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THE OPTIMAL CONTROL PROBLEM FOR OUTPUT MATERIAL FLOW ON CONVEYOR BELT WITH INPUT ACCUMULATING BUNKER

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The article is devoted to the synthesis of optimal control of conveyor belt with the accumulating input bunker. Much attention is given to the model of the conveyor belt with a constant speed of the belt. Simulation of the conveyor belt is carried out in the one-moment approximation using partial differential equations. The conveyor belt is represented as a distributed system. The used PDE-model of the conveyor belt allows to determine the state of the flow parameters for a given technological position as a function of time. We consider the optimal control problem for flow parameters of the conveyor belt. The problem consists in ensuring the minimum deviation of the output material flow from a given target amount. The control is carried out by the material flow amount, which comes from the accumulating bunker into the conveyor belt input. In the synthesis of optimal control, we take into account the limitations on the size of the accumulating bunker, as well as on both max and min amounts of control. We construct optimal control of the material flow amount coming from the accumulating bunker. Also, we determine the conditions to switch control modes, and estimate time period between the moments of the switching.

Keywords: conveyor; production line; subject of labour; PDE-model of production; parameters of the state of the production line; technological position; transition period; production control systems; optimal control; Pontryagin function; Lagrange function; differential constraints; accumulating bunker; distributed system.

Introduction

There are two fundamentally different methods to control the output flow of the conveyor belt. The first method is to regulate the conveyor belt speed [1–5]. The second method is to use the accumulating bunker at the conveyor input [6–8]. Output flow control is performed with a certain delay by changing the amount of material at the conveyor input. As a rule, the second method is carried out for a constant speed of the belt. The method to regulate the conveyor belt speed is used to reduce consumption of the energy [9,10]. This is due to the fact that in most cases the conveyor systems function in modes significantly different from the normative ones. Time-varying flow amount at the conveyor input has significant influence on uneven load of the belt along the transport route in the case of unregulated drive conveyor [11]. Regulation of the belt speed gives the transport system an ability to function in the normative mode such that the electricity consumption for transporting the rock of a unit mass is minimum. According to DIN 22101 (Germany) [10], the energy consumption for the belt conveyor is expected to be reduced. At the same time, the potential risks of failure of the conveyor belt elements are significantly increased. Indeed, the frequent transition from one mode of the belt speed to another [12] leads to significant financial costs. In transition modes, a change in the speed of the conveyor belt leads to belt tension, which is the main reason for the belt breaking in the splicing region [12]. In order to design a transport system, it is necessary to take into account other risks that arise as a result of the functioning of the conveyor belt in a transition mode: slip of the belt around the drive pulley, leakage of material away from the belt, engine overheating. Along with the potential risks of destroying the transport system, an important problem is the dynamic analysis of the transport systems both with a mode to regulate the belt speed and without such a mode [1]. The dynamic analysis is difficult, since a conveyor with a rock moving along the transport route is a distributed system with a number of limitations. The most important limitations are the maximum specific linear load of the conveyor belt and the maximum amount of the transported mass [13]. The conveyor system is statistically uncertain. Statistical uncertainty consists in the uncertainty of the value of material flow to the input of the conveyor (uncertainty of the boundary conditions) which requires using the probabilistic methods for calculating the conveyor line [14]. We focus on the construction of an optimal control of the material flow of the main conveyor belt equipped with an input accumulating bunker. In the transport system that moves a rock to the port terminal, the material flow at the conveyor output should vary depending on the loading capacities of vessels, as well as the schedule of dry cargo loading. This is achieved due to the fact that the material flow enters the accumulating input bunker. Control of the flow amount that enters the conveyor belt input allows to form the material flow required at the transport system output. In order to construct optimal control of the material flow on the main conveyor belt, we assume that the conveyor belt speed is constant. There is no ability to regulate the conveyor belt speed. The output material flow can be provided by the presence of an accumulating bunker and a system to control the material flow that enters the transport system input. A spiral belt conveyor can be used as an accumulating input equipment. The use of such accumulating types of equipment is justified for the organization of technological routes that require simultaneous accumulation and movement of products in the production process with vertical and horizontal directions. The material flow from the accumulating device to the input of the main conveyor is carried out by adjusting the belt speed in the spiral conveyor.

1. **Problem Statement**

We construct a distributed model of the main conveyor belt and determine optimal control of the material flow on the main conveyor belt equipped with an accumulating bunker. To this end, we consider the following individual problems.

1. Construct a model of a distributed transport system with an input accumulating bunker.
2. Construct a program of optimal control of flow at the conveyor belt input with an input accumulating bunker.
3. Determine the optimal value of the capacity of an input accumulating bunker and the dependence of the optimal capacity value on the length of the transport system.
4. Calculate the duration of the transition period during which the conveyor belt is --------- filled with rock along the entire transportation route. Determine the delay time, which is fixed by the time interval between the time of arrival of the element on the conveyor belt input and the time of its exit from the conveyor belt output.

2. **Model of Conveyor**

Conveyor system is a type of production system with flow method of production organization. A distinctive feature of conveyor systems is that the elements move along the transport route with the same speed equal to the conveyor belt speed. The model of the production line in one-moment approximation can be represented as follows [15, p.67], [16, p.936]:

$$\frac{\partial [\chi]_0 (\mathrm{t}, \mathrm{S})}{\partial t} + \frac{\partial [\chi]_1 (\mathrm{t}, \mathrm{S})}{\partial S} = 0, \quad (1)$$

$$[\chi]_1 (\mathrm{t},\mathrm{S}) = [\chi]_{1\psi} (\mathrm{t},\mathrm{S}) \quad (2)$$

with the initial condition

$$[\chi]_0 (0,\mathrm{S}) = \Psi(\mathrm{S}), \quad (3)$$

and the boundary conditions at the input of the production line

$$[\chi]_1 (\mathrm{t},0) = \lambda(\mathrm{t}), \quad (4)$$

where S_d is a coordinate of the technological position for the final operation; $[\chi]_0 (\mathrm{t},\mathrm{S})$,$[\chi]_1 (\mathrm{t},\mathrm{S})$ are a distribution density and a tempo of processing of labor subjects at the time t at the technological position, characterized by the coordinate $S \in (0;S_d)$; $\Psi(\mathrm{S})$ is an initial distribution of labor subjects along the technological route; $[\chi]_{1\psi} (\mathrm{t},\mathrm{S})$ is a given normative tempo of processing labor subjects at technological positions as defined in the technological production documentation; $\lambda(\mathrm{t})$ is a tempo of entry of labor subjects into the input of the production line. Conveyor is a type of production line. The main feature of a conveyor simulation for an industrial enterprise is that labor subjects move at the same speed along the conveyor belt. Therefore, we can write system of equations (1) – (4) in the following form [5]:

$$\frac{\partial [\chi]_0 (\mathrm{t}, \mathrm{S})}{\partial t} + \frac{\partial [\chi]_1 (\mathrm{t}, \mathrm{S})}{\partial S} = \delta(\mathrm{S}) \lambda(\mathrm{t}), \quad (5)$$

$$[\chi]_1 (\mathrm{t},\mathrm{S}) = a(t)[\chi]_0 (\mathrm{t},\mathrm{S}), \quad (6)$$

$$[\chi]_0 (0, \mathrm{S}) = \mathrm{H}(\mathrm{S}) \Psi(\mathrm{S}), \quad \int_{-\infty}^{\infty} \delta(S) dS = 1, \quad \mathrm{H}(\mathrm{S}) = \begin{cases} 0, & \text{if } S < 0; \\ 1, & \text{if } S \geqslant 0. \end{cases} \quad (7)$$

The flow parameters $[\chi]_0 (\mathrm{t},S)$ and $[\chi]_1 (\mathrm{t},S)$ are related to each other by the factor ($a = a(t) \frac{meter}{hour}$, which determines the conveyor belt speed. The right-hand side of equation (5), i.e. $\delta(\mathrm{S})\lambda(\mathrm{t})$, takes into account the source of material entered for the first technological operation ($S = 0$), and $\delta(\mathrm{S})$ is the delta function. The intensity of the rock receipt to the conveyor belt is represented by the function $\lambda(\mathrm{t}) \frac{ton}{hour}$ characterizing the line power. At the initial time $t = 0$ ($hour$) the material is distributed along the conveyor belt with a linear density $[\chi]_0 (0, \mathrm{S}) \frac{ton}{meter}$. The function $\delta(\mathrm{S})$ determines the point of the material receipt to the conveyor belt: $S = 0$. System of equations (5), (6) is closed with respect to the flow parameters $[\chi]_0 (\mathrm{t},S)$ and $[\chi]_1 (\mathrm{t},S)$. Condition (6) reflects the functioning of the conveyor belt having condition (2) in system of equations (1) – (4). Note that condition (2) for the simulation of production lines is approximated in the one-moment description [17]. Precision of the approximation is determined by the number N_m of labor subjects that are

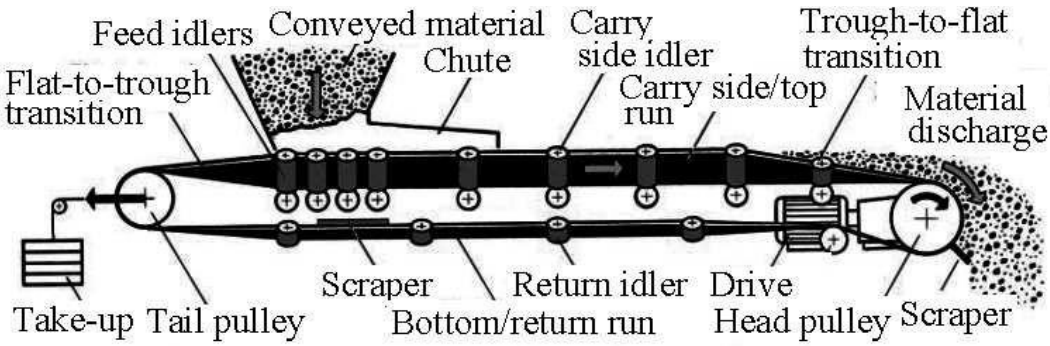

Fig. 1. Schematic diagram of the conveyor belt [12]

in inter-operational reserves before each of m technological operations [18]. For $N_m \to \infty$ in an approximate equality, equation (2) becomes an exact equality. Therefore, condition (6) allows to construct an exact solution to system of equations (5) – (7) with respect to the flow parameters $[\chi]_0(\mathrm{t},S)$ and $[\chi]_1(\mathrm{t},S)$. Let us divide the technological route with a length S_d into M sections with lengths $\Delta S_m = S_m - S_{m-1}$, $S_0 = 0$, and integrate equation (5) within a section of length ΔS_m:

$$\int\limits_{S_{m-1}}^{S_m} \frac{\partial [\chi]_0(\mathrm{t,S})}{\partial t} dS + \int\limits_{S_{m-1}}^{S_m} \frac{\partial [\chi]_1(\mathrm{t,S})}{\partial S} dS = \int\limits_{S_{m-1}}^{S_m} \delta(\mathrm{S})\,\lambda(\mathrm{t})\, dS = \begin{cases} \lambda(\mathrm{t}), & \text{if } S_{m-1} = 0; \\ 0, & \text{if } S_{m-1} \neq 0. \end{cases} \quad (8)$$

Since

$$\int\limits_{S_{m-1}}^{S_m} \frac{\partial [\chi]_0(\mathrm{t,S})}{\partial t} dS = \frac{\partial}{\partial t} \int\limits_{S_{m-1}}^{S_m} [\chi]_0(\mathrm{t,S})\, dS = \frac{dN_m}{dt}, \quad \int\limits_{S_{m-1}}^{S_m} [\chi]_0(\mathrm{t,S})\, dS = N_m, \quad (9)$$

$$\int\limits_{S_{m-1}}^{S_m} \frac{\partial [\chi]_1(\mathrm{t,S})}{\partial S} dS = [\chi]_1(\mathrm{t,S_m}) - [\chi]_1(\mathrm{t,S_{m-1}}),$$

$$\int\limits_{S_{m-1}}^{S_m} \frac{\partial [\chi]_1(\mathrm{t,S})}{\partial S} dS = [\chi]_1(\mathrm{t,S_m}) - [\chi]_1(\mathrm{t,S_{m-1}}),$$

(10)

equation (8) can be represented in the following form:

$$\frac{dN_1}{dt} = \lambda(\mathrm{t}) - [\chi]_1(\mathrm{t,S_1}), \quad \frac{dN_m}{dt} = [\chi]_1(\mathrm{t,S_{m-1}}) - [\chi]_1(\mathrm{t,S_m}), \quad [\chi]_1(\mathrm{t},0) = 0. \quad (11)$$

The condition $[\chi]_1$ (t,0) = 0 means that, if there is no source of material receipt, i.e. $\lambda(t) = 0$, then the material flow at the conveyor belt input is zero. If the section ΔS_m corresponds to the m-th technological operation of the production line, then equations (10), (11) determine the state of the interoperational stocks before the m-th technological operation. System of equations (11) clearly demonstrates how the intensity $\lambda(t)$ of the source of supply of materials and its location affect the state of interoperational stocks along the technological route of the production line. A schematic diagram of the main conveyor belt with an accumulating bunker at the input is shown in Fig. 1 [12]. The flow of the material (for example, a rock) should enter the conveyor belt from the accumulating bunker with the intensity necessary to provide the required specified flow at the output. We supply system of equations (5) – (7) with the following equation simulating work process of the accumulating bunker:

$$\frac{dN_0(t)}{dt} = \lambda_{\rm b}(t) - \lambda(t), \quad N_0(0) = N_{0_{st}}, \quad 0 \leqslant N_0(t) \leqslant N_b, \quad 0 \leqslant \lambda \leqslant \lambda_{\max}, \quad (12)$$

where $N_0(t)$ is the current number of materials in the accumulating bunker with capacity N_b. The flow of materials at the accumulating bunker input $\lambda_{\rm b}(t)$ is known. Also, assume that the required flow $\sigma(t)$ is set at the output of the transport system. The required flow is determined by the shipping schedule of the rock to the consumer. Let us represent system of equations (5) – (7), (11) in the dimensionless form. In this case, the states of the conveyor parameters are described by dimensionless variables [5]:

$$\tau = \frac{t}{T_d}, \quad \xi = \frac{S}{S_d}, \quad \gamma(\tau) = \frac{\lambda(t)}{\Theta}\frac{T_d}{S_d}, \quad \gamma_{\rm b}(\tau) = \frac{\lambda_{\rm b}(t)}{\Theta}\frac{T_d}{S_d}, \quad (13)$$

$$\theta_0(\tau,\xi) = \frac{[\chi]_0(t,S)}{\Theta}, \quad \psi(\xi) = \frac{\Psi(S)}{\Theta}, \quad n_0(\tau) = \frac{N_0(t)}{S_d\Theta}, \quad \vartheta(\tau) = \sigma(t)\frac{T_d}{S_d\Theta}, \quad (14)$$

$$g(\tau) = \frac{a(t)T_d}{S_d}, \ \Theta = \max\left\{\Psi(S), \frac{\lambda(t)}{a(t)}\right\}, \ a(t) \neq 0, \ \delta(\xi) = S_d\,\delta(S), \ H(\xi) = H(S). \quad (15)$$

If the control program allows the conveyor to stop ($a(t) = 0$), then $\Theta = \max\{\Psi(S), [\chi]_{0\max}\}$, where $[\chi]_{0\max}$ is a maximum permissible running load per belt. Note that at the dimensionless value $n_0(\tau) = 1{,}0$, $\Theta = [\chi]_{0\max}$, the accumulating bunker contains amount of material that allow to fill the conveyor belt with the maximum permissible load $N_0(t) = S_d\Theta$ along the entire length of the belt. Taking into account the introduced notation, we write balance equation (13) – (14) in the dimensionless form [5]:

$$\frac{\partial\theta_0(\tau,\xi)}{\partial\tau} + g(\tau)\frac{\partial\theta_0(\tau,\xi)}{\partial\xi} = \delta(\xi)\,\gamma(\tau), \quad (16)$$

$$\theta_0(\tau_0,\xi) = H(\xi)\psi(\xi), \quad (17)$$

$$\frac{dn_0(t)}{dt} = \gamma_{\rm b}(\tau) - \gamma(\tau), \quad n_0(0) = n_{0_{st}}, \quad 0 \leqslant n_0(t) \leqslant n_b, \quad 0 \leqslant \gamma(\tau) \leqslant \gamma_{\max}. \quad (18)$$

The solution to system of equations (16) – (17) is as follows [5]:

$$\theta_0(\tau,\xi) = \left(H(\xi) - H\Big(\xi - G(\tau)\Big)\right)\frac{\gamma\big(G^{-1}(G(\tau)-\xi)\big)}{g\big(G^{-1}(G(\tau)-\xi)\big)} + H\Big(\xi - G(\tau)\Big)\psi\Big(\xi - G(\tau)\Big)$$

$$\int g(\tau)d\tau = G(\tau), \quad G^{-1}(G(\tau)) = \tau.$$

,

For the conveyor belt speed $g(\tau) = g_0$, we have $G(\tau) = g_0\tau$, therefore

$$\theta_0(\tau,\xi) = \left(\mathrm{H}(\xi) - H(-\xi + g_0\tau)\right)\frac{\gamma(\tau - \xi/g_0)}{g_0} + H(\xi - g_0\tau)\psi(\xi - g_0\tau) \quad . \tag{19}$$

Expression (19) determines the state of the density of the material $\theta_0(\tau,\xi)$ distribution along the transport route ξ at an arbitrary time τ. Let us consider the functioning of the transport system for time $\tau - \frac{1{,}0}{g_0} \geqslant 0$. Speed switching modes of the conveyor belt are not taken into account. Using equation (19), we reduce system of equations (16) – (18) to the form:

$$\theta_0(\tau,\xi) = \frac{\gamma(\tau - \xi/g_0)}{g_0}, \quad \frac{dn_0(t)}{dt} = \gamma_{\mathrm{b}}(\tau) - \gamma(\tau), \tag{20}$$

$$n_0(0) = n_{0st}, \quad 0 \leqslant n_0(t) \leqslant n_b, \quad 0 \leqslant \gamma(\tau) \leqslant \gamma_{\max} \tag{21}$$

In order to determine the linear density $\theta_0(\tau,\xi)$ at a time τ at an arbitrary point ξ of the route, it is necessary to know the value of the input material flow $\lambda(\mathrm{t})$ on the conveyor belt at the time $\tau_\xi = \tau - \xi/g_0$ that is fixed by the measuring-weighing equipment of the conveyor belt. The relationship between the values of the linear density $\theta_0(\tau,\xi)$ at arbitrary points of the transport route ξ_1 and ξ_2 at a constant speed of the conveyor belt was studied in detail in [19]. The flow of material at the input and output of the conveyor belt can be determined from (20)

$$\theta_1(\tau,1) = g_0\,\theta_0(\tau,1) = \gamma(\tau - 1/g_0), \quad \theta_1(\tau,0) = g_0\,\theta_0(\tau,0) = \gamma(\tau). \tag{22}$$

3. **Optimal Control Problem for Material Flow Coming from Accumulating Bunker**

We consider the conveyor as an object of the control whose motion is described by system of differential equations (16), (20):

$$\frac{\partial\theta_0(\tau,\xi)}{\partial\tau} + g(\tau)\frac{\partial\theta_0(\tau,\xi)}{\partial\xi} = \delta(\xi)\,u(\tau), \quad \tau \in [0,\tau_k] \tag{23}$$

$$\frac{dn_0(t)}{dt} = \gamma_{\mathrm{b}}(\tau) - u(\tau), \quad \gamma(\tau) = u(\tau). \tag{24}$$

Control $u(\tau)$ is carried out by regulating the intensity $\gamma(\tau)$ of the supply of materials from the accumulating bunker (Fig. 1). We select the control quality criterion from the condition of a minimum of the integral at the time interval $\tau \in [0,\tau_k]$

$$\int_0^{\tau_k} |\theta_1(\tau,1) - \vartheta(\tau)|d\tau = A + \int_{\tau_0}^{\tau_k} |u(\tau - \tau_0) - \vartheta(\tau)|d\tau =$$
$$= A + \int_0^{\tau_k - \tau_0} |u(\tau) - \vartheta(\tau + \tau_0)|d\tau \to \min. \tag{25}$$

Taking into account (22), we write equation (23) in the form

$$\theta_1(\tau,1) = u(\tau - \tau_0), \quad \tau_0 = 1/g_0 \tag{26}$$

and use the result in quality criterion (25)

$$A = \int_0^{\tau_0} |\gamma(\tau - \tau_0) - \vartheta(\tau)|d\tau = \int_{-\tau_0}^{0} |\gamma(\tau) - \vartheta(\tau + \tau_0)|d\tau = const.$$

The presence of the constant A in the quality criterion indicates that the output parameters of the conveyor are uncontrollable in the time interval $\tau \in [0,1/g_0]$. In the general form, the optimal control problem for the output flow of the conveyor belt can be formulated as follows [20,21]: determine the optimal control of the intensity of the material supply at the input of the conveyor belt from the accumulating bunker, that is a minimum of the integral

$$\int_0^{\tau_k-\tau_0} |u(\tau)-\vartheta(\tau+\tau_0)|d\tau \to \min, \quad \tau_0 = 1/g_0 \tag{27}$$

in the time interval $\tau \in [0,\tau_k]$ with the differential connections

$$\frac{dn_0(t)}{dt} = \gamma_b(\tau) - u(\tau), \tag{28}$$

under constraints on the phase variable (21)

$$0 \leqslant n_0(t) \leqslant n_b, \quad n_0(0) = n_{0st}, \tag{29}$$

under constraints on the control, and with initial conditions

$$0 \leqslant u(\tau) \leqslant \gamma_{\max}, \quad u_{\min} < \vartheta_{\min}, \quad \vartheta_{\max} < u_{\max}, \quad 0 \leqslant u(\tau) \leqslant \gamma(\tau) = g_0\vartheta_{\max}. \tag{30}$$

The Pontryagin function, Lagrange function, and the conjugate system have the form [20]:

$$H = -\psi_0|u(\tau)-\vartheta(\tau+\tau_0)| + \psi_1\big(\gamma_b(\tau) - u(\tau)\big) \to \max, \tag{31}$$

$$L = H + \mu_1 n_0 + \mu_2(n_b - n_0), \quad \mu_1 \geqslant 0,\ \mu_1 n_0 = 0, \quad \mu_2 \geqslant 0,\ \mu_2(n_b - n_0) = 0,$$

$$\frac{d\psi_1}{dt} = -\frac{\partial L}{\partial n_0}, \quad \psi_1(\tau_k - \tau_0) = 0. \tag{32}$$

4. Synthesis of the Optimal Control

We assume that the materials enter the accumulating bunker with a constant intensity $\gamma_b(\tau) = 1,0$. The form of the function $\vartheta(\tau)$ is defined as [22] $\vartheta(\tau) = 1,0 + \sin(\pi\tau)$.

In the absence of phase constraints (29), taking into account (32), it should be $\psi_1(\tau) = \text{const} = 0$, and the Pontryagin function has the form

$$H = -|u(\tau)-\vartheta(\tau+\tau_0)| \to \max. \tag{33}$$

Therefore, we assume that the solution takes the following form:

$$u(\tau) = \vartheta(\tau+\tau_0). \tag{34}$$

For the obtained control, we can write dynamics of the change in the state of the stock of materials in the accumulating bunker

$$\frac{dn_0(t)}{dt} = \gamma_b(\tau) - u(\tau) = 1,0 - \big(1,0 + \sin\big(\pi(\tau+\tau_0)\big)\big) = -\sin\big(\pi(\tau+\tau_0)\big). \tag{35}$$

Using Laplace transform

$$\sin\big(\pi(\tau+\tau_0)\big) \div \frac{\pi\cos\pi(\tau+\tau_0) + p\sin\pi(\tau+\tau_0)}{p^2+\pi^2}, \tag{36}$$

we obtain equation (35) in the form

$$N_0(p) = \frac{n_{0st}}{p} - \frac{\pi\cos(\pi\tau_0)}{p(p^2+\pi^2)} - \frac{\sin(\pi\tau_0)}{p^2+\pi^2}. \quad (37)$$

Hence, we can write solution (35) as

$$n_0(t) = n_{0st} + \frac{\cos\big(\pi(\tau+\tau_0)\big) - \cos(\pi\tau_0)}{\pi}. \quad (38)$$

Expression (38) determines the solution to optimal control problem (27) – (30) in the absence of phase constraints (29), Fig. 2. It is obvious that if condition (29) satisfies

$$0 \leqslant n_{0st} + \frac{\cos\big(\pi(\tau+\tau_0)\big) - \cos(\pi\tau_0)}{\pi} \leqslant n_b, \quad (39)$$

then the solution with phase constraints (29) coincides with (38). Now, consider the case for which condition (39) at time t is not satisfied at time τ. The maximum of Pontryagin function (31) can be reached at finite values (see Table 1). This table shows the control values for which the Pontryagin function takes the maximum value. Let us consider in details the possible cases.

Pontryagin function (31) has the form

$u(\tau) - \vartheta(\tau+\tau_0) > 0$, $H = \psi_1 \gamma_b(\tau) + \vartheta(\tau+\tau_0) - u(\tau)\psi_1 + 1) \to \max$,

Table 1 Variants of controls

	$u(\tau) < \vartheta(\tau+\tau_0)$	$u(\tau) = \vartheta(\tau+\tau_0)$	$u(\tau) > \vartheta(\tau+\tau_0)$
$\psi_1 < -1$	–	–	$u_{\max}$
$\psi_1 = -1$	–	$u(\tau) = \vartheta(\tau+\tau_0)$	$u(\tau) > \vartheta(\tau+\tau_0)$
$-1 < \psi_1 < 0$	–	$u(\tau) = \vartheta(\tau+\tau_0)$	–
$\psi_1 = 0$	–	$u(\tau) = \vartheta(\tau+\tau_0)$	–
$0 < \psi_1 < 1$	–	$u(\tau) = \vartheta(\tau+\tau_0)$	–
$\psi_1 = 1$	$u(\tau) < \vartheta(\tau+\tau_0)$	$u(\tau) = \vartheta(\tau+\tau_0)$	–
$\psi_1 > 1$	$u_{\min}$	–	–

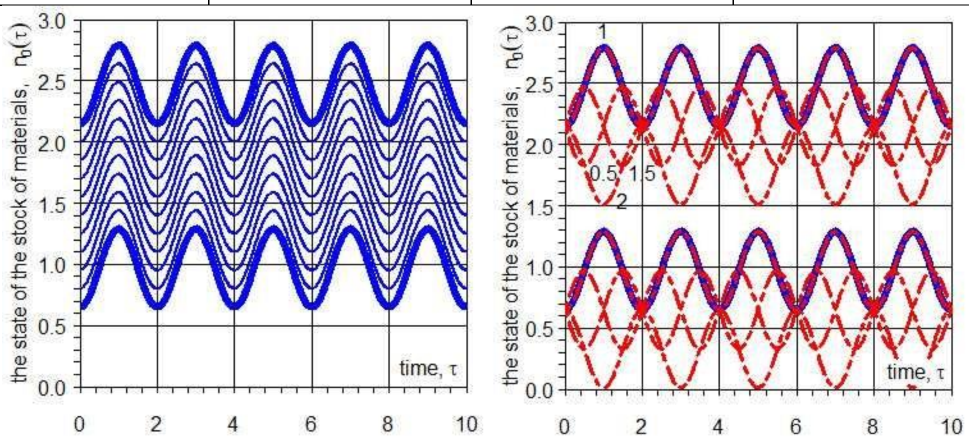

Fig. 2. Dynamics of the change in the amount of material in the bunker $n_0(\tau)$ (a – for initial states $n_{0st} = 0{,}65 + 0{,}15i$, $i = 0 \ldots 10$; b – for $\tau_0 = \{0{,}5,\ 1{,}0,\ 1{,}5,\ 2{,}0\}$; $\tau_0 = 1/g_0$)

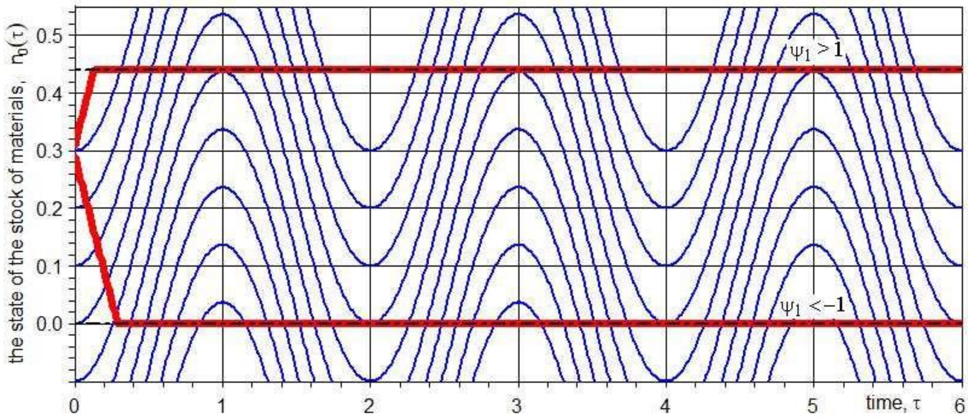

Fig. 3. Dynamics of the change in the amount of material in the bunker for the case $\psi_1 \in [-\infty;-1,0]$

$$u(\tau) - \vartheta(\tau + \tau_0) = 0,\ H = \psi_1(\gamma_b(\tau) - \vartheta(\tau + \tau_0)) \rightarrow \max,\ u(\tau) - \vartheta(\tau + \tau_0) < 0,$$

$$H = \psi_1 \gamma_b(\tau) - \vartheta(\tau + \tau_0) - u(\tau)\ \psi_1 - 1) \rightarrow \max.$$

1) $\psi_1 < -1 \rightarrow u(\tau) = u_{\max}$. Movement begins with the size of the control $u(\tau) = u_{\max}$ (Table 1), and parameters $\psi_1(0) = \psi_{10}$, $n_0(0) = n_{0st}$, $\gamma_b(\tau) = 1{,}0$:

$$\frac{dn_0(t)}{dt} = 1 - u(\tau), \quad \frac{d\psi_1(t)}{dt} = -\mu_1 + \mu_2$$

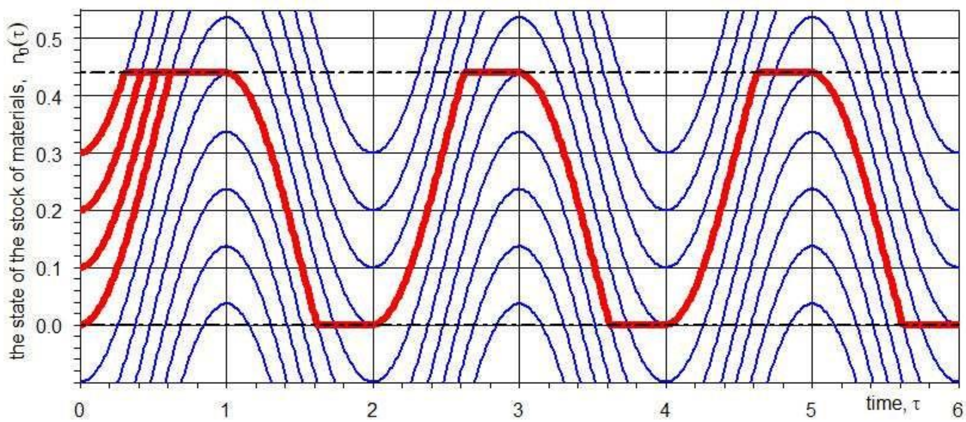

Fig. 4. Dynamics of the change in the amount of material in the bunker $n_0(\tau)$ when switching control $\{u(\tau) = \vartheta(\tau + \tau_0);\ u(\tau) = 1{,}0\}$ at phase constraints

Solution to the system of equations is given by

$$n_0(\tau) = n_{0st} = 0, \qquad \psi_1(t) = \psi_{10} - \mu_1\tau,$$

if the value of stocks in the bunker is the lower limit, and

$$n_0(\tau) = n_{0st} + \tau - u_{\max}\tau, \quad \psi_1(\tau) = \psi_{10}, \quad 0 < \tau \leqslant \tau_1 = \frac{n_{0st}}{u_{\max} - 1}$$

otherwise. For $\tau > \tau_1$, the value of the phase variable $n_0(\tau)$ reaches the lower

limit $n_0(\tau_1)$ =0 and remains $n_0(\tau) = n_{0st} = 0$, $\psi_1(\tau) = \psi_{10} - \mu_1(\tau - \tau_1)$.
Condition $\psi_1(\tau_k - \tau_0) = 0$ (32) is not met (Fig. 3). It contradicts the assumption of the existence of a solution and the maximum principle [20,21].

2) $\psi_1 > 1 \rightarrow u(\tau) = u_{\min}$. Similarly to the previous case, condition $\psi_1(\tau_k - \tau_0) = 0$ (32) is not met (Fig. 3).

3) $-1 < \psi_1 < 0$ and $0 < \psi_1 < 1$. It contains a valid solution for $u(\tau) = \vartheta(\tau + \tau_0)$. In fact, for $u(\tau) > \vartheta(\tau + \tau_0)$ we have $u(\tau) = u_{\min}$, but $u(\tau) = u_{\min} < \vartheta(\tau + \tau_0)$. We obtain a contradiction with the initial condition. The case $\psi_1 = 1$ also leads to optimal control $u(\tau) = \vartheta(\tau + \tau_0)$.

4) $\psi_1 = -1$. The initial control $u(0)$ must be such that the lower limit is not reached first. Otherwise, the phase coordinate $n_0(\tau)$ remains at the lower limit. This gives the following condition on the control: $u < \gamma_b(\tau) = 1$.

5) $\psi_1 = 1$. In this case, for the same reasons, the initial control $u(0)$ must be such that the upper limit is not reached first. Otherwise, the phase coordinate $n_0(\tau)$ remains at the upper limit. This gives the following condition on the control: $u > \gamma_b(\tau) = 1$. A family of phase trajectories is shown in (Fig. 4 – 6). The family of phase trajectories meets criterion of the control quality (27). Fig. 4 demonstrates the control algorithm. The control $u(\tau) = \vartheta(\tau + \tau_0)$ is used at the start of the conveyor belt. Then, $u(\tau) = 1{,}0$. It makes possible to ensure a constant amount of materials in the bunker $n_0(\tau) = n_b$. The bunker is completely filled. The excess amount of materials is fed to the input of the conveyor belt. The supply of materials exceeds requirements, $u(\tau) > \vartheta(\tau + \tau_0)$. A further increase in the demand for the input flow results in the phase variable $n_0(\tau)$ coming off the phase constraint $n_0(\tau) = n_b$. The amount of material in the bunker is reduced. The control

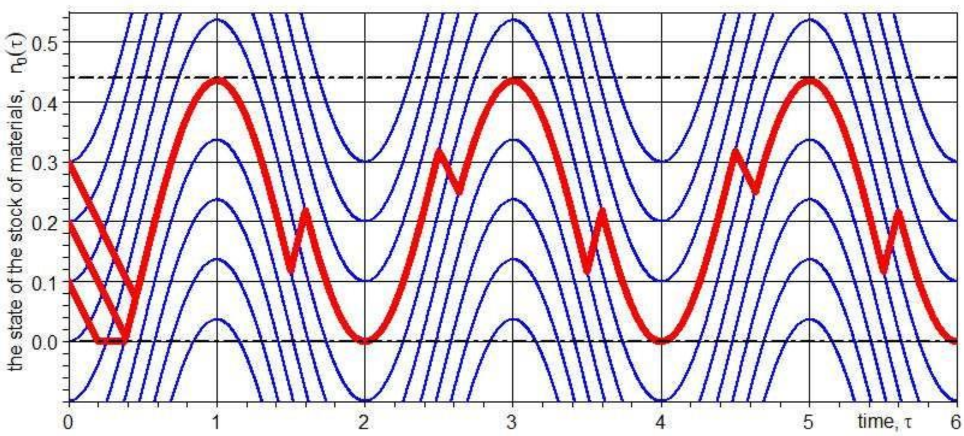

Fig. 5. Dynamics of the change in the amount of material in the bunker $n_0(\tau)$ for the controls $\{u(\tau) = 1{,}5; u(\tau) = \vartheta(\tau + \tau_0); u(\tau) = 0; u(\tau) = \vartheta(\tau + \tau_0)\}$

$u(\tau) = \vartheta(\tau + \tau_0)$ is supported until the lower limit $n_0(\tau) = 0$ is reached. Then, the control $u(\tau) = 1{,}0)$ is used. All material entering the bunker is fed to the input of the conveyor belt, $u(\tau) < \vartheta(\tau+\tau_0)$. Such control is maintained until the demand for the material reaches the exit point from the constraint. The cycle is repeated. Finally, the control algorithm can be formulated as follows. For constraints $u(\tau) = 1{,}0$ and $u(\tau) = \vartheta(\tau + \tau_0)$ beyond the constraints. Note that the phase constraints change the conjugate variable $\psi_1(t)$, $\frac{d\psi_1(t)}{dt} \neq 0$. Fig. 4 demonstrates the control algorithm when the switching points of the control are such that allow to avoid reaching the upper and lower limits for the phase variable $n_0(\tau)$. The initial movement is carried out from points $n_0(\tau)$ = {0,1; 0,2; 0,3} with the constant initial intensity $u(\tau)$ = 1,5 of the input of materials to the conveyor input. It provides an output for $n_0(\tau)$ onto the phase trajectory, which touches the constraint at the top point. The control $u(\tau) = \vartheta(\tau + \tau_0)$ is maintained until it is advisable to make a transition to the phase trajectory, which touches the constraint at its lowest point. The transition to the phase trajectory is performed with control $u(\tau) = 0$. The new phase path $u(\tau) = \vartheta(\tau + \tau_0)$ is controlled by the next switching point. The control algorithm can be formulated as follows. We use $u(\tau)$ = 1,5 to go to the phase trajectory, which touches the upper limit, and $u(\tau)$ = 0,0) for the transition to the phase trajectory, which touches the lower limit. Control between transitions is supported by $u(\tau) = \vartheta(\tau + \tau_0)$. We draw attention to the fact that for $\psi_1(0) = 0$ the phase trajectory is also sustained $\psi_1(t) = 0$. The control algorithm that determines the behavior of the phase variable $n_0(\tau)$ in Fig. 6 is similar to the algorithm that determines the behavior of the phase variable $n_0(\tau)$ in Fig. 5. The difference is that all transitions are performed under optimal control $u(\tau) = 1$). The control algorithm $\{u(\tau) = \vartheta(\tau + \tau_0); u(\tau = 1,0)\}$is similar to the algorithm in Fig. 4. With the same control chosen for phase trajectories $u(\tau) = \vartheta(\tau + \tau_0)$ and transitions $u(\tau = 1)$, the control switching points are arranged such that to avoid reaching the upper and lower limits. Let us define the costs that characterize the transition from a phase trajectory that touches the upper limit to a phase trajectory that touches the lower limit. Let us define the equation of the trajectory we want to go:

$$n_{01}(\tau) = n_{01}(\tau_1) + \int_{\tau_1}^{\tau} (\gamma_b(\tau) - \vartheta(\xi + \tau_0))\, d\xi.$$

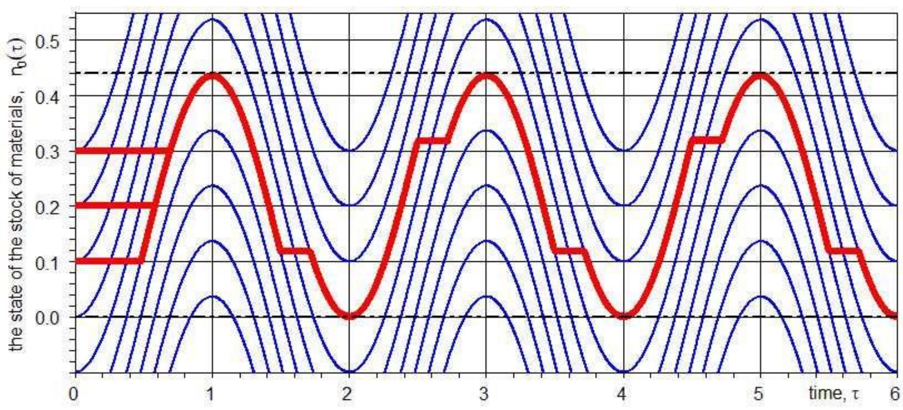

Fig. 6. Dynamics of the change in the amount of material in the bunker $n_0(\tau)$ at the controls $\{u(\tau) = \vartheta(\tau + \tau_0);\ u(\tau) = 1,0\}$

The transition is carried out along the trajectory

$$n_{02}(\tau) = n_{02}(\tau_1) + \int_{\tau_1}^{\tau} (\gamma_b(\tau) - u(\xi))\, d\xi,$$

which is determined by the control $u(\tau)$. At any instant in time, these trajectories are valid $n_{02}(\tau_1) - n_{01}(\tau_1) = \Delta n_0(\tau_1) = \text{const}$. This allows us to write

$$n_{02}(\tau) - n_{01}(\tau) = n_{02}(\tau_1) + \int_{\tau_1}^{\tau} (\gamma_{\text{b}}(\tau) - u(\xi))\, d\xi - n_{01}(\tau_1) - \int_{\tau_1}^{\tau} (\gamma_{\text{b}}(\tau) - \vartheta(\xi + \tau_0))\, d\xi = 0,$$

$$|\Delta n_0(\tau_1)| = \int_{\tau_1}^{\tau} |u(\xi) - \vartheta(\xi + \tau_0)|\, d\xi = \text{const},$$

since the transition is carried out both in the forward and reverse direction. The last expression is a consequence of the given quality criterion (27). The arbitrariness of the choice of the moment of time determines the arbitrariness of the choice of control switching points, which determines the set of solutions to the problem.

Conclusions and Recommendations

The article analyzes the PDE-model of the conveyor transport system and synthesizes a family of optimal control of the flow of materials coming from the accumulating bunker to the input of the conveyor transport system. The criterion of the quality of the control of the output flow from the conveyor belt is determined, and the optimal control problem for the transport system is formulated. The analysis of admissible solutions to the control problem is carried out. The results presented in the paper allows to make the following conclusions:

– a system to control the output flow on the conveyor belt from an accumulating bunker at the input can have a large number of algorithms;

– the switching points of the optimal control are determined from a large set of feasible solutions;

– the set of admissible optimal controls is determined by the size of the accumulating bunker.

Prospect for further research is the synthesis of optimal control for the conveyor system with input and output accumulating bunker.

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