# A tale of two Rawlsian criteria

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## Abstract

This work considers optimization problems under Rawls and maximin with multiple discount factors criteria. It proves that though these criteria are different, they have the same optimal value and solution.

**Keywords**: Maximin principle, Ralws criterion, Ramsey criterion.
**JEL** classification numbers: C61, D11, D90.

## 1 Introduction

Consider the following classical question: given a stock of a renewable resource, what would be the best inter-temporal exploitation of it, considering the welfare of both current and future generations?

The famous Ramsey criterion, which uses a constant discount rate and is used largely in research into economic dynamics, is criticized for its weak weighting parameters for generations in the distant future. The evaluation of each utilities stream is quasi-determined by a finite number of generations. This raises the

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concerns that following the Ramsey criterion, the economy does not leave enough resource for the future.

In the classical work "Theory of justice", Rawls [17] assumes that if one is hidden behind a *veil of ignorance*, with total lack of information about the condition into which she[1] will be born, the economic agent should choose the maximization of the least favoured generation. Specifically, given a inter-temporal consumption streams, her evaluation criterion of inter-temporal utilities streams should be

$$U(c_0, c_1, c_2, \dots) = \inf_{s \geq 0} u(c_s),$$

where $u(c_t)$ is the utility of the $t^{th}$ generation, given $c_t$ as the consumed resource.

We can consider the Rawls's question in another way: the economic agent may be ambiguous about what is the "good" discount factor to choose in evaluating utilities streams. Her set of possible discount factors is $(0, 1)$. Having total lack of information, for a given consumption stream $\{c_s\}_{s=0}^{\infty}$, she should evaluate it as[2]

$$U(c_0, c_1, c_2, \dots) = \inf_{\delta \in (0,1)} \left[ (1 - \delta) \sum_{s=0}^{\infty} \delta^s u(c_s) \right].$$

This criterion can also be considered as an application of Rawls's spirit in the configuration where disagreements exists between people in the economy about how to discount the future. The social planer choose a criterion that maximizes the least favoured person.

Naturally, this raises the question of the behaviour of the economy under the Rawls criteria. The first Rawls criterion is well studied in the seminar contributions of Arrow [2], Solow [15] and Calvo [4]. The result is clear: the behaviour of the economy depends strongly on the initial stock. If the stock of a renewable resource is below the *golden rule* (the level of stock allowing a maximal level of constant consumption), the optimal exploitation strategy is to ensure that the stock remains

[1] We use female pronouns as a convenient default.

[2] For the axiomatic foundation and discussion about the importance of the normalizing term $1 - \delta$, see Chambers & Echenique [5] and Drugeon & al [9]. Observe that for any $0 < \delta < 1$, we have $(1 - \delta) \sum_{s=0}^{\infty} \delta^s = 1$.

constant over time. In the case of abundant stock of a renewable resource, which is higher than the *golden rule*, there is an infinite number of solutions and every optimal path converges decreasingly to this level.

The purpose of this work is to study the same question under the second Rawls criterion. We prove that for a low level of resource stock (under the *golden rule*), the unique solution is to keep the stock constant through time. Moreover, the solutions under the two criteria coincide. For the case where the resource is abundant, the solution under the first criterion is the one under the second, and the value functions are equal.

This work is organized as follows. Section 2 introduces the two Rawlsian problems and the main properties of the first one and solves the second one. Section 3 discusses different criterion studied in the literature.

# 2 The two Rawlsian criteria

## 2.1 Fundamentals

Denote by $u$ the instantaneous utility function and $f$ the regeneration function of the renewable resource. These two functions are supposed to be strictly increasing and concave. The concavity of the utility function is strict. To simplify the presentation, suppose that $f'(0) > \frac{1}{\delta}$ and $f'(\infty) < 1$.

Denote by $\overline{x}$ the golden rule, the capital accumulation corresponding to the maximum level of constant consumption: this value $\overline{x}$ is solution to the equation $f'(x) = 1$.

For any given capital stock $x_0 \geq 0$, denote by $\Pi(x_0)$ the set of feasible paths of stock $\{x_s\}_{s=0}^{\infty}$: for any $s$, $0 \leq x_{s+1} \leq f(x_s)$.

For each discount rate $0 < \delta < 1$, it is well known in dynamic programming literature[3] that the optimal capital accumulation path corresponding to $\delta$ is monotonic

[3]See Stokey, Lucas with Prescott [16].

and converges to $x^\delta$, the solution to the equation $f'(x) = \frac{1}{\delta}$.

For each feasible stock path $\mathbf{x} = \{x_s\}_{s=0}^{\infty}$, the inter-temporal evaluation of the corresponding consumption path $\{c_s\}_{s=0}^{\infty}$ with $c_s = f(x_s) - x_{s+1}$ for any $s \geq 0$, is given as

$$\nu(\mathbf{x}) = \inf_{s \geq 0} u(c_s).$$

## 2.2 The classical Rawls criterion

The famous Rawls criterion, embedded in the optimal growth context, can be considered as the following optimization problem, which is well studied in Arrow [2], Solow [15] and Calvo [4]. The economic agent solves:

$$\max \left[ \inf_{s \geq 0} u(c_s) \right],$$

under the constraint $c_t + x_{s+1} \leq f(x_s)$ for all $s$, with $x_0 > 0$ given.

The Lemma 2.1 establishes the foundation for the existence of optimal solution and fundamental properties of the value function.

Lemma **2.1.** i) *For any $x_0 \geq 0$, the set of feasible paths $\Pi(x_0)$ is compact in product topology.*

ii) *The function $\nu$ is upper semi-continuous for the product topology.*

iii) *There exists $\boldsymbol{x}^* \in \Pi(x_0)$ such that*

$$\nu\big(\boldsymbol{x}^*\big) = \max_{\boldsymbol{x} \in \Pi(x_0)} \nu\big(\boldsymbol{x}\big).$$

Proposition 2.1 gives the behaviour of the optimal path, which depends strongly on the initial condition, with the golden rule $\overline{x}$ as the critical threshold.

Proposition 2.1. i) *Consider the case $0 \leq x_0 \leq \overline{x}$. The problem has unique* *solution* $\boldsymbol{x}^* = (x_0, x_0, \ldots)$ *and*

$$\begin{aligned}\max_{\boldsymbol{x} \in \Pi(x_0)} \nu(\boldsymbol{x}) &= \nu(\boldsymbol{x}^*) \\ &= u\left(f(x_0) - x_0\right).\end{aligned}$$

ii) *Consider* $x_0 > \overline{x}$. *The problem has an infinite number of solutions which all converge to* $\overline{x}$. *And*

$$\max_{\boldsymbol{x} \in \Pi(x_0)} \nu(\boldsymbol{x}) = u\left(f(\overline{x}) - \overline{x}\right).$$

For initial capital stock $x_0$ smaller than $\overline{x}$, the optimal choice is to remain in the *status quo*. The unique solution $\mathbf{x}^*$ satisfies $x_s^* = x_0$ for any $s \geq 0$. The optimal value is $u\left(f(x_0) - x_0\right)$. For $x_0$ bigger than $\overline{x}$, there exists an infinite number of solution, every optimal stock path converges to $\overline{x}$ and the optimal value is $u\left(f(\overline{x}) - \overline{x}\right)$.

## 2.3 The second Rawlsian criterion and the equivalence between the two criteria

In [9], Drugeon & al consider the optimization problem with multiple discount factors under the *maximin* criteria. Let $\mathcal{D} = [\underline{\delta}, \overline{\delta}]$ representing the set of possible discount factors, the economic agent solves:

$$\max \min_{\delta \in \mathcal{D}} \left[(1-\delta) \sum_{s=0}^{\infty} \delta^s u(c_s)\right]$$

s.c $c_s + x_{s+1} \leq f(x_s)$ for any $s$,

$x_0$ is given.

For each feasible stock path $\mathbf{x} = \{x_s\}_{s=0}^{\infty}$, let $c_s = f(x_s) - x_{s+1}$ for any $s \geq 0$ and

$$\hat{\nu}(\mathbf{x}) = \inf_{0<\delta<1} \left[(1-\delta) \sum_{s=0}^{\infty} \delta^s u(c_s)\right].$$

As the functions $u$ and $f$ satisfy the standard conditions in growth theory, for each discount factor $\delta$, the optimal path of the Ramsey problem corresponding to $\delta$ converges monotonically to $x^\delta$ the solution to

$$f'(x) = \frac{1}{\delta}.$$

Moreover, it is easy to verify that

$$\lim_{\delta \to 0} x^\delta = 0,$$
$$\lim_{\delta \to 1} x^\delta = \overline{x}.$$

Proposition 2.2 gives a detailed description of the optimal path under the multiple discount factors and the second Rawlsian criterion.

PROPOSITION 2.2. *Assume that* $0 < \underline{\delta} \leq \overline{\delta} < 1$. *Denote by* $\chi^*$ *the unique optimal path for the* maximin *problem.*

i) *For* $x_0 \leq x^{\underline{\delta}}$, $\chi^*$ *coincides with the optimal path of the Ramsey problem with discount factor* $\underline{\delta}$, *is increasing and converges to* $x^{\underline{\delta}}$.

ii) *For* $x^{\underline{\delta}} \leq x_0 \leq x^{\overline{\delta}}$, *for any* $s$, $x_s^* = x_0$. *The optimal path* $\chi^*$ *coincides with the optimal solution of Ramsey problem with discount factor* $\delta$ *satisfying* $x^\delta = x_0$.

iii) *For* $x_0 \geq x^{\overline{\delta}}$, $\chi^*$ *coincides with the optimal path of the Ramsey problem with discount factor* $\overline{\delta}$, *is decreasing and converges to* $x^{\overline{\delta}}$.

Figure 1, taken from Drugeon & al [9], provides an illustration of the dependence of optimal paths in initial condition. The functions $\varphi_{\underline{\delta}}$ and $\varphi_{\overline{\delta}}$ represent respectively the optimal policy functions for the Ramsey problems with the discount factors $\underline{\delta}$ et $\overline{\delta}$.

By technical difficulties relying with the fixed point arguments, Drugeon & al [9] assume that $\mathcal{D}$ is a closed set belonging to $(0,1)$: $0 < \underline{\delta} \leq \overline{\delta} < 1$. Intuitively, under the result in Proposition 2.2 we can hope that for $\mathcal{D} = (0,1)$: $\underline{\delta}$ converges to zero,

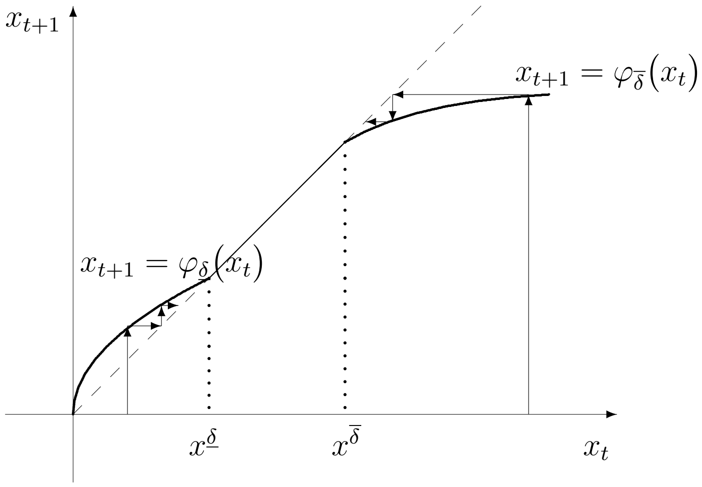


Figure 1: The optimal policy function in multiple discount factors configuration

and $\overline{\delta}$ converges to 1, the two Ralwsian problems have the same value function: for $\mathscr{D} = (0,1)$, we get $\max_{\mathbf{x}\in\Pi(x_0)} \nu(\mathbf{x}) = \max_{\mathbf{x}\in\Pi(x_0)} \hat{\nu}(\mathbf{x})$.

PROPOSITION 2.3. *For any $x_0 \geq 0$,*

i) *We have*

$$\max_{\chi\in\Pi(x_0)} \inf_{s\geq 0} u\big(f(x_s) - x_{s+1}\big) = \max_{\chi\in\Pi(x_0)} \left[\inf_{0<\delta<1} (1-\delta)\sum_{s=0}^{\infty} u\big(f(x_s) - x_{s+1}\big)\right].$$

ii) *For $0 \leq x_0 \leq \overline{x}$, the two Rawlsian problems have the same solution $\boldsymbol{x}^* = (x_0, x_0, x_0, \dots)$.*

iii) *For $x_0 > \overline{x}$, every solution under the first Rawlsian criterion is a solution under the second one.*

*Proof.* $(i)$ To facilitate the exposition, for each $0<\delta<1$, denote by $\{x_s(\delta)\}_{s=0}^{\infty}$ the optimal path of Ramsey problem corresponding to the discount factor $\delta$.

Observe that for any feasible path of stock $\{x_s\}_{s=0}^{\infty}$ belonging to $\Pi(x_0)$:

$$\inf_{s\geq 0} u\big(f(x_s) - x_{s+1}\big) \leq \inf_{0<\delta<1} \left[(1-\delta)\sum_{s=0}^{\infty} \delta^s u\big(f(x_s) - x_{s+1}\big)\right].$$

This implies

$$\max_{\chi \in \Pi(x_0)} \min_{s \geq 0} u\big(f(x_s) - x_{s+1}\big) \leq \max_{\chi \in \Pi(x_0)} \left[ \inf_{0<\delta<1} (1-\delta) \sum_{s=0}^{\infty} \delta^s u\big(f(x_s) - x_{s+1}\big) \right].$$

Now we will prove the converse inequality.

Consider first the case $0 < x_0 < \overline{x}$. Fix $0 < \underline{\delta} < \overline{\delta} < 1$ such that $x^{\underline{\delta}} < x_0 < x^{\overline{\delta}}$.

Define $\chi^* = (x_0, x_0, \dots)$, which is the unique optimal path for the *maximin* criterion with the set of discount rates $\mathscr{D} = [\underline{\delta}, \overline{\delta}]$. For any feasible path $\chi \neq \chi^*$, following Drugeon & al [9], we have

$$\begin{aligned}
\inf_{0<\delta<1} \left[ (1-\delta) \sum_{s=0}^{\infty} u\big(f(x_s) - x_{s+1}\big) \right] &\leq \inf_{\underline{\delta} \leq \delta \leq \overline{\delta}} \left[ (1-\delta) \sum_{s=0}^{\infty} u\big(f(x_s) - x_{s+1}\big) \right] \\
&< \inf_{\underline{\delta} \leq \delta \leq \overline{\delta}} \left[ (1-\delta) \sum_{s=0}^{\infty} u\big(f(x_s^*) - x_{s+1}^*\big) \right] \\
&= u\big(f(x_0) - x_0\big) \\
&= \max_{\chi \in \Pi(x_0)} \left[ \inf_{s \geq 0} u\big(f(x_s) - x_{s+1}\big) \right].
\end{aligned}$$

This implies that the two Rawlsian problems have the same maximum value and unique solution $\chi^*$.

Now consider the case $x_0 > \overline{x}$. The idea of the proof is that for any $\delta$, the sequence $\{x_s(\delta)\}_{s=0}^{\infty}$ converges to $x^\delta$ with a speed that is sufficiently high and independent with the choice of $\delta$.

We prove that for any $\epsilon > 0$, there exists $T(\epsilon)$ such that for any $T \geq T(\epsilon)$, any $0 < \delta < 1$, we have

$$x^\delta < x_T(\delta) < \overline{x} + \epsilon.$$

For each $0 < \delta < 1$, consider a time $s$ satisfying $x_0 \geq x_1(\delta) \geq \dots \geq x_{s+1}(\delta) \geq \overline{x} + \epsilon$. Observe that $f'(\overline{x} + \epsilon) < 1$. Let $f'(\overline{x} + \epsilon) = 1 - \epsilon_1$, with $\epsilon_1 > 0$.

By Euler equations, we have

$$
\begin{aligned}
u'\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) &= \delta u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) f'(x_{s+1}(\delta)) \\
&\leq u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) f'(x_{s+1}(\delta)) \\
&\leq u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) f'(\overline{x} + \epsilon) \\
&\leq u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) - \epsilon_1 u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) \\
&\leq u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) - \epsilon_2,
\end{aligned}
$$

for $\epsilon_2 = \epsilon_1 u'\big(f(x_0)\big)$, since $f(x_0) \geq f(x_{s+1}(\delta)) - x_{s+2}(\delta)$. Observe that $\epsilon_2$ does not depend on $\delta$.

We then deduce

$$
\begin{aligned}
\epsilon_2 &\leq u'\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) - u'\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) \\
&= u''(\xi) \left[\big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big) - \big(f(x_s(\delta)) - x_{s+1}(\delta)\big)\right] \\
&= (-u''(\xi)) \left[\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) - \big(f(x_{s+1}(\delta)) - x_{s+2}(\delta)\big)\right],
\end{aligned}
$$

with some $f(x_{s+1}(\delta)) - x_{s+2}(\delta) \leq \xi \leq f(x_s(\delta)) - x_{s+1}(\delta)$. This implies

$$
x_{s+1}(\delta) - x_{s+2}(\delta) \leq f(x_s(\delta)) - f(x_{s+1}(\delta)) - \frac{\epsilon_2}{-u''(\xi)}.
$$

As $x_{s+1}(\delta) \geq \overline{x} + \epsilon$, it is easy to verify that

$$
\begin{aligned}
f(\overline{x} + \epsilon) - x^\epsilon &\leq f(x_{s+1}(\delta)) - x_{s+2}(\delta) \\
&\leq \xi \\
&\leq f(x_s(\delta)) - x_{s+1}(\delta) \\
&\leq f(x_0).
\end{aligned}
$$

Let

$$
a = \sup_{f(\overline{x}+\epsilon) - x^\epsilon \leq \xi \leq f(x_0)} (-u''(\xi)),
$$

and

$$\epsilon_3 = \frac{\epsilon_2}{a}.$$

The value $\epsilon_3$ is strictly positive and is independent with respect to $\delta$. Moreover,

$$\begin{aligned} x_{s+1}(\delta) - x_{s+2}(\delta) &\leq f(x_s(\delta)) - f(x_{s+1}(\delta)) - \epsilon_3 \\ &\leq f'(x_{s+1}(\delta))(x_s(\delta) - x_{s+1}(\delta)) - \epsilon_3 \\ &\leq x_s(\delta) - x_{s+1}(\delta) - \epsilon_3. \end{aligned}$$

Hence for $T(\epsilon)$ big enough such that $x_0 - T(\epsilon)\epsilon_3 < 0$, we have $x_T(\delta) < \overline{x} + \epsilon$ for any $T \geq T(\epsilon)$ and for any $0 < \delta < 1$. Otherwise we will have $x_T(\delta) - x_{T+1}(\delta) \leq 0$ for some $T \geq T(\epsilon)$: a contradiction[4].

By the independence of $T(\epsilon)$ in respect to $\delta$, combining with result that for $s \geq T(\epsilon)$, we have $x^\delta \leq x_s(\delta) \leq \overline{x} + \epsilon$, we get the following inequality:

$$\begin{aligned} &\lim_{\delta \to 1} \left[ (1-\delta) \sum_{s=0}^{\infty} \delta^s u\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) \right] \\ &= \lim_{\delta \to 1} \left[ (1-\delta) \sum_{s=0}^{T(\epsilon)} \delta^s u\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) \right] \\ &+ \lim_{\delta \to 1} \left[ \delta^{T(\epsilon)+1} (1-\delta) \sum_{s=T(\epsilon)+1}^{\infty} \delta^{s-T(\epsilon)-1} u\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) \right] \\ &= \lim_{\delta \to 1} \left[ \delta^{T(\epsilon)+1} (1-\delta) \sum_{s=T(\epsilon)+1}^{\infty} \delta^{s-T(\epsilon)-1} u\big(f(x_s(\delta)) - x_{s+1}(\delta)\big) \right] \\ &\leq \lim_{\delta \to 1} u\big(f(\overline{x} + \epsilon) - x^\delta\big) \\ &= u\big(f(\overline{x} + \epsilon) - \overline{x}\big). \end{aligned}$$

[4]It is well known that the solution of Ramsey problem converges monotonically to the steady state.

For any feasible path $\chi \in \Pi(x_0)$,

$$\begin{aligned}
\inf_{0<\delta<1}\left[(1-\delta)\sum_{s=0}^{\infty} u\big(f(x_s)-x_{s+1}\big)\right] &\leq \inf_{0\leq\delta\leq 1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(x_s(\delta))-x_{s+1}(\delta)\big)\right] \\
&\leq \lim_{\delta\to 1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(x_s(\delta))-x_{s+1}(\delta)\big)\right] \\
&\leq u\big(f(\overline{x}+\epsilon)-\overline{x}\big).
\end{aligned}$$

Since $\epsilon > 0$ is chosen arbitrarily, this implies

$$\inf_{0<\delta<1}\left[(1-\delta)\sum_{s=0}^{\infty} u\big(f(x_s)-x_{s+1}\big)\right] \leq u\big(f(\overline{x})-\overline{x}\big).$$

We then have

$$\max_{\mathbf{x}\in\Pi(x_0)} \nu(\mathbf{x}) = \max_{\mathbf{x}\in\Pi(x_0)} \hat{\nu}(\mathbf{x}) = u\big(f(\overline{x})-\overline{x}\big).$$

For a solution of the problem with the second Rawlsian criterion, take for example the sequence $\hat{\chi} \in \Pi(x_0)$ such that $\hat{x}_s = \overline{x}$ for any $s \geq 1$. For each $\delta$,

$$(1-\delta)\sum_{s=0}^{\infty} u\big(f(\hat{x}_s)-\hat{x}_{s+1}\big) = (1-\delta)u\big(f(x_0)-\overline{x}\big)+\delta u\big(f(\overline{x})-\overline{x}\big).$$

Since $x_0 > \overline{x}$, the function $(1-\delta)u\big(f(x_0)-\overline{x}\big)+\delta u\big(f(\overline{x})-\overline{x}\big)$ is strictly decreasing in respect to $\delta$. This implies

$$\begin{aligned}
\inf_{0<\delta<1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(\hat{x}_s)-\hat{x}_{s+1}\big)\right] &= \lim_{\delta\to 1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(\hat{x}_s)-\hat{x}_{s+1}\big)\right] \\
&= u\big(f(\overline{x})-\overline{x}\big).
\end{aligned}$$

$(ii)$ This property is proven using the same the arguments as part $(i)$.

$(iii)$ Consider some feasible path $\mathbf{x}^*$ which is a solution of the problem under first Rawls criterion. Since $u\big(f(x_s^*)-x_{s+1}^*\big) \geq u\big(f(\overline{x})-\overline{x}\big)$ for any $s \geq 0$, for any

$0 < \delta < 1$,

$$(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(x_s^*) - x_{s+1}^*\big) \geq u\big(f(\overline{x}) - \overline{x}\big).$$

This implies

$$\begin{aligned}\inf_{0<\delta<1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(x_s^*) - x_{s+1}^*\big)\right] &\geq u\big(f(\overline{x}) - \overline{x}\big) \\ &= \max_{\mathbf{x}\in\Pi(x_0)}\inf_{0<\delta<1}\left[(1-\delta)\sum_{s=0}^{\infty}\delta^s u\big(f(x_s) - x_{s+1}\big)\right].\end{aligned}$$

Hence $\mathbf{x}^*$ is a solution of the problem under second Rawls criterion. The proof is completed. QED

# 3 Discussions

## 3.1 Rawls criteria and ambiguity aversion

In recent decades, a large body of literature has risen in decision theory, enlarging the world of Savage [18], where the famous *sure-thing princple* is not satisfied. The seminar contribution of Gilboa & Schmeidler [11] considers the behaviour under which the economic agent maximizes the worst scenario. This allows us to make a link to the Rawlsian criteria. Assume that the economic agent must choose a time discounting system to evaluate the inter-temporal consumption streams. The set of possible time discounting is $\Delta = (\pi_0, \pi_1, \pi_2, \dots)$ such that $\pi_s > 0$ for any $s$ and $\sum_{s=0}^{\infty}\pi_s = 1$. Behind the *veil of ignorance*, every time discounting system is possible. Hence, the criterion under *ambiguity aversion* is

$$\begin{aligned}U(c_0, c_1, c_2, \dots) &= \inf_{\pi\in\Delta}\left[\sum_{s=0}^{\infty}\pi_s u(c_s)\right] \\ &= \inf_{s\geq 0} u(c_s),\end{aligned}$$

which is the first Ralws criterion.

Now assume that the economic agent is just ambiguous about the set of time discounting systems satisfying the usual properties as *impatience*, and *stability*. Let $\mathscr{D}$ be that set. In Chambers & Echenique [5], we found that:

$$\mathscr{D} = \{\pi \in \Delta \text{ such that } \exists \delta \in (0,1) : \pi_s = (1-\delta)\delta^s \text{ for all } s \geq 0\}.$$

The criterion is then the second Rawlsian one.

### 3.2 Disussion about some criteria

The Ramsey criterion is criticized about putting privileges for the generations in present and close future. In another way, other criteria, for example the lim inf take into account only the distant future. As a way to reconcile these to extremes, Chichilnisky in [6], [7] proposes a criterion satisfying her *No-dictatorship* of present and of future. Her criterion is a convex combination of a Ramsey part and a lim inf part[5]. The weakness of this criterion is that, being the convex sum of two parts which are continuous in respect to different topologies, the optimization problem under this criterion generally has no solution. It is always difficult taking into account at the same time the efficiency and the equality.

As a response for this challenge, Alvarez-Cuadrado & Van Long [1] consider the convex combination between a Ramsey part and a Rawlsian part, in the continuous time configuration. They give a detailed description of the behaviour of the economy [6]. Another approach belongs to Asheim & Ekeland [3], who consider the markovian solutions of the problem under Chichilnisky's criterion, and prove that the lim inf part has no effect on the optimal choice.

The *overtaking* criterion of Gale satisfies the two *non-dictatorship* properties of Chichilnisky, but this criterion is not complete. If we focus only on the *good programs*, as in Dana & Le Van [8], the optimal path converges to the *golden rule*.

[5]For a discussion about Chichilnisky's criterion, see Alvarez-Cuadrado & Van Long [1].
[6]For the discrete time configuration, see Ha-Huy & Nguyen [12].

As an attempt to avoid the non-completeness problem, Le Van & Morhaim [14] consider the Ramsey problem and study the properties of the solution when the discount rate converges to 1. They prove that the sequence of solutions converge to the solution of problem under Gale's criterion.

### 3.3 Technical concerns

The result for the first Rawlsian criterion is based only on the concavity of the function $f$, and does not impose any condition on the utility function $u$. However, in order to apply results in dynamic programming literature, for solving the problem under the second Rawlsian criterion, we must assume the concavity property for utility function.

And, consider the case where $f'(\infty) \geq 1$. Under this assumption, $\overline{x} = \infty$. For the two Rawlsian criteria, for any initial stock of resource, the only solution is to remains constant. The only remark is that since the feasible paths could be unbounded, we must assume conditions ensuring the determination of value function and its continuity. For the details, curious readers can refer to the article of Le Van & Morhaim [13], with the most important condition being *tail insensitivity* property.

And if $f'(0) \leq 1$, every feasible path converges to zero, the two problems become trivial.

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