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Abstract. There is an increasing literature proposing reserve requirements for financial stability. This study assesses their effects on the probability of bank failure and compares them to those of capital requirements. To this purpose a banking model is considered that is subject to legal reserve requirements. In general, higher reserve requirements promote risk-taking as either borrowers or banks have an incentive to choose riskier assets, so banks' probability of failure rises. Borrowers' moral hazard problem augments the adverse effects. They are mitigated when allowing for imperfectly correlated loan-default as higher interest revenues from non-defaulting loans curb losses from defaulting loans.

JEL codes: E43, E58, G21, G28

Key words: reserve requirements, liquidity regulation, capital requirements, bank failure, default correlation

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1. Introduction

At the centre of the discussion on financial stability is the question of how much capital and reserves banks should be required to hold. A system with a buffer too small might be crisis-prone and in need of regular bail-outs. On the other hand, too much might render unprofitable large parts of the banking business. In this context, capital requirements are one of the most important instruments and address banks' balance sheets directly. Yet they comprise just another of two distinct well-known regulatory instruments. Reserve requirements have traditionally been used, though not necessarily with a sole financial stability objective.

Both requirements concern the composition of banks' balance sheets. Changing one of them simply has a mirror image effect as the other, since both are defined as ratios between asset and liability categories. For both requirements, an increase is associated with a balance sheet tightening. The similarity of the effects of changes in either of the two regulatory requirements on banks' balance sheets motivates a joint assessment of their effects. The effects of capital requirements on lending rates and financial stability have been analysed extensively in the literature (see Gambacorta and Karmakar, 2018; Repullo and Suarez, 2013; Covas and Fujita, 2010; VanHoose, 2007, for an overview). In contrast, reserve requirements have not yet been assessed in this context. Against this background, the aim of this study is to evaluate the effects of reserve requirements on the probability of bank failure.

As reserve requirements are the key element in characterising a fractional banking system, they have hence been analysed from the perspective of an instrument for monetary policy for a long time (see for instance Day, 1986; Vernon, 1990; Davis and Toma, 1995; Haslag and Young, 1998; Faig and Gagnon, 2008; Carpenter and Demiralp, 2012; Dutkowsky and VanHoose, 2013; Hendrickson, 2017; Armenter and Lester, 2017). The experience of emerging market economies has induced a change in the perception of the usefulness of reserve requirements (Hoffmann and Löffler, 2014). The recent literature highlights the importance of reserve requirements as a means to foster financial stability, which is motivated by the fact that reserve requirements operate directly on the narrow credit channel defined by the supply reaction of bank credit to a change in available funds (Calomiris and Kahn, 1991; Stein, 1998; Diamond and Rajan, 2011; Calomiris et al., 2015).

A series of recent theoretical papers has assessed the ability of reserve requirements to promote financial stability for which most of them rely on dynamic stochastic general equilibrium (DSGE) models. In this context, Glocker and Towbin (2012); Mimir et al. (2013); Ireland (2014); De Carvalho et al. (2013); Bustamante and Hamann (2015); Primus (2017); Agénor et al. (2018); Adrian and Boyarchenko (2018); Imhof et al. (2018); Yang and Yi (2019); Mimir and Sunel (2019); Silva Vinhado and Divino (2019), among others, find that reserve (or liquidity) requirements have the ability to contain the degree of procyclicality of the financial system and by this to foster financial stability. The argument is that reserve requirements can serve as a countercyclical tool to manage the credit cycle in a broad context, limiting the excessive leverage of borrowers in the upswing and operating as a liquidity buffer in the downswing.

The empirical literature generally confirms these findings. In particular, Montoro and Moreno (2011); Tovar et al. (2012); Glocker and Towbin (2015); Fungáčová et al. (2016); Becker et al. (2017); Crespo Cuaresma et al. (2019); Dassatti Camors et al. (2019) provide empirical evidence highlighting that spikes in reserve requirements are likely to exert a downward pressure on loans and upward pressure on lending rates. Kashyap and Stein (2012) stress the importance of reserve requirement policies to augment the possibilities of central banks to achieve financial stability. They argue that reserve requirements could be considered as a Pigouvian tax used to internalise the externalities generated by a high short-term debt exposure of financial intermediaries.[1]

Common to this literature is (i) their perception of reserve requirements as an adequate instrument for financial stability purposes, (ii) their focus on the macroeconomy and (iii) the absence of an explicit measure for financial stability. In line with most studies, Barroso et al. (2017), Andries et al. (2018) and Alper et al. (2018) find that reserve requirements affect credit growth, most importantly, they document an effect on risk-taking by banks. They stress that banks that are more affected by

[1]There is also a growing empirical literature assessing the usefulness of reserve requirements in attenuating foreign capital inflows (see for instance Brei and Moreno, 2018, for an overview) and there is also increased interest on this issue in the context of measuring financial market stress. A potentially important reason for reserve holding in excess of required reserves is the precautionary saving motive. This renders feasible the use of (excess) reserves to capture financial market stress (van Roye, 2011; Glocker and Kaniovski, 2014).

countercyclical reserve requirements avoid riskier firms, showing that higher reserve requirements are likely to mitigate bank risk-taking. However, in contrast, the results of Ely et al. (2019), Dassatti Camors et al. (2019) and Jiménez et al. (2017) point towards the opposite. They document a "search-for-yield" effect (that is, a positive risk-taking response) to the tightening of reserve requirements.[2] This raises questions about the (i) presence of a risk-shifting effect and (ii) the ability of reserve requirements to promote financial stability.

We analyse under which circumstances reserve requirements promote financial stability, which we assess by means of the probability of bank failure. To that purpose, we consider a banking model of Cournot competition within a perfectly competitive market for loans. Banks have zero intermediation costs, are funded with fully insured deposits and equity capital. We introduce a central authority which, apart from the interest rate, sets legal reserve requirements. We extend the model along two dimensions. The first dimension addresses the risk environment. The differing results found for the effects of capital requirements on bank risk-taking are due to different modelling approaches (Hakenes and Schnabel, 2011). In the first type of models, banks solve a *portfolio problem*. They hold a portfolio of projects and choose the degree of riskiness of these projects. Given limited liability and deposit insurance, banks are subject to a risk-shifting problem (see for instance Keeley, 1990; Hellmann et al., 2000; Repullo, 2004). In the second type of models, banks solve an *optimal contracting problem* (see for instance Boyd et al., 2006; Martínez-Miera and Repullo, 2010). They extend loans to entrepreneurs who determine the risk of their projects. In this environment entrepreneurs are now subject to a risk-shifting problem which is influenced by banks' lending rates.

The second dimension addresses the role of differing degrees of loan default correlation. Martínez-Miera and Repullo (2010) have stressed the importance of higher interest revenues from non-defaulting loans in curbing losses from defaulting loans in the context of competition and bank failure. As changes in reserve requirements are likely to affect lending rates in the first place, the effect of changes in reserve

[2]In line with this Nguyen and Boateng (2015, 2019) also provide empirical evidence in favour of a "search-for-yield" effect. They find that higher reserves (in particular involuntary (excess) reserves) may promote risk-taking behaviour which can be detrimental to financial stability.

requirements might affect banking stability beyond the risk-shifting effect arising from the entrepreneur's reaction to changes in lending rates.

For ease of comparison, we also consider capital requirements as an additional regulatory element. This allows us to relate the effects of reserve and capital requirements to each other and to assess their mutual dependency.

To preview some results, we find that in general higher reserve requirements promote risk-taking as either entrepreneurs or banks have an incentive to choose riskier assets, so banks' probability of failure rises. The intuition for this is the following. To the extent that higher reserve requirements raise costs, banks and entrepreneurs try to counterbalance them by financing assets with a higher success return. These assets, however, are characterised by a higher probability of default. Hence, there is a risk-shifting effect. This effect is attenuated once imperfectly correlated loan default is allowed for. In particular, as higher reserve requirements induce a shift towards assets with a higher success return, the corresponding increase in interest payments from non-defaulting loans provides a buffer to cover losses from defaulting loans. This effect is opposite to the risk-shifting effect and hence dampens the adverse effects of higher reserve requirements on the probability of bank failure. In contrast, the adverse effects are augmented once entrepreneurs' moral hazard problem is taken into account.

Interestingly, even though changes in capital and reserve requirements have comparably similar effects on banks' balance sheets, their implications for financial stability are rather distinct. While both requirements affect loan supply and the lending rate in the same way, reserve requirements promote risk-taking, whereas capital requirements (mostly) mitigate risk-taking.

With this in mind, the results presented here are in stark contrast to those of the previously cited theoretical papers. In these studies, spikes in reserve requirements trigger increases in lending rates followed by a corresponding decline in loans. The drop therein reduces entrepreneurs' leverage, which is associated with an improvement in financial stability. This is not the case in the present context. The reason is that we take entrepreneurs' and banks' moral hazard problem into account. When higher reserve requirements raise refinancing costs, entrepreneurs and banks are now choosing assets with a higher success return. These assets, however, are characterised by a higher probability of default, rendering worse the overall conditions for financial stability.

The main policy implication from this study concerns the role of reserve requirements for procyclicality. So far, the literature argues that reserve requirements serve to attenuate excessive leverage and thus to manage the credit cycle. What previous studies ignore, though, is the possibility of a risk-shifting effect. As higher reserve requirements promote risk-taking, an increase during the upswing of the cycle could lead to an even stronger appetite for risk and thus to unintended policy outcomes. In this environment, a reduction in reserve requirements rather than an increase might attenuate the degree of procyclicality.

This paper further relates to the literature on banks' portfolio choice and liquidity holdings. Bhattacharya and Gale (1987) study an environment where banks can insure against withdrawal shocks by sharing liquid resources, but aggregate liquidity is inefficiently low because banks free-ride on each other's liquidity. Most closely related to this paper are the contributions by Kara and Ozsoy (2016); Pichler and Lutz (2017); Repullo (2005). Kara and Ozsoy (2016) study the optimal design of capital and liquidity regulations as well as the interaction between the two in a model with fire-sale externalities. Pichler and Lutz (2017) argue that bank capital requirements, outright caps on borrowing, or Pigouvian taxes on debt are no longer appropriate regulatory instruments under idiosyncratic liquidity risk. Instead, the macroprudential regulator must ensure sufficient liquidity in the banking system. Repullo (2005) studies banks' decision on the liquidity buffer that it wants to hold and the risk of its asset portfolio within an environment where deposits are randomly withdrawn and a lender of last resort (LLR). The key findings are that (i) the equilibrium choice of risk is shown to be decreasing in the capital requirement and increasing in the interest rate charged by the LLR, and (ii) the presence of an LLR does not increase the incentives to take risk.

The rest of this paper is organised as follows. Section 2 considers the effects of reserve requirements within a risk environment characterized by the optimal portfolio problem and assesses the role of imperfectly correlated loan default in an extension.

Section 3 addresses the effects of reserve requirements in a risk environment of optimal contracting and pays particular attention to the role of entrepreneurs' moral hazard problem. Section 4 considers the relevance of imperfectly correlated loan default within a risk environment characterized by the optimal contracting problem. Section 5 provides some general discussion and Section 6 concludes.

2. Portfolio problem

This section analyses the implications of reserve requirements within a model setup that is based on the idea that banks can decide upon how much risk to accept in their loan portfolio. In these models, banks explicitly choose an optimal level of risk. The level chosen, depends on, among others, the stance of prudential policy. In this kind of models, it is generally found that capital requirements tend to mitigate bank risk-taking, rendering this tool useful for prudential regulation. For our assessment on the effects of reserve requirements, we differentiate between perfect and imperfect correlation in loan defaults, which allows to evaluate the role of interest revenues from non-defaulting loans.

2.1. **Reserves, capital and bank risk-taking.** We start with a simple model which serves to highlight the basic difference between reserve and capital requirements concerning their ability to promote financial stability. Consider a model with two dates ($t = 0, 1$). Banks operate in perfectly competitive markets and raise deposits D and equity capital E at date $t = 0$ and invest the proceeds in loans L to firms that yields a stochastic gross return at date $t = 1$. We assume zero intermediation costs. Banks' deposits are insured by a government-funded deposit scheme and they are in perfectly elastic supply at a deposit rate r_D. Additionally, deposits are subject to reserve regulation, which requires banks to hold a fraction $\varsigma \in (0, 1)$ of deposits at an account at the central bank in the form of reserves $R = \varsigma D$. This amount is assumed to receive a return equal to r_R and is out of a bank's free disposal. We assume that deposits are insured at a flat premium which is zero. The balance sheet for a bank reads as follows

$$(1-\varsigma)D = (1-k)L \tag{1}$$

where $k = E/L \in (0,1)$ is the capital ratio. The fraction $(1-\varsigma)/(1-k)$ determines the loan-to-deposit ratio (L/D). Banks' equity capital is provided by bankers who require an expected rate of return $\delta > r_D$ on their investment. The rationale behind assuming a strictly positive δ is given by Holmström and Tirole (1997) and Diamond and Rajan (2000), among others. The excess cost of bank capital δ is intended to capture in a reduced-form manner distortions such as agency costs of equity, which imply a comparative disadvantage of equity financing relative to deposit financing.

Banks are managed in the interest of their shareholders, who are protected by limited liability. Each period bankers have to decide how many deposits D and how much capital E to hold. Due to limited liability, the net worth of each bank – that is, gross loan returns minus gross deposit returns plus the returns on reserves – will then be distributed to bankers if it is positive, otherwise they receive zero. Bankers maximize the expected value of this payoff discounted at the rate δ and net of their initial contribution of capital. Prudential regulation requires banks to hold some minimum of reserves and capital.

A bank receives a return $\alpha(p)$ if the investment is successful, where $1-p$ determines the success of an investment project; if not successful, the bank gets $\alpha(p) = 0$ with probability p

$$\tilde{R} = \begin{cases} \alpha(p), & \text{with probability } 1-p \\ 0, & \text{with probability } p \end{cases} \tag{2}$$

The probability of failure $p \in (0,1)$ is privately chosen by the bank. Following Allen and Gale (2001, Chapter 8), we assume that $\alpha(p)$ is concave with $\alpha'(0) > 1 + \alpha(0)$, so riskier projects have a higher success return. The net worth $\pi(p)$ of a bank per unit of loan is given by

$$\pi(p) = (1-p)\left(1 + \alpha(p) - (1+r_D)\frac{1-k}{1-\varsigma} + (1+r_R)\varsigma\frac{1-k}{1-\varsigma}\right) \tag{3}$$

The first term $1 + \alpha(p)$ is the success return of an investment project, the second term captures the payments to depositors and the third term captures the return on required reserves where the amount of required reserves is given by $\varsigma\frac{1-k}{1-\varsigma}$. In what follows we make two assumptions. First, we assume that changes in reserve and capital requirements do not induce adjustment costs. Second, we assume that the return on reserves r_R is zero; hence the success return of the asset, or put differently, the lending rate ($\alpha(p) > 0$) and the deposit rate ($r_D > 0$) are to be understood as excess returns. This simplifies the net worth equation to the following

$$\pi(p) = (1-p)\left(\alpha(p) + k - r_D\frac{1-k}{1-\varsigma}\right) \tag{4}$$

The bank competes in a Cournot fashion and chooses capital k, reserves ς and the risk level p in order to maximise the present discounted value

$$V = \max_{\{k,\varsigma,p\}}\left[-k + \frac{1}{1+\delta}\pi(p)\right] \tag{5}$$

where the volume of loans L has been normalised to unity. The first order conditions read

$$\frac{\partial V}{\partial \varsigma} = -r_D\frac{(1-p)(1-k)}{(1+\delta)(1-\varsigma)^2} < 0 \tag{6}$$

$$\frac{\partial V}{\partial k} = -1 + \frac{(1-p)(1+r_D-\varsigma)}{(1+\delta)(1-\varsigma)} < 0 \tag{7}$$

Considering the first order condition with respect to the reserve ratio ς, if $p \to 0$ then $\frac{\partial V}{\partial \varsigma} = -r_D\frac{1-k}{(1+\delta)(1-\varsigma)^2} < 0$, implying that if the loan default probability goes to zero, the bank wants to hold zero reserves. However, if $p \to 1$ then $\frac{\partial V}{\partial \varsigma} = 0$, implying that the bank accepts holding reserves if all loans default. In what follows, we exclude this boundary solution and consider $p \in [0, 1)$ only.

The inequality in the first order condition with respect to the capital ratio k applies if $\frac{1+\delta-(1-r_D)(1-p)}{\delta+p} > \varsigma$, which will always hold for reasonable values of p and ς given that $\delta > r_D$. Hence the bank holds the minimum amount of capital and reserves as required by the regulation on capital and reserves.

To better understand the first order condition with respect to capital k, note that if $\varsigma = 0$, then we have that $\frac{\partial V}{\partial k} = -1 + \frac{(1-p)(1-r_D)}{1+\delta} < 0$. If $p < 1$, then $\frac{\partial V}{\partial k} < 0$ even when $\delta = r_D$, that is, when the owners of the bank do not require a higher rate of return than depositors. This is because in this case deposits would still be a cheaper source of finance since they are covered by deposit insurance in case of bank failure.

If $p \to 0$ then $\frac{\partial V}{\partial k} = -1 + \frac{1+r_D-\varsigma}{(1+\delta)(1-\varsigma)} < 0$, implying that if the loan default probability goes to zero, the bank wants to hold zero capital. If $p \to 1$ we have that $\frac{\partial V}{\partial k} = -1$ implying that the bank refrains from holding capital when all loans default. Again, this is because in this case deposits would be a cheaper source of finance than equity capital due to deposit insurance.

The first order condition with respect to the risk level p reads

$$(1-p)\alpha'(p) - \alpha(p) = k - r_D\frac{1-k}{1-\varsigma} \tag{8}$$

Notice that the corner $p = 0$ cannot be a solution if $\alpha'(0) - \alpha(0) - \gamma > 0$ with $\gamma \equiv k - r_D\frac{1-k}{1-\varsigma}$, which holds by the assumption $\alpha'(0) > 1 + \alpha(0)$ and the corner $p = 1$ cannot be a solution because $-(\alpha(1) + \gamma) < 0$. Hence the bank will choose a probability of failure $p(\gamma) \in (0, 1)$. Taking the total differential and re-arranging gives the following comparative statics

$$\frac{dp(k,\varsigma)}{dk} = \frac{1+r_D-\varsigma}{1-\varsigma}\frac{1}{(1-p)\alpha''(p) - 2\alpha'(p)} < 0 \tag{9}$$

$$\frac{dp(k,\varsigma)}{d\varsigma} = -r_D\frac{1-k}{1-\varsigma}\frac{1}{(1-p)\alpha''(p) - 2\alpha'(p)} > 0 \tag{10}$$

Equation (9) implies that higher capital requirements mitigate banks' moral hazard problem and hence the incentive for excessive risk-taking. This result has been documented in the literature (see VanHoose, 2007, for an overview). In contrast, equation (10) implies that higher reserve requirements promote risk-taking. The intuition for this result is the following: Higher reserve requirements imply that only a part of the stock of deposits can be used for loan supply in order to make profits, however, that part of deposits which has to be hold as reserve still incurs costs equal to r_D. Banks try to compensate this loss by financing loans with a higher success return. These loans, however, are characterised by a higher probability of default, which in turn reduces the quality of banks' loan portfolios. As a consequence, higher reserve requirements do not mitigate bank risk-taking in this environment.

In this setup, the effects of changes in reserve requirements on the probability of bank failure are similar to those of changes in the deposit rate r_D. To see this, note that from equation (8) we have that

$$\frac{dp(\cdot)}{dr_D} = -\frac{1-k}{1-\varsigma}\frac{1}{(1-p)\alpha''(p) - 2\alpha'(p)} > 0 \tag{11}$$

This again highlights the prevalence of the risk-shifting effect in response to changes in the cost structure.

This simple outline serves as an example where the implications of reserve requirements stand in stark contrast to those of capital requirements as regards their effect on the stability of banks. The model setup considered here is though fairly simplistic rendering feasible the potential that its implications are too model-specific. For this the following sections consider various extensions.

2.2. **The role of imperfectly correlated loan default.** The previous section's analysis is based on the assumption of perfect correlation in loan defaults. In this case, loans' probability of default coincides with banks' probability of failure. When the risk inherent to loans is increasing in the lending rate, then lower rates reduce banks' revenues from non-defaulting loans which provide a buffer to cover loan losses. If loan defaults were not perfectly correlated, increases in reserve requirements could have an effect on bank stability beyond the standard risk-shifting effect outlined previously. In particular, as higher reserve requirements induce a shift towards assets with a higher success rate of return, the corresponding increase in interest payments from non-defaulting loans can provide a buffer to cover losses from defaulting loans, so banks would be less risky. This effect is opposite to the risk-shifting effect.

In an extension to the model outlined in Section 2.1, we now consider an environment in which firms and banks are exposed to a macroeconomic risk factor $z \sim N(0,1)$. Credit risk follows the Vasicek (2002) model, based on the Merton (1974) model of credit risk. The model considers a continuum of firms that have a project that requires a unit investment at date $t=0$ and yields a stochastic return $\tilde{R}$ at date $t=1$:

$$\tilde{R} = \begin{cases} 1+\alpha, & \text{with probability } 1-p \\ 1-\lambda, & \text{with probability } p \end{cases} \tag{12}$$

where again p is the (unconditional) probability of default (PD), λ is the loss given default (LGD) and $1+\alpha$ is the gross return of the project in case of success. The assumption of identical firms implies that they all choose the same probability of default p. Then the fraction x of projects that fail (in other words, the aggregate failure rate) is only a function of the realization of the macroeconomic risk factor z. In particular, by the law of large numbers, the aggregate failure rate x coincides with the probability of failure of a representative project i conditional on the macroeconomic risk factor z. In this respect, the probability distribution of the aggregate failure rate x is the one derived from the single-risk-factor model of Vasicek (2002) that is used subsequently. Its cumulative distribution function is given by

$$F(x) = \Phi\left(\frac{\sqrt{1-\varrho}\,\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\varrho}}\right) \tag{13}$$

where $\Phi(\cdot)$ denotes the distribution function of a standard normal random variable, p is the unconditional probability of project default, $\varrho \in [0,1]$ is the loan exposure to the macroeconomic risk factor z and x is the aggregate failure rate. $F(x)$ captures the cumulative distribution function of loan losses on a large loan portfolio. The environment of the model of Section 2.1 can be replicated in the present setup when $\varrho \to 1$ (perfectly correlated failures), as in this case the distribution of the failure rate x approaches the limit $F(x) = \Phi(-\Phi^{-1}(p)) = 1 - \Phi(\Phi^{-1}(p)) = 1 - p$, for $0 \leq x \leq 1$. The mass point at $x = 0$ implies that with probability $1 - p$ no project fails and the mass point at $x = 1$ implies that with probability p all projects fail. A detailed derivation of equation (13) together with a discussion as regards the properties of the function $F(x)$ is outlined in Section A of the Appendix.

The structure of the banking sector is similar as outlined in Section 2.1 with minor modifications to account for imperfectly correlated loan default. When a firm succeeds with its investment project, the bank gets $1 + r$ while when it fails, the bank recovers $1 - \lambda$, hence λ is the loss given default. As before, r can be considered as an excess lending rate. The bank's net worth (or available capital) per unit of loan is given by:

$$\begin{aligned} \pi(x) &= (1-x)(1+r) + x(1-\lambda) - \frac{1-k}{1-\varsigma}(1 + r_D - \varsigma) \\ &= r + k - x(\lambda + r) - r_D\frac{1-k}{1-\varsigma} \end{aligned} \tag{14}$$

where the loan default rate x is a random variable whose conditional distribution function is given by equation (13). As in Section 2.1 we normalize the stock of loans L to unity. The first term, $(1-x)(1+r)$, in equation (14) is the payoff of non-defaulted loans, the second term, $x(1-\lambda)$, is the payoff of defaulted loans, and the third term, $(1 + r_D - \varsigma)\frac{1-k}{1-\varsigma}$, is the cost of deposits taking into account reserve holdings. The bank's objective is to maximize the expected discounted value of $\max(\pi(x), 0)$ net of bankers' initial infusion of capital:

$$V = -k + \frac{1}{1+\delta} E\left[\max(\pi(x), 0)\right] = -k + \frac{1}{1+\delta} \int_{-\infty}^{\hat{x}} \pi(x) dF(x) \tag{15}$$

where $\hat{x}$ denotes the critical value of x for which $\pi(x) = 0$. The bank maximizes the expected discounted value V with respect to the amount of capital k and reserves ς; optimization yields:

$$\frac{\partial V}{\partial \varsigma} = -r_D \frac{1-k}{(1+\delta)(1-\varsigma)^2} \int_{-\infty}^{\hat{x}} dF(x) < 0 \tag{16}$$

$$\frac{\partial V}{\partial k} = -1 + \frac{1 + r_D - \varsigma}{(1+\delta)(1-\varsigma)} \int_{-\infty}^{\hat{x}} dF(x) < 0 \tag{17}$$

These two equations are equivalent to equation (6) and (7) with the implication that banks will always hold the minimum amount of capital and reserves as required by the regulation. As in Section 2.1, banks decide upon the level of risk which they are still willing to accept. In the present context, they choose the bankruptcy/default rate $\hat{x}$ which characterizes the probability of bank failure in relation to the aggregate failure rate x. It is given by the solution to $\partial V / \partial \hat{x} \stackrel{!}{=} 0$ which yields the following equilibrium condition for the bankruptcy/default rate

$$\hat{x} = \frac{1}{\lambda + r} \left(k + r - r_D \frac{1-k}{1-\varsigma} \right) \tag{18}$$

Intuitively, if $x > \hat{x}$, the liabilities of a bank are larger than its assets. In this case the bank will fail and be of zero net worth. Given that $\hat{x}$ denotes the critical value of the aggregate failure rate x for which $\pi(x) = 0$, a bank defaults if the failure rate x is larger then the bankruptcy/default rate $\hat{x}$ defined in equation (18). This implies that the probability of bank failure is given by $F_B(\hat{x}) = Pr(x \geq \hat{x}) = 1 - F(\hat{x})$. Hence, a higher bankruptcy/default rate $\hat{x}$ allows banks to accept a higher aggregate failure rate x without yet getting bankrupt; by equation (18) the bankruptcy/default rate $\hat{x}$ is increasing in the lending rate r and the capital requirement k and decreasing in the reserve requirement ς.

Finally, under perfect competition, the equilibrium lending rate r is determined by the zero net value condition $V = 0$. Otherwise the market for loans would not clear and banks would either want to expand their loan portfolio to infinity (if $V > 0$)

or to not lend at all (if $V < 0$).[3] The zero net value condition $V = 0$ gives the equilibrium lending rate r as a solution to the following equation

$$k = \frac{1}{1+\delta} \int_0^{\hat{x}} \left(k + r - x(\lambda + r) - r_D \frac{1-k}{1-\varsigma} \right) dF(x) \tag{19}$$

Using integration by parts and the equilibrium condition for the bankruptcy/default rate, equation (19) can be written more compactly in the following form

$$k = \frac{1}{1+\delta} \int_0^{\hat{x}} (\lambda + r) F(x) dx \tag{20}$$

The complete model consists of equations (13), (18) and (20) which comprises a system of three equations with three variables (r, $\hat{x}$ and $F(\hat{x})$ with $F_B(\hat{x}) = 1 - F(\hat{x})$). In what follows, we analyse the implications of changes in reserve and capital requirements on the probability of bank failure $F_B(\hat{x})$.

2.2.1. *Implications of the model.* From the definition of the probability of bank failure we have

$$\frac{dF_B}{d\varsigma} = -\frac{\Phi'(\cdot)}{\sqrt{\varrho}} \frac{\sqrt{1-\varrho}}{\Phi'(\Phi^{-1}(\hat{x}))} \frac{\partial \hat{x}}{\partial \varsigma} \tag{21}$$

$$\frac{dF_B}{dk} = -\frac{\Phi'(\cdot)}{\sqrt{\varrho}} \frac{\sqrt{1-\varrho}}{\Phi'(\Phi^{-1}(\hat{x}))} \frac{\partial \hat{x}}{\partial k} \tag{22}$$

where we used the rules of differentiation of inverse functions. Since $\Phi'(\cdot)$ is the density function of a standard normal random variable, the sign of equation (21) and (22) hence depends on the sign of $\partial \hat{x}/\partial \varsigma$ and $\partial \hat{x}/\partial k$. Considering the latter first, we denote the equilibrium interest rate with r^* and obtain from equation (18)

$$\frac{\partial \hat{x}}{\partial k} = \frac{1}{\lambda + r^*} \left((1-\hat{x}) \frac{\partial r^*}{\partial k} + \frac{1 + r_D - \varsigma}{1-\varsigma} \right) \tag{23}$$

with $\frac{\partial r^*}{\partial k} = \frac{1}{\varphi(\hat{x})} \left(1 + \delta - F(\hat{x}) \frac{1+r_D-\varsigma}{1-\varsigma} \right)$, where $\varphi(\hat{x}) \equiv \int_0^{\hat{x}} (1-x) dF(x) > 0$. Obviously, $\frac{\partial r^*}{\partial k} > 0$ if $-1 + \frac{1+r_D-\varsigma}{(1+\delta)(1-\varsigma)} F(\hat{x}) < 0$, which replicates the assumption made in equation (17) and hence $\frac{\partial r^*}{\partial k} > 0$. This, in turn, implies that $\frac{\partial \hat{x}}{\partial k} > 0$ from which follows that $dF_B(\hat{x})/dk = -dF(\hat{x})/dk < 0$. This result replicates the findings in Repullo and Suarez (2004) and Kiema and Jokivuolle (2014). Intuitively, if banks are required to increase their capital ratio, they will charge higher lending rates, so the net interest

[3] If the stock of loans L was not normalized to unity, then this condition could be derived from the first order condition of the expected discounted value V with respect to loans L.

income earned on performing loans will be higher. This implies a lower probability of failure as the bankruptcy/default rate is increasing in both the capital requirement (k) and the lending rate (r).

For reserve requirements ς we obtain

$$\frac{\partial \hat{x}}{\partial \varsigma} = \frac{1}{\lambda + r^*}\left((1-\hat{x})\frac{\partial r^*}{\partial \varsigma} - r_D \frac{1-k}{(1-\varsigma)^2}\right) \tag{24}$$

Since $\frac{\partial r^*}{\partial \varsigma} = \frac{r_D(1-k)F(\hat{x})}{\varphi(\hat{x})(1-\varsigma)^2} > 0$, we observe that $\frac{\partial \hat{x}}{\partial \varsigma} \gtreqless 0$ as the sign of the term in parentheses in equation (24) is in principle ambiguous. As a consequence, $dF_B(\hat{x})/d\varsigma = -dF(\hat{x})/d\varsigma \gtreqless 0$ implying that, as a result of an increase in reserve requirements, the probability of bank failure can increase, decrease or remain unchanged. Intuitively, if banks are required to increase their reserve ratio, they will charge higher lending rates, so that the net interest income earned on performing loans will be higher – this is captured by the first term in parentheses. Higher reserve requirements lead to higher lending rates and consequently higher revenues from non-defaulting loans. This provides a buffer against the losses from defaulting loans rendering banks less risky. In contrast to that, the second term in parentheses captures the negative cost effect which, as already outlined in Section 2.1, incentivizes banks to accept assets with a higher success return; these assets are, however, riskier, which finally implies a higher probability of bank failure. Since the bankruptcy/default rate is decreasing in the reserve requirement (ς) but increasing in the lending rate (r), a higher reserve ratio hence implies a higher probability of failure, whereas the increase in the lending rate implies a lower probability of failure. In principle, these two opposing effects render uncertain the overall impact on the probability of bank failure, however, the risk-shifting effect in equation (24) still dominates, which implies: $\frac{\partial \hat{x}}{\partial \varsigma} = \frac{-r_D(1-k)}{\varphi(\hat{x})(1-\varsigma)^2}\int_0^{\hat{x}} F(x)dx < 0$. However, we note that

$$\frac{\partial \hat{x}}{\partial k} \gg \left|\frac{\partial \hat{x}}{\partial \varsigma}\right| \quad \rightarrow \quad \left|\frac{dF_B(\hat{x})}{dk}\right| \gg \frac{dF_B(\hat{x})}{d\varsigma} \tag{25}$$

In fact, since $|d\hat{x}/d\varsigma|$ is comparably small, changes in reserve requirements trigger negligibly small changes in the probability of bank failure, which is due to the two aforementioned opposing effects. To show this, we proceed by using numerical techniques and simulate the model by varying capital and reserve requirements over a reasonable range.

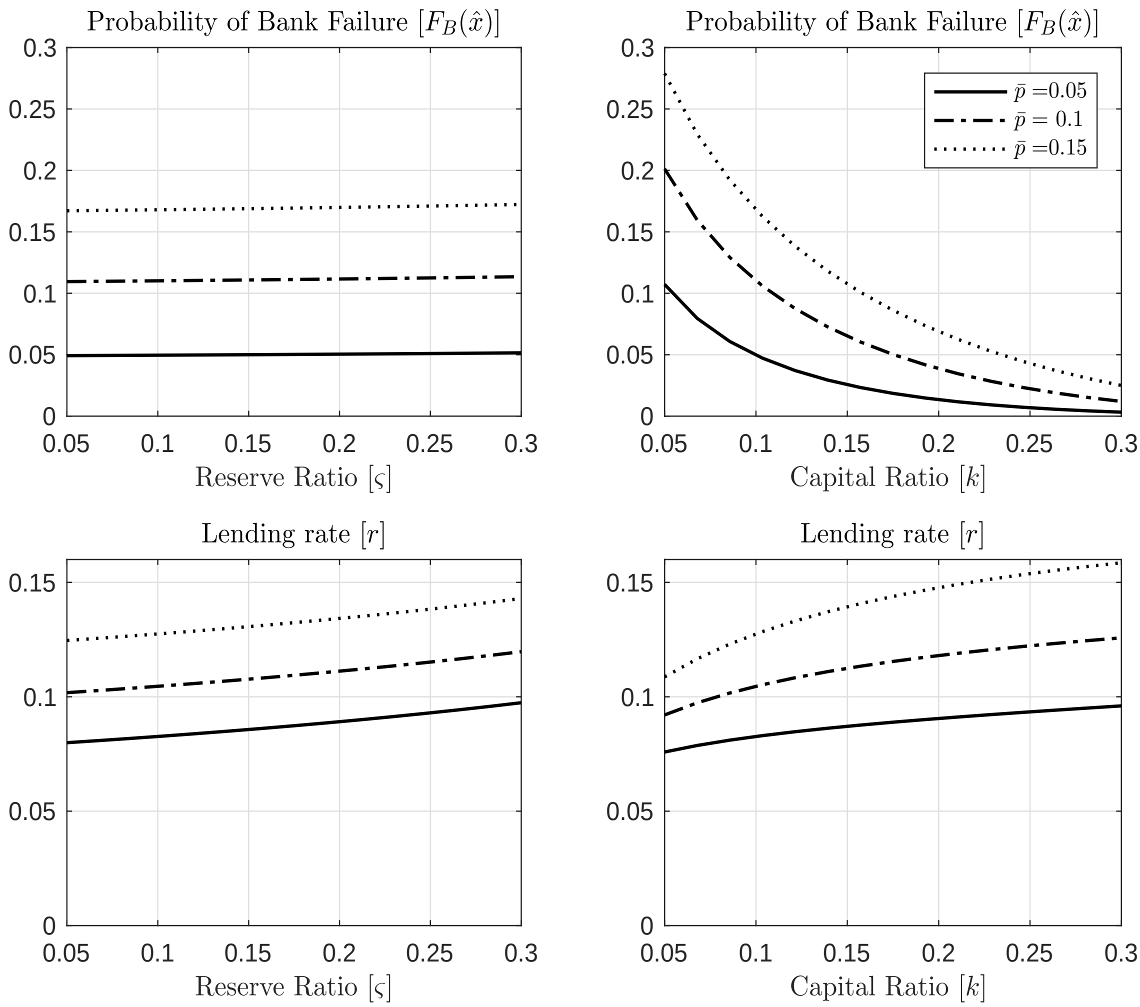
2.2.2. *Numerical results.* We follow Repullo and Suarez (2004) to calibrate the model and utilize commonly used values for the structural parameters.[4] We set the cost of bank capital δ equal to 0.1 and the exposure ϱ to the common risk factor equal to 0.5. The LGD parameter λ is set equal to 0.45 and the deposit rate r_D equal to 0.05. We set the capital and reserve ratios equal to 0.1. Finally, we are left with the probability of default p for which we choose various different values (0.05, 0.10, 0.15). Note that these parameter values are chosen for the sole purpose of illustrating the possible shapes of the relationship between the reserve and capital requirements and the risk of bank failure. They are not intended to produce realistic values of variables such as the loan rate r, the probability of loan default p or the probability of bank failure F_B.

Figure 1 shows the effects of changes in capital and reserve requirements on the probability of bank failure ($F_B(\hat{x})$) (upper subplots) and on the lending rate (r) (lower subplots). The effects are displayed for three different values of the (unconditional) probability of default (p). As can be seen, increases in capital requirements induce a decline in the probability of bank failure and an increase in the lending rate. This result is in line with Repullo and Suarez (2004). Different values of the probability of default (p) do not change the shape of the curves, though their positions.

The results are different in case of reserve requirements. An increase in reserve requirements of around ten percentage points triggers an increase in the lending rate of around two percentage points. This compares to an increase in the lending rate of similar size in case of a ten percentage points increase in capital requirements. Hence the overall effects of reserve and capital requirements on the interest rate spread are similar. The opposite though holds for the effects on the probability of bank failure. In this case the differences are significant. Higher reserve requirements induce negligibly small changes in the probability of bank failure – visually these changes cannot be recognised. In the case of capital requirements, the effects on the probability of bank failure are sizeable: a rise in capital requirements of up to fifteen percentage points triggers a decline in the probability of bank failure of around ten percentage points – the effects are weaker the smaller the probability of default (p).

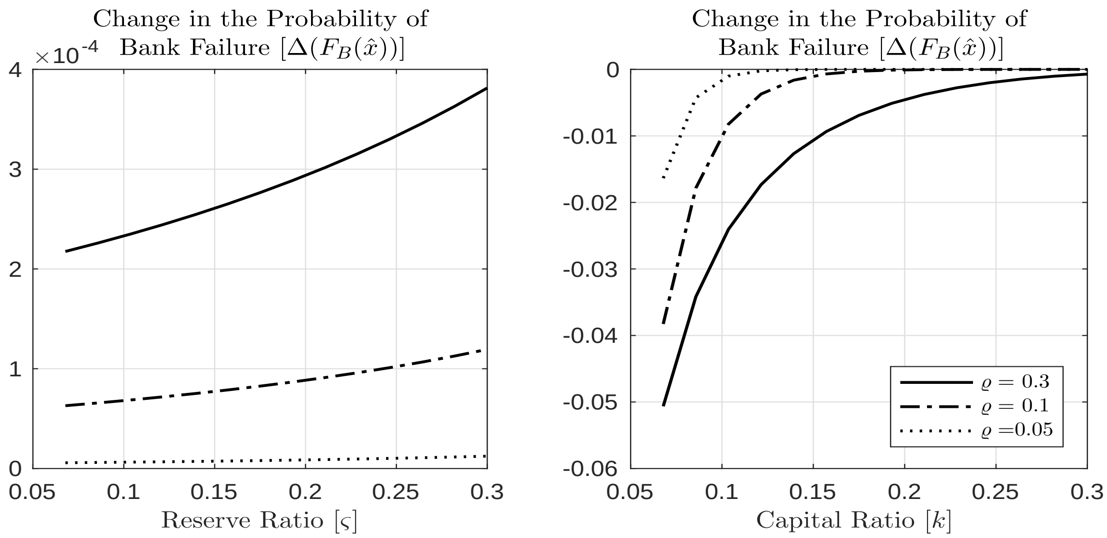
[4]The computations are carried out in Octave/Matlab. The programs are available upon request.

Figure 1. Comparative statics



The negative effects of higher reserve requirements on the probability of bank failure depend on the extent to which higher interest revenues from non-defaulting loans compensate the losses from defaulting loans. This gives rise to assessing the effects of reserve requirements on the probability of bank failure with respect to the degree of loan-default correlation – this is captured by the parameter ϱ. If $\varrho \to 1$ then loan default rates are perfectly correlated as in the set-up of Section 2.1. Figure 2 shows the effects of higher reserve and capital requirements on the change in the probability of bank failure for different values of ϱ. As can be seen, with a higher loan default correlation, rising reserve requirements lead to a larger increase in the probability of bank failure. This shows that the contribution of higher interest revenues from non-defaulting loans decreases when the degree of loan

Figure 2. Comparative statics



default correlation increases. Hence the presence of imperfectly correlated loan-default attenuates the negative effects of higher reserve requirements on financial stability as the higher interest revenues from non-defaulting loans provide a buffer for the losses from defaulting loans.

3. Optimal contracting problem

In the previous section, banks decide upon the structure of their assets by solving a portfolio problem which trades off expected returns and the risk of failure. This setup ignores the existence of a loan market. Moreover, in this environment banks can control the level of risk *directly*. Within the approach of the *optimal contracting problem*, it is now entrepreneurs who are subject to a moral hazard problem. We use a static model of Cournot competition in a market for entrepreneurial loans in which the probability of default of loans is privately chosen by the entrepreneurs.

3.1. **Reserves, capital and entrepreneurial risk-taking.**

The setup here follows the model of Boyd and De Nicoló (2005); Martínez-Miera and Repullo (2010); Hakenes and Schnabel (2011) who consider an economy with two types of risk neutral agents: entrepreneurs and banks. We assume that the return of projects of different entrepreneurs is perfectly correlated. This implies that the probability of default of their loans coincides with the probability of bank failure.

Entrepreneurs. There is a continuum of entrepreneurs who have no own resources, but have access to risky projects that require a unit investment and yield a stochastic return

$$\tilde{R}(p_i) = \begin{cases} 1 + \alpha(p_i), & \text{with probability } 1 - p_i \\ 0, & \text{with probability } p_i \end{cases} \tag{26}$$

where the probability of failure $p_i \in [0, 1]$ is chosen privately by the entrepreneur. As in Section 2.1 we assume that the success return of the project $\alpha(p_i) > 0$ is concave with $\alpha'(0) > \alpha(0)$ to get interior solutions.

To fund their projects entrepreneurs borrow from banks. Banks in turn cannot observe entrepreneurs' risk-shifting choice p_i, but take into account the best response of entrepreneurs to their choice of the lending rate r. More specifically, the entrepreneurs' choice of p_i at the beginning of the contract is unobservable for banks. Afterwards, banks observe only whether the project has been successful. In this environment, banks have no direct control over the riskiness of borrowers' projects. For any given loan rate r entrepreneur i will choose p_i in order to maximize the expected payoff from undertaking the project, which is the success return net of the interest payment, $\alpha(p_i) - r$, multiplied by the probability of success, $1 - p_i$, which implies $p(r) = \arg\max_{p_i}(1 - p_i)(\alpha(p_i) - r)$. Hence for any given loan rate r, all entrepreneurs will choose the same $p_i = p(r)\ \forall\ i$, which allows to omit the i subscript. By our previous assumptions, the entrepreneurs' objective function $(1-p)(\alpha(p)-r)$ is concave, so that $p(r)$ is obtained by solving the first-order condition

$$r = \alpha(p) - (1 - p)\alpha'(p) \tag{27}$$

For $0 \leq r < \alpha(1)$ the solution will be interior. The corner $p = 0$ cannot be a solution if $\alpha'(0) - \alpha(0) + r > 0$, which holds by the assumption $\alpha'(0) > \alpha(0)$, and the corner $p = 1$ cannot be a solution if $-\alpha(1) + r < 0$, that is for $r < \alpha(1)$. Differentiating the first-order condition (27) we get

$$p'(r) = \frac{-1}{(1 - p)\alpha''(p) - 2\alpha'(p)} > 0 \tag{28}$$

which implies that a higher lending rate promotes risk-taking by entrepreneurs.

Following Martínez-Miera and Repullo (2010), we assume that each entrepreneur is characterized by a continuous distribution of reservation utilities. Let $\Gamma(u)$ denote the measure of entrepreneurs that have a reservation utility less than or equal to u and $u(r) = \max_p(1-p)(\alpha(p) - r)$ determines the maximum expected payoff that entrepreneurs can obtain when the loan rate is r. By the envelope theorem we have $u'(r) = -(1-p(r)) < 0$ and $u''(r) = p'(r) > 0$. Entrepreneurs undertake the project at the lending rate r if the reservation utility u is smaller than or equal to $u(r)$. Hence the measure of entrepreneurs that want to borrow from the banks at the lending rate r is given by $\Gamma(u(r))$. Since each one requires a unit loan, the loan demand function is

$$L(r) = \Gamma(u(r)) \tag{29}$$

Clearly for $0 \leq r < \alpha(1)$ we have $L(r) > 0$ with $L'(r) = \Gamma'(u(r))u'(r) < 0$ and $L''(r) = \Gamma'(u(r))(u'(r))^2 + \Gamma'(u(r))u''(r) > 0$. Let $r(L)$ denote the corresponding inverse loan demand function, which satisfies $r'(L) < 0$ and $r''(L) > 0$.

Banks. The exposition of the banking sector closely follows Section 2.1. There is a continuum of banks normalized to unity. Each bank i extends loans L_i that are financed by deposits D_i and equity E_i. Additionally, banks have to hold reserves R_i. As before, we assume that deposits are insured by a government-funded deposit scheme and, to simplify the presentation, we abstract from competition in the deposit market by assuming that banks face a perfectly elastic supply of deposits at a rate equal to r_D. Aggregate deposits in the banking sector are equal to $D = \int_0^1 D_i di$, the same applies for aggregate loans $L = \int_0^1 L_i di$ and aggregate reserves $R = \int_0^1 R_i di$. We assume that banks compete for loans à la Cournot, so the strategic variable of a bank is the supply of loans L_i. Finally, we assume that a regulator imposes a minimum reserve and capital requirement ς and k, i.e. $E_i \geq kL_i$ and $R_i \geq \varsigma D_i$.

3.2. **Equilibrium.** We solve the model by backward induction and consider symmetric equilibria only. In a Nash equilibrium, each bank chooses loans and the amount of reserves and capital to maximize profits, given similar choices of the other banks and taking into account the entrepreneurs' choice of the riskiness p of the projects. In this setup, banks lend to entrepreneurs whose returns are perfectly correlated. This assumption is equivalent to the one taken in Section 2.1 on a bank portfolio composed of perfectly correlated risks. This implies that the probability of default of loans $p(r(L))$ coincides with the probability of bank failure F_B.

Banks maximize the present discounted value of their net worth $\pi(L)$ net of bankers' initial infusion of capital

$$V = \max_{\{L,k,\varsigma\}} \left[-k + \frac{1}{1+\delta}\pi(L) \right] L \tag{30}$$

and the net worth per unit of loan $\pi(L)$ is given by

$$\pi(L) = [1 - p(r(L))] \left(r(L) + k - r_D \frac{1-k}{1-\varsigma} \right) \tag{31}$$

Banks choose the profit maximizing volumes of loans L and decide upon how much capital k and reserves ς to hold per unit of loan and deposit. Importantly, though, is the fact that the aggregate supply of loans L determines the lending rate $r(L)$, which in turn determines the probability of failure chosen by the entrepreneurs as implied by equation (27). Taken together, this motivates $p(r(L))$ being implicitly defined by equations (27) and (29).

The first order conditions with respect to reserve holdings ς and capital holdings k are equivalent to equations (6) and (7). As before, banks do not hold equity capital and reserves in excess of what is required by prudential regulation. The first order condition with respect to the amount of loans is given by

$$r(L) - r_D \frac{1-k}{1-\varsigma} = \frac{1+\delta - g(L)}{g(L)} k + G(L) \tag{32}$$

where $g(L) \equiv 1 - p(r(L)) - Lp'(r(L))r'(L) > 0$ since[5] $r'(L) < 0$ and $G(L) \equiv \frac{-[1-p(r(L))]r'(L)L}{g(L))} > 0$ with $G'(L) > 0$ as shown implicitly in Boyd and De Nicoló (2005). Equation (32) defines the equilibrium lending rate $r(L)$ as a function of the cost of equity $\left(\frac{1+\delta-g(L)}{g(L)}k\right)$, the cost of deposits $\left(r_D \frac{1-k}{1-\varsigma}\right)$ and some "monopoly rents" captured by $G(L)$. Equation (32) is to be compared with equation (8) from the setup based on portfolio optimization. In contrast to equation (8), equation (32)

[5] Note that $g(L)$ can be expressed in terms of risk and loan elasticities: $g(L) = 1 + p(r(L)) \left(\eta_{p(r)}\eta_{r(L)} - 1\right)$ with $\eta_{p(r)} \equiv p'(r(L))r(L)/p(r(L)) > 0$ is the elasticity of entrepreneurs' risk-taking with respect to the lending rate and $\eta_{r(L)} \equiv -r'(L)L/r(L) > 0$ is the elasticity of the (inverse) loan demand function with respect to loans. Since $\eta_{p(r)} \geq 0$ and $\eta_{r(L)} \geq 0$, we have that $g(L) > 0$; we exclude the boundary case characterized by $\eta_{p(r)} = \eta_{r(L)} = 1 - p(r(L)) = 0$.

contains the cost of capital δ. This is because in the model outlined in Section 2.1, the volume of loans is irrelevant for banks' profits; the only decisive factor is the composition of loans in terms of the degree of risk they contain. Within the setup of the optimal contracting problem, changes in the loan volume imply changes in revenues due to the downward sloping inverse demand function $r(L)$ for loans.

3.3. **Implications of reserve and capital regulation.** We now analyse how reserve and capital regulation affect the probability of bank failure. For this we focus on the effects of prudential regulation on the degree of riskiness $p(r(L))$ of a single loan and banks' probability of default F_B. By assumption, the return of projects of different entrepreneurs is perfectly correlated. This implies that the probability of entrepreneurs' default on their loans $p(r(L))$ coincides with the probability of bank failure $F_B = p(r(L))$. We will see that (i) reserve requirements have a unique effect on the probability of bank failure and (ii) the sign of the effects of capital requirements on the probability of bank failure depends on reserve requirements.

We proceed stepwise to assess the effects of changes in reserve and capital requirements. For this, we first determine their effects on loan supply. This result will then be used to evaluate the effect on the lending rate and the degree of risk-taking by entrepreneurs. From this we can then determine the probability of bank failure. We define $\gamma(L) \equiv G'(L) - r'(L) - k\frac{g'(L)(1+\delta)}{g(L)^2}$ which satisfies $\gamma(L) > 0$ if $G'(L) - r'(L) > k\frac{g'(L)(1+\delta)}{g(L)^2}$ which we assume applies. Considering the total differential of equation (32) we find the following for the partial effects of reserve and capital requirements on loan supply

$$\frac{dL}{d\varsigma} = -\frac{r_D}{\gamma(L)}\frac{1-k}{(1-\varsigma)^2} < 0 \tag{33}$$

$$\frac{dL}{dk} = \frac{1}{\gamma(L)}\left(\frac{1-\varsigma+r_D}{1-\varsigma} - \frac{1+\delta}{g(L)}\right) < 0 \text{ if } \frac{1-\varsigma+r_D}{1-\varsigma} < \frac{1+\delta}{g(L)} \tag{34}$$

Since $r(L)$ is the inverse demand function for loans, we have that $dr(L)/d\varsigma > 0$ and $dr(L)/dk > 0$ if $\frac{1-\varsigma+r_D}{1-\varsigma} < \frac{1+\delta}{g(L)}$. Finally, taking the total differential of $F_B = p(r(L))$ implies the following for the partial effect of reserve and capital requirements for the probability of bank failure

$$\frac{dF_B}{d\varsigma} = p'(r(L))r'(L)\frac{dL}{d\varsigma} > 0 \quad (35)$$

$$\frac{dF_B}{dk} = p'(r(L))r'(L)\frac{dL}{dk} > 0 \text{ if } \frac{1-\varsigma+r_D}{1-\varsigma} < \frac{1+\delta}{g(L)} \quad (36)$$

Equation (35) highlights that the overall effect of changes in reserve requirements on the probability of bank failure depends on three components: (i) the extent to which changes in reserves trigger changes in loan supply ($dL/d\varsigma$), (ii) the extent to which changes in the loan supply affect the lending rate by means of the demand function for loans ($r'(L)$), and (iii) the extent to which changes in the lending rate affect risk-taking by entrepreneurs ($p'(r(L))$).

For a more intuitive explanation, consider an increase in the reserve requirement ς. This implies that now a larger part of the deposit volume cannot be used for loan supply. Hence higher reserve requirements render deposits more expensive inducing banks to a reduction (substitution effect). The reduction in deposits could in principle be counterbalanced with equity capital, however, this is not the case since capital is only held to the amount necessary as required by prudential regulation which is due to equation (7). Hence the decline in deposits brings about a decrease in the aggregate loan volume L which translates into an increase in the lending rate $r(L)$. This in turn promotes higher risk-taking by entrepreneurs. Hence, a tighter reserve regulation increases the risk of individual loans. Put differently, reserve requirements do not contribute to financial stability as higher reserves fail in ameliorating entrepreneurs' moral hazard problem. Importantly, the size of the adverse effects of higher reserve requirements crucially depends on entrepreneurs' risk-taking sensitivity.

In this environment, capital requirements determine the size of the effects of changes in reserve requirements on the probability of bank failure, though, they leave the sign of the effects unchanged. The opposite, in turn, applies for capital regulation as highlighted by equation (36). When $\varsigma \to 0$, then $\frac{1-\varsigma+r_D}{1-\varsigma}$ is comparably small rendering more likely $\frac{1-\varsigma+r_D}{1-\varsigma} < \frac{1+\delta}{g(L)}$. In this case, stricter capital requirements increase a bank's probability of failure. This replicates Proposition 3 in Hakenes and Schnabel (2011): an increase in the capital requirement raises capital costs, which induces banks to choose lower deposit and loan volumes. The corresponding decrease in the aggregate loan volume L translates into an increase in the lending rate and into higher risk-taking by entrepreneurs. Hence, a tighter capital regulation increases the risk of individual loans because it exacerbates the entrepreneurs' moral hazard problem. However, equation (36) highlights that this effect strongly depends on the reserve regulation. When $\varsigma \to 1$, then $\frac{1-\varsigma+r_D}{1-\varsigma}$ is comparably large with the likely consequence that $\frac{1-\varsigma+r_D}{1-\varsigma} > \frac{1+\delta}{g(L)}$. In this case $\frac{dp(r(L))}{dk} < 0$, implying that when reserve requirements are already high, increases in capital requirements have the potential to reduce banks' probability of failure.

This example highlights that, first of all, higher reserve requirements increase the probability of bank failure since they promote entrepreneurial risk taking. Secondly, the regulatory stance on reserve requirements determines not only the effectiveness of capital requirements but, even more importantly, the sign.

3.4. **Numerical results.** In the following we use numerical methods to illustrate the effects of reserve and capital requirements in an environment of optimal contracting. We follow Martínez-Miera and Repullo (2010) and utilize a simple parametrization based on the assumption of linearity for the inverse demand for loans $r(L)$ and the entrepreneurial risk-shifting function $p(r)$. This allows us to quantitatively assess the effects of changes in reserve and capital requirements on the probability of bank failure $F_B = p(r(L))$.

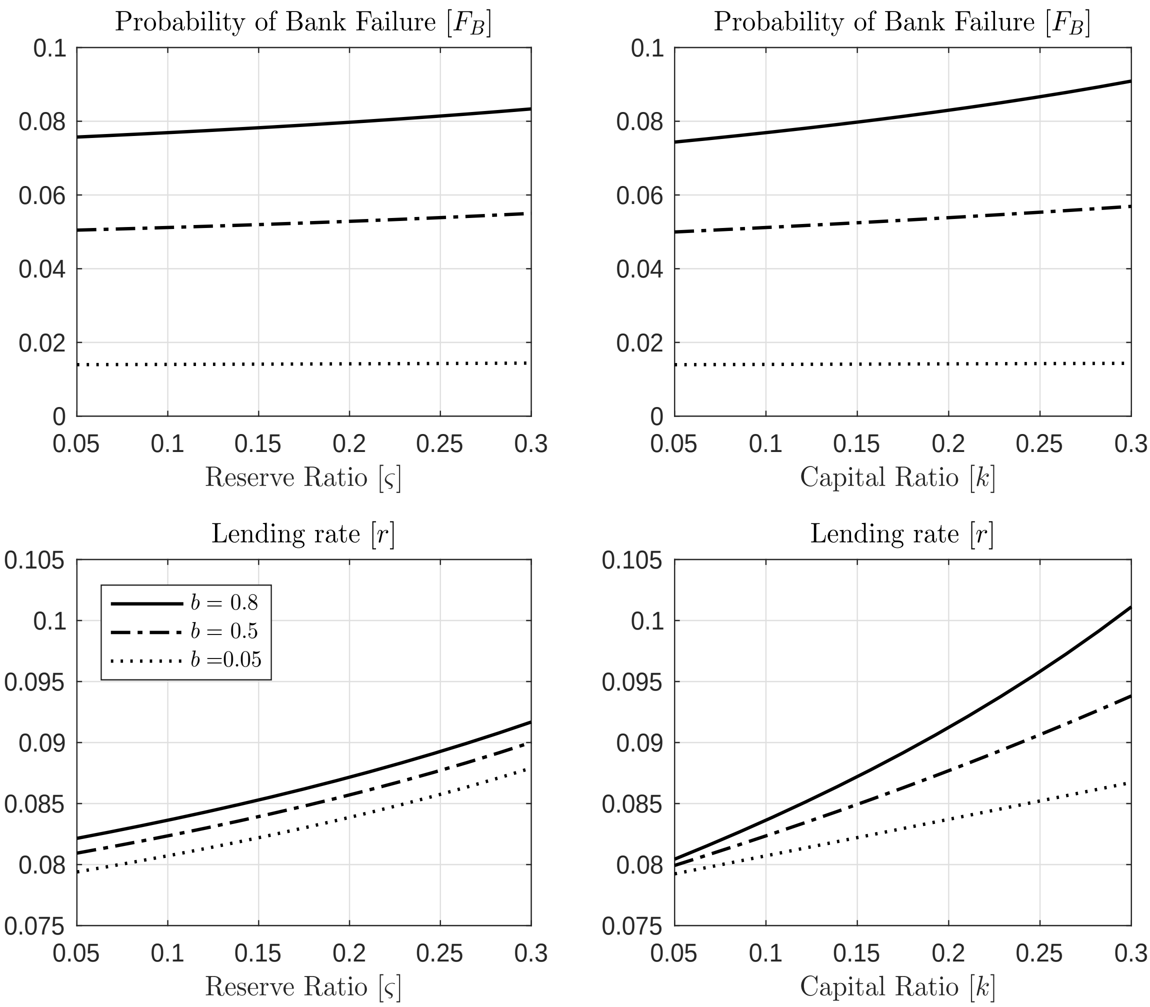
We postulate an entrepreneurial risk-shifting function $p(r)$ and an inverse loan demand function $r(L)$ of the forms

$$p(r) = a + b \cdot r \quad \text{and} \quad r(L) = c - d \cdot L \tag{37}$$

with $a > 0$, $b > 0$, $c > 0$ and $d > 0$. In this setup, the parameter a characterizes the probability of default of a project chosen by entrepreneurs when the lending rate r is equal to zero (that is, the minimum default probability of a project) and the ratio c/d gives the maximum volume of loans which occurs at a lending rate r equal to zero.[6] For the parametrization we take $a = 0.01$, $b = 0.5$, $c = 1$, and $d = 0.01$. This

[6]As highlighted in Martínez-Miera and Repullo (2010), the linear functional form for $p(r)$ can be derived from a success return specification of the form $\alpha(p) = (1 - 2a + p)/2b$ which implies the following for the expected payoff function: $u(r) = (1 - a - br)^2/2b$. Finally, noting that $L(r) = \Gamma(u(r))$, gives the following for the measure of entrepreneurs that have reservation utility less than or equal to u: $G(u) = (a + bc - 1 + \sqrt{2bu})/bd$.

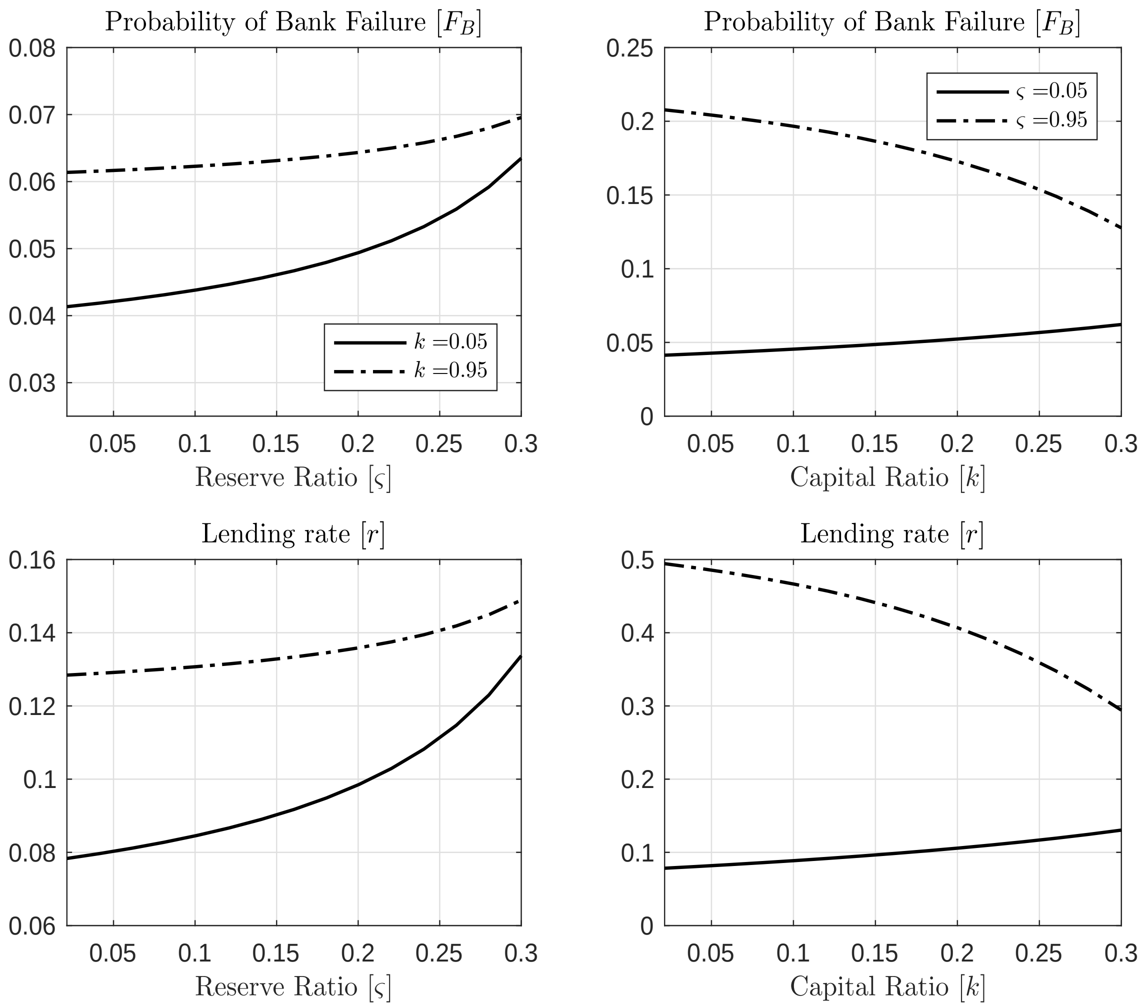
Figure 3. Comparative statics



means that the demand for loans goes from 100 to 0 as loan rates range from 0% to 100%, and that the probability of default p that corresponds to a loan rate of 2% is equal to 2%. The default value chosen for reserve and capital requirements is $\varsigma = k = 0.1$.

Figure 3 shows the effects of changes in reserve and capital requirements on the lending rate (r) and the probability of bank failure (F_B) resulting from the model which is characterized by equation (32) and the equations in (37). The simulations distinguish between different degrees of entrepreneurs' risk-taking sensitivity captured by the parameter b – higher values of b imply a higher sensitivity towards risk. The subplots in the left panel of the figure highlight that higher reserve requirements induce an increase in the probability of bank failure and the lending

Figure 4. Comparative statics



rate alike. This applies for any value of the risk-shifting parameter b. The higher entrepreneurs' risk sensitivity, the larger is the increase in the probability of bank failure induced by increases in reserve requirements. The same applies to capital requirements. If, in turn, $b = 0$, then changes in reserve and capital requirements would leave the probability of bank failure unaffected as highlighted by equations (35) and (36). Hence the effects of both reserve and capital requirements on the probability of bank failure crucially depend on entrepreneurs' risk-taking preference.

Figure 4 shows the effects of higher reserve and capital requirements on the probability of bank failure and the lending rate for concurrent values of capital and reserve requirements of a certain amount. The effects of reserve requirements are shown for two different values of capital requirements in the left panel of Figure 4. As can be seen, different values for capital requirements hardly change the shape of the curve, though its position. Most importantly, capital requirements have no ability in changing the sign of the effects of reserve requirements on the probability of bank failure. The opposite applies to capital requirements. The two subplots in the right panel of Figure 4 show that when reserve requirements are high, increases in capital requirements contribute to ameliorate entrepreneurs' moral hazard problem in the choice of risk and hence to a decrease in the probability of bank failure. The opposite applies at low values for reserve requirements. This example illustrates how the interaction between reserve and capital requirements matters for the effectiveness and even the sign of the effects of these regulatory tools.

4. The role of entrepreneurial risk-shifting and imperfectly correlated loan default

As a final attempt to assess the role of reserve regulation on bank risk-taking, we now consider its effects within a model that considers jointly the portfolio problem and the optimal contracting problem. Specifically, we merge the models considered in Sections 2.2 and 3 with minor changes. Hence the basic setup is identical to that of Martínez-Miera and Repullo (2010) except for the introduction of reserve and capital regulation as motivated in Section 2.1.

Entrepreneurs. As in Section 3, we consider a continuum of entrepreneurs who have no own resources, but have access to risky projects that require a unit investment and yields a stochastic return

$$\tilde{R}(p_i) = \begin{cases} 1+\alpha(p_i), & \text{with probability } 1-p_i \\ 1-\lambda, & \text{with probability } p_i \end{cases} \tag{38}$$

where $0 < \lambda < 1$ is an individual project's LGD and to simplify the presentation we assume that it does not depend on p_i. In contrast to Section 3.1, we now assume that project failures and consequently loan defaults are imperfectly correlated. For this we use the single risk factor model of Vasicek (2002), which introduces the probability distribution of the aggregate failure rate x given by equation (13).

Banks. The structure of the banking sector follows the setup of Section 2.2 and Section 3.1 with minor modifications to account for the different risk environment. Assuming that banks compete for loans à la Cournot, so as in Section 3, the strategic variable of a bank i is its supply of loans L_i. Bank i's net worth $\pi(L_i, L)$ per unit of loan L_i is again given by equation (14), though, it now depends on aggregate loans L, which in turn results from the dependence of lending rates on the aggregate volume of loans ($r = r(L)$). The bank fails when $\pi(L_i, L) < 0$, that is, when the default rate x is greater than the bankruptcy/default rate $\hat{x}(L)$ which is given by equation (18) and depends on loans L since the lending rate $r(L)$ is now a function of loans.[7]

As before, bank i's objective is to maximize the expected discounted value of $\max(\pi(L_i, L), 0)$, net of bankers' initial infusion of capital, which now reads

$$V_i(L_i) = L_i \cdot \left[-k + \frac{1}{1+\delta} \int_0^{\hat{x}(L)} \pi(L) dF(x; p(r(L))) \right] \tag{39}$$

where the distribution function $F(x; p(r(L)))$ of the default rate x is written so as to account for the endogenous probability of default of the loans. Thus, when bank i chooses its supply of loans L_i, it takes into account (i) the direct effect of changes in loan supply on the lending rate $r(L)$, and (ii) the indirect effect on the probability of default of the loans $p(r(L))$ and hence on the probability distribution of the default rate x.

Bank i maximizes the expected discounted value V_i with respect to the amount of capital k and reserves ς; optimization yields the same first order conditions as given by equations (16) and (17). Hence, bank i will always hold the minimum amount of reserves and capital as required by the regulation.

4.1. **Equilibrium.** In what follows, we identify the Cournot symmetric equilibrium of the model of competition in the loan market with imperfectly correlated loan default, and analyse the effect of changes in reserve and capital requirements on the equilibrium lending rate $r(L)$ and the equilibrium probability of bank failure $F_B(L) = 1 - F(\hat{x}(L); p(r(L)))$.

[7] Equivalently to Section 2.1, the bankruptcy/default rate $\hat{x}(L)$ could also be derived formally by considering $\partial V_i / \partial \hat{x}(L) \stackrel{!}{=} 0$.

The assumption of symmetry simplifies the model and implies $L_i = L$. Using equation (18) for the bankruptcy/default rate $\hat{x}(L)$ and applying integration by parts allows to re-write equation (39) in the following way

$$V(L) = L\pi(L) \tag{40}$$

with $\pi(L)$ being the net worth per unit of loan L given by

$$\pi(L) = -k + \frac{\lambda + r(L)}{1+\delta} \int_0^{\hat{x}(L)} F(x; p(r(L)))dx \tag{41}$$

The first order condition with respect to loans L yields the following first-order differential equation

$$\pi(L) = -L\pi'(L) \tag{42}$$

where $\pi'(L)$ is given by

$$\pi'(L) = \frac{r'(L)}{1+\delta}\Bigg[(1-\hat{x}(L))F(\hat{x}(L); p(r(L))) + \int_0^{\hat{x}(L)} \left(F(x; p(r(L))) + (\lambda + r(L))\frac{\partial F(x; p(r(L)))}{\partial p(r(L))} p'(r(L))\right) dx\Bigg] \tag{43}$$

and we used $\hat{x}'(L) = \frac{1-\hat{x}(L)}{\lambda + r(L)} r'(L) < 0$ as implied by equation (18). The first term in square brackets in equation (43) is positive. The sign of the term within the integral is ambiguous because $F(x; p(r(L))) > 0$, whereas $\frac{\partial F(x;p(r(L)))}{\partial p(r(L))} < 0$ (the effect on the probability distribution of the bankruptcy/default rate) and $p'(r(L)) > 0$ (risk-shifting effect). In line with Martínez-Miera and Repullo (2010), we assume that the parametrization and functional forms are such that $\pi'(L) < 0$ and $\pi''(L) < 0$, so that there is a unique symmetric equilibrium. The complication arises from the effect of the bankruptcy/default rate on the probability distribution $\left(\frac{\partial F(x;p(r(L)))}{\partial p(r(L))} < 0\right)$. To see this, assume that loan defaults are perfectly correlated ($\varrho \to 1$). This implies that $F(x; p(r(L))) = 1 - p(r(L))$ for $0 \leq x \leq 1$ and hence by using equation (18) and (41) we obtain $\pi(L) = [1 - p(r(L))]\left(r(L) + k - r_D\frac{1-k}{1-\varsigma}\right)$ which is the net worth per unit of loan as given in equation (4) of the model based on the *portfolio problem* with perfectly correlated loan default and it also corresponds to the net worth defined in equation (31) of the model based on the *optimal contracting problem* with perfectly correlated loan default. Moreover, the first order condition (42) is equal to the one given by equation (32) in Section 3 of the model with perfectly correlated loan default.

4.2. **Implications of reserve and capital regulation.** We now use the first order condition given by equation (42) to trace out the effects of reserve and capital regulation. Taking the total differential and re-arranging results in the following for the partial effect of reserve requirements on the probability of bank failure

$$\frac{dL}{d\varsigma} = -\frac{L\partial\pi'(L)/\partial\varsigma + \partial\pi(L)/\partial\varsigma}{2\pi'(L) + L\pi''(L)} \tag{44}$$

Given the previous assumptions on $\pi(L)$, we note that the denominator is negative. Since $\frac{\partial\hat{x}(L)}{\partial\varsigma} = -\frac{r_D(1-k)}{(\lambda+r(L))(1-\varsigma)^2} < 0$, we get $\partial\pi(L)/\partial\varsigma = \frac{\lambda+r(L)}{1+\delta}F(\hat{x}(L);p(r(L)))\frac{\partial\hat{x}(L)}{\partial\varsigma} = -r_D\frac{1-k}{(1+\delta)(1-\varsigma)^2}F(\hat{x}(L),p(r(L))) < 0$ as implied by equation (16), and, noting that $\partial\pi'(L)/\partial\varsigma = \partial^2\pi(L)/\partial L\partial\varsigma$, we have

$$\begin{aligned}\frac{\partial\pi'(L)}{\partial\varsigma} &= \frac{r'(L)}{1+\delta}\Bigg((1-\hat{x}(L))\frac{\partial F(\hat{x}(L);p(r(L)))}{\partial\hat{x}(L)}+ \\ &\qquad (\lambda+r(L))\frac{\partial F(\hat{x}(L);p(r(L)))}{\partial p(r(L))}p'(r(L))\Bigg)\frac{\partial\hat{x}(L)}{\partial\varsigma}\end{aligned} \tag{45}$$

of which the first term in brackets is positive whereas the second term is negative. Considering jointly $\partial\pi(L)/\partial\varsigma$ and $\partial\pi'(L)/\partial\varsigma$ within the nominator of equation (44) implies that $\frac{dL}{d\varsigma} < 0$ whenever the following condition holds

$$-\frac{F(\hat{x}(L);p(r(L)))}{Lr'(L)} - \frac{\partial F(\hat{x}(L);p(r(L)))}{\partial p(r(L))}p'(r(L)) > \frac{1-\hat{x}(L)}{\lambda+r(L)}\frac{\partial F(\hat{x}(L);p(r(L)))}{\partial\hat{x}(L)} \tag{46}$$

In case of perfectly correlated loan default ($\varrho \to 1$), the condition in equation (46) reduces to $-(1-p(r(L)))/Lr'(L)+p'(r(L)) \overset{!}{>} 0$, which is satisfied since $r'(L) < 0$ and $p'(r(L)) > 0$. Hence, the presence of imperfectly correlated loan default introduces the term $\partial F(\hat{x}(L);p(r(L)))/\partial\hat{x}(L) > 0$, which renders feasible an increase in loan supply when reserve requirements are raised. In what follows, we rule out this possibility and assume that the condition in equation (46) holds. It follows that $\frac{dL}{d\varsigma} < 0$, so that an increase in reserve requirements reduces loans. This in turn implies that $dr(L)/d\varsigma = r'(L)dL/d\varsigma > 0$, hence higher reserve requirements raise the lending rate.

Given the effects of changes in reserve requirements on loans and the lending rate, we can then identify the effect on the probability of bank failure. Banks fail whenever the default rate x is greater than the bankruptcy/default rate $\hat{x}(L)$ defined in equation (18). We denote the equilibrium amount of loans with L^*, then from the definition of the probability of bank failure we have (taking into account that $\hat{x} = \hat{x}(L)$ and $p = p(r(L))$)

$$\frac{dF_B(L^*)}{d\varsigma} = -\frac{\Phi'(\cdot)}{\sqrt{\varrho}}\left[\frac{\sqrt{1-\varrho}}{\Phi'(\Phi^{-1}(\hat{x}(L^*))}\frac{\partial \hat{x}(L^*)}{\partial \varsigma} - \frac{p'(r(L^*))r'(L^*)}{\Phi'(\Phi^{-1}(p(r(L^*)))}\frac{dL^*}{d\varsigma}\right] > 0 \tag{47}$$

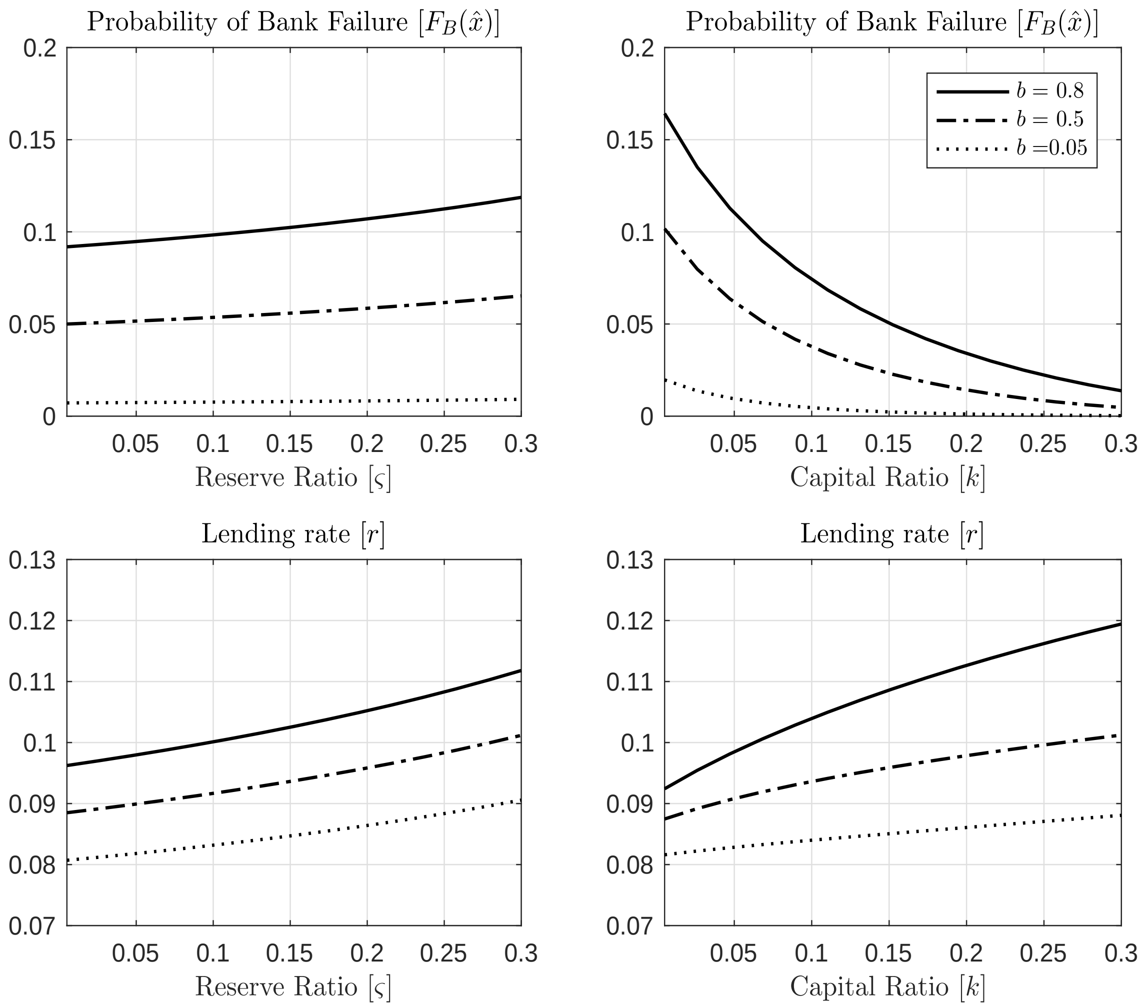
where $\frac{\partial \hat{x}(L^*)}{\partial \varsigma} < 0$ is given by equation (24). Since $\frac{dL^*}{d\varsigma} < 0$ it follows that $\frac{dF_B(L^*)}{d\varsigma} > 0$ as both terms in brackets in equation (47) are negative. Hence, higher reserve requirements increase the probability of bank failure. The first term of equation (47) is equivalent to equation (21) of the setup in Section 2.2. The second term captures the presence of entrepreneurs' risk-shifting preference ($p'(r(L)) > 0$). As can be seen, entrepreneurs' moral hazard problem augments the adverse effects of higher reserve requirements on financial stability compared to those already identified in Section 2.2. If $p'(r(L)) = 0$, then the current model reduces to the setup of Section 2.2. If in turn loan default rates are perfectly correlated ($\varrho \to 1$ with $F_B(L) \to p(r(L))$), then equation (47) reduces to $\frac{dF_B(L^*)}{d\varsigma} = p'(r(L^*))r'(L^*)\frac{dL^*}{d\varsigma} > 0$ which replicates equation (35) of the setup in Section 3.

For capital requirements, we first assume that the condition stated in equation (56) of Appendix B holds. This condition essentially implies that an increase in capital requirements triggers a decrease in loan supply. Given this, we then obtain the following partial effect on the probability of bank failure

$$\frac{dF_B(L^*)}{dk} = -\frac{\Phi'(\cdot)}{\sqrt{\varrho}}\left[\frac{\sqrt{1-\varrho}}{\Phi'(\Phi^{-1}(\hat{x}(L^*))}\frac{\partial \hat{x}(L^*)}{\partial k} - \frac{p'(r(L^*))r'(L^*)}{\Phi'(\Phi^{-1}(p(r(L^*)))}\frac{dL^*}{dk}\right] \gtreqless 0 \tag{48}$$

Since $\frac{\partial \hat{x}(L^*)}{\partial k} > 0$, as implied by equation (23), the presence of entrepreneurs' risk-shifting effect renders uncertain the overall effect of higher capital requirements on the probability of bank failure. The first term in equation (48) is equivalent to equation (22). The second term arises from entrepreneurs' risk-shifting preference and compares with equation (36) of the setup in Section 3.

FIGURE 5. Comparative statics



4.3. **Numerical results.** We again utilize numerical methods to illustrate the effects of reserve and capital requirements, now in an environment of optimal contracting combined with imperfectly correlated loan default. For this we rely on the same functional specifications for the inverse demand for loans and the entrepreneurial risk-shifting function as in Section 3.4 and focus on a quantitative assessment of the effects of changes in reserve and capital requirements on the probability of bank failure $F_B(L)$ and the lending rate $r(L)$.

Figure 5 shows the effects of changes in reserve and capital requirements on the lending rate (r) and the probability of bank failure (F_B) resulting from the model characterized by equations (13), (18), (42), the equations in (37) and the definitions for $\pi(L)$ and $\pi'(L)$ given by equations (41) and (43). The simulations distinguish between various degrees of entrepreneurs' risk-taking sensitivity which is captured by the parameter b.

The subplots in the left panel of the figure highlight that higher reserve requirements induce increases in the probability of bank failure and the lending rate alike. Higher reserve requirements induce a decline in loan supply and an increase in the lending rate, which in turn leads to a higher probability of loan default due to entrepreneurs' moral hazard problem. The final increase in the probability of bank failure is triggered by two effects (i) the higher probability of loan default $(p(r(L)))$ and (ii) the decrease in the bankruptcy/default rate $(\hat{x}(L))$.

This applies for any values of the risk-shifting parameter b. The higher entrepreneurs' moral hazard problem, the larger is the increase in the probability of bank failure induced by increases in reserve requirements. When entrepreneurs' moral hazard problem is absent $(b = 0)$, then the model replicates the setup of Section 2.2 in which higher reserve requirements still cause an increase in the probability of bank failure, however, these effects are small from a quantitative perspective (see Figure 1 for comparison) as in this case the effect from higher interest revenues of non-defaulting loans weighs stronger and hence dampens the adverse effects.

The opposite applies for capital requirements. Higher capital requirements trigger a reduction in the probability of bank failure. The size of the reduction in response to higher regulatory requirements crucially depends on the degree of the entrepreneurial risk-taking preference. When entrepreneurs' moral hazard problem is pronounced (high values of b), then increases in capital requirements trigger comparably large reductions in the probability of bank failure. This is due to the fact that when risk-taking is high, small increases in the probability of loan default $(p(r(L)))$ trigger comparably large changes in the lending rate $(r(L))$. This raises revenues to banks accruing from non-defaulting loans. In principle, this effect applies to reserve requirements, however, it is always dominated by the effect of reserve requirements on the bankruptcy/default rate as discussed in detail in Section 2.2.1.

5. Discussion

Reserve requirements relate a part of banks' assets to liabilities; in the case of capital requirements it is the opposite. Common to both is that each of the two regulatory instruments induces a contraction in loan supply (see for instance Aiyar et al., 2016; Malovaná and Frait, 2017). Despite the fact that reserve and capital requirements seem to have a similar effect on banks' balance sheets and lending rates, their effects on financial stability are different. This addresses the composition of banks' assets with respect to the risk exposure.

In what follows, we discuss a key policy implication of this study, that is, the role of reserve requirements for procyclicality. Additionally, we elaborate on a model specific aspect. We start with the latter.

5.1. **The role of monitoring.** The previous sections rely on the assumption that banks' engagement ends with the loan disbursement. In fact, banks monitor their loan portfolio to ensure that changes in borrowers' finances or circumstances do not put repayment in jeopardy. In this context, monitoring is based on the idea that changes in the risk environment are taken into account by banks by means of re-adjusting the monitoring intensity of the loan portfolio. Banks can increase the probability of getting a higher return simply by exerting a monitoring effort. Considering monitoring in relation to reserve requirements seems peculiar at first sight, however, it is a natural extension of the previous analysis. As outlined before, higher reserve requirements induce a decline in profits as part of deposits remains un-invested. This incentivizes banks to accept loans with a higher success return $\tilde{r}$ in order to compensate for the loss. These loans are, however, riskier, that is the probability of loan default p is higher. The higher share of riskier loans increases the default probability of banks. Monitoring enables banks to counteract the increase in risk in their loan portfolio. On the one hand monitoring incurs some cost, on the other it increases the probability of getting the high return $\tilde{r}$ given a particular level of probability p of loan default. Equivalently, given a particular rate of return $\tilde{r}$, monitoring allows for a decline in the probability of default. Hence, if higher reserve requirements induce an increase in monitoring, then we can expect a decline in the probability of default at a given rate of return $\tilde{r}$.

We consider this idea in Section C of the Appendix where we extend the model of Section 2.1 with monitoring. The basic results are: the more successful a bank's monitoring efforts are, that is the more likely the bank can obtain higher lending rates by means of monitoring, the more likely it is that increases in reserve requirements cause a decline in the probability of bank failure. The crucial element behind this result is the size of the elasticity of the lending rate with respect to the probability of loan default relative to the size of the elasticity of the lending rate with respect to the monitoring intensity. Further details can be found in Section C of the Appendix.

5.2. **Procyclicality.** The previous results tell a cautionary tale on the usefulness of reserve requirements as a tool to contain the degree of procyclicality of the financial system. In this respect most papers of the macroeconomic literature argue that first of all reserve requirements can serve as a countercyclical tool to manage the credit cycle in a broad context since they limit an excessive leverage of borrowers in the upswing and operate as a liquidity buffer in the downswing (see the papers cited in the Introduction). Second, reserve requirements can help to contain risk accumulation by improving the liquidity of the banking system. Regarding reserve requirements as a countercyclical tool – the first argument – what these papers ignore, is the extent to which a decline in loan supply and a corresponding increase in the lending rate might lead to a completely different outcome once banks' and entrepreneurs' moral hazard problem is taken into account. As higher reserve requirements promote risk-taking, an increase during the upswing of the cycle could lead to an even stronger appetite for risk and thus lead to unintended policy outcomes. In this environment a reduction in reserve requirements rather than an increase might attenuate the degree of procyclicality.

Regarding the second argument – reserve requirements as a liquidity buffer – Carlson (2015) describes how the aim of reserve requirements in 19th century America was to ensure that banks had sufficient liquidity to meet outflows in times of stress without reducing lending. In practice, banks generally met these requirements[8], but failed to use them as a buffer under stress. Instead, when faced with increased risk, banks would contract credit supply augmenting the overall degree of procyclicality. Reducing reserve requirements in this case might seem promising in this context at first sight, however, as Cecchetti and Kashyap (2018) argue, the impact of a reserve

[8]At that time reserve requirements in the US ranged from 15% to 25% of deposits, which compares to a current value of 2%. Nowadays in some emerging market economies reserve requirements amount to even 50% and higher.

requirement change is difficult to predict and is likely to have an inherent asymmetry. For instance, if banks are capital-impaired or increasingly sceptical about borrowers' financial conditions, then a cut in reserve requirements might simply end up in higher excess reserves rather than in an increase in loan supply. In this context, also Diamond and Kashyap (2016); Cecchetti and Kashyap (2018) remain concerned about the usability of the liquidity buffer in form of reserve requirements.

6. Conclusion

The aim of this paper is to provide a framework for analysing the impact of reserve requirements on financial stability. To this purpose, we consider a banking model that is subject to legal reserve requirements. Based on the results presented here, an increase in regulatory reserve requirements promotes risk-taking as either borrowers or banks have an incentive to choose riskier assets. Hence banks' probability of failure rises. The key elements of the analysis can be summarized as follows:

First, higher reserve requirements raise costs as only a part of the stock of deposits can be used for investment purposes. Banks try to counterbalance these higher costs by financing assets with a higher success return. These assets, however, are characterised by a higher probability of default, which in turn increases the probability of bank failure. Hence, there is a risk-shifting effect. This effect is attenuated once imperfectly correlated loan default is taken into account. In particular, as higher reserve requirements induce a shift towards assets with a higher success return, the corresponding increase in interest payments from non-defaulting loans provides a buffer to cover losses from defaulting loans. This effect is opposite to the risk-shifting effect and hence dampens the adverse effects of higher reserve requirements on the probability of bank failure. In contrast to that, the adverse effects are augmented once borrowers' moral hazard problem is taken into account. In this case, higher reserve requirements cause a decline in loan supply and an increase in the lending rate. The latter, in turn, promotes higher risk-taking by borrowers, rendering worse the overall conditions for financial stability. Importantly, the size of the adverse effects increases with borrowers' risk-shifting sensitivity. Hence, reserve requirements do not contribute to financial stability as they fail in ameliorating borrowers' and banks' moral hazard problem.

Second, even though changes in capital and reserve requirements have comparably similar effects on banks' balance sheets, their implications for financial stability are rather distinct. While both requirements affect loan supply and lending rates in the same way, reserve requirements promote risk-taking, whereas capital requirements (mostly) mitigate risk-taking.

The theoretical results presented here are in line with empirical findings. This suggests a cautionary tale of reserve requirements as a regulatory instrument for financial stability purposes. Moreover, the results also raise concerns regarding the new liquidity standards proposed by Basel III, as they share common features with reserve requirements.

As a final remark – an outlook for future research. The analysis presented here was conducted within a partial equilibrium framework. At times, this requires strong (restrictive) assumptions which might render the results less credible. A possible extension is the attempt to integrate the model building blocks presented here into a general equilibrium framework. The DSGE models as mentioned in the Introduction are particularly tempting for this purpose. In this context, the macroeconomic effects could be assessed jointly with the implications for financial stability within an environment that adequately captures the interdependencies among distinct economic agents. To the extent that reserve requirements have already been analysed in this model environment, an extension along these thoughts comprises an interesting contribution to the existing literature on this issue.

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Appendix A. Derivation of the distribution function of the default rate

This section provides details for Sections 2.2 and 4, where we consider imperfectly correlated loan default. For this we utilize the Vasicek (2002) model, which itself is based on the Merton (1974) model of credit risk.

There are many identical borrowers, indexed by i, of a continuum of measure one. The outcome of the investment project of borrower i is driven by the realization of a latent random variable y_i given by

$$y_i = \mu_i + \sqrt{\varrho}\, z + \sqrt{1-\varrho}\, \epsilon_i \tag{49}$$

where z is the macroeconomic risk factor that affects all projects, ϵ_i is an idiosyncratic risk factor that only affects the project of firm i, μ_i is a constant parameter and measures the financial vulnerability of firm i, and $\varrho \in [0, 1]$ is a parameter that determines the exposure of firm i to the macroeconomic risk factor z. The project of firm i is successful if $y_i \geq 0$; in this case, the project yields a gross return of $1+a$. If it fails, the project only yields $1-\lambda$; hence λ determines the loss given firm i defaults (loss given default, LGD). From equation (49) we have that the unconditional distribution of the latent variable satisfies $y_i \sim N(\mu_i, 1)$, so that the unconditional probability of default (p_i) of the investment project of firm i is then given by

$$p_i = Pr(y_i \leq 0) = Pr\left(\sqrt{\varrho}\, z + \sqrt{1-\varrho}\, \epsilon_i \leq -\mu_i\right) = \Phi(-\mu_i) \tag{50}$$

or equivalently $\Phi(-\mu_i) = 1 - \Phi(\mu_i)$; equation (50) implies

$$\mu_i = -\Phi^{-1}(p_i) \tag{51}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of a standard normal random variable and $\Phi^{-1}(\cdot)$ its inverse. As equation (51) points out, the parameter μ_i describes the distance-to-default of borrower i; accordingly, the probability of default of borrower i is $\Phi(-\mu_i)$. Borrower i repays the loan when $y_i \geq 0$, where $y_i = -\Phi^{-1}(p_i) + \sqrt{\varrho}\, z + \sqrt{1-\varrho}\, \epsilon_i$. Notice that for $\varrho = 0$ the macroeconomic risk factor does not play any role and we have statistically independent failures, while for $\varrho = 1$ the idiosyncratic risk factor vanishes and we have perfectly correlated failures. Conditional on the macroeconomic risk factor z, defaults are independent. In what follows, we focus on the imperfect correlation case: $\varrho \in (0, 1)$.

Consider now the continuum of firms that want to undertake their projects when the lending rate is r. By our previous argument, they all choose the same probability of failure $p_i = p$. But then the failure rate x is only a function of the realization of the macroeconomic risk factor z. Specifically, by the law of large numbers, the failure rate x coincides with the probability of default of a project of a representative firm i conditional on the realization of the macroeconomic risk factor z. Note that from equation (49) we have that $\forall i$ the distribution of the latent variable y_i conditional on the realization of the systematic risk factor z is: $y_i|z \sim N\left(\mu_i + \sqrt{\varrho}\, z, 1-\varrho\right)$, so that the conditional probability of default or *default rate* of firm i is

$$\begin{aligned} \eta(z) = Pr\left(y_i \middle|\, z \leq 0\right) &= Pr\left(\mu_i + \sqrt{\varrho}\, z + \sqrt{1-\varrho}\, \epsilon_i \middle|\, z \leq 0\right) \\ &= \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\varrho}\, z}{\sqrt{1-\varrho}}\right) \end{aligned} \tag{52}$$

where we have used equation (50) to re-write the financial vulnerability parameter μ_i as a simple non-linear transformation of the unconditional probability of default $\Phi^{-1}(p)$. Hence the default rate $\eta(z)$ is increasing in the unconditional probability of default (p) and in the realization of the macroeconomic risk factor z. The quantity $\eta(z)$ provides the loan default probability under a given scenario for the macroeconomic risk factor z. The unconditional probability of default p is the average of the default probabilities over the scenarios. The cumulative distribution function of the default rate $\eta(z)$ is given by

$$\begin{aligned}
F(x) = Pr(\eta(z) \leq x) &= Pr\left(\Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\varrho}\, z}{\sqrt{1-\varrho}}\right) \leq x\right) \\
&= 1 - Pr\left(z \leq \frac{\Phi^{-1}(p) - \sqrt{1-\varrho}\; \Phi^{-1}(x)}{-\sqrt{\varrho}}\right) \\
&= 1 - \Phi\left(\frac{\sqrt{1-\varrho}\; \Phi^{-1}(x) - \Phi^{-1}(p)}{-\sqrt{\varrho}}\right) \\
&= \Phi\left(\frac{\sqrt{1-\varrho}\; \Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\varrho}}\right) \qquad (53)
\end{aligned}$$

where we used the fact that $z \sim N(0,1)$. For $\varrho \in (0,1)$ the cumulative distribution function $F(x)$ is continuous and increasing with $\lim_{x\to 0} F(x) = 0$ and $\lim_{x\to 1} F(x) = 1$. Moreover, the mean of the distribution of the default rate $\eta(z)$ is the probability of default p of the corresponding class of loans, while the variance is entirely determined by the degree of exposure ϱ to the macroeconomic risk factor z.

Note that $\frac{\partial F(\cdot)}{\partial p} < 0$, so that changes in the probability of failure p lead to a first order stochastic dominance shift in the distribution of the failure rate p, and $\frac{\partial F(\cdot)}{\partial \varrho} \geq 0$ if and only if $p \leq \Phi\left(\sqrt{1-\varrho}\; \Phi^{-1}(p)\right)$, so changes in the correlation parameter ϱ lead to a mean-preserving spread (a second order stochastic dominance shift) in the distribution of the failure rate $\eta(z)$. Note also that when $\varrho \to 0$ (independent failures) the distribution of the failure rate approaches the limit $F(x) = 0$, for $x < p$, and $F(x) = 1$, for $x \geq p$. The single mass point at $x = p$ implies that a fraction of the projects fails with a probability of one. And when $\varrho \to 1$ (perfectly correlated failures) the distribution of the failure rate approaches the limit $F(x) = \Phi\left(-\Phi^{-1}(p)\right) = 1 - \Phi\left(\Phi^{-1}(p)\right) = 1 - p$, for $0 \leq x \leq 1$. The mass point at $x = 0$ implies that with probability $1 - p$ no project fails, and the mass point at $x = 1$ implies that with probability p all projects fail.

Appendix B. Capital requirements and bank failure - technical details

This section complements Section 4 and provides the technical details as regards the effects of changes in capital requirements on the probability of bank failure in the model characterised by the optimal contracting problem jointly with imperfectly correlated loan default.

The effects of capital requirements k on loan supply L can be characterized by the following

$$\frac{dL}{dk} = -\frac{L\partial\pi'(L)/\partial k + \partial\pi(L)/\partial k}{2\pi'(L) + L\pi''(L)} \qquad (54)$$

In comparison to equation (47), an increase in capital requirements triggers a decline in loan supply if the nominator in equation (54) is negative. Noting that $\frac{\partial \pi(L)}{\partial k} = -1 + \frac{\lambda + r(L)}{1+\delta} F(\hat{x}(L), p(r(L))) \frac{\partial \hat{x}(L)}{\partial k}$ and $\frac{\partial \hat{x}(L)}{\partial k} = \frac{1-\varsigma+r_D}{(\lambda + r(L))(1-\varsigma)}$, we hence have that $\frac{\partial \pi(L)}{\partial k} = -1 + \frac{1-\varsigma+r_D}{(1+\delta)(1-\varsigma)} F(\hat{x}(L), p(r(L))) < 0$ as implied by equation (17). Moreover, we find that

$$\text{(55)} \qquad \frac{\partial \pi'(L)}{\partial k} = \frac{r'(L)}{1+\delta} \left((1-\hat{x}(L)) \frac{\partial F(\hat{x}(L); p(r(L)))}{\partial \hat{x}(L)} + (\lambda + r(L)) \frac{\partial F(\hat{x}(L); p(r(L)))}{\partial p(r(L))} p'(r(L)) \right) \frac{\partial \hat{x}(L)}{\partial k}$$

Focusing now on the implications of that for equation (54), $dL/dk < 0$ if the numerator is negative. This is the case if the following condition is satisfied

$$\text{(56)} \qquad -\frac{\partial F(\hat{x}(L); p(r(L)))}{\partial p(r(L))} p'(r(L)) < \frac{\tilde{\zeta}}{Lr'(L)} + \frac{1-\hat{x}(L)}{\lambda + r(L)} \frac{\partial F(\hat{x}(L); p(r(L)))}{\partial \hat{x}(L)}$$

where $\tilde{\zeta} \equiv \left[-1 + \frac{1-\varsigma+r_D}{(1+\delta)(1-\varsigma)} F(\hat{x}(L); p(r(L))) \right] \frac{(1+\delta)(1-\varsigma)}{1-\varsigma+r_D} = \frac{\partial \pi(L)}{\partial k} \frac{(1+\delta)(1-\varsigma)}{1-\varsigma+r_D}$, which satisfies $\tilde{\zeta} < 0$ as implied by equation (17). The right hand side of equation (56) is positive and so is the left hand side since $\frac{\partial F(\hat{x}(L); p(r(L)))}{\partial p(r(L))} < 0$. In general, condition (56) is more restrictive than condition (46). To see this, consider the case of perfectly correlated loan default ($\varrho = 1$). Condition (56) reduces to $p'(r(L)) \overset{!}{<} \frac{\tilde{\zeta}}{Lr'(L)}$, which is in general more restrictive as the equivalent condition for reserve requirements (see Section 4.2 and also Section 3.3).

Note, combining condition (46) and condition (56) implies the following

$$\text{(57)} \qquad 0 < \frac{(1+\delta)(1-\varsigma)}{1-\varsigma+r_D}$$

which is satisfied as long $\varsigma \in [0, 1)$. This implies that if we have that $dL/d\varsigma < 0$, it follows that $dL/dk < 0$ too.

Appendix C. Monitoring in a setup of perfectly correlated loan default

This section highlights the effects of monitoring in the model outlined in Section 2.1 which is comprised by perfectly correlated loan default. The easier exposition allows to elaborate on the importance of the sensitivity of the lending rate with respect to monitoring for characterising the effects of changes in reserve and capital requirements on the probability of bank failure.

Monitoring increases the probability of obtaining a high return $r(p, m)$, but entails some cost $c(m)$ where $m \in [0, p]$ is the monitoring intensity. Within this extension, the case $m = 0$ can be associated with banks that *originate-to-distribute* and the case $m > 0$ with traditional banks that *originate-to-hold*. The monitoring cost function $c(m)$ satisfies $c(0) = c'(0)$, $c'(m) > 0$, $c''(m) > 0$, and $c'''(m) \geq 0$. We assume that monitoring is not observed by depositors; however, since deposits are fully insured, depositors do not care how much effort the bank exerts in monitoring its loan portfolio.

Each bank has an investment that yields a stochastic return $\tilde{r}$ given by

$$\tilde{r} = \begin{cases} r(p,m), & \text{with probability } 1-p+m \\ 0, & \text{with probability } p-m \end{cases} \tag{58}$$

with $r(p,m)$ being concave in both arguments and $r_m(p,m) \neq r_p(p,m)$; for simplicity we assume that $r_{m,p}(p,m) = r_{p,m}(p,m) = 0$.

The equation for the net worth of a bank reads

$$\pi(p) = (1-p+m)\left(1 + r(p,m) - (1+r_D-\varsigma)\frac{1-k}{1-\varsigma}\right) - c(m) \tag{59}$$

Banks maximize the presented discounted value, which implies the following first order conditions

$$\frac{\partial V}{\partial k} = -1 + \frac{(1-p+m)(1+r_D-\varsigma)}{(1+\delta)(1-\varsigma)} < 0 \tag{60}$$

$$\frac{\partial V}{\partial \varsigma} = -r_D\frac{(1-p+m)(1-k)}{(1+\delta)(1-\varsigma)^2} < 0 \tag{61}$$

As in Section 2.1, these two first order conditions imply that banks will always hold the minimum amount of capital k and reserves ς as required by the prudential regulation. The first order conditions with respect to the monitoring intensity m and the risk level p are

$$c'(m) = k + r(p,m) - r_D\tfrac{1-k}{1-\varsigma} + (1-p+m)r_m(p,m) \tag{62}$$

$$(1-p+m)r_p(p,m) = k + r(p,m) - r_D\tfrac{1-k}{1-\varsigma} \tag{63}$$

Combining the latter two equations and computing the total differential yields the partial effect of monitoring on the probability of bank failure

$$\frac{dp}{dm} = \frac{c''(m) - (1-p+m)r_{mm} - r_p - r_m}{(1-p+m)r_{pp} - r_p - r_m} \gtrless 0 \tag{64}$$

In what follows we assume that $c''(m) - (1-p+m)r_{mm} > r_p + r_m$ as otherwise the bank would refrain from monitoring at all[9]; with this assumption it holds that $\frac{dp(m)}{dm} < 0$. Using equation (63) and computing the total differential to isolate the effect of changes in regulatory reserve requirements on the monitoring intensity yields $\frac{dm(k,\varsigma)}{d\varsigma} = -r_D\frac{1-k}{(1-\varsigma)^2}\frac{1}{r_p - r_m}$. Combining the two partial derivatives implies the following for the probability of bank failure with respect to reserve requirements

$$\frac{dp(k,\varsigma)}{d\varsigma} = \frac{dp}{dm}\frac{dm}{d\varsigma}\begin{cases} < 0, & \text{if } r_p < r_m \\ > 0, & \text{if } r_p > r_m \end{cases} \tag{65}$$

Since $\frac{dp(m)}{dm} < 0$, the sign of the effect of reserve requirements on the probability of bank failure p is determined by $\frac{dm(k,\varsigma)}{d\varsigma}$. Hence the effects depend on the sensitivity of the monitoring intensity with respect to reserve requirements. If sizeable changes in the lending rate are triggered by small adjustments in monitoring as opposed to the risk level, then increases in regulatory reserve requirements induce a rise in the monitoring intensity and a decline in the probability of bank failure.

The intuition is the following: higher reserve requirements push up costs; banks can react to that by means of two possibilities: (i) to cut back monitoring intensity so as to save on monitoring costs and equilibrate costs or (ii) to increase monitoring

[9] This is equivalent to assuming that: $(1-p+m)\left[c''(m) - (1-p+m)r_{mm}\right] > c'(m)$.

so as to take on assets with a higher success return and a comparably low risk level p. Which possibility to choose depends on the sensitivity of the lending rate with respect to its two arguments: if lending rates react only weakly to higher monitoring, then the gain from intensifying monitoring is small, hence banks will choose the first possibility and cut monitoring all together. In this case, higher reserve requirements trigger a decline in monitoring and an increase in the probability of bank failure as banks try to compensate the loss of higher required reserve holdings by means of taking on loans with a higher success return; these loans, however, are comprised by a higher risk level. Hence the probability of bank default increases in this case. This resembles the implications of Section 2.1.

If in turn lending rates react strongly to changes in monitoring, then banks can achieve higher lending rates even without having to accept an increase in the risk level p, or equivalently, to achieve a decline in the risk level without having to accept a drop in lending rates. As a consequence, higher reserve requirements cause a decline in the probability of bank failure and hence promote bank stability; even in this case, however, the extent to which an increase in reserve requirements induces higher costs remains an essential element for the transmission mechanism.

As regards capital requirements, the partial effect of k on the monitoring intensity is given by: $\frac{dm(k,\varsigma)}{dk} = \frac{1+r_D-\varsigma}{1-\varsigma}\frac{1}{r_p-r_m}$; combining the latter expression with equation (64) yields the partial effect of capital requirements on the probability of bank failure

$$\frac{dp(k,\varsigma)}{dk} = \frac{dp}{dm}\frac{dm}{dk}\begin{cases} > 0, & \text{if } r_p < r_m \\ < 0, & \text{if } r_p > r_m \end{cases} \tag{66}$$

Obviously, if the sensitivity of lending rates with respect to the risk level p is higher than with respect to the monitoring intensity m, increases in capital requirements promote lower risk-taking by banks. Equations (65) and (66) imply that $\frac{dp(k,\varsigma)}{d\varsigma} \propto -\frac{dp(k,\varsigma)}{dk}$.