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A Simple Model of Trade with Heterogeneous Firms and Trade Policy

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Abstract

This paper builds a Ricardian-Chamberlinian two-country model with heterogeneous firms in a monopolistically competitive sector in which every new entrant faces increasing fixed costs of production. There are efficiency gaps between countries in marginal and fixed costs and a country unilaterally imposes an import tariff. It is shown that an increase in tariff increases the number of firms of the tariff imposing country while decreases the number of firms of the tariff-imposed country, possibly reverting the position of net exporter of varieties. A tariff is detrimental to the tariff-imposed country. A small tariff may be beneficial to the tariff-imposing country.

JEL Classification: D43, F12, F13

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1 Introduction

The literature on trade that takes into account firm heterogeneity has been flourishing lately, especially after the path-breaking work of Melitz (2003). It is expected that the opening of an economy to trade reallocates firms in the market since in the real world production technologies possessed by firms differ within and across industries. In such cases, the evidence provided by empirical studies is that more efficient firms would survive - and even expand in scale - whereas less efficient firms would be driven out of the market unless they raise productivity. As a result, trade liberalization would raise overall productivity. Recent developments on general equilibrium models of trade have started to provide the analytical apparatus to fill the gap between empirical observation and theory.

In this context, extensions of the Dixit-Stiglitz-Krugman love-of-variety approach\(^1\) have proved to be fairly tractable in offering a justification for the existence of firms with different productivity levels. Melitz (2003) focus on the forward-looking behavior of the firm under productivity uncertainty and analyzes the impact of trade liberalization on the productivity of firms and their reallocation in the market. In a Chamberlinian-Ricardian model with cost heterogeneity and transport costs, Venables (1987) develops a theory of trade to explain the presence of firms with varying market shares and the effects of industrial policy. The model considers asymmetric preferences over differentiated products, which also plays a role in determining market shares.\(^2\) Similar to marginal costs, heterogeneous fixed costs may be important factors determining industrial reallocations. That is the focus of Kikuchi and Shimomura (2007), who analyzes the effects on trade patterns caused by efficiency gaps in fixed costs between countries.

This note aims to clarify the role of tariffs as industrial policy when firms in the same sector feature heterogeneity in fixed costs and there is efficiency gap between countries. In a Ricardian-Chamberlinian setting, a two-country-two-sector model is built with one country unilaterally imposing an import tariff. It concludes that,\(^3\)

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\(^1\)See Helpman and Krugman (1985).

\(^2\)Similarly, Montagna (2001) extends the D-S-K model to allow for different love-of-variety levels. She clarifies how productivity heterogeneity leads to a endogenous determination of industry efficiency.
a tariff increase always increase the number of firms of the tariff-imposing country while decreases the number of the tariff-imposed country, with the possibility of reverting a country’s position of net exporter of differentiated to net importer. Also, it concludes that tariffs, when small, are beneficial to the tariff-imposing country and always detrimental to the tariff-imposed country.\(^3\) This paper provides a tractable model of monopolistic competition that includes heterogeneity of firms, positive profits and trade barriers, which enables further policy-related analyses on, for instance, trade agreements, multinational firms and other related issues without losing one important feature: simplicity.

The next section develops the model, comparative statics are performed in Section 3, Section 4 discusses trade policy, Section 5 discusses the effects of tariffs on welfare, and the last section concludes this work.

## 2 The Model

In this section the basic trade model is developed so as to include fixed cost heterogeneity, efficiency gaps between countries and tariff, while keeping tractability.

There are two countries, Home and Foreign. They are identical in terms of consumers’ preferences and factor endowments but may differ in production technologies. Labor is the only factor of production and is employed in the two sectors of the economy: a constant-returns competitive sector producing a homogeneous product and a monopolistically competitive sector producing a large number of differentiated products. The competitive sector is large and the homogeneous good is taken as numeraire.

Consumers have preferences denoted by the following utility function \(U\):

\[
U = \epsilon^{-1} D^\epsilon + Y, \quad 0 < \epsilon < 1,  
\]

where \(\epsilon\) is a parameter, \(D\) is the quantity index of the monopolistically competitive sector and \(Y\) is the consumption level of the numeraire good. This specification

\(^3\)Tariffs also represent an important source of revenue for some countries. It is well known that a small positive tariff can be welfare-enhancing even in a competitive market setting. See Helpman and Krugman (1989) for example.
implies that differentiated products are not subject to income effects. Demands for differentiated products are derived from consumer’s preferences and follow the Dixit-Stiglitz specification. Then the quantity index takes the CES form

\[ D = \left( \sum_{k=1}^{n} (d_i)^{\theta} + \sum_{k=1}^{n^*} (d_{i^*})^{\theta} \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \]

where \( n \) is the number of differentiated products produced at Home (Foreign), \( d_i \) is the demand for product \( i \), and \( 1/(1 - \theta) \) is the elasticity of substitution between every pair of products. Foreign variables and coefficients are indicated with \((*)\). The price index takes the form

\[ P = \left( \sum_{k=1}^{n} (p_i)^{\theta/(\theta - 1)} + \sum_{k=1}^{n^*} (p_{i^*})^{\theta/(\theta - 1)} \right)^{\frac{(\theta - 1)/\theta}{\theta - 1}}, \]

where \( p_i \) is the price of the \( i \)-th differentiated product produced at Home and \( p_{i^*} \) is the price of the \( i^* \)-th differentiated product produced at Foreign.

The utility maximization problem is solved in two steps. First, by minimizing the cost of attaining a given value of the quantity index \( D \) the following demands are derived:

\[ d_i = (p_i/P)^{1/(\theta - 1)} D, \quad i = 1, \ldots, n, \]

\[ d_{i^*} = (p_{i^*}/P)^{1/(\theta - 1)} D, \quad i^* = 1, \ldots, n. \]

In the second step consumers maximize utility by dividing income between differentiated and homogeneous products. Then we obtain

\[ D = P^{1/(\theta - 1)}. \]

Combining (4), (5) and (6) we obtain the following Home demand functions for the Home-produced differentiated product \( i \) and Foreign-produced differentiated
product $i^*$:

$$d_i = (p_i)^{1/((\theta-1)} \left( \sum_{k=1}^{n} (p_k)^{\theta/(\theta-1)} + \sum_{k=1}^{n^*} (p_k^*)^{\theta/(\theta-1)} \right)^{(\theta-\epsilon)/[\theta(\epsilon-1)]}$$  \quad (7)

$$d_{i^*} = (p_{i^*})^{1/((\theta-1)} \left( \sum_{k=1}^{n} (p_k)^{\theta/(\theta-1)} + \sum_{k=1}^{n^*} (p_k^*)^{\theta/(\theta-1)} \right)^{(\theta-\epsilon)/[\theta(\epsilon-1)]}.$$  \quad (8)

Note that if $\epsilon > \theta$ differentiated products are complements, thus we assume $\epsilon < \theta$.

Turning to the supply side, firms in the monopolistically competitive sector faces intraindustry heterogeneity in fixed costs, with firm $i$ at Home facing a fixed cost $\alpha(i)$ and firm $i^*$ at Foreign facing the fixed cost $\alpha^*(i^*)$. There is also international asymmetry in marginal costs with $\beta$ for Home firms and $\beta^*$ for Foreign firms. With a sufficiently large number of firms, the elasticity of demand for each product becomes $1/(1 - \theta)$.

In order to verify the effects of trade barriers in a simplified way, we assume that Home unilaterally imposes a uniform import tariff $\tau$ to Foreign differentiated products.\footnote{Bilateral trade imposition can also be considered. Note, however, that our simplification does not change the qualitative results of the model.} From the assumption of symmetry in marginal costs of differentiated products, consumer’s prices at Home for Home-produced and Foreign-produced differentiated products become, respectively,

$$p_i = \beta / \theta, \quad p_{i^*} = (1 + \tau)\beta^*/\theta,$$

and consumer’s prices at Foreign for Home-produced variety $i$ and Foreign produced variety $i^*$ become

$$p_i^* = \beta / \theta, \quad p_{i^*}^* = \beta^* / \theta.$$

Note that firms price at a markup over their marginal costs and tariffs.

Rearranging $\beta^* = a\beta$, when $n$ firms are active at Home and $n^*$ at Foreign the summation of equation (3) takes the following form for Home and Foreign,
respectively:

\[
\sum_{k=1}^{n} (p_k)^{\theta/(\theta-1)} + \sum_{k=1}^{n^*} (p_{k^*})^{\theta/(\theta-1)} = \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta-1}} \left[ n + n^* (1 + \tau)^{\frac{\theta}{\theta-1}} a^{\frac{\theta}{\theta-1}} \right]
\]

(9)

\[
\sum_{k=1}^{n} (p_k)^{\theta/(\theta-1)} + \sum_{k=1}^{n^*} (p_{k^*})^{\theta/(\theta-1)} = \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta-1}} \left[ n + n^* a^{\frac{\theta}{\theta-1}} \right].
\]

(10)

It is clear that the assumption of asymmetry in fixed costs of firms producing differentiated products plays a crucial role in determining profits. For the sake of simplification, let the efficiency ranking in fixed costs assume the form \(\alpha(i) = \mu i\) for Home firms and \(\alpha^*(i^*) = \mu^* i^*\) for Foreign firms, implying that the firm with the lowest index is the most efficient. Under this assumption firms of the same country have equal revenues from both markets but have different fixed costs. Then, the profit functions \(\pi_i\) and \(\pi_{i^*}\) for firm \(i\) at Home and \(i^*\) at Foreign, respectively, can be calculated using equations (7),(8), (9) and (10) and the pricing rule:

\[
\pi_i = (p_i - \beta) d_i + (p_i^* - \beta) d_i^* - \mu i
\]

\[
= (1 - \theta) \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta-1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta-1}} \left( n + n^* (1 + \tau)^{\frac{\theta}{\theta-1}} a^{\frac{\theta}{\theta-1}} \right) \right]^{\frac{\theta-1}{\theta}} - \mu i
\]

(11)

\[
\pi_{i^*} = (p_{i^*} (1 + \tau)^{-1} - \beta^*) d_{i^*} + (p_{i^*}^* - \beta^*) d_{i^*}^* - \mu^* n^*
\]

\[
= (1 - \theta)(1 + \tau)^{\frac{\theta}{\theta-1}} \left( \frac{\beta^*}{\theta} \right)^{\frac{\theta}{\theta-1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta-1}} \left( n + n^* (1 + \tau)^{\frac{\theta}{\theta-1}} a^{\frac{\theta}{\theta-1}} \right) \right]^{\frac{\theta-1}{\theta}} - \mu^* i^*.
\]

(12)

The number of firms is endogenously determined via free entry and exit. Note that the larger the firm index the larger the fixed cost, thus firms in both countries stop entering the market when profits break even. In this setting firm \(i\) can

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5The asymmetry in fixed costs can be interpreted as differences coming from different management skills. For example, suppose there is a ranking of competency of CEOs. Or, assume that each firm entering the market faces a higher fixed cost to develop a product different from those already in the market, that is, the cost of differentiation is higher for subsequent firms.

6We normalize Home and Foreign labor endowment to 1.
supply the same quantity as the break even firm \( n \) so as to earn positive profits \( \pi(i) = \mu n - \mu(i) \). Thus, the free entry conditions \( \pi_i = 0 \) and \( \pi_{i^*} = 0 \) imply\(^7\)

\[
(1 - \theta) \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left( n + n^*(1 + \tau)^{\frac{\theta}{\theta - 1}} a^{\frac{\theta}{\theta - 1}} \right) \right]^{\frac{\theta - 1}{\theta - 1}} \\
+ (1 - \theta) \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left( n + n^* a^{\frac{\theta}{\theta - 1}} \right) \right]^{\frac{\theta - 1}{\theta - 1}} = \mu n 
\]

(13)

\[
(1 - \theta)(1 + \tau)^{\frac{1}{\theta - 1}} \left( \frac{\beta^*}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left( n + n^*(1 + \tau)^{\frac{\theta}{\theta - 1}} a^{\frac{\theta}{\theta - 1}} \right) \right]^{\frac{\theta - 1}{\theta - 1}} \\
+ (1 - \theta) \left( \frac{\beta^*}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left[ \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}} \left( n + n^* a^{\frac{\theta}{\theta - 1}} \right) \right]^{\frac{\theta - 1}{\theta - 1}} = \mu^* n^*. 
\]

(14)

Equations (13) and (14) can be used to construct loci in \( n, n^* \) space with the number of Home firms decreasing to the number of Foreign firms and \textit{vice versa} since \( \frac{dn(n^*)}{dn} < 0, \frac{d^2n(n^*)}{dn^2} > 0, \frac{dn^*(n)}{dn} < 0, \) and \( \frac{d^2n^*(n)}{dn^2} > 0 \). Figure 1 denotes the two loci with equilibrium in the intersection of the two curves.\(^8\) Note that the curves approach the axes asymptotically.

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\(^7\)Note that under this setting there will always be active firms in both countries since the first firm will have null fixed costs.

\(^8\)Note that it is possible to calculate the number of firms when there is no active firms in the other country:

\[
\begin{align*}
 n_{[n^*=0]} &= \left[ \frac{2(1 - \theta) \left( \frac{\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}}}{\mu} \right]^{\frac{\theta(1-\epsilon)}{\theta(1-\epsilon)}} \\
 n^*_{[n=0]} &= \left[ \frac{(1 + \tau)^{\frac{1}{\theta - 1}} + 1(1 - \theta) \left( \frac{a\beta}{\theta} \right)^{\frac{\theta}{\theta - 1}}}{\mu^*} \right]^{\frac{\theta(1-\epsilon)}{\theta(1-\epsilon)}}.
\end{align*}
\]
The equilibrium number of firms is given by \( n^e \) at Home and \( n^*e \) at Foreign. According to the shape and the magnitude of the asymmetry in costs and the tariff level, the equilibrium point may have its position changed considerably and so do trade patterns. In the next section we perform some comparative statics.

3 Comparative Statics

In order to see the effects caused by heterogeneity of costs, our comparative statics analysis starts with our benchmark case with free-trade and symmetric costs. With identical countries, there are no differences in marginal costs \((a = 1)\), no differences in the fixed cost coefficient \((\mu = \mu*)\) and Home does not impose any import tariffs \((\tau = 0)\). In this case, prices do not differ nor there are technological gaps between countries. Then the shape of the curves will be symmetric and the number of firms of Home and Foreign will be equal. The equilibrium point \( e \) of Figure 1 will lie on a 45-degree line that passes through the origin.

Now suppose that Foreign firms have lower productivity in terms of marginal costs, that is, they have higher marginal cost than Home firms \((a > 1)\). From (13) and (14), it is clear that \( dn/da > 0 \) and \( dn*/da < 0 \). In that case, both loci will shift as shown in Figure 2 such that the equilibrium point changes to \( e' \).

![Figure 2 - Efficiency Gap \((a > 1)\)](image)

From our calculations it is clear that the Home marginal firm curve rotates to the right while the Foreign marginal firm curve shifts down. Now Home has a larger number of firms compared to the previous equilibrium and Foreign a smaller
number. A higher productivity in terms of marginal costs of Home firms implies in lower prices, higher demand and thus, a larger number of Home firms.

Now turn to the case of international differences in fixed costs. Suppose Home has now a smaller fixed cost coefficient than Foreign ($\mu < \mu^*$). A small decrease in $\mu$ shifts only the Home zero-profit curve outwards as shown in Figure 3.

![Figure 3 - Difference in Fixed Costs ($\mu < \mu^*$)](image)

Higher fixed cost coefficient implies in lower profits for any firm in Foreign when compared to Home firms. Note in Figure 3 that the initial equilibrium has Foreign as the net exporter of differentiated products since the equilibrium point is located above the 45-degree line. That may be the case Foreign firms have lower marginal costs than Home firms. Nevertheless, an improvement in the efficiency in fixed costs of Home firms can revert this position such that at equilibrium $e'$ Home is now the net exporter of differentiated products, even if Home firms had a disadvantage in marginal costs. The efficiency of fixed costs predominate as a determinant of trade, contrary to the prediction based in comparative advantage on prices only.\(^9\)

### 4 Trade Policy

Trade policy has long been used as a means to promote national industry. Tariffs may have the detrimental effect of raising prices but also the positive effect of raising tariff revenues and allowing national industry to take advantage of economies of scale. In this section we analyze the effects of tariff changes on the industrial structure of countries.

From (13) and (14) we find that an increase in tariffs shifts Home locus outward and Foreign locus inward (Figure 4). Since tariffs raises the price of Foreign goods, Home firms are able to obtain a larger share of the domestic market and the number of Home firms increase whereas the number of Foreign firms decreases.

**Proposition 1.** *An increase in tariff imposed by Home increases the number of Home firms and decreases the number of Foreign firms.*

Figure 4 illustrate the case which Foreign has a larger number of firms than Home \((n^e > n^e)\) before tariff rises. Thus Foreign is the net exporter of differentiated products. Note that after the tariff increase, Home becomes the net exporter \((n^e' > n^e')\) of differentiated products.

\[
\begin{align*}
\pi_n &= 0 \\
n^e &amp; e \\
n^e' &amp; n^e' \\
n &amp; \pi_n = 0
\end{align*}
\]

Figure 4 - Tariff Increase \((\tau > 0)\)

Here again we have a case in which the rise in tariff shifts the Home’s position of net importer to net exporter of differentiated products.\(^{10}\)

## 5 Welfare Analysis

This section examines the effects of tariffs on welfare. In order to simplify our analysis, we do not add tariff revenues to Home income but that does not change the qualitative results of our analysis. Under these assumptions, the welfare of representative consumers of Home and Foreign can be denoted by the following

\(^{10}\)Note that he imposition of a tariff has the effect of allowing more inefficient firms into the market.
indirect utility functions:

\[ V = \left( \frac{1 - \epsilon}{\epsilon} \right) P^t \left( \frac{1}{\theta - \tau} \right) + 1 + \frac{\mu n^2}{2} \quad (15) \]

\[ V^* = \left( \frac{1 - \epsilon}{\epsilon} \right) P^* \left( \frac{1}{\theta - \tau} \right) + 1 + \frac{\mu^* n^*^2}{2}. \quad (16) \]

The last item of the RHS of each equation denotes total profits of domestic firms. Note that total utility will be negatively affected by an increase in the price index since it lowers the demand for differentiated products. On the other hand, utility will be positively affected by an increase in the number of Home firms since profits necessarily increase. The effects of an increase in tariff imposed to Foreign products is dubious since it may increase the price index but, at the same time, increase Home profits. The overall effect depends on the magnitude of the tariff and the parameter conditions. Thus, when an increase in tariff sufficiently increases the number of Home firms or does not change the price index, there is the possibility that Home welfare increases.

We illustrate this possibility with a symmetric case. Suppose Home and Foreign are identical countries. Then under free trade the number of Home and Foreign firms is the same \((n^F = n^*F)\). A positive tariff increases the number of Home firms and decreases the number of Foreign firms. We denote the number of Foreign firms as a fraction \(\delta\) of Home firms under a positive tariff \(\tau\), that is, \(n^*t = \delta(\tau)n^t\) with \(\delta(\tau) < 1\). Then from (3), (13) and (14) we obtain the ratio of the price index and the under a positive tariff, \(P^t\), and under free trade, \(P^F\):

\[ \frac{P^t}{P^F} = \left( \frac{(1 - \theta)(1 - \delta)4}{(1 - \theta) - (1 - \theta + \tau)(1 + \tau) \left( \frac{1}{\theta - \tau} \right)^{\frac{1}{\theta - \tau}}} \times \frac{1}{1 + \delta(1 + \tau) \left( \frac{1}{\theta - \tau} \right)^{\frac{1}{\theta - \tau}}} \right)^{(1-\theta)(1-\delta)} \quad (17) \]

Equations (15) and (16) indicate that there are unambiguous welfare gains Home and welfare loss for Foreign when \(P^t/P^F \leq 1\) since a positive tariff raises total profits at Home and the price index does not increase. We prove now that under certain conditions that will be the case. Notice that the last item of equation (17) is clearly smaller than one. And, as \((1 - \theta) - (1 - \theta + \tau)(1 + \tau) \left( \frac{1}{\theta - \tau} \right)^{\frac{1}{\theta - \tau}} > 0\) for a positive tariff \(\tau\), we verify that the first item is smaller than one if \(\delta > 3/4\) no
matter the level of tariff that induces such level of $\delta$. Thus, small tariffs, which imply high values of $\alpha$, may increase the number of firms, and consequently profits, without increasing the price index.\footnote{Notice that welfare gains would be even higher have we considered tariff revenues. Moreover, the range for the level of a beneficial tariff may be even larger since welfare losses from increased tariffs can be offset by gains in profits.} This result is summarized as follows:

**Proposition 2.** A small tariff is beneficial to the tariff-imposing country.

From the same argument, we can argument that complete trade liberalization may be harmful.

A model with bilateral tariff imposition could easily be set up and tariffs could be interpreted as iceberg costs, for example. We point out that possible extensions of the model can include heterogeneity in marginal costs, a three-country setup that allows us to perform trade policy analysis in free trade agreements, or a framework with multinational firms, without losing tractability.

### 6 Concluding Remarks

This note built a two-country-two-sector model with heterogeneous firms in the differentiated goods sector, new entrants face increasing fixed costs in the differentiated products sector. There are technological differences between countries. The model provides a very tractable framework to deal with firm heterogeneity and reallocation in the market in the presence of tariffs. We found that when Home unilaterally imposes a tariff to Foreign products the number of Home firms increases, while the number of Foreign firms decreases. At Home, more firms will enter the market while more efficient firms will gain higher profits. At Foreign, less efficient firms will be driven out of the market while the remaining firms will face lower profits. Also, there exists the possibility that an increase in tariffs raises Home welfare by increasing the number of firms - and consequently, total profits - and decreasing the price index.

The introduction of cost heterogeneity allows firms to have positive profits with the marginal firm being reallocated according to changes in tariff and other parameters. A tariff increase may benefit a country but at the cost of allowing more
inefficient firms (in terms of fixed costs) to enter the market. Trade liberalization can be detrimental to a country while efficiency gains are always beneficial.

References


