The Demand for Status and Optimal Capital Taxation

Li, Fanghui and Wang, Gaowang

Shandong University, Shandong University

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Fanghui Li†
Shandong University

Gaowang Wang‡
Shandong University

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Abstract

The paper examines the famous Chamley-Judd zero capital tax theorem in model economies where agents care about their social status. We show that the limiting capital income tax is not zero in general and its sign depends only on the utility specifications. Our conclusion is robust to several important extensions: the model with multiple physical capitals, the model with both human and physical capitals, and the one with heterogeneous agents.

Keywords: Demand for Status; Capital Income Tax; Human Capital; Heterogeneous Agents.

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†Center for Economic Research, Shandong University, Jinan, China. E-mail: fanghui_li_0513@163.com.

‡Center for Economic Research, Shandong University, Jinan, China. E-mail: gaowang.wang@sdu.edu.cn. Corresponding author.

1 Introduction

One of the most startling results in dynamic optimal tax theory is the famous finding by Chamley (1986) and Judd (1985). Although working in somewhat different settings, they draw the strikingly similar conclusions: capital should not be taxed in any steady state. The Chamley-Judd result, taking convergence to a steady state as granted, is that taxes on capital should be zero in the long run.

As the most important benchmarks in the optimal tax literature, the Chamley-Judd theorem stimulates a large body of work on this topic. Economists have continued to take turns reinvestigating the Chamley-Judd results in different settings and putting forth various intuitions to interpret it. Lucas (1990) recovers the zero limiting capital tax result in a model with endogenous growth driven by endogenous learning/human capital accumulation and quantifies the welfare cost of capital taxation for the U.S. economy. In a model with both physical and human capital, Jones, Manuelli and Rossi (1997) show that the optimality of a limiting zero tax applies to both labor income and capital income, as long as the technology for accumulating human capital displays constant return to scale in the stock of human capital and goods used. Correia (1996) argues that zero capital tax hinges on a complete set of flat-rate taxes for all production factors which guarantees the perfect shifts of the long-run burden of capital taxation to other production factors. When these other factors cannot be taxed directly the optimality of the zero tax rate on capital income disappears. Atkeson, Chari and Kehoe (1996) shows that Chamley's result holds when agents are heterogeneous rather than identical, the economy's growth rate is endogenous rather than exogenous, the economy is open rather than closed, and agents live in overlapping generations rathen than forever. By incorporating exogenous shocks to the production function and government purchases, Zhu (1992) and Chari, Christiano and Kehoe (1994) generalize the Chamley-Judd result to the stochastic version. Zhu (1992) establishes that for some special utility functions, if there exists a stationary Ramsey equilibrium, the Ramsey plan prescribes a zero ex ante capital tax rate that can be implemented by setting a zero tax on capital income. However, except for those preferences, Zhu (1992) shows that the ex ante capital tax rate should vary around zero. Chari, Christiano and Kehoe (1994) perform numerical simulations and conclude that there is a quantitative presumption that the ex ante capital tax is approximately zero.

Others researchers overturns the Chamley-Judd result by introducing different mechanisms. Aiyagari (1995) shows that for the Bewley-type models with incomplete insurance market and borrowing constraints, the optimal tax rate on capital income is positive, even in the long run. The intuition behind a positive capital income tax rate is as follows: because of incomplete insurance market, there is a precautionary motive for accumulating capital. Furthermore, the possibility of being borrowing-constrainted in some future periods leads agents to accumulate more capital. Therefore, these two features lead to excess (i.e., greater-than-the-optimal level of) capital. And a positive tax rate on capital income will be needed to reduce capital accumulation and bring capital to the optimal level. In order to confirm the importance of complete taxation for zero capital tax, Correia (1996) studies a case with an additional fixed production factor that cannot be taxed by the government and shows that if the tax system is incomplete, the limiting value of optimal capital tax can be different from zero. Stiglitz (2018) constructs two overlapping generations models to deny the desirability of a zero capital tax. In one model with time separability but with non-separability between consumption and leisure, capital taxation depends on the complementarity/substitutability of leisure during work with retirement consumption. In the other two-class model with sufficiently equalitarian social welfare functions and sufficiently high productivity of educational expenditures, it derives a positive optimal capital tax. In reexamining the two models developed by Chamley (1986) and Judd (1985) respectively by assuming constant-relative-risk-aversion (CRRA) preferences, Straub and Werning (2018) establish that when the intertemporal elasticity of substitution (IES) is below one, the economy converges towards a positive limit tax. The economic intuition they provide for this result is based on the anticipatory savings effects of future tax rates: when the IES is less than one, any anticipated increase in taxes leads to higher savings today, since the substitution effect is relatively small and dominated by the income effects. To exploit such anticipatory effects, the optimum involves an increasing path for capital tax rates and converges to a positive value.

In the paper we introduce the status preferences (or wealth effects or the spirit of capitlism)[1] in the dynamic tax theory and reexamine the Chamley-Judd results on optimal capital taxation. The reason why we incorporate status preferences into the optimal tax theory is based on the following two considerations. On one hand, in the optimal growth model, Cass (1965) establishes that the net marginal product of per capital capital is equal to the time preference rate (i.e., $f'(k^{mg}) = \rho$), which is well-known as the modified golden rule level of physical capital. By incorporating the status preferences in the Cass model, Kurz (1968) and Zou (1994) derive a less marginal product of capital (i.e., $f'(k^*) = \rho - U_k/U_c < \rho = f'(k^{mg})$), and hence a higher steady state level of physical capital (i.e., $k^* > k^{mg}$). Then we want to ask whether the government should levy a capital tax for this kind of over-accumulation. On the other hand, the status preferences have been used extensively in the economics literature and proven to be useful in understanding a number of puzzles, including those of asset pricing (Bakshi and Chen, 1995; Smith, 2002; Boileau and Rebecca, 2007), savings and wealth accumulation (Cole, Mailath and Postlewaite, 1992; Zou, 1995), occupational choice (Doepke and Zilibotti, 2008), wealth distribution (Luo and Young, 2009), business cycle (Boileau and Rebecca, 2007; Karnizova, 2010), and cross-country growth differences (Kurz, 1968; Zou, 1994).

The paper introduces status preferences into optimal tax theory and derives the tax formulas for capital income in different settings. It is shown that the limiting capital tax is not zero generally and its sign depends completely on the specifications of the utility function. The indefiniteness of optimal capital income taxation is robust to different settings, including the representative-agent model with unique or multiple physical capitals, the one with human capital and physical capital, and the model with heterogeneous agents like Judd (1985).

The remainder of the paper is organized as follows. In section 2, we analyze a representative-agent model with status concerns and derive the formula for optimal capital tax. Then we extend it to the case with multiple physical capitals. In section 3, we introduce human capital into the baseline model and derive the very simliar results. In section 4, we extend

[1] The modeling strategy of putting capital/wealth into the utility function is named differently as social status/norms (Cole, Mailath and Postlewaite, 1992; Luo and Young, 2009), wealth effects (Kurz, 1968), or the spirit of capitalism (Bakshi and Chen, 1996; Boileau and Braeu, 2007; Doepke and Zilibotti, 2008; Karnizova, 2010; Smith, 2001; Zou, 1994, 1995).

the model to the case with heterogeneous agents. Finally, Section 5 offers some concluding remarks.

2 The Baseline Model with Status Concerns

2.1 Model setup

Consider a production economy with no uncertainty. An infinitely lived representative household likes consumption, leisure and capital streams $\{c_t, l_t, k_t\}_{t=0}^{\infty}$ that give higher values of

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, k_t), \tag{1}$$

where $\beta \in (0,1)$ is the time discount rate, $c_t \geq 0$, $l_t \geq 0$ and $k_t \geq 0$ are consumption, leisure and physical capital stock at time t, respectively, and $u_i > 0$, $u_{ii} < 0$, $u_{ij} \geq 0$, for $i, j \in \{c, l, k\}$ with $i \neq j$. The household is endowed with one unit of time per period that can be used for leisure l_t and labor n_t:

$$l_t + n_t = 1. \tag{2}$$

The single good is produced with labor n_t and capital k_t. Output can be consumed by households, used by the government, or used to augment the capital stock. The resource constraint is

$$c_t + g_t + k_{t+1} = F(k_t, n_t) + (1 - \delta_k) k_t, \tag{3}$$

where $\delta_k \in (0,1)$ the depreciation rate of capital and $\{g\}_{t=0}^{\infty}$ is an exogenous sequence of government purchases. We assume that a standard increasing and concave production function that exhibits constant return to scale. By Euler's theorem on homogeneous functions, linear homogeneity of F implies $F(k_t, n_t) = F_k(k_t, n_t) k_t + F_n(k_t, n_t) n_t$.

Government. The government finances its stream of purchases $\{g_t\}_{t=0}^{\infty}$ by levying flat-rate, time varying taxes on earnings from capital at rate τ_t^k and earnings from labor at rate τ_t^n. The government can also trade one-period bonds, sequential trading of which suffices to accomplish any intertemporal trade in a world without uncertainty. Let B_t be government indebtedness to the private sector, denominated in time t-goods, maturing at the beginning of period t. The government's budget constraint is

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{B_{t+1}}{R_t} - B_t, \tag{4}$$

where r_t and w_t are the market-determined rental rate of capital and the wage rate for labor, respectively, denominated in units of time t goods, and R_t is the gross rate of return on one-period bonds held from t to $t+1$. Interest earnings on bonds are assumed to be tax exempt; this assumption is innocuous for bond exchanges between the government and the private sector. We assume that the government can commit fully and credibly to future tax rates and thus evade the issue of time-consistency raised in Kydland and Prescott (1977).

Households. A representative household chooses $\{c_t, l_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ to maximizes expression (1) subject to the time allocation constraint (2) and the sequence of budget constraints

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t n_t + (1-\delta_k) k_t + b_t, \tag{5}$$

for $t \geq 0$, given k_0 and b_0. Here, b_t is the real value of one-period government bond holdings that mature at the beginning of period t, denominated in units of time t consumption.

Substituting the time allocation equation into the objective function, we construct the following Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, 1-n_t, k_t) + \lambda_t \left[(1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t n_t + b_t - \frac{b_{t+1}}{R_t} - c_t - k_{t+1} + (1-\delta_k) k_t \right] \right\}.$$

At an interior solution, the first-order conditions with respect to c_t, n_t, k_{t+1} and b_{t+1} are[2]

$$u_c(t) = \lambda_t, \tag{6}$$

$$u_l(t) = \lambda_t (1-\tau_t^n) w_t, \tag{7}$$

$$\lambda_t = \beta \left\{ u_k(t+1) + \lambda_{t+1} [(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k] \right\}, \tag{8}$$

$$\frac{\lambda_t}{R_t} = \beta \lambda_{t+1}. \tag{9}$$

From equations (6) and (7), we have

$$\frac{u_l(t)}{u_c(t)} = (1-\tau_t^n) w_t, \tag{10}$$

which displays that the marginal rate of substitution of consumption and leisure equals their (after-tax) price ratio. Combining equations (6) and (8) yields us the consumption Euler equation

$$u_c(t) = \beta \left\{ u_k(t+1) + u_c(t+1) [(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k] \right\}, \tag{11}$$

in which the demand for status ($u_k > 0$) is a new channel for savings. This savings motive can be seen more clearly from the modified no-arbitrage condition

$$R_t = \frac{(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k}{1 - \beta u_k(t+1)/u_c(t)}, \tag{12}$$

which is derived by putting equation (9) into (8). Due to $u_k, u_c > 0$, the value of $1 - \beta u_k(t+1)/u_c(t)$ belongs to $(0,1)$ and the real rate of return for savings is higher than the case without status concerns. Hence, consumers with demand for status have more stronger motive for saving.

Firms. In each period, the representative firm takes as given, rents capital and labor from households, and maximizes profits,

$$F(k_t, n_t) - r_t k_t - w_t n_t.$$

[2] Let $u_c(t)$ and $u_l(t)$ denote the time t values of the derivatives of $u(c_t, l_t)$ with respect to consumption and leisure, respectively.

The first-order conditions for this problem are

$$r_t = F_k(k_t, n_t), w_t = F_n(k_t, n_t). \tag{13}$$

In words, inputs should be employed until the marginal product of the last unit is equal to its rental price. With constant return to scale, we get the standard result that pure profits are zero.

2.2 Primal Approach to the Ramsey Problem

Definition 1 *A competive equilibrium is an allocation $\{c_t, l_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, a price system $\{w_t, r_t, R_t\}_{t=0}^{\infty}$, and a government policy $\left\{g_t, \tau_t^k, \tau_t^n, B_{t+1}\right\}_{t=0}^{\infty}$ such that (a) given the price system and the government poicy, the allocation solves both the firm's problem and the household's problem with $b_t = B_t$ for all $t \geq 0$; (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraint (4) for all $t \geq 0$; (3) the time allocation constraint (2) and the resource constraint (3) are safisfied for all $t \geq 0$.*

There are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem.

Definition 2 *Given k_0, b_0 and m_0, the Ramsey problem is to choose a competitive equilibrium that maximizes expression (1).*

We use the Primal approach to formulate the Ramsey problem by following the procedure written by Ljungqvist and Sargent (2012). For this purpose, we firstly substitute repeatedly the flow budget constraint (5) to derive the household's present-value budget constraint[3]

$$\sum_{t=0}^{\infty}\left[q_t^0 c_t + q_{t+1}^0 \frac{u_k(t+1)}{u_c(t+1)} k_{t+1}\right] = \sum_{t=0}^{\infty} q_t^0 (1-\tau_t^n) w_t n_t + \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0, \tag{14}$$

where $q_t^0 \equiv \sum_{i=0}^{t-1} R_i^{-1}$ is the Arrow-Debreu price for $t \geq 1$, with the numeraire $q_0^0 = 1$.

Let λ be a Lagrange multiplier on the household's present-value budget constraint (14). The first-order conditions for the household's problem are

$$\beta^t u_c(t) = \lambda q_t^0, \tag{15}$$

$$\beta^t u_l(t) = \lambda q_t^0 (1-\tau_t^n) w_t, \tag{16}$$

$$\lambda q_t^0 = \beta^t u_k(t+1) + \lambda q_{t+1}^0 \left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]. \tag{17}$$

Using condition (15) and the corresponding expression for $t = 0$ and the numeraire $q_0^0 = 1$, the Arrow-Debreu price q_t^0 can be expressed as

$$q_t^0 = \beta^t \frac{u_c(t)}{u_c(0)}. \tag{18}$$

[3] We impose the transversality condition $\lim_{T\to+\infty} q_T^0 b_T = 0$.

From equations (15) and (16), we obtain

$$(1-\tau_t^n)w_t = \frac{u_l\left(t\right)}{u_c\left(t\right)}, \tag{19}$$

which is essential equation (10). Substituting equation (15) into (17) yields us

$$\left[\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta\right] = \frac{u_c\left(t\right)-u_k\left(t+1\right)}{\beta u_c\left(t+1\right)}. \tag{20}$$

Substituting equations (18), (19), (20) into the present-value budget constraint (14), we derive the following implementability condition[4]

$$\sum_{t=0}^{\infty}\beta^t[u_c(t)c_t - u_l(t)n_t + \beta u_k(t+1)k_{t+1}] = u_c(0)\{[(1-\tau_0^k)r_0+1-\delta_k]k_0+b_0\} \equiv \widetilde{A}_1. \tag{21}$$

The Ramsey problem is to maximize expression (1) subject to equation (21) and the resource constraint (3). We proceed by assuming that government expenditures are small enough that the problem has a convex constraint set and that we can approach it using Lagrangian methods. In particular, let Φ be the Lagrangian multiplier on equation (21) and define

$$U(t) \equiv U(c_t, n_t, k_t, c_{t+1}, n_{t+1}, k_{t+1}, \Phi) \equiv u(c_t, 1-n_t, k_t) + \Phi[u_c(t)c_t - u_l(t)n_t + \beta u_k(t+1)k_{t+1}].$$

Then we can form the Lagrangian

$$J = \sum_{t=0}^{\infty}\beta^t\{U(t) + \theta_t[F(k_t, n_t) - c_t - g_t - k_{t+1} + (1-\delta_k)k_t]\} - \Phi\widetilde{A}_1,$$

where $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of Lagrangian multipliers. The first order conditions for this problem are

$$c_t: \quad U_4(t-1) = \beta[\theta_t - U_1(t)], t \geq 1 \tag{22}$$

$$n_t: \quad -U_5(t-1) = \beta[U_2(t) + \theta_t F_n(t)], t \geq 1 \tag{23}$$

$$k_{t+1}: \quad \theta_t - U_6(t) = \beta\{U_3(t+1) + \theta_{t+1}[F_k(t+1) + 1 - \delta_k]\}, t \geq 0 \tag{24}$$

where

$$\begin{aligned}
U_1(t) &= u_c(t) + \Phi[u_{cc}(t)c_t + u_c(t) - u_{lc}(t)n_t], \\
U_2(t) &= -u_l(t) + \Phi[-u_{cl}(t)c_t + u_{ll}\left(t\right)n_t - u_l(t)], \\
U_3(t+1) &= u_k(t+1) + \Phi[u_{ck}(t+1)c_{t+1} - u_{lk}(t+1)n_{t+1}], \\
U_4(t-1) &= \Phi\beta u_{kc}(t)k_t, \\
U_5(t-1) &= -\Phi\beta u_{kl}(t)k_t, \\
U_6(t) &= \Phi\beta[u_{kk}(t+1)k_{t+1} + u_k(t+1)].
\end{aligned}$$

Consider the special case in which there is a $T \geq 0$ for which $g_t = g$ for all $t \geq T$. Assume that there exists a solution to the Ramsey problem and that it converges to a time-invariant allocation, so that c, n and k are constant after some time. Then we have the following

[4] The derivation of the implementability condition is on appendix A.

Theorem 1 *Suppose the economy converges to an interior steady state in the model with status concerns. The limiting taxes for both capital and labor income are indefinite. That is, they could be positive, negative or zero, which are determined completely by the functional form of the utility function, namely,*

$$\tau^k \begin{cases} >0 \\ =0 \\ <0 \end{cases}, \text{if } (u_k\eta_1 - u_c\eta_3) \begin{cases} >0 \\ =0 \\ <0 \end{cases}; \tau^n \begin{cases} >0 \\ =0 \\ <0 \end{cases}, \text{if } (\eta_2 - F_n\eta_1) \begin{cases} >0 \\ =0 \\ <0 \end{cases}.$$

Proof The steady-state equations for equations (22)-(24) are

$$\theta = (1+\Phi)u_c + \Phi\underbrace{(u_{cc}c - u_{lc}n + u_{kc}k)}_{\equiv\eta_1}, \tag{25}$$

$$\theta F_n = (1+\Phi)u_l + \Phi\underbrace{(u_{cl}c - u_{ll}n + u_{kl}k)}_{\equiv\eta_2}, \tag{26}$$

$$\theta[1-\beta(F_k+1-\delta_k)] = \beta[(1+\Phi)u_k + \Phi(\underbrace{u_{ck}c - u_{lk}n + u_{kk}k}_{\equiv\eta_3})]. \tag{27}$$

From equations (25) and (26), we solve for $(1+\Phi)/\theta$ and Φ/θ as follows:

$$\frac{(1+\Phi)}{\theta} = \frac{\eta_2 - F_n\eta_1}{u_c\eta_2 - u_l\eta_1}, \frac{\Phi}{\theta} = \frac{u_cF_n - u_l}{u_c\eta_2 - u_l\eta_1}. \tag{28}$$

From equation (11), we know that

$$F_k + 1 - \delta_k = \frac{1}{\beta} - \frac{u_k}{u_c} + \tau^k F_k. \tag{29}$$

Dividing the both sides of equation (27) by θ and plugging (28) and (29) into it, we obtain

$$\tau^k = \frac{1}{u_cF_k}\underbrace{\frac{(u_cF_n - u_l)}{(u_c\eta_2 - u_l\eta_1)}}_{=\frac{\Phi}{\theta}}(u_k\eta_1 - u_c\eta_3). \tag{30}$$

From equation(28), the term $(u_cF_n - u_l)/(u_c\eta_2 - u_l\eta_1) = \Phi/\theta$ is nonnegative, because the Lagrange multiplier Φ is nonnegative, while the insatiable utility function implies that θ is strictly positive. Notice that u_c and F_k are both strictly positive. Hence the sign of the limiting capital income tax is determined completely by the sign of the term $(u_k\eta_1 - u_c\eta_3)$. To examine the optimal labor income tax, we substitute (25) into (26), rearrange the terms and obtain

$$u_cF_n - u_l = \frac{\Phi}{1+\Phi}(\eta_2 - F_n\eta_1). \tag{31}$$

Equations (10) and (13) give us

$$u_cF_n - u_l = \tau^n u_c F_n. \tag{32}$$

Combining the above two equation leads to

$$\tau^n = \frac{1}{u_c F_n} \frac{\Phi}{1+\Phi} \left(\eta_2 - F_n \eta_1\right). \tag{33}$$

Since $u_c > 0$, $F_n > 0$ and the multiplier Φ is nonnegative, the limiting optimal labor income tax depends on the value of the term in the bracket, listed in the theorem.□

Theorem 1 tells that the limiting capital income tax is in general not zero, since the term $(u_k\eta_1 - u_c\eta_3)$ is generally not equal to zero. It should be noted that the sign of the optimal capital tax rate is determined completely by the form of utility function not by the technology, since the term $(u_k\eta_1 - u_c\eta_3)$ depends only on the utility function. If the consumer cares about the utility from both social status and consumption, then more complex forces destroy the zero capital income taxation theorem.

Proposition 1 *(Chamley, 1986; Judd, 1985) If there is no status concern (i.e., $u_k = 0$), then the limiting capital income tax is zero, i.e., $\tau^k = 0$, and the corresponding labor income tax is nonnegative, i.e., $\tau^n \geq 0$.*

Proof If $u_k = 0$, then the term $u_k\eta_1 - u_c\eta_3$ equals zero and hence $\tau^k = 0$. Meanwhile, equation (33) degenerates as

$$\tau^n = \frac{1}{u_c F_n} \frac{\Phi}{1+\Phi} \left[\left(u_{cl} - F_n u_{cc}\right) c + \left(-u_{ll} + F_n u_{lc}\right) n\right] \geq 0,$$

which is nonnegative due to the assumptions $u_c > 0$, $F_n > 0$, $u_{cl} \geq 0$, $u_{cc} < 0$, $u_{ll} < 0$ and $u_{lc} \geq 0$. □

Proposition 1 replicates the zero limiting capital tax and nonnegative labor income tax results developed by Chamley (1986) and Judd (1985). Whether the limiting labor income tax equals zero depends on the initial conditions (k_0 and b_0) and the steam of government purchases $\{g_t\}_{t=0}^{\infty}$. For the large values of k_0 and low values of b_0 and $\{g_t\}_{t=0}^{\infty}$, by raising τ_0^k and thereby increasing the revenues from lump-sum taxation of k_0, the government reduces its need to rely on future distortionary taxation and hence the value of Φ falls. Acturally, the positive values of the derivative $\left(\partial J/\partial \tau_0^k = \Phi u_c(0) F_k(0) k_0 > 0\right)$ for all τ_0^k imply that the government could set τ_0^k high enough to drive Φ down to zero. This would enable the government to set $\tau_t^n = 0$ for all $t \geq 0$ and $\tau_t^k = 0$ for all $t \geq 1$. In this case, the government should raise all revenues through a time 0 capital levy, then lend the proceeds to the private sector and finance government expenditures by the interest from the loan. However, with low values of k_0 and high values of b_0 and $\{g_t\}_{t=0}^{\infty}$, the government has to use distortionary labor income tax together with time 0 capital vevy to finance government expenditures, which pushes up Φ to be positive. This point has been emphasized by Auerbach and Kotlikoff (1987) in a life cycle context and by Lucas (1990) in a model with human capital.

Corollary 1 *Suppose that (1) the utility function is additively separable in its three arguments: consumtion c, leisure l, and capital k, and (2) there is a positive correlation between the elasticity of marginal utility for consumption and the one of marginal utility for capital, i.e., $u_{cc}c/u_c = \phi u_{kk}k/u_k$, with $\phi > 0$. Then, if $\phi > 1$, then $\tau^k < 0$; if $\phi < 1$, then $\tau^k < 0$; and if $\phi = 1$, then $\tau^k = 0$. For those additively separable utility functions, the labor income tax is nonnegative in the limit.*

Proof If the utility function is additively separable with respect to its three arguments, i.e., $u(c_t, l_t, k_t) = f(c_t) + g(l_t) + h(k_t)$, then $u_{ij} = 0$, for $i \neq j \in \{c, l, k\}$. In this case, we have $\eta_1 = u_{cc}c$ and $\eta_3 = u_{kk}k$. Equation (30) turns out to

$$\tau^k = \frac{u_k}{F_k}\frac{\Phi}{\theta}(\phi - 1)\frac{u_{kk}k}{u_k} = \frac{u_{kk}k}{F_k}\frac{\Phi}{\theta}(\phi - 1),$$

which asserts the results about the limiting capital tax stated in the corollary, since we know that $F_k > 0$, $u_{kk} < 0$, $\Phi \geq 0$ and $\theta > 0$. Furthermore, the expression of the limiting labor income tax turns out to

$$\tau^n = \frac{1}{u_c F_n}\frac{\Phi}{1+\Phi}(-F_n u_{cc}c - u_{ll}n) \geq 0,$$

which shows that the limiting labor income tax is nonnegative. $\square$

Corollary 1 displays that if the marginal utility for consumption is more sensitive than the one for capital, i.e., $\phi\,(= (-u_{cc}c/u_c)\,/\,(-u_{kk}k/u_k)) > 1$, then the government should subsidy capital accumulation in the limit, namely, $\tau^k < 0$; if the marginal utility for consumption is less sensitive than the one of physical capital, i.e., $\phi < 1$, then the government should levy on capital income, namely, $\tau^k < 0$; and if they have the same degree of sensitivity, i.e., $\phi = 1$, then a zero limiting tax applies to capital inome. For additively separable utilites, the limiting labor income tax is always nonnegative.

In order to show the results displayed in Theorem 1 more clearly, we extend the baseline model to include two types of physical capitals: one, k_t with interest rate r_t and depreciation rate δ, is in the utility, and the other, κ_t with interest rate r_t^κ depreciation rate δ_κ, is not. We also assume that the production function of the economy, $F(k_t, \kappa_t, n_t)$, is linearly homogenous on three production factors. It is shown in Proposition 2 that the limiting tax rate on r_t of physical capital with status concerns is indefinite and the one on r_t^κ of other physical capitals without status concerns is zero.

Proposition 2 *(Two Types of Physical Capitals) If the steady state exists in the extended model with two types of physical capitals, then the limiting tax on capital with status concerns is indefinite and pinned down by equation (30), the limiting tax on capital without status concerns equals zero, and the limiting tax on labor income is determined by equation (33).*[5]

3 Human Capital and Physical Capital

In a model with human capital, Jones, Manuelli and Rossi (1997) show that the optimality of a limiting zero tax applies to both labor income and capital income, as long as the technology for accumulating human capital displays constant return to scale in the stock of human capital and goods used. In this section, we extend the baseline model with status

[5]Note that the partial derivatives of the prodcution function in the tax equations depend on κ. That is, $F_n = F_n(k, \kappa, n)$, $F_k = F_k(k, \kappa, n)$, and $F_\kappa = F_\kappa(k, \kappa, n)$. We omit the proof of Proposition 2, which is very similar to the proof of Theorem 1.

concerns by allowing the same human capital technology as Jones, Manuelli and Rossi (1997), and show that zero limiting tax applies to human capital but not to physical capital.

We postulate that human capital accumulation follows

$$h_{t+1} = (1-\delta_h)\, h_t + H\left(x_{ht}, h_t, n_{ht}\right), \tag{34}$$

where $\delta_h \in (0,1)$ is the rate at which human capital depreciates, and the function $H(\cdot)$ describes how new human capital is created with the flow of inputs coming from current output x_{ht}, the stock of human capital h_t, and raw labor n_{ht}. The idea that the accumulation of human captical is an internal activity using market goods as well as human capital and labor appears in Heckman (1976) and is standard in the labor economics literature.[6] Human capital is in turn used to produce efficiency units of labor e_t,

$$e_t = M\left(x_{mt}, h_t, n_{mt}\right),$$

where x_{mt} and n_{mt} are the market good and raw labor used in the process. It is assumed that both H and M are linearly homogeneous in market goods $(x_{jt}, j = h, m)$ and human capital (h_t), and twice continuously differentiable with strictly decreasing and anywhere positive marginal products of all factors.

The number of efficiency units of labor e_t replaces our earlier argument for labor n_t in the production function, $F(k_t, e_t)$. Applying Euler's theorem for the homogeneous functions H, M, and F, we have that

$$\begin{aligned} H\left(x_{ht}, h_t, n_{ht}\right) &= H_x(t)\, x_{ht} + H_h(t)\, h_t, \\ M\left(x_{ht}, h_t, n_{ht}\right) &= M_x(t)\, x_{ht} + M_h(t)\, h_t, \\ F\left(k_t, e_t\right) &= F_k(t)\, k_t + F_e(t)\, e_t. \end{aligned}$$

The household's preferences are still described by expression (1), with leisure $l_t = 1 - n_{ht} - n_{mt}$. The economy's aggregate resource constraint is

$$c_t + g_t + k_{t+1} + x_{mt} + x_{ht} = F\left(k_t, e_t\right) + (1-\delta_k)k_t. \tag{35}$$

3.1 The Ramsey Problem

The representative household maximizes the objective function (1), subject to the flow budget constraint

$$(1+\tau_t^c)\, c_t + k_{t+1} + (1+\tau_t^m)\, x_{mt} + x_{ht} + \frac{b_{t+1}}{R_t} = (1-\tau_t^k) r_t k_t + (1-\tau_t^n) w_t e_t + (1-\delta_k) k_t + b_t, \tag{36}$$

and human capital accumulation equation (34). The first-order conditions for interior solutions[7] are then

$$\frac{u_l(t)}{u_c(t)} = \frac{H_n(t)}{(1+\tau_t^c)\, H_x(t)} = \frac{(1-\tau_t^n)\, w_t M_n(t)}{(1+\tau_t^c)}, \tag{37}$$

[6] This formulation has those popular specificaitons of Heckman (1976) (with $H(x,h,n) = F(x,hn)$) and Lucas (1988) (with $H(x,h,n) = hm(n)$) as special cases.

[7] We derive the status-concerns model with both physical capital and human capital in Appendix B.

$$(1-\tau_t^n)\, w_t M_x(t) = 1+\tau_t^m, \tag{38}$$

$$\frac{u_c(t)}{(1+\tau_t^c)} = \beta\left\{u_k(t+1) + \frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}\left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1-\delta_k\right]\right\}, \tag{39}$$

$$\frac{u_c(t)}{(1+\tau_t^c)\, H_x(t)} = \beta\frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}\left[\left(1-\tau_{t+1}^n\right) w_{t+1} M_h(t+1) + \frac{H_h(t+1)+1-\delta_h}{H_x(t+1)}\right], \tag{40}$$

$$\frac{u_c(t)}{(1+\tau_t^c)} = \beta R_t \frac{u_c(t+1)}{\left(1+\tau_{t+1}^c\right)}, \tag{41}$$

$$R_t = \frac{(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k}{1-\beta u_k(t+1)\,(1+\tau_t^c)\,/u_c(t)}. \tag{42}$$

Optimality requires that the last unit of final goods has the same marginal contributions on consumption and human capital production and the last unit of time has the same marginal contributions on utility maximization, producing human capital or generating effective labor forces, shown in Equations (37). Equations (38) shows that the (net) marginal product value of the market good is equal to its marginal cost (i.e., after-tax price). Both (39) and (41) are consumption Euler equations with the new term $\beta u_k(t+1)$ due to the status concerns. Equation (42) is the modified no-arbitrage condition for portfolio choices.

The present-value budget constraint of the representative consumer is derived as

$$\sum_{t=0}^{\infty}\left\{\begin{array}{c} q_t^0\,(1+\tau_t^c)\, c_t + \\ q_{t+1}^0 \frac{u_k(t+1)\left(1+\tau_{t+1}^c\right)}{u_c(t+1)} k_{t+1} \end{array}\right\} = \left\{\begin{array}{c} \sum_{t=0}^{\infty} q_t^0\left[(1-\tau_t^n) w_t e_t - (1+\tau_t^m)\, x_{mt} - x_{ht}\right] \\ + \left[\left(1-\tau_0^k\right) r_0 + 1-\delta\right] k_0 + b_0 \end{array}\right\}. \tag{43}$$

The optimal path of the Lagrange multiplier λ_t tells that $\lambda_t = \lambda_0 \beta^{-t} q_t^0$. Substituting it into the first-order condition for consumption c_t leads to

$$q_t^0 = \frac{\beta^t u_c(t)}{u_c(0)} \frac{1+\tau_0^c}{1+\tau_t^c}. \tag{44}$$

Putting equation (44) in equation (40), we obtain

$$\frac{q_t^0}{H_x(t)} = q_{t+1}^0\left[\frac{H_h(t+1)+1-\delta_h}{H_x(t+1)} + \left(1-\tau_{t+1}^n\right) w_{t+1} M_h(t+1)\right]. \tag{45}$$

Invoking the homogeneity of M and H, and substituting equations (38), (44) and (45) into (43), we obtain the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left[u_c(t)\, c_t + \beta u_k(t+1)\, k_{t+1}\right] = \widetilde{A}, \tag{46}$$

where

$$\widetilde{A} \equiv \frac{u_c(0)}{1+\tau_0^c}\left\{\left[\frac{H_h(0)+1-\delta_h}{H_x(0)} + (1-\tau_0^n)\, F_e(0)\, M_h(0)\right] h_0 + \left[\left(1-\tau_0^k\right) r_0 + 1-\delta\right] k_0 + b_0\right\}.$$

We define

$$U\left(c_t, n_{ht}, n_{mt}, k_t, c_{t+1}, n_{ht+1}, n_{mt+1}, k_{t+1}, \widetilde{\Phi}\right) = u\left(c_t, 1 - n_{ht} - n_{mt}, k_t\right) + \widetilde{\Phi}\left[u_c\left(t\right)c_t + \beta u_k\left(t+1\right)k_{t+1}\right],$$

and formulate a Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} U\left(c_t, n_{ht}, n_{mt}, k_t, c_{t+1}, n_{ht+1}, n_{mt+1}, k_{t+1}, \widetilde{\Phi}\right) + \\ \theta_t\left[F\left(k_t, M\left(x_{mt}, h_t, n_{mt}\right)\right) + (1-\delta_k)k_t - c_t - g_t - k_{t+1} - x_{mt} - x_{ht}\right] \\ v_t\left[\left(1-\delta_h\right)h_t + H\left(x_{ht}, h_t, n_{ht}\right) - h_{t+1}\right] \end{array} \right\} - \widetilde{\Phi}\widetilde{A}_2.$$

The first-order conditions for $c_t, n_{ht}, n_{mt}, x_{ht}, x_{mt}, k_{t+1}$ and h_{t+1} are

$$\beta\left[U_1\left(t\right) - \theta_t\right] + U_5\left(t-1\right) = 0, \tag{47}$$

$$\beta\left[U_2\left(t\right) + v_t H_n\left(t\right)\right] + U_6\left(t-1\right) = 0, \tag{48}$$

$$\beta\left[U_3\left(t\right) + \theta_t F_e\left(t\right) M_n\left(t\right)\right] + U_7\left(t-1\right) = 0, \tag{49}$$

$$\theta_t = v_t H_x\left(t\right), \tag{50}$$

$$F_e\left(t\right) M_x\left(t\right) = 1, \tag{51}$$

$$U_8\left(t\right) - \theta_t + \beta\left\{U_4\left(t+1\right) + \theta_{t+1}\left[F_k\left(t+1\right) + 1 - \delta_k\right]\right\}, \tag{52}$$

$$v_t = \beta\left\{\theta_{t+1} F_e\left(t+1\right) M_h\left(t+1\right) + v_{t+1}\left[H_h\left(t+1\right) + 1 - \delta_h\right]\right\}, \tag{53}$$

where

$$U_1\left(t\right) = u_c\left(t\right) + \widetilde{\Phi} u_{cc}\left(t\right) c_t + \widetilde{\Phi} u_c\left(t\right),$$

$$U_2\left(t\right) = U_3\left(t\right) = -u_l\left(t\right) - \widetilde{\Phi} u_{cl}\left(t\right) c_t,$$

$$U_4\left(t+1\right) = u_k\left(t+1\right) + \widetilde{\Phi} u_{ck}\left(t+1\right) c_{t+1},$$

$$U_5\left(t-1\right) = \widetilde{\Phi}\beta u_{kc}\left(t\right) k_t,$$

$$U_6\left(t-1\right) = U_7\left(t-1\right) = -\widetilde{\Phi}\beta u_{kl}\left(t\right) k_t,$$

$$U_8\left(t\right) = \widetilde{\Phi}\beta\left[u_{kk}\left(t+1\right) k_{t+1} + u_k\left(t+1\right)\right].$$

3.2 Optimal Taxation

In this subsection, we examine the limiting optimal taxes on capital income, labor income, consumption and expenditures for generating effective labor force, respectively.

Capital Income Tax $\left(\tau^k\right)$. In the steady state, equation (52) turns out to

$$\beta\left[\frac{u_k + \widetilde{\Phi}\left(u_{kk}k + u_k + u_{ck}c\right)}{\theta} + \left(F_k + 1 - \delta_k\right)\right] = 1. \tag{54}$$

Substituting equation (37) into equation (39) leads to

$$\beta\left\{\frac{u_k}{u_l}\frac{H_n}{H_x} + \left[\left(1-\tau^k\right)F_k + 1 - \delta_k\right]\right\} = 1. \tag{55}$$

Combining equations (54) and (55), we obtain the limiting tax rate for capital income

$$\tau^k = \frac{1}{F_k}\left[\frac{u_k}{u_l}\frac{H_n}{H_x} - \frac{u_k + \widetilde{\Phi}\left(u_{kk}k + u_k + u_{ck}c\right)}{\theta}\right]. \tag{56}$$

Namely, the limiting capital income tax may be positve, negative or zero. From equation (47) or (49), we solve for the expressions of θ, substitute them into (56) and hence rewrite the expression for the limiting capital tax

$$\tau^k = \frac{F_e M_n \widetilde{\Phi}}{F_k u_l}\left[\frac{u_k\left(u_{cl}c + u_{kl}k\right) - u_l\left(u_{kk}k + u_k + u_{ck}c\right)}{u_l + \widetilde{\Phi}\left(u_{cl}c + u_{kl}k\right)}\right],$$

which is very similar to the formula (30) of the baseline case without human capital. The sign of the limiting capital tax depends on the numerator of the term in the square brackets, which also depends on the utility functional form not on the production technology. The formulas of nonzero capital income taxation are very similar for these two models with status concerns.

Labor Income Tax (τ^n). From (48) and (49), we know that in the steady state

$$\frac{\theta}{v} = \frac{H_n}{F_e M_n}.$$

The substitution of equation (50) into the above equation yields

$$\frac{H_n}{H_x} = F_e M_n. \tag{57}$$

Meanwhile, the first-order equation (37) of the representative consumer tells us

$$\frac{H_n}{H_x} = \left(1 - \tau^n\right) F_e M_n. \tag{58}$$

If follows immediately from equations (57) and (58) that $\tau^n = 0$. Even though the optimal tax on physical capital is indeterminate, the limiting tax on human capital (or labor income) is definitely equal to zero.

Taxing on Expenditures (τ^m, τ^c). Given $\tau^n = 0$, conditions (38) and (51) imply that $\tau^m = 0$. To derive the optimal consumption rax, we use equation (37) and $\tau^n = 0$ to get

$$1 + \tau^c = \frac{u_c}{u_l} F_e M_n.$$

From equations (47) and (49), we have

$$F_e M_n = \frac{u_l + \widetilde{\Phi}\left(u_{cl}c + u_{kl}k\right)}{u_c + \widetilde{\Phi}\left(u_{cc}c + u_c + u_{kc}k\right)}.$$

Hence, we know from the above two equations

$$1 + \tau^c = \frac{u_c}{u_l}\frac{u_l + \widetilde{\Phi}\left(u_{cl}c + u_{kl}k\right)}{u_c + \widetilde{\Phi}\left(u_{cc}c + u_c + u_{kc}k\right)},$$

which displays that the limiting constumption tax is not zero in general.

Therefore, we have proved the following

Theorem 2 *In the status-concerns model with both physical capital and human capital, the limiting capital income tax can be positive, negative or zero; the optimal tax rates on labor income and expenditures for generating effective labor are both zeros; and the limiting optimal consumption tax depends.*

4 Heterogeneous Agents

In this section we extend the model to more realistic settings with heterogeneous agents. Each agent is a point in the unit interval $[0,1]$. There are two types of agents, workers and capitalists/entrepreneurs, with exogenously given weights α and $1-\alpha$, respectively. Entrepreneurs save, trade with the government and care about their social status. They do not work. Workers work for salaries and derive utility from consumption and leisure. We use superscripts 1 and 2 to denote capitalists and workers respectively. Both capitalists and workers discount the future with a common discount factor $\beta \in (0,1)$. Firms hire labor from workers, rent capital from capitalists and produce the final goods with the linearly homogenous production technology $F\left(k_t^1, n_t^2\right)$.

The representative capitalist solves the following maximization problem:

$$\max_{\{c_t^1, k_{t+1}^1, b_{t+1}^1\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u^1(c_t^1, k_t^1),$$

$$s.t. \;\; c_t^1 + k_{t+1}^1 - (1-\delta)k_t^1 = (1-\tau_t^k)r_t k_t^1 + b_t - \frac{b_{t+1}}{R_t},$$

and the representative worker solves

$$\max_{\{c_t^2, n_t^2\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u^2(c_t^2, 1-n_t^2),$$

$$s.t., c_t^2 = (1-\tau_t^n)w_t n_t^2.$$

The government finances its expenditures $\{g_t\}_{t=0}^{\infty}$ with tax revenues and one-period bonds with a balanced budget constraint

$$g_t = \tau_t^k r_t k_t^1 + \tau_t^n w_t n_t^2 + \frac{b_{t+1}}{R_t} - b_t.$$

And the resource constraint of the economy is

$$c_t^1 + c_t^2 + k_{t+1}^1 - (1-\delta)k_t^1 + g_t = F\left(k_t^1, n_t^2\right). \tag{59}$$

From the first order conditions form the representative capitalist, we have the consumption Euler equation

$$u_c^1(t) = \beta\left\{u_k^1(t+1) + u_c^1(t+1)\left[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta)\right]\right\}, \tag{60}$$

and the no-arbitrage condition

$$R_t = \frac{(1-\tau_{t+1}^k)r_{t+1} + 1 - \delta}{1 - \beta u_k^1(t+1)/u_c^1(t)}.$$

The optimization of the representative worker is described by the static equation

$$\frac{u_l^2(c_t^2, 1-n_t^2)}{u_c^2(c_t^2, 1-n_t^2)} = (1-\tau_t^n)\, w_t = \frac{c_t^2}{n_t^2}. \tag{61}$$

The implementability condition can be derived as follows

$$\sum_{t=0}^{\infty} \beta^t [u_c^1(t) c_t^1 + \beta u_k^1(t+1) k_{t+1}^i] = u_c^1(0)\{[(1-\tau_0^k) r_0 + 1 - \delta] k_0^1 + b_0\} \equiv \widetilde{A}_3. \tag{62}$$

The Ramsey problem is to maximize a weighted sum of utilities with weights α on capitalists and $(1-\alpha)$ on workers

$$\max_{\{c_t^1, c_t^2, k_{t+1}^1, n_t^2\}} \sum_{t=0}^{\infty} \beta^t \left[\alpha u^1(c_t^1, k_t^1) + (1-\alpha)\, u^2(c_t^2, 1-n_t^2)\right],$$

subject to the imc (62), the static optimization condition of the worker (61), i.e., $u_c^2(t)\, c_t^2 = u_l^2(t) n_t^2$, and the resource constraint (59). To solve it, we construct the Lagrangian

$$\begin{aligned} L \;=\; & \sum_{t=0}^{\infty} \beta^t \left[\alpha u^1(c_t^1, k_t^1) + (1-\alpha)\, u^2(c_t^2, 1-n_t^2)\right] + \widehat{\Phi} \left[\sum_{t=0}^{\infty} \beta^t [u_c^1(t) c_t^1 + \beta u_k^1(t+1) k_{t+1}^i] - \widetilde{A}_3\right] \\ & + \sum_{t=0}^{\infty} \beta^t \mu_t \left[u_l^2(t) n_t^2 - u_c^2(t)\, c_t^2\right] + \sum_{t=0}^{\infty} \beta^t \theta_t \left[F\left(k_t^1, n_t^2\right) - c_t^1 - c_t^2 - k_{t+1}^1 + (1-\delta) k_t^1 - g_t\right], \end{aligned}$$

where $\widehat{\Phi}$, $\{\mu_t\}_{t=0}^{\infty}$ and $\{\theta_t\}_{t=0}^{\infty}$ are the Lagrange multipliers associated with (62), (61), and (59), respectively. The optimality conditions for c_t^1, c_t^2, n_t^2, and k_{t+1}^1 are:

$$\left(\alpha + \widehat{\Phi}\right) u_c^1(t) + \widehat{\Phi} \underbrace{\left[u_{cc}^1(t)\, c_t^1 + u_{kc}^1(t)\, k_t^1\right]}_{\equiv \varrho_1} = \theta_t, \tag{63}$$

$$(1-\alpha-\mu_t)\, u_c^2(t) + \mu_t \underbrace{\left[u_{lc}^2(t)\, n_t^2 - u_{cc}^2(t)\, c_t^2\right]}_{\equiv \varrho_2} = \theta_t, \tag{64}$$

$$(1-\alpha-\mu_t)\, u_l^2(t) + \mu_t \underbrace{\left[u_{ll}^2(t)\, n_t^2 - u_{cl}^2(t)\, c_t^2\right]}_{\equiv \varrho_3} = \theta_t F_n(t), \tag{65}$$

$$\beta \left\{ \left(\alpha + \widehat{\Phi}\right) u_k^1(t+1) + \widehat{\Phi} \underbrace{\left[u_{kk}^1(t+1)\, k_{t+1}^1 + u_{ck}^1(t+1)\, c_{t+1}^1\right]}_{\equiv \varrho_4} \right\} = \theta_t - \beta \theta_{t+1} \left[F_k(t+1) + 1 - \delta\right]. \tag{66}$$

Suppose that the economy converges to an interior steady state. Combining the steady state equations of (60) and (66) yields us

$$\tau^k = \frac{1}{F_k} \left[\frac{u_k^1}{u_c^1} - \frac{\alpha + \widehat{\Phi}}{\theta} u_k^1 - \frac{\widehat{\Phi}}{\theta} \varrho_4\right].$$

Solving equation (63) for $\left(\alpha + \widehat{\Phi}\right)/\theta = \left(1 - \widehat{\Phi}\varrho_1/\theta\right)/u_c^1$ and putting it into the above equation, we solve for

$$\tau^k = \frac{\widehat{\Phi}}{\theta}\frac{1}{u_c^1 F_k}\left(u_k^1\varrho_1 - u_c^1\varrho_4\right).$$

To search for the limiting labor income tax, we combine equations (64) and (65) to derive $u_l^2/u_c^2 = (\theta F_n - \mu\varrho_3)/(\theta - \mu\varrho_2)$. Substituting it into the static optimization equation of the representative worker, we obtain the formula for the limiting labor income tax

$$\tau^k = \frac{\varrho_3 - \varrho_2 F_n}{F_n}\frac{\mu}{\theta - \mu\varrho_2}.$$

Therefore, we have the following

Theorem 3 *Assume that there exists an interior steady state in the economy with heterogeneous agents. The limiting capital income tax can be positive, negative or zero, the sign of which is determined completely by the utility function. The sign of the limiting labor income tax is also indefinite.*

5 Conclusion

By introducing status preferences into the dynamic optimal tax theory, we reexamine the Chamley-Judd results on optimal capital income taxation in the different settings utilized extensively in the literature. Generally, zero limiting capital income tax does not hold in these settings with status concerns. We examine not only the representative agent models (with unique physical capital, with multiple physicla capitals, and with both human capital and physical capital), but also the heterogeneous-agent model. In each case, we derive explicitly the formula for optimal capital tax, the sign of which depends completely on the particular specification of the very general utility function but not the production side of the economy. The sign of the limiting capital income tax is indefinite, that is, the limiting capital income tax can be positive, negative or zero.

6 Mathematical Appendix

6.1 Appendix A: Derive the Baseline Model

We firstly derive the present-value budget constraint. Repeated substitutions of equation (5) from time 0 lead to

$$b_0 = \sum_{t=0}^{\infty} q_t^0 \left\{ c_t - (1-\tau_t^n)w_t n_t + \underbrace{k_{t+1} - \left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] k_t}_{\equiv x_t} \right\} + \lim_{T\to\infty} q_t^0 b_T. \tag{67}$$

Then we derive the term $\sum_{t=0}^{\infty} q_t^0 x_t$ in the above equation (67)

$$\begin{aligned}
\sum_{t=0}^{\infty} q_t^0 x_t &= \sum_{t=0}^{\infty} q_t^0 \left\{k_{t+1} - \left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] k_t\right\} \\
&= \lim_{T\to\infty} \sum_{t=0}^{T} q_t^0 \left\{k_{t+1} - \left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] k_t\right\} \\
&= \lim_{T\to\infty} \left\{\sum_{t=0}^{T} q_t^0 k_{t+1} - \sum_{t=1}^{T} q_t^0 \left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] k_t\right\} - \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 \\
&= \lim_{T\to\infty} \sum_{t=0}^{T-1} \left\{q_t^0 - q_{t+1}^0 \left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]\right\} k_{t+1} - \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + \lim_{T\to\infty} q_T^0 k_{T+1} \\
&= \sum_{t=0}^{\infty} \left\{q_t^0 - q_{t+1}^0 \left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]\right\} k_{t+1} - \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + \lim_{T\to\infty} q_T^0 k_{T+1}.
\end{aligned}$$

Substituting equations (6) and (9), we rewrite the modified no-arbitrage condition (12) as follows:

$$R_t - \left[(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k\right] = \frac{u_k(t+1)}{u_c(t+1)}.$$

Multiplying both sides of the above equation with q_{t+1}^0 and using the definition of the Arrow-Debreu price, we have

$$q_t^0 - q_{t+1}^0 \left[(1-\tau_{t+1}^k) r_{t+1} + 1 - \delta_k\right] = q_{t+1}^0 \frac{u_k(t+1)}{u_c(t+1)}. \tag{68}$$

Plugging (68) in the term $\sum_{t=0}^{\infty} q_t^0 x_t$ gives us

$$\sum_{t=0}^{\infty} q_t^0 x_t = \sum_{t=0}^{\infty} q_{t+1}^0 \frac{u_k(t+1)}{u_c(t+1)} k_{t+1} - \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + \lim_{T\to\infty} q_T^0 k_{T+1}.$$

Substituting the above result about $\sum_{t=0}^{\infty} q_t^0 x_t$ into equation (67) and imposing the transversality conditions

$$\lim_{T\to\infty} q_t^0 b_T = 0, \lim_{T\to\infty} q_T^0 k_{T+1} = 0,$$

we obtain the present-value budget constraint of the representative consumer

$$\sum_{t=0}^{\infty} \left[q_t^0 c_t + q_{t+1}^0 \frac{u_k(t+1)}{u_c(t+1)} k_{t+1}\right] = \sum_{t=0}^{\infty} q_t^0 (1-\tau_t^n) w_t n_t + \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0.$$

Then, substituting those price equations (18) and (19) into the present-value budget constraint (14) and rearranging, we have the implementability condition (21):

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t - u_l(t) n_t + \beta u_k(t+1) k_{t+1}] = u_c(0)\{[(1-\tau_0^k) r_0 + 1 - \delta_k] k_0 + b_0\} \equiv A.$$

6.2 Appendix B: Derive the Model with Human Capital

Firstly, we derive the First-Order Conditions of the representative household. The Lagrangian is constructed as follows

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^{t}\left\{\begin{array}{c} u(c_t,1-n_{ht}-n_{mt},k_t)+\mu_t\left[(1-\delta_h)\,h_t+H\left(x_{ht},h_t,n_{ht}\right)-h_{t+1}\right]+ \\ \lambda_t\left[\begin{array}{c}(1-\tau_t^k)r_tk_t+(1-\tau_t^n)w_tM\left(x_{mt},h_t,n_{mt}\right)+(1-\delta_k)k_t+b_t \\ -\left(1+\tau_t^c\right)c_t-k_{t+1}-\left(1+\tau_t^m\right)x_{mt}-x_{ht}-\frac{b_{t+1}}{R_t}\end{array}\right]\end{array}\right\},$$

where λ_t and μ_t are two Lagrange multipliers associated with the flow budget constraint and the dynamic equation of human capital. The necessary conditions are

$$\begin{aligned}
u_c\left(t\right) &= \left(1+\tau_t^c\right)\lambda_t,\left(c_t\right) && (69)\\
u_l\left(t\right) &= \mu_t H_n\left(t\right),\left(n_{ht}\right) && (70)\\
u_l\left(t\right) &= \lambda_t(1-\tau_t^n)w_tM_n\left(t\right),\left(n_{mt}\right) && (71)\\
\lambda_t &= \mu_t H_x\left(t\right),\left(x_{ht}\right) && (72)\\
(1-\tau_t^n)w_tM_x\left(t\right) &= \left(1+\tau_t^m\right),\left(x_{mt}\right) && (73)\\
\lambda_t &= \beta\left\{u_k(t+1)+\lambda_{t+1}[(1-\tau_{t+1}^k)r_{t+1}+1-\delta_k]\right\},\left(k_{t+1}\right) && (74)\\
\mu_t &= \beta\lambda_{t+1}(1-\tau_{t+1}^n)w_{t+1}M_h\left(t+1\right)+\beta\mu_{t+1}\left[H_h\left(t+1\right)+1-\delta_h\right],\left(h_{t+1}\right) && (75)\\
\frac{\lambda_t}{R_t} &= \beta\lambda_{t+1}.\left(b_{t+1}\right) && (76)
\end{aligned}$$

From equations (69)-(72), we have equation (37). Equation (73) is (38). Combining equations (69) and (74) gives us the Euler equation (39). Substituting equations (69) and (70) into (75) leads to (40). Equation (41) comes from equations (69) and (76). The no-arbitrage condition (42) comes from equations (69), (74), and (76).

Secondly, the implementability condition can be derived by the following procedure. Applying the homogeneity of H to equation (34) and solving for x_{ht}, we have

$$x_{ht}=\frac{h_{t+1}-\left[1-\delta_h+H_h\left(t\right)\right]h_t}{H_x\left(t\right)}.$$

Substitute the above expression for x_{ht} and the production technology of effective labor into the sum on the right side of equation (43), which then becomes

$$\sum_{t=0}^{\infty}q_t^0\left[(1-\tau_t^n)w_tM_x\left(t\right)x_{mt}+(1-\tau_t^n)w_tM_h\left(t\right)h_t-\left(1+\tau_t^m\right)x_{mt}-\frac{h_{t+1}-\left[1-\delta_h+H_h\left(t\right)\right]h_t}{H_x\left(t\right)}\right],$$

where we have also invoked the homogeneity of M. First-order condition (73) implies that the term multiplying x_{mt} is zero, $[(1-\tau_t^n)w_tM_x\left(t\right)-\left(1+\tau_t^m\right)]=0$. After rearranging, we are left with

$$\left[(1-\tau_0^n)w_0M_h\left(0\right)+\frac{1-\delta_h+H_h\left(0\right)}{H_x\left(0\right)}\right]h_0-$$
$$\sum_{t=1}^{\infty}h_t\left\{\frac{q_{t-1}^0}{H_x\left(t-1\right)}-q_t^0\left[(1-\tau_t^n)w_tM_h\left(t\right)+\frac{1-\delta_h+H_h\left(t\right)}{H_x\left(t\right)}\right]\right\}.$$

However, the term in braces is zero by first-order condition (45), so the sum on the right side of equation (43) simplifies to the very first term in this expression. Then substituting (44) into equation (43) gives the implementability condition (46).

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