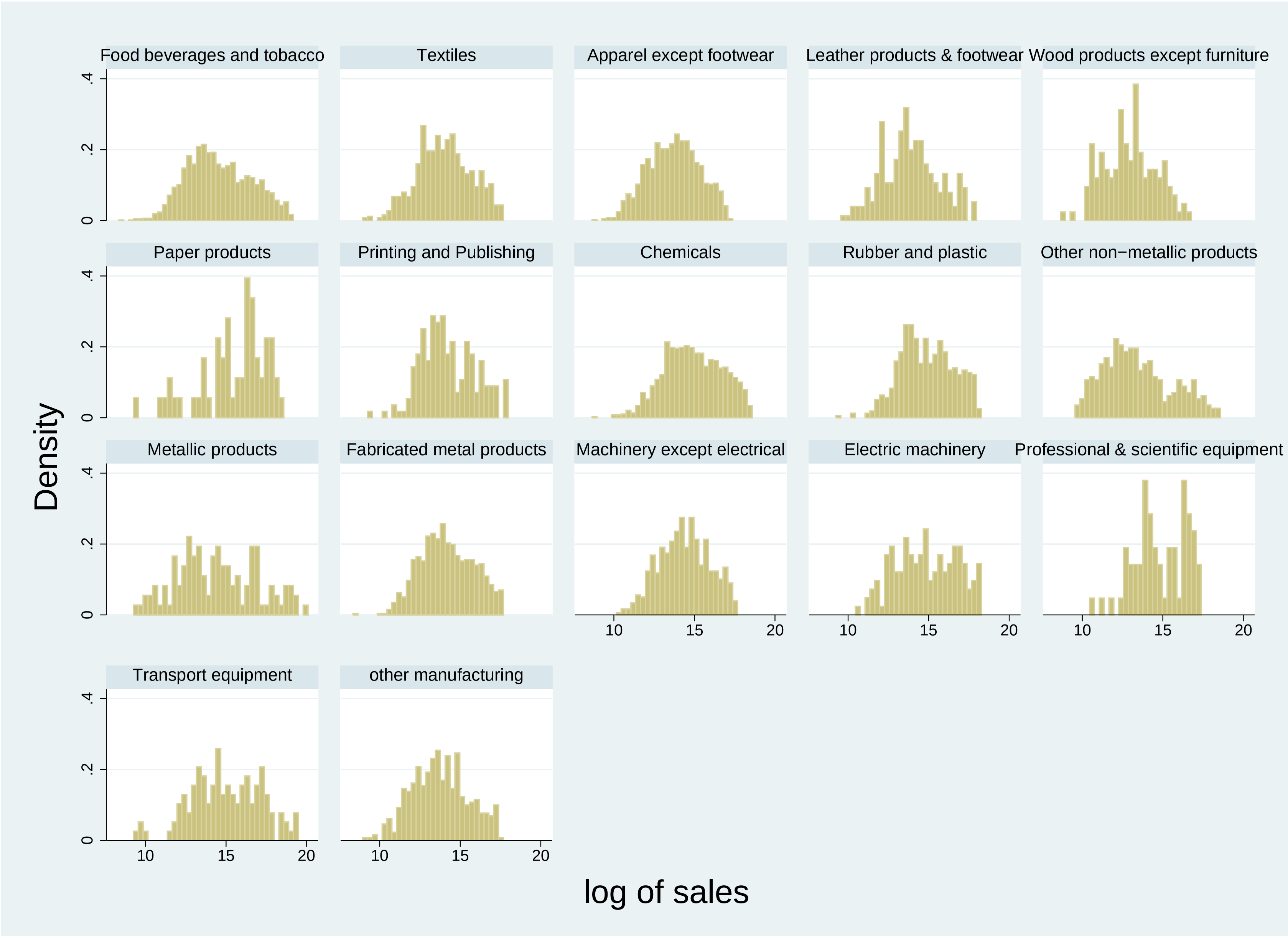
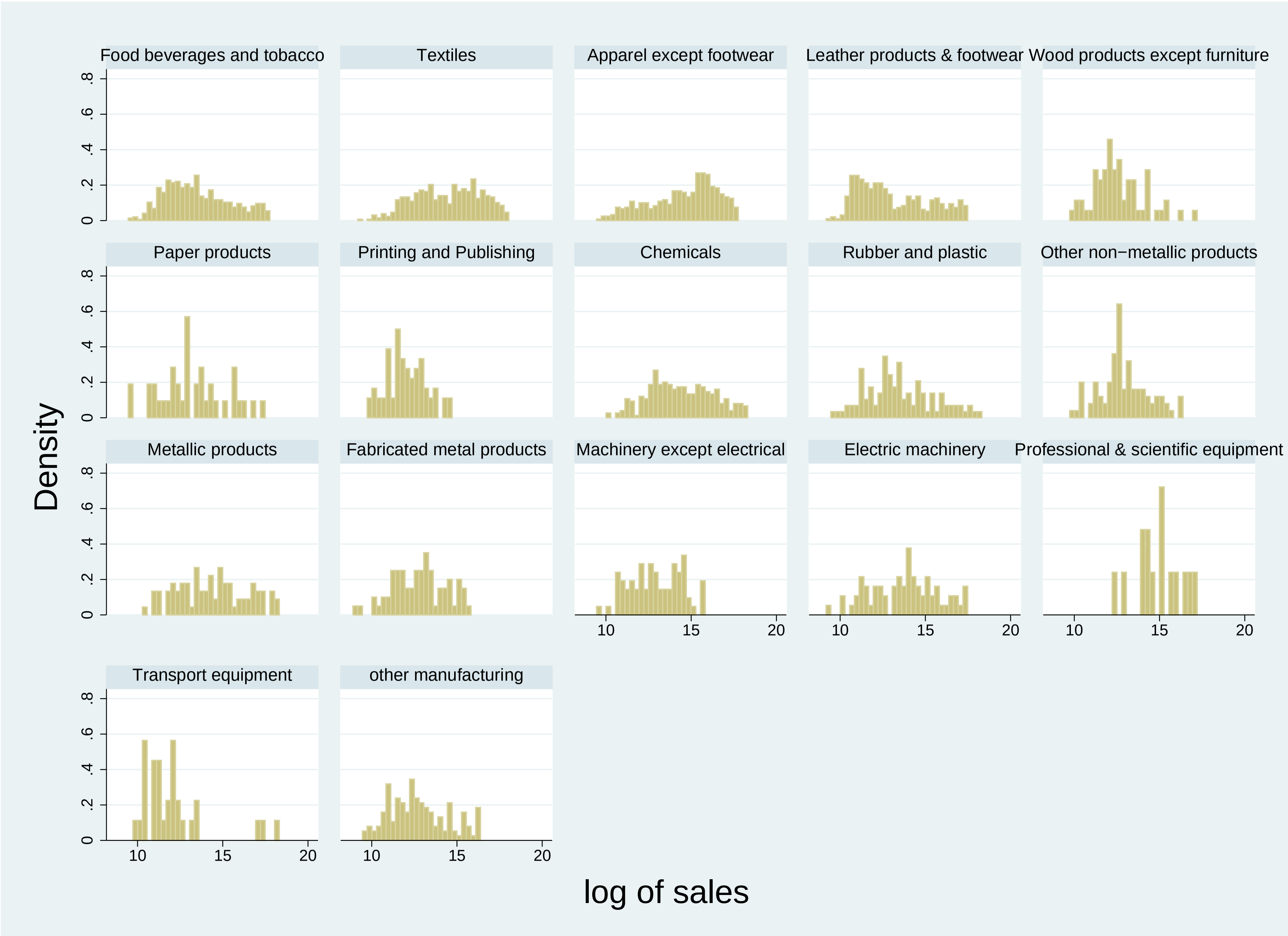


Figure XI: Distribution of log sales of firms for 17 sectors in the South Asia region



Appendices

A Closed Economy

Useful Formulas

$$\bar{r}_s = r(\tilde{\varphi}_s) = \sigma u_s f_s h_s^{\sigma_s - 1} \tag{A.1}$$

$$\bar{t}_s = t_s(\tilde{\varphi}_s) = \tau \left(u_s f_s h_s^{\sigma_s - 1} - \delta_s w f_s \right) \tag{A.2}$$

$$\frac{\partial u_s}{\partial \tau} = \frac{(1-\delta_s)}{(1-\tau)^2} \gtreqless 0 \tag{A.3}$$

$$\frac{\partial u_s}{\partial \delta_{s'}} = -\frac{\tau}{1-\tau} < 0 \qquad \text{if s=s', otherwise 0} \tag{A.4}$$

$$\frac{\partial \bar{r}_s}{\partial \delta_s} = \sigma_s f_s \left(h_s^{\sigma_s - 1} \frac{\partial u_s}{\partial \delta_s} + u_s \frac{\partial h_s^{\sigma_s - 1}}{\partial \delta_s} \right) \qquad \text{if s=s', otherwise 0} \tag{A.5}$$

$$\frac{\partial \bar{r}_s}{\partial \tau} = \sigma_s f_s \left(h_s^{\sigma_s - 1} \frac{\partial u_s}{\partial \tau} + u_s \frac{\partial h_s^{\sigma_s - 1}}{\partial \tau} \right) \tag{A.6}$$

$$\frac{\partial h_s^{\sigma_s - 1}}{\partial x} = (\sigma_s - 1) h_s^{\sigma_s - 1} \left[\frac{\partial \varphi_s^*}{\partial x} \frac{1}{\varphi_s^*} \left[\xi^s_{\tilde{\varphi}_s, \varphi_s^*} - 1 \right] \right] \tag{A.7}$$

To get $\frac{\partial \tilde{\varphi}}{\partial \varphi^*}$ apply Leibniz rule to the average productivity equation. The simplified result is:

$$\frac{\partial \tilde{\varphi}_s}{\partial \varphi_s^*} = \frac{z(\varphi_s^*)\tilde{\varphi}_s}{(\sigma - 1)(1 - Z_s(\varphi_s^*))} \left[1 - h_s^{1-\sigma} \right] \tag{A.8}$$

Elasticities

As mentioned in the paper, let $\xi^s_{x,y}$ be the elasticity of variable x with respect to y for sector s.

$$\xi^s_{\tilde{\varphi}_s,\varphi^*} = \frac{z(\varphi^*_s)\varphi^*_s}{(\sigma-1)(1-Z(\varphi^*_s)}\left[1-h_s^{1-\sigma}\right] \tag{A.9}$$

$$\xi^s_{M_s,\delta_{s'}} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \delta_{s'}}\delta_{s'}}{\left(wL+\sum_{i=1}^S T_i - q_0^G\right)} - \left[\frac{-\tau\delta_s}{(1-\delta_s\tau)} + (\sigma-1)\left(\xi_{\varphi^*_s,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi^*_s}-1\right]\right)\right] \tag{A.10}$$

$$\xi^s_{M_s,\delta_{s'}} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \delta_{s'}}\delta_{s'}}{\left(wL+\sum_{i=1}^S T_i - q_0^G\right)} \qquad \text{if s} \neq \text{s'} \tag{A.11}$$

$$\xi^s_{M_s,\tau} = \frac{\sum_{i=1}^S \frac{\partial T_i}{\partial \tau}\tau}{\left(wL+\sum_{i=1}^S T_i - q_0^G\right)} - \left[\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)} + (\sigma-1)\left(\xi_{\varphi^*,\tau}\left[\xi_{\tilde{\varphi}_s,\varphi^*_s}-1\right]\right)\right] \tag{A.12}$$

A.1 Optimal Taxes in the Closed Model

The FOCs for δ_i and τ are rewritten into:

$$\alpha_i\left[\frac{\tau\delta_i}{(1-\delta_i\tau)(1-\sigma_i)} - \xi_{\varphi^*_i,\delta_i}\right] = \tilde{\lambda}M_i\tau\delta_i f_i\left[\frac{-w}{1-\delta_i\tau} + (\sigma_i-1)\xi_{\varphi^*_i,\delta_i}(\xi_{\tilde{\varphi}_i,\varphi^*_i}-1)w\right] \tag{A.13}$$

$$\begin{aligned}\sum_{i=1}^S \alpha_i\left(\frac{-(1-\delta_i)\tau}{(1-\tau)(1-\delta_i\tau)(1-\sigma_i)} - \xi_{\varphi^*_{s'},\tau}\right) = \\ \tilde{\lambda}\sum_{i=1}^S\left[M_i\tau w f_i\left((\sigma_i-1)\xi_{\varphi^*_i,\tau}(\xi_{\tilde{\varphi}_i,\varphi^*_i}-1)\delta_i + u_i h_i^{\sigma_i-1} - \delta_i\left(\frac{1-2\tau+\delta_i\tau^2}{(1-\tau)(1-\delta_i\tau)}\right)\right)\right]\end{aligned} \tag{A.14}$$

Pareto Distribution

Assuming productivities follow a Pareto distribution, i.e:

$$Z_i(\varphi) = 1 - \left(\frac{\varphi_{min,i}}{\varphi}\right)^{k_i}$$

Under this distribution, the variables needed to solve the model can be found:

$$\tilde{\varphi}_i = \left(\frac{k_i}{k_i-(\sigma_i-1)}\right)^{\frac{1}{\sigma_i-1}} \varphi_i^* \tag{A.15}$$

$$\varphi_i^* = \left[\left(\frac{\sigma_i-1}{k_i-(\sigma_i-1)}\right)\left(\frac{f_i(1-\delta_i\tau)}{\psi f_{e,i}}\right)\right]^{1/k_i} \varphi_{min,i} \tag{A.16}$$

$$\xi_{\varphi_i^*,\delta_i} = \frac{-\tau\delta_i}{k_i(1-\delta_i\tau)} = \xi_{\varphi_i^*,\tau} \tag{A.17}$$

Using these values we use equation A.13 to find δ_i as a function of τ and parameters.

$$1-\delta_i\tau = \tilde{\lambda}(1-\tau)\rho_i wL$$

Such relation is used to find the optimal tax rate through equation A.14, leading to:

$$1-\tau = \left[\sum_{i=1}^{S}\frac{\alpha_i}{k_i}\right]\left[\tilde{\lambda}wL\sum_{i=1}^{S}\frac{\alpha_i\rho_i}{k_i}\right]^{-1} \tag{A.18}$$

This equation implied

Log-normal Distribution

Under this distribution, the variables needed to solve the model must be found through numerical methods. To solve for $\tilde{\varphi}_i$ define:

$$d_i = \frac{(log(\varphi_i^*)-m_i)}{v_i} \tag{A.19}$$

$$\Phi(x) = \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}x^2) \tag{A.20}$$

where m_i, v_i are the parameters for the lognormal distribution of productivities for sector i. The function $\Phi(x)$ is the CDF for the standard normal distribution. Using, these variables:

$$\tilde{\varphi}_i^{\sigma_i-1} = \frac{1}{1-Z_i(\varphi_i^*)}\int_{\varphi_i^*}^{\infty}\varphi^{\sigma_i-1}z(\varphi)d\varphi \tag{A.21}$$

$$= \exp\left(m_i(\sigma_i-1)+\frac{((\sigma_i-1)v_i)^2}{2}\right)\frac{\Phi((\sigma_i-1)v_i-d_i)}{\Phi(-d_i)} \tag{A.22}$$

$$= A_i g(\varphi_i^*) \tag{A.23}$$

Equation A.22 is obtained through various substitutions in the integral, as well as using the symmetry of the normal distribution.[29] The productivity cutoff φ_s^* is found by solving:

$$\frac{A_i g_i(\varphi_i^*)}{(\varphi_i^*)^{\sigma-1}} = \frac{\psi f_{e,i}}{(1-\delta_i\tau)\Phi(-d_i)f_i} + 1 \tag{A.24}$$

In order to solve for the optimal rates we must find a formula for $\xi_{\varphi_i^*,\delta_i}$. This is accomplish by using A.7,A.9 and the ZP and FE conditions.

$$\begin{aligned} \xi_{\varphi_i^*,\delta_i} &= \frac{\psi f_{e,i}}{X_i(1-\sigma_i)}\left(\frac{\tau\delta_i}{1-\tau\delta_i}\right) && \text{(A.25)} \\ X_i &= \psi f_{e,i} + (1-\delta_i\tau)\Phi(-d_i)f_i && \text{(A.26)} \end{aligned}$$

Using the above formula, equations A.13 result in the following relationship:

$$\frac{1}{(1-\tau)\rho_i\lambda wL} = \frac{\psi f_{e,i} + \Phi(-d_i)f_i}{X_i} - \frac{\psi f_{e,i}\phi(-d_i)}{X_i\Phi(-d_i)v_i}\xi_{\varphi_i^*,\delta_i} \tag{A.27}$$

while equation A.14 can be simplified to:

$$\begin{aligned} &\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1}\left(\frac{\tau}{(1-\tau)X_i}\right)(\psi f_{e,i} + (1-\delta_i)\Phi(-d_i)f_i) \\ &\qquad = \tilde{\lambda}\tau\sum_{i=1}^{S}M_i w f_i\left[\delta_i\left(-\frac{(\psi f_{e,i}+\Phi(-d_i)f_i)}{X_i} + \frac{\tau}{1-\tau} + \frac{\psi f_{e,i}\phi(-d_i)}{X_i\Phi(-d_i)v_i}\xi_{\varphi_i^*,\delta_i}\right) + u_i h_i^{\sigma-1}\right] \end{aligned}$$

which simplifies to:

$$1-\tau = \left[\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1}\right]\left[\tilde{\lambda}wL\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i}\left(\frac{\psi f_{e,i}+\Phi(-d_i)f_i}{X_i}\right)\right]^{-1} \tag{A.28}$$

Thus the solution to the problem is found by solving the system of $S+1$ equations given by A.27 and A.28.

[29]The step by step derivation can be provided upon request

B Open Model Equilibrium with Asymmetric Countries

The world consists of N countries whose households have the same utility function form but the parameters (σ, α) are allowed to vary across countries. Firms can export their products by paying an iceberg trade cost θ_s^{ij} in which i is the destination country and j is the source country and. s is the industry. Will keep this notation for the remaining variables in which there is a need to specify the flows. Companies in j that want to export to country i have to pay a fixed cost $f_{ex,s}^{ij}$. We assume that wages across countries are the same which is justified by using a homogeneous good that is freely traded and use this as the numeraire. Since elasticities of substitutions can be heterogeneous across countries, it implies that the markup charged by firms is different in each country leading to the pricing decision rule:

$$p_s^{ij}(\varphi) = \theta_s^{ij} \frac{w}{\rho_s^i \varphi}$$

Let $\pi_{d,s}^j(\varphi)$ be the domestic profit of firms in j selling domestically and $\pi_{ex,s}^{ij}(\varphi)$ represents the profits of the firm from exporting into i.

$$\pi_{d,s}^j(\varphi) = (1 - \tau^j) \left(\frac{r_{d,s}^j(\varphi)}{\sigma_s^j} - u_s^j w f_s^j \right)$$
$$\pi_{d,s}^{ij}(\varphi) = (1 - \tau^j) \left(\frac{r_{ex,s}^{ij}(\varphi)}{\sigma_s^i} - u_s^j w f_{ex,s}^{ij} \right)$$

B.1 Equiibrium and Aggregation

Let $\varphi_{d,s}^j$ be the cutoff productivity to enter the j domestic market while $\varphi_{ex,s}^{ij}$ is the cutoff productivity of the marginal firm that decides to serve the market in country i. Unlike many Melitz type models, the export cutoff productivity is different depending on the destination country. Furthermore, if a country decides to serve a particular market it does not necessarily imply that it will serve all the other markets. Nonetheless, conditions will be imposed to ensure that $\varphi_{ex}^{ij} > \varphi_{d,s}^j \quad \forall i \neq j$. Using $\tilde{\varphi}()$ (equation 2.7) we can define the average productivity of all firms producing and selling in j as $\tilde{\varphi}_d^j = \tilde{\varphi}^j(\varphi_d^j)$ and, the productivity of the firms exporting by $\tilde{\varphi}_{ex}^{ij} = \tilde{\varphi}^i(\varphi_{ex}^{ij})$

Let $i \neq j$ then the number of firms (in sector s) that produce in country j be M_s^j and the amount of firms that export into i is represented by $M_{ex,s}^{ij}$. Thus, the total number of varieties in industry

s available to consumers in country j is given by $M^j_{tot} = M^j + \sum_{i \neq j} M^{ji}_{ex}$. Thus, the average total productivity in j and the price index is :

$$
\begin{aligned}
\tilde{\varphi}^j_s &= \left[\frac{1}{M^j_{tot,s}}\left(M^j_s\left(\tilde{\varphi}^j_s\right)^{\sigma^j_s-1} + \sum_{i\neq j}\left(\left(\theta^{ji}_s\right)^{-1}\tilde{\varphi}^{ji}_{ex,s}\right)^{\sigma^j_s-1}\right)\right] \\
\mathbb{P}^j_s &= \left[\frac{1}{1-Z^j_s(\varphi^j_{d,s})}\int_{\varphi^j_{d,s}}^{\infty} p_s(\varphi)^{1-\sigma^j_s} M^j_s z^j_s(\varphi) + \sum_{i\neq j}\frac{1}{1-Z^i_s(\varphi^{ji}_{ex,s})}\int_{\varphi^{ji}_{ex,s}}^{\infty} p^{ji}_{ex,s}(\varphi)^{1-\sigma^j_s} M^{ji}_{ex,s} z^i_s(\varphi)\right]^{\frac{1}{1-\sigma^j_s}} \\
\mathbb{P}^j_s &= \left(M^j_{tot,s}\right)^{\frac{1}{1-\sigma^j_s}} p_s(\tilde{\varphi}^j_{tot,s})
\end{aligned}
$$

Now, the aggregate and average functions for firm revenues and profits are given by:

$$
\begin{aligned}
R^j_s &= M^j_s r^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} M^{ij}_{ex,s} r^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s}) \\
\Pi^j_s &= M^j_s \pi^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} M^{ij}_{ex,s} \pi^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s}) \\
\bar{r}^j_s &= r^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} \mathrm{p}^{ij}_{ex,s} r^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s}) \\
\bar{\pi}^j_s &= \pi^j_{d,s}(\tilde{\varphi}^j_{d,s}) + \sum_{i\neq j} \mathrm{p}^{ij}_{ex,s} \pi^{ij}_{ex,s}(\tilde{\varphi}^{ij}_{ex,s})
\end{aligned}
$$

in which $\mathrm{p}^{ij}_{ex} = \dfrac{1-Z^j_s(\varphi^{ij}_{ex,s})}{1-Z^j_s(\varphi^j_{d,s})}$ is the conditional probability of a firm drawing a productivity that allows them to serve market i from country j. Also, $\mathrm{p}^{ij}_{ex} M^j_s = M^{ij}_{ex,s}$. The above formulas are used to find the average profit as a function of $\varphi^j_{d,s}$ (productivity that generates zero profit from domestic operations) and $\varphi^{ij}_{ex,s}$ (productivity that generates zero profit of exporting to i).

$$
\bar{\pi}^j_s = (1-\delta^j_s\tau^j)w\left[f^j_s\left(\left(\frac{\tilde{\varphi}^j_{d,s}}{\varphi^j_{d,s}}\right)^{\sigma^j_s-1} - 1\right) + \sum_{i\neq j}\mathrm{p}^{ij}_{ex} f^{ij}_{ex,s}\left(\left(\frac{\tilde{\varphi}^{ij}_{ex,s}}{\varphi^{ij}_{ex}}\right)^{\sigma^i_s-1} - 1\right)\right] \tag{B.1}
$$

to solve or $\varphi^j_{d,s}$ the export cutoffs must be expressed as functions of such variable:

$$
\varphi^{ij}_{ex,s} = \left[\left(\frac{\sigma^i_s f^{ij}_{ex,s}}{\sigma^j_s f^j_s}\right)\frac{Y^j_s}{Y^i_s}\frac{M^i_{tot,s}}{M^j_{tot,s}}\right]^{\frac{1}{\sigma^i_s-1}}\left(\frac{\varphi^j_{d,s}}{\tilde{\varphi}^j_{tot,s}}\right)^{\frac{\sigma^j_s-1}{\sigma^i_s-1}}\tilde{\varphi}^i_{tot,s}\theta^{ij}_s \tag{B.2}
$$

where $Y_s = \alpha_s(wL + \sum \Pi_i^\tau)$ is the income spend in sector s by consumers, in which we assume that taxes collected by the government are redistributed to their citizens. Plugging this formula into equation B.1 gives rise to zero profit condition for the open economy asymmetric model. The fixed entry (equation FEC) remains the same. The export cutoff formula depends on the total number of firms in the destination country as well as the country where the firms is located. The number of firms for sector s in country j is:

$$M_s^j = \frac{\alpha_s^j(wL^j + \sum_{s=1}^S \Pi_s^{\tau,j})}{\sigma_s^j\left(\frac{\bar{\pi}_s^j}{1-\tau^j} + u_s^j f_s^j\right) + wu_s^j \sum_{i\neq j} p_{ex,s}^{ij} f_{ex,s}^{ij}\left(\sigma_s^j + (\sigma_s^i - \sigma_s^j)\frac{\tilde{\varphi}_{ex,s}^{ij}}{\varphi_{ex,s}^{ij}}\right)} \tag{B.3}$$

Thus, for each sector, in each country, we solve 2 equations ZPC = FE and B.3 with N auxiliary equations (B.2). This leads to a system of $N \times S \times (N+2)$ equations that are solved simultaneously to give rise to the equilibrium of the model. In the case of Pareto distributions, the system of equations can be reduced to $N \times S \times 2$ as the ratio $\tilde{\varphi}_{ex}/\varphi_{ex}$ is constant.

C Proposition Proofs

C.1 Proof of Proposition 2.1

For any non-degenerate distribution the mean of the random variable is greater than the minimum value of the support. Thus $\tilde{\varphi} > \varphi^*$ which implies $h > 1 \implies h^{-1} < 1$. Raising both sides of the inequality by the positive number $\sigma - 1$ is use to show that $1 - h^{1-\sigma}$ is greater than zero. Thus equation A.9 consist of positive factors and hence greater than zero.

For the second part, assume that productivities follow a Pareto distribution with $x_{min,s} = \varphi_{min,s}$ and shape parameter k_s.Then

$$\begin{aligned}
\tilde{\varphi}_s &= \left[\frac{k_s}{k_s - (\sigma_s - 1)}\right]^{\frac{1}{\sigma_s - 1}} \varphi_s^* \\
\frac{\partial \tilde{\varphi}_s}{\partial \varphi_s^*} &= \left[\frac{k_s}{k_s - (\sigma_s - 1)}\right]^{\frac{1}{\sigma_s - 1}}
\end{aligned}$$

Using the above equations it is clear that $\xi_{\tilde{\varphi},\varphi^*}$ is exactly one.

C.2 Proof of Proposition 2.2

Assume the government budget constraint is binding and therefore the number of firms in equilibrium is: $M_s = \dfrac{wL}{\sigma_s u_s f_s h_s^{\sigma_s - 1}}$. Let $s \neq s'$, then the binding budget assumption implies that equation A.11 is equal to zero for any distribution of productivities.

Now assume that $s = s'$ for some $s' \in S$. For a any productivity distribution, equation $A.10$ simplifies to:

$$\xi_{M_s,\delta_{s'}} = -\left[\frac{-\tau\delta_s}{(1 - \delta_s\tau)} + (\sigma_s - 1)\left(\xi_{\varphi_s^*,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)\right]$$

Proposition 2.1 says that $\xi^P_{\tilde{\varphi},\varphi^*} \equiv 1$, therefore:

$$\xi_{M_s,\delta_{s'}} - \xi^P_{M_s,\delta_{s'}} = -(\sigma_s - 1)\left(\xi_{\varphi_s^*,\delta_{s'}}\left[\xi_{\tilde{\varphi}_s,\varphi_s^*} - 1\right]\right)$$

The term $(\sigma - 1)\xi_{\varphi^*_{s'},\delta_{s'}}$ is less than zero since the productivity cutoff is negatively related to the depreciation rate for its sector. Using the appropriate assumptions on $\xi_{\tilde{\varphi}_s,\varphi_s^*}$ gives the inequalities between both elasticities.

It remains to show that the elasticity spawn from a Pareto distribution is greater than zero. The formula for such elasticity is:

$$\xi^P_{M_{s'},\delta_{s'}} = \frac{\tau\delta_{s'}}{1-\delta_{s'}\tau}$$

by assumption, $\delta_s\tau < 1$ for all sectors, and hence $\xi^P_{M_s,\delta_s}$ is positive.

C.3 Proof of Proposition 2.3

Only the first bullet point is proved as the second one follows a similar argument. Under a binding government constraint, equation A.12 simplifies to:

$$\begin{aligned}
\xi_{M_s,\tau} &= -\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)} - (\sigma_s - 1)\left(\xi_{\varphi^*,\tau}\left[\xi_{\tilde{\varphi}_s,\varphi^*_s} - 1\right]\right) \\
\xi^P_{M_s,\tau} &= -\frac{(1-\delta_s)\tau}{(1-\tau)(1-\delta_s\tau)}
\end{aligned}$$

If $\delta_s \leq 1$, then clearly $\xi^P_{M_s,\tau} \leq 0$, with strict inequality if $\delta_s < 1$. Since $\xi_{\varphi_{s'},\delta_{s'}} = \xi_{\varphi_{s'},\tau}$ (this is shown in the next proof), I use a similar argument for the proof of proposition 2.2 to establish the inequalities between ξ_M and ξ^P_M. Assuming $\xi_{\tilde{\varphi},\varphi^*} < 1$ and proposition 2.2, the following equality is obtained:

$$\xi_{M_s,\tau} < \xi^P_{M_s,\tau} \leq 0$$

On the other hand, if $\xi_{\tilde{\varphi},\varphi^*} < 1$ then $\xi_{M_s,\tau} > \xi^P_{M_s,\tau}$; and therefore the sign of the elasticity of firms to taxes under a distribution that is not Pareto is indeterminate. The exception being $\delta = 1$, which then implies such elasticity to be positive since $\xi^P_{M,\tau} = 0$

C.4 Proof of Proposition 3.1

The first step is to show the following equality between elasticities

Claim: $\xi_{\varphi^*_i,\delta_i} = \xi_{\varphi^*_i,\tau}$

Proof. The ZPC and FEC conditions imply that the equilibrium φ^*_s must solve the equation:

$$h_s^{\sigma-1} = \frac{\psi F_{e,s}}{(1-Z_s(\varphi^*_s))(1-\delta_s\tau)f_s} + 1$$

Take the derivative with respect to τ as well as δ_s. The ratio of such derivatives is:

$$\frac{\frac{\partial h_s^{\sigma-1}}{\partial \tau}}{\frac{\partial h_s^{\sigma-1}}{\partial \delta_s}} = \frac{z_s(\varphi_s^*)\frac{\partial \varphi^*}{\partial \tau}(1-\delta_s\tau) + (1 - Z_s(\varphi_s^*))\delta_s}{z_s(\varphi_s^*)\frac{\partial \varphi^*}{\partial \delta_s}(1-\delta_s\tau) + (1 - Z_s(\varphi_s^*))\tau}$$

By equation A.7:

$$\frac{\frac{\partial h_s^{\sigma-1}}{\partial \tau}}{\frac{\partial h_s^{\sigma-1}}{\partial \delta_s}} = \left(\frac{\partial \varphi_s^*}{\partial \tau}\right)\left(\frac{\partial \varphi_s^*}{\partial \delta_s}\right)^{-1}$$

Set the last two equation equal to each other and rearrange to obtain:

$$\begin{aligned} \tau\left(\frac{\partial \varphi_s^*}{\partial \tau}\right) &= \delta_s\left(\frac{\partial \varphi_s^*}{\partial \delta_s}\right) \\ \xi_{\varphi_s^*,\tau} &= \xi_{\varphi_s^*,\delta_s} \end{aligned}$$

□

After proving the above claim, the FOCs (eq. 3.4 and 3.5) are re-written into:

$$\alpha_{s'}\left(\frac{\tau\delta_{s'}}{(1-\delta_{s'}\tau)(1-\sigma_i)} - \xi_{\varphi_{s'}^*,\delta_{s'}}\right) = \tilde{\lambda}M_{s'}\left(\xi_{M_{s'},\delta_{s'}}\bar{t}_{s'} + \frac{\partial \bar{t}_{s'}}{\partial \delta_{s'}}\delta_{s'}\right) \tag{C.1}$$

$$\sum_{i=1}^{S}\alpha_i\left(\frac{-(1-\delta_i)\tau}{(1-\tau)(1-\delta_i\tau)(1-\sigma_i)} - \xi_{\varphi_{s'}^*,\tau}\right) = \tilde{\lambda}\left[\sum_{i=1}^{S}M_i\left(\xi_{M_i,\tau}\bar{t}_i + \frac{\partial \bar{t}_{i'}}{\partial \tau}\tau\right)\right] \tag{C.2}$$

Adding equation C.1 across all sectors and using the equality of the claim results in:

$$\begin{aligned} \sum_{i=1}^{S}\alpha_i\left(\frac{\tau(1-\delta_i\tau)}{(1-\delta_i\tau)(1-\tau)(1-\sigma_i)}\right) &= \tilde{\lambda}\sum_{i=1}^{S}M_i\left[(\xi_{M_i,\delta_i} - \xi_{M_i,\tau})\,\bar{t}_i + \left(\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i - \frac{\partial \bar{t}_i}{\partial \tau}\tau\right)\right] \\ \sum_{i=1}^{S}\frac{\alpha_i\tau}{(1-\tau)(1-\sigma_i)} &= \tilde{\lambda}\sum_{i=1}^{S}M_i\left[\left(\frac{\tau}{1-\tau}\bar{t}_i\right) + \left(\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i - \frac{\partial \bar{t}_i}{\partial \tau}\tau\right)\right] \end{aligned} \tag{C.3}$$

Next, the remainder derivatives are computed:

$$\begin{aligned}
\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i &= \tau\delta_i w f_i\left(\frac{\partial u_i}{\partial \delta_i}h_i^{\sigma_i-1}+\frac{\partial h_i^{\sigma_i-1}}{\partial \delta_i}u_i-1\right)\\
\frac{\partial \bar{t}_i}{\partial \tau}\tau &= \tau w f_i\left[\left(\frac{\partial u_i}{\partial \tau}h_i^{\sigma_i-1}+\frac{\partial h_i^{\sigma_i-1}}{\partial \tau}u_i\right)\tau+u_i h_i^{\sigma_i-1}-\delta_i\right]\\
\frac{\partial \bar{t}_i}{\partial \delta_i}\delta_i-\frac{\partial \bar{t}_i}{\partial \tau}\tau &= \tau w f_i\left[h_i^{\sigma_i-1}\left(\frac{\partial u_i}{\partial \delta_i}\delta_i-\frac{u_i}{\tau}\tau\right)+u_i\left(\frac{\partial h_i^{\sigma_i-1}}{\partial \delta_i}\delta_i-\frac{\partial h_i^{\sigma_i-1}}{\partial \tau}\tau\right)-u_i h_i^{\sigma_i-1}\right]\\
&= \tau w f_i\left[h_i^{\sigma-1}u_i\left(\frac{-\tau}{1-\tau}\right)+0-u_i h_i^{\sigma_i-1}\right]\\
&= \tau w f_i\left(h_i^{\sigma_i-1}u_i\frac{-1}{1-\tau}\right)
\end{aligned}$$

Replacing terms in equation C.3 gives the formula for λ

$$\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1} = \tilde{\lambda}\left[\sum_{i=1}^{S}-M_i\bar{t}_i+\frac{\alpha_i(wL)}{\sigma_i}\right]$$

$$\tilde{\lambda} = \frac{\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i-1}}{wL\sum_{i=1}^{S}\frac{\alpha_i}{\sigma_i}-p_0^G q_0^G} \tag{C.4}$$

C.5 Proof of Proposition 3.4

1. Pareto Economy: Assume $k_i=\bar{k},\sigma_i=\bar{\sigma}\quad i\in S$, then $1-\tau=\left(\tilde{\lambda}wL\bar{\rho}\right)^{-1}$. From the optimality equation for δ:

$$\delta_i=\frac{1-\tilde{\lambda}\bar{\rho}wL(1-\tau)}{\tau}=\frac{0}{\tau}=0 \qquad \forall i$$

   The equation above is valid since $\tau>0$.

2. Log-normal Economy: Assume sectors are completely symmetric, hence no sector subscript will be needed for the model parameters. Equation A.28 implies:

$$\begin{aligned}
1-\tau &= \frac{1}{\rho\tilde{\lambda}wLA}\\
A &= \frac{\psi F_e+\Phi(-d)f}{X}
\end{aligned}$$

Replacing $(1-\tau)$ in equation A.27, leads to:

$$\frac{1}{A} = A - \frac{\psi F_e \phi(-d)}{X\Phi(-d)v}\xi_{\tilde{\varphi}^*,\delta} = A - B$$

There are 3 possible case for δ, with each determining is A if above, below, or equal to 1. We show that cases of $\delta \neq 0$ produce a contradiction.

Case 1: Assume $\delta > 0$. This implies $A > 1$ and $1/A < 1$. Using the formula for the elasticity, we can see that $B < 0$. Hence, the equality can't hold as the LHS is less than one, while the RHS is greater than 1.

Case 2: Assume $\delta < 0$. Just as the above case, the equality can't hold since $A < 1, 1/A > 1$ and $B > 0$.

Case 3: Assume $\delta = 0$. In this case, $A = 1 \implies 1/A = 1$. Since $\delta = 0$, the elasticity $\xi_{\tilde{\varphi}^*,\delta}$ is equal to 0. Hence, the equality holds as $1 = 1$. Therefore, the only solution to the optimal tax rate problem is $\delta = 0$ for all sectors.

C.6 Proof of Proposition 3.5

Dividing A.28 and A.14:

$$\frac{1-\tau^{log}}{1-\tau^P} = \left[\frac{\sum \dfrac{\alpha_i}{\sigma_i - 1}}{\sum \dfrac{\alpha_i}{k_i}}\right] \times \left[\left(\sum_{i=1}^{S} \frac{\alpha_i}{\sigma_i}\frac{\sigma_i - 1}{k_i}\right) \div \sum_{i=1}^{S} \frac{\alpha_i}{\sigma_i}\left(\frac{\psi f_{e,i} + \Phi(-d_i) f_i}{X_i}\right)\right] \tag{C.5}$$

The first factor of the above equation is greater than one since $k > \sigma - 1$ for all sectors. The second factor is also greater than one since $\delta\tau < 1$. Therefore $\tau^{log} < \tau^P$.