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IS INFLATION Fiscally DETERMINED? *

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Abstract

This paper examines the relationship between fiscal variables and inflation for 46 countries from 1960–2017 using a linear identity that links inflation to fiscal and monetary variables and economic growth. The results indicate that inflation is affected by both monetary and fiscal policies. However, the relation between inflation and fiscal variables disappears when monetary policy is based on commitment strategies. We conclude that fiscal determinacy of inflation is only possible when central banks practice poorly structured discretion.

JEL classification: E31, E63, H63, E43
Key words: inflation, fiscal policy, monetary policy, public debt, panel VAR GMM

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I INTRODUCTION

In recent decades, many countries have attempted to bolster weak economic growth using fiscal policy. As a result, significantly increased public debt, macroeconomic imbalances, and more particularly the effects on price level, have become a source of public concern. Nonetheless, fears of an inflation outburst resulting from high indebtedness can only be justified if the existence of a relationship between inflation and fiscal policy is verified.

The literature usually links inflation to monetary policy factors. For example, the monetarist view suggests that inflation is the result of too much money chasing too few goods and often considers the demand for money, interest rates, and exchange rates. In general, the conduct of monetary policy is thought to be a major determinant of inflation dynamics. That is why the recent low levels of inflation are viewed as the result of changed approaches to monetary policy, which, since the 1980s, considers a broader range of factors apart from money supply and uses instruments in addition to open market operations. Even so, this perspective was recently challenged, especially with the long-lasting zero bound episodes. Furthermore, the difficulty encountered by some central banks in achieving their inflation targets and the inability of dynamic stochastic general equilibrium (DSGE) models to forecast the recent crisis called into question their effectiveness in providing efficient policy tools.

Fiscal determinacy of price level, on the other hand, has not always been successfully demonstrated. The first theoretical contribution in this respect was of Sargent and Wallace (1981). They highlighted that governments running persistent deficits would sooner or later have to finance those deficits through money creation (seigniorage), thus, producing higher inflation, especially when GDP growth rates are lower than interest rates. This study paved the way for several others, such as those on the fiscal theory of monetary policy. Although this theory does not exclude the impact of monetary variables, it also posits that, if future surpluses fail to adjust to growing public debt (through more taxes or less expenditure), then price level would increase to satisfy the present value government debt equation. Empirical studies have not had much success in uncovering a strong and statistically significant
connection between fiscal policy and inflation.

Therefore, we study the link between fiscal variables and inflation rate for a sample of 46 countries, over 1960–2017, using a linearized equation derived from the household budget constraint. This equation links the inflation rate to both fiscal and monetary policy variables, in addition to economic growth rates. Our results indicate that, overall, inflation is affected by both monetary and fiscal policies. However, the link between fiscal variables and inflation disappears if we exclusively consider countries implementing monetary policy based on commitment strategies.

II LITERATURE REVIEW

II.1 A traditional approach

Sargent and Wallace (1981) were the pioneers in establishing a link between fiscal policy and price level. They derived the fiscal determinacy of prices from the government intertemporal budget constraint, expressed as follows:

\[ G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}) \]  

(1)

where \( G_t \) is government expenditures on goods, services, and transfers at time \( t \), and \( T_t \) is the tax revenue. \( i_{t-1} \) is the interest rate and \( B_t \) is interest-bearing debt. Thus, \( i_{t-1}B_{t-1} \) represents interest payments on total outstanding debt, and \( B_t - B_{t-1} \) denotes new issues of interest-bearing debt.

This equation shows that government expenditures can be funded by either taxes, new-issued debt, or printing new currency (expressed by the change in outstanding stock of non-interest-bearing debt \( H_t - H_{t-1} \)). Based on this relation, Sargent and Wallace (1981) provided an expression for inflation rate depending on the stock of interest-bearing government debt per capita. This model is based on the assumption that fiscal policy dominates mone-
They concluded that if interest rates on bonds are higher than the economy’s growth rates, then the real stock of bonds will grow faster than the size of the economy. However, since the demand for bonds places an upper limit on the stock of bonds, eventually, the financing of both principal and interest will be through seigniorage. In other words, the decisions of the fiscal authority can induce printing more money and, therefore, inflation.

II.2 The government debt valuation equation

Based on the assumption of a dominant fiscal policy, a new strand in literature has emerged, known as the fiscal theory of the price level (FTPL). The central equation of the FTPL is the government debt valuation equation, which equates the real value of government debt to the expected present value of future fiscal surpluses.

\[
\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j})
\]

(2)

where \(B_{t-1}\) is one-period nominal debt issued at \(t-1\) and due at \(t\), \(p_t\) is the price level, \(s_t\) is the real primary government surplus including seigniorage, and \(m_{t,t+j}\) is the discount factor.

The derivation of this equation can be found in several studies, such as Sims (1994) and Woodford (1994 and 1995). It is usually obtained from the intertemporal government budget constraint, with the government debt (\(B_{t-1}\) in the LHS) expressed in nominal terms and the present value of primary surpluses (RHS) in real terms. Nonetheless, in Cochrane (2005), this relation is presented as an asset pricing model, based on a different logic. The underlying idea is that nominal debt, including the monetary base, is a claim on future government primary surpluses (in the same way a stock is a claim on future earnings).\(^2\)

\(^1\)That is, the fiscal authority independently sets its budget, announcing current and future deficits and surpluses. The amount of revenue to raise is then determined based on those decisions. The monetary authority then finances the discrepancy between the revenue demanded by the fiscal authority and the number of bonds that can be sold to the public with seigniorage. Leeper (1991) defines this as an active fiscal policy, passive monetary policy scenario, where monetary policy is constrained by the fiscal authority’s actions and simply reacts to government debt shocks.

\(^2\)As underlined by Cochrane (2018), bond prices are determined by the market depending on bond yields and future streams of expected primary surpluses.
II.3 Models of the fiscal theory of monetary policy

The FTPL faced many empirical challenges and inferring concrete policy implications from its formal expression also proved to be difficult. Further, just like fiscal determinacy of prices was subject to the condition that economic growth rates be below interest rates (Sargent and Wallace 1981), Bassetto et al. (2018) showed that the FTPL is not a robust equilibrium selection criterion when interest rates are persistently below growth rates. Therefore, it appeared—after several theoretical and empirical attempts—that whether the FTPL holds depends on other economic forces that were not considered in the valuation equation. Consequently, there was a need for a more comprehensive theoretical framework, including assumptions that are closer to the actual economic dynamics. Such a framework was suggested by Sims (2011) and is a neo-Keynesian style model with long-term debt and sticky prices. It includes eight equations: a Taylor-principle policy rule equation defining the target nominal interest rate as the function of inflation and output; a Fisher-rule equation; a relation describing the real market value of debt (nominal perpetuities); a perpetuity yield equation based on the expectations hypothesis; a continuous-time Phillips curve equation; and a primary surpluses equation expressed as a function of output growth (pro-cyclical).3

One important implication of this formulation is that, even in an active fiscal/passive monetary policy equilibrium, monetary variables still have a powerful effect on both output and inflation. His conclusions, through impulse response functions and the analysis of Cochrane (2018), are as follows. On the one hand, contractionary monetary policies lead to a drop in inflation at first but then produce exactly the opposite effect with a time lag.4 Fiscal variables can be considered a part of this dynamic in the sense that interest rate changes affect the market value of bond prices. More precisely, when interest rates are high, the real value of government debt appears to be greater than its real market value for investors. Consequently, the demand for government debt increases at the expense of demand for goods

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3The derivation and solution of this model are provided by Cochrane (2018).
4This has been dubbed “the stepping on a rake” phenomenon.
and services, which leads to lower aggregate demand and, thereby, lower prices. On the other hand, the model also shows that an expansionary fiscal shock creates a boom in consumption and an upward unanticipated spurt in inflation rate. In that case, inflation is fiscally determined through the output transmission channel.

More recently, Cochrane (2019a) studied the fiscal roots of inflation based on the following log linearized identity:

$$v_t + r^n_{t+1} - \pi_{t+1} + g_{t+1} = s_{t+1} + v_{t+1}$$

(3)

where $v_t$ represents the market value of debt, $r^n_t$ the nominal returns on the government debt portfolio with a maturity length of $n$, $\pi_t$ the inflation rate, $g_t$ the GDP growth, $s_t$ the real primary surplus. This identity shows that unexpected inflation less the unexpected nominal return on government bonds must equal the innovation in the sum of future surpluses in GDP less the sum of future real bond returns. Therefore, it links unexpected inflation rate to both monetary and fiscal variables, in addition to GDP growth. The main results of the author’s structural shocks analysis are as follows. A positive monetary policy shock\(^5\) is super-Fisherian and raises inflation immediately. A negative fiscal shock\(^6\) induces a protracted inflation. The disinflation resulting from a recession shock corresponds entirely with a decline in discount rates leading to higher debt.

II.4 Public debt and fiscal space

Some empirical studies (see the following subsection) found that fiscal determinacy of prices was present in developing economies but not in advanced economies. The most popular explanation is that governments of advanced economies have access to more funds and need not resort to the monetary authorities, because they have a better capacity to repay their debt. Investors are willing to lend more to governments with better economic perspectives

\(^5\)Defined as a shock to interest rates not accompanied by a movement in future primary surpluses.

\(^6\)Defined as a shock to future primary surpluses not accompanied by a movement in interest rates.
and solid expectations of future budget surpluses. This insight led to the emergence of a
new concept in the literature: that of fiscal space. The most widely-used definition of fiscal
space is the one provided by Heller (2005, p. 3)—fiscal space reflects the availability of
budgetary room that allows a government to provide resources for a desired purpose without
any prejudice to the sustainability of a government’s financial position. In other words, the
availability of fiscal space implies that a government can find the resources to finance desired
expenditures and service its debt obligations. Therefore, the definition of fiscal space is
closely linked to a government’s fiscal sustainability and its potential to expand its financing
capacity.

Even though fiscal space refers to the same concept in most macroeconomic policy dis-
cussions, concrete measures of fiscal space used in the empirical literature differ significantly.
One commonly used approach is the fiscal gap approach, which is based on the idea of esti-
mating the difference between a given level of public debt or fiscal balance and a benchmark
level considering the sustainability level (Ostry et al. 2010, Ghosh et al. 2013).

The sustainable debt level is estimated in different ways. In some cases, it is considered as
the mean level of debt for a given group of countries. Another approach is the signal approach
suggested by Kaminsky et al. (1998). In some other cases, a forward-looking approach is
used by computing the present value of future primary balances, as done in the government
debt valuation equation. Such a methodology was popularized by Bohn (1998 and 2008),
in which fiscal solvency is considered fulfilled when primary balance reacts positively to an
increase in public debt.

In other studies (e.g., Buiter 1985, Buiter et al. 1993, Auerbach et al. 2011), an index of
fiscal sustainability is derived based on the projections of future balances, depending on the

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7 The debt threshold that maximizes the ratio of percentage of correctly classified debt crises in total crises observations divided by the percentage of falsely classified crises in non-crises observations.

8 Some of the criticisms against Bohn’s methodology is that it does not consider the endogenous relation between debt and interest rate, and it also does not rule out the possibility of an infinitely increasing debt. These issues were addressed in the models of Ostry et al. (2010) and Ghosh et al. (2013), where the interest payment schedule depends on debt level and the response of the primary balance to public debt is nonlinear. In these models, the interest rate becomes infinite beyond the debt limit as there is no finite rate that would compensate creditors for the probability of default (which becomes equal to unity).
macroeconomic outlook and forecasts of discount rate. Similarly, some publications by the IMF, such as the Fiscal Monitor, present a measure of fiscal adjustment by a country, defined as the distance between the 2011 cyclically-adjusted primary balance and the one needed to reduce the general government debt ratio to a sustainable level.\footnote{Equivalent to 60\% of GDP in advanced economies and 40\% of GDP in emerging and low-income economies by 2030 (or to 2012 levels, if these are lower than the 60\% and 40\% benchmarks). For Japan, a net debt target of 80\% of GDP is assumed.}

Finally, Aizenman and Jinjarak (2010) suggested an alternative fiscal space measure called the de facto fiscal space, defined as the inverse of the number of tax-years needed to repay the debt. This ratio requires estimating the de facto tax base corresponding to the realized tax collection averaged across multiple years to smooth for business cycle fluctuations. In Aizenman et al. (2019), this fiscal space measure is used to examine fiscal cyclicality across a large sample of countries. The authors found that lower fiscal space coincides with procyclical government spending.

II.5  Empirical literature on fiscal determinacy of prices

The results of the existing empirical studies of the fiscal policy-inflation relationship are mixed. There is no conclusive evidence so far in favor of or against the existence of a fiscal determinacy of price level. Some of these studies (Table 1) focused on the link between fiscal deficits and inflation, while others examined the relationship between fiscal surpluses and public debt or used a different approach.

For example, Catao et al. (2005) examined the existence of a relationship between inflation and fiscal deficits for a large sample of countries, but could only verify it for the high-inflation, developing economies. Fischer et al. (2002) also concluded that fiscal deficits are the main drivers of high inflation, but only in high-inflation countries.\footnote{More specifically, they found that a reduction in the fiscal balance by 1 percent of GDP leads to an increase in the inflation rate by 4.2 percent in this specific group.} Bohn (1998) followed a different approach. He examined the U.S. deficit and debt processes and showed that primary surplus responds positively to the debt-to-GDP ratio. The author concluded that
the fiscal authority acts in a Ricardian fashion.

Following Bohn’s approach, Bajo-Rubio et al. (2009) used cointegration analysis between the primary surplus over GDP and public debt-to-GDP, complemented with Granger causality tests (for a sample of 11 E.U. countries). They concluded that there was no clear evidence supporting the FTPL, as primary surpluses responded positively to debt in almost all cases. Canzoneri et al. (2001) and Creel et al. (2006) reached a similar conclusion. Conversely, some other studies (e.g., Favero et al. 2005) showed the presence of alternating Ricardian and non-Ricardian regimes.

There are also other empirical studies that validate the FTPL. For instance, Loyo (1999) argued that the Brazilian policy in the late 1970s and early 1980s was non-Ricardian and that the FTPL provided a persuasive explanation for Brazil’s high inflation during that time. Tanner et al. (2003) found evidence of fiscal dominance in Brazil for some important periods. Fan et al. (2013) investigated whether the FTPL could explain the U.K. inflation in the 1970s. They found evidence that fiscal policy was non-Ricardian and money growth was entirely endogenous in that period. They concluded that government expenditure was the only driving force for inflation and that the 1970s inflation outburst resulted from an increased level of expenditure, unmatched in the previous decades.

Most of these studies used the government intertemporal budget constraint as the starting point to determine both the approach for the study and the variables. For instance, Bohn (1998) and Canzoneri et al. (2001) focused on the relationship between public debt and primary balances (using a regression model), based on the idea that an adjustment between the two would be an indication of a Ricardian regime. However, as pointed out by Bajo-Rubio et al. (2009), a fall in the public debt ratio following a decrease in the expected value of future surpluses could be caused by a price increase and not a change in fiscal surpluses. Therefore, the presence of a link between fiscal balances and public debt is compatible with both a monetary dominant (MD) and a fiscally dominant (FD) regime. To address this, the author included Granger causality tests between primary surpluses and debt in addition to
the cointegration analysis.

Another way to account for effects of price level would be using the government debt valuation equation. This is a more direct way of verifying the FTPL. However, it would be difficult to draw any conclusions based on this method, for many reasons. First, the variable for the present value of future primary surpluses would be hard to measure and analyze.\footnote{Judgment could be biased by business cycle fluctuations as primary surpluses are likely to fall during recessions and increase afterwards} Second, a change in inflation could be the reflection of a change in the discount rate, not a movement in fiscal variables. Further, as indicated by Cochrane (2019b), the choice of the discount rate would have a significant impact on the results obtained. Finally, another critique to the previous approaches in the empirical literature is that most of them exclusively examine fiscal variables without considering interactions with monetary policy, in a single model\footnote{In studies on switching regimes, often two separate models are considered.}, as done in Sims (2011). In this respect, the approach used by Cochrane (2019a) is innovative because it uses a single linearized relation to examine links between inflation and both fiscal and monetary variables, in addition to the business cycle (based on U.S. data).

We use a similar approach, but with a different derivation of the linear relation of inflation, using variables expressed in real terms. Moreover, as opposed to the methodology used in Cochrane (2019a), we do not derive the value of primary surpluses from the linear identity and instead, use actual data to construct a panel vector autoregression (VAR) model. We then examine the effects of a fiscal policy shock, monetary policy shock, and recessionary shock on our variables, for the sample of 46 countries from 1960–2017. Finally, we further extend our study by examining how the choice of a monetary policy regime and the level of fiscal space affect the links between our variables.
III THEORETICAL MODEL

Based on the theoretical framework similar to Woodfords (1994), we use the following comprehensive dynamic consumer budget constraint (in nominal terms):

\[ p_t c_t + B_t - B_{t-1} = R_{t-1} B_{t-1} + p_t y_t - T_t \]  \hspace{1cm} (4)

where \( p_t \) is the price level, \( c_t \) the consumption, \( B_t \) the portfolio of assets (assumed to consist only of government bonds), \( R_t \) the interest rate on government bonds, \( y_t \) the household income, and \( T_t \) taxes. This relation states that the sum of consumption and assets purchases is equal to the sum of interest earnings on assets held from the previous period and income less taxes.

Assuming that the economy’s total income is \( y_t = g_t + c_t \) with \( g_t \) being government expenditures, we obtain

\[ B_t - B_{t-1} - R_{t-1} B_{t-1} = G_t - T_t \]  \hspace{1cm} (5)

For \( a \) being the percent change in the value of debt, we have \( B_t = (1 + a) B_{t-1} \). Therefore,

\[ B_{t-1} (a - R_{t-1}) = G_t - T_t = -PS_t \]  \hspace{1cm} (6)

Since the change in public debt also depends on government bond yields, we assume the following expression (for a given coefficient \( k \)):

\[ a = k R_{t-1} \]

Therefore,

\[ B_{t-1} = \frac{PS_t}{R_{t-1} (1 - k)} \]  \hspace{1cm} (7)

Rescaling by real GDP \( (y_t) \), and with \( \theta_t \) being GDP growth, we have

\[ \frac{B_{t-1}}{y_{t-1} \theta_t} = \frac{PS_t}{y_t R_{t-1} (1 - k)} \]  \hspace{1cm} (8)
For $B_{yt-1}$ representing the term $B_{t-1}$ rescaled by GDP and $PS_{yt}$ the term $PS_t$ rescaled by GDP, we obtain

$$B_{yt-1} = \frac{\theta_t PS_{yt}}{R_{t-1} (1 - k)}$$

(9)

Since the term representing bond yields ($R_{t-1}$) depends on debt maturity, if we assume that there is only one category of bonds with maturity ($n$), the term structure equation implies that:

$$R_t^n = i_t i_{t+1} i_{t+2} \ldots i_{t+n-1}.$$  

where $i_t$ is the annualized short-term interest rate. Replacing in Equation (9)

$$B_{yt-1} = \frac{\theta_t PS_{yt}}{\prod_{j=t}^{n} i_j^{1/n} (1 - k)}$$

(10)

Dividing this equation by $p_t$, 

$$\frac{B_{yt-1}}{p_t} = \frac{\theta_t}{\prod_{j=t}^{n} i_j^{1/n} (1 - k)} \frac{PS_{yt}}{p_t}$$

(11)

with the output-adjusted real primary surplus being $ps_{yt} = \frac{PS_{yt}}{p_t}$, and inflation corresponding to $\pi_t = \frac{ps_t}{p_{t-1}}$, we obtain

$$\frac{B_{yt-1}}{\pi_t p_{t-1}} = \frac{\theta_t ps_{yt}}{\prod_{j=t}^{n} i_j^{1/n} (1 - k)}$$

(12)

Considering real debt over GDP, $b_{yt-1} = \frac{B_{yt-1}}{p_t}$

Then,

$$\frac{b_{yt-1}}{\pi_t} = \frac{\theta_t ps_{yt}}{\prod_{j=t}^{n} i_j^{1/n} (1 - k)}$$

(13)

Taking logs,

$$\log (b_{yt-1}) - \log (\pi_t) = \log (\theta_t) + \log (ps_{yt}) - \frac{1}{n} \log \left( \prod_{j=t}^{n} i_j \right) - \log (1 - k)$$

(14)

In this equation, the inflation level appears to be affected by both fiscal and monetary
policies, in addition to growth. As in Cochrane (2019a), we expect a positive relationship between inflation and public debt, and a negative relationship with the fiscal balance. According to his model and empirical findings, higher fiscal deficits should indeed lead to higher inflation. Moreover, the response of inflation to interest rate is assumed to be positive, consistent with the neo-Fisherian view and the conclusions in Sims (2011) and Cochrane (2018).

Finally, we expect a negative relationship between inflation and GDP growth rate. This result contradicts the conventional view based on Keynesian and neo-Keynesian frameworks, according to which this relation should be positive (the AD-AS model, Phillips Curve). However, our assumption is consistent with the findings of several empirical studies and theoretical frameworks. First, the model developed by Stockman (1981) established that an increase in inflation rate results in a lower steady-state output level, by reducing the purchasing power of money balances and, thereby, the demand for goods and capital.\textsuperscript{13} Second, most money and endogenous growth models concluded that inflation rate reduces both the return on capital and growth rate in the long-run (Aratawari et al. 2016, Vaona 2012).

Empirically, several studies revealed an overall negative effect of inflation on growth and many of them also detected the presence of non-linearity in the relationship. Kormendi et al. (1985) were among the first to shift the common belief about this relationship (from a positive to a negative one), based on a study using data from 47 countries over 1950–1977. Fischer (1993) found that inflation negatively affects output growth through the channels of investment and productivity growth. These effects were found to be particularly prominent for a high inflation rate. Gomme (1993) also concluded that a rise in inflation leads to lower growth.\textsuperscript{14} Using pooled cross-section time-series regressions for a number of countries, De Gregorio (1992) observed that inflation has been an important factor in inhibiting growth.

\textsuperscript{13}Assume that a part of investment projects is financed through cash.
\textsuperscript{14}The author applied Lucas endogenous growth framework combined with a cash-in-advance exchange technology. The described mechanism is as follows: a rise an inflation reduces the marginal value of the last unit of consumption in a given period. Since this value is equivalent to the cost of the last unit of work, people are induced to work less. Reduced labor results in a slower rate of capital accumulation.
in Latin America. Likewise, Andres et al. (1997) obtained a significant negative effect of inflation on economic growth over the long run. Their main policy message stated that reducing inflation by 1% could raise output by between 0.5% and 2.5%.

IV DATA

This study is conducted using a sample of 46 countries (Appendix 1) from 1960 to 2017. We use yearly data from several sources for the variables specified in the theoretical model (a detailed description of variables and data sources is provided in Appendix 2). For further scrutiny, in addition to this dataset, we use the dataset for monetary policy classification compiled by Cobham (2018) and data related to government revenues to determine the fiscal space level for our sample countries, as explained in this section.

IV.1 Relationships between the main variables

The correlation coefficients between the five main variables for the whole sample are provided in Table 2. The highest coefficient is the correlation between interest rates and inflation, with a positive sign (45%). Conversely, correlation with the fiscal variables is very weak—negative with the fiscal balance and positive with public debt. Finally, GDP growth rate does not appear to be strongly correlated with any of the variables under investigation. The highest coefficient is -17% with the variable of public debt over GDP.

Using the model described in the previous section, forecast error variance decomposition of inflation is generated using the Cholesky decomposition based on an active fiscal policy/passive monetary policy setting. Therefore, the following ordering of the variables is adopted: the primary balance over GDP (most exogenous), public debt over GDP, GDP growth rate, short-term interest rates, and inflation. The results clearly indicate that a significant share of the forecast error variance of inflation can be explained by exogenous shocks to the interest rates (between 10% at the beginning and 21% at the end of the observation
period) or past inflation (88% at period 1) (see Figure 1). The share of shocks to the primary balance, although not null, is very small (varies between 1.4% and 2.7%). Therefore, fiscal policy does affect inflation but not to the same extent as monetary policy. The impact of public debt on inflation is almost absent (around 0.6%).

Using the same model to generate forecast error variance decomposition of public debt (Figure 2), we do not find evidence of a contribution of inflation to public debt (limited to 0.5%) either. Public debt is significantly affected by past debt (around 70%) and a growing contribution of the primary balance shocks (as expected) (reaching 28.8%), but other variables are insignificantly related. Interestingly, the variance decomposition of a one-period lagged public debt shows a more significant contribution of growth (close to 10% at the 20th period).

IV.2 Dataset for monetary policy frameworks

As stated in the introduction, some analysts attribute the generalized low level of inflation to a major change in monetary policy frameworks (MPF). This shift in monetary policy thinking by setting up rules or targets (money, credit, exchange rate, interest rates, and inflation) aimed at addressing the dynamic inconsistency issue (Kydland et al. 1997, Calvo 1978). A consensus in this literature is that commitment is a better policy because it generates lower average inflation in the long run (Barro et al. 1983, Rogoff 1985). The gains from commitment are the direct consequence of the role that expectations play in shaping economic conditions. More specifically, inflation targeting regimes are thought to have contributed to lower inflation levels and higher monetary policy credibility.

To verify this assumption, we use Cohbam’s classification (2018) for our sample—he defined a monetary policy framework as a combination of objectives, Constraints, and conven-

15Dynamic inconsistency refers to changes in the decisions of monetary authorities and the absence of commitment to a single optimal policy. Especially when output is below the optimal level, monetary authorities have an incentive in moving away from an announced target to generate a “surprise” inflation and, thereby, a short-term increase in growth. Hence, it is usually based on the idea that the relationship between inflation and GDP growth is positive.
tions for monetary authorities. A distinction between different frameworks was made based on whether the monetary authorities publish targets for some objectives and whether such targets exist for monetary aggregates, exchange rates, inflation, or other variables. Based on this definition and some additional criteria, Cobham identified 32 different categories (see Appendix 3). Some of these categories are not represented in our sample and Cobham’s database does not include some of our sample countries (we aggregated them within the ”No national framework” category). Furthermore, the data are available only from 1980 to 2016.

If there is no fiscal dominance in our sample, then the choice of monetary policy frameworks should affect the way inflation reacts to fiscal variables. More specifically, if monetary policy is the main driver of inflation, we expect that the inflation-fiscal policy relation will be weakened in commitment monetary policy regimes, and more particularly, inflation targeting.

**IV.3 Estimation of the fiscal space level**

Observing levels of debt alone is not sufficient to draw conclusions about a government’s solvency. Therefore, we use the concept of fiscal space in our analysis to account for a government’s capacity to repay contracted debt. We use a definition that is closer to the one suggested by Aizenman and Jinjarak (2010). We first calculate the ratio of public debt divided by total government revenues. This measure reflects the number of years of revenue needed to repay the outstanding public debt on a given date. Fiscal space is then defined as the inverse of that ratio. The most recent available value (2016), the average, some descriptive statistics of the calculated measure, in addition to the correlation coefficients with fiscal balance, are given for all sample countries in Appendix 4.17

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16Constraints and conventions include “rules or disciplines to which authorities are subject (voluntarily or involuntarily), the nature of the financial and monetary markets and institutions, the understanding of key macroeconomic relationships, and the political environment”(Cobham 2018).

17To reduce gaps in data, missing values have been filled using the closest available values.
To make the best use of the information in our dataset, we study the inflation-fiscal variable relationship using the panel VAR model that can be expressed as:

\[ z_t = \theta^1 z_{t-1} + \ldots + \theta^j z_{t-j} + w_t \tag{15} \]

where

\[
\begin{pmatrix}
  p_{st} \\
  b_{yt} \\
  \theta_t \\
  i_t \\
  \pi_t
\end{pmatrix} \quad \begin{pmatrix}
  \theta^1 \\
  \vdots \\
  \theta^j
\end{pmatrix} =

\begin{pmatrix}
  \theta^j_{11} & \theta^j_{12} & \theta^j_{13} & \theta^j_{14} & \theta^j_{15} \\
  \theta^j_{21} & \theta^j_{22} & \theta^j_{23} & \theta^j_{24} & \theta^j_{25} \\
  \theta^j_{31} & \theta^j_{32} & \theta^j_{33} & \theta^j_{34} & \theta^j_{35} \\
  \theta^j_{41} & \theta^j_{42} & \theta^j_{43} & \theta^j_{44} & \theta^j_{45} \\
  \theta^j_{51} & \theta^j_{52} & \theta^j_{53} & \theta^j_{54} & \theta^j_{55}
\end{pmatrix}

and

\[
\begin{pmatrix}
  v_{1t} \\
  v_{2t} \\
  v_{3t} \\
  v_{4t} \\
  v_{5t}
\end{pmatrix}
\]

The estimation is made using the methodology suggested by Sigmund et al. (2017) for panel VAR generalized method of moments (GMM) (the details are provided in Appendices 5 and 6), and after applying forward orthogonal transformation to the variables of study (see Appendix 7). The methodology of Sigmund et al. (2017) is useful to eliminate the Nickell bias (Nickell 1981), according to which the correlation between error terms and regressors present in panel VAR ordinary least squares (OLS) estimations leads to inconsistent estimates.

The results indicate that public debt is persistent with a significant autoregressive coefficient (Table 3). The variables of economic growth, interest rate, and inflation also have significant own coefficients. The inflation equation shows that the only significant variable in determining inflation is the interest rate, with a positive and high coefficient. Fiscal variables’ coefficients are negative but not statistically significant. The OLS estimation, on the other hand, shows that both fiscal policy and monetary policy variables significantly affect the variables in this study (Table 4). Moreover, primary balance reacts positively to debt consistent with our expectations, while debt is affected negatively by primary balance (as
cumulative budget deficits lead to a growing debt). The variable of growth has a negative effect on both fiscal balance and public debt.

V.1 Inflation’s interdependence with the variables of study

Using generalized impulse response functions, we examine the impact of a one-standard deviation innovation in all variables on inflation (Figure 3). We note a positive and very significant response to interest rates (1 to 1). A positive shock in the primary balance also induces a positive response in inflation (higher surplus implies higher inflation), but of a smaller magnitude (0.3 units). These first results indicate that inflation is affected by both fiscal and monetary policy. However, the response of inflation to public debt is very small and negative (less than -0.1 at period 1). Similarly, a shock to growth also leads to a very limited, negative response in inflation.

Conversely, we also observe responses of all variables to an innovation in inflation (see Figure 4). We note that this shock induces a positive and very significant response in short-term interest rates (0.05). Public debt to GDP (in log terms) responds negatively over the long run (but with a magnitude limited to -0.01). The responses of other variables are negligible.

V.2 Fiscal policy shocks

A 1 standard deviation (SD) positive innovation in the primary balance (Figure 5) induces a positive but poor response in growth and interest rates (less than 0.02 units). This suggests that fiscal policy does not affect monetary policy. On the other hand, a significant and negative response in public debt over GDP (in log terms) is generated over the long run (reaching -0.1 at the end of the period). However, the response in the inflation rate is the most notable. After a small positive jump, inflation becomes negative immediately in the second period (which is consistent with findings of the previous subsection) reaching the value of -0.15. Therefore, a policy that increases taxation or reduces government expenditure would eventually lead to lower future debt and lower inflation. Similarly, an expansionary fiscal
policy induces the opposite effect as applying a negative shock to primary balance leads to a positive response in the inflation rate, suggesting that fiscal deficits are inflationary.

Nevertheless, when we split the data using the classification of monetary policy frameworks provided by Cobham (2018) (for the period 1980-2016), we find that the relationship between inflation and fiscal balance varies across monetary policy regimes. First, the correlation coefficients, although not very high, are very different. The highest level of correlation is 23%, in the case of unstructured discretionary regimes. Second, when we decompose the generalized impulse response functions of inflation to a fiscal shock based on monetary policy frameworks (Figure 6), we notice that the negative response in inflation after the second period is only seen in unstructured or loosely structured discretionary regimes (and with a magnitude lower than the response obtained for the entire sample in Figure 5), while no response is noted for other monetary policy frameworks. Mixed target regimes do show a positive response in the long run, but due to the small number of observations (3 countries in the 1980s), it is difficult to draw any conclusions. We find a similar result when estimating the GMM model for inflation based on the different frameworks (Table 5).

In the following step of the study, to see how fiscal space affects the relationship between inflation and fiscal variables, we split our sample into two fiscal space categories based on the middle quartile. S1 is the subsample of higher fiscal space countries, that need few years to repay their debt and S2 is the subsample of economies with lower fiscal space, that need many years to repay their debt. The results (Table 6) indicate that the correlation between inflation and fiscal balance is positive with higher correlation for economies with the lowest levels of fiscal space.

Using generalized impulse response functions (Figure 7), a strong positive (up to +0.6 units), then negative (to -0.4), response in inflation to a positive shock to the primary balance is noted for the S2 subsample (economies with low fiscal space) while in the case of S1, the response in inflation is very weak and positive (+0.2). The implication is that countries with lower levels of fiscal space are more at risk of inflationary pressures when increasing
deficits. Focusing on the upper and lower 25th quartiles also corroborates this conclusion, as no response is obtained for the upper quartile (Figure 7).

Table 7 shows an estimated regression model of inflation using GMM (with lagged variables as instruments) and OLS. We can see that for subsamples with high fiscal space, the coefficient of the fiscal balance is positive but not very high (around 5 in the GMM specification and 1 based on OLS for the upper 50th quantile). In the case of subsamples with low fiscal space, that coefficient is negative, statistically significant, and a much higher magnitude (-103.8 for the 25th lower quartile in the GMM model).

Finally, focusing on the public debt variable, the correlation coefficient of inflation with public debt (Table 6) is negative in the case of S1 (meaning that higher debt for countries with enough revenues has a negative effect on inflation), and positive and significant for S2 (public debt is associated with high inflation in countries with low fiscal space). The generalized impulse response functions show a positive response of inflation to a shock to public debt. By fiscal space categories, this relationship appears once again to hold only for the lower fiscal space countries (Figure 8).

V.3 Monetary policy shocks

A positive 1 SD innovation in short-term interest rates induces a positive and notable response in the inflation rate (Figure 9). No response is obtained for primary balances, which implies that monetary policy does not affect fiscal policy. The response of other variables is negligible. The correlation coefficient between inflation and short-term interest rates is also positive and very high (see Table 6).

As in the previous subsection, we decompose the generalized impulse response functions of inflation following a monetary policy shock by MPF (Figure 10). The most notable responses are those of unstructured discretion and exchange rate regimes. The correlation coefficient between inflation and interest rates is also stronger in the case of unstructured discretion (88%, Figure 14). Interestingly, short-term interest rates do not appear to be a sig-
significant determinant of inflation in inflation targeting regimes (see Table 5, Figure 14). The impulse response functions by categories of fiscal space show that the response of inflation is stronger in the lower fiscal space countries (Figure 11). This result is also corroborated by the regression model discussed previously (Table 7) where the coefficient of the interest rate appears to be much more significant in case of countries with lower fiscal space.

V.4 Recessionary shocks

In the final step, we apply a -1 unit shock to growth to see the impact of a recessionary episode on the variables (Figure 12). The response of inflation (modeled on the right axis) is strong and becomes negative in the second period (to -0.8), but quickly resumes to the initial level (consistent with a Phillips curve as the growth level also adjusts). No response is shown for primary balance and interest rate. Public debt responds positively over the medium term (which is consistent with our expectations). In all, growth does not appear to significantly affect our variables, except for public debt.

The growth rate could, however, indirectly affect inflation, through its impact on the relationship between fiscal variables and inflation. As pointed out by Sargent et al. (1981), fiscal policy affects price level when interest rates are greater than the growth rate and, thereby, the cumulative debt burden gradually surpasses the demand for bonds by investors. To check this assumption, we split the data: observations where growth is greater than the interest rate and observations with growth lower than the interest rate (for which we expect to see a stronger relationship between fiscal policy and inflation). Results (Figure 13) clearly corroborate this idea as no response from inflation is obtained in the case where growth is greater than the interest rate. As interest rates have been significantly lower than growth in the recent years, this could be one explanation to the low inflation level.
V.5 The absence of fiscal dominance

Previous empirical studies, notably Bohn (1998), Bajo-Rubio et al (2009) and Canzoneri et al. (2001), rejected fiscal determinacy of price level and concluded that governments act in a Ricardian fashion because of the presence of an adjustment of primary surpluses to changes in public debt. We find a similar result when we examine the response of fiscal balances to a shock to public debt and reciprocally, as in Canzoneri et al. (2001), public debt reacts negatively to a positive shock to primary balances (Figure 5). Such a finding is an indication of the absence of fiscal dominance since, as stated by Tanner et al. (2003), the primary surplus adjusts to limit debt growth when the regime is monetary dominant (MD), so that monetary policy can be conducted independently of fiscal financing requirements. Moreover, we are unable to obtain a strong direct relation between public debt and inflation, when considering the entire sample (see Forecast error variance decomposition of both debt and inflation in Section 4).

However, when examining the relationship between fiscal balances and inflation, we find a significant negative relationship between both variables (Section 5.2). This suggests that fiscal deficits are inflationary, as shown in Fan et al. (2013). Still, further analysis shows that this relation is not robust across all sample countries and periods. It only holds in cases when unstructured or loosely structured discretion is used. In monetary regimes based on targets, our results indicate that inflation is not sensitive to fiscal policy (and is also less sensitive to other shocks).

Such an outcome is not surprising and does not contradict the conclusions from previous discussions on monetary policy (Kydland et al. 1977, Barro et al. 1983) according to which inflation’s sensitivity to shocks is reduced when central banks operate under commitment. The main justification is that such regimes require independence of monetary authorities. Therefore, in these particular cases, governments cannot force central banks to finance their deficits by generating excess liquidity or pressure central banks to keep interest rates low to lower the governments’ borrowing costs. In the language of Leeper (1991), fiscal policy is
passive and monetary policy is active, such that there is no fiscal dominance. Consequently, we conclude that fiscal determinacy of price level is only possible when central banks practice unstructured/loosely structured discretion.

VI CONCLUSION

In this paper, we attempted to verify the presence of a relationship between fiscal policy and inflation. We used a theoretical and analytical framework similar to the one suggested by Cochrane (2019a), which allowed us to examine the links between inflation, fiscal and monetary variables, and GDP growth simultaneously, in contrast to the previous empirical studies that only focused on the links between debt and primary balance or between inflation and some fiscal variables.

For our sample (46 countries, from 1960–2017), we found that inflation is affected by both monetary and fiscal policy. A clear and positive relation between inflation and interest rate was uncovered in all the steps of our study (GIRF showed a 1 unit response in inflation to a 1 SD innovation in interest rates). Simultaneously, inflation rate also appeared to be negatively affected by fiscal balance, suggesting that an expansionary fiscal policy is inflationary. These findings are consistent with those of Cochrane (2019a), in which (covering U.S. data), the relationship between interest rates and inflation was also found to be strong and positive and that between fiscal balances and inflation was found to be significant and negative. However, in the first step, we could not draw any conclusions about the existence of a direct link between public debt over GDP and inflation. While impulse response functions showed a positive response in inflation to increasing debt, this connection did not appear clearly in the variance decomposition analysis.

We then further extended our analysis by examining how the choice of a monetary policy regime affects the links between our variables. Interestingly, we found that the relationship between fiscal balances and inflation only holds when unstructured or loosely structured
discretion is used. In monetary regimes based on targets, our results indicate that inflation is not sensitive to fiscal policy (and is also less sensitive to other shocks). We, thereby, conclude on the absence of fiscal dominance. Based on these findings, fiscal determinacy of price level is only possible when central banks do not operate under commitment. The most significant implication is that running persistent deficits will not lead to unexpected higher inflation, if monetary policy is resilient and credible enough to generate well-anchored inflation expectations.

Next, we examined how fiscal space (defined as the inverse of the number of years of revenue needed to repay the outstanding public debt on a given date) affects the relationship of inflation with fiscal and monetary policy. We found that inflation is more sensitive to variations in both fiscal balance and interest rate when fiscal space is low. Finally, although we expected a significant relation between growth and inflation based on the theoretical model, the variable of growth did not appear to significantly affect any of the variables of study (except for debt). Nevertheless, growth could play an indirect role in inflation dynamics since its level determines the repayment capacity of government debt, which affects the probability of fiscal dominance. We were able to verify this because, as predicted by Sargent and Wallace (1981), the relationship between fiscal policy and inflation is apparent only when growth is lower than the interest rate.
References


Canzoneri, M. B., R. E. Cumby, and B. T. Diba (2001). Is the price level determined by the needs of fiscal solvency?


<table>
<thead>
<tr>
<th>Paper</th>
<th>Sample</th>
<th>Methodology</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catao &amp; Terrones (2005)</td>
<td>107 countries over 1960–2001</td>
<td>ARDL model</td>
<td>Inflation and fiscal deficits are only correlated in the case of high-inflation, developing economies</td>
</tr>
<tr>
<td>Bohn (1998)</td>
<td>US data 1916-1995</td>
<td>OLS regressions by periods+ a non linear model for PB (based on debt)</td>
<td>The fiscal authority acts in a ricardian fashion (PB adjusts)</td>
</tr>
<tr>
<td>Bajo-Rubio et al. (2009)</td>
<td>11 EU countries 1970-2005</td>
<td>Cointegration analysis and Granger causality tests</td>
<td>No evidence of FTPL (PB adjust to debt)</td>
</tr>
<tr>
<td>Canzoneri et al. (2001)</td>
<td>postwar U.S. data 1951-1995</td>
<td>VAR, IRF</td>
<td>Evidence of a ricardian regime</td>
</tr>
<tr>
<td>Tanner &amp; Ramos (2003)</td>
<td>Brazil 1991-2000</td>
<td>VAR, IRF and Granger causality tests</td>
<td>Fiscal dominance for the case of Brazil for some important periods</td>
</tr>
<tr>
<td>Creel &amp; Le Bihan (2006)</td>
<td>France, Germany, Italy, the UK and the US data 1963-2001</td>
<td>VAR, IRF (same approach as Canzoneri) + separation between structural/cyclical PB</td>
<td>FTPL non valid</td>
</tr>
<tr>
<td>Fan &amp; Minford (2013)</td>
<td>The U.K. in the 1970s</td>
<td>ARIMA model for inflation, ADF and cointegration tests</td>
<td>Behaviour of inflation can be explained by the FTPL (gov. expenditures)</td>
</tr>
</tbody>
</table>
TABLE 2

Correlation coefficients between main variables

<table>
<thead>
<tr>
<th></th>
<th>$ps_y$</th>
<th>$\log(b_y)$</th>
<th>$\pi$</th>
<th>$i$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ps_y$</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log(b_y)$</td>
<td>-15%</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>4%</td>
<td>1%</td>
<td>45%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>$i$</td>
<td>4%</td>
<td>1%</td>
<td>45%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>15%</td>
<td>-17%</td>
<td>-1%</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $ps_y$: primary surplus over GDP, $\log(b_y)$: log of public debt over GDP, $\pi$: inflation rate, $i$: short-term interest rate, $\theta$: growth rate of GDP per capita

TABLE 3

Panel VAR GMM Estimation

<table>
<thead>
<tr>
<th></th>
<th>$ps_y$</th>
<th>$\log(b_y)$</th>
<th>$\theta$</th>
<th>$i$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag 1 $ps_y$</td>
<td>8.1820</td>
<td>-1.0367</td>
<td>0.2123</td>
<td>-2.2192</td>
<td>-11.4538</td>
</tr>
<tr>
<td></td>
<td>(10.7477)</td>
<td>(0.6839)</td>
<td>(0.4580)</td>
<td>(3.4129)</td>
<td>(8.5006)</td>
</tr>
<tr>
<td>Lag 1 $\log(b_y)$</td>
<td>-0.3483</td>
<td>0.9448***</td>
<td>0.0004</td>
<td>0.1029</td>
<td>-0.1038</td>
</tr>
<tr>
<td></td>
<td>(0.4910)</td>
<td>(0.0284)</td>
<td>(0.0248)</td>
<td>(0.1571)</td>
<td>(0.1534)</td>
</tr>
<tr>
<td>Lag 1 $\theta$</td>
<td>-3.3278</td>
<td>-0.0786</td>
<td>0.4696*</td>
<td>0.9481</td>
<td>0.9424</td>
</tr>
<tr>
<td></td>
<td>(4.7177)</td>
<td>(0.2211)</td>
<td>(0.2077)</td>
<td>(1.4744)</td>
<td>(0.6138)</td>
</tr>
<tr>
<td>Lag 1 (i)</td>
<td>-0.3201</td>
<td>0.0947</td>
<td>-0.0026</td>
<td>0.8783**</td>
<td>3.8592**</td>
</tr>
<tr>
<td></td>
<td>(1.8159)</td>
<td>(0.2080)</td>
<td>(0.1238)</td>
<td>(0.2925)</td>
<td>(1.2843)</td>
</tr>
<tr>
<td>Lag 1 $\pi$</td>
<td>0.033</td>
<td>-0.0176</td>
<td>0.0027</td>
<td>-0.0005</td>
<td>0.1168*</td>
</tr>
<tr>
<td></td>
<td>(0.2301)</td>
<td>(0.0277)</td>
<td>(0.0152)</td>
<td>(0.0319)</td>
<td>(0.0501)</td>
</tr>
</tbody>
</table>

Notes: *** p<0.001, **p<0.01, *p<0.05
(P-value between brackets)

$ps_y$: primary surplus over GDP, $\log(b_y)$: log public debt/GDP, $\theta$: growth rate, (i): interest rate, $\pi$: inflation rate.
Forward orthogonal transformation is applied to the variables.
Hansen test of overidentified restrictions: chi2(1375)=35.2, proba >chi2=1
## TABLE 4

Panel VAR OLS Estimates

<table>
<thead>
<tr>
<th>Lag</th>
<th>$ps_y$</th>
<th>$\log(b_y)$</th>
<th>$\theta$</th>
<th>$i$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag 1 $ps_y$</td>
<td>0.825</td>
<td>-0.704</td>
<td>0.271</td>
<td>-0.154</td>
<td>-8.726</td>
</tr>
<tr>
<td></td>
<td>(71.7)</td>
<td>(-7.5)</td>
<td>(4.13)</td>
<td>(-1.817)</td>
<td>(-5.77)</td>
</tr>
<tr>
<td>Lag 1 $\log(b_y)$</td>
<td>-8.6E-05</td>
<td>1.007</td>
<td>0.0013</td>
<td>0.0056</td>
<td>-0.073</td>
</tr>
<tr>
<td></td>
<td>(-0.604)</td>
<td>(859)</td>
<td>(15.4)</td>
<td>(5.3)</td>
<td>(-3.899)</td>
</tr>
<tr>
<td>Lag 1 $\theta$</td>
<td>-0.007</td>
<td>-0.408</td>
<td>0.244</td>
<td>-0.0136</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(-2.054)</td>
<td>(-14.71)</td>
<td>(12.56)</td>
<td>(-0.54)</td>
<td>(1.85)</td>
</tr>
<tr>
<td>Lag 1 (i)</td>
<td>-0.001</td>
<td>0.021</td>
<td>0.043</td>
<td>0.848</td>
<td>3.06</td>
</tr>
<tr>
<td></td>
<td>(-0.61)</td>
<td>(1.567)</td>
<td>(4.47)</td>
<td>(68)</td>
<td>(14.047)</td>
</tr>
<tr>
<td>Lag 1 $\pi$</td>
<td>0.00065</td>
<td>0.00056</td>
<td>-0.0013</td>
<td>-0.002</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>(4.223)</td>
<td>(0.445)</td>
<td>(-1.5)</td>
<td>(-1.89)</td>
<td>(6.7)</td>
</tr>
</tbody>
</table>

**Notes:** *(t-statistics between brackets)*

$ps_y$: primary surplus over GDP, $\log(b_y)$: log public debt/GDP, $\theta$: growth rate of GDP per capita, (i): short-term interest rate, $\pi$: inflation rate

R-squared: 0.6797, Adj R-squared: 0.6791

## TABLE 5

Estimated GMM model for inflation by Monetary Policy framework

<table>
<thead>
<tr>
<th></th>
<th>Whole sample</th>
<th>Monetary Targets</th>
<th>Inflation Targeting</th>
<th>Exchange rate Targets</th>
<th>Discretionary regimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ps_y$</td>
<td>-18.6</td>
<td>0.71</td>
<td>-0.145</td>
<td>10.3</td>
<td>-14.5</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(0.18)</td>
<td>(0.24)</td>
<td>(0.76)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>$\log(b_y)$</td>
<td>-0.2</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.16</td>
<td>-0.24</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(0.82)</td>
<td>(0.5)</td>
<td>(0.74)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>9.3</td>
<td>0.022</td>
<td>0.62</td>
<td>-10.82</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(0.92)</td>
<td>(0.005)</td>
<td>(0.76)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>(i)</td>
<td>4.3</td>
<td>0.72</td>
<td>0.008</td>
<td>-0.97</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(0.907)</td>
<td>(0.79)</td>
<td>(0)</td>
</tr>
<tr>
<td>J-stat (p-value)</td>
<td>0.35</td>
<td>0.02</td>
<td>0.79</td>
<td>0.98</td>
<td>0.37</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.10</td>
<td>0.24</td>
<td>-8.49</td>
<td>-804</td>
<td>0.45</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.09</td>
<td>0.17</td>
<td>-8.58</td>
<td>-816</td>
<td>0.45</td>
</tr>
</tbody>
</table>

**Notes:** *(P-value between brackets)*

$ps_y$: primary surplus over GDP, $\log(b_y)$: log public debt/GDP, $\theta$: Growth rate of GDP per capita, (i): short-term interest rate, $\pi$: inflation rate (dependent variable)
### TABLE 6
Correlation coefficients by fiscal space categories

<table>
<thead>
<tr>
<th>Correlation</th>
<th>S1</th>
<th>S2</th>
<th>Global</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation/ fiscal balance</td>
<td>7%</td>
<td>21%</td>
<td>23%</td>
</tr>
<tr>
<td>Inflation/ public debt</td>
<td>-13%</td>
<td>42%</td>
<td>47%</td>
</tr>
<tr>
<td>Inflation/short-term interest rate</td>
<td>90%</td>
<td>71%</td>
<td>76%</td>
</tr>
</tbody>
</table>

**Notes:** S1: High fiscal space (50th upper percentile); S2: low fiscal space (50th lower percentile)

### TABLE 7
Estimated inflation regression model by fiscal space categories

<table>
<thead>
<tr>
<th></th>
<th>GMM estimation</th>
<th>OLS estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Global S1 S2</td>
<td>25th up q.</td>
</tr>
<tr>
<td>(ps_y)</td>
<td>-18.6 5.4 -43</td>
<td>4.84 -103.8 -4.55</td>
</tr>
<tr>
<td></td>
<td>(0) (0.04) (0)</td>
<td>(0) (0.02) (0)</td>
</tr>
<tr>
<td>(log(b_y))</td>
<td>-0.2 0.08 -0.4</td>
<td>0.04 -1.3 -0.10</td>
</tr>
<tr>
<td></td>
<td>(0) (0.22) (0)</td>
<td>(0.58) (0.01) (0)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>9.3 -11 20.3</td>
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<tr>
<td></td>
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<tr>
<td>(i)</td>
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<td>(0) (0) (0)</td>
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<td>0.28 0.97</td>
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<td>R-squ.</td>
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<td>0.28 -0.73 0.20</td>
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</table>

**Notes:** (\(P\)-value between brackets)

S1: subsample of the upper 50th percentile (High fiscal space); S2: subsample of the lower 50th percentile (Low fiscal space); 25th up q.: subsample of the upper 25th percentile (Highest fiscal space quartile); 25th low q.: subsample of the lower 25th percentile (Lowest fiscal space quartile).

**Variables**
- \(ps_y\): primary surplus over GDP,
- \(log(b_y)\): log public debt/GDP,
- \(\theta\): growth of GDP per capita,
- (i): short-term interest rate,
- \(\pi\): inflation rate.
FIGURE 1: Variance decomposition of inflation using Cholesky (d.f. adjusted) factors

$ps/y$: primary surplus over GDP, $b/y$: log of public debt over GDP, $\pi$: inflation rate, $i$: short-term interest rate, $\theta$: growth rate of GDP per capita

FIGURE 2: Variance decomposition of public debt using Cholesky (d.f adjusted) factors

$ps/y$: primary surplus over GDP, $b/y$: log of public debt over GDP, $\pi$: inflation rate, $i$: short-term interest rate, $\theta$: growth rate of GDP per capita
FIGURE 3: Response of inflation to a one SD shock to all variables

(Inflation $\pi$ plotted on the right axis)

FIGURE 4: Response of all variables to a one SD innovation in inflation

(Inflation $\pi$ plotted on the right axis)

FIGURE 5: Response of all variables to a positive fiscal policy shock

(Inflation $\pi$ plotted on the right axis)

FIGURE 6: Response of inflation to a positive fiscal policy shock, by MPF

FIGURE 7: Response of inflation to a positive fiscal policy shock, by fiscal space category

FIGURE 8: Response of inflation to a positive shock to public debt, by fiscal space category

(Inflation $\pi$ plotted on the right axis)
FIGURE 9: Response of all variables to a positive monetary policy shock (contractionary policy)

FIGURE 10: Response of inflation to a positive monetary policy shock, by MPF

FIGURE 11: Response of inflation to a positive monetary policy shock, by fiscal space category

FIGURE 12: Response of all variables to a recessionary shock

FIGURE 13: Response of inflation to a positive fiscal policy shock

FIGURE 14: Correlation coefficients between inflation, fiscal balance, and interest rates (by MPF)
APPENDIX 1: SAMPLE COUNTRIES

<table>
<thead>
<tr>
<th>Group</th>
<th>Advanced Countries</th>
<th>Low-Income and Developing Countries</th>
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</thead>
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<tr>
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<tr>
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<tr>
<td>United States</td>
<td></td>
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</tbody>
</table>

APPENDIX 2: A GENERAL OVERVIEW OF DATA

Based on the theoretical model, we use the following variables in the empirical study. Because \( \log(1 + x) \approx x \) for small values of \( x \), in some cases, variables are considered at level:

- Level of Primary Balance/GDP (%): data for primary balances are retrieved from the dataset of Mauro et al. (2013). Missing data are completed from various databases such as the OECD, the World Bank and the website http://moxlad-staging.herokuapp.com/home/es for Latin American economies.

- Log of public debt to GDP: we use the underlying dataset of the paper Mauro et al. (2013). Missing data are then completed from various sources such as the website "tradingeconomics.com" and the database of Reinhart et al. (2019).

- GDP growth (%): data of GDP per capita are extracted from the World Bank database.

- Short term interest rates: data are collected from several sources, more specifically from the IFS, Eurostat, the OECD database and in some cases from central banks’ websites. For very large values, \( \log(1+i) \) is used.
• Inflation rate: data are calculated from the Consumer Price Index (with 2010 as the base year), taken from the World Bank database. Missing data are completed based on Reinhart et al. (2009). For double-digit values, log(1+inflation) is used instead.

Yearly evolution of data shows that the level of indebtedness has overall been increasing over the recent years. By revenue groups (using the IMF classification), economies affected the most by this increase are advanced economies (see figure below); whereas the most extreme levels are observed for low-income developing economies in the 1980s (scaled on the right axis). Both developing and emerging economies have been deleveraging in the recent decade.

Inflation levels became very low for all countries after 1995 (see figure below, emerging economies and total average are modeled on right axis). The most extreme levels have been observed in the 1970s hyperinflation episode, especially for advanced economies. In emerging economies, the maximum level of inflation is observed for the year 1990 due to very high values noted for Argentina, Brazil and Peru. At the same time, public debt to GDP reached its highest level in the 1980s. However, in the recent decades, the level of Public debt to GDP is lower than the high peaks of the 1980s or the year 2002 (Argentina’s debt crisis). Regarding developing economies, excessively high levels of debt accompanied by high inflation were observed in the 1980s; whereas in the recent years, the levels of both public debt to GDP and inflation are relatively low.

**APPENDIX 3: CLASSIFICATION OF MONETARY POLICY FRAMEWORKS**

Cobham (2018) identified 32 different monetary policy frameworks, aggregated into the following 9 broad categories (based on target variables):

• Direct controls: multiple exchange rates and/or controls on direct lending, interest rates, etc.
- Fixed exchange rates: exchange rate fixed by intervention, some or no monetary instruments in use or pure currency board (domestic currency 100% backed by foreign currency, no monetary instruments in use)

- Exchange rate target

- Monetary target

- Inflation target

- Mixed targets: monetary targets and exchange rate fixes or targets, monetary dominant or use of three full targets (or fixes) (money, ER and IT), whichever dominant

- Unstructured, loosely structured discretion: ineffective set of instruments and incoherent mix of objectives

- Well structured discretion

- No national framework: this category encompasses cases where a different sovereign currency is used (such as dollarization) in addition to membership in a currency union (euro)

Using this classification for our sample (see following figure), we can see that the IT regime has spread significantly after the 1990s, while monetary target frameworks disappeared and exchange rate targets and discretionary regimes became less frequent.
APPENDIX 4: ESTIMATED FISCAL SPACE

Table(a) provides the most recent available value (2016), the average and some descriptive statistics of the calculated fiscal space measure. We note that in 2016, Japan has the lowest fiscal space with the greatest number of years required to repay the debt, but not the largest fiscal deficit (observed in Brazil) (see figure below). From a simple observation of data, the link between the fiscal balance and fiscal space does not seem obvious, since both groups of countries running substantial budget deficits and those with high surpluses can have either low or high fiscal space.

However, correlation coefficients between fiscal space and the primary balance appear to be significant in many countries (see table (b)), even though the correlation sign varies (it is negative in cases like Japan, Australia, Brazil and the USA, and positive in some other cases, such as Belgium, Mexico and Pakistan, etc). We also note that, after the 2007 crisis, the fiscal space measure has worsened for advanced economies in particular (for example Japan, from 3.4 to 6.9 or the USA, from 1.8 to 2.9), reflecting the use of fiscal stimulus after the recession. In contrast, emerging and developing economies, which suffered from a deteriorating fiscal stance during the 1980s debt crisis, have had a better fiscal space indicator over the last decade.
Table (a): Inverse of fiscal space ratio (in years): descriptive statistics by country

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<th></th>
<th>2016 Value</th>
<th>Average</th>
<th>Min</th>
<th>Max</th>
<th>Stand. Deviation</th>
<th>Number of observations</th>
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Notes: Fiscal space is defined as the sum of total government revenues divided by public debt. The inverse of this measure reflects the number of years of revenue needed to repay the outstanding of public debt at a given date.
Table (b): Correlation coefficients between the fiscal balance and the inverse of the fiscal space ratio (The number of years of revenue needed to repay the debt)

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APPENDIX 5: PANEL VAR GMM MODELS
(METHODOLOGY OF SIGMUND & FERSTL, 2017)

In this paper, we use the methodology provided by Sigmund and Ferstl (2017) for panel VAR models. As classical OLS-based regression methods cannot be applied for panel data models because of the Nickell bias (Nickell, 1981), authors suggested the use of generalized method of moments for panel VAR models. More specifically, they provided an extension to the work of Anderson and Hsiao (1982), for the first difference GMM estimator, using lags of the endogenous variables as instruments (Holtz-Eakin et al., 1988; Arellano and Bond, 1991) and for the system GMM estimator (Blundell and Bond, 1998).

Sigmund and Ferstl (2017) use the PVAR model of Holtz-Eakin et al. (1988) and extend it to allow for \( p \) lags of \( m \) endogenous variables, \( k \) predetermined variables and \( n \) strictly exogenous variables, such that:

\[
y_{i,t} = (I_m - \sum_{l=1}^{p} A_l) \mu_i + \sum_{l=1}^{p} A_l y_{i,t-l} + B x_{i,t} + C s_{i,t} + \varepsilon_{i,t}
\]

with \( I_m \) an \( m \times m \) identity matrix, \( y_{i,t} \) an \( m \times 1 \) vector of endogenous variables, \( y_{i,t-l} \) an \( m \times 1 \) vector of lagged endogenous variables, \( x_{i,t} \) a \( k \times 1 \) vector of predetermined variables, \( s_{i,t} \) a \( n \times 1 \) vector of strictly exogenous variables, \( \varepsilon_{i,t} \) the idiosyncratic error vector. \( A_l \) \( (m \times m) \), \( B \) \( (m \times k) \) and \( C \) \( (m \times n) \) are parameter matrices. \( \mu_i \) is an individual error component representing the fixed effects.

First, fixed effects are removed by transforming this relation into its first difference or applying the forward orthogonal transformation (suggested by Arellano and Bover (1995) to minimize data losses resulting from data gaps). Based on the first difference representation, the derived first difference GMM moment conditions are as follows:

\[
E(\Delta \varepsilon_{i,t} y_{T,i,j}^T) = 0 \quad j \in \{1, \ldots, T-2\}
\]

\[
E(\Delta \varepsilon_{i,t} x_{T,i,j}^T) = 0 \quad j \in \{1, \ldots, T-1\}
\]

\[
E(\Delta \varepsilon_{i,t} \Delta s_{T,i,t}^T) = 0
\]

Considering that

\[
q_{i,t}^T := (y_{T,i,t-p-1}^T, \ldots, y_{T,i,1}^T, x_{T,i,t-1}^T, \ldots, x_{T,i,1}^T, \Delta s_{T,i,t}^T)
\]

Then, after stacking the model over time, moment conditions for each \( i \) are:

\[
E[Q_i^T (\Delta E_i)] = 0
\]

Considered as equivalent to the sample average \( g(\Phi) = \frac{1}{N} \sum_{i=1}^{N} g_i(\Phi) \) where:

\[
g_i(\Phi) = (Q_i \otimes I_{m \times m})(\text{vec}(\Delta E_i))
\]

The number of moment conditions depends on the value of \( p, m, k \) and \( n \). Two solutions are suggested to reduce this number: one is fixing a maximal lag length after which no further instruments are used, or starting with a different minimal lag. This idea can be applied to lagged endogenous variables and to predetermined variables. The other is collapsing the instruments such that the first difference GMM moment conditions become:
\[ E[\sum_{j=1}^{T-2} (\Delta \varepsilon_{i,t} y_{i,j}^T)] = 0 \]
\[ E[\sum_{j=1}^{T-1} (\Delta \varepsilon_{i,t} x_{i,j}^T)] = 0 \]
\[ E(\Delta \varepsilon_{i,t} \Delta s_{i,t}^T) = 0 \]

And \( Q_t \) is reduced to a \((T - 2) \times (T - 2)\) matrix. The GMM estimator is derived by minimizing the function:

\[
\Pi(\Phi) = \left( \sum_{i=1}^{N} Z_i^T \text{vec}(\Delta Y_i - [\Delta Y_i, -1\Delta X_i \Delta S_i] \Phi) \right)^T \Lambda^{-1}_Z \left( \sum_{i=1}^{N} Z_i^T \text{vec}(\Delta Y_i - [\Delta Y_i, -1\Delta X_i \Delta S_i] \Phi) \right)
\]

where \( \Phi \) is defined as \([A \ B \ C]\), an \( m \times (p + k + n) \) matrix of parameters, \( Z_i \) is equal to \( Q_i \otimes I_{m \times m} \) and \( \Lambda_Z \) is the GMM weighting matrix. This matrix is defined, as proposed in the relevant literature through a one-step or a two-step estimation procedure. In the one-step estimation procedure, it is defined as in Binder et al. (2005)

\[
\Lambda_Z = \left[ \sum_{i=1}^{N} Q_i^T D D^T Q_i \right] \otimes I_{m \times m}
\]

\( D \) serves as a \((T - 1) \times T\) linear transformation matrix such that for any matrix \( V_i \), \( DV_i = \Delta V_i \) The two-step estimation uses the residuals of the one-step estimation as \( \Delta E_i \)

In addition, Sigmund and Ferstl (2017) address the case of a system GMM estimator, which is considered to perform better as the first difference GMM estimator. The additional moment conditions in this case are:

\[ E[\varepsilon_{i,t} + (I - \sum_{j=1}^{p} A_j) \mu_i (y_{i,t-1} - y_{i,t-2})^T] = 0, \quad t \in 3, 4, \ldots T \]
\[ E[\varepsilon_{i,t} + (I - \sum_{j=1}^{p} A_j) \mu_i (x_{i,t} - x_{i,t-1})^T] = 0, \quad t \in 2, 3, \ldots T \]
\[ E[\varepsilon_{i,t} + (I - \sum_{j=1}^{p} A_j) \mu_i s_{i,t}^T] = 0, \quad t \in 2, 3, \ldots T \]

The estimator is then derived similarly as in the previous case. Authors also provide a framework of structural analysis functions for PVAR models such as orthogonal and generalized impulse response functions, with a GMM-specific bootstrap method for estimating confidence intervals, and forecast error variance decomposition.

**APPENDIX 6: GENERALIZED IMPULSE RESPONSE FUNCTIONS**

Suppose we have the following standard VAR(2) model:

\[ Z_t = \theta_0 + \theta Z_{t-1} + \varepsilon_t \] (16)

With \( Z_t = \begin{bmatrix} b_t \\ p_t \end{bmatrix} \), \( \theta_0 = \begin{bmatrix} \theta_{b0} \\ \theta_{p0} \end{bmatrix} \), \( \theta = \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \) and \( \varepsilon_t = \begin{bmatrix} \varepsilon_{bt} \\ \varepsilon_{pt} \end{bmatrix} \)

Backward iteration yields the following expression:
In this case, \( W \) can be stated as follows:

\[
Z_t = \theta_0 + \theta (\theta_0 + \theta Z_{t-2} + \varepsilon_{t-1}) + \varepsilon_t
\]

\[
Z_t = \theta_0 + \theta \times \theta_0 + \theta^2 Z_{t-2} + \theta \times \varepsilon_{t-1} + \varepsilon_t
\]

Similarly

\[
Z_t = \theta_0 + \theta \times \theta_0 + \theta^2 (\theta_0 + \theta Z_{t-3} + \varepsilon_{t-2}) + \theta \times \varepsilon_{t-1} + \varepsilon_t
\]

\[
Z_t = \theta_0 + \theta \times \theta_0 + \theta^2 \times \theta_0 + \theta^3 Z_{t-3} + \theta^2 \varepsilon_{t-2} + \theta \times \varepsilon_{t-1} + \varepsilon_t
\]

\[
Z_t = \theta_0 \times (1 + \theta + \theta^2) + \theta^3 Z_{t-3} + (\theta^2 \varepsilon_{t-2} + \theta \times \varepsilon_{t-1} + \varepsilon_t)
\]

More generally:

\[
Z_t = \theta_0 \times (1 + \ldots + \theta^n) + \theta^{n+1} Z_{t-(n+1)} + \sum_{i=0}^{n} \theta^i \varepsilon_{t-i}
\]

If the stability condition holds, then \( A_{11}^{n+1} \) would tend towards 0 as \( n \) approaches infinity. This leads to the MA representation of the VAR model:

\[
Z_t = \mu + \sum_{i=0}^{n} \theta^i \varepsilon_{t-i}
\]

Impulse response functions express the response of one endogenous variable to an impulse in another endogenous variable. Based on the MA representation of the VAR model, IRF can be stated as follows:

\[
IRF(k, r) = \frac{\partial Z_{t+k}}{\partial (\varepsilon_t)_r} = \theta^k e_r
\]

where \( k \) is the number of periods after the shock to the \( r \)th component of and \( er \) is a \( 2 \times 1 \) vector with 1 in the \( r \)th column and 0 otherwise. In the VAR model used so far, the disturbances terms are correlated since they incorporate the contemporaneous effects of all the endogenous variables. The corresponding structural VAR model can be expressed as follows:

\[
\begin{bmatrix}
1 & A_{12} \\
A_{21} & 1
\end{bmatrix}
\begin{bmatrix}
b_t \\
p_t
\end{bmatrix}
= \begin{bmatrix}
\phi_{w0} \\
\phi_{p0}
\end{bmatrix} + \begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
b_{t-1} \\
p_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
w_{bt} \\
w_{pt}
\end{bmatrix}
\]

(18)

Considering that \( A = \begin{bmatrix} 1 & A_{12} \\ A_{21} & 1 \end{bmatrix} \), \( \phi_0 = \begin{bmatrix} \phi_{w0} \\ \phi_{p0} \end{bmatrix} \), \( \phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \) and \( W_t = \begin{bmatrix} w_{bt} \\ w_{pt} \end{bmatrix} \)

Then

\[
AZ_t = \phi_0 + \phi Z_{t-1} + W_t
\]

(19)

In this case, \( W_t \) is a vector of uncorrelated white-noise disturbances.

Also

\[
Z_t = A^{-1} \phi_0 + A^{-1} \phi Z_{t-1} + A^{-1} W_t
\]

(20)

Such that: \( \theta_0 = A^{-1} \phi_0, \theta = A^{-1} \phi \) and \( \varepsilon_t = A^{-1} W_t \)

Thus \( \varepsilon_t = A^{-1} W_t = \begin{bmatrix} 1 & A_{12} \\ A_{21} & 1 \end{bmatrix}^{-1}
\begin{bmatrix}
w_{bt} \\
w_{pt}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{1-A_{12} A_{21}} & \frac{-A_{12}}{1-A_{12} A_{21}} \\ \frac{1}{1-A_{12} A_{21}} & \frac{1}{1-A_{12} A_{21}}
\end{bmatrix}
\begin{bmatrix}
w_{bt} \\
w_{pt}
\end{bmatrix}
\]

And

\[
\varepsilon_{bt} = \frac{w_{bt}-A_{12} w_{pt}}{1-A_{12} A_{21}}
\]

\[
\varepsilon_{pt} = \frac{-A_{21} w_{bt} + w_{pt}}{1-A_{12} A_{21}}
\]

It follows that these error terms are composites of the two shocks from the two variables of the model. Because of this feedback effect, it is difficult to estimate the structural model.
But after estimating the reduced-form VAR model, it is possible to decompose $\varepsilon_t$ by finding a matrix $A$ such that $\varepsilon_t = A^{-1}W_t$. Since the objective is to isolate a disturbances vector that would have a diagonal variance covariance matrix, one way is using the Cholesky decomposition on the variance covariance matrix of $\varepsilon_t$ to find a lower triangular matrix $A$, such that $AA' = E(\varepsilon\varepsilon')$. Conversely, the variance covariance matrix of $W_t$ will be diagonal because and therefore $E(ww') = AE(\varepsilon\varepsilon')A' = A\Sigma \varepsilon A' = I$.

Using the structural model, we obtain the orthogonal impulse response function: $OIRF(k, r) = \frac{\partial Z_{i,t+k}}{\partial \varepsilon_{i,t,r}} = \theta_{k} \Sigma_{\varepsilon}^{-1} \delta_{r}$, such that $B_{k} = \theta_{k} A^{-1}$. The main limit of using the Cholesky-decomposition is its dependence on the ordering of the variables. To remedy this issue, Pesaran and Shin (1998) suggested an alternative approach which is the Generalized IRF. They described IR as the outcome of a conceptual experiment in which the effect over time of a hypothetical vector of shocks $\delta$ hitting the economy at time $t$ is compared with a base-line profile at time $t + k$ given the economy’s history. Therefore the disturbances vector comprises both the shocks expected to hit the economy before $t$ and the vector $\delta$. Formally:

$$GIRF(k, \delta, \Omega_{t-1}) = E[Z_{i,t+k}|\varepsilon_{i,t,r} = \delta_{r}, \Omega_{t-1}] - E[Z_{i,t+k}|\Omega_{t-1}]$$

With $\Omega_{t-1}$ being the set of available information about economic history at time $t-1$. The idea of Pesaran and Shin (1998) approach is choosing to shock only one element (the $r$th element) and integrate the effects of other shocks using the historically observed distribution of the errors. The $\Sigma_{\varepsilon}$ being the variance covariance matrix of $\varepsilon_t$, GIRF are thus expressed as:

$$GIRF(k, r, \Sigma_{\varepsilon}) = E[Z_{i,t+k}|\varepsilon_{i,t,r} = \delta_{r}, \Sigma_{\varepsilon}] - E[Z_{i,t+k}|\Sigma_{\varepsilon}]$$

(21)

As pointed out by Koop et al. (1996), if the vector of random shocks is considered as jointly normally distributed, then the conditional expectation of the shocks is a linear function of $\delta$:

$$E(\varepsilon_{i,t,r} = \delta_{r}) = (\sigma_{1r}, \sigma_{2r}, \ldots, \sigma_{mr})' = \Sigma_{\varepsilon}^{-1} \delta_{r}$$

(22)

The generalized impulse response of the effect of a shock to the $r$th equation at time $t$ on $Z_{t+k}$ is given by:

$$\theta_{k} E(\varepsilon_{i,t,r} = \delta_{r}) = (\theta_{k} \Sigma_{\varepsilon}^{-1/2})(\delta_{r}/\sqrt{\sigma_{rr}})$$

By setting $\delta_{r} = \sqrt{\sigma_{rr}}$, and considering that $\sigma_{r,r}$ is the $r$-th diagonal element of $\Sigma_{\varepsilon}$, we obtain the following expression:

$$GIRF(k, r, \Sigma_{\varepsilon}) = \theta_{k} \Sigma_{\varepsilon}(\sigma_{r,r})^{-1/2}$$

(23)

**APPENDIX 7: FORWARD ORTHOGONAL TRANSFORMATION**

The forward orthogonal transformation was suggested by Arellano and Bover (1995) to minimize data losses due to data gaps. Based on the following panel VAR model that includes fixed effects:

$$y_{i,t} = \mu_{i,t} + \sum_{l=1}^{p} A_l y_{i,t-l} + B x_{i,t} + C s_{i,t} + \epsilon_{i,t}$$

(24)
where $y_{i,t}$ is a vector of endogenous variables for the $i_{th}$ cross-section at time $t$, $y_{i,t-1}$ a vector of lagged endogenous variables, $x_{i,t}$ a vector of predetermined variables potentially correlated with past error terms, $s_{i,t}$ a vector of strictly exogenous variables independent from error terms and $\epsilon_{i,t}$ a vector of i.i.d. disturbance terms. When the first difference transformation is used, we get

$$\Delta y_{i,t} = \sum_{l=1}^{p} A_l \Delta y_{i,t-l} + B \Delta x_{i,t} + C \Delta s_{i,t} + \Delta \epsilon_{i,t} \quad (25)$$

On the other hand, if forward orthogonal transformation is applied, variables are replaced by the following expression

$$y_{i,t+1}^L = c_{i,t} \left( y_{i,t} - \frac{1}{T_{i,t}} \sum_{s > t} y_{i,s} \right) \quad (26)$$

Where $c_{i,t} = \sqrt{\frac{T_{i,t}}{T_{i,t} + 1}}$