Optimal Growth Policies in a Two-Sector Model with Financial Market Imperfections

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Abstract

This paper studies the effects of fiscal policy on growth in an environment where heterogeneous entrepreneurs face collateral borrowing constraints and human capital accumulation is influenced by public spending. In such an environment, the paper first shows that positive public spending on human capital improves growth. The paper then explores the role of a policy mix that consists of capital income tax and capital subsidy to productive entrepreneurs in addressing growth. The theoretical analysis shows that when the productivity distribution is heavy-tailed, the government can utilize the capital subsidy instrument as a non-linear function of the idiosyncratic productivity to achieve growth maximizing policy mix.

JEL Classification: E10, E22, E44, H21, O16

Keywords: Heterogeneity; Financial Frictions; Growth Policies

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1 Introduction

This paper studies the effects of fiscal policy on growth in an environment where heterogeneous entrepreneurs face borrowing constraints and human capital is accumulated and influenced by public spending. More specifically, in the presence of collateral borrowing constraints, the amounts of funds that one can borrow is limited by his own wealth. As a result, heterogeneous producers with high productivity are unable to borrow sufficiently to extend their production, hence, capital is not efficiently allocated and there are losses in aggregate productivity and growth. [1] This paper addresses the question of whether and how in such environment fiscal policy can be utilized to improve aggregated marginal product of capital and growth.

To that end, this paper constructs a tractable human capital based endogenous growth model with a continuum of heterogeneous entrepreneurs, a representative worker, and the government. Entrepreneurs own private firms that produce goods with heterogeneous productivity, accumulate personal wealth and face collateral borrowing constraints. The representative worker accumulates human capital by using his own efforts, the existing human capital stock and public services related to human capital formation. The government finances public spending by capital income tax and also intervenes into the capital input market by providing capital subsidy to active firms. Capital market equilibrium implies that there exists a threshold of entrepreneurs who are active in producing goods and the marginal product of capital of those entrepreneurs at the productivity cut-off will determine the equilibrium capital rental rate of the economy. The government, therefore, can influence the capital rental rate by changing the capital subsidy policy rate.

The paper then focuses its analysis on the balanced growth path equilibrium. It shows that when human capital formation is affected by public spending, fiscal policy with positive public spending on human capital, in general, yields strictly higher growth than when there is no public spending. Intuitively, in the presence of financial frictions, the aggregated marginal product of capital is higher than the capital rental rate because the former is equal to the average marginal product of

[1] In the absence of borrowing constraints, only producers with the highest productivity produce goods, while all others just save and lend capital. Hence, capital is efficiently allocated to the most productive producers and aggregate measured productivity is the first-best level.

capital of all active productive entrepreneurs while the latter is equal to that of entrepreneurs at the productivity cut-off. Because human capital or effective labor is used by only active entrepreneurs, the aggregated marginal product of capital will be improved when there is more human capital accumulation. In the meantime if the government finance public spending on human capital by capital taxation, the taxation cost of this fiscal policy is incurred by savers who would just receive lower capital rental rate. The paper particularly shows that any fiscal policy with positive public spending on human capital financed only by the capital income tax instrument will attain higher growth than when there is no public spending.

The paper proceeds to explore the effects of the fiscal policy mix that consists of both the capital income tax rate and capital subsidy rate on growth. It derives the rule that the policy mix obeys to attain growth maximization and then identifies the condition when the capital subsidy instrument are necessary to achieve growth maximizing fiscal policy. The theoretical prediction is that when the degree of productivity heterogeneity is high or the idiosyncratic productivity distribution is heavy-tailed the government needs to utilize both capital income tax and capital subsidy instruments to achieve growth maximizing fiscal policy. The reason is that when the productivity distribution is heavy-tailed the productivity cut-off is low, therefore equilibrium capital rental rate is sufficiently low if there is no capital subsidy. Consequently, the government is unable to collect sufficient tax revenues just from capital income in order to spend on human capital formation. By contrast, by appropriately utilizing the capital subsidy as a function of the idiosyncratic productivity, the government can affect the capital rental rate, hence, its tax revenues to sufficiently spend on human capital formation to attain growth maximization. The paper demonstrates that there exists such a capital subsidy instrument as a *non-linear* function of the idiosyncratic productivity. Intuitively, this policy subsidizes active entrepreneurs at the productivity cut-off to increase the capital market's rental rate and then impose sufficiently high capital income rate to all capital savers obtain sufficient tax revenue. We discuss other properties that the policy mix satisfies in details in the main text.

This paper is related to two branches of literature. One is the literature studying the interactions between entrepreneurship, financial frictions, and productivity (see e.g. Buera et al 2011; Buera and Shin 2013; Midrigan and Xu 2014, Moll 2014). [2] The other is the endogenous growth theory. This paper contributes to the literature by incorporating financial frictions and heterogeneous agents in lines with Moll (2014) into a human capital based endogenous growth model developed by Uzawa (1965) and Lucas(1988) and shows that growth can be affected by fiscal policy. [3] This model, however, differs from the conventional Uzawa-Lucas framework by incorporating the productive role of public services in the spirit of Barro (1990) in the process of accumulating human capital. This paper also shares with Mino (2015, 2016) the idea of addressing endogenous growth in the presence of productivity heterogeneity and financial frictions. However, Mino (2015, 2016) addresses the question in the AK and Marshall-Arrow-Romer capital externality based endogenous growth model while this paper focuses on the role of human capital formation on endogenous growth.

This paper is also close to Itskhoki and Moll (2019) that studies Ramsey policies in a one-sector growth model with financial frictions and productivity heterogeneity. The differences are that Itskhoki and Moll (2019) does not incorporate human capital accumulation and associated public spending; it focuses on the optimal transfer policies in a steady state equilibrium framework. This paper addresses pro-growth policy on the balanced-growth path equilibrium in a human-capital based endogenous growth framework. In the absence of human capital accumulation, Itskhoki and Moll (2019) shows that it is always optimal to subsidize entrepreneurs by taxing workers. By contrast, this paper shows that when human capital is accumulated and influenced by public spending, conclusions of Itskhoki and Moll (2019) may no longer hold. Moreover, this paper also considers subsidy policy instruments that are function of idiosyncratic productivity, therefore, it is able to address more general policy function forms in the balanced-growth paths.

This paper is organized as follows. Section 2 sets up the model economy. Section 3 analyzes the effects of fiscal policy in a balanced-growth path equilibrium with an emphasis on the role of the policy mix in addressing growth maximizing. Section 4 concludes.

[2] See Buera et al. (2015) for a thorough review of this literature.

[3] Moll (2014) introduces productivity persistence into the previous work of Angeletos (2007) and Kiyotaki and Moore (2012), and shows that self-financing can undo capital mis-allocation and reduce the long-run steady state TFP losses when the shocks are sufficiently persistent.

2 The Model Setting

This is a two-sector growth model with a unit measure of heterogeneous entrepreneurs, a continuum of representative workers and the government. Each entrepreneur indexed by his productivity, z, and wealth, a. The representative worker accumulates human capital by using his own efforts, the existing human capital stock and also receives public services related to human capital formation from the government. The government collects capital income tax to finance its spending on human capital and capital subsidy.

2.1 Entrepreneurs

Each entrepreneur owns a private firm that employs n^d units of effective labor at the wage rate, $w(t)$ and rents k^d units of physical capital at the rental rate, $r(t)$, to produce homogeneous final goods as,

$$y = f(z, k, n) = (zk)^{\alpha} n^{1-\alpha}, \quad \alpha \in (0, 1) \tag{2.1}$$

where z denotes idiosyncratic productivity and can be interpreted as individual entrepreneurial ability.

We assume that idiosyncratic productivity z is $i.i.d$ over time as well as across entrepreneurs with the following Pareto distribution function $Q(z)$, [4]

$$Q(z) = \begin{cases} 1 - \left(\frac{1}{z}\right)^{\varphi} & z \geq 1 \\ 0 & z < 1 \end{cases}, \quad \varphi > 1 \tag{2.2}$$

where φ is the shape parameter. Smaller φ corresponds to a heavier tail of the productivity distribution, i.e., a higher fraction of very productive entrepreneurs, therefore, implying a higher degree of productivity heterogeneity among entrepreneurs.

The assumption of Pareto distribution for productivity can be justified by empirical work. In particular, Axtell (2001) shows that the Pareto distribution well captures the entire firm size distribution for the US firms using 1997 data. Luttmer (2007) constructed a stochastic balanced growth model in which the stationary firm size follows a Pareto distribution. The Pareto distribution is also widely used to

[4] The law of large numbers implies that the population share of type z entrepreneurs is stationary and deterministic.

describe the productivity distribution of firms in the literature of trade with heterogeneous firms (Melitz, 2003 and Eaton et al., 2011).

The entrepreneur faces the following borrowing collateral constraint:

$$k \leq \lambda a, \quad \lambda \geq 1 \tag{2.3}$$

which states that the amount of capital an entrepreneur can borrow is limited by his personal assets, and the maximum borrowing leverage ratio λ that reflects the financial development level. In particular, $\lambda = 1$ expresses financial autarky where entrepreneurs are completely capital self-financed whereas $\lambda = \infty$ denotes perfect financial markets where entrepreneurs can borrow freely regardless of the personal wealth. [5]

Each time, the entrepreneur maximizes the following profit obtained from his private firm subject to the technology (2.1) and his borrowing constraint (2.3),

$$\pi(z, k, n) \equiv f(z, k, n) - wn - [1 - \eta(z)]rk$$

where $\eta(z)$ denotes the capital subsidy rate policy and is a function of idiosyncratic productivity.

While z is idiosyncratic productivity for each private firm, we assume that in order to receive subsidy firms have obligations to report their productivity to the government who know about the productivity distribution. We also rule out the case of strategic lying by implicitly assuming prohibitive penalties in cases of dishonesty for firms. The optimization conditions of this problem imply that his capital and effective labor demands and profits are function of personal wealth; and there is a productivity cutoff for active entrepreneurs $\underline{z}$ as follows: [6]

$$\Pi(a, z) = \max\{z\pi - [1 - \eta(z)]r, 0\}\,\lambda a$$

$$k^d(a, z) = \lambda a \cdot \mathbb{1}_{\{z \geq \underline{z}\}} \tag{2.4}$$

$$n^d(a, z) = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} k^d(a, z) \tag{2.5}$$

$$\underline{z}\pi = [1 - \eta(\underline{z})]\, r, \text{ where } \pi \equiv \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \text{ and } \eta(\underline{z}) \equiv \eta(z = \underline{z}) \tag{2.6}$$

[5] See Buera and Shin (2013) and Moll (2014) for further discussions of this borrowing constraint and Nguyen (2019a) for the case where the maximum borrowing leverage ratio is endogenous.

[6] See the Appendix A for derivation.

Besides profits, each entrepreneur also receives capital income from his personal assets. Hence, his personal wealth $a(t)$ evolves as follows:

$$\dot{a} = \Pi(a, z) + (1 - \tau)ra - c_e \tag{2.7}$$

where τ denotes the tax rate on capital/asset income. [7] Entrepreneurs maximize the following expected sum of discounted utilities subject to the budget constraint (2.7),

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c_e(t) dt$$

This in turn implies the optimal consumption rule, $c_e(t) = \rho a(t)$. [8] Hence,

$$\dot{a} = s(z)a, \quad \text{where} \;\; s(z) = \lambda \max\{z\pi - [1 - \eta(z)]r, 0\} + (1 - \tau)r - \rho$$

2.2 The Representative Worker

The representative workers has the following preferences,

$$\int_0^\infty e^{-\rho t} \log c_w(t) dt, \tag{2.8}$$

where c_w denotes the worker's consumption.

At each time, the worker has one unit of non-leisure time and human capital stock, h. He then divides a fraction u of his non-leisure time to supply uh units of effective labor. The left fraction of time $1 - u$ is used to increase his level of human capital stock via as the following technology:

$$\dot{h} = b(1 - u) \left[\phi h^\sigma + (1 - \phi) g^\sigma\right]^{\frac{1}{\sigma}}, \quad \phi \in [0, 1], \sigma \leq 1 \tag{2.9}$$

where h the existing human capital stock and g is the level of public spending on human capital formation that are provided by the government.

Human capital formation in this model should be interpreted in broad terms that include education and training, wealth improvements, and also having and raising children. Hence, the main difference in this model setup from the existing literature is that human capital formation in (2.9) is influenced by the level of public

[7] For simplicity, I assume a flat rate for capital income, similar theoretical results can be obtained when τ is a function of a.

[8] See the Appendix A for details.

spending/services on human capital. [9] This assumption is motivated the fact that in many countries the government usually subsidizes the expenses of having and raising children and covers the compulsory education and public spending often accounts for a lion share of total spending on education and health, particularly at the early ages. [10]The assumption that public services is an input of human capital formation is also close to Barro (1990) where public services is an input of the aggregate production function. In this model, $1-u$, can be interpreted as the flow of the private investment level, h can be interpreted as the number of teachers and the existing knowledge stock while g is the flow of public services related to human capital.

We assume that the worker is hand-to-mouth and consume all his labor incomes. This assumption can be justified by the facts that in the U.S. Top 20% controls about 95% of the total US financial assets (Wolff, 2017) and that around one-third of all households in the America, Canada, Germany and the U.K, live hand to mouth based on 2010 data (Kaplan et al., 2014). [11] Hence, the budget constraint of the hand-to-mouth worker can be expressed as follows:

$$c_w = uhw \tag{2.10}$$

The representative worker maximizes the sum of discounted utilities (2.8) subject to the budget constraint (2.10) and human capital accumulation (2.9) while taking g as given.

After substituting the budget constraint into the instantaneous utility function and denoting the current-value costate variable by μ, the current-value Hamiltonian for the representative worker' optimization problem can be expressed as:

$$H\left(u, h, \lambda\right) = \log\left[wuh\right] + \mu b(1-u)\left[\phi h^{\sigma} + (1-\phi) g^{\sigma}\right]^{\frac{1}{\sigma}}$$

[9]When $\phi = 1$, so $\dot{h} = b(1-u)h$, namely the human capital formation does not depend on public spending this specification reverts to the original Uzawa-Lucas setup.

[10]See Nguyen (2019b) for strong and significant correlations between public spending per capita on education and health and human capital in OECD countries.

[11]An important implication of this assumption is that since workers do not participate in the capital markets, their behavior is not directly affected by the the process of capital accumulation.

The F.O.Cs of this optimization state:

$$\frac{1}{u} = \mu b \left[\phi h^{\sigma} + (1-\phi) g^{\sigma}\right]^{\frac{1}{\sigma}}$$
$$\frac{1}{h} + \mu(1-u) b \phi h^{\sigma-1} \left[\phi h^{\sigma} + (1-\phi) g^{\sigma}\right]^{\frac{1}{\sigma}-1} = \rho\mu - \dot{\mu}$$
$$\lim_{t\to\infty} e^{-\rho t} \mu h = 0$$

These F.O.Cs together with the evolution equation for h imply that optimal allocating time to work, $u(t)$, will obey the following differential equations:

$$\frac{\dot{u}}{u} - \frac{b\left[\phi h^{\sigma} + (1-\phi) g^{\sigma}\right]^{\frac{1}{\sigma}}}{h} u + \rho = 0 \tag{2.11}$$

Notes that since workers are borrowing constrained and human capital is accumulated and affected by public spending, g_e, fiscal policy can influence workers' decisions on working hours and private investment on human capital via the above equation.

2.3 The Government

We assume that each time the government have balanced budget. Namely, at each time t, the government collects asset income taxes and spends all on public services related to human capital formation and on capital subsidy. That is:

$$\iint \tau r a \phi(a,z) da dz = g + \iint \left[\lambda \eta(z) \cdot \mathbb{1}_{\{\underline{z} \geq z\}}\right] r a \phi(a,z) da dz \tag{2.12}$$

where $\phi(a,z)$ is the joint distribution of productivity and wealth.

2.4 The Aggregate Equilibrium Dynamics

An equilibrium in this economy is sequences of quantities and factor prices such that (1) workers and entrepreneur maximize their utilities subject to corresponding budget constraints taking as given equilibrium prices, (2) the government budget constraint (2.12) balances and (3) the factor markets clear as follows,

$$\iint k^d(a,z)\phi(a,z) da dz = \iint a\phi(a,z) da dz \equiv k \tag{2.13}$$
$$\iint n^d(a,z)\phi(a,z) da dz = uh \tag{2.14}$$

Substituting capital demand into (2.13) and recall that z is $i.i.d$ we obtain $1 = \lambda [1 - G(\underline{z})]$, hence the productivity cut-off $\underline{z} = \lambda^{\frac{1}{\varphi}}$.

The aggregate output, y, is then obtained by summing the amounts of final goods produced by all active entrepreneurs as: [12]

$$y = Ak^{\alpha}(uh)^{1-\alpha} \tag{2.15}$$

where A denotes measured TFP and is equal to $\left(\frac{\varphi}{\varphi-1}\underline{z}\right)^{\alpha}$.

The dynamics physical and human capital, u, can the level of public spending on human capital can be summarized as:

$$\frac{\dot{k}}{k} = \alpha A k^{\alpha-1}(uh)^{1-\alpha} - [\rho + ((\tau - \lambda\tilde{\eta}))r] \tag{2.16}$$

$$\frac{\dot{h}}{h} = \frac{b\left[\phi h^{\sigma} + (1-\phi)g^{\sigma}\right]^{\frac{1}{\sigma}}}{h}(1-u) \tag{2.17}$$

$$\frac{\dot{u}}{u} = \frac{b\left[\phi h^{\sigma} + (1-\phi)g^{\sigma}\right]^{\frac{1}{\sigma}}}{h}u - \rho \tag{2.18}$$

$$g = \alpha y \left[\frac{\tau - \lambda\tilde{\eta}}{1-\eta(\underline{z})}\frac{\varphi - 1}{\varphi}\right] \tag{2.19}$$

where $\tilde{\eta} \equiv \int_{\underline{z}}^{\infty} \eta(z)g(z)dz$.

Finally, the distortions due to financial frictions and also the effects of government's subsidy are expressed in the rental rate equation as,

$$r = \frac{1}{1-\eta(\underline{z})}\frac{\varphi - 1}{\varphi}\bar{r} \tag{2.20}$$

where $\bar{r} = \alpha A k^{\alpha-1}(uh)^{1-\alpha}$ is the aggregated marginal product of capital implied from the aggregated production (2.15). When there is no subsidy, capital rental rate is equal to the marginal product of capital of marginal entrepreneurs (those with productivity cut-off), i.e. $\frac{\varphi-1}{\varphi}\alpha A k^{\alpha-1}(uh)^{1-\alpha}$, so is lower than $\bar{r}$ in general. The rental rate depends on the shape parameter φ that represents the degree of heterogeneity. When entrepreneurs are relatively more heterogeneous (lower φ), marginal producers are relatively more distant from the average active producers so rental rate is relatively lower.

[12] See the Appendix B for details.

3 Analysis

We focus our analysis on balanced growth pate equilibrium and address the effects of the policy mix on growth rate.

3.1 Balanced Growth Path Equilibrium

Define the physical human capital ratio κ as $\frac{k}{h}$ as and a *balanced-growth path equilibrium* of this economy is established when,

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{h}}{h} = \gamma, \quad \text{for all t}$$

where γ denotes the balanced-growth rate.

Substituting κ and γ into (2.16), (2.17), (2.18) and note that under a balanced-growth path equilibrium, u and κ stay constant over time, we obtain the following 3 equations for 3 variables, κ, γ and u as,

$$\begin{aligned}
&\left[1 - \frac{\tau - \lambda\tilde{\eta}}{1 - \eta(\underline{z})}\frac{\varphi - 1}{\varphi}\right] \alpha A \left(\frac{\kappa}{u}\right)^{\alpha - 1} - \rho = \gamma \\
&b(1-u)\left[\phi + (1-\phi)\left(\frac{g}{h}\right)^{\sigma}\right]^{\frac{1}{\sigma}} = \gamma \\
&-bu\left[\phi + (1-\phi)\left(\frac{g}{h}\right)^{\sigma}\right]^{\frac{1}{\sigma}} + \rho = 0
\end{aligned}$$

Consequently, the balanced-growth equilibrium fraction of working hour, u and physical-human capital ratio κ can be obtained as:

$$u = \frac{\rho}{\gamma + \rho} \tag{3.21}$$

$$\kappa = \frac{\rho}{\gamma + \rho}\left[\frac{\alpha A}{\gamma + \rho}(1 - \theta)\right]^{\frac{1}{1-\alpha}} \tag{3.22}$$

where

$$\theta \equiv \frac{\tau - \lambda\tilde{\eta}}{1 - \eta(\underline{z})}\frac{\varphi - 1}{\varphi} \tag{3.23}$$

Substitute u, κ, g from (3.21), (3.22), (2.19) into (2.17) to obtain the equation that determine the balanced-growth rate γ as,

$$(\gamma + \rho)^{\frac{1}{1-\alpha}}\left[\left(\frac{\gamma + \rho}{b}\right)^{\sigma} - \phi\right]^{\frac{1}{\sigma}} = \rho\,(1-\phi)^{\frac{1}{\sigma}}\,(\alpha A)^{\frac{1}{1-\alpha}}\,\Theta(\tau, \eta) \tag{3.24}$$

where

$$\Theta(\tau,\eta) \equiv \theta\,(1-\theta)^{\frac{\alpha}{1-\alpha}} \tag{3.25}$$

Lucas-meets-Moll: We first consider a special case where public spending does not affect human capital accumulation, namely $\phi = 1$. This is Lucas-meets-Moll case when human capital accumulation technology becomes $b(1-u)h$ as in Lucas (1988). Consequently, the equation (3.24) reduces to

$$(\gamma+\rho)^{\frac{1}{1-\alpha}} \left[\left(\frac{\gamma+\rho}{b}\right)^{\sigma} - 1\right]^{\frac{1}{\sigma}} = 0 \tag{3.26}$$

It is then straightforward that the economy converges to balanced growth path equilibrium with the growth rate $\gamma_L = b - \rho$ that is similar as in Lucas (1988) and is independent of the policy mix (τ,η) governing the value of θ. In other words, the government is unable to influence the growth rate when human capital formation does not depend on related public spending. We then focus our analysis for $\phi < 1$, namely when human capital formation is influenced by public spending, g.

No Pubic Spending on Human Capital: Suppose the policy mix (τ,η) is such that there is no public spending on education. Since (2.19) can be re-written as $g = \theta\alpha y$ this policy mix implies $\theta = 0$. Therefore, the equation (3.24) becomes,

$$(\gamma+\rho)^{\frac{1}{1-\alpha}} \left[\left(\frac{\gamma+\rho}{b}\right)^{\sigma} - \phi\right]^{\frac{1}{\sigma}} = 0 \tag{3.27}$$

which, under the assumption that $b\phi^{\frac{1}{\sigma}} > \rho$, implies that the economy will attain a positive balanced growth path rate as, [13]

$$\gamma_0 = b\phi^{\frac{1}{\sigma}} - \rho \tag{3.28}$$

From the definition of θ in (3.23) there are two types of policy mix (τ,η) that results zero public spending on human capital formation. One is no government policy, namely $\tau = 0$ and $\eta(z) = 0, \forall z$. The other is that the government just conducts transfer policy between active and inactive entrepreneurs without spending on human capital formation such that $\tau = \lambda\tilde{\eta}$. Both types of the two policy mix lead to the same balanced growth path rate.

[13] We will impose the assumption that $b\phi^{\frac{1}{\sigma}} - \rho > 0$ for further analysis. The assumption implies that the model economy attains a positive balanced growth path rate even in the absence of any fiscal policy.

Positive Pubic Spending on Human Capital: Next, we are going to investigate the effects of (τ, η) on growth rate when public spending on human capital formation is positive. Before proceeding, it should be well noted that in the literature of public finance and endogenous growth, welfare maximizing , not growth maximizing, is usually considered as the main objective of any benevolent government. There are two main reasons for us to focus on growth rate in this model. First, unlike other growth models with heterogeneity such as and Itskhoki and Moll (2018) this is an endogenous growth model and on the balanced-growth-path, assets together with other variables such as capital stock grows at the BGP rate over time, hence, the (stationary) distribution of individual entrepreneurs by assets does not exist. Consequently, it is not feasible to conduct welfare and efficiency analysis in a Ramsey policy sense based on an aggregate weighed welfare function for heterogeneous entrepreneurs.

Second, after the seminal work of Barro (1990) there have been intensive research and debate about the difference between welfare-maximizing and growth maximizing policy. In particular, Barro (1990) theoretically constructs an endogenous growth model where the flow of public spending financed by distortionary capital taxation, is a productive input in the aggregate production function. Barro (1990) then shows that maximizing the long-run growth is equivalent to maximizing welfare. In other words, growth maximizing fiscal policy that is financed by distortionary taxation can be rationalized. Futagami et al. (1993) extends Barro (1990) to incorporate the accumulation of productive public capital stock and shows than the growth maximizing tax rate is higher than the welfare maximizing tax rate. Misch et al. (2013) constructs an endogenous growth model with various combinations of assumptions related to the roles of public and private capital then numerically shows that while growth maximizing and welfare maximizing tax rates can be either equivalent or different from each other, they translate into relatively small differences in growth rates. In particular, for models with flow of public services in lines with Barro (1990) the differences in welfare levels is relatively small, hence growth maximization can be appropriate second-best policy for the benevolent government. Because we focus on analytical tractability in a model where public spending on human capital formation is flow as in Barro (1990) we are going to assume that the government chooses the tax rate that maximizing growth. [14]

[14] Another reason that growth focusing policy often appears on many government policy agenda

We then obtain the following Proposition abouth balanced growth rate in the presence of public spending on human capital formation.

Proposition 1. *For any policy mix* (τ, η) *that yields* $0 < \theta < 1$, *the economy attains a higher growth than* γ_0 *in* (3.28).

In other words, this Proposition states that when there is positive public spending on human capital formation, namely $\theta > 0$ and the level of public spending is not too high, namely $\theta < 1$ then the economy achieves a higher growth than the case where the government does not have public spending on human capital. It is worth mentioning from the definition of θ in (3.23) that the value of θ can be greater than 1. One possibility is *negative* subsidy or tax on capital rental against active entrepreneurs, namely $\eta(z) < 0$. When the tax rate on capital rental is sufficiently high θ can be greater than one and the implied growth rate can be lower than (3.28).

Proof. Let us denote $O(\gamma)$ as $(\gamma+\rho)^{\frac{1}{1-\alpha}}$ and $P(\gamma)$ as $\left[\left(\frac{\gamma+\rho}{b}\right)^{\sigma} - \phi\right]^{\frac{1}{\sigma}}$ the LHS of (3.24) can be expressed as $G(\gamma) \equiv O(\gamma) \cdot P(\gamma)$.

Since $0 < \alpha < 1$, it is straightforward that $\frac{dO(\gamma)}{d\gamma} > 0$, namely $O(\gamma)$ is an increasing function of γ. At the same time, it is possible to show that

$$\begin{aligned} \frac{d}{d\gamma}P(\gamma) &= \frac{1}{b}\left[\left(\frac{\gamma+\rho}{b}\right)^{\sigma} - \phi\right]^{\frac{1}{\sigma}-1}\left(\frac{\gamma+\rho}{b}\right)^{\sigma-1} \\ \frac{d^2}{d\gamma^2}P(\gamma) &= \frac{\phi(1-\sigma)}{b}\left[\left(\frac{\gamma+\rho}{b}\right)^{\sigma} - \phi\right]^{\frac{1}{\sigma}-2}\left(\frac{\gamma+\rho}{b}\right)^{\sigma-2} \end{aligned} \tag{3.29}$$

which implies that $P(\gamma)$ is an increasing convex function of γ. Consequently, the LHS of (3.24), $G(\gamma)$, is an increasing convex function of γ.

Notice that $G(\gamma_0) = 0$, where γ_0 is the growth rate when there is no public spending on human capital and is defined as in (3.28). Therefore, $G(\gamma) > G(\gamma_0) = 0 \; \forall \gamma > \gamma = 0$. At the same time, when $\phi < 1$ the RHS of (3.24) is independent of γ and is strictly positive for $0 < \theta < 1$. As a result, the horizontal line representing the RHS of (3.24) interacts with the curve representing the LHS of (3.24) at a unique point with the horizontal coordinate greater than γ_0. Equivalently, this implies that there exists a unique solution greater than γ_0 for the equation (3.24).

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is from the practical fiscal policy making perspective. Particularly, growth maximizing policy is sometimes considered as the second-best policy since it is difficult to estimate the preferences of economic agents such as households in order to conduct welfare maximizing policy.

$\square$

Proposition 1 and its proof imply other theoretical predictions that is worth mentioning here. The first one is that a higher value of measured TFP A leads to a higher RHS of (3.24), hence a higher value for the solution γ of this equation. Since the value of measured TFP depends positively on the level of financial development captured by the value of the parameter λ, this in turn implies that an economy with a higher measured TFP level thanks to deeper financial markets will have a higher balanced growth path rate.

The other one is that the balanced growth path rate, in general, depends on the value of the RHS of (3.24), which in turn depends on the the policy mix (τ, η). The policy mix determines the value of θ, hence the value of $\Theta(\tau, \eta)$. As discussed above, in the presence of the capital subsidy instrument, $\eta(z)$, the value of θ can be greater than one so the value of $\Theta(\tau, \eta)$ can be negative, consequently the implied growth rate can be lower than γ_0. However, in one special case where the government just utilizes one capital income instrument τ, namely $\eta(z) = 0\ \forall z$, then we obtain the following results.

Corollary 1. *The economy attains a higher growth than* γ_0 *in* (3.28) *when public spending on human capital formation is just financed by capital income instrument.*

Proof. When $\eta(z) = 0\ \forall z$, the definition of θ in (3.23) implies that $\theta = \tau^{\frac{\varphi-1}{\varphi}} < 1$. Consequently, the definition of Θ in (3.25) implies that the value of RHS of (3.24) is positive and the solution of this equation is greater than γ_0. $\square$

3.2 Growth-Maximizing Policy

In this secttion, we will explore further the effects on the policy mix on growth with an emphasis on growth maximizing policy. Notice that a higher value for $\Theta(\tau, \eta)$will lead to a higher value for the RHS of (3.24), therefore a higher balanced growth path rate. Moreover,

$$\begin{aligned}\frac{\partial}{\partial \theta}\Theta(\tau, \eta) &= (1-\theta)^{\frac{\alpha}{1-\alpha}-1}\left[1 - \frac{\theta}{(1-\alpha)}\right] \\ &= (1-\theta)^{\frac{\alpha}{1-\alpha}-1}\left[1 - \frac{\tau - \lambda\tilde{\eta}}{1-\eta(\underline{z})}\frac{\varphi - 1}{\varphi(1-\alpha)}\right]\end{aligned} \tag{3.30}$$

Consequently, we obtain the following Lemma.

Lemma 1. *The growth maximizing policy mix* (τ, η) *obeys the following rule:*

$$\frac{\tau - \lambda\tilde{\eta}}{1 - \eta(\underline{z})} = \frac{\varphi(1-\alpha)}{\varphi - 1} \tag{3.31}$$

where $\tilde{\eta} \equiv \int_{\underline{z}}^{\infty} \eta(z)g(z)dz$.

Proof. The proof is straightforward from (3.30). □

So far, we have not specified any properties for the policy mix (τ, η). For further analysis, we assume that capital income tax τ is positive and less than one the subsidy policy function $\eta(z)$ has the following properties,

Property 1. *(i) Feasibility:* $\eta(z) < 1 \ \forall z$.

(ii) Fairness: $\left[\frac{z}{\underline{z}} - \frac{1-\eta(z)}{1-\eta(\underline{z})}\right]$ *is an increasing function of* z *for* $z \geq \underline{z}$.

The *Feasibility* property is straightforward. To understand the latter, expressing profit of the active entrepreneur as: $\Pi(z, a) = \alpha A k^{\alpha-1}(uh)^{1-\alpha}\frac{\varphi-1}{\varphi}\left[\frac{z}{\underline{z}} - \frac{1-\eta(z)}{1-\eta(\underline{z})}\right] a$. Consequently, the *Fairness* property requires that for a given level of wealth, an active entrepreneur who has higher productivity will obtain higher *net* profit.

We then obtain the following Proposition about the growth maximizing policy mix,

Proposition 2. *There always exists the growth maximizing policy mix* (τ, η) *that satisfies the Property 1 and the optimal policy rule (*3.31*).*

Proof. First, consider that case that $\frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$. In this case, by setting $\eta(z) = 0, \ \forall z$ then the optimal policy rule (3.31) implies: $\tau = \frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$. Consequently, it is straightforward that there exists a capital income tax rate, τ that satisfies this condition.

Second, when $\frac{\varphi(1-\alpha)}{\varphi-1} > 1$, consider a subsidy policy function $\eta(z)$ for $z \geq \underline{z}$ that satisfies the following properties:

$$\frac{1-\eta(z)}{1-\eta(\underline{z})} = \left(\frac{z}{\underline{z}}\right)^{\mu}, \text{ where } \mu > 0 \text{ and } \eta(\underline{z}) \text{ are to be determined} \tag{3.32}$$

This policy function then implies,

$$\int_{\underline{z}}^{\infty} (1 - \eta(z))\, g(z)dz = (1 - \eta(\underline{z})) \int_{\underline{z}}^{\infty} \left(\frac{z}{\underline{z}}\right)^{\mu} g(z)dz$$

which after some algebra is equivalent to: $\lambda \int_{\underline{z}}^{\infty} \eta(z) g(z) dz = \frac{\varphi \eta(\underline{z}) - \mu}{\varphi - \mu}$. Substituting this result into the equation (3.31), we obtain,

$$\eta(\underline{z}) = \frac{\tau + \frac{\mu}{\varphi-\mu} - \frac{\varphi(1-\alpha)}{\varphi-1}}{\frac{\varphi}{\varphi-\mu} - \frac{\varphi(1-\alpha)}{\varphi-1}} \tag{3.33}$$

It is straightforward that when $\frac{\varphi}{\varphi-\mu} > \frac{\varphi(1-\alpha)}{\varphi-1}$ or $\mu > \frac{1-\varphi\alpha}{1-\alpha}$ then $\eta(\underline{z}) < 1$ for any $\tau < 1$. Moreover, if τ is set such that $\tau \geq \frac{\varphi(1-\alpha)}{\varphi-1} - \frac{\mu}{\varphi-\mu}$ then $0 \leq \eta(\underline{z}) < 1$. Notice that (3.32) implies $\eta(z) < \eta(\underline{z}) \forall z$ hence, $\eta(z)$ satisfies the *Feasibility* property.

Next, substituting (3.32) into the *Fairness* property we have, $\left[\frac{z}{\underline{z}} - \frac{1-\eta(z)}{1-\eta(\underline{z})}\right] = \frac{z}{\underline{z}}\left[1 - \left(\frac{z}{\underline{z}}\right)^{\mu-1}\right]$, which is an increasing function of z for $z \geq \underline{z}$ if $\mu < 1$.

As a result, if we set the parameter μ belongs to $\left(\frac{1-\varphi\alpha}{1-\alpha}, 1\right)$, capital income tax τ belongs to $\left(\frac{\varphi(1-\alpha)}{\varphi-1} - \frac{\mu}{\varphi-\mu}, 1\right)$, $\eta(z)$ and $\eta(\underline{z})$ as in (3.32) and (3.33) then the policy mix $(\tau, \eta(z))$ is a growth maximizing policy.

$\square$

Intuitively, this Proposition states that when the level of productivity heterogeneity is not too high, specifically, $\varphi \geq \frac{1}{\alpha}$, or equivalently $\frac{\varphi(1-\alpha)}{\varphi-1} \leq 1$ then the growth maximizing policy can be implemented by appropriately using the capital income tax instrument. The reason is that as φ is sufficiently high, the equation (2.20) implies that rental rate is sufficiently high even in the absence of any capital subsidy. Hence, the government is able to rely just on one tax instrument, the capital income tax rate τ, to collect sufficient level of revenue to finance human capital public investment in order to attain growth maximizing fiscal policy.

By contrast, when entrepreneurs are highly heterogeneous the capital rental rate, which is equal to the marginal product of capital of entrepreneurs at the productivity cut-off, is low. This Proposition then specifically demonstrates that when $\varphi < \frac{1}{\alpha}$ growth maximizing policy can not be implemented without utilizing the capital subsidy instrument because the government is unable to collect sufficient revenue to conduct growth maximizing fiscal policy. The Proposition then shows that there exists a feasible growth maximizing policy mix (τ, η) where an active entrepreneur who has higher productivity will obtain higher net profit for a given level of wealth. The capital subsidy policy of the growth maximizing policy mix is a *non-linear* function of the idiosyncratic productivity z and basically utilizes the heterogeneity nature of entrepreneurs to satisfy both the Fairness and Feasibility properties.

It should, in the meantime, be noted that it is not trivial to choose a policy mix that satisfy both the optimal rule (3.31) and Property 1. For instance, it is infeasible to set the same flat subsidy rate $\bar{\eta}$ for all active entrepreneurs because in this case, (3.31) implies that $1 < \tau$, which is a contradiction. However, if the government only subsidize a subset of active entrepreneurs by a flat rate $\bar{\eta}$ then the Fairness property is violated because there is *jump* in the net profit functions of active entrepreneurs.

4 Concluding remarks

This paper constructs a two-sector endogenous growth model with human capital accumulation and productivity heterogeneity to address the effects of fiscal policy on growth, especially growth maximizing policies. It shows that fiscal policies can influence growth and that growth maximizing policy may depend on the productivity distribution. There are several possible extensions from this paper. One is to have more tax instruments and to allow dynamic inter-temporal government budget constraints. Another is to add more objectives to the fiscal policies such as reducing distortions in the capital markets and optimizing entrepreneurs and workers' welfare in richer settings.

A Appendix A: Entrepreneurs' Optimization

Firms' Problem: Each entrepreneur maximizes the profit of his private firm subject to the technology (2.1) and his borrowing constraint (2.3), hence the profit of an entrepreneur can be expressed as follows:

$$\Pi(a,z) = \max_{k,n} \{f(z,k,n) - wn - (1-\eta(z))rk\} \quad s.t. \quad k \leq \lambda a$$

The first order condition of this problem with respect to labor requires: $(1-\alpha)(zk)^{\alpha}n^{-\alpha} = w$, which then implies labor demand as, $n^d = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}(zk)$. After substituting into the technology equation (2.1) we then obtain the following production function:

$$F(z,k) = \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} zk$$

Consequently, an entrepreneur's profits can be expressed as:

$$\Pi(a,z) = \max_{k} \left\{ \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} zk - (1-\eta_k(z))rk \right\} \quad s.t. \quad k \leq \lambda a$$

This entrepreneur's profit is linear in capital input, hence implying capital demand, labor demand and the productivity cutoff as in (2.4).

Entrepreneur's Consumption Rule: Let $V(a,z,t)$ be the value function of this optimality problem, then the Hamilton-Jacobi-Bellman equation is set as follows,

$$\rho V(a,z,t) = \max_{c_e(t)} \left\{ log(c_e(t)) + \frac{1}{dt}\mathbb{E}_{a,z}\left[dV(a,z,t)\right] \quad \text{s.t.} \quad (2.7) \right\}$$

This optimal rule can be confirmed as follows. First guess that the value function takes the form $V(a,z,t) = B\left[v(z,t) + loga\right]$, where B is an undermined constant. Substitute this form back to the above Hamilton-Jacobi-Bellman equation and take first order condition to obtain $c = a/B$. Substitute back in and then apply the envelop theorem to obtain $B = 1/\rho$.

B Appendix B: Aggregation

In this economy, the aggregate output denoted by y can be obtained by summing the amounts of homogenous final goods produced by all active entrepreneurs, i.e., entrepreneurs with idiosyncratic productivity higher than the cut-off $\underline{z}$:

$$\begin{aligned}
y =& \iint f(z,k,n)\phi(a,z)dadz = \iint \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z\lambda a g(z)\psi(a)dadz \\
=& \frac{\pi}{\alpha}\lambda \int a\psi(a)da \int zg(z)dz = \frac{\pi}{\alpha}\lambda k \int_{\underline{z}}^{\infty} zg(z)dz \\
=& \frac{\pi}{\alpha}\lambda X k, \quad \text{where} \quad X \equiv \int_{\underline{z}}^{\infty} zg(z)dz
\end{aligned} \tag{B.34}$$

Substituting the optimal labor demand (2.5) into the labor market clearing condition (2.14), we obtain

$$uh = \iint n^d(a,z)\phi(a,z)dadz = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{1-\alpha}} \lambda X k$$

which then implies that:

$$\pi = \alpha\left(\lambda X\right)^{\alpha-1} k^{\alpha-1}(uh)^{1-\alpha} \tag{B.35}$$

Plugging this equation back to $(B.34)$ we obtain the aggregate output as follows:

$$y = \left(\lambda X\right)^{\alpha} u^{1-\alpha} k^{(1+\alpha-1)}(h)^{1-\alpha} = Ak^{\alpha}(uh)^{1-\alpha}$$

where A is the endogenous measured TFP

$$A(t) \equiv \left(\lambda X\right)^{\alpha} = \left(\frac{\int_{\underline{z}}^{\infty} zg(z)dz}{(1-G(\underline{z}))}\right)^{\alpha} = \mathbb{E}\left[z|z \geq \underline{z}\right]^{\alpha}$$

Substituting capital demand from (2.4) into (2.13) and recall that z is $i.i.d$ we obtain the following capital market equilibrium equation

$$1 = \lambda\left[1 - G(\underline{z})\right] \tag{B.36}$$

which in turn determines the productivity cut-off $\underline{z} = \lambda^{\frac{1}{\varphi}}$.

The wage rate, w, can be obtained by substituting $(B.35)$ into the definition of π (2.6):

$$w = (1-\alpha)Ak^{\alpha}(uh)^{-\alpha}$$

Substituting $(B.35)$ into (2.6) to obtain the capital rental return rate after subsidy, r, as:

$$r = \frac{1}{1-\eta(\underline{z})}\frac{\underline{z}}{\mathbb{E}\left[z|z \geq \underline{z}\right]}\alpha A k^{\alpha-1}(uh)^{1-\alpha} \tag{B.37}$$

The dynamic equation of the aggregate capital stock is then derived by first aggregating wealth of all entrepreneurs

$$
\begin{aligned}
\frac{\dot{k}}{k} &= \frac{1}{k}\iint \dot{a}g(z)\psi(a)dadz \\
&= \frac{1}{k}\iint \left(\lambda \max\left\{z\pi - (1-\eta(z))r, 0\right\} + (1-\tau)r - \rho\right) ag(z)\psi(a)dadz \\
&= \int_0^\infty \left(\lambda \max\left\{z\pi - (1-\eta(z))r, 0\right\} + (1-\tau)r - \rho\right) g(z)dz
\end{aligned}
$$

and then dividing entrepreneurs into the inactive group ($z < \underline{z}$) and the active group ($z \geq \underline{z}$), therefore

$$
\begin{aligned}
\frac{\dot{k}}{k} &= r - r\int \tau\psi(a)da - \rho + \int_{\underline{z}}^\infty \lambda\left\{z\pi - (1-\eta(z))r\right\} g(z)dz \\
&= r - r\int \tau\psi(a)da - \rho + \pi\lambda\int_{\underline{z}}^\infty zg(z)dz - r - r\lambda\int_{\underline{z}}^\infty \eta(z)g(z)dz \\
&= \pi\lambda X - \rho - r\left(\int \tau\psi(a)da - \lambda\int_{\underline{z}}^\infty \eta(z)g(z)dz\right) \\
&= \alpha\left(\lambda X\right)^{\alpha-1} k^{\alpha-1}(uh)^{1-\alpha}\lambda X - \rho - r\left(\int \tau\psi(a)da - \lambda\int_{\underline{z}}^\infty \eta(z)g(z)dz\right) \\
&= \alpha A k^{\alpha-1}(uh)^{1-\alpha} - \rho - r\left(\tau - \lambda\tilde{\eta}\right)
\end{aligned}
$$

where the third and forth equal signs are implied by the definition of X in (2.15), and π in $(B.35)$ and the effective capital subsidy rate policy, $\tilde{\eta}$ are defined as follows,

$$
\tilde{\eta} \equiv \int_{\underline{z}}^\infty \eta(z)g(z)dz \tag{B.38}
$$

The budget constraint of the government becomes

$$
\begin{aligned}
g &= \iint \left[\tau - \lambda\eta(z)\cdot \mathbb{1}_{\{\underline{z}\geq z\}}\right] ra\phi(a,z)dadz \\
&= (\tau - \lambda\tilde{\eta})rk = \alpha y\left[\frac{\tau - \lambda\tilde{\eta}}{1-\eta(\underline{z})}\frac{\varphi - 1}{\varphi}\right]
\end{aligned}
$$

References

Angeletos, G.M. (2007) “Uninsured Idiosyncratic Investment Risk and Aggregate Saving”, *Review of Economic Dynamics*, 10, 1-30.

Axtell, R. L. (2001) “Zipf Distribution of U.S. Firm Sizes”, *Science*, 293, 1818-1820.

Barro, R. (1990) “Government Spending in a Simple Model of Endogenous Growth”, *Journal of Political Economy* 98(5), 103-S125.

Buera, F., J. P. Kaboski, and Y. Shin (2011) “Finance and Development: A Tale of Two Sectors”, *American Economic Review* 101 (5), 1964-2002.

Buera, F., J. P. Kaboski, and Y. Shin (2015) “Entrepreneurship and Financial Frictions: A Macro-Development Perspective”, *Annual Review of Economics*, 7, 409-436

Buera, F. and Y. Shin (2013) “Financial Frictions and the Persistence of History: A Quantitative Exploration”, *Journal of Political Economy* 121(2), 221-272.

Futagami, K., Y. Morita, and A. Shibata (1993), “Dynamic analysis of an endogenous growth model with public capital”, *Scandinavian Journal of Economics* 95, 607-625.

Itskhoki, O. and B. Moll (2018) “Optimal Development Policies with Financial Frictions”, Econometrica, forthcoming.

Kiyotaki, N. and J. Moore (2012) “Liquidity, Business Cycles, and Monetary Policy”, *NBER Working Papers* 17934.

Lucas R (1988) “On the mechanics of economic development”, *Journal of Monetary Economics* 22(1), 342

Luttmer, E. (2007) “Selection, Growth, and the Size Distribution of Firms”, *Quarterly Journal of Economics*, 122(3), 1103-1144.

Melitz, M. J. (2003) “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity”, *Econometrica*, 71(6), 1695-1725.

Midrigan, V. and D. Y. Xu (2014) "Finance and Misallocation: Evidence from Plant-Level Data", *American Economic Review*, 104(2), 422-458.

Misch, F. , Gemmell, N. , Kenller, R. (2013), "Growth and welfare maximization in models of public finance and endogenous growth", *Journal of Public Econconomic Theory*, 15(6), 939-967.

Mino, K. (2015) "A Simple Model of Endogenous Growth with Financial Frictions and Firm Heterogeneity", *Economics Letters*, 127, 20-23.

Mino, K. (2016) "Fiscal Policy in a Growing Economy with Financial Frictions and Firm Heterogeneity", *The Japanese Economic Review*, 67(1), 3-30.

Moll, B. (2014) "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?", *American Economic Review*, 104 (10), 3186-3221.

Nguyen, Q.H. (2019a) "Growth Model with Financial Deepening and Productivity Heterogeneity", *The Japanese Economic Review*, 70(1), 123-140.

Nguyen, Q.H. (2019b) "Financial Deepening in a Two-Sector Endogenous Growth Model with Productivity Heterogeneity", *The B.E. Journal of Macroeconomics*, forthcoming.

Uzawa, H. (1965) "Optimum technical change in an aggregative model of economic growth", *International Economic Review*, 6, 1831.