Effects of Minimum Wage on Automation and Innovation in a Schumpeterian Economy

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Abstract

This study explores the effects of minimum wage on automation and innovation in a Schumpeterian growth model. We find that raising the minimum wage decreases the employment of low-skill workers and has ambiguous effects on innovation and automation. Specifically, if the elasticity of substitution between low-skill workers and high-skill workers in production is less (greater) than unity, then raising the minimum wage leads to an increase (a decrease) in automation and innovation. We also calibrate the model to aggregate data to quantify the effects of minimum wage on the macroeconomy.

JEL classification: E24, O30, O40
Keywords: minimum wage, unemployment, innovation, automation

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1 Introduction

Does raising the minimum wage provide incentives for firms to allocate resources to innovation and the automation of the production process? Or does the decrease in low-skill production labor as a result of raising the minimum wage lead to a reallocation of high-skill labor from innovation and automation to the production of goods and services? We find that both scenarios are possible. Which scenario occurs crucially depends on a structural parameter that determines the elasticity of substitution between low-skill workers and high-skill workers in production.

Specifically, we consider a Schumpeterian growth model in which the production of goods requires both low-skill workers and high-skill workers whereas the automation process and the innovation process require only high-skill workers. Within this growth-theoretic framework, we find that raising the minimum wage decreases the employment of low-skill workers and has ambiguous effects on automation and innovation. Specifically, the effects of minimum wage on automation and innovation depend on the elasticity of substitution between low-skill workers and high-skill workers in production. If this elasticity of substitution is less (greater) than unity, then raising the minimum wage leads to an increase (a decrease) in automation and innovation.

The intuition of the above results can be explained as follows. Because the minimum wage is binding in the low-skill labor market but not in the high-skill labor market, raising the minimum wage reduces low-skill employment but does not affect high-skill employment. The decrease in low-skill production workers leads to a decrease (an increase) in high-skill production workers if the two types of workers are gross complements (substitutes) in which case the amount of high-skill workers for automation and innovation increases (decreases). Finally, we calibrate the model to aggregate data in the US economy to simulate the quantitative effects of minimum wage on unemployment, capital intensity, automation, innovation, economic growth and social welfare.

This study relates to the literature on innovation and economic growth. The seminal study by Romer (1990) develops the first R&D-based growth model in which the creation of new products drives economic growth. Then, subsequent studies by Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) develop the Schumpeterian growth model in which the quality improvement of products drives economic growth. In this literature, some studies, such as Askenazy (2003), Meckl (2004), Agenor and Lim (2018) and Chu, Kou and Wang (2019), introduce minimum wage into variants of the R&D-based growth model to explore the relationship between unemployment and innovation.[1] This study differs from these previous studies by introducing automation into the analysis and analyzing the relationship between minimum wage and automation. If we set aside automation in the model, then our result relates to previous studies on minimum wage and innovation by showing that the elasticity of substitution between low-skill workers and high-skill workers in production determines the effect of minimum wage on innovation.

This study also relates to the literature on automation and economic growth.[2] The

[1] There are other approaches of incorporating unemployment into the R&D-based growth model; see Mortensen and Pissarides (1998) for search frictions, Parello (2010) for efficiency wage, Peretto (2011) for wage bargaining, and Ji *et al.* (2016) and Chu *et al.* (2016, 2018) for trade unions.

[2] See Aghion *et al.* (2017) for a comprehensive discussion of this literature.

seminal study in this literature is Zeira (1998), who develops a growth model with capital-labor substitution. Subsequent studies by Zeira (2006), Peretto and Seater (2013), Aghion *et al.* (2017), Acemoglu and Restrepo (2018) and Hemous and Olson (2018) introduce this capital-labor substitution into variants of the R&D-based growth model to explore the relationship between automation and innovation.[3] This study complements these interesting studies by introducing minimum wage into the Schumpeterian growth model with automation in Chu, Cozzi, Furukawa and Liao (2019) to explore the relationship between unemployment and automation. Prettner and Strulik (2019) develop a variety-expanding R&D-based growth model with unemployment driven by fair wage as in Akerlof and Yellen (1990) to analyze the effect of automation on unemployment. Instead, we focus on the effect of minimum wage on the relationship between unemployment and automation, which turns out to be ambiguous and depends on the elasticity of substitution between low-skill workers and high-skill workers in production.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 explores the effects of minimum wage. Section 4 concludes.

2 A Schumpeterian growth model with automation and minimum wage

The Schumpeterian growth model originates from Aghion and Howitt (1992). Chu, Cozzi, Furukawa and Liao (2019) incorporate capital-labor substitution as in Zeira (1998) into the Schumpeterian growth model with an automation-innovation cycle. We generalize their production function to allow for a non-unitary elasticity of substitution between low-skill workers and high-skill workers in production and introduce minimum wage into the model to explore its effects on unemployment, automation and innovation.

2.1 Household

The utility function of the representative household is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{1}$$

where c_t is the household's consumption of final good (numeraire) and the parameter $\rho > 0$ determines the rate of subjective discounting. The household maximizes (1) subject to the following asset-accumulation equation:

$$\dot{a}_t + \dot{k}_t = r_t a_t + (R_t - \delta)k_t + w_{h,t}H + \overline{w}_{l,t}l_t + b_t\left(L - l_t\right) - \tau_t - c_t. \tag{2}$$

a_t is the value of assets owned by the household. r_t is the real interest rate. k_t is the amount of physical capital owned by the household. $R_t - \delta$ is the rental price of capital net of depreciation. The household has $H + L$ members. Each of H members supplies one unit of

[3] See Chu, Cozzi, Furukawa and Liao (2019) for a discussion of these studies.

high-skill labor and earns the high-skill wage rate $w_{h,t}$, which is above the minimum wage and determined as an equilibrium outcome in the high-skill labor market. Each of L members supplies one unit of low-skill labor. Employed low-skilled workers l_t earn the low-skill wage rate $\overline{w}_{l,t}$, which is determined by the minimum wage set by the government. Unemployed low-skill workers $L - l_t$ receive an unemployment benefit $b_t < \overline{w}_{l,t}$. The household pays a lump-sum tax τ_t to the government. Dynamic optimization yields the Euler equation as

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

Also, the no-arbitrage condition $r_t = R_t - \delta$ holds.

2.2 Final good

Competitive firms produce final good y_t using the following Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods:

$$y_t = \exp\left(\int_0^1 \ln x_t(i) di\right). \tag{4}$$

$x_t(i)$ denotes intermediate good $i \in [0,1]$. Profit maximization yields the conditional demand function for $x_t(i)$ as

$$x_t(i) = \frac{y_t}{p_t(i)}, \tag{5}$$

where $p_t(i)$ is the price of $x_t(i)$.

2.3 Unautomated intermediate goods

There is a unit continuum of industries $i \in [0,1]$ that produce differentiated intermediate goods. If an industry is not automated, then the production process uses low-skill labor $l_t(i)$ and high-skill labor $h_{x,t}(i)$. The production function is given by

$$x_t(i) = z^{n_t(i)} \left\{(1-\beta)\left[l_t(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left[h_{x,t}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{\varepsilon}{\varepsilon-1}}, \tag{6}$$

where the parameter $\varepsilon \in (0,\infty)$ is the elasticity of substitution between $l_t(i)$ and $h_{x,t}(i)$. From cost minimization, the conditional demand functions for $l_t(i)$ and $h_{x,t}(i)$ are given by

$$\overline{w}_{l,t} = \frac{(1-\beta)\xi_t(i) z^{n_t(i)}}{\left[l_t(i)\right]^{\frac{1}{\varepsilon}}} \left\{(1-\beta)\left[l_t(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left[h_{x,t}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{1}{\varepsilon-1}}, \tag{7}$$

$$w_{h,t} = \frac{\beta\xi_t(i) z^{n_t(i)}}{\left[h_{x,t}(i)\right]^{\frac{1}{\varepsilon}}} \left\{(1-\beta)\left[l_t(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left[h_{x,t}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{1}{\varepsilon-1}}, \tag{8}$$

where ξ_t is the Lagrange multiplier from the cost minimization problem. Using (7) and (8), we obtain $l_t(i)/h_{x,t}(i) = \{[\beta/(1-\beta)](\overline{w}_{l,t}/w_{h,t})\}^{-\varepsilon}$. We substitute this relative labor demand function into (6) to derive

$$l_t(i) = \frac{x_t(i)}{z^{n_t(i)}} \left(\frac{\overline{w}_{l,t}}{1-\beta} \frac{1}{\psi_t} \right)^{-\varepsilon}, \tag{9}$$

$$h_{x,t}(i) = \frac{x_t(i)}{z^{n_t(i)}} \left(\frac{w_{h,t}}{\beta} \frac{1}{\psi_t} \right)^{-\varepsilon}, \tag{10}$$

where we have defined the following transformed variable:

$$\psi_t \equiv \left[(1-\beta) \left(\frac{\overline{w}_{l,t}}{1-\beta} \right)^{1-\varepsilon} + \beta \left(\frac{w_{h,t}}{\beta} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Using (9) and (10), we find that the marginal cost of production for the leader in an unautomated industry i is given by $\psi_t/z^{n_t(i)}$. Aghion and Howitt (1992) and Grossman and Helpman (1991) assume that the markup ratio is given by the quality step size, due to limit pricing between current and previous quality leaders. Here we follow Howitt (1999) and Dinopoulos and Segerstrom (2010) to assume that previous quality leaders exit the market and need to pay a re-entry cost. In this case, the unconstrained profit-maximizing monopolistic price would be infinite, so we consider price regulation as in Evans *et al.* (2003) to impose a policy constraint on the markup ratio such that

$$p_t(i) \leq \mu \frac{\psi_t}{z^{n_t(i)}}. \tag{11}$$

To maximize profit, the industry leader chooses $p_t(i) = \mu\psi_t/z^{n_t(i)}$. In this case, the wage payment in an unautomated industry is

$$\overline{w}_{l,t} l_t(i) + w_{h,t} h_{x,t}(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{12}$$

and the amount of monopolistic profit in an unautomated industry is

$$\pi_t^l(i) = p_t(i)x_t(i) - [\overline{w}_{l,t} l_t(i) + w_{h,t} h_{x,t}(i)] = \frac{\mu - 1}{\mu} y_t. \tag{13}$$

2.4 Automated intermediate goods

If an industry is automated, then production uses capital as in Zeira (1998). The production function is

$$x_t(i) = \frac{A}{Z_t} z^{n_t(i)} k_t(i), \tag{14}$$

where $A > 0$ is a relative productivity parameter and Z_t captures an erosion effect of new technologies that reduce the adaptability of existing physical capital. Given the productivity level $z^{n_t(i)}$, the marginal cost function of the leader in an automated industry i is $Z_t R_t/[Az^{n_t(i)}]$. Due to price regulation, the monopolistic price $p_t(i)$ is once again a markup μ over the marginal cost $Z_t R_t/[Az^{n_t(i)}]$ such that

$$p_t(i) = \mu \frac{Z_t R_t}{Az^{n_t(i)}}. \tag{15}$$

The capital rental payment in an automated industry is

$$R_t k_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{16}$$

and the amount of monopolistic profit in an automated industry is

$$\pi_t^k(i) = p_t(i) x_t(i) - R_t k_t(i) = \frac{\mu - 1}{\mu} y_t. \tag{17}$$

2.5 Automation-innovation cycle

This section derives the equilibrium condition that supports an automation-innovation cycle, which can be explained as follows. When an industry becomes automated, it uses capital as the factor input. In order for the automation to reduce the marginal cost of production, we need the following condition to hold: $Z_t R_t / A < \psi_t$. Then, when an automated industry becomes unautomated, it uses the two types of workers as factor inputs. In order for the innovation to reduce the marginal cost of production, we need the following condition to hold: $\psi_t / z < Z_t R_t / A$. Combining these two conditions yields $\psi_t / z < Z_t R_t / A < \psi_t$. In Lemma 1, we derive the steady-state equilibrium expression for this condition, in which $g_y \equiv \dot{y}_t / y_t$ denotes the steady-state growth rate of output.

Lemma 1 *The steady-state equilibrium condition for the automation-innovation cycle is*

$$\frac{1}{z} < \left[\frac{\mu}{A} \left(g_y + \rho + \delta \right) \right]^{\frac{1}{1-\theta}} < 1.$$

Proof. See Appendix A. ■

2.6 Innovation and automation

Equations (13) and (17) imply $\pi_t^l(i) = \pi_t^l$ and $\pi_t^k(i) = \pi_t^k$. Therefore, we follow the standard treatment to focus on the symmetric equilibrium in which $v_t^l(i) = v_t^l$ and $v_t^k(i) = v_t^k$.[4] The no-arbitrage condition that determines the value v_t^l of an unautomated invention is

$$r_t = \frac{\pi_t^l + \dot{v}_t^l - (\alpha_t + \lambda_t) v_t^l}{v_t^l}, \tag{18}$$

which equates the interest rate to the rate of return on v_t^l given by the sum of profit π_t^l and capital gain $\dot{v}_t^l$ minus expected capital loss $(\alpha_t + \lambda_t) v_t^l$, where α_t is the arrival rate of

[4] See Cozzi *et al.* (2007) for a microfoundation of the symmetric equilibrium in the Schumpeterian model.

automation and λ_t is the arrival rate of innovation. Similarly, the no-arbitrage condition that determines the value v_t^k of an automation is

$$r_t = \frac{\pi_t^k + \dot{v}_t^k - \lambda_t v_t^k}{v_t^k}, \tag{19}$$

which equates the interest rate to the rate of return on v_t^k given by the sum of profit π_t^k and capital gain $\dot{v}_t^k$ minus expected capital loss $\lambda_t v_t^k$, where λ_t is the arrival rate of innovation. The condition in Lemma 1 ensures that the previous automation becomes obsolete when the next innovation arrives.

Competitive entrepreneurs perform innovation in industry i by employing high-skill labor $h_{r,t}(i)$. The arrival rate of innovation in industry i is given by

$$\lambda_t(i) = \varphi_t h_{r,t}(i), \tag{20}$$

where $\varphi_t \equiv \varphi h_{r,t}^{\eta-1}$. The aggregate arrival rate of innovation is $\lambda_t = \varphi h_{r,t}^{\eta}$, where $h_{r,t}$ denotes aggregate R&D labor, and the parameter $\eta \in (0,1)$ captures the intratemporal duplication externality in Jones and Williams (2000).[5] In a symmetric equilibrium, the free-entry condition of R&D becomes

$$\lambda_t v_t^l = w_{h,t} h_{r,t} \Leftrightarrow \varphi v_t^l = w_{h,t} h_{r,t}^{1-\eta}. \tag{21}$$

Competitive entrepreneurs also perform automation in industry i by employing high-skill labor $h_{a,t}(i)$. The arrival rate of automation in industry i is given by

$$\alpha_t(i) = \phi_t h_{a,t}(i), \tag{22}$$

where $\phi_t \equiv \phi(1-\theta_t) h_{a,t}^{\eta-1}$ and θ_t is the endogenous share of automated industries at time t. As in Chu, Cozzi, Furukawa and Liao (2019), the term $1-\theta_t$ in ϕ_t captures an increasing difficulty effect of automation under which more industries that are already automated make the next automation more difficult.[6] The aggregate arrival rate of automation is $\alpha_t = \phi h_{a,t}^{\eta}$, where $h_{a,t}$ denotes aggregate automation labor and we have used the condition that $h_{a,t}(i) = h_{a,t}/(1-\theta_t)$. In a symmetric equilibrium, the free-entry condition of automation becomes

$$\alpha_t v_t^k = w_{h,t} h_{a,t}/(1-\theta_t) \Leftrightarrow \phi(1-\theta_t) v_t^k = w_{h,t} h_{a,t}^{1-\eta}. \tag{23}$$

2.7 Government

We assume that the government sets the minimum wage as a certain percentage γ of average wage income, where $\gamma > 0$ is the minimum-wage policy instrument. We will show that the minimum wage $\overline{w}_{l,t}$ is binding in the low-skill labor market if γ is sufficiently large. The government collects a lump-sum tax τ_t to finance the unemployment benefit subject to the balanced-budget condition given by

$$\tau_t = b_t \left(L - l_t\right). \tag{24}$$

[5] Davidson and Segerstrom (1998) show that constant returns to scale in multiple R&D actitivities can lead to equilibrium instability and perverse comparative statics. Our model features innovation and automation, so the decreasing returns to scale in innovation and automation helps to ensure equilibrium stability.

[6] Otherwise, $h_{a,t}(i) = h_{a,t}/(1-\theta_t)$ would become unbounded as $\theta_t \to 1$.

2.8 Aggregation

Aggregate technology Z_t is defined as

$$Z_t \equiv \exp\left(\int_0^1 n_t(i)di \ln z\right) = \exp\left(\int_0^t \lambda_\omega d\omega \ln z\right). \tag{25}$$

Differentiating the log of Z_t in (25) with respect to time yields the growth rate of technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z. \tag{26}$$

Substituting (6) and (14) into (4) yields the following aggregate production function:

$$\ln y_t = \int_0^{\theta_t} \ln\left[\frac{A}{Z_t} z^{n_t(i)} k_t(i)\right] di + \int_{\theta_t}^1 \ln\left\{z^{n_t(i)}\left[(1-\beta)\left[l_t(i)\right]^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left[h_{x,t}(i)\right]^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right\} di$$

$$\Longrightarrow y_t = \left(\frac{Ak_t}{\theta_t}\right)^{\theta_t}\left\{\frac{Z_t\left[(1-\beta)\, l_t^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_{x,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{1-\theta_t}\right\}^{1-\theta_t}, \tag{27}$$

where we have used $k_t(i) = k_t/\theta_t$, $l_t(i) = l_t/(1-\theta_t)$ and $h_{x,t}(i) = h_{x,t}/(1-\theta_t)$. The share θ_t of automated industries determines the degree of capital intensity in the aggregate production function. The evolution of θ_t is determined by

$$\dot{\theta}_t = \alpha_t(1-\theta_t) - \lambda_t\theta_t, \tag{28}$$

where $\alpha_t = \phi h_{a,t}^\eta$ and $\lambda_t = \varphi h_{r,t}^\eta$ are respectively the arrival rates of automation and innovation. Using (2), one can derive the familiar law of motion for capital as follows:[7]

$$\dot{k}_t = y_t - c_t - \delta k_t. \tag{29}$$

From (9), (10) and (16), the capital and labor shares of income are respectively

$$R_t k_t = \frac{\theta_t}{\mu} y_t, \tag{30}$$

$$\overline{w}_{l,t} l = \frac{(1-\theta_t)\, y_t}{\mu}(1-\beta)^\varepsilon \left(\frac{\overline{w}_{l,t}}{\psi_t}\right)^{1-\varepsilon}, \tag{31}$$

$$w_{h,t} h_{x,t} = \frac{(1-\theta_t)\, y_t}{\mu}\beta^\varepsilon\left(\frac{w_{h,t}}{\psi_t}\right)^{1-\varepsilon}. \tag{32}$$

[7]In Appendix B, we provide the detailed derivations.

2.9 Decentralized equilibrium

The equilibrium is a time path of allocations $\{a_t, k_t, c_t, y_t, x_t(i), l_t(i), k_t(i), h_{x,t}(i), h_{r,t}(i), h_{a,t}(i)\}$ and a time path of prices $\{r_t, R_t, \overline{w}_{l,t}, w_{h,t}, p_t(i), v_t^l(i), v_t^k(i)\}$ such that the following conditions hold in each instance:

- the household maximizes utility taking $\{r_t, R_t, \overline{w}_{l,t}, w_{h,t}\}$ as given;
- competitive final-good firms produce y_t to maximize profit taking $p_t(i)$ as given;
- each monopolistic intermediate-good firm i produces $x_t(i)$ and chooses $\{l_t(i), h_{x,t}(i), k_t(i), p_t(i)\}$ to maximize profit taking $\{\overline{w}_{l,t}, w_{h,t}, R_t\}$ as given;
- competitive entrepreneurs choose $\{h_{r,t}(i), h_{a,t}(i)\}$ to maximize expected profit taking $\{w_{h,t}, v_t^l(i), v_t^k(i)\}$ as given;
- the market-clearing condition for final good holds such that $y_t = c_t + \dot{k}_t + \delta k_t$;
- the market-clearing condition for capital holds such that $\int_0^{\theta_t} k_t(i)di = k_t$;
- the market-clearing condition for high-skill labor holds such that $\int_0^1 h_{r,t}(i)di + \int_{\theta_t}^1 h_{a,t}(i)di + \int_{\theta_t}^1 h_{x,t}(i)di = h_{r,t} + h_{a,t} + h_{x,t} = H$;
- the minimum wage in the low-skill labor market implies $\int_{\theta_t}^1 l_t(i)di = l_t < L$;
- the value of inventions is equal to the value of the household's assets such that $\int_0^{\theta_t} v_t^k(i)di + \int_{\theta_t}^1 v_t^l(i)di = a_t$; and
- the government balances the fiscal budget.

2.10 Steady-state equilibrium allocation

From (13) and (17), the amount of monopolistic profit in both automated and unautomated industries is

$$\pi_t^l = \pi_t^k = \frac{\mu - 1}{\mu} y_t. \tag{33}$$

The balanced-growth values of an innovation and an automation are respectively

$$v_t^l = \frac{\pi_t^l}{\rho + \alpha + \lambda} = \frac{\pi_t^l}{\rho + \phi h_a^\eta + \varphi h_r^\eta}, \tag{34}$$

$$v_t^k = \frac{\pi_t^k}{\rho + \lambda} = \frac{\pi_t^k}{\rho + \varphi h_r^\eta}. \tag{35}$$

Substituting (34) and (35) into the free-entry conditions in (21) and (23) yields

$$\frac{\varphi h_a^{1-\eta}}{\phi(1-\theta) h_r^{1-\eta}} = \frac{\rho + \phi h_a^\eta + \varphi h_r^\eta}{\rho + \varphi h_r^\eta},$$

which can be reexpressed as

$$\frac{\varphi}{\phi}+\left(\frac{h_a}{h_r}\right)^{\eta}=\left(\frac{h_r}{h_a}\right)^{1-\eta}+\left(\frac{h_r}{h_a}\right)^{1-2\eta}\frac{\phi}{\varphi+\rho/h_r^{\eta}}. \tag{36}$$

This equation shows that there is a positive relationship between h_a and h_r if $\eta \leq 1/2$; see Figure 1 for an illustration.

We make use of (32) to obtain

$$w_{h,t}h_{x,t}=\frac{(1-\theta_t)\,y_t}{\mu}\frac{\beta^{\varepsilon}\left(w_{h,t}/\overline{w}_{l,t}\right)^{1-\varepsilon}}{(1-\beta)^{\varepsilon}+\beta^{\varepsilon}\left(w_{h,t}/\overline{w}_{l,t}\right)^{1-\varepsilon}}. \tag{37}$$

Based on (31) and (32), we can derive $w_{h,t}/\overline{w}_{l,t}=\left[\beta/\left(1-\beta\right)\right]\left(l_t/h_{x,t}\right)^{1/\varepsilon}$. Substituting this condition into (37) and using (23), (33) and (35), we obtain

$$\phi\left(\mu-1\right)=\frac{\beta\left(\rho+\varphi h_r^{\eta}\right)h_a^{1-\eta}}{\left(1-\beta\right)l^{\frac{\varepsilon-1}{\varepsilon}}\left(H-h_a-h_r\right)^{\frac{1}{\varepsilon}}+\beta\left(H-h_a-h_r\right)}, \tag{38}$$

where we have used the market-clearing condition for high-skill labor $h_x+h_a+h_r=H$. Equation (38) shows that for any given amount of low-skill labor l, there is a negative relationship between h_a and h_r.

Low-skill labor l in (38) is still an endogenous variable. To solve for l, we use the following rule that sets the minimum wage as a percentage γ of the labor share of output per capita:

$$\overline{w}_{l,t}=\gamma\frac{1-\theta_t}{\mu}\frac{y_t}{H+L}, \tag{39}$$

where $(1-\theta_t)/\mu$ is the labor income share. Substituting (5), (6) and $\xi_t(i)=p_t(i)/\mu$ into (7) and then the resulting expression into (39) yields

$$l=\min\left\{\frac{H+L}{\gamma}\frac{\left(1-\beta\right)l^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\beta\right)l^{\frac{\varepsilon-1}{\varepsilon}}+\beta\left(h_x\right)^{\frac{\varepsilon-1}{\varepsilon}}},L\right\}. \tag{40}$$

In summary, (36), (38), (40) and $h_x+h_a+h_r=H$ together solve for the steady-state equilibrium allocation $\{h_r,h_a,h_x,l\}$. We can substitute $h_x=H-h_a-h_r$ into (40) to obtain the following implicit function:

$$l(h_x)=l(H-h_a-h_r). \tag{41}$$

Then, we substitute (41) into (38) to obtain

$$\phi\left(\mu-1\right)=\frac{\beta\left(\rho+\varphi h_r^{\eta}\right)h_a^{1-\eta}}{\left(1-\beta\right)\left[l(H-h_a-h_r)\right]^{\frac{\varepsilon-1}{\varepsilon}}\left(H-h_a-h_r\right)^{\frac{1}{\varepsilon}}+\beta\left(H-h_a-h_r\right)}, \tag{42}$$

which continues to feature a negative relationship between h_a and h_r as shown in the proof of Lemma 2. Therefore, the equilibrium allocation $\{h_r,h_a\}$ is unique; see Figure 1 for an illustration. Finally, we obtain $\{h_x,l\}$ using $h_x=H-h_a-h_r$ and (40).

Lemma 2 *The steady-state equilibrium allocation* $\{h_r, h_a, h_x, l\}$ *is unique.*

Proof. See Appendix A. ■

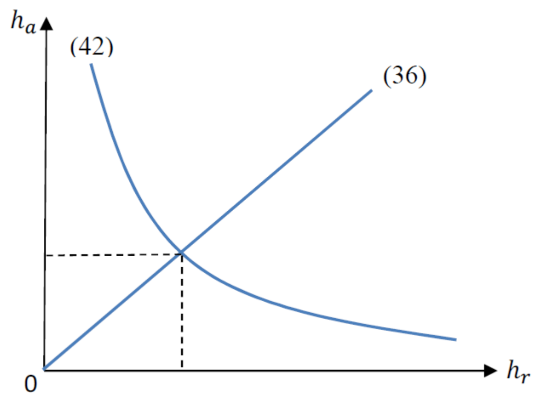

Figure 1

3 How minimum wage affects R&D and automation

In the proof of Proposition 1, we show that if γ is sufficiently large, then the minimum wage is binding in the low-skill labor market and causes unemployment such that $l < L$. Intuitively, a binding minimum wage gives rise to an excess supply of low-skill workers and causes their employment level to be below full employment. Then, any further increase in the minimum-wage policy instrument γ reduces the level of low-skill employment such that

$$\frac{dl}{d\gamma} < 0. \tag{43}$$

Intuitively, raising the minimum wage reduces the demand for low-skill workers l and their employment level. Given that the employment of labor-skill labor is already below full employment (i.e., $l < L$), any increase in the minimum wage γ would increase the unemployment rate u that is given by

$$\underset{+}{u(\gamma)} = \frac{1}{H+L}[L - \underset{-}{l(\gamma)}]. \tag{44}$$

As for the effects of the minimum wage on the allocation of high-skill workers, we need to consider two cases for the elasticity of substitution between low-skill workers and high-skill workers in production. If $\varepsilon > 1$, then the right-hand side (RHS) of (38) is decreasing in l. In this case, an increase in l must be accompanied by an increase in h_a and h_r and a decrease in h_x; see Figure 2 for an illustration. Conversely, if $\varepsilon < 1$, then the RHS of (38) is increasing in l. In this case, an increase in l must be accompanied by a decrease in h_a and h_r and an increase in h_x; see Figure 2 for an illustration. We summarize the above results as follows:

$$h_a = h_a(l)\,;\; h_{a,l} \equiv \frac{dh_a}{dl} \gtrless 0 \;\text{ if } \varepsilon \gtrless 1,$$

$$h_r = h_r(l)\,;\; h_{r,l} \equiv \frac{dh_r}{dl} \gtrless 0 \;\text{ if } \varepsilon \gtrless 1,$$

$$h_x = h_x(l)\,;\; h_{x,l} \equiv \frac{dh_x}{dl} \lessgtr 0 \;\text{ if } \varepsilon \gtrless 1.$$

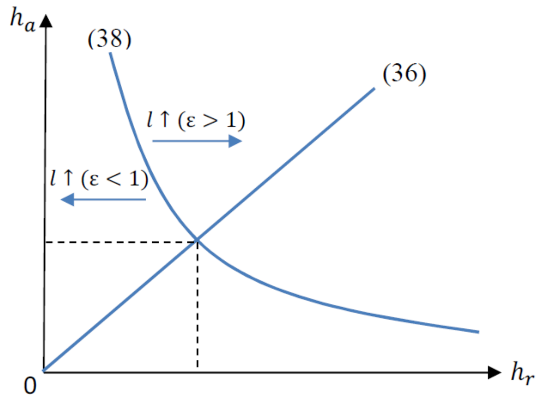

Figure 2

Therefore, if the elasticity of substitution between low-skill workers and high-skill workers in production is less than unity (i.e., $\varepsilon < 1$), then we obtain

$$\underbrace{\frac{dh_x}{dl}}_{+}\underbrace{\frac{dl}{d\gamma}}_{-} < 0, \quad \underbrace{\frac{dh_a}{dl}}_{-}\underbrace{\frac{dl}{d\gamma}}_{-} > 0, \quad \underbrace{\frac{dh_r}{dl}}_{-}\underbrace{\frac{dl}{d\gamma}}_{-} > 0. \tag{45}$$

In other words, the decrease in low-skill production workers l (due to the higher minimum wage) leads to a decrease in high-skill production workers h_x given the gross complementarity between the two types of workers. As a result, the amount of high-skill workers for automation h_a and R&D h_r increases.

If the elasticity of substitution between low-skill workers and high-skill workers in production is greater than unity (i.e., $\varepsilon > 1$), then we obtain

$$\underbrace{\frac{dh_x}{dl}}_{-}\underbrace{\frac{dl}{d\gamma}}_{-} > 0, \quad \underbrace{\frac{dh_a}{dl}}_{+}\underbrace{\frac{dl}{d\gamma}}_{-} < 0, \quad \underbrace{\frac{dh_r}{dl}}_{+}\underbrace{\frac{dl}{d\gamma}}_{-} < 0. \tag{46}$$

In this case, the opposite effects prevail that the decrease in low-skill production workers l (due to the higher minimum wage) leads to an increase in high-skill production workers h_x given the gross substitutability between the two types of workers. As a result, the amount of high-skill workers for automation h_a and R&D h_r decreases.

Finally, we explore the effects of minimum wage on economic growth. The steady-state equilibrium growth rate of aggregate technology Z_t is

$$g_z(\gamma) = \lambda(\gamma)\ln z = [h_r(\gamma)]^{\eta}\varphi \ln z. \tag{47}$$

Given that y_t and k_t grow at the same rate on the balanced growth path, the aggregate production function in (27) implies that the steady-state equilibrium growth rate of output y_t is also

$$g_y(\gamma) = g_z(\gamma) = [h_r(\gamma)]^{\eta}\varphi \ln z. \tag{48}$$

Therefore, whether the equilibrium growth rate is increasing or decreasing in the minimum wage also depends on the elasticity of substitution between low-skill workers and high-skill workers in production. We summarize all the above results in Proposition 1.

Proposition 1 *An increase in the minimum wage has the following effects: (a) a negative effect on the employment of low-skill workers; (b) a positive effect on the unemployment rate; (c) a negative effect on high-skill production workers and a positive effect on automation, R&D and economic growth if the elasticity of substitution between low-skill workers and high-skill workers in production is less than unity; and (d) a positive effect on high-skill production workers and a negative effect on automation, R&D and economic growth if the elasticity of substitution between low-skill workers and high-skill workers in production is greater than unity.*

Proof. See Appendix A. ■

3.1 Quantitative analysis

In this section, we calibrate the model to aggregate data of the US economy in order to provide a quantitative illustration on the effects of the minimum wage. The model could feature scale effects as in Aghion and Howitt (1992). We sidestep this issue by normalizing high-skill labor H to unity. Then, the model features the following structural parameters $\{\varepsilon, \rho, \mu, \eta, \delta, \beta, z, \varphi, \phi, A, L\}$ and a policy variable γ. We determine their parameter values as follows.

We consider two values for the substitution elasticity $\varepsilon \in \{0.5, 2.5\}$ that corresponds to the range of empirical estimates reported in Katz and Autor (1999).[8] We set the discount rate ρ to 0.05 and the markup ratio μ to 1.05. We follow Jones and Williams (2000) to set the intratemporal duplication externality parameter η to 0.5. As for the capital depreciation rate δ, we calibrate its value using an investment-capital ratio of 0.0768 in the US. We set the distribution parameter β between high-skill and low-skill workers to 0.634, which corresponds to a value of 0.366 for the intensity of low-skill labor in Ben-Gad (2008). We calibrate the quality step size z using a long-run technology growth rate of 0.0125 in the US. We calibrate the R&D productivity parameter φ using an innovation arrival rate of one-third

[8] The substitution elasticity ε is more likely to be greater than unity according to recent estimates, see for example Ben-Gad (2008) and Acemoglu and Autor (2011); however, $\varepsilon < 1$ is still possible empirically.

as in Acemoglu and Akcigit (2012). We calibrate the automation productivity parameter ϕ using a labor-income share of 0.60 in the US. For the parameter A, we choose a value that satisfies the condition for the automation-innovation cycle in Lemma 1. We calibrate the low-skill members L using the unemployment rate of 0.06 in the US. Finally, we calibrate the value of γ using the skill premium $w_{h,t}/\overline{w}_{l,t} = 1.974$ in 2008 in the US; see Acemoglu and Autor (2011). We summarize the parameter values in Table 1.

Table 1: Calibration

ε	ρ	μ	η	δ	β	z	φ	ϕ	A	L	γ
0.500	0.050	1.050	0.500	0.064	0.634	1.039	1.311	1.030	0.136	1.087	0.761
2.500	0.050	1.050	0.500	0.064	0.634	1.039	1.254	0.985	0.136	1.379	0.794

In the rest of this section, we simulate the effects of the minimum wage γ on the output growth rate g_y, the unemployment rate u, labor allocations $\{h_r, h_a, h_x, l\}$, the share θ of automated industries and the steady-state level of social welfare U.[9] Figure 3 simulates the effects of the minimum wage γ when the elasticity of substitution between low-skill workers and high-skill workers in production is 0.5 (i.e., $\varepsilon < 1$). In this case, Figure 3a and 3b show that raising the minimum wage γ has a positive effect on the growth rate of output and the unemployment rate. The increase in the unemployment rate is due to the decrease in low-skill production labor as shown in Figure 3f. As for the positive effect on economic growth, it is due to the positive effect of γ on innovation labor in Figure 3c, which in turn is due to the negative effect of γ on high-skill production labor in Figure 3e. Figure 3d shows that raising γ also has a positive effect on automation labor, which in turn leads to the positive effect on the share of automated industries in Figure 3g. Finally, Figure 3h shows that raising the minimum wage γ has a negative effect on social welfare,[10] which is mainly driven by the decrease in the level of output as a result of the reduction in low-skill production labor despite the increase in the growth rate.

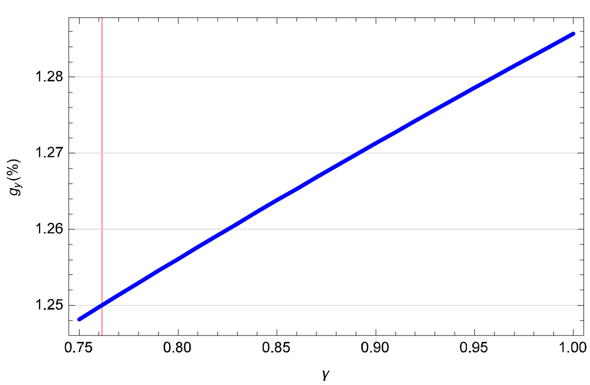

Figure 3a: Effect of γ on g_y ($\varepsilon = 0.5$)

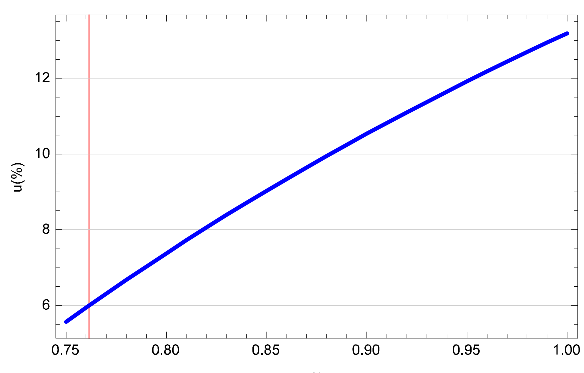

Figure 3b: Effect of γ on u ($\varepsilon = 0.5$)

[9] See Appendix C for the derivation of the steady-state level of social welfare.

[10] The welfare changes are expressed in the usual equivalent variation in consumption.

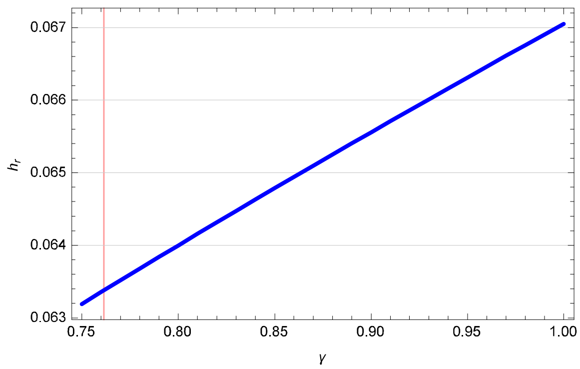

Figure 3c: Effect of γ on h_r $(\varepsilon = 0.5)$

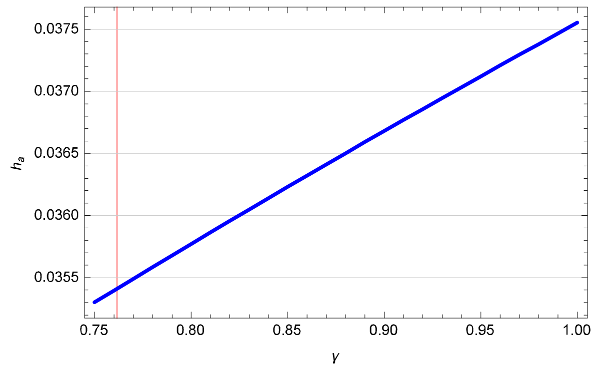

Figure 3d: Effect of γ on h_a $(\varepsilon = 0.5)$

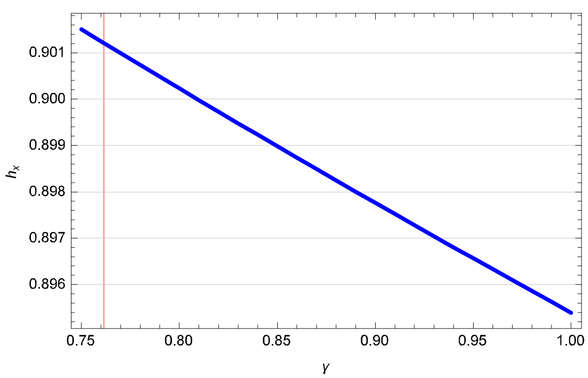

Figure 3e: Effect of γ on h_x $(\varepsilon = 0.5)$

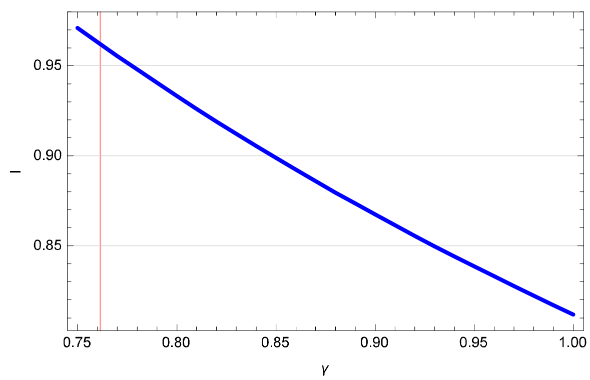

Figure 3f: Effect of γ on l $(\varepsilon = 0.5)$

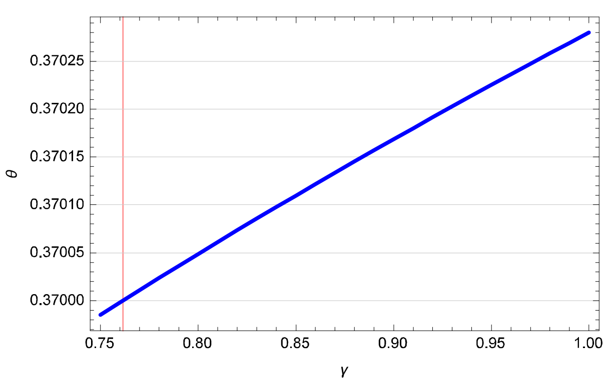

Figure 3g: Effect of γ on θ $(\varepsilon = 0.5)$

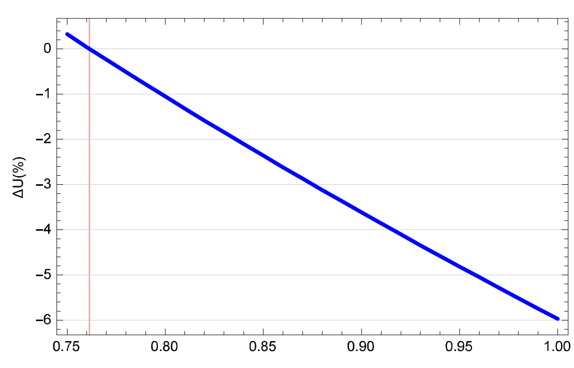

Figure 3h: Effect of γ on U $(\varepsilon = 0.5)$

Figure 4 simulates the effects of the minimum wage γ when the elasticity of substitution between low-skill workers and high-skill workers in production is 2.5 (i.e., $\varepsilon > 1$). In this case, Figure 4a and 4b show that raising the minimum wage γ continues to have a positive effect on the unemployment rate but now a negative effect on the growth rate of output. As before, the increase in the unemployment rate is due to the decrease in low-skill production labor as shown in Figure 4f. As for the negative effect on economic growth, it is due to the negative effect of γ on innovation labor in Figure 4c, which in turn is due to the now positive effect of γ on high-skill production labor in Figure 4e. Figure 4d shows that raising γ has a negative effect on automation labor, which in turn leads to the negative effect on the share of automated industries in Figure 4g. Finally, Figure 4h shows that raising the minimum wage γ continues to have a negative effect on social welfare, which is now driven by the decrease in the growth rate of output in addition to the decrease in the level of output (as a result of the reduction in low-skill production labor).

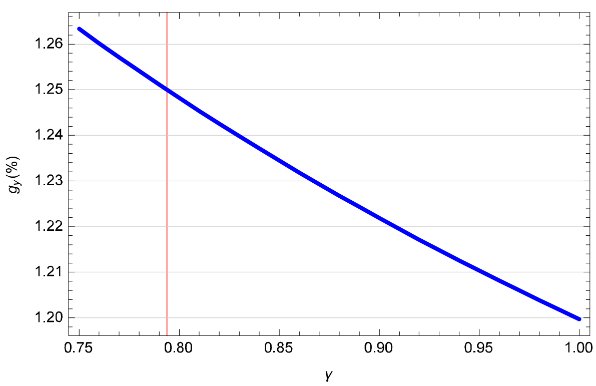

Figure 4a: Effect of γ on g_y ($\varepsilon = 2.5$)

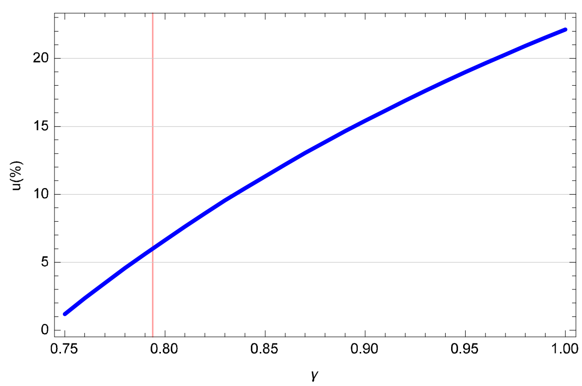

Figure 4b: Effect of γ on u ($\varepsilon = 2.5$)

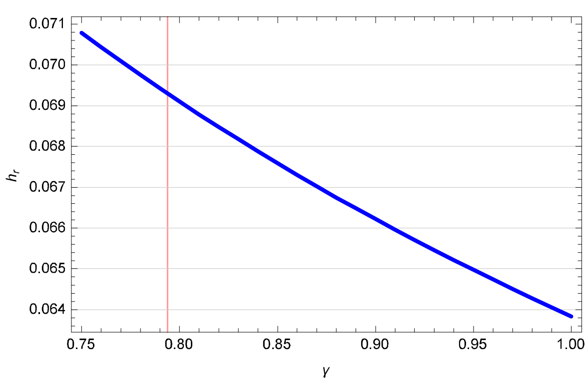

Figure 4c: Effect of γ on h_r ($\varepsilon = 2.5$)

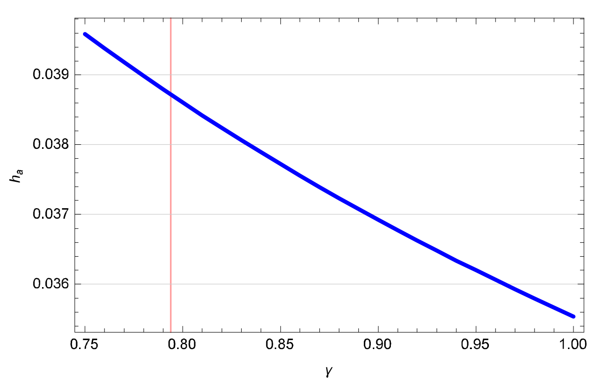

Figure 4d: Effect of γ on h_a ($\varepsilon = 2.5$)

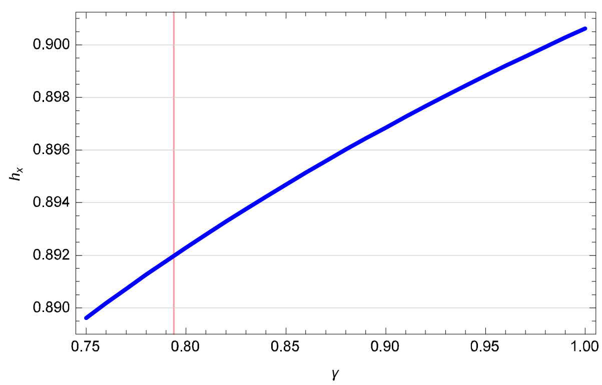

Figure 4e: Effect of γ on h_x ($\varepsilon = 2.5$)

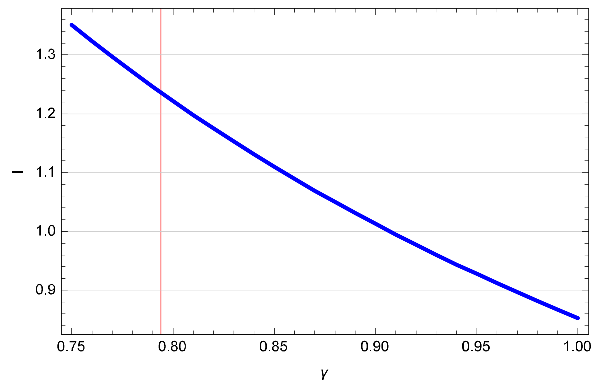

Figure 4f: Effect of γ on l ($\varepsilon = 2.5$)

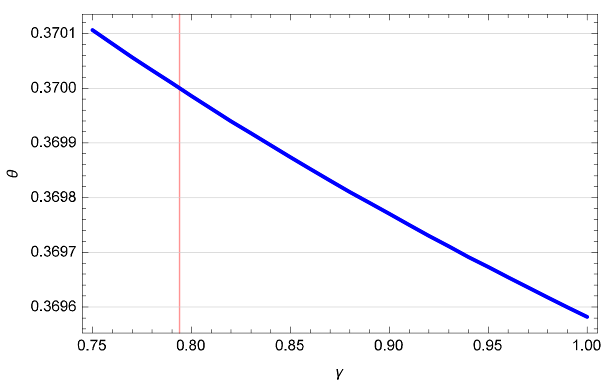

Figure 4g: Effect of γ on θ ($\varepsilon = 2.5$)

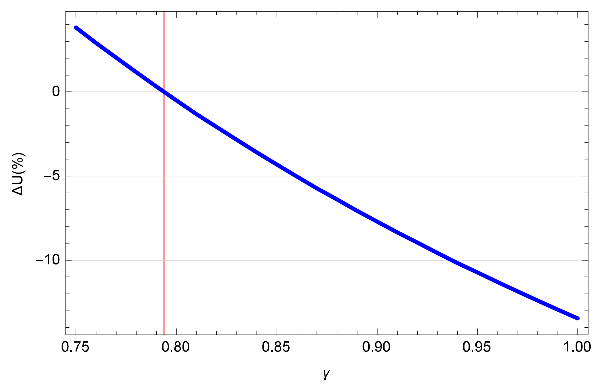

Figure 4h: Effect of γ on U ($\varepsilon = 2.5$)

4 Conclusion

In this study, we have explored the effects of minimum wage in a Schumpeterian growth model with automation. We find that raising the minimum wage has ambiguous effects on innovation and automation, which crucially depend on the elasticity of substitution between low-skill workers and high-skill workers in the production process. In an economy in which the two types of workers are gross complements (substitutes), raising the minimum wage would have a positive (negative) effect on innovation and automation. Therefore, the elasticity of substitution between low-skill and high-skill workers is an important factor that empirical studies should take into account when evaluating the effects of minimum wage on innovation and automation.

References

[1] Acemoglu, D., and Akcigit, U., 2012. Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, 10, 1-42.

[2] Acemoglu, D., and Autor, D., 2011. Skills, tasks and technologies: Implications for employment and earnings. *Handbook of Labor Economics* (edited by O. Ashenfelter and D. Card), 4, 1043-1171.

[3] Acemoglu, D., and Restrepo, P., 2018. The race between man and machine: Implications of technology for growth, factor shares and employment. *American Economic Review*, 108, 1488-1542.

[4] Agenor, P.-R., and Lim, K., 2018. Unemployment, growth and welfare effects of labor market reforms. *Journal of Macroeconomics*, 58, 19-38.

[5] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica*, 60, 323-351.

[6] Aghion, P., Jones, B., and Jones, C., 2017. Artificial intelligence and economic growth. NBER Working Paper No. 23928.

[7] Akerlof, G., and Yellen, J., 1990. The fair wage-effort hypothesis and unemployment. *Quarterly Journal of Economics*, 105, 255-283.

[8] Askenazy, P., 2003. Minimum wage, exports and growth. *European Economic Review*, 47, 147-164.

[9] Ben-Gad, M., 2008. Capital-skill complementarity and the immigration surplus. *Review of Economic Dynamics*, 11, 335-365.

[10] Chu, A., Cozzi, G., and Furukawa, Y., 2016. Unions, innovation and cross-country wage inequality. *Journal of Economic Dynamics and Control*, 64, 104-118.

[11] Chu, A., Cozzi, G., Furukawa, Y., and Liao, C., 2019. Should the government subsidize innovation or automation?. MPRA Working Paper No. 94240.

[12] Chu, A., Kou, Z., and Liu, X., 2018. Labor union and the wealth-income ratio. *Economics Letters*, 167, 29-35.

[13] Chu, A., Kou, Z., and Wang, X., 2019. Dynamic effects of minimum wage on growth and innovation in a Schumpeterian economy. MPRA Paper No. 94822.

[14] Cozzi, G., Giordani, P., and Zamparelli, L., 2007. The refoundation of the symmetric equilibrium in Schumpeterian growth models. *Journal of Economic Theory*, 136, 788-797.

[15] Davidson, C., and Segerstrom P., 1998. R&D subsidies and economic growth. *RAND Journal of Economics*, 29, 548-577.

[16] Dinopoulos, E., and Segerstrom, P., 2010. Intellectual property rights, multinational firms and economic growth. *Journal of Development Economics*, 92, 13-27.

[17] Evans, L., Quigley, N., and Zhang, J., 2003. Optimal price regulation in a growth model with monopolistic suppliers of intermediate goods. *Canadian Journal of Economics*, 36, 463-474.

[18] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43-61.

[19] Hemous, D., and Olsen, M., 2018. The rise of the machines: Automation, horizontal innovation and income inequality. SSRN Working Paper No. 2328774.

[20] Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy*, 107, 715-730.

[21] Ji, L., Chang, J., and Huang, C., 2016. Unionization, market structure, and economic growth. *Southern Economic Journal*, 82, 935-951.

[22] Jones, C., and Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth*, 5, 65-85.

[23] Katz, L., and Autor, D., 1999. Changes in the wage structure and earnings inequality. *Handbook of Labor Economics* (edited by O. Ashenfelter and D. Card), 3, 1463-1554.

[24] Meckl, J., 2004. Accumulation of technological knowledge, wage differentials, and unemployment. *Journal of Macroeconomics*, 26, 65-82.

[25] Mortensen, D., and Pissarides, C., 1998. Technological progress, job creation, and job destruction. *Review of Economic Dynamics*, 1, 733–753.

[26] Parello, C., 2010. A Schumpeterian growth model with equilibrium unemployment. *Metroeconomica*, 61, 398-426.

[27] Peretto, P., 2011. Market power, growth, and unemployment. *Frontiers of Economics and Globalization*, 11, 493-525.

[28] Peretto, P., and Seater, J., 2013. Factor-eliminating technological change. *Journal of Monetary Economics*, 60, 459-473.

[29] Prettner, K., and Strulik, H., 2019. Innovation, automation, and inequality: Policy challenges in the race against the machine. GLO Discussion Paper 320.

[30] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.

[31] Segerstrom, P., Anant, T., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review*, 80, 1077-91.

[32] Zeira, J., 1998. Workers, machines, and economic growth. *Quarterly Journal of Economics*, 113, 1091-1117.

[33] Zeira, J., 2006. Machines as engines of growth. CEPR Discussion Paper No. 5429.

Appendix A: Proofs

Proof of Lemma 1. Using the no-arbitrage condition $r = R - \delta$ and the Euler equation $r = g_y + \rho$, we can reexpress the equilibrium condition that supports a cycle of automation and innovation as

$$\frac{1}{z} < \frac{Z}{A}\left(\frac{g_y + \rho + \delta}{\psi}\right) < 1. \tag{A1}$$

We substitute (5), (6), (11) and (27) into (A1) to derive

$$\frac{1}{z} < \left(\frac{1}{A}\right)^{\frac{1}{1-\theta}}\left(\frac{\theta y}{k}\right)^{\frac{\theta}{1-\theta}}\left[\mu\left(g_y + \rho + \delta\right)\right] < 1. \tag{A2}$$

From capital income $Rk = \theta y/\mu$, the steady-state capital-output ratio is given by

$$\frac{k}{y} = \frac{\theta}{\mu R} = \frac{\theta}{\mu\left(r + \delta\right)} = \frac{\theta}{\mu\left(g_y + \rho + \delta\right)}. \tag{A3}$$

Substituting (A3) into (A2) yields the steady-state equilibrium condition for the automation-innovation cycle. ■

Proof of Lemma 2. From (36), it is easy to verify that there is a positive relationship between h_a and h_r if $\eta \leq 1/2$. Moreover, we reexpress (41) as

$$l\left(h_x\right) = l\left(H - h_a - h_r\right), \tag{A4}$$

where

$$l_{h_x} \equiv \frac{dl}{dh_x} = -\frac{\left[\beta\left(\varepsilon - 1\right)/\varepsilon\right]\left[\left(H - h_a - h_r\right)l\right]^{\frac{-1}{\varepsilon}}}{\left(1-\beta\right)l^{\frac{-2}{\varepsilon}} + \left(\beta/\varepsilon\right)\left(H - h_a - h_r\right)^{\frac{\varepsilon-1}{\varepsilon}}l^{\frac{-(1+\varepsilon)}{\varepsilon}}}. \tag{A5}$$

Equation (A5) shows that l is monotonically decreasing (increasing) in h_x if $\varepsilon > 1 (< 1)$. We make use of (42) and (A5) to derive

$$\frac{dh_a}{dh_r} = -\frac{\left[\left(1-\beta\right)\left(H - h_a - h_r\right)^{\frac{1}{\varepsilon}}l^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left(H - h_a - h_r\right)\right]\eta\varphi h_r^{\eta-1} + \Phi\left(\rho + \varphi h_r^{\eta}\right)}{\left(\rho + \varphi h_r^{\eta}\right)\left\{\left[\left(1-\beta\right)\left(H - h_a - h_r\right)^{\frac{1}{\varepsilon}}l^{\frac{\varepsilon-1}{\varepsilon}} + \beta\left(H - h_a - h_r\right)\right]\left(1-\eta\right)/h_a + \Phi\right\}}, \tag{A6}$$

where

$$\Phi \equiv \frac{\left[\left(1-\beta\right)/\varepsilon\right]\left(H - h_a - h_r\right)^{\frac{1-\varepsilon}{\varepsilon}}l^{\frac{\varepsilon-1}{\varepsilon}}\Delta}{\left(1-\beta\right)l^{\frac{-2}{\varepsilon}} + \left(\beta/\varepsilon\right)\left(H - h_a - h_r\right)^{\frac{\varepsilon-1}{\varepsilon}}l^{\frac{-(1+\varepsilon)}{\varepsilon}}} + \beta, \tag{A7}$$

$$\Delta \equiv \left(1-\beta\right)l^{\frac{-2}{\varepsilon}} + \frac{\beta}{\varepsilon}\left(H - h_a - h_r\right)^{\frac{\varepsilon-1}{\varepsilon}}l^{\frac{-(1+\varepsilon)}{\varepsilon}}\left[1 - \left(\varepsilon - 1\right)^2\right]. \tag{A8}$$

Equations (A7) and (A8) show $\Phi > 0$ and $\Delta \geq 0$ if $\varepsilon \leq 2$. Therefore, (42) features a negative relationship between h_a and h_r if $\varepsilon \leq 2$. Based on (36) and (42), we obtain that the equilibrium allocation $\{h_r, h_a\}$ is unique. From (A5), we know that l is monotonically decreasing in h_x or increasing in h_x. Using this condition and $h_x = H - h_a - h_r$, we obtain that the equilibrium allocation $\{h_x, l\}$ is also unique. ■

Proof of Proposition 1. We make use of (36), (38) and $h_x = H - h_a - h_r$ to derive

$$h_{a,l} \equiv \frac{dh_a}{dl} = \left(\frac{\Omega}{\Theta}\right) \frac{(\varepsilon-1)(1-\beta)}{\varepsilon (l/h_x)^{1/\varepsilon}}, \tag{A9}$$

$$h_{r,l} \equiv \frac{dh_r}{dl} = \left(\frac{\Pi}{\Theta}\right) \frac{(\varepsilon-1)(1-\beta)}{\varepsilon (l/h_x)^{1/\varepsilon}}, \tag{A10}$$

$$h_{x,l} \equiv \frac{dh_x}{dl} = -\left(\frac{dh_a}{dl} + \frac{dh_r}{dl}\right), \tag{A11}$$

where

$$\Omega \equiv \left[\frac{\eta}{h_r} + \frac{1-\eta}{h_a} + \frac{1-2\eta}{h_a}\left(\frac{h_a}{h_r}\right)^{\eta} \frac{\phi h_r^{\eta}}{\varphi h_r^{\eta} + \rho} + \left(\frac{h_r}{h_a}\right)^{1-\eta} \frac{\rho\eta\phi h_r^{\eta-1}}{(\varphi h_r^{\eta}+\rho)^2}\right] > 0,$$

$$\Pi \equiv \left(\frac{h_r}{h_a}\right)\left[\frac{\eta}{h_r} + \frac{1-\eta}{h_a} + \frac{1-2\eta}{h_a}\left(\frac{h_a}{h_r}\right)^{\eta} \frac{\phi h_r^{\eta}}{\varphi h_r^{\eta}+\rho}\right] > 0,$$

$$\Theta \equiv \left[(1-\beta) h_x^{\frac{1}{\varepsilon}} l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x\right]\left[\frac{\eta\varphi h_r^{\eta-1}\Pi}{\rho+\varphi h_r^{\eta}} + \frac{(1-\eta)\Omega}{h_a}\right] + (\Pi+\Omega)\left[\frac{1-\beta}{\varepsilon}\left(\frac{h_x}{l}\right)^{\frac{1-\varepsilon}{\varepsilon}} + \beta\right] > 0.$$

It is helpful to note that we set $\eta \leq 1/2$ and $\varepsilon \leq 2$ so that the steady-state equilibrium allocation $\{h_r, h_a, h_x, l\}$ is unique. Equations (A9) and (A10) show that both h_a and h_r are increasing (decreasing) in l if $\varepsilon > 1(< 1)$. Given this result, it is easy to verify that there is a negative (positive) relationship between h_x and l if $\varepsilon > 1(< 1)$. Based on (40), we take the differentials of l with respect to γ to obtain

$$\frac{dl}{d\gamma} = -\frac{\left[(1-\beta) l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x^{\frac{\varepsilon-1}{\varepsilon}}\right]^2}{(1-\beta)(H+L)\left\{(1-\beta) l^{\frac{-2}{\varepsilon}} + (\beta/\varepsilon) h_x^{\frac{\varepsilon-1}{\varepsilon}} l^{\frac{-(1+\varepsilon)}{\varepsilon}} \underbrace{\left[1 + (\varepsilon-1)(l/h_x) h_{x,l}\right]}_{\equiv\Lambda}\right\}}. \tag{A12}$$

We substitute (A11) into Λ and then use the sufficient conditions of the unique equilibrium (i.e, $\eta \leq 1/2$ and $\varepsilon \leq 2$) to obtain

$$\Theta\Lambda = \left[(1-\beta) h_x^{\frac{1}{\varepsilon}} l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x\right]\left[\frac{\eta\varphi h_r^{\eta-1}\Pi}{\rho+\varphi h_r^{\eta}} + \frac{(1-\eta)\Omega}{h_a}\right] + (\Pi+\Omega)\left\{\beta + \frac{1-\beta}{\varepsilon}\left(\frac{h_x}{l}\right)^{\frac{1-\varepsilon}{\varepsilon}}\left[1-(\varepsilon-1)^2\right]\right\} > 0.$$

As a result, (A12) shows that there is a negative relationship between l and γ. Given this result, we make use of (44) to derive that there is a positive relationship between u and γ. Combining (A12) and (A9)-(A11), we obtain that both h_a and h_r are decreasing (increasing) in γ if $\varepsilon > 1(\varepsilon < 1)$ and h_x is increasing (decreasing) in γ if $\varepsilon > 1(\varepsilon < 1)$. Finally, we use (48) to obtain that g is decreasing (increasing) in γ if $\varepsilon > 1(\varepsilon < 1)$. ∎

Appendix B: The capital-accumulation equation

Using (2) and $\tau_t = b_t\,(L - l_t)$, we obtain

$$\dot{a}_t + \dot{k}_t = r_t a_t + (R_t - \delta)k_t + \overline{w}_{l,t} l_t + w_{h,t} H - c_t. \tag{B1}$$

Given $a_t = \theta_t v_t^k + (1-\theta_t)\, v_t^l$, we derive $\dot{a}_t = \theta_t \dot{v}_t^k + v_t^k \dot{\theta}_t + (1-\theta_t)\, \dot{v}_t^l - v_t^l \dot{\theta}_t$. Substituting (28) into this condition, we obtain

$$\dot{a}_t = \theta_t \dot{v}_t^k + v_t^k \left[\alpha_t(1-\theta_t) - \lambda_t \theta_t\right] + (1-\theta_t)\, \dot{v}_t^l - v_t^l \left[\alpha_t(1-\theta_t) - \lambda_t \theta_t\right]. \tag{B2}$$

Substituting (B2) and $a_t = \theta_t v_t^k + (1-\theta_t)\, v_t^l$ into (B1), we obtain

$$\begin{aligned} & \theta_t \dot{v}_t^k + v_t^k \left[\alpha_t(1-\theta_t) - \lambda_t \theta_t\right] + (1-\theta_t)\, \dot{v}_t^l - v_t^l \left[\alpha_t(1-\theta_t) - \lambda_t \theta_t\right] + \dot{k}_t \\ = \; & r_t \left[\theta_t v_t^k + (1-\theta_t)\, v_t^l\right] + (R_t - \delta)k_t + \overline{w}_{l,t} l_t + w_{h,t} H - c_t. \end{aligned} \tag{B3}$$

Using (18) and (19) yields

$$\begin{aligned} \dot{k}_t \; = \; & -\alpha_t\,(1-\theta_t)\, v_t^k + \theta_t \pi_t^k + (1-\theta_t)\, \pi_t^l \\ & -\lambda_t v_t^l + R_t k_t - \delta k_t + \overline{w}_{l,t} l_t + w_{h,t} H - c_t. \end{aligned} \tag{B4}$$

Moreover, we make use of (13), (17), (30), (31) and (32) to derive

$$\dot{k}_t = y_t - c_t - \delta k_t - \alpha_t\,(1-\theta_t)\, v_t^k - \lambda_t v_t^l + w_{h,t} h_{a,t} + w_{h,t} h_{r,t}. \tag{B5}$$

Substituting (21) and (23) into (B5), we obtain

$$\dot{k}_t = y_t - c_t - \delta k_t. \tag{B6}$$

Appendix C: The welfare function

The steady-state level of social welfare U can be expressed as

$$\rho U = (\ln c_0) + \frac{g_y}{\rho}. \tag{C1}$$

The law of motion capital is $\dot{k}_t = y_t - c_t - \delta k_t$. Using this condition, one can derive the following steady-state consumption-output ratio:

$$\frac{c}{y} = 1 - (g_y + \delta)\frac{k}{y}. \tag{C2}$$

Substituting (C2) into (C1) and using (27), the steady-state level of social welfare U can be re-expressed as

$$\rho U = \ln\left[1 - (g_y + \delta)\frac{k}{y}\right] + \theta \ln A + \theta \ln\left(\frac{k}{\theta}\right) + (1-\theta)\ln\left\{\frac{\left[(1-\beta)\, l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{1-\theta}\right\} + \frac{g_y}{\rho}, \tag{C3}$$

where Z_0 is normalized to unity. The steady-state capital-output ratio and the capital-technology ratio are respectively

$$\frac{k}{y} = \frac{\theta}{R\mu} = \frac{\theta}{\mu(r+\delta)} = \frac{\theta}{\mu(g_y+\rho+\delta)}, \tag{C4}$$

$$\frac{k}{Z} = \frac{\theta\left[(1-\beta)\, l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{A(1-\theta)}\left(\frac{A}{\theta}\frac{k}{y}\right)^{\frac{1}{1-\theta}}. \tag{C5}$$

Substituting (C4) and (C5) into (C3), we obtain

$$\rho U = \ln\left[1 - (g_y + \delta)\frac{k}{y}\right] + \left(\frac{\theta}{1-\theta}\right)\ln\left(\frac{A}{\theta}\frac{k}{y}\right) + \ln\left\{\frac{\left[(1-\beta)\, l^{\frac{\varepsilon-1}{\varepsilon}} + \beta h_x^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}{1-\theta}\right\} + \frac{g_y}{\rho}, \tag{C6}$$

where we have used $Z_0 = 1$.