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Abstract

Using a static model of firm behaviour with imperfect competition on the product and labour markets, we quantify the effect of firm heterogeneity in total factor productivity, product and labour market power, wages and output prices on the aggregate labour share. In particular, we suggest a new decomposition of the aggregate labour share in terms of the first moments of the joint distribution of these variables across firms, providing a bridge between the micro and the macro approach to functional distribution. We provide an application of our method to the UK manufacturing sector, using firm-level data for the period 1998-2014. The analysis confirms that heterogeneity matters: in an economy populated only by representative firms, the labour share would be 10 percentage points lower. However, and contrary to a common narrative focussing on increasing disparities between firms, the observed decline in the aggregate labour share over the period is driven entirely by the decline in the labour share of the representative firm, mostly due to an increasing disconnect between average productivity and real wages. Changes in the dispersion of firm-level variables have contributed to slightly contain this decline.

KEYWORDS: labour share, firm heterogeneity, market power, firm level data.

JEL CLASSIFICATION: D33, E25, L10, D20, D42, D43.

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1 Introduction

A marked decrease in the labour share over the recent decades has been documented in many countries. Updating the data collected by Karabarbounis and Neiman (2014), the IMF shows that in advanced economies the labour share decreased from around 75% in the first half of the 1970s to less than 40% in the first half of the 2010s (IMF, 2017). A downward trend, although of smaller magnitude, is also observed for European countries (Dimova, 2019). There is considerable debate about the causes underlying the documented decline in the labour share, ranging from capital-augmenting technological change; a decline in the price of capital relative to labour, capital accumulation, globalisation, deregulation of product and labour markets, an increase in firms' product and labour market power, financial deepening, monetary policy, the rise of "superstar firms", and even an increase in the cost of housing.

Most studies have adopted either a macro or a representative agent perspective, linking the aggregate labour share (at the national, regional or industry level) to aggregate values of those determinants, with a smaller number of studies looking at the determinants of the labour share at the level of the individual firm, and a few others focusing on compositional issues, that is explaining the decline in the aggregate labour share with an increase in the relative importance of firms with a lower than average labour share.

In this paper we propose a novel approach. Starting from a static model of firm behaviour with CES production functions and imperfect competition in the product and labour markets, we offer a decomposition formula for the aggregate labour share that depends on the distribution of the individual determinants of firm heterogeneity (in particular, we focus on total factor productivity, product and labour market power, wages and output prices). This methods builds up from the micro level, without requiring the existence of an aggregate production function. The full characterisation of the labour share can be approximated by a parsimonious characterisation in terms of the mean, variance and covariance of all those variables. Hence, we are able to generalise the three approaches described above, provide a quantification of the overall effect of heterogeneity, and look into the relative importance of the different sources of heterogeneity. While our method is more demanding on data than a mere statistical decomposition, due to its reliance on theory it can go further, and provide new insights on the drivers of the labour share.

The theoretical results shows the aggregate labour share is affected by the joint distribution of all firm-level variables. We distinguish between a "direct effect" and an "indirect effect" of firm heterogeneity on the aggregate labour share. The former is independent of the correlation structure between firm characteristics. In particular, when the elasticity of substitution between capital and labour is below 1 – the empirically relevant case – a *ceteris paribus* increase in the dispersion of productivity or monopsony power increases the aggregate labour share, while a *ceteris paribus* increase in the dispersion of real wages or product market power decreases it. As for what concerns the "indirect effect", other things being equal an increase in the correlation between labour market power and TFP, or between product market power and wages increases the aggregate labour share, whereas an increase in the correlation between product market power and labour market power, product market power and TFP, labour market power and wages, or TFP and wages decreases it.

Another important theoretical lesson is derived for the case of a Cobb-Douglas production function (where the elasticity is equal to 1, and the most common empirical assumption). We show that, unless models assume *heterogeneity* in product or labour market power, heterogeneity in other variables does not affect the labour share. As such, models assuming Cobb-Douglas production functions might severely constrain the potential determinants of the labour share. Relatedly, since an aggregate production function cannot be derived from the micro level production functions under imperfect competition (regardless of its shape), firm heterogeneity is by definition invisible to a model based on an aggregate production function.

We then apply this new decomposition method to firm level data covering UK manufacturing between 1998 and 2014. Implementing the method requires estimates for the production function (including TFP), measures of product and labour market power and wages and output prices. This is not difficult, given the extensive literature on these issues. In terms of results, we show that the labour share of a "representative", average firm, is roughly 10 percentage points lower than the one actually observed in the data. In other words, firm heterogeneity increases the *level* of the aggregate labour share, in our data. Empirically as well as theoretically therefore, "heterogeneity matters". Second, we find that this wedge between the aggregate labour share and the average labour share is due largely to two dimensions of heterogeneity, namely TFP and, to a lower extent, labour market power. Heterogeneity in wages and product market power has little effect on this wedge. This is likely to reflect the fact that TFP and labour market power are more difficult to arbitrate across firms (e.g. due to organisational knowledge specific to the firm or geographical amenities, respectively). Third, we find that the aforementioned wedge between average and aggregate labour share remained fairly constant over time.The fall in the aggregate labour share observed over the period is therefore attributable by and large to changes in the characteristics of the average firm: in particular, to an increased pay-productivity gap, and to a lesser extent to increased market power. Interestingly, and differently from a common narrative hightlighting the role of "superstar firms", we find that changes in firm heterogeneity overall contributed to slightly *reduce* the fall in the aggregate labour share.

The remaining of the paper is structured as follows. Section 2 presents a brief review of the literature. Section 3 presents a simple model of firm optimisation, where firms have a CES technology with constant returns to scale, with imperfect competition both in the product and the labour markets, and derives our theoretical results and main decomposition formula. Section 4 describes the data and our empirical strategy, while Section 5 describes our main findings. Section 6 summarises and concludes.

2 Literature

2.1 The determinants of the aggregate labour share

Abstracting from measurement issues, we can divide the existing empirical work on the labour share in three categories: (i) studies based on aggregate data, at the national, regional or industry level, where the outcome variable is the aggregate labour share; (ii) studies based on micro data, either at the firm or at the establishment level, where the outcome variable is the individual-level labour share, and (iii) studies where the aggregate labour share is analysed as an average of the individual-level labour shares, for instance by means of a shift-share analysis.[1]

Analyses of the aggregate labour share typically consider only aggregate variables – that is, totals or averages – as controls.[2] This might sound natural but as our contribution shows, the whole distribution of these variables matters. The aggregate approach is sometimes either explicitly or implicitly justified with reference to an aggregate production function at the country/industry level. Our criticism simply reflects the fact that aggregate production functions typically do not exist (see below). Thus, macro studies generally fall short of establishing causal relationships in the data. Only when a theoretical model of firm behaviour predicts an unambiguous association between variables at the micro level, irrespective of other firm's characteristics, this association can be safely tested using aggregate data. This is for instance the case of Azmat et al. (2012), where they test the prediction that privatization is associated with a lower labour share on cross-country industry-level OECD data.

Studies that analyse composition effects (e.g. Valentinyi and Herrendorf, 2008; Abdih and Danninger, 2017) can offer valuable insights about the dynamics of the aggregate labour share, but are often more descriptive in nature as they typically do not dig into what caused the shift in the composition of firms, nor they model firms' behaviour. For instance, Hopenhayn et al. (2018) point to the decline in population growth, which reduces firm entry rates and shifts the distribution of firms

[1]The imperfect measurement of capital and labour income is recognised as a potential confounder, although its importance is debated. Koh et al. (2018) suggest that the observed decline in the labour share is mostly explained by unaccounted intangible investments in R&D. This is however contrasting with Corrado et al. (2009), who show that a proper measurement of intangibles would point to a stronger increase in labour productivity, with a correspondingly stronger decline in the labour share. Elsby et al. (2013) refine the treatment of self-employment income and show that this slightly reduces the decline in the labour share, in the US. Karabarbounis and Neiman (2014) consider the case of using higher depreciation to account for less durable capital such as computers and software, but find similar trends in gross and net labour share, worldwide.

[2]A comprehensive review of the papers adopting the macro approach is outside the scope of this work. It is however interesting to consider what this literature has identified as the main determinants of the fall in the aggregate labour share. Zeira (1998); Acemoglu (2003); Brynjolfsson and McAfee (2014) and Acemoglu and Restrepo (2018) point to (capital augmenting) technological change as a main driver, with Autor and Salomons (2018) and Eden and Gaggl (2018) focusing in particular on the role of automation. Piketty (2014); Piketty and Zucman (2014) and Glover and Short (2019) stress the role of capital accumulation. Harrison (2002); Bentolila and Saint-Paul (2003); Acemoglu (2003) and Karabarbounis and Neiman (2014) point to the decline in the price of capital relative to labour, while Hergovich and Merz (2018) and León-Ledesma and Satchi (2018) stress increased factor substitutability between capital and labour, and Grossman et al. (2018) bring the attention to a slowdown in productivity. González and Trivín (2017) point to increased asset prices, which lower investment – an explanation which is however at odds with the emphasis on capital deepening as a driver of the decline in the labour share (see above). Harrison (2002); Lee and Jayadev (2005); Guscina (2006); Daudey and García-Peñalosa (2007); Jayadev (2007); IMF (2007) and Elsby et al. (2013), among others, focus on globalisation and its implications in terms of the balance of power between capital and labour. Deregulation of product and labour markets, including privatisation policies, de-unionisation and the decline of employment-protection policies, is emphasised by Bassanini and Duval (2006); Annett (2006); Bental and Demougin (2010); Stiglitz (2012); Barkai (2016); Ciminelli et al. (2018); Dizon and Lim (2018); Dimova (2019) and Pak and Schwellnus (2019), among others. Blanchard and Giavazzi (2003), in an influential theoretical work, take into account the general equilibrium effects of deregulation policies and show that workers lose from product market deregulation but gain as consumers, and they eventually gain even from labour market deregulation, although only in long run, due to lower unemployment. Weil (2017) generically refer to the disempowerment of labour and the consequential reduction in the labour share, linked to practices such subcontracting, franchising, and a global supply chain, as *financialisation*. Furceri et al. (2018) point to financial globalisation and the liberalisation of international capital flows. Cantore et al. (2018) find an empirical relation between the decrease in the labour share and monetary policy *easing*, pointing to a new theoretical puzzle as this is inconsistent with a broad range of standard models. Rognlie (2015) and Gutierrez Gallardo (2017) point to the increase in the cost of housing and the related increase in the value of capital and in real estate profits.

Our framework considers most of those determinants, in terms of their effects on firm-level variables. Technological change, given an elasticity of substitution between capital and labour smaller than 1, has a negative impact on the labour share in our model, as well as between-sectors and within-sectors shifts to relatively more capital-intensive technologies, changes in the relative price of capital with respect to labour (as brought about by a decreasing bargaining power of workers connected to globalisation and/or a weakening of labour market institutions, e.g. unions, collective bargaining and other industrial relations, minimum wages, employment protection and unemployment benefits), increased product and labour market power of firms. Only capital deepening *per se*, that is an increase in the K/L ratio at given technologies, has no effects on the labour share, in our model.

towards older firms with a lower labour share, in the US, but they do not offer an explanation of why older firms have a lower labour share.[3]

Microeconometric studies are more causal in nature, but they generally do not derive implications for the aggregate labour share, or make the explicit or implicit assumption that what is relevant at the micro level is also relevant at the macro level, again a sort of aggregate production function type of argument. Studies that follow this approach include Siegenthaler and Stucki (2015), who look at the determinants of the firm-level labour share on a panel of Swiss firms. They conclude that the most important factor in driving down the labour share is the diffusion of information and communication technologies (ICT). The aggregate labour share however remained fairly constant due to slow technological progress and sectoral reallocation towards industries with above-average labour share. Perugini et al. (2017) find a negative effect of internationalisation (in terms of export propensity, offshoring and foreign direct investment) on the firm-level labour share, using balance sheet data for six EU countries. De Loecker and Eeckhout (2018) document a rise in markups in the US from around 20% in 1980 to around 60% in the mid 2010s, well exceeding the rise in overhead costs. They link this to the decline in the labour share, mostly to the benefit of profits. An increase in product market power, coupled with a decline in rent sharing with employees, is also foud in the UK (Bell et al., 2018).

An interesting paper combining a microeconometric analysis with a decomposition exercise is Böckerman and Maliranta (2012). They disaggregate changes in the labour share into changes in wages and changes in labour productivity, then distinguish between changes in wages and labour productivity that occur in continuing plants, changes in the market shares of continuining plants, and the levels of wages and labour productivity in entering and exiting firms. They finally relate these components to other plant-level variables, showing on Finnish manufacturing data that international trade is a major driver of those shifts.

We also analyse the aggregate labour share as a weighted average of individual-level labour shares, but we characterise the behaviour of individual firms and map it directly into the aggregate outcome. In our analysis it is the joint distribution of firm-level characteristics that matters for the aggregate labour share. We can therefore explain the dynamics of the aggregate labour share in terms of changes in the moments of this joint distribution. Our approach thus provides a bridge between the different perspective considered above.

Some theoretical models of firm behaviour take firm heterogeneity into account and have clearcut implications in terms of the aggregate labour share. This is the case of the theory of superstar firms (Autor et al., 2017a,b), where the driving force is an increasing "winner takes most" feature of contemporary markets, and of the model proposed by Aghion et al. (2019), where the driving force is a reduction in the cost of spanning multiple markets, leading to the selection of more productive firms characterised by a lower labour share, with an initial outburst of growth, followed by a low-innovation, low-growth regime. Consistently with the superstar firms narrative, Kehrig and Vincent (2017) find that the labour share has increased in most plants, but the reallocation of production towards hyper-productive, low labour share plants has caused the aggregate labour share to decline, in the US.

With respect to those papers, we follow a static, partial equilibrium approach; on the other hand,

[3] Using macro data, Short and Glover (2017) point to a decreased ability of older workers to extract their marginal product of labour as a wage.

we are able to fully characterise and quantify the impact of the different dimensions of heterogeneity on the aggregate labour share, offering a comprehensive and novel decomposition method.

A paper closely related to our work is Mertens (2019). He develops a parsimonious theory of firm behaviour where three factors can affect the firm-level labour share: product market power, labour market power, and the output elasticity of labour, reflecting the importance of labour in production. Using German firm-level data, Mertens shows that his framework accounts for 94% of the observed variation in the labour share in manufacturing, between 1995 and 2014. Product and labour market power however account for only 30% of this explained change, leaving the remaining 70% to generic changes in production processes. Our theoretical framework is slightly more elaborated than his, allowing us to identify more determinants, at the cost of using a specific functional form for the production function, albeit quite general. In particular, we remain agnostic about the nature of imperfect competition in both the product and the labour market and characterise it following a reduced-form approach where a negatively sloped product demand curve and a positively sloped labour supply curve introduce a wedge between marginal costs and marginal revenues in the optimal firms' plans. This wedge is assumed to be constant irrespective of what other firms do.

2.2 Firm heterogeneity

Our focus is on between-firm heterogeneity, as opposed to within-firm heterogeneity. The literature has long recognised that some firms are more productive than others (e.g. Bernard et al., 2003; Foster et al., 2008; Hsieh and Klenow, 2009; Syverson, 2011; Aiello and Ricotta, 2015; Bartelsman and Wolf, 2017) and pay higher wages, for equally skilled workers (e.g. Dunne et al., 2004; Abowd et al., 1999; Goux and Maurin, 1999; Abowd et al., 2002; Gruetter and Lalive, 2009; Holzer et al., 2011) – see also the comprehensive review of the evidence in Van Reenen (2018).

More recently, a new generation of papers has shown that these between-firm wage differentials account for most of the overall wage inequality, and they have generally widened over time – see Barth et al. (2016) and Song et al. (2018) for the US; Faggio et al. (2010) for the UK; Card et al. (2013) for Germany; Håkanson et al. (2015) for Sweden; Card et al. (2016) for Portugal; and Elhanan Helpman and Oleg Itskhoki and Marc-Andreas Muendler and Stephen J. Redding (2017) and Alvarez et al. (2018) for Brazil.[4] Evidence across OECD countries show that the productivity gap between firms at the technology frontier and the rest has risen since the mid-2000s (Andrews et al., 2016), as well as the prevalence of and the resources sunk in "zombie" firms (McGowan et al., 2017); between-firm wage dispersion has also increased substantially, with most of the between-firm wage variance being driven by differences in pay across firms within sectors rather than by differences in average wages across sectors (Berlingieri et al., 2017). Also, Hartman-Glaser et al. (2019) make the point that as volatility of productivity has increased, the owners of the firm require an increased risk premium. Moreover, uncertainty about future productivity levels delays exit and increases the importance of mega-firms. Both factors lower the labour share.

The availability of firm-level data has allowed researchers to assess the dispersion of product market power – see among others De Loecker and Warzynski (2012) for the US, Tamminen and Chang

[4]Both Card et al. (2013) and Song et al. (2018) show that the increase in between-firm wage heterogeneity is mostly due to increased worker sorting / assortative worker-firm matching (high-wage workers becoming increasingly likely to work in high-wage firms) and segregation / assortative worker-worker matching (high-wage workers becoming increasingly likely to work with each other), with little role for an increase of firm fixed effect.

(2013) for Finland, Forlani et al. (2016) for Belgium. The general agreement is that product market power has increased *and* has become more dispersed among firms (Epifani and Gancia, 2011; De Loecker and Eeckhout, 2018; De Loecker et al., 2018).[5] In addition to a marked increase in the average markup (see previous section), De Loecker and Eeckhout (2018) also document a substantial increase in its dispersion, with the median markup remaining roughly constant, and the 90th percentile increasing from 1.5 to 2.3. They relate the decrease in the aggregate labour share to the increase in average market power; however, they do not make any connection with its increasing dispersion.

A smaller number of papers look at labour market power, and they also find significant heterogeneity. Ransom and Oaxaca (2010) infer the elasticity of labour supply at the firm level – a measure of monopsonistic power – from the elasticity of the quit rates with respect to wages, and find for the US elasticities between 2.4 and 3 for men and between 1.5 and 2.5 for women. (Hirsch et al., 2010) for Germany and Weber (2015) for the US compute labour supply elasticities directly, using large linked employer–employee datasets, and also find considerable variation, between 1.9 and 3.7 for Germany and lognormally distributed with an average of 1.08 for the US.

Other papers look jointly at product and labour market power. Dobbelaere and Mairesse (2013) estimate production functions for different French manufacturing industries and compute firm specific price-cost markups and elasticities of labour supply as a wedge between the factor elasticities and their corresponding shares in revenues. They find considerable dispersion in both parameters. Félix and Portugal (2017) follow a similar approach for Portugal, while also decomposing the impact of the estimated labour supply elasticity on wages within an efficient bargaining setting. They estimate an average price-cost markup of 1.2, with a standard deviation of .3, and an average wage elasticity of labour supply of 3.3, with a standard deviation of 4.2. They also show that heterogeneity in monopsonistic power affects heterogeneity of wages across firms, with a one unit increase in a firm's labour supply elasticity being associated with an increase in earnings between 5 and 16 percent. Card et al. (2016) also link wage heterogeneity to labour market power, in terms of a random utility model of worker preferences which leads to firm-specific labour supply elasticities.[6]

Finally, the substantial heterogeneity in relative price variation, as measured typically by sectoral inflation and inflation persistence, is well documented (see, among others Blinder et al., 1998; Bils and Klenow, 2004; Lünnemann and Mathä, 2004; Bilke, 2005; Clark, 2006; Altissimo et al., 2009; Boivin et al., 2009; Wolman, 2011; Duarte and Restuccia, 2016; Kato and Okuda, 2017).

While remaining agnostic about the causes of between-firm heterogeneity, we look at the evidence of increased dispersion in wages, productivity, product and labour market power and relative inflation, and relate it to the observed changes in the labour share, for the UK manufacturing sector. As already anticipated, we find that between-firm heterogeneity is an important determinant of the aggregate labour share, particularly heterogeneity in total factor productivity and labour market power. However, its contribution has remained fairly constant over time, and therefore cannot explain the observed decline in the aggregate labour share.

[5] Fernández et al. (2015) show that in Spain heterogeneity in markups has increased significantly in some sectors (professional services, telecommunications, accommodation and food, utilities) after the Great Recession, while it has decreased in others (manufacturing).

[6] Another paper is Hornstein et al. (2011), which in the context of a search model obtain smaller wage dispersion.

3 Model

A well-studied though often neglected result from the neoclassical theory of production is that when input and output prices and quantities are heterogeneous across firms, or when firms differ in terms of fundamental factors like total factor productivity, aggregation of firms' technologies into a single production function is not possible (Green, 1964; Fisher, 1969; Zambelli, 2004; Felipe and McCombie, 2014). Thus, under firm heterogeneity the aggregate labour share cannot be computed with reference to an optimal production plan of a "representative firm", using aggregates of input and output prices and factors. Instead, it must be computed adding up labour costs and value added across firms. Here we use a simple neoclassical model of firm behaviour in order to characterise the relationship between the distribution of firms' characteristics and the aggregate labour share in the economy, in a partial equilibrium setting.

3.1 Setup

First, let us define the firm level labour share, upon which all the analysis is built. This is:

$$\lambda_i \equiv \frac{w_i L_i}{p_i Y_i} \tag{1}$$

where w_i are wages, L_i is the level of employment, p_i is output price, and Y_i is real value added, for a given firm i.[7]

The aggregate labour share, defined as aggregate labour costs over aggregate value added, can then be expressed as a weighted average of λ_i:

$$\lambda \equiv \frac{\sum_i w_i L_i}{\sum_i p_i Y_i} = \sum_i \lambda_i \delta_i \tag{2}$$

where $\delta_i = \dfrac{p_i Y_i}{\sum_i p_i Y_i}$ corresponds to the share of aggregate value added produced by firm i.

Our aim is to characterise λ in terms of firms' choices. Since the latter depends on λ_i, which in turns depends on $\frac{Y_i}{L_i}$, we need assumptions about technology, market structure and firm's behaviour which enables us to find the optimal $\frac{Y_i}{L_i}$ ratio for firms. Our starting point is a *value added* production function (i.e. a mathematical relation between capital, labour and value added).[8] In particular, we

[7]In practice, workers are heterogeneous (e.g. in terms of skills, type of contract, or hours worked). However, most datasets, including ours, only report the total number of employees. Therefore, because of necessity rather than desire, the theory assumes workers are homogeneous within the firm. In the empirical analysis, $w_i L_i$ is taken to be the reported total labour costs, which means w_i is defined as the average wage per worker.

[8]The existence of a value added production function hinges on some assumption about the underlying gross output production function (which relates capital, labour and intermediate inputs to gross output), as Bruno (1978) demonstrated. In particular, the elasticity of substitution between intermediate inputs and the rest of inputs (in our case, capital and labour) must be either zero (i.e. a Leontief) or infinity (i.e. a linear production function). Alternatively, a value added production function is well defined when the relative price of intermediate inputs to output is constant. Unfortunately, because of the multiple non-linearities in our model, we were unable to test the elasticity of substitution of the gross output production function linked to our model (a nested CES). Regarding the price condition, we do not observe the price of intermediary inputs, and so cannot test this assumption either. For further details on the gross output production function associated with our model, see Appendix A.

assume a CES production function:

$$Y_i = A_i \left(\alpha L_i^{\rho} + (1-\alpha) K_i^{\rho}\right)^{\frac{1}{\rho}} \tag{3}$$

where $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution between capital and labour (hence: $\rho < 1$). Notice firms have the same technology in terms of elasticities (ρ and α), but they might have heterogeneous total factor productivity (TFP), A_i. A justification for the assumptions in equation (3) is presented later, once the main result is obtained.

We assume firms have a certain degree of *monopolistic power* in the pricing of the final good. Importantly, the degree of market power might be heterogeneous across firms. Formally, firms face an inverse demand function for their good given by $p_i(Y_i) = f(\eta_i^Y, \Theta_i^Y)$, where η_i^Y corresponds to the own-price elasticity of output demand, and Θ_i^Y refers to arbitrary characteristics of the product Y_i, idiosyncratic to firm i, which are valuable to consumers. Similarly, we assume firms have some degree of *monopsony power* in the labour market, which could also be heterogeneous across firms (for example, because of some non-pecuniary location effects valued by workers). Formally, firms face an inverse labour supply function given by $w_i(L_i) = g(\eta_i^L, \Theta_i^L)$, where η_i^L is the own-price elasticity of the labour supply, and Θ_i^L represents idiosyncratic firm characteristics, valuable for workers. The role of Θ_i^Y and Θ_i^L is to permit heterogeneous prices and wages even when firms have the same level of market power, or when they have no market power at all. The latter is not unknown to the literature, both in the case of firms with an homogeneous final good (Dahlby and West, 1986; Hosken and Reiffen, 2004) and homogeneous labour (Rosen, 1987; Hamermesh, 1999).

With the above assumptions in place, the profit function of the firm is $\Pi_i(L_i, K_i) = p_i(Y_i)Y_i - w_i(L_i)L_i - r_i K_i$. The first order condition with respect to labour is given by:

$$\frac{\partial Y_i}{\partial L_i} \equiv \alpha A_i^{\rho}(Y_i)^{1-\rho}(L_i)^{\rho-1} = \left(\frac{w_i}{p_i}\right)\frac{\chi_i^L}{\chi_i^Y} \tag{4}$$

where $\chi_i^L = 1 + \frac{1}{\eta_i^L}$ and $\chi_i^Y = 1 + \frac{1}{\eta_i^Y}$. The term $\frac{\chi_i^L}{\chi_i^Y}$ represents the wedge between the real wage and the marginal product of labour when markets are not perfectly competitive. The higher labour and/or product market power are, the higher this ratio is. Conversely, in the case of perfectly competitive product and labour markets (i.e. $\eta_i^L = \infty$ and $\eta_i^Y = -\infty$), $\frac{\chi_i^L}{\chi_i^Y} = 1$. Note that profit maximisation requires $|\eta_i^Y| > 1$, so that χ_i^Y is always positive.

From equation (4) we obtain the optimal $\frac{L_i}{Y_i}$ as a function of the firm characteristics:

$$\frac{L_i}{Y_i} = \left(\frac{\alpha w_i \chi_i^Y}{p_i \chi_i^L}\right)^{\frac{1}{1-\rho}} A_i^{\frac{\rho}{1-\rho}} \tag{5}$$

This is then replaced into the formula for the firm level labour share (equation 1), leading to:

$$\lambda_i = \left(\frac{\alpha \chi_i^Y}{\chi_i^L}\right)^{\frac{1}{1-\rho}} \left(\frac{A_i p_i}{w_i}\right)^{\frac{\rho}{1-\rho}} \tag{6}$$

A few insights are worth pointing out here. First, λ_i does not explicitly depend on the size of the firm (either in terms of K_i or L_i). This property emanates from the fact that the CES function is homothetic. This means it has a linear expansion path, which is to say, optimal K_i/L_i and L_i/Y_i ratios are constant. However, a correlation between λ_i and firm size might be observed in practice, provided the other determinants of λ_i (TFP, market power, wages or prices) do depend on the size of the firm. In effect, there is evidence of such correlation, not the least because bigger firms tend to be more productive and have more market power (e.g. Autor et al., 2017a; Schwellnus et al., 2018). Additionally, in our framework, wages and prices do depend on L_i whenever there is imperfect competition.

Second, the effect on the labour share of all parameters but market power depends on the sign of ρ. For instance, a *ceteris paribus* increase in TFP increases (decreases) λ_i if ρ is positive (negative). Meanwhile, both higher monopoly power (i.e. a decrease in χ_i^Y) and higher monopsony power (i.e. an increase in χ_i^L) lower λ_i. In the limiting case of $\rho = 0$ (Cobb-Douglas), only market power affects λ_i.[9]

Third, there is a close relationship between the pay-productivity disconnect (with productivity understood as TFP) and the labour share. In particular, the latter changes whenever a given increase in TFP does not translate into a similar increase in the real wage (i.e when $\frac{A_i p_i}{w_i}$ falls). Again, the final effect depends on ρ. Further analysis of the effect of individual firm level variables on the firm level and aggregate labour share is presented in Appendix C, where the relationship with the theory of superstar firms is also briefly discussed.

Finally, notice we do not model firms' choice of capital, as it is not needed in our framework. This does not mean capital is necessarily fixed. Rather, we remain agnostic about the precise capital accumulation mechanism (for instance, in addition to the first order optimality condition for capital, firms might take into account adjustment costs to the capital stock).

3.2 Heterogeneity and the aggregate labour share

Ultimately, we are interested in the effects of firm heterogeneity on the aggregate labour share. Replacing the individual firm labour share λ_i into equation (2) leads to the following expression for the aggregate labour share:

$$\lambda = \sum_i \left(\frac{\alpha \chi_i^Y}{\chi_i^L} \right)^{\frac{1}{1-\rho}} \left(\frac{A_i p_i}{w_i} \right)^{\frac{\rho}{1-\rho}} \delta_i \tag{7}$$

We measure firm heterogeneity with respect to an hypothetical "average" firm. More specifically, for given relative weights $\{\omega_i\}$ we define $\bar{A} = \sum_i \omega_i A_i$, $\bar{w} = \sum_i \omega_i w_i$, $\bar{p} = \sum_i \omega_i p_i$, $\bar{\chi}^Y = \sum_i \omega_i \chi_i^Y$, $\bar{\chi}^L = \sum_i \omega_i \chi_i^L$. This is, we compute a weighted average of all heterogeneous parameters in the model, which then define the parameters of the benchmark firm.

It is natural to weight variables by some measure of firm size. Whilst employment might seem a reasonable option, there is often significant capital-labour variability at a similar employment level (something which is true in our data too). Since a given level of value added can be achieved

[9] In particular, in the Cobb-Douglas case the labour share is equal to $\frac{\alpha \chi_i^Y}{\chi_i^L}$. Perfect competition yields the familiar result that $\lambda_i = \alpha$.

with different capital and labour combinations, we consider value added a more suited weighting variable. In effect, value added (or sales) is also one often used in the literature to aggregate firms (e.g. De Loecker and Eeckhout, 2018; De Loecker et al., 2018, in the context of mark-ups). Notice however that the method itself is agnostic regarding the weights chosen. What is needed is that heterogeneity is quantified with respect to a given counterfactual, just as the variance is computed with respect to a mean. As long as there is heterogeneity in a given dimension (except capital alone, as Proposition 1 below states), such decomposition is **always possible**.

Having defined weighted averages for every variable we can then re-write the aggregate LS as:

$$\lambda = \lambda^{HOM} \sum_i \left(\frac{\chi_i^Y}{\bar{\chi}^Y}\right)^{\frac{1}{1-\rho}} \left(\frac{\bar{\chi}^L}{\chi_i^L}\right)^{\frac{1}{1-\rho}} \left(\frac{A_i}{\bar{A}}\right)^{\frac{\rho}{1-\rho}} \left(\frac{\bar{w}}{w_i}\right)^{\frac{\rho}{1-\rho}} \left(\frac{p_i}{\bar{p}}\right)^{\frac{\rho}{1-\rho}} \delta_i \tag{8}$$

where λ^{HOM} is the labour share of the counterfactual firm, and defined as:

$$\lambda^{HOM} = \left(\frac{\bar{\alpha}\bar{\chi}^Y}{\bar{\chi}^L}\right)^{\frac{1}{1-\rho}} \left(\frac{\bar{A}\bar{p}}{\bar{w}}\right)^{\frac{\rho}{1-\rho}} \tag{9}$$

Equation (8) is our decomposition formula, which shows that *any* form of heterogeneity affects the aggregate labour share, with the exception of capital *alone*. If firms differ only with respect to capital, their labour shares are identical (see equation 6).[10] The proof can be trivially seen in equation (2), once we assume $\lambda_i = \lambda^{HOM}$.

The following proposition summarises the CES result:

Proposition 1. *Assume firms have identical CES technologies (i.e. α and ρ are the same across firms), and $\rho \neq 0$ (i.e. technology is not Cobb-Douglas). Then, it is true that:*

(i) heterogeneity in wages, price dynamics, TFP or market power affects the aggregate labour share (directly and through δ_i);

(ii) heterogeneity in capital affects the aggregate labour share (through δ_i) only if other forms of heterogeneity are also present.

Notice the decomposition formula is purely descriptive of the optimal production plans of the different firms, reflecting the partial equilibrium nature of the model. Yet, provided we can produce an estimate for each element in equation (8), this is sufficient for our purposes. The drawback of this partial equilibrium approach is, of course, that we cannot provide a deeper understanding of why heterogeneity in wages and prices occurs in the first place.

This result can be contrasted with the Cobb-Douglas case, where the aggregate LS is:

$$\lambda = \frac{\alpha\bar{\chi}^Y}{\bar{\chi}^L} \sum_i \left(\frac{\chi_i^Y}{\bar{\chi}^Y}\right) \left(\frac{\bar{\chi}^L}{\chi_i^L}\right) \delta_i \tag{8'}$$

[10] Incidentally, this is exactly the case where an aggregate production function exists, namely when firms only differ in their size. Because they have identical K/L ratios, it is possible to mechanically redistribute factor of productions among them without altering factor prices (abstracting from competition considerations). Equivalently, it is possible to combine all firms into one big firm; the production function of this firm "becomes" the aggregate production function of the economy.

This highlights that for firm heterogeneity to affect the aggregate labour share if the technology is Cobb-Douglas, there must be *heterogeneous* imperfect competition. With perfect competition (where an exact aggregate production function exists), $\lambda = \alpha$, a well-known property of a Cobb-Douglas production function. The following corollary summarises the result:

Corollary 1. *Assume firms have identical Cobb-Douglas technologies (i.e. α is the same). If market power is homogeneous across firms (including the limit case of perfect competition), then firm heterogeneity is irrelevant for the aggregate labour share: the labour share is identical across firms (with perfect competition, it is equal to α). On the other hand, with heterogeneous market power, firm heterogeneity of any dimension affects the aggregate labour share. In particular, heterogeneity in capital, wages, prices and TFP affect the labour share indirectly through δ_i.*

The above result is very simple but makes an important point, given the extensive use of Cobb-Douglas production functions with perfect competition in the literature: even when firms are heterogeneous along many dimensions (including TFP), and an aggregate production function hence does not exist, in competitive markets, the aggregate labour share only depends on technology.

On the other hand, a CES enables a richer set of determinants for the labour share, hence our choice. However, it might seem odd that in our CES analysis we assume the production function to be homogeneous across firms (i.e. common parameters ρ and α). This is necessary as allowing heterogeneity in ρ impedes decomposition, and allowing heterogeneity in α greatly complicates estimation (see Appendix A for details).

Our last remark is on the choice of a value added production function. Using a gross output production function does not yield a decomposable formula for the aggregate labour share, except when the conditions suggested by Bruno (1978) are fulfilled. This is, when a value added production function exists, as assumed here. Appendix A provides further insights on these points.

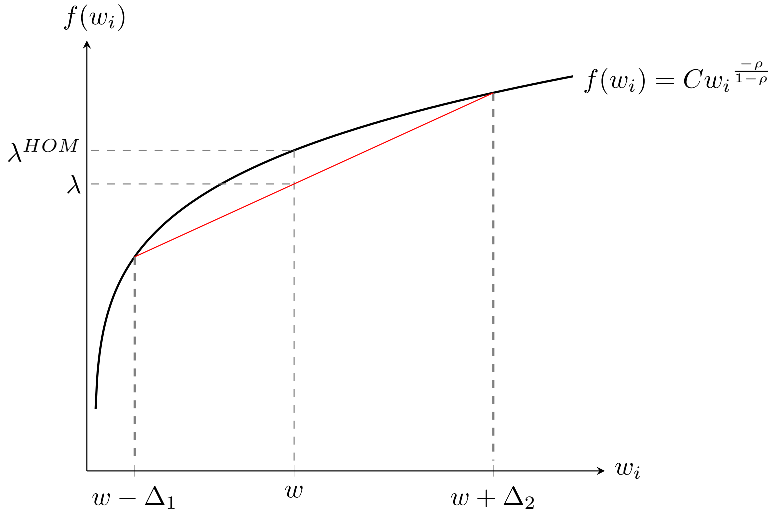
3.3 Exercise: a mean-preserving increase in the dispersion of one variable

To better illustrate the implications of equation (8), we now consider the most simple scenario possible, namely an economy where only one dimension of heterogeneity is present. First, we focus on wages. Then, the result is extended to other forms of heterogeneity.

For simplicity, we consider only two (types of) firms $i = \{1, 2\}$. We start from a situation where the two firms are identical, with wage w. Since the LS does not depend on the firm's size, both firms (and the aggregate economy) have the same labour share, λ. Now, consider an exogenous *value added-weighted* mean-preserving spread in wages. This is a change in wages such that their weighted average (using value added as weights) yields the same original average, w. Mathematically, for new wages $w_1 = w + \Delta_1$ and $w_2 = w - \Delta_2$, this is true if $\Delta_1 = \Delta_2 \frac{\delta_2}{\delta_1}$, where δ_i represents the firm's share of value added in the economy with this new set of wages.[11]

[11]One might suggest here that the aggregate demand for labour in the two scenarios has not been restricted to be the same. However, the labour supply has not been restricted either (in fact, nothing has been said about the source of the change in wages). Being our model a partial equilibrium one, we assume any resource constraints are fulfilled. In other words, wages represent an equilibrium.

Figure 1: A VA-weighted mean-preserving spread of wages, $\rho < 0$.



In this setting, each firm's LS is (equation 8):

$$\lambda_i = Cw_i^{\frac{-\rho}{1-\rho}} \tag{10}$$

where $C = \left(\frac{\alpha\chi^Y}{\chi^L}\right)^{\frac{1}{1-\rho}} (Ap)^{\frac{\rho}{1-\rho}}$ (identical across firms).

This function is strictly concave and increasing in wages for $\rho < 0$, and strictly convex and decreasing in wages for $\rho > 0$. The case of $\rho < 0$ is depicted in figure 1. The aggregate LS is a weighted mean of the individual LS, with weights equal to δ_i (equation 2). Jensen's inequality ensures that the aggregate LS is lower the bigger the dispersion in wages, Δ. In other words, starting from a situation of firm homogeneity, an increase in the dispersion of wages such that the counterfactual firm does not change leads to a fall in the aggregate LS, if the elasticity of substitution between capital and labour is lower than one. Again, notice the limiting case of the Cobb-Douglas, where dispersion in wages *alone* does not change the aggregate LS, which is constant over w_i.

The reason why the aggregate LS falls in the example above is nothing else than Jensen's inequality, given the shape of the LS function. But why does the LS function depend on ρ? To understand this, let's first look at the first derivative, and explain why the LS is increasing in wages for $\rho < 0$, and decreasing for $\rho > 0$. Consider first the case of $\rho < 0$, where there is relatively low degree of substitution between capital and labour. Starting from a given wage w, an increase in such wage by Δ produces a fall in employment and in value added. Yet, because of low substitution between K and L, such fall in output is relatively significant. Thus, L/Y falls (because of CRS), but not so much. In fact, precisely because of this low substitution, the firm labour share actually increases (recall the labour share is $\frac{w}{p}\frac{L}{Y}$). This is, the "price effect" outweighs the "quantity effect". Conversely, if $\rho > 0$ (high substitution), L/Y falls considerably more, in which case the quantity effect dominates and the labour share falls. In the Cobb-Douglas case, these two effects cancel out.

Let's now look at the second derivative, and explain why the LS is concave in wages for $\rho < 0$, and convex for $\rho > 0$. Consider again the case of $\rho < 0$. As we said, an increase in the wage from w by Δ lowers L/Y by relatively little. As we further increase wages by Δ, L/Y falls again, but because of decreasing marginal product of labour, the overall change in Y gets smaller, and therefore L/Y falls (again because of CRS) in an increasing fashion, as employment just cannot raise output fast enough. In turn, the price effect of higher w, which always outweighs the quantity effect for $\rho < 0$, is less capable of rising the labour share. This effect plateaus in the limit (i.e. as $w \xrightarrow{\infty}$); hence its concavity. The argument is the same for the case of $\rho > 0$. Recall that when $\rho > 0$ the LS is decreasing with wages, as the quantity effect outweighs the price effect. Yet, because of decreasing marginal product of labour, such outweighing looses force with w_i and it plateaus in the limit; hence its convexity.

The above example of wage heterogeneity also holds in the case of an unweighted mean-preserving spread of wages (i.e. where $\Delta_1 = \Delta_2 = \Delta$). The only difference is that the counterfactual wage that produces an equivalent level to that of the new (heterogeneous) aggregate LS is no longer w but $w - \Delta(\delta_2 - \delta_1)$. This level is lower (higher) than w for $\rho < 0$ ($\rho > 0$), only strengthening the result. Furthermore, it can be shown that the same conclusion arises for changes from an already heterogeneous economy, under plausible circumstances.

A similar analysis holds for other firm-level variables. In the end, all depends on the shape of the equivalent of function $f(\cdot)$ in Figure 1. For most dimensions, this shape depends on the value of ρ. In particular, product market power exhibits the same behaviour than wages. Namely, for $\rho < 0$ ($\rho > 0$), an increase in product market power heterogeneity lowers (raises) the aggregate LS. The relationship is the opposite for TFP; for $\rho < 0$ ($\rho > 0$), an increase in the dispersion of productivity leads to an increase (decrease) in the aggregate LS. This contrasting relationship for wages and TFP makes sense. Recall that the firm level LS depends on the pay-productivity disconnect. So if wages and productivity change in tandem, the effect on the firm LS is muted. This must be reflected also in the aggregate LS. Note the relationship with the theory of superstar firms (Autor et al., 2017b). That theory predicts that because some firms become much more productive thanks to new technologies, the aggregate labour share falls. Hence the theory says both that (i) the average productivity goes up, and (ii) the dispersion in productivity goes up. We show that these two things have different implications with respect to the aggregate labour share. An increase in the average productivity pushes the aggregate labour share down (see equation 6), while the corresponding increase in its dispersion reduces this effect.

Finally, labour market power is an exceptional case because the function $f(\chi_i^L)$ is strictly convex (and decreasing) for every ρ (even for the Cobb-Douglas case of $\rho = 0$; see footnote 9). This means that an increase in the dispersion of labour market power *always* raises the aggregate LS. The intuition of this particular case is also evident. Recall that higher χ_i^L means more monopsonistic power by the firm. Since the labour share represents the proportion of firm's value accruing to workers, it is reasonable to expect that this proportion falls with χ_i^L, regardless of the degree of complementarity between capital and labour. This is why $f(\chi_i^L)$ is strictly convex and decreasing for every ρ. Conversely, product market power affects directly the "size of the pie" (value added), and thus its final effect on the labour share does depend on the degree of complementarity between capital and labour.[12]

[12] It must be noted here that a *ceteris paribus* change in market power is implausible. In our particular setting, this would require that Θ^L and Θ^Y exactly compensate the changes in η^L and η^Y, respectively, in order to keep wages and prices identical across firms.

3.4 Distributional characterisation

Proposition 1 is very general. In particular, it does not quantify how heterogeneity affects the aggregate labour share: the summation term in equation (8) is obscure enough for this to be seen. In order to shed more light on the issue, we approximate each of the fractions inside the summation term in equation (8) by means of a second-order Taylor expansion around the respective weighted average. For each $z = \{\chi^Y, \chi^L, A, w, p\}$, this approximation is:

$$\left(\frac{z_i}{\bar{z}}\right)^{\phi} \approx 1 + \phi\left(\frac{\Delta_{z,i}}{\bar{z}}\right) + \frac{\phi(\phi-1)}{2}\left(\frac{\Delta_{z,i}}{\bar{z}}\right)^2 \tag{11}$$

where $\bar{z}$ is the weighted mean of the respective variable, and $\Delta z_i = z_i - \bar{z}$ is the deviation from that mean. After dropping all interaction terms of order higher than two, equation (8) can be approximated by:[13]

$$\begin{aligned}
\lambda \approx \lambda^{HOM} \sum_i \delta_i \Bigg[& 1 + \frac{1}{1-\rho}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right) - \frac{1}{1-\rho}\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right) + \frac{\rho}{1-\rho}\left(\frac{\Delta A_i}{\bar{A}}\right) - \frac{\rho}{1-\rho}\left(\frac{\Delta w_i}{\bar{w}}\right) + \frac{\rho}{1-\rho}\left(\frac{\Delta p_i}{\bar{p}}\right) \\
& + \frac{\rho}{2(1-\rho)^2}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)^2 + \frac{2-\rho}{2(1-\rho)^2}\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right)^2 + \frac{\rho(2\rho-1)}{2(1-\rho)^2}\left(\frac{\Delta A_i}{\bar{A}}\right)^2 + \frac{\rho}{2(1-\rho)^2}\left(\frac{\Delta w_i}{\bar{w}}\right)^2 + \frac{\rho(2\rho-1)}{2(1-\rho)^2}\left(\frac{\Delta p_i}{\bar{p}}\right)^2 \\
& - \frac{1}{(1-\rho)^2}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right) + \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)\left(\frac{\Delta A_i}{\bar{A}}\right) - \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)\left(\frac{\Delta w_i}{\bar{w}}\right) \\
& + \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)\left(\frac{\Delta p_i}{\bar{p}}\right) - \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right)\left(\frac{\Delta A_i}{\bar{A}}\right) + \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right)\left(\frac{\Delta w_i}{\bar{w}}\right) \\
& - \frac{\rho}{(1-\rho)^2}\left(\frac{\Delta\chi_i^L}{\bar{\chi}^L}\right)\left(\frac{\Delta p_i}{\bar{p}}\right) - \frac{\rho^2}{(1-\rho)^2}\left(\frac{\Delta A_i}{\bar{A}}\right)\left(\frac{\Delta w_i}{\bar{w}}\right) + \frac{\rho^2}{(1-\rho)^2}\left(\frac{\Delta A_i}{\bar{A}}\right)\left(\frac{\Delta p_i}{\bar{p}}\right) \\
& - \frac{\rho^2}{(1-\rho)^2}\left(\frac{\Delta w_i}{\bar{w}}\right)\left(\frac{\Delta p_i}{\bar{p}}\right) \Bigg]
\end{aligned}$$

This can be simplified further. First, notice that when $\bar{z}$ is defined using value added as weights, $\sum_i \delta_i \Delta z_i = 0$. Thus, the first four terms in the parenthesis above (representing the weighted sum of all deviations from the weighted average) are zero. Second, notice that $\sum_i \delta_i (\Delta z_i)^2 = \mathrm{Var}(z)$ and $\sum_i \delta_i \Delta x_i \Delta z_i = \mathrm{Cov}(x, z)$, with both defined as value added weighted measures, and not in the standard, unweighted fashion. Then, we can restate our decomposition formula solely in terms of variances and covariances or, equivalently, in terms of correlations (r) and coefficient of variations (CV), both of which are dimensionless and scale invariant:

[13] For instance, terms like $\frac{\Delta A_i}{\bar{A}} \frac{\Delta\chi_i^Y}{\bar{\chi}^Y} \frac{\Delta\chi_i^L}{\bar{\chi}^L}$ and $\frac{\Delta A_i}{\bar{A}} \left(\frac{\Delta\chi_i^Y}{\bar{\chi}^Y}\right)^2$ are dropped. In our empirical analysis, this omitted residual is never above 5% of the total value.

$$
\begin{aligned}
\lambda \approx \lambda^{HOM} \Big[1 \\
&+ \frac{\rho}{2(1-\rho)^2}\mathrm{CV}^2(\chi^Y) + \frac{2-\rho}{2(1-\rho)^2}\mathrm{CV}^2(\chi^L) + \frac{\rho(2\rho-1)}{2(1-\rho)^2}\mathrm{CV}^2(A) + \frac{\rho}{2(1-\rho)^2}\mathrm{CV}^2(w) + \frac{\rho(2\rho-1)}{2(1-\rho)^2}\mathrm{CV}^2(p) \\
&- \frac{1}{(1-\rho)^2}\mathrm{r}(\chi^Y,\chi^L)\mathrm{CV}(\chi^Y)\mathrm{CV}(\chi^L) + \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^Y,A)\mathrm{CV}(\chi^Y)\mathrm{CV}(A) - \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^Y,w)\mathrm{CV}(\chi^Y)\mathrm{CV}(w) \\
&+ \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^Y,p)\mathrm{CV}(\chi^Y)\mathrm{CV}(p) - \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^L,A)\mathrm{CV}(\chi^L)\mathrm{CV}(A) + \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^L,w)\mathrm{CV}(\chi^L)\mathrm{CV}(w) \\
&- \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^L,p)\mathrm{CV}(\chi^L)\mathrm{CV}(p) - \frac{\rho^2}{(1-\rho)^2}\mathrm{r}(A,w)\mathrm{CV}(A)\mathrm{CV}(w) + \frac{\rho^2}{(1-\rho)^2}\mathrm{r}(A,p)\mathrm{CV}(A)\mathrm{CV}(p) \\
&- \frac{\rho^2}{(1-\rho)^2}\mathrm{r}(w,p)\mathrm{CV}(w)\mathrm{CV}(p) \Big] \qquad (12)
\end{aligned}
$$

This final equation reflects that it's the joint distribution of all the variables (in terms of averages, variances and covariances, or, as in our preferred terminology, coefficient of variations and correlations) that affects the aggregate labour share. On the one hand, there is the "direct effect" of dispersion (the coefficient of variation), which has the same direction as that of unidimensional heterogeneity explored in section 3.3, depending on the value of ρ (the exception is, again, monopsony power, with the same direction throughout). On the other hand, there is the "indirect effect" of dispersion acting through the correlation structure. The direction of this effect depends on the sign of the correlation, as well as ρ. Importantly, heterogeneity matters even if all variables are orthogonal to each other, i.e if all correlations are zero. Moreover, changes in the correlation structure also affects the labour share, without necessarily arising from changes in dispersion itself.

Our result is presented in the following proposition.

Proposition 2. *If firms have a CES technology with identical α and ρ and constant returns to scale, the aggregate labour share is approximately given by equation (12). Hence, the effect of firm heterogeneity on the labour share depends on the joint distribution of all firm-level variables, and for most of the variables, on ρ. The total effect can be separated in two components, a "direct effect" unaffected by the correlation structure among variables, and an "indirect effect" that depends on the correlation structure. The direct effect is such that, for the empirically relevant case (i.e. when the elasticity of substitution between capital and labour is smaller than 1, that is to say, $\rho < 0$), and other things being equal, an increase in the dispersion of productivity or monopsony power increases the aggregate labour share, while an increase in the dispersion of wages or product market power decreases it. The indirect effect holds that ceteris paribus changes in the correlation structure affect the labour share. In particular, for $\rho < 0$, an increase in the correlation between labour market power and TFP, or between product market power and wages increases the aggregate labour share, whereas an increase in the correlation between product market power and labour market power, product market power and TFP, labour market power and wages, or TFP and wages decreases it.*

The implications of Proposition 2 are somewhat difficult to visualise, as any change in firm level variables will typically trigger a change in the market shares, δ_i. This in turns will cause a change in λ^{HOM}, which refers to the hypothetical labour share of a weighted average firm. Moreover, coefficients of variation and correlations will change too (since they are weighted by δ_i). Hence, the *ceteris paribus* clause typically won't hold in simple thought experiments.

However, the above approximation allows us to generalise the unidimensional heterogeneity exercise presented in section 3.3 beyond two (types of) firms, to the case of many heterogeneous firms. In particular, in the case of nominal wage heterogeneity only, equation (12) simplifies to:

$$\lambda \approx \lambda^{HOM}\left[1 + \frac{\rho}{2(1-\rho)^2}\mathrm{CV}^2(w)\right] \tag{12'}$$

We see here that an increase in firm heterogeneity (defined in terms of the coefficient of variation) lowers the aggregate labour share when $\rho < 0$, as predicted in section 3.3. Similar parallels exist for the other firm dimensions.[14]

4 Data[15]

4.1 Sample

Equation (12) provides a model-based decomposition of the aggregate labour share in terms of firm heterogeneity vis-a-vis a counterfactual firm. We apply this decomposition to the manufacturing sector in Great Britain (UK without Northern Ireland), for the 1998-2014 period.[16] We focus on the manufacturing sector because value added is very imperfectly measured in other sectors, where intermediate inputs are less clearly identified – see for instance the discussion in Autor et al. (2017a).

We use data from the 3rd edition of the Annual Respondent Database (ARD), which contains a census of all enterprises with at least 250 employees, plus a sample of all those firms with less than 250 employees.[17] The dataset has information both at the plant and "reporting unit" level. The latter is the smallest unit that contains detailed financial information needed for the analysis (like labour costs, investment, and so on), and so it is our working definition of firm. Still, most of firms only have one plant (for example, 97% in 2014).

[14] In section 3.3 we showed the effects of a mean-preserving increase in wage dispersion, that is a case where λ^{HOM} remains constant while $\mathrm{CV}(w)$ increases. We also showed that the result still holds (and is indeed reinforced) if the increase in wage dispersion happens around the unweighted mean, rather than the weighted one. Equation 12′ quantifies the effect. Recall that for keeping the *weighted* mean (hence, λ^{HOM}) constant, we need to have $\Delta_1 = \Delta_2 \frac{\delta_2}{\delta_1}$: the increase in wages at firm 1 is bigger than the decrease at firm 2. To keep the *unweighted* mean constant we have to further lower wages at firm 2. This effectively lowers λ^{HOM}. Hence, the aggregate labour share falls because of both a reduction in λ^{HOM} and an increase in $\mathrm{CV}(w)$, when $\rho < 0$.

[15] Unfortunately, we are unable to provide the data underlying our results because this can only be accessed through the UK Data Service's secure lab. Nevertheless, we plan to make our code public, so anyone with access to the dataset can reproduce our results. Information about the dataset and how to access it can be found at http://doi.org/10.5255/UKDA-SN-7989-4

[16] Although our analysis is for Geat Britain only, for simplicity to the reader, we refer to our sample as the UK. The approximation might still be valid enough, given that during the sample period, manufacturing's GVA in Northern Ireland has been constantly below 3% of the UK-wide level, according to ONS data.

[17] ARD covers the Non-Financial Business Economy of Great Britain, between 1998 and 2014. In terms of SIC07 codes, all sectors are included except O (Public administration, defence and compulsory social security), T (mainly activities of households as employers of domestic personnel), U (activities of extraterritorial organisations), sections 01.1 to 01.5 (inclusive) of Agriculture, section 65.3 of Financial and Insurance activities, any educational activity carried out by the public sector in P, section 86.2 (medical and dental practice activities) and any other public provision of human health and social work activities in Q. The coverage is around two-thirds of the GB gross value added. The sample does not cover self-employees (formally called sole proprietors or traders), unless they are registered with the UK tax authority, HMRC (which is not necessary for businesses below a given income threshold). For further details, see ONS (2012).

Several sample selection procedures were made. First, as suggested by Schwellnus et al. (2017), sub-sector 19 in the SIC07 classification ("manufacturing of coke and refined petroleum") was dropped, because of the noise introduced by the volatility of oil prices. Second, firms with less than 10 employees were dropped. This is because for small firms (and particularly for firms with 1 or 2 employees, the bulk of those dropped) the level of wages might not so much be associated with market mechanism, as both capital income and labour income can be used to reward the firm's owners (combination which might depend on the tax system). This might distort the computation of the labour share in ways unrelated to the theory.[18] Third, non-profit and other non-market oriented firms were excluded, as these are less likely to be characterised by profit maximising behaviour. Fourth, firms with missing information (e.g. no investment data, needed to compute capital stocks) were also dropped. Fifth, outliers in terms of top and bottom 0.5% percentiles, computed independently for different variables (including firm level labour share, Y/L and L/K), were discarded. The final sample used contains 115,150 observations, covering around 38,000 unique firms. In any case, all the analysis presented here is carried out using turnover-based sampling weights, in order to represent the whole sector as good as possible.

4.2 Variables not in ARD

Although ARD is a very rich dataset, in terms of our needs it only contains information on number of employees, total labour costs (including pension funds contributions) and value added (the latter either directly available, or computed using gross output and intermediaries, when missing). Therefore, we need to either add or produce our own estimates for the remaining terms, namely firm-level prices, TFP, production function parameters (α and ρ), and product and factor market power. We also need to impute the capital stock of the firm.

Prices

Our theory is build upon firm-level prices; however, no price information is available in ARD. Instead, we use the most disaggregated industry-level producer price index available (4 digits), provided by the Office for National Statistics.

There are, of course, differences between the industry price index and the firm price level, p_i. In practice however, using the former is not only our only option but it is, theoretically speaking, more informative. To see this, notice that in the theory, p_i refers to the price of a unit of physical output (or "real" value added), e.g. apples. This poses two difficulties. On the one hand, the unit of measurement of prices is arbitrary. For instance, if we were to measure p_i in terms of a kilo of apples instead of a unit of apples, the price level would change. On the other hand, comparing the price level of different goods (e.g. apples with cars) is uninformative. Because of these issues, price heterogeneity among heterogeneous goods can only be made sense if the price of goods are

[18] Two things must be mentioned here. First, regarding the actual threshold of 10 employees, ARD is based on stratified sampling, using industry, region and employment size as strata. The latter uses 0-9 employees band as one cell for sampling. Hence, it is natural to exclude the whole band together. Second, firms with less than 10 employees tend not to be sampled in consecutive years. This means their capital stock cannot be imputed, nor be used in the production function estimation (see Appendix D). These issues and other information about sampling in ARD can be found in ONS (2012).

normalised to a common unit. This is precisely the goal of a price index.[19]

To see this more clearly, define as $p_{j,0}$ the price *level* in sector j in the base period of the index, and $\pi_j = p_j/p_{j,0}$ as the sectoral price index. We do not observe p_i but the π_j to which the firm belongs. Thus, in all the theoretical analysis presented so far, we could replace p_i with π_j. In other words, when we talk about price heterogeneity, we are referring to a *relative* price heterogeneity, in the sense that it is the *change* in the measure which is informative, and not the level. In effect, by construction, price heterogeneity is zero in the base year. Again, this is the only type of heterogeneity which is informative when dealing with heterogeneous goods.

Now, the replacement of p_i with π_j induces a second change. If we are dividing p_i by $p_{j,0}$, we must also multiply it somewhere else. As it turns out, it is the TFP term which captures the extra $p_{j,0}$ because, as shown later, TFP is estimated from a regression where value added is deflated by the same price index. Thus, when we replace p_i with π_j, we are also replacing A_i with $\tilde{A}_i$, where $\tilde{A}_i = A_i p_{j,0}$. This transformation is not a problem. In fact, TFP suffers from the same problem than prices. Notice that the units of measurement of TFP are directly related to those of output. Thus, if we are talking about heterogeneous goods like apples and cars, comparing A_i *levels* across firms is just as uninformative as comparing p_i across firms. Thus, TFP must also be normalised in order to be comparable. And $p_{j,0}$ is precisely such a normalisation.

In summary, by replacing firm level prices with that of an industry-level index (in terms of the equations, by replacing p_i with π_j and A_i with $\tilde{A}_i$), we achieve an empirical implementation of the general theoretical result, where the *interpretation* of price and TFP heterogeneity is informative. For notation economy though, we keep using p_i and A_i.

Estimation of TFP, α and ρ

A very important element of the model not available in the sample is a measure of firms' total factor productivity, A_{it}. Additionally, we do not observe the production function parameters α and ρ. We thus estimate a CES value added production function (eq. 3) in the data. As extensively noted in the literature (e.g. Olley and Pakes, 1996), it is necessary to account for the potential endogeneity of employment which, being a variable factor, might respond to contemporary unobserved shocks to TFP. we follow the dynamic panel method proposed by Blundell and Bond (2000). In this method, unobserved TFP is assumed to follow an AR(1) with parameter θ, and the model is then θ-differentiated, and estimated with GMM.[20] This dynamic panel approach is preferred to the, also common, control function method, because the latter is more demanding on the data, reducing the sample size.

Full details of the estimation method are presented in Appendix D. Here we just highlight that the estimated elasticity of substitution (for the manufacturing sector as a whole) is 0.46 ($\hat{\rho} = -1.18$), significant at the 1% confidence level. This elasticity implies capital and labour are gross substitutes,

[19] Notice that the two issues are independent. If firm-level prices were available, we could construct a price index based on firm level prices. As these are not available, we use industry level prices as a proxy.

[20] The resulting model is highly non-linear (see equation (27) in Appendix D), and GMM does not converge on our data (in effect, most of the literature estimates Cobb-Douglas production functions, which are log-linear in the parameters). We therefore consider a translog production function, which is a non-linear approximation of the CES around an elasticity of substitution equal to 1. According to Monte Carlo simulations in Lagomarsino (2017), the bias of a second order (i.e. non-linear) Taylor approximation of a two-input CES is neglectible for $\rho > -1$, and it is still relatively small at $\rho = -2$. Our main estimates situate ρ around -1.18.

a result that is generally consistent with other firm-level evidence (an example using UK data is Barnes et al., 2008).

Importantly, the firm-level capital stock is not available in the data, and yet it is required for estimating the production function. We therefore impute capital using a combination of the perpetual inventory method and information from the capital stock for the whole sector, obtained from the Office for National Statistics. See Appendix D for further details.[21]

Finally, having estimated α and ρ, we can use the production function to compute $\hat{A}_{it}$ as a residual.[22] This can be done also for observations not used in the estimation of the production function (for example, because of missing data in a given year). This means that the final sample used for the decomposition is larger than the one used in the regression. For details, see Appendix D.

Market power

Labour and product market power are defined in terms of labour supply and output demand elasticities, respectively. As these are not directly observable, we calibrate χ_i^L and χ_i^Y using proxies. For labour market power, we start by measuring the employment share of each firm in the local labour market they are situated. Importantly, this share is computed for each occupational group, after which a weighted average is produced for each firm. The aim of this occupation-adjusted share is to reflect different occupational composition of firms vis-a-vis that of the local labour market (e.g. a firm employing mostly high skill workers in a local labour market with mostly low skill workers has more market power than a firm mostly employing low skill workers in the same local labour market).[23] The local labour market is understood to be a "travel to work area" (TTWA).[24] The final measure of hiring concentration ranges between 0 and 1.

We then need to map the measure of monopsony power derived above (which we denote as s_{it}) into the labour supply elasticity faced by the firm, η_{it}^L. The method we use is relatively simple. Notice that the elasticity of supply is a number that goes between 0 and ∞. Therefore, any relationship between s_{it} and η_{it}^L must be such that, in competitive markets, $s_{it} \approx 0$, whereas in complete monopsony power, $s_{it} \approx 1$. Albeit there are several functional forms producing such relationship, a flexible one is

$$\eta_{it}^L = -c_1 \left(\frac{1}{ln\,(1 - s_{it})} \right)^{c_2}$$

[21] Collard-Wexler and De Loecker (2016) show that measurement errors in the capital stock introduce a downward bias in the estimates of the production function parameters. To deal with the problem, they suggest a hybrid IV-Control function approach that instruments capital with lagged investment. However, the method relies on log-linearity and is therefore not directly applicable outside a Cobb-Douglas setting.

[22] As explained in Appendix D, it is impossible to identify the shock to value added. This is therefore included in the computation of $\hat{A}_{it}$. This introduces a bias in the latter, which is constant as long as the variance of the shock to value added is also constant. For further details, see also footnote 35.

[23] Unfortunately, ARD does not contain information on the skill level of the workers employed. These are instead imputed from the Annual Survey of Hours and Earnings (ASHE). In particular, we compute the share of workers in each of the nine occupation groups (SOC2010 major groups), in a given industry (SIC07 division), and year. Then, we assign this share to firms in ARD in that given industry-year cluster. Total employment for each occupation group in the local labour market is also computed from ASHE.

[24] The official documentation from the Office for National Statistics defines a TTWA as "[i]n concept, a self-contained labour market area is one in which all commuting occurs within the boundary of that area. In practice, it is not possible to divide the UK into entirely separate labour market areas as commuting patterns are too diffuse. TTWAs have been developed as approximations to self-contained labour markets reflecting areas where most people both live and work." More details about the definition and methodology for computing the TTWA can be found in https://ons.maps.arcgis.com/home/item.html?id=379c0cdb374f4f1e94209e908e9a21d9.

This is flexible because it is possible to shape this function by changing the positive terms c_1 and c_2, holding the end points mentioned above fixed. These constants are chosen in order to match the scarce evidence available in the literature about η^L_{it} at the firm level. In effect, we match two empirical properties of η^L_{it}. First, Manning (2003) estimates an average firm level elasticity of supply for the UK of around 0.75. Second, Webber (2015) provides a characterisation of the distribution of this elasticity for the US, from where it is possible to calibrate with decent fit a log-normal distribution of this elasticity.[25] For lack of a better alternative, we assume the UK also follows this distribution, but scaled to match the UK average estimated by Manning (2003).[26] This enable us to compute c_1 and c_2 (found to be 0.01 and 0.37 respectively).

Regarding product market power, our theory defines this in terms of the firm's elasticity of demand. Albeit this is also unobserved, there is a direct relationship between this elasticity and the mark-up (price over marginal cost). In particular, under monopolistic competition, if the mark-up of a firm is μ, with $\mu \geq 1$, its elasticity of demand is $\eta^L = -\frac{\mu}{\mu-1}$. We compute the firm-level mark-up as the sales to total variable costs' ratio, which approximates marginal costs with average (e.g. Branston et al., 2014; De Loecker et al., 2018).[27]

4.3 Theoretical versus empirical decomposition

Before moving forward, an important issue must be dealt with. The decomposition formulas are built upon the optimisation behaviour of firms. Thus, they refer to the *predicted* labour share of firms, as given by equation (6). However, the objective of the decomposition is to characterise *observed* labour shares (in terms of observed value added and labour costs). Naturally, there will be differences between these two. There are a multitude of reasons why predicted and observed values can be different. For a start, in terms of our theory, firms' optimisation process might be more complex (e.g. dynamic rather than static), or firms might face constraints that lead to misallocation of resources (i.e. firms are not efficient). There is substantial evidence in the literature for this. Even if the theory is a good enough approximation to reality, discrepancies might arise from imprecise or inconsistent estimates for the variables and parameters of the model (e.g. market power, or TFP). Additionally, variables in the data could be measured with errors. Last but not least, the stochastic nature of firms' production means the latter is subject to idiosyncratic shocks (e.g. productivity), which will always deviate the observed values from the predicted ones. This is an irreducible source of theory-data mismatch, at least at the firm level.

The discrepancy between predicted and observed labour share introduces an extra term into the decomposition.[28] To see this, let us define $\tau_{it} \equiv \dfrac{\lambda^{obs}_{it}}{\lambda_{it}}$, which captures the divergence between the

[25] In particular, we fit a log-normal distribution using the percentiles presented in Table 6 in Webber (2015).

[26] Notice Manning (2003) derives an elasticity for the whole economy. In consequence, we apply this method *before* removing other sectors and firms from our sample. This is, we use the maximum sample available in ARD.

[27] Unfortunately, we can not implement the mark-up estimation method put forward by De Loecker and Warzynski (2012), where mark-ups are derived from the first order condition of the *gross output* production function with respect to a fully variable and competitive input (in our case, intermediate inputs, as labour is subject to imperfect competition). As already mentioned before, estimating a gross output function consistent with our framework (that is a nested CES, where the value added production function is one of the inputs, and materials is the other), proved impossible.

[28] An approach where this term would not show up is when one of the variables of the model is *not* computed using an optimality condition, but as a residual. For example, we could measure labour market power implicitly, as the value that makes the rest of the measured variables fit that equation (e.g. as in Brummund, 2012). This however confounds any "true" discrepancy with the measure of labour market power.

observed and predicted labour share for firm i in period t, where the latter is given by equation (6). Now, consider the following identity regarding the aggregate labour share (for notation economy, time index is omitted throughout):

$$\lambda^{obs} \equiv \sum_i \lambda_i^{obs}\, \delta_i^{obs}$$

where $\delta_i^{obs} = \frac{p_i Y_i^{obs}}{\sum p_i Y_i^{obs}}$. This is the empirical counterpart to the labour share definition in equation (2). The connection between the model and data is done precisely through τ_i. This is, we can write:

$$\lambda^{obs} = \sum_i \lambda_i\, \tau_i\, \delta_i^{obs} \tag{13}$$

Replacing the predicted firm level labour share λ_i by its components (equation 6), and introducing the counterfactual λ^{HOM}, produces:

$$\lambda^{obs} = \lambda^{HOM} \sum_i \left(\frac{\chi_i^Y}{\bar{\chi}^Y}\right)^{\frac{1}{1-\rho}} \left(\frac{\bar{\chi}^L}{\chi_i^L}\right)^{\frac{1}{1-\rho}} \left(\frac{A_i}{\bar{A}}\right)^{\frac{\rho}{1-\rho}} \left(\frac{\bar{w}}{w_i}\right)^{\frac{\rho}{1-\rho}} \left(\frac{p_i}{\bar{p}}\right)^{\frac{\rho}{1-\rho}} \left(\frac{\tau_i}{\bar{\tau}}\right) \delta_i^{obs} \tag{14}$$

where

$$\lambda^{HOM} = \bar{\tau} \left(\frac{\alpha \bar{\chi}^Y}{\bar{\chi}^L}\right)^{\frac{1}{1-\rho}} \left(\frac{\bar{A}\bar{p}}{\bar{w}}\right)^{\frac{\rho}{1-\rho}} \tag{15}$$

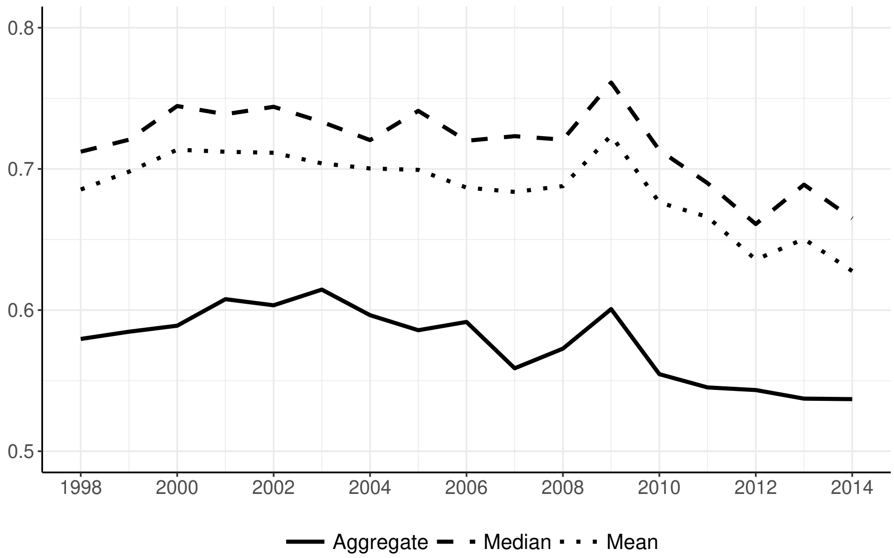
and $\bar{\tau} \equiv \sum_i \omega_i \tau_i$, a weighted average of the discrepancy term. The introduction of $\bar{\tau}$ in the above equation is useful because $\frac{\tau_i}{\bar{\tau}}$ can be redefined as $1 + \frac{\Delta_{\tau_i}}{\bar{\tau}}$, which is similar in structure to the second order Taylor approximation of the other terms inside the summation. This ensures the final decomposition is defined in terms of correlations and coefficients of variation only, as was equation (12) before.

With the above modification, and looking at the ratio w/p to further simplify the notation (this has also the advantage to identify a real wage term in the expression for λ^{HOM}), we obtain the final "empirical" decomposition formula, used in the subsequent analysis:

$$\begin{aligned}
\lambda \approx \lambda^{HOM} \Bigg[1 &+ \frac{\rho}{2(1-\rho)^2}\mathrm{CV}^2(\chi^Y) + \frac{2-\rho}{2(1-\rho)^2}\mathrm{CV}^2(\chi^L) + \frac{\rho(2\rho-1)}{2(1-\rho)^2}\mathrm{CV}^2(A) + \frac{\rho}{2(1-\rho)^2}\mathrm{CV}^2\left(\frac{w}{p}\right) \\
&- \frac{1}{(1-\rho)^2}\mathrm{r}(\chi^Y,\chi^L)\mathrm{CV}(\chi^Y)\mathrm{CV}(\chi^L) - \frac{\rho^2}{(1-\rho)^2}\mathrm{r}(A,\frac{w}{p})\mathrm{CV}(A)\mathrm{CV}\left(\frac{w}{p}\right) \\
&+ \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^Y,A)\mathrm{CV}(\chi^Y)\mathrm{CV}(A) - \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^L,A)\mathrm{CV}(\chi^L)\mathrm{CV}(A) \\
&- \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^Y,\frac{w}{p})\mathrm{CV}(\chi^Y)\mathrm{CV}\left(\frac{w}{p}\right) + \frac{\rho}{(1-\rho)^2}\mathrm{r}(\chi^L,\frac{w}{p})\mathrm{CV}(\chi^L)\mathrm{CV}\left(\frac{w}{p}\right) \\
&+ \frac{1}{1-\rho}\mathrm{r}(\chi^Y,\tau)\mathrm{CV}(\chi^Y)\mathrm{CV}(\tau) - \frac{1}{1-\rho}\mathrm{r}(\chi^L,\tau)\mathrm{CV}(\chi^L)\mathrm{CV}(\tau) \\
&+ \frac{\rho}{1-\rho}\mathrm{r}(A,\tau)\mathrm{CV}(A)\mathrm{CV}(\tau) - \frac{\rho}{1-\rho}\mathrm{r}(\frac{w}{p},\tau)\mathrm{CV}\left(\frac{w}{p}\right)\mathrm{CV}(\tau)\Bigg]
\end{aligned} \tag{16}$$

The only difference with the theoretical counterpart is the addition of the last four terms, and the introduction of $\bar{\tau}$ in λ^{HOM}. Notice that heterogeneity in τ itself does not affect the labour share, unless it is correlated with other factors. Also, if the discrepancy is constant across firms (i.e. $\mathrm{CV}(\tau) = 0$), there is no difference between the theoretical and empirical decomposition.

Figure 2: Aggregate, mean and median labour share



Source: our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more, ARD data.

5 Results[29]

5.1 Descriptive analysis of the labour share

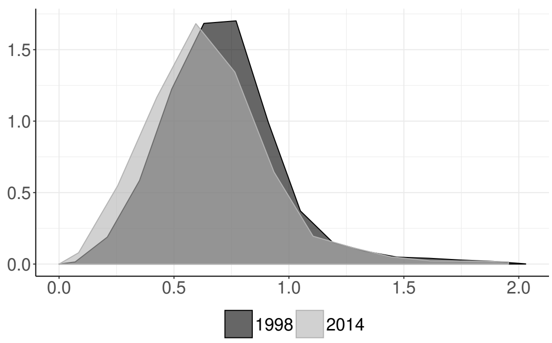
Before proceeding with the decomposition, it is useful to describe the labour share in our sample. Figure 2 presents different metrics for the latter, covering UK manufacturing, between 1998 and 2014. Starting with the aggregate labour share, we see a net fall over the period, from 0.58 in 1998 to 0.53 in 2014.[30] It's interesting to notice an initial period of increase in the labour share (peaking at 0.61 in 2003), and a subsequent fall, with a minor interruption during the financial crisis. Figure 2 also presents the (unweighted) mean and median labour share, which are above the aggregate labour share, highlighting that firms with higher value added (our measure of firm size) have a lower labour share. This is consistent with other findings in the literature (e.g. Autor et al., 2017a; Schwellnus et al., 2018).

Since the aggregate labour share is defined in terms of a weighted sum of firm level labour share (eq. 2), changes in the labour share can be due to changes in the magnitude of the firm level labour

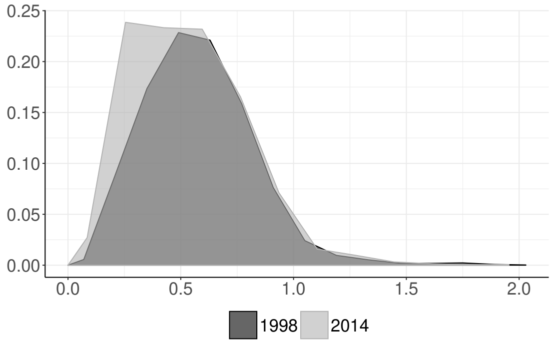
[29] In order to produce standard errors for the estimated variables (e.g. TFP), we bootstrap the whole estimation procedure (i.e. the imputation of capital, the estimation of the production function, and the decomposition), with 1,000 repetitions. Bootstrap is actually needed in order to compute the correct standard errors for the parameters of the production function, given that capital is a generated regressor. To compute the confidence intervals presented in this section we use the *percentile method* (e.g. see Efron and Tibshirani, 1986). This takes the point estimates as the center of the interval, rather than the bootstrap average. Because of the non-linearities involved in the imputation process, a bias might emerge when adding normally distributed variability to the estimations via bootstrap. In practice, the two means have a correlation above 0.98, for every variable. The major discrepancy arises with the mean of TFP, which is 14% higher in the bootstrap case. Trends are however the same.

[30] The labour share in manufacturing, computed from national accounts, shows an increase in the labour share between 1998 and 2009, and a fall thereafter, with the 2014 level being roughly the same as that in 1998. The level is also around 0.10 points higher in the national accounts. There is however no reason why they should be the same. For instance, the sample used here focuses only on firms with more than 10 employees (with smaller firms tending to have a higher labour share).

Figure 3: Unweighted and (value added) weighted distribution of the labour share, 1998 and 2014



(a) Unweighted



(b) (Value added) weighted

Source: our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more.

shares, in the distribution of weights across different labour share levels, or both. Figure 3 compares the distribution of the unweighted and (value added) weighted labour share at the beginning and at the end of our period. Panel (a) shows that the sample distribution of firms' labour share in 2014 has more mass at lower levels than in 1998. Similarly, Panel (b) shows that in 2014 more value added was produced by firms with lower labour share than in 1998.[31]

To identify the effect of changes in the composition of weights on the aggregate labour share, we compute the ratio between the weighted (aggregate) and the unweighted (average) labour share. This ratio is a measure of the correlation between λ_i and δ_i. If this correlation is positive (negative), the ratio will be above (below) one. If the two variables are uncorrelated, the ratio is one: unweighted and weighted labour share are the same.[32] As said earlier, smaller firms have higher labour share, so this correlation is negative. There is no trend in this variable (results not shown), suggesting that most of the change in the aggregate labour share is due to the fall in the level of the labour share across the firm size spectrum.

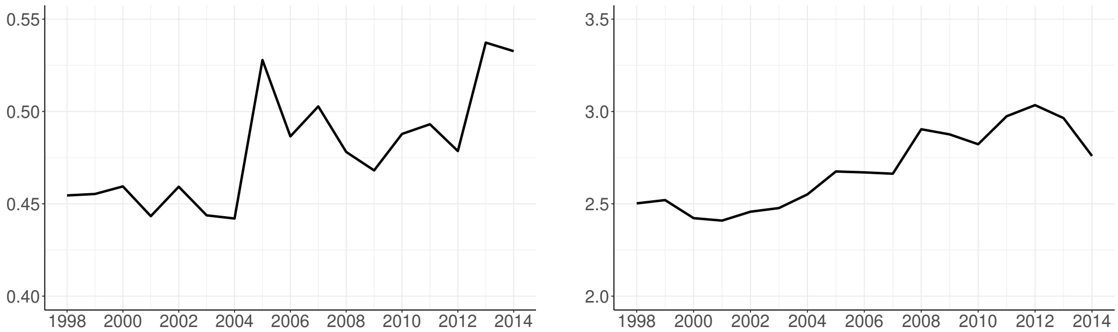
Importantly, as Panel (a) in Figure 3 show, the fall in the level of the labour share has not been an homogeneous phenomenon. In effect, the upper tail of the distribution barely changed between the two years. This reflects an increase in the dispersion of the labour share. Figure 4 documents this change, computed either as a coefficient of variation or a p90/p10 ratio. Dispersion changed particularly after 2003. There seems to be, actually, a relatively strong inverse relation between the aggregate labour share and the dispersion of firm level labour share (correlation of -0.70 or higher).

5.2 Decomposition of the aggregate labour share

Equation (14) decomposes the aggregate labour share in terms of λ^{HOM} (i.e. the labour share of a counterfactual "representative" firm) and $\sum$ (i.e. a quantification of firms' multidimensional dispersion with respect to that counterfactual firm). The decomposition for the manufacturing sector as a whole is depicted in Figure 5 (see Appendix E for an equivalent analysis at the sub-sectoral level).

[31] Notice the labour share is always positive, because the (few) observations with negative value added are removed

Figure 4: Dispersion in the labour share



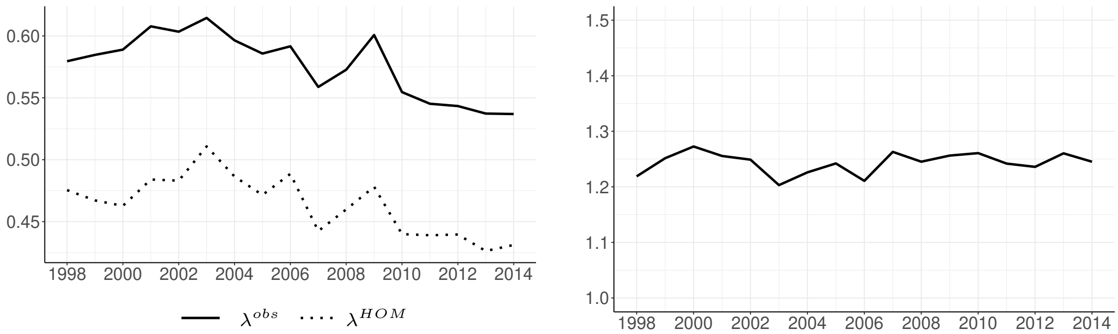
(a) Coefficient of variation

(b) P90/P10 ratio

Source: our calculation based on ARD data.

Sample: UK manufacturing firms with 10 employees or more.

Figure 5: Decomposition of the aggregate labour share



(a) Aggregate labour share (λ^{obs}) and counterfactual labour share (λ^{HOM})

(b) Heterogeneity component, $\sum$

Source: our calculation based on ARD data.

Sample: UK manufacturing firms with 10 employees or more.

Two things are important to notice. First, the level of the aggregate labour share is significantly different when compared with the level of the counterfactual, average firm. This is, if all firms were identical, the labour share would be significantly smaller. This result supports the theoretical result of the paper, and echoes the result in the example presented in Figure 1, namely that heterogeneity matters. Second, as Panel (b) shows, the role of heterogeneity has been relatively stable over the period. This is, changes in firm heterogeneity has not been a major driver of the movements of the aggregate labour share observed over the period. Importantly, this does not mean firm heterogeneity has not changed. As shown later, changes have partly offsetted each other.

To quantify the role of firm heterogeneity vis-a-vis that of λ^{HOM} in movements of λ^{obs}, we carry out a simple growth accounting decomposition of the equation $\lambda^{obs} = \lambda^{HOM} \sum$. Such decomposition is given by

$$g_{\lambda^{obs}} = g_{\lambda^{HOM}} + g_{\sum} + \text{interaction effect}$$

where g_Z stands for the growth rate of factor Z, over a given period. Table 1 presents the result of this exercise for two sub-periods, using 2003 (the year that the labour share reached it highest level) as threshold, as well as for the entire period. In all cases we see that the bulk of the change in the labour share has been due to λ^{HOM}. Meanwhile, $\sum$ has partly counteracted the effect of the former, reducing the decrease from 9% to 7%.

Table 1: Disaggregation of the growth rate of the aggregate labour share (λ^{obs})

Period	$g_{\lambda^{obs}}$	$g_{\lambda^{HOM}}$	$g_{\sum}$	Interaction
			%	
1998-2003	6.04	7.43	-1.29	-0.10
2003-2014	-12.63	-15.58	3.50	-0.54
1998-2014	-7.36	-9.31	2.16	-0.20

Note: $g_{\lambda^{obs}} = g_{\lambda^{HOM}} + g_{\sum} + \text{interaction effect}$.
Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more.

5.3 Decomposition of the labour share of the representative firm

The homogeneous labour share (eq. 15) can be further analysed by looking at its constituent elements. Figure 6 presents the evolution of the different variables for the whole manufacturing sector (again, see Appendix E for a sub-sector level analysis).[33] Recall that these represent weighted averages of the sector's firms. Panel (a) shows a fairly unstable but overall increase in TFP over the period (trend interrupted by a 2008-9 dip). Real wages (Panel b) show a stable pre-2008

from the sample (as they cannot be used in the estimation of the production function).

[32] The mathematical characterisation of this ratio is presented in Appendix B.

[33] 2007 presents an unusual behaviour, with significantly more missing observations in the original dataset, particularly for small firms (this issue is to be resolved in the 4th edition of the dataset, unavailable at the moment of producing this paper). This issue with the sample implies weighted averages are exaggerated in 2007. The sign and magnitude of the correlation between the weights and each variable determines the effect this has on the particular variable. In the figure, we observe particularly high values for productivity and real wages (which might suggest the Great Recession hit in 2008, whereas it did so towards the end of that year, and particularly in 2009). There is also an artificial fall in labour market power and τ in that year. Importantly, this problem does not affect the decomposition, which focuses in changes over larger periods.

growth, with a subsequent dip (particularly in 2009). Interestingly, such growth rate has slowed down post-2008, a trend consistent with ONS aggregate data. Product market power (Panel c) has increased over the period (recall lower χ^Y means more product market power), albeit also not in a steady fashion.[34] Labour market power (Panel d) fell in the early years of the period, and subsequently increased post-2008 (recall lower χ^L means less less market power for the firm). The sharp fall in 2007 is artificial (see footnote 33). Lastly, except for 2007, the discrepancy term $\bar{\tau}$ (Panel e) is fairly stable, meaning this is unlikely to drive any of the results.[35]

Table 2 presents the growth rates of each variable in Figure 6, over the subperiods of interest. As equation (15) indicates, the effect of these variables on λ^{HOM} is mediated by ρ. In order to see the final effect of each of these variables on λ^{HOM}, we carry out a growth accounting decomposition of equation (15). This decomposition is given by

$$g_{\lambda^{HOM}} = \left(\frac{\rho}{1-\rho}\right) g_{\bar{A}} - \left(\frac{\rho}{1-\rho}\right) g_{\bar{w}/\bar{p}} + \left(\frac{1}{1-\rho}\right) g_{\bar{\chi}^Y} - \left(\frac{1}{1-\rho}\right) g_{\bar{\chi}^L} + g_{\bar{\tau}} + \text{interaction effect} \quad (17)$$

Table 2: Growth rates of the components of λ^{HOM}

Period	$g_{\bar{A}}$	$g_{\bar{w}/\bar{p}}$	$g_{\bar{\chi}^Y}$	$g_{\bar{\chi}^L}$	$g_{\bar{\tau}}$
			%		
1998-2003	8.54	16.24	1.46	-4.02	0.91
2003-2014	35.72	14.05	-4.87	7.96	-1.71
1998-2014	47.32	32.57	-3.48	3.62	-0.81

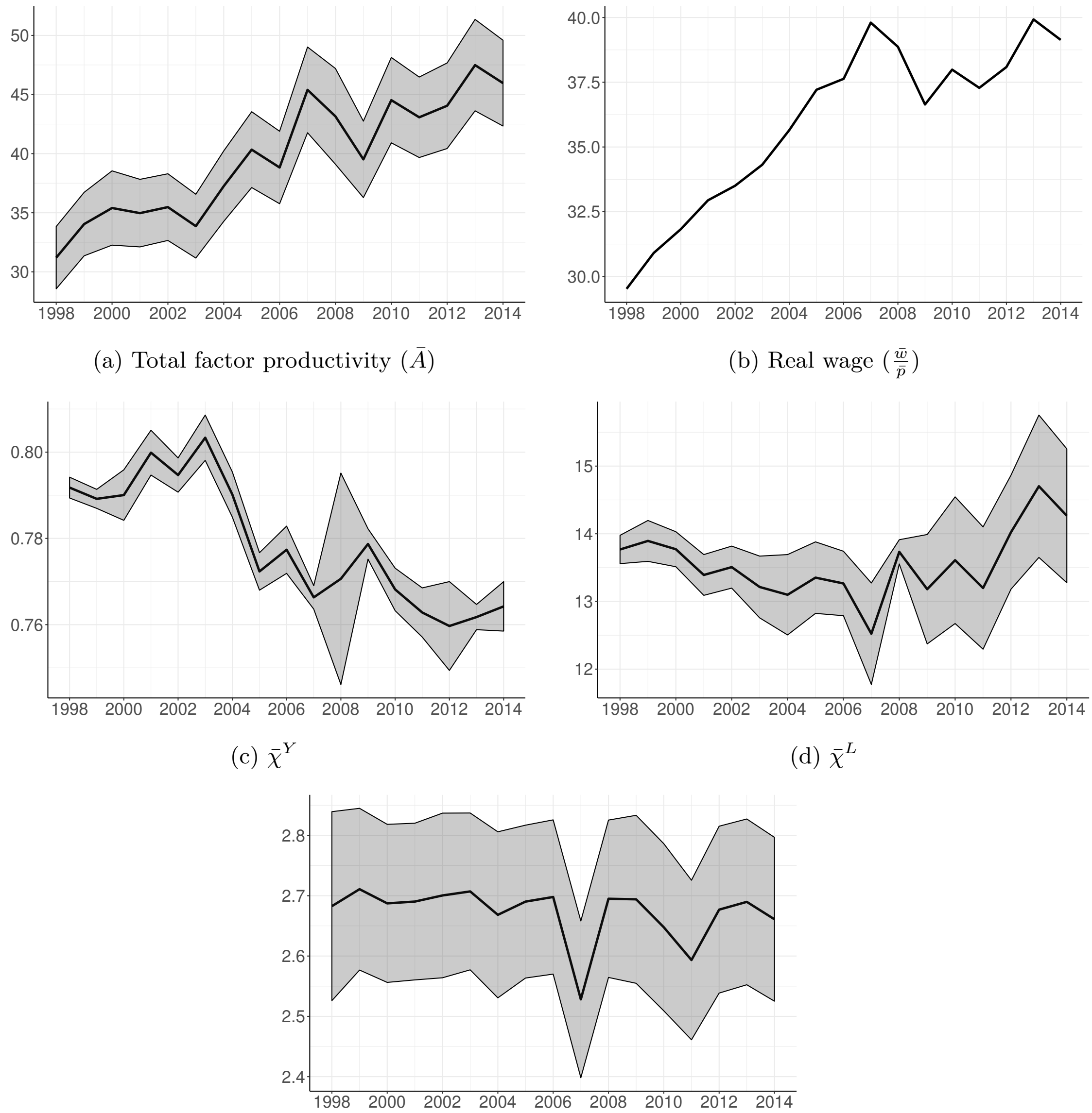
Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more.

Table 3 shows the resulting contribution of each component of λ^{HOM}. It can be seen that real wages did not grow as fast as productivity did, and the gap between the two can explain most of the actual change in λ^{HOM}. In particular, we see that the pay-productivity disconnect is the key driver in both sub-periods. Given that the second sub-period is longer, and what was observed previously regarding the slower growth in real wages post recession, part of the blame is on the latter. In fact, if we impose the same annual growth rate of real wages observed between 1998 and 2003 (3.1%) for the post-crisis period, no pay-productivity disconnect would have emerged over the

[34] De Loecker and Eeckhout (2018) also document a mild increase in mark-ups for the UK, although with a different timing than the one described here. However, the difference between their method and ours are major. They do not use micro-data but balance-sheet data, covering sectors beyond manufacturing; they assume a Cobb-Douglas gross output production function; and they use sales rather than value added to compute national level averages. A similar method and data to the latter is applied by Haldane et al. (2018), who report an increase in mark-ups in UK manufacturing, starting around 2005.

[35] At first, the level of $\bar{\tau}$ might appear to be relatively high. Recall this is computed as the ratio between the observed and predicted labour share across firms. Thus, $\bar{\tau}$ around two suggests predicted λ_i is around half of the observed labour share. This is however not necessarily true. As Appendix D shows, $\hat{A}_{it}$ contains both the shock to TFP and the shock to value added (terms ξ_{it} and ϵ_{it} in equations (25) and (26), respectively). While the latter has zero mean in terms of the *logarithm* of value added (again, see equation 25), it does not do so around value added itself. This bias is captured by the level of $\hat{A}_{it}$ (bias that should be constant as long as the variance of ϵ_{it} is constant). It can be shown that $E(\hat{A}_{it}|\Phi_t) = A_{it}e^{\frac{\sigma^2}{2}}$, where σ^2 is the variance of ϵ_{it}. The magnitude of such bias is unknown because the two shocks cannot be empirically identified, and thus σ^2 cannot be estimated. The sign however is evidently positive; TFP is overestimated. Furthermore, since the predicted labour share (equation 6) contains $\hat{A}_{it}$ to the power of $\frac{\rho}{1-\rho}$, and ρ is estimated to be -1.18, such bias is lowering the predicted labour share, which in turns raises τ_i and therefore $\bar{\tau}$. Again, as long as σ^2 is constant over time, such bias is only a level effect, without affecting trends and therefore the decomposition exercise.

Figure 6: Evolution of the components of λ^{HOM}



(a) Total factor productivity ($\bar{A}$)

(b) Real wage ($\frac{\bar{w}}{\bar{p}}$)

(c) $\bar{\chi}^{Y}$

(d) $\bar{\chi}^{L}$

(e) $\bar{\tau}$

Source: Our calculation based on ARD data.

Sample: UK manufacturing firms with 10 employees or more.

Note: 95% confidence intervals are displayed as a shadowed area (except for real wages, which are observed).

period, virtually muting any change in λ^{HOM}, *ceteris paribus*.

Table 3: Determinants of the growth rate of λ^{HOM}

Period	$g_{\lambda^{HOM}}$	$\bar{A}$	$\bar{w}/\bar{p}$	$\bar{\chi}^Y$	$\bar{\chi}^L$	$\bar{\tau}$	Interaction
				%			
1998-2003	7.43	-4.62	8.79	0.67	1.85	0.91	-0.17
2003-2014	-15.58	-19.34	7.61	-2.23	-3.65	-1.71	3.74
1998-2014	-9.31	-25.61	17.63	-1.60	-1.66	-0.81	2.73

Note: Effects are attributed as per eq. (17).
Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more.

Table 3 also indicates that (product and labour) market power contributed to the fall in the labour share. However, as commented earlier in relation to Figure 6, there is a marked difference in the effects of market power within the whole period. Between 1998 and 2003, both product and labour market power fell, albeit only slightly (reflected in higher and lower $\bar{\chi}^Y$ and $\bar{\chi}^L$, respectively). The second sub-period is characterised by a marked reversal of this initial timid trend. By 2014, both measures of market power are significantly higher than in 1998, jointly pushing for a 3.09% fall in the aggregate labour share.[36]

5.4 Firms' heterogeneity

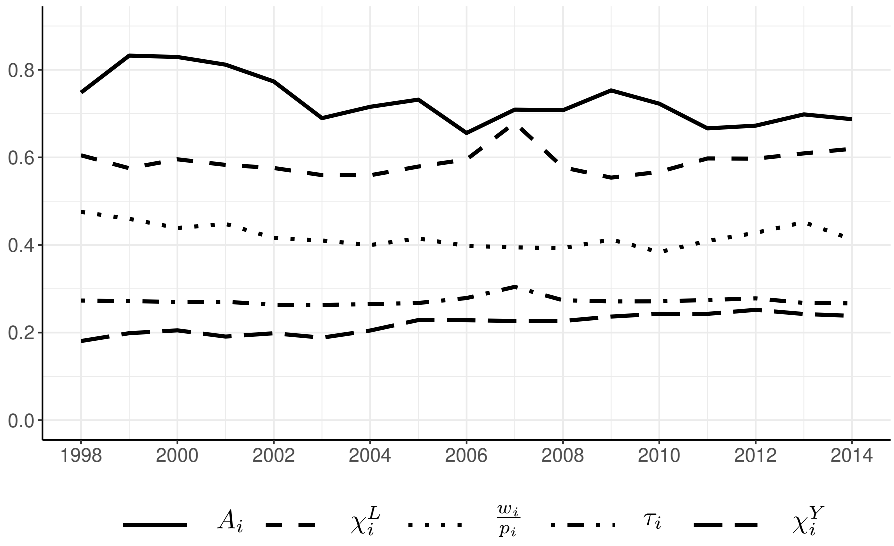
The last decomposition exercise focuses on the heterogeneity component, $\sum$. As equation (16) shows, this is a function of the coefficient of variation and the correlation among variables (this is, the structure of the joint distribution of heterogeneity). Before carrying out this decomposition, it is then interesting to evaluate these elements.

Figure 7 characterises firm heterogeneity in the five different dimensions under study, measured by their coefficient of variation – which, being dimensionless, can be compared across variables.[37] In terms of levels, TFP and labour market power have the highest variation across firms. (Deflated) wages are somewhere in the middle, with product market power and the "discrepancy" component having the lowest variability across firms. It should not be surprising that TFP varies more than wages, given that the latter is a much more "structural" process, driven by market trends relatively common across firms, and regulated by legal contracts; conversely, TFP might be quite idiosyncratic to the firm's conditions, slow to reproduce elsewhere. What's more interesting is the relatively high variability of labour market power vis-a-vis that of product market power. The logic might

[36] The documented increase in product market power contributing to a lower labour share seems consistent with the "winner-take-most" literature. To relate more to that literature, we compute changes in market shares and market concentration (which are not necessarily related to mark-ups, our measure of product market power). In particular, we compute the gross output-based Herfindahl-Hirschman Index (HHI) for the 218 4-digit SIC07 sub-industries available in the data. We find that this index rose in two thirds of these sub-industries, between 1998 and 2014. If we aggregate the subindustry HHI up to the division level (2-digit SIC07, 22 divisions in total), properly weighted, we observe that 52% of divisions had a higher HHI index in 2014 than in 1998. Finally, the aggregated manufacturing-level HHI index also went up over the period, from 0.10 to 0.12. So, even if concentration rose in most sectors, the change is relatively small. Moreover, the level itself is relatively low, according to traditional interpretations of the HHI index. This evidence seem broadly consistent with results from other studies like Valletti et al. (2017) and Bell and Tomlinson (2018) (minding the differences in terms of sample, indicator, and period). Overall, the "winner-take-most" phenomenon is only weakly present in UK manufacturing, if present at all.

[37] Recall these coefficients are computed using value added as weights.

Figure 7: Evolution of the dispersion of firm-level variables



Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more, ARD data.

be the same though. Product market power is computed in terms of mark-ups, which depend on the prices of final goods and variable inputs. Prices adjust easily and they move based on fairly common trends. Conversely, labour market power (calculated as local labour market shares) reflect the spatial heterogeneity of firms, with all the geographical idiosyncrasies involved. Spatial mobility of firms is a slow process. Finally, it is good news that the term describing the mismatch between the data and the theory has relatively low variability.

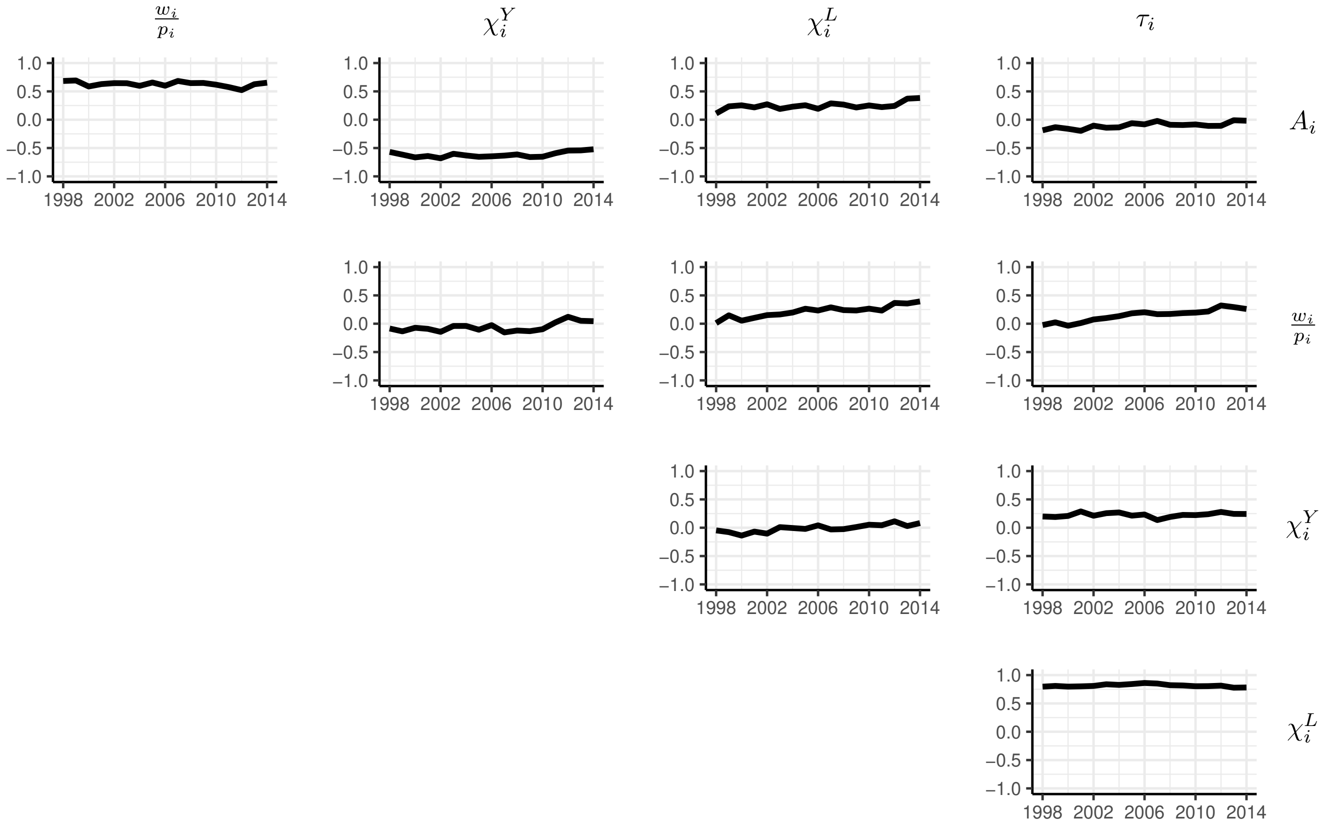
In terms of how heterogeneity evolved over time, Figure 7 shows there are no drastic changes over the period. TFP and real wages moved towards less heterogeneity (with some oscillation over the period), whereas product market power heterogeneity increased over the period; labour market power remained relatively stable, except for the artificial jump in 2007 mentioned earlier.[38]

The second important element of firm heterogeneity refers to the correlation among variables across firms over time, presented in Figure 8.[39] Most of them are fairly close to zero. Noticeable exceptions are the positive correlation between TFP and real wages and the negative correlation between product market power and TFP (suggesting that firms with higher TFP have higher product market product). The other very high correlation is between labour market power and τ_i, perhaps suggesting measurement errors associated with the former. Interestingly, product market power is

[38] The decline in TFP heterogeneity might be surprising in light of papers like Andrews et al. (2016). They document an increasing productivity gap between "top" firms (defined, roughly speaking, as those at the top 5% of the TFP distribution, within 2-digit industries) and the rest, from 2001 onwards, and using firm-level data for 14 OECD countries. This gap reflects TFP stagnation in most firms together with rapid TFP growth among "top" firms. Nevertheless, CV(A) as presented in Figure 7 is not best equipped to show this phenomenon, because it is weighted by firm's size. Thus, to explore the proposed productivity gap in our data, we look at the evolution of TFP at different percentiles of the TFP distribution (also computed at the 2-digit SIC07). On the one hand, we find the ratio between percentiles P_{50} and P_{10} is virtually unchanged over the period, as productivity was pretty much stagnant at these percentiles. On the other hand, the ratio between P_{95} and P_{50} shows an initial compression between 1998 and 2004, increasing later on, peaking in 2013. In any case, the change is not substantial, suggesting the productivity gap between "top" and the rest of firms, although existent, has not changed dramatically in the last twenty years, as Andrews et al. (2016) suggest for a wider sample of countries.

[39] Recall these correlations are computed using value added as weights.

Figure 8: Evolution of correlation among different dimensions of firm heterogeneity



Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more, ARD data.

virtually uncorrelated with real wages, reflecting perhaps a low bargaining power of workers, as they are unable to capture rents by firms.

Regarding the evolution of these correlations over time, the most noticeable change concerns the correlation between labour market power and real wages, increasing from almost zero in 1998 to 0.40 in 2014 (positive correlation means firms with higher wages have greater market power). The strengthening link between these two variables seems counterintuitive, but it might be reflecting the fact that larger firms tend to have higher wages and labour market power. In fact, the correlation of value added with real wages and labour market power went up from 0.43 and 0.19 in 1998 to 0.59 and 0.48 in 2014, respectively.

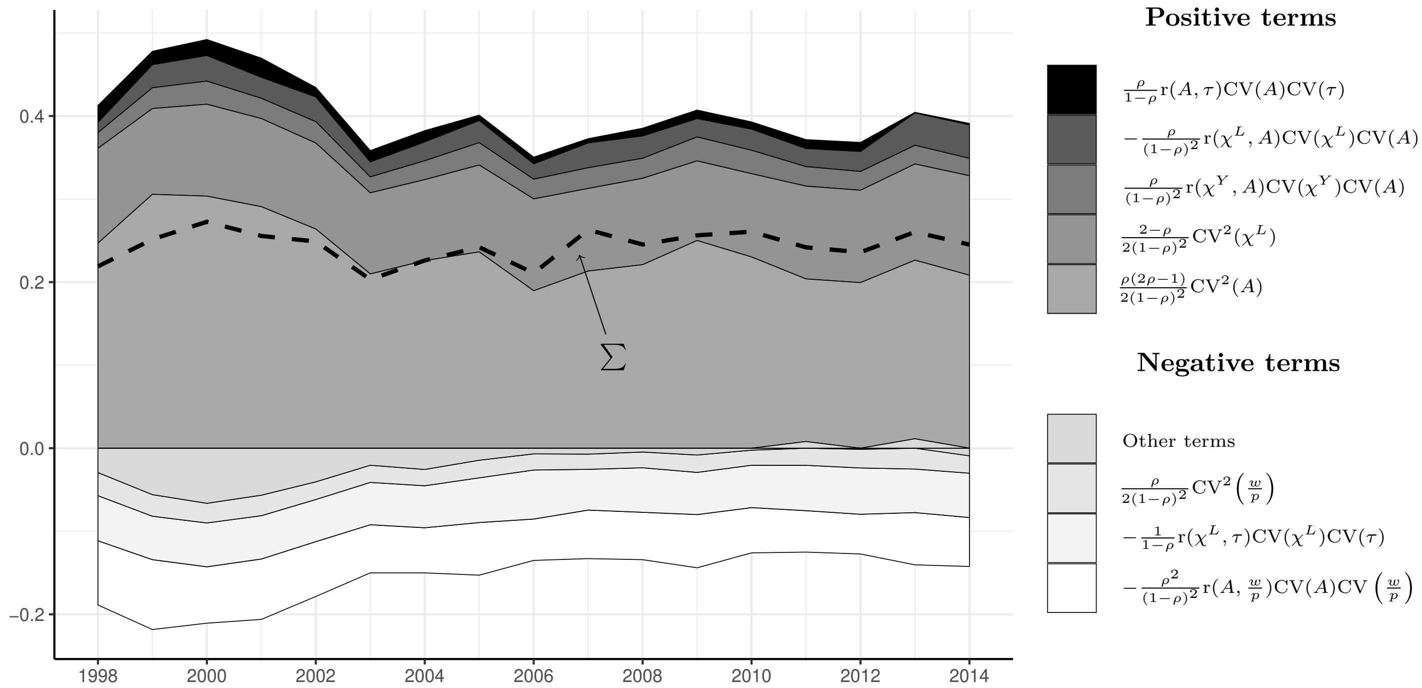
Having provided some background evidence regarding the structure of the joint distribution of firms' characteristics, we can now proceed to the decomposition of $\sum$, which Table 1 above suggested had a minor role in the observed labour share. Two issues are of particular interest here. First, which are the most relevant sources of firm heterogeneity? Second, how have these sources changed over time? Figure 9 helps answering these two questions by presenting the evolution of the different components of $\sum$. The graph is a stacked area plot, meaning the sum of all terms (or equivalently, the difference between the positive and negative totals), yields $\sum$. For ease of visualisation, $\sum$ is centered around zero, whereas, as equation (16) shows, this moves around one.

What this figure shows is that the bulk of the effect of firm heterogeneity on the aggregate labour share is due to two elements, namely TFP and labour market power. Figure 7 has already shown these are the dimensions with the highest variability. Figure 9 shows that they also have the biggest impact on the labour share, taking into account the effect of the the elasticity of substitution parameter ρ. Variability in the real wage is of second order of importance (and its effect goes in the other direction), whereas variability in product market power is completely irrelevant (its value averages -0.006 over the period), as it is that of τ_i, which in itself does not affect λ (see equation 16). In terms of correlations, the size of the effect again mimics those seen in Figure 8. The correlation of TFP with deflated wages, and labour market power with τ_i are the most significant, followed by that of TFP with both product and labour market power. Recall the latter correlation is not significantly high, but its combination with $\mathrm{CV}(\chi^L)$ and $\mathrm{CV}(A)$ (both high), pushed the effect up.

It is worth point it out that the terms not explicitly mentioned in the decomposition (included in "Other terms") are mostly irrelevant for the labour share. Crucially, this component includes every other term excluded from the approximation in equation (16), and therefore, acts as an empirical test for the validity of the decomposition. It is therefore revealing to see that our approximation is sufficient for capturing the bulk of the changes in $\sum$. Naturally, it is impossible to extrapolate this conclusion to every empirical application of the method, but it is our suspicion the method is in general good enough for its purpose.

Another relevant result from the decomposition presented in Figure 9 refers to what in our theoretical section was defined as the "direct effect" versus the "indirect effect" of heterogeneity on the labour share. Results suggest that the direct effects (the terms containing coefficients of variation alone) are much more relevant than the indirect effects (the terms containing a correlation term). This is at least true for TFP and labour market power, with large direct effects that moreover go in the same direction, and smaller indirect effects that moreover offset each other. For real wages, direct and indirect effects are roughly similar in size, both operating on the negative direction.Product

Figure 9: Stacked area plot for the decomposition of $\sum$



Notes: As equation (16) shows, $\sum$ is centered around one. For ease of visualisation, here we center it around zero. Positive (negative) terms are those above (below) zero, thereby increasing (decreasing) $\sum$. "Other terms" encompasses all terms in equation (16) inside $\sum$ not listed in the plot. It also considers all higher order terms not part of the approximation. Adding up all positive and negative terms yields $\sum$.
Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more.

market power is the exception, in that the indirect effect is much more relevant than the direct effect. This is due to the interesting combination of a relatively low coefficient of variation and a very high correlation between product market power and TFP (which itself has a high coefficient of variation).

To conclude this section, we can see what type of heterogeneity matters the most for the aggregate labour share (TFP and χ_i^L), what matters the least (χ_i^Y and τ_i), and what matters in between (wages and prices). Naturally, TFP and labour market power are grounded in idiosyncratic processes (e.g. organisational knowledge specific to the firm, or geographical amenities, respectively), more difficult to arbitrate across firms, and therefore with higher and more persistent heterogeneity. Conversely, prices are flexible, meaning less variation between firms and lower persistence of firm differentials. Luckily for our analysis, the term capturing the discrepancy between the data and our theory is irrelevant for our results.

6 Conclusions

This paper presented a novel approach to study the aggregate labour share, without relying on an aggregate production function. The method is based on a simple, yet powerful enough model of firm behaviour, which allows for a detailed decomposition of the aggregate labour share in terms of different dimensions of firm heterogeneity (TFP, product and labour market power, wages and output prices). The method characterises the aggregate economy by means of a weighted average firm, and quantifies heterogeneity with respect to such average. The main theoretical result presents the conditions under which firm heterogeneity affects the labour share. The role of the joint distribution of firm-level variables is captured in the decomposition formula in terms of the coefficient of variation for each variable and the correlation among variables. Importantly, the paper shows that firm heterogeneity matters in ways that are invisible when using models based on an aggregate production function. In this sense, our model provides a bridge between the micro and the macro approach to the analysis of the labour share.

To prove the value of the method, we applied the decomposition to a firm level dataset from the UK manufacturing sector, covering the 1998-2014 period. Descriptively speaking, the data indicates that the aggregate labour shares fell around 7% over the period, something that seems related mostly to a generalised fall in the firm level labour share across the firm size spectrum. Albeit the distribution of the labour share moved towards the left, the upper tail remained stable, implying an increase in the dispersion of the labour share.

The decomposition exercise produced two results. First, firm heterogeneity has a significant impact on the aggregate labour share: the weighted average labour share is around ten points lower than the aggregate labour share. Second, the fall in the aggregate labour share (7.3% over the period) is mostly accounted for by changes in the weighted average labour share. Indeed, the fall in the weighted average labour share is even bigger (9.3 %), indicating that the change in the dispersion of the firm-level determinants of the labour share has softened the downward trend.

Then, we provide further insights on the drivers of the observed fall in the weighted average labour share. We show that the pay-productivity gap widened over the period (and particularly after 2003), which alone can explain 90% of the change in the weighted average labour share (8.3 out of

9.3 percentage points). Firm market power (in the product and labour market) grew somewhat over the period too (particularly after the Great Recession), also contributing to the lower labour share.

Lastly, we look deeper into the factors that produce the wedge between the weighted average labour share and the aggregate labour share. This is, we look at what type of heterogeneity matters. The analysis reveals that TFP and labour market power are the two key sources of heterogeneity driving the wedge. The least relevant dimension is product market power heterogeneity (which is fairly low), with wages and price dispersion somewhere in between. This result seems intuitive enough. TFP and labour market power reflect phenomena which are much more difficult to arbitrate across space and time (e.g. because of some organisational knowledge specific to the firm, or the reduced mobility of workers across space). Conversely, product market power and real wages are rooted in prices, which by definition can adjust much quicker across space and time. Different degrees of persistence matter.

Some issues remain to be solved. In particular, even though our analysis benefits from relatively low degrees of (bi-variate) correlation across variables, our approach is still that of partial equilibrium. To get a more fundamental grasp of the deep drivers of our results, a general equilibrium analysis would be needed, something we leave for future research.

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A Alternative production functions and the aggregate labour share

The assumption of a constant returns to scale CES value added production function with homogeneous parameters is not arbitrary, but chosen for necessity and parsimony. To see this, consider the consequences for the decomposition formula of alternative assumptions. First, let us recall our main assumptions. The production function is

$$Y_i = A_i \left(\alpha L_i^{\rho} + (1-\alpha) K_i^{\rho}\right)^{\frac{1}{\rho}}$$

This implies the firm level labour share is

$$\lambda_i = \left(\frac{\alpha \chi_i^Y}{\chi_i^L}\right)^{\frac{1}{1-\rho}} \left(\frac{A_i p_i}{w_i}\right)^{\frac{\rho}{1-\rho}}$$

and the aggregate labour share is

$$\lambda = \sum_i \left(\frac{\alpha \chi_i^Y}{\chi_i^L}\right)^{\frac{1}{1-\rho}} \left(\frac{A_i p_i}{w_i}\right)^{\frac{\rho}{1-\rho}} \delta_i$$

In the case of a Cobb-Douglas production function (i.e. when $\rho = 0$), the latter is given by

$$\lambda = \sum_i \left(\frac{\alpha \chi_i^Y}{\chi_i^L}\right) \delta_i$$

Thus, there is no explicit role for productivity and real wages (and thus for the pay productivity disconnect) in the aggregate labour share; only market power affects the latter. This result holds even if there are non constant returns to scale.

Regarding the assumption of constant returns to scale (CRS) for the general CES, this is necessary for a decomposition to be possible. In effect, for a CES like the following:

$$Y_i = A_i \left(\alpha L_i^{\rho} + (1-\alpha) K_i^{\rho}\right)^{\frac{\nu}{\rho}}$$

it can be shown that the firm level labour share is

$$\lambda_i = \left(\frac{\alpha \nu \chi_i^Y}{\chi_i^L}\right)^{\frac{\nu}{\nu-\rho}} \left(\frac{A_i p_i}{w_i}\right)^{\frac{\rho}{\nu-\rho}} L_i^{\frac{\rho(\nu-1)}{\nu-\rho}}$$

This is, the labour share depends on the actual *level* of employment (except, of course, under CRS). To be consistent with our framework, where we replace value added and employment by their optimal values in terms of TFP, wages, prices and market power, we need to replace L_i above with its solution in terms of these same variables. This solution is a highly non-linear function, obtained from the combination of the FOC of profits with respect to labour and capital. The final expression for the firm's labour share is then not a neat, multiplicative function of each variable, and thus it is not possible to decompose using our method.

What if technologies are heterogeneous? If α varies across firms, it is still possible to achieve a clear decomposition. In one sense, α is like any other variable inside λ. It would be possible to compute a counterfactual $\bar{\alpha}$ and add it to λ^{HOM}. The problem is empirical. The model is already too complicated for it to be estimated, and adding non-linear firm specific parameters would not improve things (see equation 27 in Appendix D). Meanwhile, heterogeneity in ρ denies the possibility to arrive at a decomposition formula altogether. More precisely, there is no way to write terms like $\frac{A_i}{\bar{A}}$ and separate λ into a counterfactual firm and dispersion with respect to it, as in equation (8).

One might also question the use of a value added production function as a starting point, rather than, for instance, a CES gross output production function. To see this, consider such a function:

$$Q_i = B_i \left(\alpha L_i^\gamma + \beta M_i^\gamma + (1-\alpha-\beta) K_i^\gamma\right)^{\frac{1}{\gamma}} \tag{18}$$

where Q_i is gross output and M_i is intermediary inputs. The first order condition with respect to L_i is:

$$\frac{\partial Q_i}{\partial L_i} \equiv \alpha B_i^\gamma (Q_i)^{1-\gamma} (L_i)^{\gamma-1} = \left(\frac{w_i}{p_i}\right) \frac{\chi_i^L}{\chi_i^Y}$$

From here, we obtain a formula for optimal $\frac{Q_i}{L_i}$, given by:

$$\frac{Q_i}{L_i} = \left(\frac{w_i \chi_i^L}{\alpha B_i^\gamma p_i \chi_i^Y}\right)^{\frac{1}{1-\gamma}}$$

Repeating for intermediary inputs, we obtain:

$$\frac{Q_i}{M_i} = \left(\frac{p_i^M}{\beta B_i^\gamma p_i \chi_i^Y}\right)^{\frac{1}{1-\gamma}}$$

where p_i^M is the price of intermediary inputs. Now, the labour share for the firm is given by:

$$\lambda_i \equiv \frac{w_i L_i}{p_i Q_i - p_i^M M_i} = \frac{w_i}{p_i \frac{Q_i}{L_i} - p_i^M \frac{M_i}{L_i}}$$

where nominal value added is defined as $p_i Q_i - p_i^M M_i$. Combining the above results, it is possible to show that λ_i is:

$$\lambda_i = \left(\frac{\alpha}{\chi_i^L}\right)^{\frac{1}{1-\gamma}} \frac{w_i^{\frac{\gamma}{1-\gamma}}}{(\beta p_i)^{\frac{\gamma}{\gamma-1}} \left(\chi_i^Y\right)^{\frac{1}{\gamma-1}} - \beta^{\frac{1}{1-\gamma}} \left(p_i^M\right)^{\frac{\gamma}{\gamma-1}}}$$

It is evident that the above is unhelpful in achieving a decomposition of the aggregate labour share, not even in the special cases of a Cobb-Douglas or Leontief gross output production function ($\gamma = 0$ or $\gamma = -\infty$, respectively). An alternative is to specify the production function in equation (18) as a nested CES, as follows:

$$Q_i = B_i \left(b Y_i{}^\gamma + (1-b) M_i{}^\gamma\right)^{\frac{1}{\gamma}} \tag{19}$$

where $Y_i = A_i \left(\alpha L_i^\rho + (1-\alpha) K_i^\rho\right)^{\frac{1}{\rho}}$, this is, a CES of labour and capital. Bruno (1978) shows that, if this gross output production function is either linear or Leontief (i.e. $\gamma = -\infty$ or $\gamma = 1$), Y_i corresponds to real value added (where both output and inputs are deflated). In other words,

$Y_i = f(L_i, K_i)$ is a value added production function. Such is the implicit assumption in our model, one which unfortunately cannot be tested, due to the complexity of applying the dynamic panel method (or the control function method) to the production function in equation (19).

It must be noted here that, albeit it is tempting to estimate equation (19) using value added directly as an input in equation (19) (rather than the nested CES with capital, labour and intermediary inputs) this is spurious. Value added is *defined* as gross output minor intermediary inputs (properly deflated). Estimating such equation will produce (and does produce, in our data) $\hat{B}_i \approx 1$, $\hat{b} \approx 0.5$ and $\hat{\gamma} \approx 1$. The latter might suggest the conditions for the existence of a value added production function set out in Bruno (1978) are met. Yet, the problem with this approach is that when using value added as an input, the existence of such production function is actually imposed in the equation. In effect, replacing the estimates given above into equation (19) leads to $Q_i \approx Y_i - M_i$, which is the identity for the definition of real value added. In other words, we are merely estimating an identity.

B Statistical decomposition

The aggregate labour share is defined as a weighted average of firms' labour share:

$$\lambda^{obs} = \sum_i \delta_i \lambda_i$$

where δ_i is the total economy's share of value added of firm i. As the sample size grows, sample moments converge to population moments (ultimately, if we were to have a census of all firms, the two would be the same, provided no other issues like measurement errors exist). One such moment is $E(\delta\lambda)$, for which the Law of Large Numbers states that

$$\lim_{N\to\infty} \frac{\lambda^{obs}}{N} = \mathrm{E}(\delta\lambda)$$

Using the formulas from the covariance, and replacing population moments with sample equivalent, it is trivial to show that

$$\lambda^{obs} = \hat{\mathrm{E}}(\lambda) + N\hat{\mathrm{Cov}}(\delta, \lambda) \tag{20}$$

where $\hat{\mathrm{E}}(\lambda)$ is the observed unweighted average labour share. Since $\hat{\mathrm{E}}(\delta) = \frac{1}{N}$, it follows that

$$\lambda^{obs} = \hat{\mathrm{E}}(\lambda) + \hat{\mathrm{E}}(\lambda)\hat{\mathrm{Corr}}(\delta, \lambda) \tag{21}$$

Therefore, the weighted over the unweighted average labour share is:

$$\frac{\lambda^{obs}}{\hat{\mathrm{E}}(\lambda)} = 1 + \hat{\mathrm{Corr}}(\delta, \lambda) \tag{22}$$

This ratio is smaller the more negative the correlation between firm size (in terms of value added) and labour share is, *ceteris paribus*.

C Effects of changes at one specific firm

C.1 Effects on the firm level labour share

How the firm level labour share reacts to changes in prices, wages, technology, market power and capital depends on ρ. By taking the partial derivative of (6) with respect to each variable, it is immediate to identify the effects of the different (firm level) variables, which are summarised in Tables A1 and A2, for $\rho < 0$ and $\rho > 0$ respectively. For simplicity, the two forms of market power are combined in one term, with $\chi_i = \frac{\chi_i^L}{\chi_i^Y}$.

Table A1: Determinants of the firm level labour share, $\rho < 0$.

		Sign of the effect
$\lambda_i'(A_i)$		–
$\lambda_i'(K_i)$		0
$\lambda_i'(w_i)$	originated by a change in Θ_i^L	+
$\lambda_i'(p_i)$	originated by a change in Θ_i^Y	–
$\lambda_i'(\chi_i)$	direct effect	–
$\lambda_i'(\chi_i)$	indirect effect through prices	–
$\lambda_i'(\chi_i)$	indirect effect through wages	–
$\lambda_i'(\chi_i)$	overall effect	–

Table A2: Determinants of the firm level labour share, $\rho > 0$.

		Sign of the effect
$\lambda_i'(A_i)$		+
$\lambda_i'(K_i)$		0
$\lambda_i'(w_i)$	originated by a change in Θ_i^L	–
$\lambda_i'(p_i)$	originated by a change in Θ_i^Y	+
$\lambda_i'(\chi_i)$	direct effect	–
$\lambda_i'(\chi_i)$	indirect effect through prices	+
$\lambda_i'(\chi_i)$	indirect effect through wages	+
$\lambda_i'(\chi_i)$	overall effect	?

Note that when analysing the *ceteris paribus* effects of a change in wages or output price on λ_i, we are assuming that they were originated by a change in the idiosyncratic preference parameters Θ_i^L and Θ_i^Y, and not in a change in market power χ_i. Market power, on the other hand, has two effects: a *direct* effect, and an *indirect* effect through prices and wages. The direct effect of market power on the labour share is negative (see the first term of equation 6). Said differently, a lower $|\eta_i^Y|$ or η_i^L, which are equivalent to a higher market power χ_i for the firm, translate into a lower labour share. When $\rho < 0$, the indirect effect goes in the same direction, so that the overall effect of market power is unambiguously negative. Note also that the amount of capital employed, given the CRS assumption, does not affect λ_i.

Table A1 or variants obtained with different theoretical assumptions informed most of the micro-econometric investigations on the determinants of the firm-level LS.

C.2 Effects on the aggregate labour share

The effects of a variation in one firm level variable on the aggregate labour share, keeping all other firms unchanged, depend among other things on whether the firm considered has an above-average or below-average labour share. Recall that the aggregate labour share is a weighted average of the individual labour shares (equation 2), and write it as

$$\lambda = \lambda_J(1 - \delta_i) + \lambda_i \delta_i \tag{2'}$$

where $\lambda_J = \sum_{j \neq i} \lambda_j \frac{1-\delta_j}{1-\delta_i}$ is the average labour share excluding firm i.

The change in the aggregate labour share following from changes in the production plans at firm i is therefore given by:

$$\lambda'(\cdot) = (\lambda_i - \lambda_J)\delta_i'(\cdot) + \lambda_i'(\cdot)\delta_i \tag{23}$$

The direction of change in λ depends therefore on (i) how the labour share changes at firm i, (ii) how the relative weight of firm i changes, and (iii) whether the labour share at firm i is above or below the average.

Note that equation (23) does not depend on firm i having the same production function (e.g the same α and ρ) of any other firm j. What originates the change in λ is simply a change in λ_i for a given firm i, and a change in the weight of that individual firm in the whole economy: no changes occur to any other firm, so how the labour share is determined in those other firms does not matter. These simple results provide the basics for understanding the effects of simultaneous changes taking places at different firms. Each change in firm level variables pulls in a direction as identified by Table A1 and A2, and the overall effect is nothing else than a simple composition of all these individual effects.

We know from Tables A1 and A2 how λ_i reacts to changes in firm level variables. As for what concerns the weights, these depend, keeping all other firms fixed, on the value added of firm i. From equation (3) we know that value added Y_i grows with A_i, L_i and K_i, while it is easy to show from equation (4) that employment at firm i expands as productivity A_i, prices p_i (keeping market power constant) and capital K_i increase, and contracts as wages w_i (keeping market power constant), and market power χ_i (keeping wages fixed) increase. Putting all this together, by the envelop theorem we get that the relative weight of firm i in the economy increases with A_i, K_i and p_i, and decreases with w_i, keeping market power constant.

The latter has an ambiguous effect on market share. When $\rho < 0$, the partial derivative of Ω_i with respect to χ_i is negative: the direct effect of market power is thus to reduce market share. However, an increase in market power originates either from a decrease in $|\eta_i^Y|$ (output is more rigid) or from a decrease in η_i^L (labour supply is more rigid), translating either into a higher price or into lower wages. Both increase production and hence market share, counteracting the negative direct effect.

The overall effects of changes in the characteristics of one firm on the aggregate labour share are reported in Tables A3 and A4, for $\rho < 0$ and $\rho > 0$ respectively.

Table A3: Effects of changes in the characteristics of one firm on the aggregate labour share, $\rho < 0$.

		δ_i'	λ_i'	λ'	
				$\lambda_i > \lambda_J$	$\lambda_i < \lambda_J$
A_i		+	−	?	−
K_i		+	0	+	−
w_i	originated by a change in Θ_i^L	−	+	?	+
p_i	originated by a change in Θ_i^Y	+	−	?	−
χ_i	direct + indirect effect	?	−	?	?

Table A4: Effects of changes in the characteristics of one firm on the aggregate labour share, $\rho > 0$.

		δ_i'	λ_i'	λ'	
				$\lambda_i > \lambda_J$	$\lambda_i < \lambda_J$
A_i		+	+	+	?
K_i		+	0	+	−
w_i	originated by a change in Θ_i^L	−	−	−	?
p_i	originated by a change in Θ_i^Y	+	+	+	?
χ_i	direct + indirect effect	?	?	?	?

Note that, in the empirically relevant case of $\rho < 0$, an increase in the productivity of a top firm, with a lower-than-average labour share to start with, unequivocally translates into a decrease in the aggregate labour share (last column of table A3). This is consistent with the theory of the superstar firms (Autor et al., 2017b). Interestingly, the same is not true for an increase in market power, as the effects on the market share are indeterminate.

D Estimation of value added production function

As said in the main text, TFP and the production function parameters are not observed in the data. We draw from the abundant literature on estimating production functions in order to compute these missing terms.

The point of departure in this analysis is the fact that A_{it} is not observed by the econometrician, but might be observed by firms. Thus, if firms choose inputs based on their productivity level, a simple estimation of TFP using least squares would suffer from endogeneity. Since the seminal paper by Olley and Pakes (1996), several techniques have been put forward to address the endogeneity problem that might exist when estimation productivity using micro data.[40] Here we follow the dynamic panel approach (e.g. Blundell and Bond, 2000), where endogeneity is eliminated by assuming TFP follows an AR(1) process with parameter θ, and then the main model is θ-differentiated. The dynamic panel approach has some advantages with respect to other methods, including less stringent

[40] For a survey, see Ackerberg et al. (2007).

data requirement, expanding the sample size.[41] In short, the method works as follows. We start with our production function, extended to an econometric notation:

$$Y_{it} = e^{\omega_{it}} \left(\alpha L_{it}^{\rho} + (1-\alpha) K_{it}^{\rho}\right)^{\frac{1}{\rho}} e^{\epsilon_{it}} \tag{24}$$

where for convenience we have defined $A_{it} \equiv e^{\omega_{it}}$, and where ϵ_{it} is a idiosyncratic *iid* shock to output. As the rest of the literature, we assume ω_{it} follows a first-order Markov process. This is, $\omega_{it} = E\left[\omega_{it}|\omega_{it-1}\right] + \xi_{it}$, where ξ_{it} is an idiosyncratic *iid* shock to productivity, known to the firm. Taking logs of (24), we get:

$$y_{it} = \frac{1}{\rho} \ln\left(\alpha L_{it}^{\rho} + (1-\alpha) K_{it}^{\rho}\right) + \omega_{it} + \epsilon_{it} \tag{25}$$

The common assumption in the literature about the informational setting is that capital is a state variable (in the sense that it is chosen in period $t-1$), whereas labour is a flexible factor (in the sense that it can be chosen in period t). This informational structure is relevant because under the assumption that a firm knows its shock to productivity (ξ_{it}), labour is correlated with the unobserved (by the econometrician) error term, and hence endogenous in equation (25). Non-linear least squares would then yield inconsistent results.

To move further, we align with the literature by assuming ω_{it} follows an AR(1):

$$\omega_{it} = \theta\omega_{it-1} + \xi_{it} \tag{26}$$

If we combine equations (24) and (26) (i.e. if we "θ-differentiate" the production function), we get:

$$y_{it} = \frac{1}{\rho} \ln\left(\alpha L_{it}^{\rho} + (1-\alpha) K_{it}^{\rho}\right) + \theta \left[y_{it-1} - \frac{1}{\rho} \ln\left(\alpha L_{it-1}^{\rho} + (1-\alpha) K_{it-1}^{\rho}\right)\right] + \xi_{it} + (\epsilon_{it} - \theta\epsilon_{it-1}) \tag{27}$$

Thus, "θ-differencing" the model eliminates unobserved productivity from the equation. The above can then be estimated using GMM.

It is important to notice here that the above model is highly non-linear. In effect, most of the literature estimates Cobb-Douglas production functions, which are log-linear in parameters. Unfortunately, in our data GMM does not converge. Hence, in practice, we estimate a translog production function, which is an approximation of the CES around an elasticity of substitution

[41] Another common approach is the control function method, based on Olley and Pakes (1996) and Levinsohn and Petrin (2003). This semi-parametric method is based on stronger assumptions and requires greater data availability than the dynamic panel approach. The latter is the case because the control function method relies on past values of investment (in the case of Olley and Pakes, 1996), or intermediary inputs (in the case of Levinsohn and Petrin, 2003) as instruments, which in our sample are only available when a firm is selected for the survey (say, in period t). Conversely, we can implement the dynamic panel approach using past values of employment and capital as instruments, without need for investment or intermediary inputs. ARD has a companion dataset with the universe of firms for all years, including minimal data like employment (dataset built from administrative tax registry data). Similarly, using the perpetual inventory method, we can compute capital in $t-1$ for every firm sampled in t. Therefore, we are able to extend the sample with past values of the key instruments, thereby expanding the sample size. In any case, for comparative purposes, we also estimated the production function with the control function approach, using intermediate inputs as proxy. Unfortunately, it yielded invalid results (in terms of parameter outside the theoretical domain).

equal to 1.[42] This production function is:

$$y_{it} \approx \ln(A_{it}) + \alpha \ln(L_{it}) + (1-\alpha)\ln(K_{it}) + \frac{\rho\alpha(1-\alpha)}{2}\ln^2(L_{it}) \\ -\rho\alpha(1-\alpha)\ln(L_{it})\ln(K_{it}) + \frac{\rho\alpha(1-\alpha)}{2}\ln^2(K_{it}) + \mu_{it}$$

The final model estimated by GMM is obtained by "θ-differencing" the equation above, with instruments $\{\ln(K_{it}), \ln^2(K_{it}), \ln(K_{it-1}), \ln^2(K_{it-1}), \ln(L_{it-1}), \ln^2(L_{it-1}), \ln(L_{it-1})\ln(K_{it-1})\}$. Estimation produces the following values, all significant at the 1%:[43] $\hat{\alpha} = 0.38$, $\hat{\rho} = -1.18$, $\hat{\theta} = 0.92$, and a constant of 3.8.

Finally, having estimated α and ρ, we can use equation (24) to compute $\hat{A}_{it}$ as a residual, for every firm and period. Importantly, since the productivity shock (ξ_{it}) cannot be identified separately from the idiosyncratic shock to value added (ϵ_{it}), $\hat{A}_{it}$ also includes the realised shock to value added, $\hat{\epsilon}_{it}$. In effect, from equation (24) we can see that $\hat{A}_{it} = e^{c+\hat{\omega}_{it}+\hat{\epsilon}_{it}}$ (where c is the constant term in the regression). This means that $\hat{A}_{it}$ is a biased predictor of A_{it}. Nevertheless, as long as the variance of ϵ_{it} is constant over time, such bias is constant too, not affecting the decomposition, which focuses on changes over time.[44] Notice equation (24) allows us to compute the "realised" value of A even for observations not part of the regression sample (for example, because of missing data in a given year). We follow this approach, and "extrapolate" $\hat{A}_{it}$ whenever possible. Around 50% of final observations used in the analysis are extrapolated.

Capital stock

Having outlined the estimation procedure for the production function, we should mention that firm-level capital stock is not available in the dataset. Nonetheless, firms report information on their capital expenditures (investment) for a variety of assets like buildings, vehicles, and so on. One method often used to compute capital at the firm level is the perpetual inventory method. Whilst this is a good approximation for firms that are *observed* to be born during the sample period (i.e. for those which are sampled during their first year of existence), for firms that do not (in our sample, 99.99% of firms), the level of capital may be greatly underestimated with such method.

Instead, we follow the strategy proposed by Chen and Plotnikova (2014), which estimates capital at the firm level using the aggregate level of capital stocks in the manufacturing sector (obtained from the Office for National Statistics). First, we select a few "proxy" variables, which are likely to be positively correlated with unobserved firm-level capital, and are observed both at the firm and at the aggregate level. We use intermediate inputs and employment. Then, we estimate the "structural relationship" between these proxies and capital (based on an assumed stability of their

[42] As commented earlier, Monte Carlo simulations in Lagomarsino (2017) show that the non-linear translog used here is a very good approximation of the underlying CES, for ρ close to or below 1, as it is our case (see end of appendix).

[43] Since capital is a generated regressors (see next subsection), standard errors are based on bootstrap estimates, with 1000 replications.

[44] See footnote 35 for further details.

joint distribution).[45] This relationship is given by the following formula:

$$K_{it} = \left(\frac{L_{it}}{L_t}\right)^a \left(\frac{M_{it}}{M_t}\right)^{1-a} K_t \tag{28}$$

where L_t, M_t, and K_t represent the observed values of employment, intermediate inputs and capital in the whole of manufacturing sector in year t; parameter a accounts for the relative importance of each proxy in the structural relationship. This parameter is assumed constant over time.

In practice, a is unknown. Furthermore, this cannot be estimated from equation (28), since K_{it} is also unknown. The solution is to combine equation (28) with that of capital accumulation, namely $K_{it} = (1-\delta)K_{it-1} + I_{it}$, where I_{it} is firm level investment (available in the dataset), and d is the depreciation rate of the capital stock in manufacturing. This leads to the following equation:

$$I_{it} = \left(\frac{L_{it}}{L_t}\right)^a \left(\frac{M_{it}}{M_t}\right)^{1-a} K_t - (1-\delta)\left(\frac{L_{it-1}}{L_{t-1}}\right)^a \left(\frac{M_{it-1}}{M_{t-1}}\right)^{1-a} K_{t-1} \tag{29}$$

The above can be estimated using GMM. Results for the whole manufacturing sector yield $\hat{a} = 0.42$, significant at 1%.[46] With this value is then possible to impute capital at the firm level using equation (28). Notice this imputation allows for extrapolation from the estimation sample to firms which are not observed in consecutive years (condition required by the regression), or which are sampled only in one year. The extrapolation is valid as long as the "structural relationship" does not depend on properties of the sample selection (for instance, firm size).

E Decomposition results for manufacturing sub-sectors

The main text presented the decomposition analysis for the whole manufacturing sector. Here, we repeat the main exercise for manufacturing sub-sectors, defined as 2 digit SIC07 (divisions). Instead of assuming a common production function across sub-sectors, we estimate the (translog) production function for each division separately.

Unfortunately, only 13 out of 23 sub-sectors produced meaningful results (in terms of parameters within the theoretical boundaries), suggesting not every sub-sector might be represented by a CES/translog production function.[47] Overall, the 13 sub-sectors cover 62% of the total observations (firm-years) available across the manufacturing sector, and used in the main text.

Table A5 presents the decomposition for each sub-sector's labour share. Additionally, the table presents an extra row ("combined sub-sectors") with the decomposition of an aggregate series of λ^{obs}, computed from a weighted average of sub-sectors' λ^{obs}, using value added as weights. For comparison, another row is added with the results for the whole manufacturing sector presented

[45] In principle, this relationship is not testable, since we lack capital data at the firm level. However, the correlation between capital estimated with this method and the perpetual inventory method, for firms observed to be born in the sample period, is 0.56. This is a significantly high correlation, considering that new firms are likely to be significantly different that established firms, e.g. in terms of their investment patters.

[46] The depreciation rate is assumed to be 4.58%, the average for the 1998-2014 period, according to Office for National Statistics data for the UK.

[47] In particular, a meaningful result is one where ρ is not greater than 1 (for which the elasticity of substitution is properly defined), and where α is between 0 and 1 (otherwise, one factor of production would have negative marginal product).

in the main text. Finally, to give a sense of the importance of different sub-sectors, the table includes an extra column with the 2014's share of value added of each sub-sector with respect to all manufacturing.

The overall picture is the same as in our results for the whole manufacturing sector, namely that firm heterogeneity has not been a major driver of the labour share. This is true both for sub-sectors individually and for their combination. The latter decomposition is also quite similar to the results for manufacturing as a whole. Still, some disparity is observed in $\sum$ across sub-sectors, both in terms of direction of change and magnitude, with most of the effect of heterogeneity going against the observed change in the labour share (just like in the main results). Notice also that the labour share went up in some sub-sectors, albeit fell in most of them.

Finally, Table A6 shows the decomposition of λ^{HOM} across sub-sectors (similar to Table 3).[48] In line with results at the aggregate level, the key driver of the homogeneous labour share (and thus of the sub-sector labour shares) is the disconnect between pay and productivity. In most sub-sectors, productivity grew faster than real wages. Exceptions are sub-sectors 26 and 27 ("manufacture of computer, electronic and optical products" and "manufacture of electrical equipment", respectively), where wages grew faster than productivity, and sub-sector 33 ("repair and installation of machinery and equipment"), where both productivity and wages shrank over the period, the former more than the latter.

Regarding market power, there are differences with respect to the results for the whole sector. Whereas in the latter both product and market power had an equally minor role in λ^{HOM}, in most sub-sectors the contribution of labour market power is significantly greater than that of product market power. In fact, in some sub-sectors the change in labour market power is high enough to make a significant difference to the sub-sector's λ^{HOM}. For instance, in sub-sector 26, the pay-productivity disconnect changed very little over the period; it is $\bar{\chi}^L$ which defines the bulk of the change. In particular, the labour market power of firms in this sub-sector fell importantly over the period.[49]

Overall, sub-sector and industry-wide results differ only where the latter masks heterogeneity in the former. As Table A6 reveals, this is particularly relevant for labour market power, which contribution contains both large positive and negative values. Conversely, variables like TFP and real wages have the same sign in all sub-sectors but one (sub-sector 33).

[48] Unlike in Table A5, no decomposition is shown for the "combined" sub-sectors because this requires an estimate of ρ, which was only estimated at the sub-sector levels.

[49] Recall from the growth accounting decomposition that $g_{\bar{\chi}^L}$ is multiplied by $-\left(\frac{1}{1-\rho}\right)$. The estimated ρ for this sub-sector is negative, which, combined with a fall in $\bar{\chi}^L$ (i.e. a fall in firms' labour market power) yields the positive contribution of this variable to λ^{HOM}, as Table A6 shows.

Table A5: Contribution to changes in the sub-sectoral labour share (λ^{obs}), 1998-2014

Sub-sector	λ^{obs}	λ^{HOM}	$\sum$	Interaction	Sub-sector's share of value added (2014)
				%	
13 (Manufacture of textiles)	-9.21	-9.38	0.19	-0.02	1.4
14 (Manufacture of wearing apparel)	-25.02	-33.28	12.38	-4.12	0.4
16 (Manufacture of wood and products of wood and cork, excl. furniture)	-10.28	-15.61	6.32	-0.99	1.8
17 (Manufacture of paper and paper products)	-8.67	-9.40	0.81	-0.08	2.4
18 (Printing and reproduction of recorded media)	-5.59	-6.09	0.54	-0.03	3.2
23 (Manufacture of other non-metallic mineral products)	-10.45	-7.08	-3.63	0.26	3.6
25 (Manufacture of fabricated metal products, excl. machinery and equip.)	-11.47	-13.84	2.75	-0.38	11.1
26 (Manufacture of computer, electronic and optical products)	4.16	7.35	-2.97	-0.22	5.4
27 (Manufacture of electrical equipment)	1.84	8.81	-6.40	-0.56	3.1
28 (Manufacture of machinery and equipment n.e.c.)	-6.46	-7.39	1.00	-0.07	8.4
29 (Manufacture of motor vehicles, trailers and semi-trailers)	-19.49	-21.27	2.27	-0.48	10.5
30 (Manufacture of other transport equipment)	-11.22	-19.61	10.43	-2.04	6.6
33 (Repair and installation of machinery and equipment)	10.44	11.12	-0.61	-0.07	3.9
Combined sub-sectors	-8.75	-9.39	0.69	-0.05	61.9
All manufacturing	-7.36	-9.31	2.16	-0.20	100

Note: $g_{\lambda^{obs}} = g_{\lambda^{HOM}} + g_{\sum} +$ interaction effect.
Source: Our calculation based on ARD data.
Sample: UK manufacturing firms with 10 employees or more. Sub-sectors represent 2 digit (division) SIC07 codes. Sub-sectors which estimates were spurious and thus omitted are 10 ("manufacture of food products"), 11 ("manufacture of beverages"), 12 ("manufacture of tobacco products"), 15 ("manufacture of leather and related products"), 20 ("manufacture of chemicals and chemical products"), 21 ("manufacture of pharmaceutical products"), 22 ("manufacture of rubber and plastic products"), 31 ("manufacture of furniture") and 32 ("other manufacturing"). Sub-sector 19 ("manufacture of coke and refined petroleum") is omitted from main analysis and thus also omitted here.

Table A6: Contribution to changes in sub-sectoral λ^{HOM}, 1998-2014

Sub-sector	λ^{HOM}	$\bar{A}$	$\bar{w}/\bar{p}$	$\bar{\chi}^{Y}$	$\bar{\chi}^{L}$	$\bar{\tau}$	Interaction
				%			
13	-9.38	-33.27	24.61	-2.24	-2.33	0.54	3.31
14	-33.28	-135.16	65.08	-3.77	26.50	-34.21	48.28
16	-15.61	-22.44	7.68	-2.21	4.85	-6.97	3.49
17	-9.40	-28.04	14.11	-0.61	1.26	-4.88	8.76
18	-6.09	-19.68	11.57	-1.62	-3.38	8.87	-1.84
23	-7.08	-13.25	9.68	0.84	-0.12	-5.19	0.96
25	-13.84	-19.79	9.11	-3.73	-7.29	3.90	3.96
26	7.35	170.52	-148.98	-10.04	81.55	-7.60	-78.10
27	8.81	-7.83	17.43	0.20	10.21	-11.53	0.33
28	-7.39	-34.00	23.65	-0.88	1.43	-3.29	5.70
29	-21.27	-86.78	55.69	0.45	-15.63	3.04	21.97
30	-19.61	-14.03	0.49	-1.75	-0.61	-7.56	3.87
33	11.12	4.93	-0.30	0.12	17.37	-14.92	3.90

Note: Effects are attributed as per eq. (17). The extreme behaviour of sub-sector 14 is due to a significant reduction in the sample size available, from 340 firms in 1998 to 56 firms in 2018. Meanwhile, large numbers in sub-sector 26 reflect the rapid fall in this sector's output prices, between 1998 and 2005.
Source: Our calculation based on ARD data.
Sample: See Table A5.