The term structure and the expectations hypothesis: a threshold model

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a Threshold Model

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Abstract

The expectations hypothesis implies that rational investors can predict future changes in interest rates by simply observing the yield spread. According to Mishkin (1990) the expectations theory can also be reformulated in terms of the ability of the spread to predict future inflation. Unfortunately, although appealing, the theory has found little empirical support. Time-varying term premia and changing risk perception have been advocated to rationalize the aforementioned weak empirical evidence. In this work we suggest that the time-varying nature of term premia makes single-equation models inappropriate to analyse the informative content of the term structure. In particular, when the deviations between the expected and the actual spread are large, which occurs in times of soaring term premia volatility, linear models fail to support the expectations theory. Within a threshold model for term premia, we provide evidence that the yield spread contains valuable information to predict future interest rates changes once the risk-averse attitude of economic agents is appropriately considered. Empirical results show that the predictive ability of the yield spread is contingent on the level of uncertainty as captured by the size of monetary policy surprise.

JEL classification: C01, C30, E43, G12.

Keywords: Expectations Hypothesis, Term Premia, Threshold Models.
1. Introduction

Forecasting future interest rates has always been a major concern of both economists and policymakers. Understanding the dynamics of interest rates is important for financial economists, because, for instance, the price of derivative securities depends on market’s yields; it is equally essential for macroeconomists, as long as aggregate-spending decisions depend on long term interest rates, while the opportunity cost of holding money is represented by short term rates. In addition, understanding the relationship between short and long term rates is relevant for policymakers. Although the monetary authority might be interested in influencing long term interest rates, monetary policy actions can only affect the short end of the term structure; moreover, the Treasury can eventually perform active debt management, since the maturity structure of public debt affects the government budget.

In this paper we attempt to rationalize the empirical failure of the expectations hypothesis by exploiting the potentiality offered by non linear modelling. In particular, this study investigates whether threshold effects are relevant in the empirical analysis of the expectations theory. Working with U.S. post-war data in a multiple regime framework we examine the informative content of the U.S. term structure. Evidence suggests that non linearity can account for the empirical failure of the expectations hypothesis; in particular, we find that the predictive ability of the yield curve’s slope to anticipate future movements in short rates is contingent on both the level and the volatility of term premia. Threshold estimates indicate that the yield spread returns an accurate prediction of future short term rates when the absolute value of the term premium is low.

The interest in non linear models in general, and in threshold models in particular, is motivated by the fact that the expectations hypothesis (henceforth EH) has been usually rejected by linear models. The empirical failure of the EH in single-equation models has been often attributed to the presence of a time-varying term premium (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tzavalis and Wickens, 1997). In addition, McCallum (1994, 2006) has shown the intrinsic inability of linear models to corroborate the EH simply considering a time-varying first-order autoregressive term premium coupled with a monetary policy rule that allows for interest rate smoothing. Hence, we believe that the time-varying nature of the term premium might be considered a significant source of asymmetry in the empirical analysis of the expectations theory. In this vein, this contribution extends and complements the strand of literature pioneered by Campbell and Shiller (1991). Although the slope of the term structure gives a forecast in the right direction of long term changes in short rates the predictive ability of linear models is usually modest. Therefore, since linear models can be viewed as constrained non linear models, detection of non linearity can improve the ability of the spread to anticipate the evolution of future interest rates. Results highlight
an asymmetric reaction of short term rates to changes in the yield spread; moreover, the response of interest rates to the spread dynamics appears to be conditioned on the level of the term premium. We thus provide significant evidence that the informative content of the term structure is related to the uncertainty about the future conduct of monetary policy.

It has been noted by Laurent (1988, 1989), Bernanke and Blinder (1992), that the yield spread, i.e. the difference between long and short term interest rates describes the expectations regarding the incoming monetary policy stance. The monetary authority can easily influence the short end of the yield curve, while long term rates are generally market-driven and do not react hastily to policy interventions. Furthermore, the long term rate can act as a proxy for the equilibrium short term rate, i.e. a Wicksellian natural rate; so that, the yield spread can be thought as a measure of the relative tightness of policy. In this paper, we focus on the unexpected component of the yield spread, i.e. the term premium, which is regarded as a measure of monetary policy surprise. It has been shown that the yield spread can be decomposed into two elements, an expectations-based component and a term premium, which may be thought as the sum of a liquidity premium and a risk premium (Campbell and Shiller, 1991; Hamilton and Kim, 2002; Favero, Kaminska, and Soderstrom, 2005; Rudebusch, Sack, and Swanson, 2007). In this paper we assume that the term premium is not only time-varying, but also regime dependent; in particular, we allow the term premium to determine distinct regimes, i.e. different states of the world, in which the EH is examined.

The multiple regime model allows us to assess whether the Mankiw and Miron (1986) view, that the increased unpredictability of short term rates affects the predictive power of the spread, is supported by data. Moreover, the threshold model is also suitable to verify the idea put forward by Campbell (1995) who argues that the term structure is more informative about future interest rates movements when the variability of the expected changes in short rates is higher than the variability of the term premium. Finally, the two-regime framework provides a useful apparatus to test the thesis put forward by Thornton (2004); he points out that also the relative variance of short to long rates is empirically relevant for the success of the EH. Our empirical findings suggest that both the volatility of short rates and the relative variability of the theoretical spread to the term premium matter in the empirical analysis of the expectations hypothesis.

The rest of the paper is organized as follows. In the next Section we discuss a selected survey of the literature. Section 3 describes data. In Section 4 we discuss the empirical model and provide preliminary evidence of non linearity in the expectations equation. In Sections 5 we present the empirical results. Section 6 concludes.
2. Literature Review

The expectations hypothesis states that the long term yield can be expressed as the average of expected future short term yields\(^1\). The expectations theory, however, has always found little empirical support. Campbell and Shiller (1991) conclude "we thus see an apparent paradox: the slope of the term structure almost always gives a forecast in the wrong direction for the short term change in the yields on longer bonds, but gives a forecast in the right direction for long term changes in short term rates". The weak empirical support for the expectations hypothesis\(^2\) has thus inspired numerous empirical studies.

Campbell and Shiller (1991) and Hardouvelis (1994) suggest that the empirical failure may be due to an over-reaction of long rates to the expected change in short rates. In addition, Hardouvelis (1994) believes that large measurement errors can account for the forecast in the wrong direction of long term rate prediction. Fama (1986), Cook and Hahn (1989), Lee (1995), Tsavalis and Wickens (1997), among others, argue that a time-varying term premium correlated with the spread can account for the empirical failure of EH. Froot (1989), however, indicates that a violation of the rationality principle, rather than a time-varying risk premium, is one of the main reasons underlying the rejection of EH. McCallum (1994) warns that the rejection of EH might be simply due to a misspecification of the equation used to test the theory. In particular, he believes, as we do, that single-equation models may be inappropriate to test the EH. He points out the traditional Campbell-Shiller equation is misleading to think in terms of the predictive power of the spread. He shows that estimates of the slope coefficients are inherently lower than one when allowing for both a time-varying risk premium (first-order stationary autoregressive process) and a specific monetary policy rule that features interest rate smoothing and responds to the spread dynamics.

Mankiw and Miron (1986) provide with a suggestive explanation of the inability of the spread to predict future movements in interest rates. They show that the slope of the yield curve seems to have substantial predictive power to anticipate future short term rate dynamics before the creation of the Fed. Between 1890 and 1915 the high predictability of short term rates was due to a clear mean reverting behaviour displayed by the short term policy rate; however, after 1915, the interest rate smoothing policy pursued by the monetary authority has enhanced the difficulty of forecasting short term rates, reducing the predictive power of the spread. The random walk path followed by the

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\(^1\) The expectations hypothesis implies both that the forward rate equals the future spot rate and that the expected holding period return is constant, i.e. equal, on bonds of all maturities.

\(^2\) Thornton (2003) has introduced a suitable terminology to distinguish the empirical testing of the expectations hypothesis. The regression for predicting short term rate changes over the life of the long term bond is labelled the conventional test of EH. This test returns a positive estimation of the slope coefficient although less than one; the conventional test thus gives a forecast in the right direction for long term changes in short rates. The equation to predict long term rates is called the contrarian test, since it returns negative estimates of the slope coefficient, i.e. a forecast in the wrong direction. In this paper we focus only on the conventional test of the EH.
short term policy rate has largely affected the predictability of short term rates. Mankiw and Miron admit they cannot fully explain the failure of the EH though; however, they believe that time-varying risk premia, change in risk perception, adjustments in relative asset supplies, measurement errors, and, finally, near rational, rather than rational, expectations can play a role in explaining the empirical rejection of EH. Koo and Thornton (2004) find that, if financial panics in 1907 – 1908 are properly taken into account, the evidence in favour of EH is not significantly different before and after the foundation of the Federal Reserve. They show that the apparent support for the EH theory before 1915 is merely due to some extreme observations. In line with Mankiw and Miron (1986), Campbell (1995) points out that, in the regression for predicting short term rate over the life of the long term bond, changing rational expectations about long bond returns act like a measurement error; this kind of measurement error biases the coefficient downward toward zero. The magnitude of the bias depends on the relative variances of the two components of the spread. In particular, the larger the variance of the theoretical spread relative to the variance of the term premium, the smaller the size of the bias, i.e. the closer the estimated slope coefficient to one. In such a situation, agents are supposed to be well informed about future interest rate changes, since the variance of the term premium, which is the unexpected component of the yield spread, is relatively small.

Rudebusch (1995) finds that the ability of the term structure to predict future changes in short rates is quite good at very short horizons, i.e. shorter than about one month. However, he argues that at horizons longer than two years there is some evidence that the predictive power increases. The EH has also been examined in different monetary regimes. Hardouvelis (1988) using weekly data shows that the spread carries substantial predictive power between October 1979 and October 1992. In the same vein, Simon (1990) finds that the slope of the term structure significantly anticipates future changes in interest rates during the non-borrowed reserve operating procedure. Roberds, Runkle, and Whiteman (1996) provide evidence regarding the informative content of the yield curve using daily data for settlement Wednesdays. Thornton (2005) believes these results are contrary to common wisdom. He rationalizes these tricky empirical findings and concludes: “these results are anomalous in that they suggest that the funds rate is more predictable (1) during periods when the Fed is targeting monetary aggregates than when it is explicitly targeting the federal funds rate and (2) on days when there are large idiosyncratic shocks to the federal funds rate. I argue that the funds rate should be more predictable when the Fed is explicitly targeting it.... In addition, I show that settlement Wednesday changes in the funds rate can, at best, account for a very modest improvement in the market’s ability to predict the funds rate”.

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On the other hand, the predictive power of the spread has been also examined in respect of its ability to anticipate future inflation. In 1977 Robert Shiller and Jeremy Siegel provide an empirical assessment of the so-called Gibson paradox. Using British data, Gibson (1923) found a strong positive correlation between the (log) series of a price index and long term interest rates. Using spectral techniques, Shiller and Siegel confirm the aforementioned relation.

Mishkin (1990) investigates whether the term structure of interest rates helps to forecast future inflation. He finds that interest rates on bonds with maturities less than or equal to six months do not provide any relevant information about the future path of inflation. However, the short end of the term structure seems to contain substantial information about the term structure of real interest rates. Conversely, for maturities between nine and twelve months, the slope of the term structure appears to carry information about future inflation but not about the real term structure. In line with the prevalent view, Mishkin provides also evidence that an inverted yield curve reflects expectations of a declining rate of inflation. Estrella and Mishkin (1997) analyse the predictive content of the spread regarding the future level of both output and inflation. Estimates for U.S. indicate that at very short horizon the ability of the spread to anticipate future movements in inflation is either negligible or absent. The predictive accuracy of the spread increases at long horizons; in particular the slope of the term structure is informative about future inflation over horizons of three to five years. Estrella and Mishkin have also documented the ability of the spread to predict future economic growth.

A recent strand of research has focused on the ability of term premium, rather than of the spread, to predict future movements in economic activity. The term premium is derived from a decomposition of the spread into an expectations-based factor and a risk premium. Hamilton and Kim (2002) show that both components are informative for predicting real GDP growth. Their findings suggest that a decrease in the term premium predicts slower GDP growth. Favero, Kaminska, and Soderstrom (2005) obtain similar results. Ang, Piazzesi, and Wei (2006) find that the coefficient of the expectational component is larger in magnitude than the estimated coefficient of the term premium, which is not statistically significant. Finally, in spite of existing evidence, Rudebusch, Sack, and Swanson (2007) provide evidence of inverse relation between term premia and business cycle, in that respect a decline in term premia tends to be a stimulus for economic activity. They rationalize this atypical result concluding “we only speculate that our empirical findings may reflect a heterogeneous population in which a decline in the term premium makes financial markets conditions more accommodative for certain classes of borrowers”. 
3. Data

All data employed in the analysis have monthly frequency. The entire sample goes from January 1964 to June 2007. United States ZCB yields data from January 1964 and December 1998 are from either the McCulloch-Kown database (3-month, 6-month, and 10-year) or from the Fama-Bliss dataset (1-, 2-, 3-, 4-, 5-year)\(^3\). From January 1999 to June 2007 all yields data are from the Datastream database (ZCB yields). The effective federal funds rate series is from the Federal Reserve Economics Database (FRED). Below we plot the federal funds rate, the 3-month, and the 60-month yields from January 1964 to June 2002, the range over which the empirical analysis is performed.

![Figure 1](image)

Rather than in yields’ level, we are interested in the spreads. We thus compute the spreads between long term yields and the 3-month yield. Campbell and Shiller (1991) show that the yield spread can be decomposed into the expected change in short term rates and a term premium:

$$i_t^n - i_t^m = \left( \frac{m}{n} \sum_{q=0}^{n-m} E_{t+q} i_{t+q}^m - i_t^m \right) + tp_{t}^{n,m} \tag{1}$$

where \(n\) denotes the long term maturity and \(m\) indicates the short term maturity (\(m = 3\)). The term premium associated to the combination of maturities \((n, m)\) is \(tp_{t}^{n,m}\). The first element on the RHS is the expectations-based component, otherwise known as the theoretical, or perfect foresight, spread.

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\(^3\) McCulloch data are available from the Gregory R. Duffee web page; while the Fama-Bliss yields data are from Cochrane and Piazzesi (AER, 2005).
Table 1 shows some descriptive statistics about the spread and its two components when the short term rate is \( m = 3 \) months\(^4\).

<table>
<thead>
<tr>
<th>sample jan64-sep97</th>
<th>long term maturity (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>obs 405</td>
<td></td>
</tr>
<tr>
<td>spread</td>
<td>mean 0.2306 0.4216 0.6446 0.8116 1.0064 1.3204</td>
</tr>
<tr>
<td></td>
<td>st.dev. 0.2348 0.4656 0.7447 0.9395 1.1693 1.3977</td>
</tr>
<tr>
<td>thsprn</td>
<td>mean 0.0058 0.0174 0.0339 0.0538 0.0820 -0.0690</td>
</tr>
<tr>
<td></td>
<td>st.dev. 0.5492 0.9302 1.4742 1.8200 2.1443 2.4942</td>
</tr>
<tr>
<td>tp</td>
<td>mean 0.2463 0.4221 0.6185 0.7545 0.9159 1.3697</td>
</tr>
<tr>
<td></td>
<td>st.dev. 0.6017 0.9926 1.5284 1.8281 2.0629 2.4257</td>
</tr>
</tbody>
</table>

**Table 1**

The mean of both the spread and the term premium is increasing with maturity \( n \); whereas the mean of the theoretical spread is almost constant\(^5\). The standard deviation of all variables is increasing with maturity. According to Campbell (1995), the empirical failure of the expectations theory can be rationalized by noting that the standard deviations of the expected changes in the interest rates (theoretical spread) are smaller relative to the standard deviations of the term premia. In our data set this occurs for medium-short horizons \( n \leq 36 \). Term premia reflect the unexpected future changes in interest rates, i.e. the unpredictable evolution of the yield curve, which is regarded to be a measure of investors’ risk aversion; in addition, when agents are well informed about future movements in interest rates the variability of term premia should be lower than the variability of the perfect foresight spread. Consistently with this idea, Mankiw and Miron (1986), among others, suggest that the empirical failure of the expectations hypothesis can be attributed to the increased unpredictability of interest rates after the creation of the Federal Reserve System.

In Figure 2 we plot the yield spreads (left diagram) and the term premia (right diagram). Shaded areas indicate periods of NBER recession\(^6\). The spreads tend to be negative immediately before recessions. This fact is consistent with the prevalent view that an inverted, or flat, yield curve anticipates a decline in economic activity; since a flat yield curve is supposed to reflect agents’ expectations of a severe tightening in the monetary policy conduct. On the contrary, term premia tend to rise before recessions, denoting an accentuated risk-averse attitude of investors in bad states of the world.

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\(^4\) The sample ends in September 1997 which is the most recent available observation for the theoretical spread obtained by rolling over the 10-year bond.

\(^5\) Similar results are obtained by Campbell (1995).

All the time series are covariance stationary. The results of both the Augmented Dickey-Fuller (ADF), the Kwiatkowski-Phillips-Schmidt-Shin (KPSS), and the Phillips-Perron (PP) tests are reported in Table 2. The top panel of the table reports the probability values of the ADF test. In the central panel the LM statistics values of the KPSS test are displayed. In the bottom panel we provide the results of the Phillips-Perron test. Both the ADF and the Phillips-Perron tests lead to the rejection of the unit root hypothesis; while, the null hypothesis of stationarity cannot be rejected by the KPSS test\(^7\).

<table>
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<th>sample jan64-sep02</th>
<th>long term maturity (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>obs 465</td>
</tr>
<tr>
<td>spread</td>
<td>ADF</td>
</tr>
<tr>
<td>thspr</td>
<td>ADF</td>
</tr>
<tr>
<td>tp</td>
<td>ADF</td>
</tr>
<tr>
<td>spread</td>
<td>KPSS</td>
</tr>
<tr>
<td>thspr</td>
<td>KPSS</td>
</tr>
<tr>
<td>tp</td>
<td>KPSS</td>
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<tr>
<td>spread</td>
<td>PP</td>
</tr>
<tr>
<td>thspr</td>
<td>PP</td>
</tr>
<tr>
<td>tp</td>
<td>PP</td>
</tr>
</tbody>
</table>

The short term maturity is always m = 3 months. Exogenous: *Intercept, **Intercept and Trend; a18 lags; bestimated with fixed spectral autoregressive least squares.

\(^7\) To match the monthly frequency of data, the rule of thumb selected number of lags in the auxiliary regression is 12. The automatic lag selection based on different criteria (Akaiake, Schwarz, Hannan-Quinn) is roughly consistent with our choice. Unit root test results obtained with the automatic lag selections are similar. The KPSS critical values are 0.739 (1%), 0.463 (5%), and 0.347 (10%) when the intercept is included in the model. Only in one case (120, 3) we also include the time trend in the KPSS auxiliary regression. The KPSS test critical values if also the trend is added are 0.216, 0.146, and 0.119 at 1%, 5%, and 10% significance levels respectively.
4. Interest Rates Prediction: Methodology

The expectations hypothesis establishes a relationship between long term \((n)\) and short term \((m)\) interest rates. The theory asserts that the long term yield can be expressed as an average of expected future spot rates as implied by Equation (1). We can also say that forward rates are unbiased predictor of future spot rates.

\[
i_t^n = \left(\frac{m}{n}\right) \sum_{q=0}^{m} E_t i_{t+mq}^m + tp_t^{n,m}
\]

(2)

where \(i_t^n\) and \(i_t^m\) are the long term and short term yields with maturity \(n\) and \(m\) respectively \((m < n)\). The ratio \((m/n)\) should be an integer. The operator \(E_t\) represents rational expectations conditional to the information available at time \(t\). The pure version of the expectations hypothesis implies a null term premium \((tp_t^{n,m} = 0)\); while the traditional version of EH assumes that \(tp_t^{n,m}\) is simply constant over time. If the expectations hypothesis holds, i.e. the \(n\)-period yield equals the average of the market’s expectations for the \(m\)-period rates, therefore the expected holding period returns are equal on bonds of all maturities. Developing the summation and ignoring the constant premium we get:

\[
i_t^n = \left(\frac{m}{n}\right) i_t^m + \left(\frac{m}{n}\right) \sum_{q=0}^{m} E_t i_{t+mq}^m
\]

(3)

Let \(i_t^{n-m}\) denote the forward (implicit) rate from time \(t+m\) to \(n\) (the life of the associated artificial bond thus lasts \(n-m\) periods):

\[
i_t^n = \left(\frac{m}{n}\right) i_t^m + \left(\frac{n-m}{n}\right) E_t i_{t+m}^{n-m}
\]

(4)

Both sides of (4) must be equal in absence of arbitrage opportunities. Equation (4) simply states that a maturity strategy (LHS) must generate the same rate of return of a roll-over strategy (RHS). The spread, which is a measure of the term structure slope, is obtained on the LHS by subtracting the short yield from both sides.

The following regressing equation captures the predictive ability of the spread to anticipate future variations in the short yields movements over the life of the long term bond:
\[
\sum_{q=1}^{n-m} \left( \frac{n-mq}{n} \right) (i_{t+mq}^m - i_{t+mq(q-1)}^m) = \alpha + \beta (i_t^m - i_t^m) + \varepsilon_t
\]  

(5)

Henceforth this equation will be referred to as the Campbell-Shiller equation. The above model is equivalent to the following:

\[
\left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E^m_{t+mq} - i_t^m = \alpha + \beta (i_t^m - i_t^m) + \varepsilon_t
\]  

(5')

According to Thornton (2004), we label Equation (5) the conventional test. The LHS of (5) is the theoretical, or perfect foresight, spread. The expectations hypothesis holds if the estimated coefficients \( \alpha \) and \( \beta \) are zero and one respectively, i.e. if the actual spread is equal to the theoretical spread. The above regression generates \( n \)-period overlapping errors, causing OLS residuals to be serially correlated; residuals thus follow a moving average stochastic process. In order to deal with the non-spherical disturbances issue, Hansen-Hodrick (1980) and Newey-West (1987)\(^3\) have suggested a consistent estimate of the variance-covariance matrix.

Unfortunately, in the financial literature the expectation hypothesis has found weak empirical support; the estimated slope coefficient \( \hat{\beta} \) in (5) is almost always below unity. The presence of a time-varying risk premium is widely acknowledged to be a potential cause of the EH failure (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tzavalis and Wickens, 1997; Hejazi and Li, 2000).

In this paper we estimate a threshold model (Hansen, 2000) in which we use the term premium to separate regimes. Threshold (THR) models are a special case of Markov switching models in which the probability of switching regime is known ex ante. A natural approach to modelling non linear economic relationships seems thus to define different states of the world, or regimes, and to allow for the possibility that the behaviour of economic and financial variables depends upon the regime that occurs at any different point in time. Regime switching models à la Markov imply that the transition between regimes occurs with a certain probability that needs to be estimated. THR models can be considered a deterministic version of Markov switching models, in which the transition between regimes occurs whenever the threshold variable assumes a certain identified value. In this sense, we say that Markov switching models nest threshold models.

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The main reason behind our methodological choice stands in the key difference between threshold modelling and a structural change model. Although conceptually similar, the difference between these model is important. In a standard structural change (SC) model the sample is split at one point in time, hence regimes are defined temporally. In this study, instead, we focus on the possibility that regimes can switch back and forth depending on the value of a threshold variable. Threshold modelling is a flexible tool which allows to capture switches in regime that occur frequently over time, as it might happen in bond markets; interest rates dynamics are highly responsive to a complex sequence of small macroeconomic and financial shocks, so that agents reformulate continuously their expectations as soon as new information become available. SC and THR are different also from a statistical point of view, since structural change models usually imply a time trend, either in the explanatory or in the threshold variable (or both), that affects the distribution of the threshold variable, which is, instead, stationary in THR models.

In the next Section we estimate a threshold model for term premia with two regimes:

\[
\begin{align*}
\left\{ \frac{m}{n} \sum_{q=0}^{n-m} E_i t_{t+mq}^m \right. & \left. \cdot i_{t}^m \left( \alpha + \beta \left( i_{t}^m \cdot i_{t}^m \right) \right) + \varepsilon_t, \quad \text{if } th \leq \hat{\tau} \\
\left. \frac{m}{n} \sum_{q=0}^{n-m} E_i t_{t+mq}^m \right. & \left. \cdot i_{t}^m \left( \alpha + \beta \left( i_{t}^m \cdot i_{t}^m \right) \right) + \varepsilon_t, \quad \text{if } th > \hat{\tau} \\
\end{align*}
\]  

(6)

The threshold variable \(th\) is either the term premium \(tp_{t}^{m,n}\) or its absolute value. The term premium is computed as the unexpected change in short term rates, so that it captures the risk adverse attitude of economic agents. The term premium is obtained by subtracting the short term yield from both sides of Equation (2). Equation (1) shows that the yield spread can be decomposed into an expectational component and a term premium\(^9\).

The threshold methodology allows us to match two salient features outlined in the empirical literature regarding the EH. On the one hand, we allow the term premium to be time-varying; in particular, the term premium is also assumed to be regime-dependent \(tp(\hat{\tau}, t, n, m)\). On the other hand, we follow Mankiw and Miron (1986), who put forward the idea of using some measure for uncertainty to separate regimes. They suggest that the predictive ability of the spread is conditional on the agents’ capability of anticipating future movements in short rates; in particular, they argue that the short term rate has become a martingale, and thus unpredictable, after the founding of the Federal Reserve System: “the Fed has announced to stabilize -or even to peg- the interest rates”.

\(^9\) It is possible to demonstrate that the term premium is a function of the future path of the stochastic discount factor (or pricing kernel) used to price any asset in the economy. The stochastic discount factor provides with a measure of the intertemporal marginal rate of substitution.
If we assume that parameters are not constant over time, we can split the entire sample into two or more sub-samples; each sub-sample corresponds to a specific regime, determined by the estimated value of a selected threshold variable \((th)\). It is reasonable to assume that agents respond asymmetrically to shocks provided they expect different scenarios; as well as it is plausible to analyse differently the response of financial variables to macroeconomic news in good or bad times. Technically, threshold estimates are obtained by minimizing the sum of squared residuals in both regimes; computationally the minimization process takes the form of a grid search. Once the threshold value is determined, the estimation process is equivalent to estimating a regression with deterministic dummy variables. The threshold effect is simply denoted by the difference of parameter estimates in sub-regimes.

Hansen has proposed a statistical test to check for the presence of a threshold. Consider the null hypothesis \(H_0: \tau_0 = \tau\), where \(\tau_0\) is the true value of the threshold, while \(\tau\) an arbitrarily fixed value. To test the hypothesis on the threshold we use the following \(F\) statistics:

\[
LR_n(\tau) = n \frac{S(\tau) - S(\hat{\tau})}{S(\hat{\tau})} \tag{7}
\]

where \(n\) is the number of observations, and \(S(\tau)\) is the concentrated sum of squared residuals. The likelihood ratio statistics has a non-standard distribution which depends on \(\eta^2\). In particular, \(\eta^2\) is equal to unity in case of homoscedasticity; while in case of heteroscedasticity it must be estimated (Hansen, 2000). Test results are valid asymptotically; so that \(n\) needs to be greater than 100. The sequence of the likelihood ratio is then used to obtain 95% confidence intervals.

Consider now the Campbell-Shiller equation (5). In the top panel of Figure 3 we plot the likelihood ratio sequence against the threshold variable \((np)^{n,m}\). The left diagram refers to the pair of maturities \((60, 3)\); the right diagram show results for the combination \((120, 3)\). In both cases the \(LR\) sequence breaks the 5% critical value line suggesting the presence of a breakpoint, i.e. one threshold. Results for the other couples of maturities \((n, 3)\) are similar; test results reveal the presence of two regimes.

![Figure 3](image-url)
Figure 4 plots the $F$ sequence to test for the presence of a threshold. The null hypothesis of linearity is rejected against the alternative of one threshold if the $F$ sequence exceeds the 5% critical value line. In both cases, i.e. for both couples of maturities (60, 3) and (120,3), there is clear evidence of non linearity.

In this paper, asymmetry can be regarded as any non-constant effect of the yield spread on the theoretical spread. In testing for non linearity, Andrews (1993) points out that searching over a grid invalidates standard statistical inference: assessing the significance of the threshold with the usual $t$-statistics is actually not feasible. Hansen (1996) has proposed a method to overcome this issue. He suggests making inference using $p$-values obtained by estimating an asymptotic distribution of the test statistic through bootstrapping. In this work, we use 1000 bootstrap replications to simulate the asymptotic distribution. In Table 3 we report the bootstrap probability values. The null hypothesis of absence of threshold effect (linearity) is rejected in favour of a multiple regime model. The threshold variables tested for non linearity in the Campbell-Shiller equation are both the term premium and its absolute value, respectively in the first and in the second row of Table 3.

<table>
<thead>
<tr>
<th>Test for Non Linearity</th>
<th>long term maturity ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$tp_{<em>,n}^{</em>,n}$</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$</td>
<td>p_{<em>,n}^{</em>,n}</td>
</tr>
</tbody>
</table>

Table 3

In the following part of this Section we provide some further evidence of non linearity in the equation to test the EH. Parameter instability in Equation (5) would eventually provide a justification for estimating a non linear econometric model. Hansen (1992) has proposed a test to check for parameter constancy in linear models which does not require any prior knowledge of the timing of the structural break; this feature makes the test appealing compared to the popular Chow
test. In addition, the Hansen test is not subject to criticism regarding both the CUSUM and CUSUM of squares proposed by Brown, Durbin, and Evans (1975). In particular, the former has been criticized for being a trivial test to detect instability in the intercept of a model; while the latter suffers from poor asymptotic power. The test proposed by Hansen has local optimal power. The variables in the linear model (5) must be weakly dependent process, that is they cannot contain any deterministic or stochastic trend$^{10}$. A linear model returns efficient estimates when the disturbance term is a zero mean process $E(e_i | x_i) = 0$ with constant variance $E(e_i^2) = \sigma^2$. In addition, zero covariance between noise and the explanatory variables is necessary to presume the model is correctly specified $E(x_i' e_i) = 0$. The Hansen test statistics are based on the cumulative sums of the aforementioned first-order conditions. The test is used to check for both individual (Li) and joint (Lc) parameter stability. Asymptotic critical values for the individual stability test are provided by Hansen (1992)$^{11}$. The null hypothesis of joint parameter constancy implies that the first-order conditions are zero-mean, i.e. the cumulative sum tends to be distributed around zero. The non-standard distribution is tabulated by Hansen (1992)$^{12}$.

There are three explanatory variables in the Campbell-Shiller equation (5) including both the constant and the errors variance. Test results are displayed in Table 4. The calculated statistics associated to the expectations hypothesis equation are extremely high for any couple of maturities (n, m); the null hypothesis of parameter constancy is decisively rejected. The Hansen test thus suggests clear parameter instability.

<table>
<thead>
<tr>
<th>Hansen Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(120, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
<tr>
<td>(60, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
<tr>
<td>(36, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
<tr>
<td>(24, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
<tr>
<td>(12, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
<tr>
<td>(6, 3)</td>
</tr>
<tr>
<td>spread</td>
</tr>
<tr>
<td>var</td>
</tr>
<tr>
<td>joint Lc</td>
</tr>
</tbody>
</table>

sample jan64-sep02

---

$^{10}$ As shown in Section 3 all the variables are covariance stationary.

$^{11}$ The 5% critical value is 0.47; while the 10% is 0.353. Large values of the test statistics implies a violation of the first-order conditions, and thus lead to the rejection of the null hypothesis of parameter stability.

$^{12}$ At 5% significance level the critical value is 1.01, while the 10% critical value is 0.846. The null hypothesis of joint parameter stability is rejected if the test statistics exceeds the critical values.
Finally, Strikholm and Terasvirta (2005) have proposed a testing procedure to determine the optimal number of regimes within a threshold analysis. In this work it seems reasonable to restrict the choice between two or, at most, three regimes, since both the spreads and its components are strictly stationary time series. The idea of the test is to compare the linear model with smoothed transitions regressions that allow for either two or three regimes\textsuperscript{13}. Test results are supportive of the two-regime specification against both linearity and three-regime model, as shown in Table 5.

<table>
<thead>
<tr>
<th>Strikholm - Terasvirta Test</th>
<th>Number of Regimes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 vs 2</td>
</tr>
<tr>
<td>((n, m))</td>
<td></td>
</tr>
<tr>
<td>(120, 3)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(60, 3)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(36, 3)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(24, 3)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(12, 3)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(6, 3)</td>
<td>(0.996)</td>
</tr>
</tbody>
</table>

\textit{Table 5}

5. Empirical Results

In this Section we present some further evidence regarding the instability over time of the linear equation to test the expectations hypothesis; then we present the threshold estimates. We believe that when the deviations between the expected and the actual spread are large, which occurs when also the variance of the term premium shoots exponentially, single equation models fail to support the expectations theory. We thus propose a non linear model to analyse the informative content of the term structure. The empirical analysis of the expectations hypothesis is performed within a threshold framework in which regimes are determined by the level of the term premium. We recall that the term premium is proxy for monetary policy surprise, since it measures the unexpected component of the yield spread. According to Campbell and Shiller (1991), the term premium is

\textsuperscript{13} The testing procedure implies a sequential comparison of the sum of squared residuals obtained by logistic-STR models with different number of regimes. In particular, if the one-regime specification is rejected against the alternative two-regime specification, we proceed to test the two-regime model against the three-regime model, and so on. Smooth transition regressions are based on the logistic function. The test considers the first-order Taylor approximation of the logistic function around the parameter that governs the transition between regimes. The test has been performed both with and without the Taylor approximation obtaining similar results. The practical implementation of the test requires specification of the parameters. In testing two regimes against linearity we set the threshold value in the logistic function equal to the mean of the series. This value has been chosen to make the test independent of the threshold estimate. After estimating the threshold model, we also performed the test with the estimated threshold value \( \hat{T} \) obtaining similar results. In both cases the non linear framework has been preferred to the null of linearity. Without loss of generality, the strategy followed for testing two against three regimes has been to set, threshold \( c_1 \) and threshold \( c_2 \) respectively equal to the 33\% and the 66\% quantiles in the three-regime logistic-STR specification. Following Strikholm and Terasvirta we use an asymptotic \( F \) approximation of the \( \chi^2 \) test statistics.
computed as the unexpected change in the long term interest rate, which is also the unanticipated change in the yield spread:

\[ tp_{1}^{e} = i_{1}^{e} - \left( \frac{m}{n} \sum_{q=0}^{n-m} E_{t}i_{t+mq}^{m} \right) \]  

(8)

For instance, in early 1980s, when the economy was suffering from soaring inflation due to effects of the oil shocks, term premia were unusually large; moreover, GDP growth was weak and the rate of growth of industrial production became negative. Hence, the increased uncertainty affecting the economy was also reflected in financial markets affecting the risk-averse attitude of both investors and consumers. In general, large negative values of the output gap, due to substantial deviations of the actual GDP from its potential, induce the monetary authority to change preferences in conducting monetary policy, since it becomes socially optimal to include unemployment among the final targets. The enhanced complexity of the macro scenario contributes to the changing behaviour of the monetary authority which, in turn, affects the ability of agents to anticipate accurately the future dynamics of the term structure. Large term premia reflect both the market inability of forecasting future monetary policy stances and market participants’ difficulty in anticipating the future dynamics of the term structure slope.

We estimate the Campbell-Shiller Equation (9) by means of a rolling procedure to highlight the time-varying pattern of the slope coefficient\(^{14}\) (\( \beta \)).

\[ \left( \frac{m}{n} \sum_{q=0}^{n-m} E_{t}i_{t+mq}^{m} - i_{t}^{m} \right) = \alpha + \beta (i_{t}^{n} - i_{t}^{m}) + \varepsilon_{i} \]  

(9)

Each regression is estimated by OLS (with Newey-West corrected standard errors) on samples with 60 monthly observations, i.e. five years. Figure 5 shows the rolling estimation of the slope coefficient and the associate probability values of the t-test (null hypothesis \( \hat{\beta} = 0 \)) for the pairs of maturities (60, 3) and (120, 3). Results for other combination \((n, m)\) confirm the variable path of the slope coefficient. We recall that the expectation hypothesis holds if \( \beta = 1 \) and \( \alpha = 0 \); nevertheless, following standard practice, we mainly focus on the slope coefficient \( \beta \). The rolling estimated slope tends to fluctuate around one; deviations from the value implied by the EH are substantial though, that is, the variance of the estimated coefficient is quite large over time.

\(^{14}\) Equation (9) here is Equation (5) in Section 4.
These results, coupled with the statistical tests performed in Section 4, suggest the presence of non-linearity in the empirical equation used to test the expectations hypothesis. In particular, the slope estimates tend to be statistically not significant when both the level and the volatility of term premia are large.

There is broad consensus on the role played by that time-varying term premia in explaining the empirical failure of the expectation hypothesis (Fama, 1986; Cook and Hahn, 1989; Campbell, 1995; Lee, 1995; Tzavalis and Wickens, 1997). In the expectation model the term premia effect is captured partially by the residual term and partially by the intercept of the model. Consequently, a change in the level of term premia, due for instance to changing conditions of the economy, affects the empirical assessment of the expectations theory; the term premia effect cause a shift in the intercept of the model, which in turns generates a bias in the slope estimate; we thus suggest that linear models are not appropriate to test the EH.

In order to deal with this evidence we suggest estimating the Campbell-Shiller equation with a threshold model; hence, we allow the term premium to be, not only time-varying, but also regime-dependent ($tp_{t,n}^m(\hat{t})$); in particular, regimes are determined by the value of the term premium as shown below:

\[ tp_{t,n}^m(\hat{t}) \]

---

15 Equation (9) has been estimated in the entire sample (between 1964 and 2002); then we have performed the Chow breakpoint test performed to check for structural breaks. The test has failed to reject the null hypothesis of absence of structural break for the dates when the term premium displays highest local volatility. In addition, residuals obtained by the OLS estimation of (9) are both serially correlated and heteroscedastic; the Newey and West correction help to deal with this problem.

16 Mankiw and Miron (1986) point out that the uncertainty regarding the future path of interest rates can explain the empirical failure of the expectations theory. In particular, they argue that the random walk behaviour of the short term rate, due to the interest rate smoothing policy of the Federal Reserve, affect the predictability of short rates. They identify the breakpoint with the creation of the Fed in 1914.
The choice of the term premium as threshold variable ($th = tp^{n,m}_r$) allows us to cluster volatility as well, since the variability of term premia is strictly associated to the level (as shown in Figure 6). The threshold methodology provides us with a useful framework to separate periods with low uncertainty from periods characterized by high uncertainty. We recall that the term premium captures the agents’ sentiment towards risk and is also a proxy for (excess) bonds returns\textsuperscript{17}.

Unfortunately the term premium is not known in advance and is not observable; therefore it is necessary to make a reasonable forecast\textsuperscript{18}, i.e. to obtain the threshold estimate. Once the estimated threshold is obtained, economic agents are better informed about which regime will occur in the future after observing the evolution of the threshold variable. Ideally, after obtaining the threshold estimate, agents are able to distinguish with certainty regimes on the basis of the observable dynamics followed by the selected threshold variable. Threshold modelling is appealing because it acts as an uncertainty reducer. Figure 6 plots term premia and the respective volatility\textsuperscript{19} for the pairs of maturities (60, 3) and (120, 3). Volatility seems to be closely related to the level of term premia.

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
\frac{m}{n} \sum_{q=0}^{n-m} E_i i_t^{m} - i_t^m = \alpha + \beta (i_t^n - i_t^m) + \varepsilon_t, & \text{th} \leq \hat{\tau} \\
\frac{m}{n} \sum_{q=0}^{n-m} E_i i_t^{m} - i_t^m = \alpha + \beta (i_t^n - i_t^m) + \varepsilon_t, & \text{th} > \hat{\tau}
\end{array} \right.
\end{aligned}
\end{equation}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Figure 6}
\end{figure}

\textsuperscript{17} Term premia are perfectly correlated with (log) excess bond returns as computed by Cochrane and Piazzesi (AER, 2005). Term premia reflect market participants’ incapability of anticipating the future evolution of interest rates.

\textsuperscript{18} Few methods are available to achieve the same objective: Markov switching models, structural change multiple-break models, and threshold models. The choice of threshold modelling is motivated by the fact that we do not think it is appropriate to constrain different regimes to be continuous time periods. The threshold framework provides with more flexibility in characterizing agent’s behaviour with respect to the other aforementioned models. As explained above, threshold modelling in this paper is employed to distinguish the high uncertainty regime from the low uncertainty regime.

\textsuperscript{19} Volatility is computed as the squares of the first differences.
Estimation results of model (10) are reported in the second and third columns of Table 6. The left panel of Table 6 shows the single-regime (entire sample) slope estimates of the traditional Campbell-Shiller equation; the central panel reports the threshold estimates when the threshold variable is the term premium; finally, in the right column the estimates are obtained from threshold model in which the absolute value of the term premium determines the regime shifting.

| Interest Rates Prediction | Linear | Threshold: \( t_{p, t}^n \) | Threshold: \( |t_{p, t}^n| \) |
|---------------------------|--------|-----------------------------|-----------------------------|
| \((n, m)\) | \( \begin{array}{l} \text{obs} \\ \text{R}^2 \\ \text{(p-val)} \end{array} \) | \( \begin{array}{l} \tau \text{ reg} \\ \text{R}^2 \\ \text{(p-val)} \end{array} \) | \( \begin{array}{l} \tau \text{ reg} \\ \text{obs} \\ \beta \end{array} \) | \( \begin{array}{l} \text{R}^2 \\ \text{(p-val)} \end{array} \) |
| \((120,3)\)* | \begin{array}{lll} 400 & 0.5846 & 0.107 \\ (0.001) \end{array} | \begin{array}{lll} 1.223 & 1 & 196 \\ 0.800 & 0.753 & 0.000 \end{array} | \begin{array}{lll} 2.183 & 1 & 227 \\ 0.746 & 0.475 & 0.000 \end{array} |
| \((60,3)\) | \begin{array}{lll} 465 & 0.6592 & 0.127 \\ (0.000) \end{array} | \begin{array}{lll} 1.460 & 1 & 275 \\ 0.683 & 0.384 & 0.000 \end{array} | \begin{array}{lll} 3.801 & 1 & 415 \\ 0.423 & 0.202 & 0.000 \end{array} |
| \((36,3)\) | \begin{array}{lll} 465 & 0.4761 & 0.059 \\ (0.010) \end{array} | \begin{array}{lll} 1.184 & 1 & 270 \\ 0.676 & 0.2405 & 0.000 \end{array} | \begin{array}{lll} 2.762 & 1 & 371 \\ 0.346 & 0.206 & 0.000 \end{array} |
| \((24,3)\) | \begin{array}{lll} 465 & 0.3800 & 0.036 \\ (0.032) \end{array} | \begin{array}{lll} 0.907 & 1 & 268 \\ 0.641 & 0.218 & 0.000 \end{array} | \begin{array}{lll} 1.972 & 1 & 349 \\ 0.351 & 0.183 & 0.000 \end{array} |
| \((12,3)\) | \begin{array}{lll} 465 & 0.3252 & 0.027 \\ (0.032) \end{array} | \begin{array}{lll} 0.537 & 1 & 276 \\ 0.528 & 0.148 & 0.000 \end{array} | \begin{array}{lll} 0.924 & 1 & 313 \\ 0.285 & 0.341 & 0.000 \end{array} |
| \((6,3)\) | \begin{array}{lll} 465 & 0.0716 & 0.001 \\ (0.700) \end{array} | \begin{array}{lll} 0.403 & 1 & 276 \\ 0.383 & 0.115 & 0.000 \end{array} | \begin{array}{lll} 1.964 & 1 & 458 \\ 0.350 & 0.038 & 0.001 \end{array} |

The estimated value of the threshold variable \( (\tau) \), the joint20 goodness of fit \( (j-R^2) \), the number of observations \( (\text{obs}) \), the goodness of fit in each regime \( (R^2) \), the slope estimated coefficient \( (\beta) \) and the associated \( p \)-values of the \( t \)-test (in parenthesis) are shown for each regime \( (\text{reg}) \).

**Table 6**

The estimated slope coefficient in the entire sample (left panel) tends to increase with maturity \( n \). In the single regime setting, at the very short end the spread is not informative about future movements of short term interest rates; results substantially improve in the threshold setting, as long as in both regimes \( \hat{\beta} \) estimates increase and become statistically significant. The joint goodness of fit is much

---

20 The joint goodness of fit \( (j-R^2) \) is computed as “one minus the ratio between the sum of the residual sum of squares in both regimes and the total sum of squares in the single regime”. The goodness of fit measures the proportion of the variability in the dependent variable which can be explained by the explanatory variables.
higher in the threshold model than in the single equation model; in addition, $R^2$ is also higher in each sub-regime than in the single regime.

The advantages of examining the EH in a multiple regime framework are evident. The estimated slope coefficient gets significantly closer to one in both regimes when the threshold variable is the term premium. When the absolute values of the term premium discriminate regimes, at short and medium horizons ($n = 36, 24, 12, 6$) the slope coefficient is statistically significant only in the regime characterized by moderate uncertainty; while, regime 2 estimates of the slope turn out to be not significant. Evidence thus highlights a clear asymmetric effect in the empirical analysis of the expectations theory.

These results can also be interpreted consistently with the hypothesis that interest rate unpredictability affects the empirical corroborations of the EH (Mankiw and Miron, 1986). Separating regimes on the basis of the term premium allows investors to identify two distinct states of the world, each characterized by a specific level of uncertainty; in particular, in both states the range of values assumed by the term premium is bounded and term premia volatility limited (Table 8). Hence, the investors’ forecast ability improves in both regimes, since in every regime the variability of term premia is lower than in the single regime. This is true only when regimes are determined by the level of term premia though (central column of Table 6). However, this no longer holds when the absolute level of term premia drives the regime switching; as pointed out before, for medium-short maturities only below the estimated threshold the slope coefficient is significant, i.e. when prediction errors ($tp_{t}^{n,m}$) are low in absolute value.

In Figure 7 we plot the estimated slope coefficients $\hat{\beta}$ against maturity together with the 95% confidence interval bands. In left part of Figure 7 we report the linear model $\hat{\beta}$ estimates; while in the second and third columns there are the plots of the slope coefficients in the first and second regime respectively. The second and the third diagrams in the top panel report the regime one and two $\hat{\beta}$ estimates obtained in the threshold model when the term premium is the threshold variable. In the bottom part of Figure 7 the second and the third diagrams plots regime one and two $\hat{\beta}$ estimates obtained when the threshold variable is the absolute value of the term premium. In the single regime model at the very short end the yield spread does not have any predictive power\textsuperscript{21}; while in the two-regime framework the term structure appears to be informative about future

\textsuperscript{21} Rudebusch (1995) documents significant predictive power of the spread at very short horizons, i.e. lower than two months. Our analysis is not comparable with his study since we do not consider maturities shorter than 3 months; moreover, data used in this paper have different frequency.
interest rates movements also at short horizons. Empirical evidence suggests that the predictive power of the spread increases with maturity in both the single and in the threshold regimes\(^{22}\).

![Graphs showing predictive power of spread with maturity]

In the single regime the slope coefficient never reaches \textit{one} though. Threshold estimates of the slope coefficient are quite close to \textit{one}; so that, in each regime the predictive power of the yield spread is substantial. In particular, the 95% confidence interval at medium-long horizons contains the expectations hypothesis value \textit{one} (the horizontal dashed line).

The advantage of employing a threshold model can be shown by a kernel density estimate\(^{23}\) of the short term rate \((m = 3)\) distribution. In Figure 8 the shape of the density estimate makes clear the convenience of adopting a multiple regime model, since the estimated empirical distribution of the 3-month yield in regime one is peaked (central panel); that is short term interest rates are easily predictable. The left panel plots the kernel density for the 3-month yield in the entire sample, while the right panel shows the empirical distribution in regime 2, characterized by high values of the term premium. The standard deviation is much lower in regime one (below the estimated threshold) than in the other regimes; whilst the kurtosis is much higher. In regime one, characterized by low values of the term premium the empirical distribution of the short term rate is leptokurtic.

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\(^{22}\) In regime two when the threshold is the absolute value of the term premium this result does not hold.

\(^{23}\) The Epanechnikov kernel density estimation is performed with automatic bandwidth selection. These results are obtained by focusing on the pair of maturities \((60, 3)\); similar results hold focusing on other couples of maturities \((n, 3)\). In the entire sample the standard deviation of the 3-month yields is 2.62, the kurtosis is 5. In regime one (below the estimated threshold) the std. deviation is 2.08 and the kurtosis is 7.18. In regime 2 (above the estimated threshold) the std. deviation is 2.90 and the kurtosis is 3.80.
A further investigation on the usefulness of adopting a threshold model is provided by the statistical results of the Wald test. In the two regime framework the probability of rejecting the null hypothesis of $\hat{\beta} = 1$ occurs less frequently, as show in Table 7.

<table>
<thead>
<tr>
<th>Wald Test</th>
<th>$H_0$: $\beta = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>linear $th = tp_{i,m}$</td>
</tr>
<tr>
<td></td>
<td>(p-val) reg. (p-val)</td>
</tr>
<tr>
<td>(120,3)*</td>
<td>(0.0355) 1 (0.0002)</td>
</tr>
<tr>
<td></td>
<td>2 (0.6925)</td>
</tr>
<tr>
<td>(60,3)</td>
<td>(0.0434) 1 (0.7948)</td>
</tr>
<tr>
<td></td>
<td>2 (0.1901)</td>
</tr>
<tr>
<td>(36,3)</td>
<td>(0.0085) 1 (0.0154)</td>
</tr>
<tr>
<td></td>
<td>2 (0.0897)</td>
</tr>
<tr>
<td>(24,3)</td>
<td>(0.0011) 1 (0.0002)</td>
</tr>
<tr>
<td></td>
<td>2 (0.0892)</td>
</tr>
<tr>
<td>(12,3)</td>
<td>(0.0000) 1 (0.0010)</td>
</tr>
<tr>
<td></td>
<td>2 (0.0020)</td>
</tr>
<tr>
<td>(6,3)</td>
<td>(0.0000) 1 (0.0018)</td>
</tr>
<tr>
<td></td>
<td>2 /</td>
</tr>
</tbody>
</table>

sample jan64-sep02; *jan64-mar97, p-values in parenthesis

Finally, as a further robustness check we run a rolling estimate of the Campbell-Shiller equation in both regimes. The following figures show the time-varying behaviour of the slope coefficient in each regime (regime one in the left panel; regime two in the right panel). Estimates are obtained by estimating a rolling OLS (Newey-West corrected) on sequential samples of 50 observations. The plot of $\hat{\beta}_i$ over time is smooth and stands closely around one (horizontal solid line). Furthermore, in each regime the slope coefficients are statistically significant as opposed to the rolling $\hat{\beta}_i$ estimates obtained in the single regime setting (Figure 6). Figure 9.a is obtained from rolling the
Campbell-Shiller regression for the maturity couple (60, 3); while, Figure 9.b (panel below) focuses on the maturity pair (120, 3). Results are similar for other couples of maturities (n, 3).

![Figure 9.a](image_url)

![Figure 9.b](image_url)

Table 8 shows some descriptive statistics of the estimated slope coefficient, obtained with rolling regressions. The bottom part reports statistics computed over the entire sample (January 1964 - June 2002). The top part reports statistics worked out within each threshold regime. For any combination of maturities (n, m), the mean of the estimated slope coefficient is very close to unity. Moreover, the variability of \( \hat{\beta}_i \) is definitely lower in the sub-samples. In particular, the standard deviation of the rolling \( \hat{\beta}_i \) estimates in the sub-regimes is approximately one-tenth of the standard deviation computed in the single regime model.
<table>
<thead>
<tr>
<th>long term maturity (n)</th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>reg 1 mean(β)</td>
<td>0.9836</td>
<td>0.9792</td>
<td>0.9814</td>
<td>0.9871</td>
<td>0.9954</td>
<td>1.0104</td>
</tr>
<tr>
<td>std(β)</td>
<td>0.0472</td>
<td>0.0496</td>
<td>0.0529</td>
<td>0.0518</td>
<td>0.0578</td>
<td>0.0434</td>
</tr>
<tr>
<td>reg 2 mean(β)</td>
<td>0.7904</td>
<td>0.9628</td>
<td>0.9930</td>
<td>0.9651</td>
<td>0.9748</td>
<td>0.9848</td>
</tr>
<tr>
<td>std(β)</td>
<td>0.4327</td>
<td>0.0530</td>
<td>0.0701</td>
<td>0.0776</td>
<td>0.0689</td>
<td>0.0494</td>
</tr>
<tr>
<td>linear mean(β)</td>
<td>0.4151</td>
<td>0.5671</td>
<td>0.6323</td>
<td>0.8385</td>
<td>0.9929</td>
<td>0.9401</td>
</tr>
<tr>
<td>std(β)</td>
<td>0.6603</td>
<td>0.4427</td>
<td>0.5608</td>
<td>0.7087</td>
<td>0.6486</td>
<td>0.5103</td>
</tr>
</tbody>
</table>

Linear model rolling 60 obs; regimes 1 and 2 rolling 50 obs

Table 8

A financial interpretation rationalizes our empirical results. Campbell argues that a large bias downward of the estimated slope coefficient is due to a small variance of the rationally expected changes in short rates relative to the variance of the term premium. That is, the expectations theory holds when investors are well informed about future movements in short rates.

According to Campbell and Shiller (1991) we can decompose the spread into into an expectational component (theoretical spread) and the term premium:

$$i^n_t - i^m_t = \left[ \frac{m}{n} \sum_{q=0}^{n-m} E_t \Delta i_{t+mq} - i^m_t \right] + tp_i^{n,m} =$$

$$= thsp_i^{n,m} + tp_i^{n,m}$$

The variance of the spread depends on the variance of both its components plus twice their covariance:

$$\text{var}\{i^n_t - i^m_t\} = \text{var}\{thsp_i^{n,m}\} + \text{var}\{tp_i^{n,m}\} + 2 \cdot \text{cov}\{thsp_i^{n,m}, tp_i^{n,m}\} =$$

$$= \text{var}\{i^n_t\} + \text{var}\{i^m_t\} - 2 \cdot \text{cov}\{i^n_t, i^m_t\}$$

As shown in the bottom panel of Table 9, in the single regime model for $n \leq 36$, the standard deviation of the expectational component is lower than the standard deviation of the term premium, the slope estimate is thus strongly biased downward (far below unity, as shown in the right panel of Table 6). In the threshold framework this result is inverted. In each regime the variability of the term premium, as measured by its standard deviation, is much lower than the variability of the

---

24 The covariance between the theoretical spread and the term premium is negative. At short maturities ($n = 12, 6$) the covariance is close to zero, but still negative. The covariance between long and short term rates is positive. The covariance is a negative function of the distance between maturities ($n - m$). The variance of short term yields is generally larger than the variance of long term yields.
theoretical spread. The threshold model allows reducing the uncertainty associated to the volatility of term premia. The augmented predictive power of the spread in both regimes follows directly from the lower level of uncertainty that characterizes each regime. This story is also consistent with the idea put forward by Mankiw and Miron (1986), who suggest the high interest rate predictability leads to small downward bias of the slope estimates. In particular, they attribute to the random walk behaviour of the short term rate, due to the interest rate smoothing policy adopted by the Federal Reserve, has affected the empirical support for the EH.

<table>
<thead>
<tr>
<th>Standard Deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>long term maturity (n)</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>reg 1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>reg 2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>linear</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Table 9

Thornton (2003) points out that the conventional test of the expectations hypothesis tends to generate large estimates of the slope coefficient depending on the relative variance of the short term to the long term rate, suggesting that the uncertainty is largely connected to the conduct of the Federal Reserve. In particular, the more volatile short rates are relative to long rates, the closer the estimated $\hat{\beta}$ to one. The uncertainty affecting the economy, in our model captured by the level of term premia, influences the empirical validation of the expectations hypothesis. The rationale works as follows. Suppose that an exogenous unanticipated inflationary shock hits the economy generating a massive response by long rates; unexpected important variations in long rates increases volatility, which in turn bias the slope estimate downward. In Table 10 for any combination of maturities ($n,$ 3) we show the values of the ratio between the variances of the short and the long rates. The bottom panel refers to the single regime (January 1964 – June 2002). In the top panels we report the variance ratio for the threshold models, both when the threshold is the term premium, and when the threshold is the absolute value of the term premium. The relative variance increases with maturity $n$ both in the entire sample and in each regime determined by the level of the term premium; whereas, in regimes split by the absolute value of the term premium the relative variance is increasing with maturity only below the threshold (regime one, that is when the term premium is low in absolute
value); the pattern it is irregular across maturities above the threshold (regime two). These results are consistent with the estimated magnitude of the slope coefficients reported in Table 6.

<table>
<thead>
<tr>
<th>Relative Variance</th>
<th>long term maturity (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>$t'<em>{p</em>{r,n}}$</td>
<td></td>
</tr>
<tr>
<td>reg 1</td>
<td>1.0123</td>
</tr>
<tr>
<td>reg 2</td>
<td>0.9420</td>
</tr>
<tr>
<td>$l'<em>{p</em>{r,n}}$</td>
<td></td>
</tr>
<tr>
<td>reg 1</td>
<td>0.9896</td>
</tr>
<tr>
<td>reg 2</td>
<td>1.1384</td>
</tr>
<tr>
<td>linear</td>
<td>0.9766</td>
</tr>
</tbody>
</table>

Table 10

The predictability, or the unpredictability, of short term rates, as measured by the unexpected change in the spread (the term premium) affects the empirical testing of the expectations hypothesis. The crucial assumption of regime-dependent behaviour of the term premium seems to explain this empirical fact. An example will clarify the underlying principle. Suppose the economy is hit by a negative supply shock, like a sudden increase in the oil price; suppose further that the effects on inflation are expected to be long-lasting; hence, it follows also a sharp and permanent increase in long term rates. The yield curve becomes steeper and term premia tend to increase. The shock affecting the interest rates dynamics also increases the probability of switching regime. Market participants rationally forecast the transition to the new regime and, in such enriched informative context, are able to improve interest rates forecasts. For example, forward looking risk-averse agents know that large values of the term spread are associated with increasing short term rates; therefore they may anticipate future monetary policy tightening. Threshold models for risk premia provide with a technical framework that works as uncertainty reducer; the final effect is to diminish the unpredictability of interest rates. In sum, risk-averse agents rationalize their attitude towards risk and exploit financial information more effectively. In this sense, the main advantage of the threshold approach is to provide the empirical framework that is consistent with completeness in financial markets. In each state of the world agents know exactly the macroeconomic and financial environment in which they are playing.

In the next Section we provide evidence that non linear modelling is also suitable to analyse the informative content of the term structure for predicting future inflation.
6. Concluding Remarks

This paper deals with the empirical failure of the expectations hypothesis; we suggest that single-equation models might be inappropriate to analyse the expectations theory. We thus provide evidence to support the view that non-linear models, threshold models in particular, are useful to examine the informative content of the term structure of interest rates. Economists have provided evidence suggesting that time-varying term premia may be responsible of the weak empirical support for the expectation hypothesis. In the expectations equation the term premia effect is captured partially by the residual component and partially by the intercept of the model. A shift in the intercept generates a bias in the estimation of the slope coefficient thus affecting the effectiveness of linear model to test the EH.

We propose a multiple regime framework to analyse the predictive power of the yield spread. Regimes are assumed to be a function of term premia, which provide with a measure of both the unexpected stance of monetary policy and agents’ attitude towards risk. We thus suggest that the expectations model may well be informative in a framework that exploits the countercyclical behaviour of term premia. We extend previous works by Campbell and Shiller (1991) providing evidence that the predictive ability of the yield spread is contingent to the level of uncertainty that reigns in the economy. Results suggest that the informative content of the slope of the term structure increase substantially once the risk-averse attitude of economic agents is taken into account. Our results do suggest the presence of important asymmetric effects also in the prediction of future short term interest rates.
References


