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Multi-criteria of Bike Purchasing Using Fuzzy Choquet Integral

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Abstract. A bike purchasing is a multi-criteria decision-making problem including both quantitative and qualitative main and sub-criteria. This paper shows that when these criteria include interactions between each others, Choquet integral presents an excellent tool for the solution of this multi-criteria decision making problem. An example is given to illustrate the proposed model.

Keywords: Choquet integral; hierarchy; multi-criteria decision; fuzzy measure.

1 Introduction

The multi-criteria decision between alternatives is a problem including both quantitative and qualitative criteria. The conventional approaches to multi-criteria decision problem tend to be less effective in dealing with the vague or imprecise nature of the linguistic assessment. Under many situations, the values of the qualitative criteria are often imprecisely defined for the decision-makers. Choquet integral has been used for the solution of multiple criteria decision-making problems in the literature. Marichal et al. [6] analyzed an ordinal sorting procedure (TOMASO) for the assignment of alternatives to graded classes and present a freeware constructed from this procedure. Meyer et al. [7] presented a multiple criteria decision support approach in order to build a ranking and suggest a best choice on a set of alternatives. The aggregation is performed through the use of a fuzzy extension of the Choquet integral. Demirel et al. [3] used it for warehouse location selection. This paper proposes a multi-criteria decision-making method using fuzzy Choquet integral for the selection of bike model. We first determine the main and sub-criteria and the hierarchy for the bike selection problem, then make a multi-criteria evaluation of the bike models to illustrate how the generalized Choquet integral is used to do this. The Choquet integral is a flexible aggregation operator being introduced by Sugeno [10] and it is the generalization of the weighted average method, the Ordered Weighted Average (OWA) operator, and the max–min operator [4].
As it is explained before, these methods do not take the interactions among the bike selection attributes into account and cannot handle vague and incomplete information. The methods trying to consider the interactions, like ANP, are very tedious since they require a huge amount of calculations. The Choquet integral is an excellent multi-attribute tool for the problems having interactive attributes under fuzziness. Selection of bike according to our requirements, is an important partial task of optimization of logistic systems. Some of the qualities are in one bike but some are in another model of bike. In this kind of problems, our decision are based on different kind of criterias. Vlachopoulou et al. [12] developed a geographic decision support system, enabling the manager to use qualitative and quantitative criteria in order to classify the best one.
In this study we consider different criteria due to multistage characteristics of bikes which distinguishes models of bikes from other models, where some criterias of information are given for each model of bike in the manual copy of bike. This paper is organized as follows. Section 2 introduces some definitions and formulations associated with Choquet integral. The steps of the methodology used for bike selection are also given in Section 2. Bike selection criteria are defined in the Section 3. An application to a bike selection problem is presented in Section 4.

2 Choquet integral and the steps of the methodology

A fuzzy integral is a sort of general averaging operator that can represent the notions of importance of a criterion and interactions among criteria. To define fuzzy integrals, a set of values of importance is needed. This set is composed of the values of a fuzzy measure. So, a value of importance for each subset of attributes is needed.

The success of a Choquet integral depends on an appropriate representation of fuzzy measures, which captures the importance of individual criterion or their combination. We use, the generalized Choquet integral proposed by Auephanwiriyahul et al. in [1], in which measurable evidence is represented in terms of intervals, whereas fuzzy measures are real numbers. It is an extension of the standard Choquet integral. In contrast to [1], [11] proposes another generalization that involves linguistic expressions as well as information fusion between criteria to overcome vagueness and imprecision of linguistic terms in questionnaires. Our methodology follows Tsai and Lu's [11] approach to [1].

In the following, some definitions are given to explain the basics of Choquet integral [8]:

Let I be the set of attributes (or any set in a general setting). A set function $\mu : P(I) \to [0,1]$ is called a fuzzy measure if it satisfies the following three axioms:

1. $\mu(\emptyset) = 0$;
2. $\mu(I) = 1$;
3. $\mu(B) = \mu(C)$ if $B, C \subset I$ and $B \subset C$.

Therefore, in a problem where $card(I) = n$, a value for every element of $P(I)$ including 2^n values is needed. Assuming that the values of the empty set and of the maximal set are fixed, $(2^n - 2)$ values or coefficients to define a fuzzy measure are needed. So, there is clearly a trade-off between complexity and accuracy. However, the complexity can be significantly reduced in order to guarantee that fuzzy measures are used in practical applications. A fuzzy integral is a sort of weighted mean taking into account the importance of every coalition of criteria.

2.1 Fuzzy feature components

Fuzzy data is a data type with imprecision or with a source of uncertainty not caused by randomness, but due to ambiguity. Examples of fuzzy data types can easily be found in natural language. It is generally more convenient and useful in describing fuzzy data to use LR-type fuzzy numbers [9]. Zimmermann [13, Subsubsection 5.3.2] defined the LR-type fuzzy numbers.

Let L (and R) be decreasing, shape functions from $\Re^+ = [0, \infty)$ to $[0,1]$ with $L(0) = 1$; $L(x) < 1$ for all $x > 0$; $L(x) > 0$ for all $x < 1$; $L(1) = 0$ or ($L(x) > 0$ for all x and $L(+\infty) = 0$). An LR-type TFN X has the following membership function

$$\mu_X(x) = \begin{cases} L\left(\frac{m_2 - x}{\alpha}\right) & \text{for } x \le m_2, \\ 1 & \text{for } m_2 \le x \le m_3, \\ R\left(\frac{x - m_3}{\beta}\right) & \text{for } x \ge m_3, \end{cases}$$

where $\alpha = m_2 - m_1 > 0$ and $\beta = m_4 - m_3 > 0$ are called the left and right spreads, respectively. Symbolically, X is denoted by $(m_1, m_2, m_3, m_4)_{LR}$.

The LR-type TFN is very general and allows one to represent the different types of information. For example, the LR-type TFN $X = (0, m, m, 0)_{LR}$ with $m \in \Re = (-\infty, \infty)$ is used to denote a real number X and the LR-type TFN $X = (0, a, b, 0)_{LR}$ with $a, b \in \Re$ and $a < b$ is used to denote an interval X.

$$l = \int_0^1 L^{-1}(w)dw \text{ and } r = \int_0^1 R^{-1}(w)dw.$$

For an LR-type TFN $X = (m_1, m_2, m_3, m_4)_{LR}$, if $L(x) = R(x) = 1 - x$ then X is called a TFN, denoted by $X = (m_1, m_2, m_3, m_4)_T$, i.e.

$$\mu_X(x) = \begin{cases} 1 - \frac{m_2 - x}{\alpha} & \text{for } x \le m_2 (\alpha > 0), \\ 1 & \text{for } m_2 \le x \le m_3, \\ 1 - \frac{x - m_3}{\beta} & \text{for } x \ge m_3 (\beta > 0). \end{cases}$$

Obviously, $l = r = 1/2$. In LR-type TFNs, the TFNs are most commonly used. In the rest of paper we use TFN and denoted by $X = (m_1, m_2, m_3, m_4)$, i.e.

$$\mu_X(x) = \begin{cases} 1 - \frac{m_2 - x}{\alpha} & \text{for } x \leq m_2 (\alpha > 0), \\ 1 & \text{for } m_2 \leq x \leq m_3, \\ 1 - \frac{x - m_3}{\beta} & \text{for } x \geq m_3 (\beta > 0). \end{cases}$$

where $\alpha = m_2 - m_1 > 0$ and $\beta = m_4 - m_3 > 0$.

The methodology is based on the following eight steps [11]:

Step 1. Given criterion i, respondents' linguistic preferences for the degree of importance, perceived performance levels of alternative bikes, and tolerance zone are surveyed.

Step 2. In view of the compatibility between perceived performance levels and the tolerance zone, trapezoidal fuzzy numbers are used to quantify all linguistic terms in this study. Given respondent t and criteria i, linguistic terms for the degree of importance is parameterized by $\tilde{A}_i^t = (a_{i1}^t, a_{i2}^t, a_{i3}^t, a_{i4}^t)$, perceived performance levels by $\tilde{p}_i^t = (p_{i1}^t, p_{i2}^t, p_{i3}^t, p_{i4}^t)$, and the tolerance zone by $\tilde{e}_i^t = (e_{i1L}^t, e_{i2L}^t, e_{i3U}^t, e_{i4U}^t)$. In this case study, $t = 1$, $i = 1, 2, ..., n_j$, $j = 1, 2, 3, 4, 5$, $n_1 = 2$, $n_2 = 2$, $n_3 = 7$, $n_4 = 3$, $n_5 = 3$; where n_j represents the number of criteria in dimension j.

Step 3. Average $\tilde{A}_i^t, \tilde{p}_i^t$ and $\tilde{e}_i^t$ into $\tilde{A}_i$, and $\tilde{e}_i$, respectively using Eq. (1).

$$\tilde{A}_i = \frac{\sum_{t=1}^k \tilde{A}_i^t}{k} = \left(\frac{\sum_{t=1}^k a_{i1}^t}{k}, \frac{\sum_{t=1}^k a_{i2}^t}{k}, \frac{\sum_{t=1}^k a_{i3}^t}{k}, \frac{\sum_{t=1}^k a_{i4}^t}{k} \right). \tag{1}$$

Step 4. Normalize the bike value of each criterion using Eq. (2).

$$\tilde{f}_i = \bigcup_{\alpha \in [0,1]} \tilde{f}_i^\alpha = \bigcup_{\alpha \in [0,1]} [f_{i,\alpha}^-, f_{i,\alpha}^+], \tag{2}$$

where $f_i \in F(S)$ is a fuzzy-valued function. $F(S)$ is the set of all fuzzy-valued functions f,

$$f_i^\alpha = [f_{i,\alpha}^-, f_{i,\alpha}^+] = \frac{\tilde{p}_i^\alpha - \tilde{e}_i^\alpha + [1, 1]}{2},$$

$\tilde{p}_i^\alpha$ and $\tilde{e}_i^\alpha$ are $\alpha-$level cuts of $\tilde{p}_i$ and $\tilde{e}_i$ for all $\alpha = [0, 1]$,

Step 5. Find the bike value of dimension j using Eq. (3),

$$(C)\int \tilde{f} d\tilde{g} = \bigcup_{\alpha \in [0,1]} \left[(C)\int f_\alpha^- dg_\alpha^-, (C)\int f_\alpha^+ dg_\alpha^+ \right], \tag{3}$$

where $\tilde{g}_i : P(S) \to I(R^+), \tilde{g}_i = [g_i^-, g_i^+], \tilde{g}_i^\alpha = [g_{i,\alpha}^-, g_{i,\alpha}^+], \tilde{f}_i : S \to I(R^+)$, and $[f_i^-, f_i^+]$ for $i = 1, 2, 3, ..., n_j$.
To be able to calculate this bike value, a λ value and the fuzzy measures $g(A_{(i)})$, $i = 1, 2, ..., n$, are needed. These are obtained from the following Eqs. (4)–(6) ([5], [10]):

$$g(A_{(n)}) = g(\{S_{(n)}\}) = g_n, \tag{4}$$

$$g(A_{(i)}) = g_i + g(A_{(i+1)}) + \lambda g_i g(A_{(i+1)}), \text{ where } 1 \leq i < n \tag{5}$$

$$1 = g(S) = \begin{cases} \frac{1}{\lambda} \left\{ \prod_{i=1}^n [1 + \lambda g(A_i)] - 1 \right\} & \text{if } \lambda \neq 0 \\ \sum_{i=1}^n g(A_i) & \text{if } \lambda = 0 \end{cases} \tag{6}$$

where, $A_i \cap A_j = \emptyset$ for all $i, j = 1, 2, 3, ..., n$ and $i \neq j$, and $\lambda \in (-1, \infty]$.
Let μ be a fuzzy measure on $(I, P(I))$ and an application $f : I \to \Re^+$. The Choquet integral of f with respect to μ is defined by:

$$(C)\int_I f d\mu = \sum_{i=1}^{n} (f(\sigma(i)) - f(\sigma(i-1)))\mu(A_{(i)}) \tag{7}$$

where σ is a permutation of the indices in order to have $f(\sigma(1)) \leq ... \leq f(\sigma(n))$, $A_{(i)} = \{\sigma(i), ..., \sigma(n)\}$ and $f(\sigma(0)) = 0$, by convention.

It is shown in [8] that under rather general assumptions over the set of alternatives X, and over the weak orders $\succeq_i$, there exists a unique fuzzy measure μ over I such that:

$$\forall x, y \in X, \qquad x \succeq y \iff u(x) \geq u(y), \tag{8}$$

where

$$u(x) = \sum_{i=1}^{n} \left[u_{(i)}(x_{(i)}) - u_{(i-1)}(x_{(i-1)})\right] \mu(A_{(i)}), \tag{9}$$

which is simply the aggregation of the monodimensional utility functions using the Choquet integral with respect to μ.

Step 6. Aggregate all dimensional performance levels of the bike alternatives into overall performance levels, using a hierarchical process applying the two-stage aggregation process of the generalized Choquet integral. This is represented in Eq. (10). The overall performance levels yields a fuzzy number, $\tilde{V}$.

$$\left.\begin{array}{l} maincriterion_{(1)} = (C)\int f dg \\ \vdots \\ maincriterion_{(m)} = (C)\int f dg \end{array}\right\rangle V = (C)\int maincriterion \ \ dg \tag{10}$$

Step 7. Assume that the membership of $\tilde{V}$ is $\mu_{\tilde{V}}(x)$; defuzzify the fuzzy number $\tilde{V}$ into a crisp value v using Eq. (11) and make a comparison of the overall performance levels of bikes.

$$F(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4}. \tag{11}$$

Step 8. Compare weak and advantageous criteria among the bikes using Eq. (2).

3 Bike purchasing criteria

In this study, five main criteria and seventeen sub-criteria of these main criteria are used for the purchase of bike. These criteria were selected from the authorized dealer of bikes. Moreover, while determining these criteria and their hierarchy, the opinions of experts in the logistics sector are also taken into account. The definitions of the main and sub-criteria are summarized as follows:

3.1 Price

Price is one of the factors that highly affects purchasing of the bike. Under the price criterion, two sub-criteria are defined: purchase price and resale price. Purchase price is the criterion that changes with respect to bike. Resale price is the criteria which shows the worth of bike at any time. Purchase price and resale price of bike is vary for different bikes.

3.2 Comfortable

This criterion is concerned with the comfortability of bike. In this criteria we defined two sub-criteria; seating capacity and seating comfortabilty on bike and about the suspension of bike.

3.3 Bike power

It is a main criterion defining the power of bike. This criterion is composed of six sub-criteria: Engine power with units in cc, gear system, speed, brakes, engine sound, pick of bike and road grip of bike. Existence of engine power has an importance based on the availability of different engine power types in the market. Gear system is a criterion that defines the smoothness of bike during the change of gear. Speed is also a very important criteria due to the different speed limits for different bikes. Brakes and road grip of bike play an important role in the safety issues. Engine sound is related to the noise pollution and sensitivity of rider and pick of bike plays role in the racing of bikes.

3.4 Spare parts

It is a main criterion that defines the quality of parts of bike, availability of spare parts in market and the cost of spare parts. Quality of parts of bike is concern with the physical quality of bike and quality of parts vary in bikes. Some models of bike are new and some are very old models that is the reason of availability of spare parts of bike in the market. Cost of spare parts also vary for bikes.

3.5 Fuel

Main criterion concern with the fuel consumption of bike. This criteria is divided into three sub-criterias; Supreme petrol consumption, High octane petrol consumption and engine oil consumption. Engine of all bikes are designed for different level of supreme petrol consumption and high octane petrol consumption is use to improve the quality of engine. Capacity of engine oil is different for different bikes.

4 Application

A customer decide to purchase a bike, wants to decide which bike is suitable according to his requirement. Authorized dealer of bikes presented the criteria to decide the bike purchasing. This dealer is dealing in four different models of bike, name of these models are CD70, CD100, CG125 and DLX125. Customer confirmed the criteria and sub-criteria and decide on using the evaluation scale in Table 1.

Table 1
The relationship between trapezoidal fuzzy numbers and degrees of linguistic importance in a Seven-linguistic-term scale is selected from Delgado et al. [2].

Low/high levels		Trapezoidal fuzzy numbers
Label	Linguistic terms	
VL	Very low	$(0.001, 0.01, 0.02, 0.07)$
L	Low	$(0.04, 0.1, 0.18, 0.23)$
SL	Slightly low	$(0.17, 0.22, 0.36, 0.42)$
M	Middle	$(0.32, 0.41, 0.58, 0.65)$
SH	Slightly high	$(0.58, 0.63, 0.8, 0.86)$
H	High	$(0.72, 0.78, 0.92, 0.97)$
VH	Very high	$(0.93, 0.98, 0.98, 0.999)$

To express the sub-criteria easier, the symbols in Table 2 were generated. Customer decided the individual importance of main and sub-criteria. That authorized dealer made each bike linguistic evaluation of all sub-criterias. Table 3 gives these results. Trapezoidal fuzzy numbers are used to quantify the linguistic terms in Table 4(a,b). The tolerance zones in table 4(a) are obtained in that way: the first two numerical values of the lower linguistic value of a tolerance zone in Table 3 are combined with the last two numerical values of the upper linguistic value of the same tolerance zone.

Table 2
The criteria of bike purchasing and their symbols.

Criteria	The Symbol of each criterion
Price	P
Purchase price	Pp
Resale price	Rp
Comfortable	C
Seats	Se
Suspension	Su
Bike Power	E
Engine power units in cc	Uc
Gear system	Gs
Speed	Sp
Breaks	Br
Engine sound	Es
Pick of bike	Pb
Road grip	Rg
Spare Parts	S
Spare parts quality	Sp
Availability of spare parts	Sa
Price of spare parts	Pr
Fuel	F
Supreme petrol consumption	Ps
High octane usage	Ph
Engine oil usage	Po

Consider the tolerance zone [L,VH]. The corresponding numerical values of L and VH are (0.04,0.1, 0.18,0.23) and (0.93,0.98,0.98,0.999) respectively. Then the combined tolerance zone is (0.04,0.1,0.98, 0.999). Table 4(a,b) present the compromised evaluations by using Table 1. Table 5 gives the evaluation results by the generalized Choquet Integral for $\alpha = 0$. For the sub-criteria, Eq. (2) is used while Eq. (3) is for the main criteria. For example, the value [0.305,0.64] of "model CD70 and sub-criterion Sp " is obtained in that way:

$$f,\ \bar{f}_1^0 = [f_{1,0}^-, f_{1,0}^+] = \frac{[0.58,0.86]-[0.58,0.97]+[1,1]}{2} = [0.305, 0.64]$$

For the value $[0.4539, 0.9987]$ of "model CD70 and main-criterion S" is obtained in that way:
The other normalized discrepancies between for model CD70 and main criterion S at $\alpha = 0$ are $\bar{f}_1^0 = [0.305, 0.64]$, $\bar{f}_2^0 = [0.4655, 0.999]$ and $\bar{f}_3^0 = [0.0205, 0.595]$,respectively. Their corresponding degrees of individual importance are $\bar{g}_1^0 = [0.58, 0.97]$, $\bar{g}_2^0 = [0.001, 0.999]$ and $\bar{g}_3^0 = [0.04, 0.999]$, respectively. First, the sequence $f_{i,0}^-$ is sorted, where $i = 1, 2$ and 3, as follows:

$$\begin{array}{lllll} f_{S_3}^- = 0.0205 & \leq & f_{S_1}^- = 0.305 & \leq & f_{S_2}^- = 0.4655 \\ g_{S_3}^- = 0.04 & & g_{S_1}^- = 0.72 & & g_{S_2}^- = 0.93 \\ g_1^- = 0.04 & & g_2^- = 0.72 & & g_3^- = 0.93 \end{array}$$

By solving the following equation for λ the fuzzy measures $g(A_{(i)})$, $i = 1, 2, \ldots, n$ are obtained as follows:

$$1 = g(S) = \frac{1}{\lambda}\{[(1 + \lambda 0.04)(1 + \lambda 0.72)(1 + \lambda 0.93)] - 1\}$$

That is,

$$\lambda = -0.9724.$$

The fuzzy measures are,

$$g^-(A_{(3)}) = g_3^- = 0.93,$$
$$g^-(A_{(2)}) = g_2^- + g^-(A_{(3)}) + \lambda g_2^- g^-(A_{(3)}) = 0.9989,$$
$$g^-(A_{(1)}) = g_1^- + g^-(A_{(2)}) + \lambda g_1^- g^-(A_{(2)}) = 1.0.$$

Tables 6(a,b) and 8(a,b) summarize the whole fuzzy measures and λ values, which are calculated in the same way.
The aggregated Choquet integral values for the main criterion S are calculated as in the following. Tables 5 and 7 include the normalized discrepancies and bike model values (Choquet integrals).

$$(C)\int f^-_{\alpha=0} dg^-_{\alpha=0} = 1.0(0.0205) + 0.9989(0.305 - 0.0205) + 0.93(0.4655 - 0.305) = 0.4539.$$
$$(C)\int f^+_{\alpha=0} dg^+_{\alpha=0} = 0.9987.$$

That is,

$$(C)\int \bar{f} d\bar{g} = [0.4539, 0.9987].$$

Table 3
Individual importance of criteria the tolerance zones, and each bike linguistic evaluation.

Criteria		Individual importance of criteria	The Tolerance zone	Linguistic evaluation			
				CD70	CD100	CG125	125DLX
P		VH					
	Pp	M	[M,VH]	M	SH	VH	VH
	Rp	VH	[VL,VH]	VH	SH	H	VL
C		SL					
	Se	M	[M,H]	M	SH	H	H
	Su	SH	[SH,VH]	SH	H	SH	VH
E		L					
	Uc	SH	[M,VH]	M	SH	VH	VH
	Gs	SH	[M,VH]	M	SH	H	VH
	Sp	M	[L,VH]	L	M	H	VH
	Br	VL	[VL,VH]	VL	H	M	VH
	Es	SL	[L,VH]	M	L	VH	H
	Pb	M	[L,H]	SL	L	H	SL
	Rg	SL	[L,H]	L	H	M	SH
S		VH					
	Sp	H	[SH,H]	SH	H	SH	H
	Sa	VH	[VL,VH]	VH	L	SH	VL
	Pr	L	[L,VH]	L	SL	SH	VH
F		VL					
	Ps	VL	[VL,VH]	VL	L	H	VH
	Ph	L	[VL,M]	VL	L	SL	M
	Po	L	[VL,M]	VL	L	SL	M

In Table 9(a,b), using the calculation for Choquet integral as above, the overall bike model values are obtained. The defuzzified overall values of alternative bike models using generalized Choquet Integral are also given in the same table.

For Bike model CD70, the overall Choquet integral value at $\alpha = 0$ are found as follows:

$$\lambda = -0.9956.$$
$$g^{-}(A_{(5)}) = 0.93$$
$$g^{-}(A_{(4)}) = 0.9989$$
$$g^{-}(A_{(3)}) = 0.9998$$
$$g^{-}(A_{(2)}) = 1.0$$
$$g^{-}(A_{(1)}) = 1.0$$

and finally,

$$(C)\int \bar{f} d\bar{g} = [0.453, 0.9988].$$

Table 4(a)
Individual importance of criteria and tolerance zone in trapezoidal fuzzy number from table 3 and table 1.

Criteria		Individual importance of criteria	The combined Tolerance zone
P		(0.93,0.98,0.98,0.999)	
	Pp	(0.32,0.41,0.58,0.65)	(0.32,0.41,0.98,0.999)
	Rp	(0.93,0.98,0.98,0.999)	(0.001,0.01,0.98,0.999)
C		(0.17,0.22,0.36,0.42)	
	Se	(0.32,0.41,0.58,0.65)	(0.32,0.41,0.92,0.97)
	Su	(0.58,0.63,0.8,0.86)	(0.58,0.63,0.98,0.999)
E		(0.04,0.1,0.18,0.23)	
	Uc	(0.58,0.63,0.8,0.86)	(0.32,0.41,0.98,0.999)
	Gs	(0.58,0.63,0.8,0.86)	(0.32,0.41,0.98,0.999)
	Sp	(0.32,0.41,0.58,0.65)	(0.04,0.1,0.98,0.999)
	Br	(0.001,0.01,0.02,0.07)	(0.001,0.01,0.98,0.999)
	Es	(0.17,0.22,0.36,0.42)	(0.04,0.1,0.98,0.999)
	Pb	(0.32,0.41,0.58,0.65)	(0.04,0.1,0.92,0.97)
	Rg	(0.17,0.22,0.36,0.42)	(0.04,0.1,0.8,0.86)
S		(0.93,0.98,0.98,0.999)	
	Sp	(0.72,0.78,0.92,0.97)	(0.58,0.63,0.92,0.97)
	Sa	(0.93,0.98,0.98,0.999)	(0.001,0.01,0.98,0.999)
	Pr	(0.04,0.1,0.18,0.23)	(0.04,0.1,0.98,0.999)
F		(0.001,0.01,0.02,0.07)	
	Ps	(0.001,0.01,0.02,0.07)	(0.001,0.01,0.98,0.999)
	Ph	(0.04,0.1,0.18,0.23)	(0.001,0.01,0.58,0.65)
	Po	(0.04,0.1,0.18,0.23)	(0.001,0.01,0.58,0.65)

Table 4(b)
Perceived level of each criteria in trapezoidal fuzzy number from table 3 and table 1.

Criteria		Perceived level of different bike models			
		CD70	CD100	CG125	125DLX
P					
	Pp	(0.32,0.41,0.58,0.65)	(0.58,0.63,0.8,0.86)	(0.93,0.98,0.98,0.999)	(0.93,0.98,0.98,0.999)
	Rp	(0.93,0.98,0.98,0.999)	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)	(0.001,0.01,0.02,0.07)
C					
	Se	(0.32,0.41,0.58,0.65)	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)	(0.72,0.78,0.92,0.97)
	Su	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)	(0.58,0.63,0.8,0.86)	(0.93,0.98,0.98,0.999)
E					
	Uc	(0.32,0.41,0.58,0.65)	(0.58,0.63,0.8,0.86)	(0.93,0.98,0.98,0.999)	(0.93,0.98,0.98,0.999)
	Gs	(0.32,0.41,0.58,0.65)	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)	(0.93,0.98,0.98,0.999)
	Sp	(0.04,0.1,0.18,0.23)	(0.32,0.41,0.58,0.65)	(0.72,0.78,0.92,0.97)	(0.93,0.98,0.98,0.999)
	Br	(0.001,0.01,0.02,0.07)	(0.72,0.78,0.92,0.97)	(0.32,0.41,0.58,0.65)	(0.93,0.98,0.98,0.999)
	Es	(0.32,0.41,0.58,0.65)	(0.04,0.1,0.18,0.23)	(0.93,0.98,0.98,0.999)	(0.72,0.78,0.92,0.97)
	Pb	(0.17,0.22,0.36,0.42)	(0.04,0.1,0.18,0.23)	(0.72,0.78,0.92,0.97)	(0.17,0.22,0.36,0.42)
	Rg	(0.04,0.1,0.18,0.23)	(0.32,0.41,0.58,0.65)	(0.32,0.41,0.58,0.65)	(0.58,0.63,0.8,0.86)
S					
	Sp	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)	(0.58,0.63,0.8,0.86)	(0.72,0.78,0.92,0.97)
	Sa	(0.93,0.98,0.98,0.999)	(0.04,0.1,0.18,0.23)	(0.58,0.63,0.8,0.86)	(0.001,0.01,0.02,0.07)
	Pr	(0.04,0.1,0.18,0.23)	(0.17,0.22,0.36,0.42)	(0.58,0.63,0.8,0.86)	(0.93,0.98,0.98,0.999)
F					
	Ps	(0.001,0.01,0.02,0.07)	(0.04,0.1,0.18,0.23)	(0.72,0.78,0.92,0.97)	(0.93,0.98,0.98,0.999)
	Ph	(0.001,0.01,0.02,0.07)	(0.04,0.1,0.18,0.23)	(0.17,0.22,0.36,0.42)	(0.32,0.41,0.58,0.65)
	Po	(0.001,0.01,0.02,0.07)	(0.04,0.1,0.18,0.23)	(0.17,0.22,0.36,0.42)	(0.32,0.41,0.58,0.65)

Table 5
Evaluation results by the generalized Choquet integral for $\alpha = 0$.

Criteria		Individual importance of criteria	The normalized discrepancy $\tilde{f}_i = [f_i^-, f_i^+]$ and bike value $\left[(C)\int f^- dg^-, (C)\int f^+ dg^+\right]$			
			CD70	CD100	CG125	125DLX
P		(0.93,0.999)	(0.4442,0.9987)	(0.2905,0.9293)	(0.3941,0.9843)	(0.1496,0.7327)
	Pp	(0.32,0.65)	(0.1605,0.665)	(0.2905,0.77)	(0.4655,0.8395)	(0.4655,0.8395)
	Rp	(0.93,0.999)	(0.4655,0.999)	(0.2905,0.9295)	(0.3605,0.9845)	(0.001,0.5345)
C		(0.17,0.42)	(0.2419,0.6562)	(0.3372,0.7437)	(0.3175,0.7603)	(0.4275,0.7846)
	Se	(0.32,0.65)	(0.175,0.665)	(0.305,0.77)	(0.375,0.825)	(0.375,0.825)
	Su	(0.58,0.86)	(0.2905,0.64)	(0.3605,0.695)	(0.2905,0.64)	(0.4655,0.7095)
E		(0.04,0.23)	(0.1506,0.733)	(0.2662,0.8105)	(0.4295,0.9621)	(0.4479,0.9579)
	Uc	(0.58,0.86)	(0.1605,0.665)	(0.2905,0.77)	(0.4655,0.8395)	(0.4655,0.8395)
	Gs	(0.58,0.86)	(0.1605,0.665)	(0.2905,0.77)	(0.3605,0.825)	(0.4655,0.8395)
	Sp	(0.32,0.65)	(0.0205,0.595)	(0.1605,0.805)	(0.3605,0.965)	(0.4655,0.9795)
	Br	(0.001,0.07)	(0.001,0.5345)	(0.3605,0.9845)	(0.1605,0.8245)	(0.4655,0.999)
	Es	(0.17,0.42)	(0.1605,0.805)	(0.0205,0.595)	(0.4655,0.9795)	(0.3605,0.965)
	Pb	(0.32,0.65)	(0.1,0.69)	(0.035,0.595)	(0.375,0.965)	(0.1,0.69)
	Rg	(0.17,0.42)	(0.09,0.595)	(0.23,0.805)	(0.23,0.805)	(0.36,0.91)
S		(0.93,0.999)	(0.4539,0.9986)	(0.2765,0.6932)	(0.3009,0.9293)	(0.2784,0.7568)
	Sp	(0.72,0.97)	(0.305,0.64)	(0.375,0.695)	(0.305,0.64)	(0.375,0.695)
	Sa	(0.93,0.999)	(0.4655,0.999)	(0.0205,0.6145)	(0.2905,0.9295)	(0.001,0.5345)
	Pr	(0.04,0.23)	(0.0205,0.595)	(0.0855,0.69)	(0.2905,0.91)	(0.4655,0.9795)
F		(0.001,0.07)	(0.1258,0.5345)	(0.1453,0.6144)	(0.26,0.7287)	(0.3351,0.8366)
	Ps	(0.001,0.07)	(0.001,0.5345)	(0.0205,0.6145)	(0.3605,0.9845)	(0.4655,0.999)
	Ph	(0.04,0.23)	(0.1755,0.5345)	(0.195,0.6145)	(0.26,0.7095)	(0.335,0.8245)
	Po	(0.04,0.23)	(0.1755,0.5345)	(0.195,0.6145)	(0.26,0.7095)	(0.335,0.8245)

Table 6(a)
Fuzzy measure for $\alpha = 0$.

CD70		CD100	
$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$	$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$
λ=-0.84	λ=-0.9994	λ=-0.84	λ=-0.9994
$g^-(A_{(2)})$=0.93	$g^+(A_{(2)})$=0.999	$g^-(A_{(2)})$=0.93	$g^+(A_{(2)})$=0.999
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=0.5387	λ=-0.9123	λ=0.5387	λ=-0.9123
$g^-(A_{(2)})$=0.58	$g^+(A_{(2)})$=0.65	$g^-(A_{(2)})$=0.58	$g^+(A_{(2)})$=0.65
$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=1.0
λ=-0.9242	λ=-0.9992	λ=-0.9242	λ=-0.9992
$g^-(A_{(7)})$=0.17	$g^+(A_{(7)})$=0.42	$g^-(A_{(7)})$=0.001	$g^+(A_{(7)})$=0.07
$g^-(A_{(6)})$=0.658	$g^+(A_{(6)})$=0.797	$g^-(A_{(6)})$=0.58	$g^+(A_{(6)})$=0.461
$g^-(A_{(5)})$=0.885	$g^+(A_{(5)})$=0.972	$g^-(A_{(5)})$=0.849	$g^+(A_{(5)})$=0.812
$g^-(A_{(4)})$=0.943	$g^+(A_{(4)})$=0.997	$g^-(A_{(4)})$=0.886	$g^+(A_{(4)})$=0.974
$g^-(A_{(3)})$=0.965	$g^+(A_{(3)})$=0.998	$g^-(A_{(3)})$=0.944	$g^+(A_{(3)})$=0.997
$g^-(A_{(2)})$=0.999	$g^+(A_{(2)})$=1.0	$g^-(A_{(2)})$=0.985	$g^+(A_{(2)})$=0.999
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.9724	λ=-0.9999	λ=-0.9724	λ=-0.9999
$g^-(A_{(3)})$=0.93	$g^+(A_{(3)})$=0.999	$g^-(A_{(3)})$=0.72	$g^+(A_{(3)})$=0.97
$g^-(A_{(2)})$=0.9989	$g^+(A_{(2)})$=1.0	$g^-(A_{(2)})$=0.732	$g^+(A_{(2)})$=0.977
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=396.95	λ=4.601	λ=396.95	λ=4.601
$g^-(A_{(3)})$=0.04	$g^+(A_{(3)})$=0.23	$g^-(A_{(3)})$=0.04	$g^+(A_{(3)})$=0.23
$g^-(A_{(2)})$=0.715	$g^+(A_{(2)})$=0.703	$g^-(A_{(2)})$=0.715	$g^+(A_{(2)})$=0.703
$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999	$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999

Table 6(b)
Fuzzy measure for $\alpha = 0$.

CG125		125DLX	
$g^{-} = (A_{(i)})$	$g^{+} = (A_{(i)})$	$g^{-} = (A_{(i)})$	$g^{+} = (A_{(i)})$
λ=-0.84	λ=-0.9994	λ=-0.84	λ=-0.9994
$g^{-}(A_{(2)})$=0.32	$g^{+}(A_{(2)})$=0.999	$g^{-}(A_{(2)})$=0.32	$g^{+}(A_{(2)})$=0.65
$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0	$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0
λ=0.5387	λ=-0.9123	λ=0.5387	λ=-0.9123
$g^{-}(A_{(2)})$=0.32	$g^{+}(A_{(2)})$=0.65	$g^{-}(A_{(2)})$=0.58	$g^{+}(A_{(2)})$=0.65
$g^{-}(A_{(1)})$=0.999	$g^{+}(A_{(1)})$=1.0	$g^{-}(A_{(1)})$=0.999	$g^{+}(A_{(1)})$=1.0
λ=-0.9242	λ=-0.9992	λ=-0.9242	λ=-0.9992
$g^{-}(A_{(7)})$=0.17	$g^{+}(A_{(7)})$=0.42	$g^{-}(A_{(7)})$=0.001	$g^{+}(A_{(7)})$=0.07
$g^{-}(A_{(6)})$=0.659	$g^{+}(A_{(6)})$=0.797	$g^{-}(A_{(6)})$=0.321	$g^{+}(A_{(6)})$=0.674
$g^{-}(A_{(5)})$=0.784	$g^{+}(A_{(5)})$=0.929	$g^{-}(A_{(5)})$=0.729	$g^{+}(A_{(5)})$=0.812
$g^{-}(A_{(4)})$=0.872	$g^{+}(A_{(4)})$=0.991	$g^{-}(A_{(4)})$=0.918	$g^{+}(A_{(4)})$=0.891
$g^{-}(A_{(3)})$=0.985	$g^{+}(A_{(3)})$=0.999	$g^{-}(A_{(3)})$=0.944	$g^{+}(A_{(3)})$=0.985
$g^{-}(A_{(2)})$=0.999	$g^{+}(A_{(2)})$=0.999	$g^{-}(A_{(2)})$=0.966	$g^{+}(A_{(2)})$=0.999
$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0	$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0
λ=-0.9724	λ=-0.9999	λ=-0.9724	λ=-0.9999
$g^{-}(A_{(3)})$=0.72	$g^{+}(A_{(3)})$=0.999	$g^{-}(A_{(3)})$=0.04	$g^{+}(A_{(3)})$=0.23
$g^{-}(A_{(2)})$=0.732	$g^{+}(A_{(2)})$=0.999	$g^{-}(A_{(2)})$=0.732	$g^{+}(A_{(2)})$=0.977
$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0	$g^{-}(A_{(1)})$=1.0	$g^{+}(A_{(1)})$=1.0
λ=396.95	λ=4.601	λ=396.95	λ=4.601
$g^{-}(A_{(3)})$=0.001	$g^{+}(A_{(3)})$=0.07	$g^{-}(A_{(3)})$=0.001	$g^{+}(A_{(3)})$=0.07
$g^{-}(A_{(2)})$=0.057	$g^{+}(A_{(2)})$=0.374	$g^{-}(A_{(2)})$=0.057	$g^{+}(A_{(2)})$=0.374
$g^{-}(A_{(1)})$=0.999	$g^{+}(A_{(1)})$=0.999	$g^{-}(A_{(1)})$=0.999	$g^{+}(A_{(1)})$=0.999

Table 7

Evaluation results by the generalized Choquet integral for $\alpha = 1$.

Criteria		Individual importance of criteria	The normalized discrepancy $\bar{f}_i = [f_i^-, f_i^+]$ and bike value $[(C)\int f^- dg^-, (C)\int f^+ dg^+]$			
			CD70	CD100	CG125	125DLX
P		(0.98,0.98)	(0.4943,0.977)	(0.325,0.891)	(0.441,0.9516)	(0.2138,0.6674)
	Pp	(0.41,0.58)	(0.215,0.585)	(0.325,0.695)	(0.5,0.785)	(0.5,0.785)
	Rp	(0.98,0.98)	(0.5,0.985)	(0.325,0.895)	(0.4,0.955)	(0.015,0.505)
C		(0.22,0.36)	(0.2954,0.585)	(0.3833,0.674)	(0.368,0.6836)	(0.4741,0.7214)
	Se	(0.41,0.58)	(0.245,0.585)	(0.355,0.695)	(0.43,0.755)	(0.43,0.755)
	Su	(0.63,0.8)	(0.325,0.585)	(0.4,0.645)	(0.325,0.585)	(0.5,0.675)
E		(0.1,0.18)	(0.2070,0.6572)	(0.3102,0.7313)	(0.4744,0.9063)	(0.4885,0.9013)
	Uc	(0.63,0.8)	(0.215,0.585)	(0.325,0.695)	(0.5,0.785)	(0.5,0.785)
	Gs	(0.63,0.8)	(0.215,0.585)	(0.325,0.695)	(0.4,0.755)	(0.5,0.785)
	Sp	(0.41,0.58)	(0.06,0.54)	(0.215,0.74)	(0.4,0.91)	(0.5,0.94)
	Br	(0.01,0.02)	(0.015,0.505)	(0.4,0.955)	(0.215,0.785)	(0.5,0.985)
	Es	(0.22,0.36)	(0.215,0.74)	(0.06,0.54)	(0.5,0.94)	(0.4,0.91)
	Pb	(0.41,0.58)	(0.15,0.63)	(0.09,0.54)	(0.43,0.91)	(0.15,0.63)
	Rg	(0.22,0.36)	(0.15,0.54)	(0.305,0.74)	(0.305,0.74)	(0.415,0.85)
S		(0.98,0.98)	(0.4969,0.977)	(0.3499,0.6409)	(0.3484,0.8898)	(0.3549,0.6889)
	Sp	(0.78,0.92)	(0.355,0.585)	(0.43,0.645)	(0.355,0.585)	(0.43,0.645)
	Sa	(0.98,0.98)	(0.5,0.985)	(0.06,0.585)	(0.325,0.895)	(0.015,0.505)
	Pr	(0.1,0.18)	(0.06,0.54)	(0.12,0.63)	(0.325,0.85)	(0.5,0.94)
F		(0.01,0.02)	(0.1494,0.5049)	(0.1944,0.5849)	(0.3207,0.6805)	(0.4158,0.7888)
	Ps	(0.01,0.02)	(0.015,0.505)	(0.06,0.585)	(0.4,0.955)	(0.5,0.985)
	Ph	(0.1,0.18)	(0.215,0.505)	(0.26,0.585)	(0.32,0.675)	(0.415,0.785)
	Po	(0.1,0.18)	(0.215,0.505)	(0.26,0.585)	(0.32,0.675)	(0.415,0.785)

Table 8(a)
Fuzzy measure for $\alpha = 1$.

CD70		CD100	
$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$	$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$
λ=-0.9706	λ=-0.9852	λ=-0.9706	λ=-0.9852
$g^-(A_{(2)})$=0.98	$g^+(A_{(2)})$=0.98	$g^-(A_{(2)})$=0.98	$g^+(A_{(2)})$=0.98
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.1548	λ=-0.8189	λ=-0.1548	λ=-0.8189
$g^-(A_{(2)})$=0.63	$g^+(A_{(2)})$=0.8	$g^-(A_{(2)})$=0.63	$g^+(A_{(2)})$=0.58
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.9656	λ=-0.997	λ=-0.9656	λ=-0.997
$g^-(A_{(7)})$=0.22	$g^+(A_{(7)})$=0.36	$g^-(A_{(7)})$=0.01	$g^+(A_{(7)})$=0.02
$g^-(A_{(6)})$=0.716	$g^+(A_{(6)})$=0.7318	$g^-(A_{(6)})$=0.634	$g^+(A_{(6)})$=0.373
$g^-(A_{(5)})$=0.91	$g^+(A_{(5)})$=0.948	$g^-(A_{(5)})$=0.878	$g^+(A_{(5)})$=0.737
$g^-(A_{(4)})$=0.937	$g^+(A_{(4)})$=0.992	$g^-(A_{(4)})$=0.912	$g^+(A_{(4)})$=0.949
$g^-(A_{(3)})$=0.976	$g^+(A_{(3)})$=0.995	$g^-(A_{(3)})$=0.961	$g^+(A_{(3)})$=0.992
$g^-(A_{(2)})$=0.999	$g^+(A_{(2)})$=1.0	$g^-(A_{(2)})$=0.99	$g^+(A_{(2)})$=0.998
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.9949	λ=-0.9985	λ=-0.9949	λ=-0.9985
$g^-(A_{(3)})$=0.98	$g^+(A_{(3)})$=0.98	$g^-(A_{(3)})$=0.78	$g^+(A_{(3)})$=0.92
$g^-(A_{(2)})$=0.999	$g^+(A_{(2)})$=0.999	$g^-(A_{(2)})$=0.802	$g^+(A_{(2)})$=0.935
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=47.23	λ=12.92	λ=47.23	λ=12.92
$g^-(A_{(3)})$=0.1	$g^+(A_{(3)})$=0.18	$g^-(A_{(3)})$=0.1	$g^+(A_{(3)})$=0.18
$g^-(A_{(2)})$=0.672	$g^+(A_{(2)})$=0.779	$g^-(A_{(2)})$=0.672	$g^+(A_{(2)})$=0.779
$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999	$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999

Table 8(b)
Fuzzy measure for $\alpha = 1$.

CG125		125DLX	
$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$	$g^- = (A_{(i)})$	$g^+ = (A_{(i)})$
λ=-0.9706	λ=-0.9852	λ=-0.9706	λ=-0.9852
$g^-(A_{(2)})$=0.41	$g^+(A_{(2)})$=0.98	$g^-(A_{(2)})$=0.41	$g^+(A_{(2)})$=0.58
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.1548	λ=-0.8189	λ=-0.1548	λ=-0.8189
$g^-(A_{(2)})$=0.41	$g^+(A_{(2)})$=0.58	$g^-(A_{(2)})$=0.63	$g^+(A_{(2)})$=0.58
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.9656	λ=-0.997	λ=-0.9656	λ=-0.997
$g^-(A_{(7)})$=0.22	$g^+(A_{(7)})$=0.36	$g^-(A_{(7)})$=0.01	$g^+(A_{(7)})$=0.02
$g^-(A_{(6)})$=0.716	$g^+(A_{(6)})$=0.732	$g^-(A_{(6)})$=0.416	$g^+(A_{(6)})$=0.588
$g^-(A_{(5)})$=0.843	$g^+(A_{(5)})$=0.889	$g^-(A_{(5)})$=0.793	$g^+(A_{(5)})$=0.737
$g^-(A_{(4)})$=0.919	$g^+(A_{(4)})$=0.891	$g^-(A_{(4)})$=0.941	$g^+(A_{(4)})$=0.833
$g^-(A_{(3)})$=0.99	$g^+(A_{(3)})$=0.98	$g^-(A_{(3)})$=0.961	$g^+(A_{(3)})$=0.969
$g^-(A_{(2)})$=0.999	$g^+(A_{(2)})$=0.998	$g^-(A_{(2)})$=0.977	$g^+(A_{(2)})$=0.996
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=-0.9949	λ=-0.9985	λ=-0.9949	λ=-0.9985
$g^-(A_{(3)})$=0.78	$g^+(A_{(3)})$=0.98	$g^-(A_{(3)})$=0.1	$g^+(A_{(3)})$=0.18
$g^-(A_{(2)})$=0.802	$g^+(A_{(2)})$=0.984	$g^-(A_{(2)})$=0.802	$g^+(A_{(2)})$=0.935
$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0	$g^-(A_{(1)})$=1.0	$g^+(A_{(1)})$=1.0
λ=47.23	λ=12.92	λ=47.23	λ=12.92
$g^-(A_{(3)})$=0.01	$g^+(A_{(3)})$=0.02	$g^-(A_{(3)})$=0.01	$g^+(A_{(3)})$=0.02
$g^-(A_{(2)})$=0.157	$g^+(A_{(2)})$=0.247	$g^-(A_{(2)})$=0.157	$g^+(A_{(2)})$=0.247
$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999	$g^-(A_{(1)})$=0.999	$g^+(A_{(1)})$=0.999

Table 9(a)
Overall evaluation results by the generalized Choquet.

Criteria		$(C)\int \bar{f} d\bar{g}$	
		CD70	CD100
		(0.453,0.4967,0.9767,0.9988)	**(0.2976,0.3568,0.8864,0.9292)**
P		(0.4442,0.4943,0.977,0.9987)	(0.2905,0.325,0.891,0.9293)
	Pp	(0.1605,0.215,0.585,0.665)	(0.2905,0.325,0.695,0.77)
	Rp	(0.4655,0.5,0.985,0.999)	(0.2905,0.325,0.895,0.9295)
C		(0.2419,0.2954,0.585,0.6562)	(0.3372,0.3834,0.674,0.7437)
	Se	(0.175,0.245,0.585,0.665)	(0.305,0.355,0.695,0.77)
	Su	(0.2905,0.325,0.585,0.64)	(0.3605,0.4,0.645,0.695)
E		(0.1506,0.207,0.6572,0.733)	(0.2662,0.3102,0.7313,0.8105)
	Uc	(0.1605,0.215,0.585,0.665)	(0.2905,0.325,0.695,0.77)
	Gs	(0.1605,0.215,0.585,0.665)	(0.2905,0.325,0.695,0.77)
	Sp	(0.0205,0.06,0.54,0.595)	(0.1605,0.215,0.74,0.805)
	Br	(0.001,0.015,0.505,0.5345)	(0.3605,0.4,0.955,0.9845)
	Es	(0.1605,0.215,0.74,0.805)	(0.0205,0.06,0.54,0.595)
	Pb	(0.1,0.15,0.63,0.69)	(0.035,0.09,0.54,0.595)
	Rg	(0.09,0.15,0.54,0.595)	(0.23,0.305,0.74,0.805)
S		(0.4539,0.4969,0.977,0.9986)	(0.2765,0.3499,0.6409,0.6932)
	Sp	(0.305,0.355,0.585,0.64)	(0.375,0.43,0.645,0.695)
	Sa	(0.4655,0.5,0.98,0.999)	(0.0205,0.06,0.585,0.6145)
	Pr	(0.0205,0.06,0.54,0.595)	(0.0855,0.12,0.63,0.69)
F		(0.1258,0.1494,0.5049,0.5345)	(0.1453,0.1944,0.5849,0.6144)
	Ps	(0.001,0.015,0.505,0.5345)	(0.0205,0.06,0.585,0.6145)
	Ph	(0.1755,0.215,0.505,0.5345)	(0.195,0.26,0.585,0.6145)
	Po	(0.1755,0.215,0.505,0.5345)	(0.195,0.26,0.585,0.6145)

Table 9(b)
Overall evaluation results by the generalized Choquet.

Criteria		$(C)\int \bar{f} d\bar{g}$	
		CG125	125DLX
		(0.3895,0.4427,0.9502,0.9884)	**(0.3025,0.4699,0.7889,0.8157)**
P		(0.3941,0.441,0.952,0.9843)	(0.1496,0.2138,0.667,0.7327)
	Pp	(0.4655,0.5,0.785,0.8395)	(0.4655,0.5,0.785,0.8395)
	Rp	(0.3605,0.4,0.955,0.9845)	(0.001,0.015,0.505,0.5345)
C		(0.3175,0.368,0.684,0.7603)	(0.4275,0.474,0.721,0.7846)
	Se	(0.375,0.43,0.755,0.825)	(0.375,0.43,0.755,0.825)
	Su	(0.2905,0.325,0.585,0.64)	(0.4655,0.5,0.675,0.7095)
E		(0.4295,0.4744,0.906,0.9621)	(0.4479,0.4885,0.901,0.9579)
	Uc	(0.4655,0.5,0.785,0.8395)	(0.4655,0.5,0.785,0.8395)
	Gs	(0.3605,0.4,0.755,0.825)	(0.4655,0.5,0.785,0.8395)
	Sp	(0.3605,0.4,0.91,0.965)	(0.4655,0.5,0.94,0.9795)
	Br	(0.1605,0.215,0.785,0.8245)	(0.4655,0.5,0.985,0.999)
	Es	(0.4655,0.5,0.94,0.9795)	(0.3605,0.4,0.91,0.965)
	Pb	(0.375,0.43,0.91,0.965)	(0.1,0.15,0.63,0.69)
	Rg	(0.23,0.305,0.74,0.805)	(0.36,0.415,0.85,0.91)
S		(0.3009,0.3484,0.8899,0.9293)	(0.2784,0.3549,0.6889,0.7568)
	Sp	(0.305,0.355,0.585,0.64)	(0.375,0.43,0.645,0.695)
	Sa	(0.2905,0.325,0.895,0.9295)	(0.001,0.015,0.505,0.5345)
	Pr	(0.2905,0.325,0.85,0.91)	(0.4655,0.5,0.94,0.9795)
F		(0.26,0.3207,0.68,0.7287)	(0.3351,0.416,0.788,0.8366)
	Ps	(0.3605,0.4,0.955,0.9845)	(0.4655,0.5,0.985,0.999)
	Ph	(0.26,0.32,0.675,0.7095)	(0.335,0.415,0.785,0.8245)
	Po	(0.26,0.32,0.675,0.7095)	(0.335,0.415,0.785,0.8245)

From Table 9(a,b), the defuzzified overall values of alternative bikes using Choquet Integral are obtained as 0.7313, 0.618, 0.6927 and 0.594 in Table 10. This means that the ranking order from the best to worst is CD70, CG125, CD100 and DLX125. This ranking order is same as the actual sale of these models in Pakistan. The best bike CD70 has the largest weight for price and spare parts, while DLX125 is for comfortable, bike power and fuel. Largest value of each criteria is highlighted with mark of * in Table 10. There are four, zero, six and fourteen largest values of CD70, CD100, CG125 and DLX125 respectively. The number of * values in any model will not decide the best model of bike. CD70 is the best model, because price and spare parts criteria have largest values, comfortable, bike power and fuel criteria have smallest value (in Table 10), according to our individual importance of criteria (in Table 3). Similar comments can be written by analyzing Table 10 and Table 3.

Table 10
Defuzzified all values of table 9(a,b).

Criteria		Defuzzified $(C)\int \bar{f} d\bar{g}$			
		CD70	**CD100**	**CG125**	**DLX125**
		0.7313*	**0.618**	**0.6927**	**0.594**
P		0.7285*	0.6089	0.6928	0.4409
	Pp	0.4064	0.5201	0.6475*	0.6475*
	Rp	0.7374*	0.61	0.675	0.2639
C		0.4447	0.5346	0.5324	0.6019*
	Se	0.4175	0.5312	0.5962*	0.5962*
	Su	0.4601	0.5251	0.4601	0.8975*
E		0.4369	0.5295	0.6931	0.6989*
	Uc	0.4064	0.5201	0.6475*	0.6475*
	Gs	0.4064	0.5201	0.5851	0.6475*
	Sp	0.3039	0.4801	0.6588	0.7212*
	Br	0.2639	0.675	0.4963	0.7374*
	Es	0.4801	0.3039	0.7212*	0.6589
	Pb	0.3925	0.315	0.67*	0.3925
	Rg	0.3438	0.52	0.52	0.6338*
S		0.7317*	0.4901	0.6171	0.5198
	Sp	0.3039	0.32	0.4801*	0.375
	Sa	0.3039	0.4801	0.6589*	0.61
	Pr	0.7374*	0.6589	0.675	0.721
F		0.3287	0.3848	0.4975	0.5941*
	Ps	0.275	0.3224	0.4911	0.6299*
	Ph	0.3575	0.3575	0.4911	0.5899*
	Po	0.3575	0.4438	0.6211	0.6461*

References

[1] S. Auephanwiriyahul, J.M. Keller and P.D. Gader, Generalized Choquet fuzzy integral fusion. Information Fusion, 3(2002), 69–85.

[2] M. Delgado, F. Herrera, E. Herrera-Viedma and L. Martnez, Combining numerical and linguistic information in group decision making. Information Sciences, 107(1998), 177–194.

[3] T. Demirel, N. C. Demirel and C. Kahraman, Multi-criteria warehouse location selection using Choquet integral, Expert Systems with Applications 37(5) (2010), 3943-3952

[4] M. Grabisch, T. Murofushi and M. Sugeno, Fuzzy measures and integrals. New York: Physica-Verlag 2000.

[5] K. Ishii and M. Sugeno, A model of human evaluation process using fuzzy integral. International Journal of Man–Machine Studies, 22(1)(1985), 19–38.

[6] J.-R. Marichal, P. Meyer and M. Roubens, Sorting multi-attribute alternatives: The TOMASO method. Computers and Operations Research, 32(2005), 861–877.

[7] P. Meyer and M. Roubens, On the use of the Choquet integral with fuzzy numbers in multiple criteria decision support. Fuzzy Sets and Systems, 157(2006), 927–938.

[8] F. Modave and M. Grabisch, Preference representation by the Choquet integral: The commensurability hypothesis. Proceedings 7th international conference on information processing and management of uncertainty in knowledge-based systems (IPMU), Paris, France, July 1998.

[9] S.F. Schnatter, On statistical inference for fuzzy data with applications to descriptive statistics. Fuzzy Sets and Systems, 50 (1992), 143–165.

[10] M. Sugeno, Theory of fuzzy integrals and its applications. Ph.D. thesis, Tokyo Institute of Technology, Tokyo.1974

[11] H.-H. Tsai, and I.-Y. Lu, The evaluation of service quality using generalized Choquet integral. Information Sciences, 176(6) (2006), 640–663.

[12] M. Vlachopoulou, G. Silleos, and V. Manthou, Geographic information systems in warehouse site selection decisions. International Journal of Production Economics, 71 (2001), 205–212.

[13] H.J. Zimmermann, Fuzzy Set Theory and Its Applications. Kluwer, Dordrecht 1991.