Continuous time debt dynamics and fiscal policy for full-employment: A Keynesian approach by mathematics and simulation

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Abstract

We present a continuous time version of a dynamic analysis of debt-to-GDP ratio, and examine the effects of a fiscal policy which realizes full-employment from a state of under-employment or with deflationary GDP gap. We show that the larger the extra growth rate of real GDP by a fiscal policy is, the smaller the debt-to-GDP ratio at the time when full-employment is realized is, and a fiscal policy for full-employment can reduce the debt-to-GDP ratio. Therefore, full-employment can be realized by an aggressive fiscal policy with smaller debt-to-GDP ratio than before the fiscal policy. An increase in the government expenditure may induce a rise of the interest rate. Since the higher the interest rate is, the larger the debt-to-GDP ratio is, we need an appropriate monetary policy which maintains the low interest rate. Also we show that even if the marginal propensity to consume is very small, an aggressive fiscal policy can realize full-employment without increasing debt-to-GDP ratio.

Keywords: fiscal policy, full-employment, debt-to-GDP ratio, continuous time debt dynamics

JEL Classification No.: E62.

1 Introduction

Watts and Sharpe (2016) presented a discrete time version of dynamic analysis of debt-to-GDP ratio, and showed that an aggressive fiscal policy can reduce the debt-to-GDP ratio. Generalizing their model we present a continuous time version of a dynamic analysis of debt-to-GDP ratio, and examine the effects of a fiscal policy which realizes full-employment from a state of under-employment or with deflationary GDP gap[1]. Under-employment state arises due to aggregate demand shortage.

Using a general continuous time model of debt dynamics we consider time required to realize full-employment, and examine the debt-to-GDP ratio at the time when full-employment is realized. The government increases its expenditure to accelerate the economic growth until full-employment is realized. The extra growth rate of the government expenditure over the ordinal growth rate (the growth rate of the full-employment real GDP) depends on the target growth rate of real GDP over ordinal growth, the share of the government expenditure in real GDP, and the magnitude of multiplier effects. Also we show that even if the marginal propensity to consume is very small, an aggressive fiscal policy can realize full-employment without increasing debt-to-GDP ratio.

In the next section we consider a steady state of continuous time debt dynamics, and analyze the effects of a fiscal policy to realize full-employment. In Section 3 we present some graphical simulations based on plausible assumptions of variables.

Let g be the growth rate of the full-employment real GDP, ρ be the extra growth rate of real GDP over g by a fiscal policy (the growth rate of real GDP is $g + \rho$) in a state of under-employment, and γ be the extra growth rate of the government expenditure over g by a fiscal policy (the growth rate of the government expenditure is $g + \gamma$). The main results are as follows.

1. The larger the value of ρ is, the faster the full-employment state is realized. (Figure 1)

2. The larger the value of ρ is, the smaller the debt-to-GDP ratio at the time when full-employment is realized, that is, the more aggressive the fiscal policy is, the smaller the debt-to-GDP ratio at the time when full-employment is realized is. (Figure 3)

    The reason for this result is as follows. The smaller the value of ρ is, the longer the time we need to realize full-employment is. On the other hand, as shown in 5 below (Proposition 1), the share of the government expenditure in real GDP at the time when full-employment is realized does not depend on ρ. Therefore, when ρ is small, the accumulated budget deficit including burden of interest is large.

3. When the value of ρ is larger than the critical value, the fiscal policy to realize full-employment reduces the debt-to-GDP ratio. (Figure 4)

4. By a fiscal policy, first the debt-to-GDP ratio increases, and then it decreases. (Figure 5 and 6)

[1]In another paper we have presented a mathematical analysis and simulations of fiscal policy for full-employment using a discrete time version of debt dynamics.

5. The share of the government expenditure in real GDP at the time when full-employment is realized does not depend on the values of ρ and γ. (Proposition 1)

6. Even if the marginal propensity to consume is very small, an aggressive fiscal policy can realize full-employment without increasing debt-to-GDP ratio (Subsection 3.10).

The main conclusion of this paper is that full-employment can be realized by an aggressive fiscal policy with smaller debt-to-GDP ratio than before the fiscal policy.

An increase in the government expenditure may induce a rise of the interest rate. Since the higher the interest rate is, the larger the debt-to-GDP ratio is (Subsection 3.9), we need an appropriate monetary policy which maintains the low interest rate.

2 Continuous time debt dynamics

We consider a continuous time version of debt dynamics. The variables are as follows.

c: marginal propensity to consume (including marginal propensity to import), $0 < c < 1$,
τ: tax rate, $0 < \tau < 1$,
$\beta = 1 - c(1-\tau),\ 0 < \beta < 1$,
$Y(0)$: real GDP at time 0,
$Y(t)$: real GDP at time t, $t \geq 0$,
$Y_m(0)$: full-employment real GDP at time 0,
$Y_m(t)$: full-employment real GDP at time t, $t \geq 0$,
$\zeta = \frac{Y_m(0)}{Y(0)}$, $\zeta > 1$,
$\tilde{t}$: the time at which full-employment is realized, $\tilde{t} > 0$,
$G(0)$: government expenditure at time 0,
$G(t)$: government expenditure at time t,
$T(0)$: tax revenue at time 0,
$T(t)$: tax revenue at time t,
$\alpha = \frac{G(0)}{Y(0)}$,
$B(0)$: government budget surplus at time 0,
$B(t)$: government budget surplus at time t,
$b(0) = \frac{B(0)}{Y(0)}$,
$b(t) = \frac{B(t)}{Y(t)}$,
$D(0)$: government debt at time 0,
$D(t)$: government debt at time t,
$d(0) = \frac{D(0)}{Y(0)}$,
$d(t) = \frac{D(t)}{Y(t)}$,
d^*: the steady state value of $d(t)$,
g: the growth rate of the full-employment real GDP, $g > 0$

ρ: the extra growth rate of real GDP by a fiscal policy, $\rho > 0$
γ: the extra growth rate of the government expenditure by a fiscal policy, $\gamma > 0$
r: interest rate.

The unit of time is a year. We assume $g + \rho > r$.

2.1 A steady state

First we examine a steady state of debt dynamics. At the steady state

$$Y(t) = e^{gt}Y(0),\ G(t) = e^{gt}G(0),\ T(t) = e^{gt}T(0).$$

Thus,

$$B(t) = T(t) - G(t) = e^{gt}B(0).$$

The derivative of $D(t)$ with respect to t is

$$D'(t) = rD(t) - B(t).$$

$D(t)$ is calculated as

$$\begin{aligned}
D(t) =& e^{rt}D(0) - \int_0^t e^{r(t-s)}B(s)ds = e^{rt}D(0) - \int_0^t e^{r(t-s)}e^{gs}B(0)ds \\
=& e^{rt}D(0) - e^{rt}B(0)\int_0^t e^{(g-r)s}ds = e^{rt}D(0) - e^{rt}B(0)\left[\frac{e^{(g-r)s}}{g-r}\right]_0^t \\
=& e^{rt}D(0) - e^{rt}B(0)\frac{e^{(g-r)t} - 1}{g-r}.
\end{aligned}$$

Since $Y(t) = e^{gt}Y(0)$,

$$\frac{D(t)}{Y(t)} = e^{(r-g)t}\frac{D(0)}{Y(0)} - e^{(r-g)t}\frac{B(0)}{Y(0)}\frac{e^{(g-r)t} - 1}{g-r}.$$

Therefore, the debt-to-GDP ratio at time t is obtained as follows.

$$d(t) = e^{(r-g)t}d(0) - e^{(r-g)t}b(0)\frac{e^{(g-r)t} - 1}{g-r}.$$

At the steady state

$$d(t) = d(0) = d^*.$$

Then,

$$d^* = \frac{0}{1 - e^{(r-g)t}}\left[b(0)\frac{1 - e^{(r-g)t}}{r-g}\right] = \frac{b(0)}{r-g}. \tag{1}$$

2.2 Fiscal policy for full-employment

We assume that there exists a deflationary GDP gap, that is, $Y(0)$ is smaller than the full-employment real GDP, $Y_m(0)$, at time 0. Then, $\zeta > 1$. Since $Y_m(t)$ increases at the rate g,

$$Y_m(t) = e^{gt} Y_m(0).$$

The government increases the growth rate of its expenditure from g to $g + \gamma$ to increase the growth rate of GDP from g to $g + \rho$ so as to realize full-employment. Then,

$$Y(t) = e^{(g+\rho)t} Y(0).$$

Suppose that at time $\tilde{t}$

$$e^{(g+\rho)\tilde{t}} Y(0) = e^{g\tilde{t}} Y_m(0),$$

that is, full-employment is realized at $\tilde{t}$. Then, we have

$$e^{\rho\tilde{t}} = \zeta.$$

$\tilde{t}$ is obtained as follows.

$$\tilde{t} = \frac{\ln \zeta}{\rho}. \tag{2}$$

The larger the value of ρ is, the faster the full-employment state is realized.

Since $G(t)$ increases at the rate $g + \gamma$,

$$G(t) = e^{(g+\gamma)t} G(0).$$

We examine the relation between ρ and γ. The increase in real GDP over the ordinary growth is brought by the *multiplier effect* of an increase in the government expenditure over the ordinary growth. Therefore, we have the following relation

$$\frac{1}{\beta}\left[e^{(g+\gamma)\tilde{t}} - e^{g\tilde{t}}\right] G(0) = \left[e^{(g+\rho)\tilde{t}} - e^{g\tilde{t}}\right] Y(0).$$

This means

$$\frac{1}{\beta}\left(e^{\gamma\tilde{t}} - 1\right) G(0) = \left(e^{\rho\tilde{t}} - 1\right) Y(0).$$

And so

$$\frac{\alpha}{\beta}\left(e^{\gamma\tilde{t}} - 1\right) = e^{\rho\tilde{t}} - 1,$$

or

$$e^{\gamma\tilde{t}} = \frac{\beta}{\alpha}\left(e^{\rho\tilde{t}} - 1\right) + 1.$$

Since $\zeta = e^{\rho\tilde{t}}$,

$$e^{\gamma\tilde{t}} = \frac{\beta}{\alpha}(\zeta - 1) + 1.$$

Thus,

$$\gamma \frac{\ln \zeta}{\rho} = \ln \left[\frac{\beta}{\alpha}(\zeta - 1) + 1 \right].$$

This means

$$\gamma = \frac{\rho \ln \left[\frac{\beta}{\alpha}(\zeta - 1) + 1 \right]}{\ln \zeta}. \tag{3}$$

$B(t)$ is the sum of the budget surplus growing by g from $B(0)$ and the budget surplus brought by the fiscal policy. It is written as

$$B(t) = e^{gt} B(0) + \tau \left(e^{(g+\rho)t} - e^{gt} \right) Y(0) - \left(e^{(g+\gamma)t} - e^{gt} \right) G(0).$$

The derivative of $D(t)$ with respect to t is

$$D'(t) = rD(t) - B(t) = rD(t) - e^{gt} B(0) - \tau \left(e^{(g+\rho)t} - e^{gt} \right) Y(0) + \left(e^{(g+\gamma)t} - e^{gt} \right) \alpha Y(0).$$

Therefore,

$$\begin{aligned} D(t) =& e^{rt} D(0) - B(0) \int_0^t e^{(t-s)r} e^{gs} ds - \tau Y(0) \int_0^t e^{(t-s)r} \left(e^{(g+\rho)s} - e^{gs} \right) ds \\ &+ \alpha Y(0) \int_0^t e^{(t-s)r} \left(e^{(g+\gamma)s} - e^{gs} \right) ds \\ =& e^{rt} D(0) - e^{rt} B(0) \int_0^t e^{(g-r)s} ds - e^{rt} \tau Y(0) \int_0^t \left(e^{(g+\rho-r)s} - e^{(g-r)s} \right) ds \\ &+ e^{rt} \alpha Y(0) \int_0^t \left(e^{(g+\gamma-r)s} - e^{(g-r)s} \right) ds. \end{aligned}$$

Since

$$Y(t) = e^{(g+\rho)t} Y(0),$$

we get

$$\begin{aligned} d(t) =& e^{(r-g-\rho)t} d(0) - e^{(r-g-\rho)t} b(0) \int_0^t e^{(g-r)s} ds \\ &- e^{(r-g-\rho)t} \tau \int_0^t \left(e^{(g+\rho-r)s} - e^{(g-r)s} \right) ds + e^{(r-g-\rho)t} \alpha \int_0^t \left(e^{(g+\gamma-r)s} - e^{(g-r)s} \right) ds \\ =& e^{(r-g-\rho)t} d(0) - e^{(r-g-\rho)t} b(0) \left[\frac{e^{(g-r)s}}{g-r} \right]_0^t - e^{(r-g-\rho)t} \tau \left[\frac{e^{(g+\rho-r)s}}{g+\rho-r} - \frac{e^{(g-r)s}}{g-r} \right]_0^t \\ &+ e^{(r-g-\rho)t} \alpha \left[\frac{e^{(g+\gamma-r)s}}{g+\gamma-r} - \frac{e^{(g-r)s}}{g-r} \right]_0^t \\ =& e^{(r-g-\rho)t} d(0) - e^{(r-g-\rho)t} b(0) \left[\frac{e^{(g-r)t} - 1}{g-r} \right] - e^{(r-g-\rho)t} \tau \left[\frac{e^{(g+\rho-r)t} - 1}{g+\rho-r} - \frac{e^{(g-r)t} - 1}{g-r} \right] \\ &+ e^{(r-g-\rho)t} \alpha \left[\frac{e^{(g+\gamma-r)t} - 1}{g+\gamma-r} - \frac{e^{(g-r)t} - 1}{g-r} \right]. \end{aligned}$$

Thus,

$$
\begin{aligned}
d(t) =& e^{(r-g-\rho)t}d(0) - b(0)\left[\frac{e^{-\rho t} - e^{(r-g-\rho)t}}{g-r}\right] \qquad (4)\\
&- \tau\left[\frac{1-e^{(r-g-\rho)t}}{g+\rho-r} - \frac{e^{-\rho t} - e^{(r-g-\rho)t}}{g-r}\right] + \alpha\left[\frac{e^{(\gamma-\rho)t} - e^{(r-g-\rho)t}}{g+\gamma-r} - \frac{e^{-\rho t} - e^{(r-g-\rho)t}}{g-r}\right].
\end{aligned}
$$

Let $t = \tilde{t}$. Then,

$$
\begin{aligned}
d(\tilde{t}) =& e^{-\rho\tilde{t}}e^{(r-g)\tilde{t}}d(0) - e^{-\rho\tilde{t}}b(0)\left[\frac{1-e^{(r-g)\tilde{t}}}{g-r}\right] \qquad (5)\\
&- e^{-\rho\tilde{t}}\tau\left[\frac{e^{\rho\tilde{t}} - e^{(r-g)\tilde{t}}}{g+\rho-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right] + e^{-\rho\tilde{t}}\alpha\left[\frac{e^{\gamma\tilde{t}} - e^{(r-g)\tilde{t}}}{g+\gamma-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right]\\
=& \frac{1}{\zeta}\left\{e^{(r-g)\tilde{t}}d(0) - b(0)\left[\frac{1-e^{(r-g)\tilde{t}}}{g-r}\right] - \tau\left[\frac{\zeta - e^{(r-g)\tilde{t}}}{g+\rho-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right]\right.\\
&\left.+\alpha\left[\frac{e^{\gamma\tilde{t}} - e^{(r-g)\tilde{t}}}{g+\gamma-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right]\right\}.
\end{aligned}
$$

From (5),

$$
\begin{aligned}
d(\tilde{t}) - d(0) =& \frac{1}{\zeta}\left\{\left[e^{(r-g)\tilde{t}} - \zeta\right]d(0) - b(0)\left[\frac{1-e^{(r-g)\tilde{t}}}{g-r}\right]\right. \qquad (6)\\
&\left.-\tau\left[\frac{\zeta - e^{(r-g)\tilde{t}}}{g+\rho-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right] + \alpha\left[\frac{e^{\gamma\tilde{t}} - e^{(r-g)\tilde{t}}}{g+\gamma-r} - \frac{1-e^{(r-g)\tilde{t}}}{g-r}\right]\right\}.
\end{aligned}
$$

Because $e^{(r-g)\tilde{t}} - \zeta = e^{(r-g)\tilde{t}} - e^{\rho\tilde{t}} < 0$ by $g + \rho > r$ or $r - g < \rho$, (6) is decreasing with respect to $d(0)$. γ is obtained from (3), and $\tilde{t}$ is obtained from (2).

$\alpha = \frac{G(0)}{Y(0)}$ is the share of the government expenditure in real GDP at time 0. GDP grows at the rate $g+\rho$, on the other hand the government expenditure grows at the rate $g+\gamma$, and $\gamma > \rho$. The larger the values of ρ and γ are, the smaller the time necessary for realization of full-employment is. The value of α at $\tilde{t}$ is denoted by

$$
\alpha(\tilde{t}) = \frac{G(\tilde{t})}{Y(\tilde{t})} = \frac{e^{(g+\gamma)\tilde{t}}}{e^{(g+\rho)\tilde{t}}}\alpha = e^{(\gamma-\rho)\tilde{t}}\alpha.
$$

From (2) and (3), we get

$$
\alpha(\tilde{t}) = e^{\left(\frac{\ln\left[\frac{\beta}{\alpha}(\zeta-1)+1\right]}{\ln\zeta}-1\right)\rho\frac{\ln\zeta}{\rho}}\alpha = \frac{\frac{\beta}{\alpha}(\zeta-1)+1}{\zeta}\alpha.
$$

This is constant, that is, it does not depend on ρ and γ. We have shown the following result.

Proposition 1. *The share of the government expenditure in real GDP at the time when full-employment is realized does not depend on the values of ρ and γ.*

3 Graphical simulations

We present some simulation results. Assume the following values for the variables.

$c = 0.5$, $\tau = 0.25$, $\alpha = 0.3$, $g = 0.025$, $r = 0.015$, $b(0) = -0.015$ and $\zeta = 1.15$.

We assume that g and r are constant, and $g > r$[2]. However, in Subsection 3.9 we examine a case where $r > g$. We do not assume that $d(0)$ and $b(0)$ have steady state values described in (1). But, in Subsection 3.11 we consider a case where $d(0)$ and $b(0)$ have steady state values.

3.1 Relation between ρ and $\tilde{t}$

In addition to the above assumptions we assume $d(0) = 0.45$. Figure 1 represents the relation between ρ and $\tilde{t}$. As (2) suggests, the larger the value of ρ is, the smaller the value of $\tilde{t}$ is, that is, the faster the full-employment state is realized. Therefore, the more aggressive the fiscal policy is, the faster full-employment is realized. For example, when $\rho = 0.05$, $\tilde{t} \approx 2.7$, when $\rho = 0.1$, $\tilde{t} \approx 1.4$.

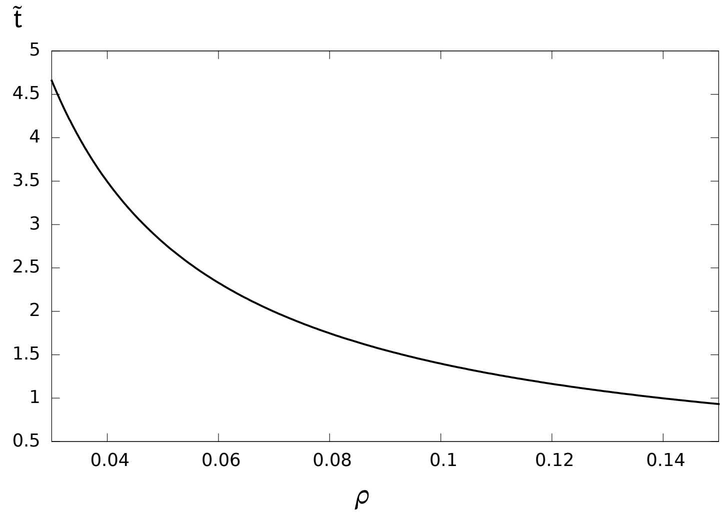

Figure 1: The relation between ρ and $\tilde{t}$

3.2 Relation between ρ and γ

Again we assume $d(0) = 0.45$. Figure 2 represents the relation between the value of ρ and the value of γ according to (3). The larger the value of ρ is, the larger the value of γ is. For

[2]In Mitchell et al. (2019) (pp. 357-358) it is stated that when $g > r$, there exists a stable steady state value of the debt-to-GDP ratio. Also see Wray (2016).

example, when $\rho = 0.05$, $\gamma \approx 0.1$, when $\rho = 0.1$, $\gamma \approx 0.19$.

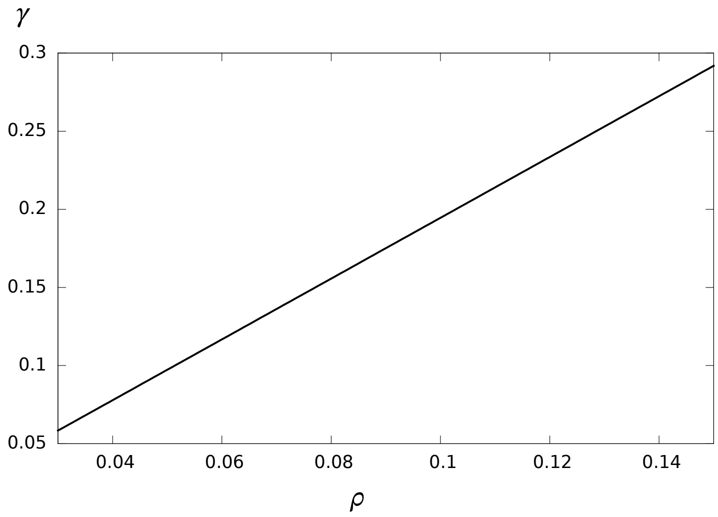

Figure 2: The relation between ρ and γ

3.3 Relation between ρ and $d(\tilde{t})$

We assume $d(0) = 0.45$. Figure 3 represents the relation between ρ and $d(\tilde{t})$ according to (5). The larger the value of ρ is, the smaller the value of $d(\tilde{t})$ is, that is, the smaller the debt-to-GDP ratio at the time when full-employment is realized.

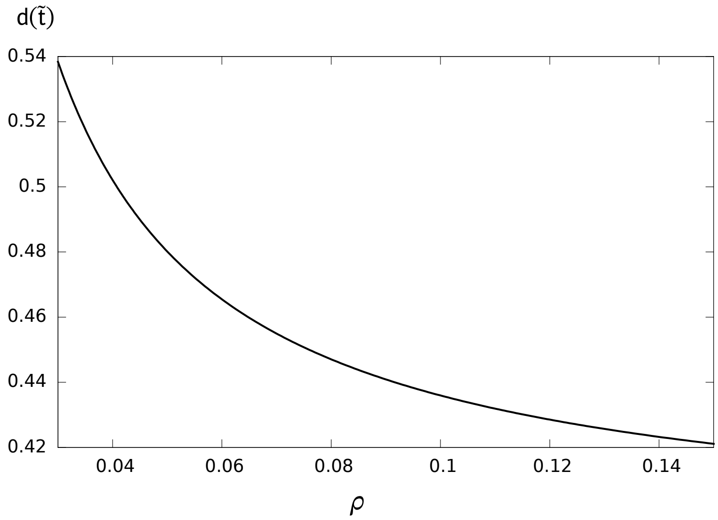

Figure 3: The relation between ρ and $d(\tilde{t})$

3.4 Relation between ρ and $d(\tilde{t}) - d(0)$

We assume $d(0) = 0.45$. Figure 4 represents the relation between ρ and $d(\tilde{t}) - d(0)$, which is the difference between the debt-to-GDP ratio at $\tilde{t}$ and that at $t = 0$, according to (6). The larger the value of ρ is, the smaller the value of $d(\tilde{t}) - d(0)$ is. If ρ is larger than about 0.072, the debt-to-GDP ratio at $t = \tilde{t}$ is smaller than that at $t = 0$, that is, the aggressive fiscal policy to realize full-employment reduces the debt-to-GDP ratio.

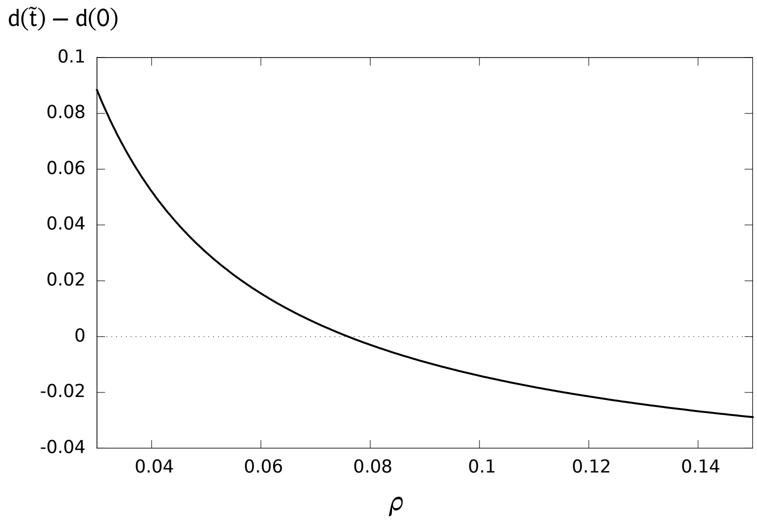

Figure 4: The relation between ρ and $d(\tilde{t}) - d(0)$

3.5 Relation between t and $d(t)$

We assume $d(0) = 0.45$ and $\rho = 0.085$. Figure 5 represents the relation between the time (t) and the value of $d(t)$ according to (4). First $d(t)$ increases, then it decreases.

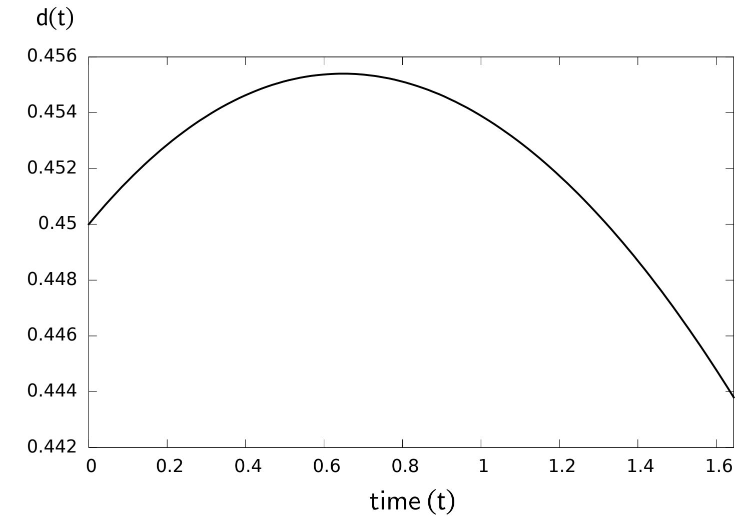

Figure 5: The relation between the time and $d(t)$

3.6 Relation between t and $d(t) - d(0)$

Again we assume $d(0) = 0.45$ and $\rho = 0.085$. Figure 6 represents the relation between the time (t) and the value of $d(t) - d(0)$. First $d(t) - d(0)$ increases, then it decreases.

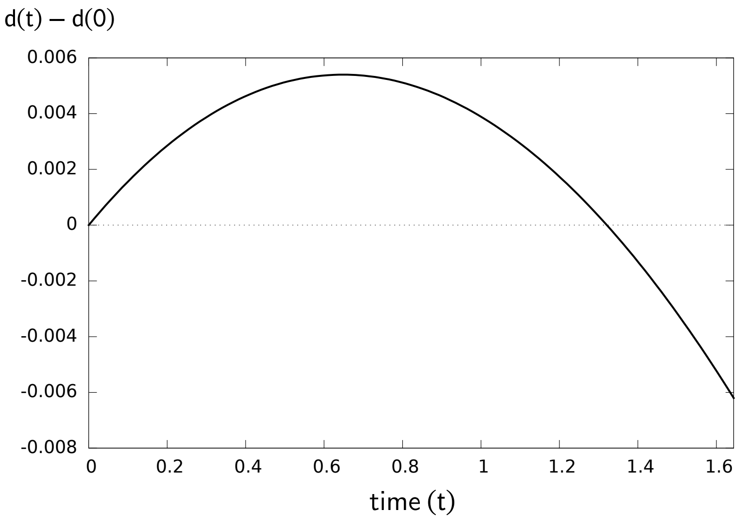

Figure 6: The relation between the time and $d(t) - d(0)$

3.7 Relation between $d(0)$ and $d(\tilde{t})$

We assume $\rho = 0.085$. Figure 7 represents the relation between the value of $d(0)$ and the value of $d(\tilde{t})$ according to (5). By (5) it is a straight line whose slope is smaller then one.

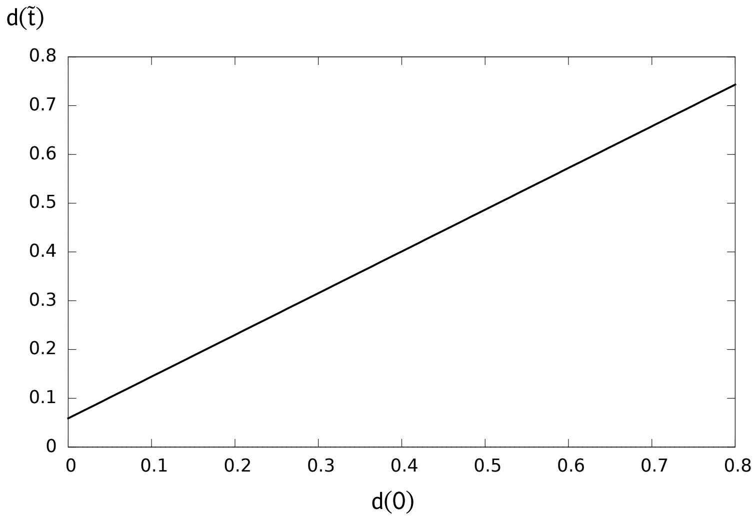

Figure 7: The relation between $d(0)$ and $d(\tilde{t})$

3.8 Relation between $d(0)$ and $d(\tilde{t}) - d(0)$

Again we assume $\rho = 0.085$. Figure 8 represents the relation between the value of $d(0)$ and the value of $d(\tilde{t}) - d(0)$ according to (6). By (6), since $e^{(r-g)\tilde{t}} < \zeta (= e^{\rho\tilde{t}})$, it is a straight line whose slope is negative.

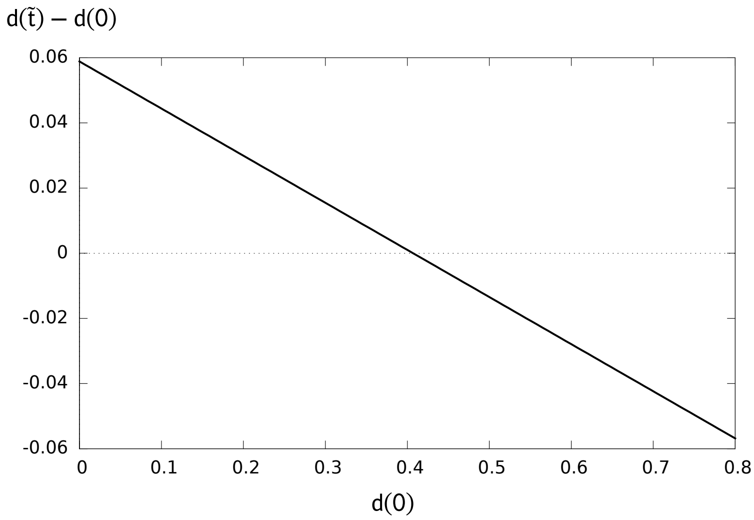

Figure 8: The relation between $d(0)$ and $d(\tilde{t}) - d(0)$

3.9 Relation between ρ and $d(\tilde{t}) - d(0)$ with low and high interest rates

We assume $r = 0.035$. The values of other variables are the same as those in the previous cases. In Figure 9 we compare the relation between ρ and $d(\tilde{t}) - d(0)$ in the case of low interest rate and that in the case of high interest rate.

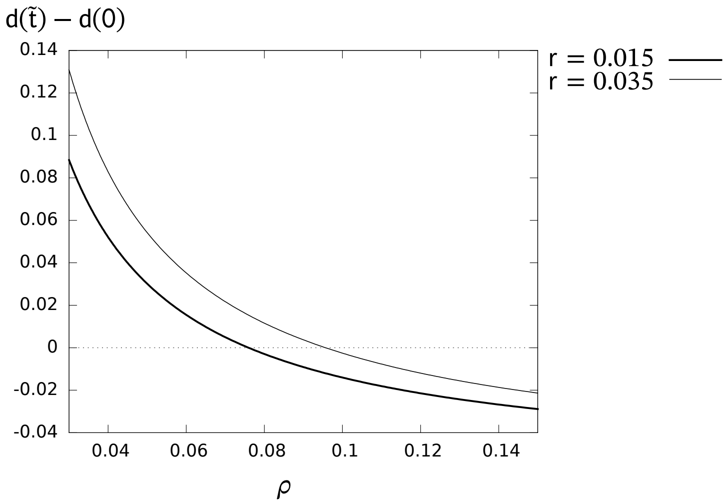

Figure 9: The relation between ρ and $d(\tilde{t}) - d(0)$ with low and high interest rates

With higher interest rate the debt-to-GDP ratio at the time when full-employment is realized is less likely smaller than that at time 0 than the case with low interest rate.

3.10 Relation between ρ and $d(\tilde{t}) - d(0)$ with very small marginal propensity to consume

We assume $c = 0.01$. The values of other variables are the same as those in the previous cases. In Figure 10 we compare the relation between ρ and $d(\tilde{t}) - d(0)$ in this case and that when $c = 0.5$.

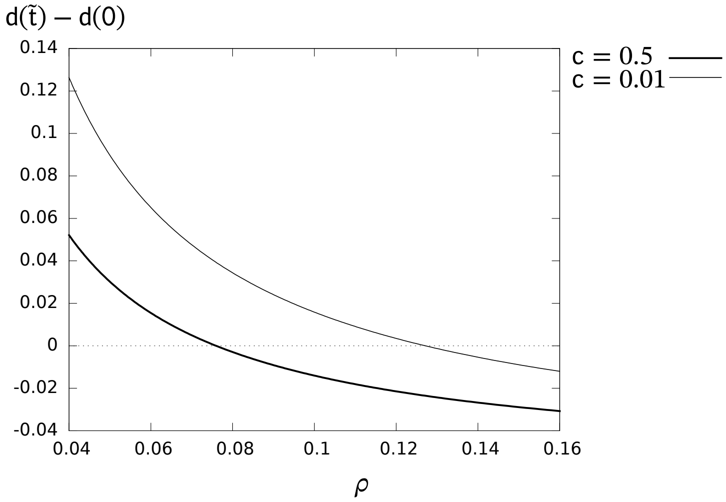

Figure 10: The relation between ρ and $d(\tilde{t}) - d(0)$ in the case where $c = 0.01$ and the case where $c = 0.5$

Even if marginal propensity to consume is very small, an aggressive fiscal policy can reduce the debt-to-GDP ratio at the time when full-employment is realized.

3.11 Relation between ρ and $d(\tilde{t}) - d(0)$ when $d(0)$ and $b(0)$ have the steady state values

We assume $b(0) = (r - g)d(0)$. The values of other variables are the same as those in the previous cases. In Figure 11 we compare the relation between ρ and $d(\tilde{t}) - d(0)$ in this case and that $b(0) = -0.015$.

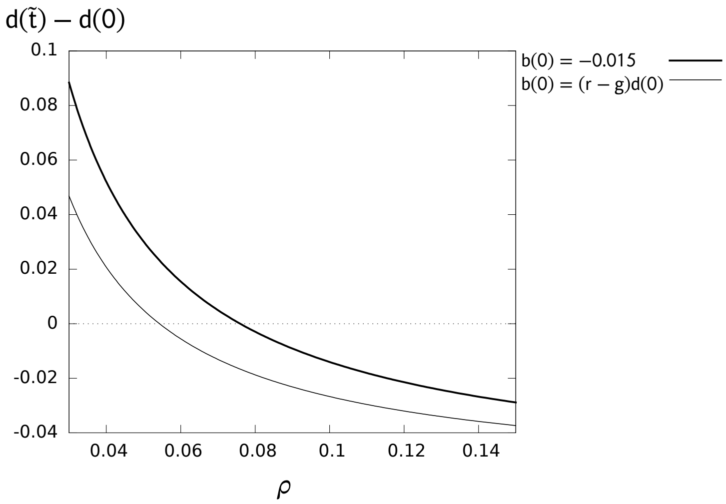

Figure 11: The relation between ρ and $d(\tilde{t}) - d(0)$ in the case where $b(0) = (r - g)d(0)$ and the case where $b(0) = -0.015$

If $d(0)$ and $b(0)$ have the steady state values, the debt-to-GDP ratio at the time when full-employment is realized is more likely smaller than that at period 0 than the case where $b(0) = -0.015$.

4 Concluding Remark

We have presented a mathematical analysis and simulations of a fiscal policy which realizes full-employment from an under-employment state without increasing the debt-to-GDP ratio than before the fiscal policy. Even if the marginal propensity to consume is small, by an appropriate fiscal policy we can realize full-employment without increasing the debt-to-GDP ratio.

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References

W. Mitchell, L. R. Wray, and M. Watts. *Macroeconomics*. Red Gbole Press, 2019.

M. Watts and T. Sharpe. The immutable laws of debt dynamics. *Journal of Post Keynsian Economics*, 36:59–84, 2016.

L. R. Wray. *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems, 2nd ed.* Palgrave Macmillan, 2016.