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Abstract

We propose a tractable algorithm to solve stochastic growth models of structural change. Under general conditions, structural change implies an unbalanced growth path. This property prevents the use of local solution techniques when uncertainty is introduced, and requires the adoption of global methods. Our algorithm relies on the Parameterized Expectations Approximation and we apply it to a stochastic version of a three-sector structural transformation growth model with Stone-Geary preferences. We use the calibrated solution to show that in this class of models there exists a tension between the long- and the short-run properties of the economy. This tension is due to the non-homothetic components of the various types of consumption, which are needed to fit long-run structural change, but imply a counterfactually high volatility of services, and counterfactually low volatilities of manufacturing and agriculture in the short-run.

JEL Classification: C63, L16, O41.
Keywords: Structural Change, Stochastic Growth, Parameterized Expectations Approximation.

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1 Introduction

Models of structural change attracted a great deal of attention in the last decades. Indeed, the reallocation of resources along economic development out of agriculture into manufacturing first, and into services later, appears as a robust empirical observation that occurs together with economic growth in cross-country data. By its nature, structural change is a long run phenomenon, so that its macroeconomic effects are observed over large time spans. For this reason, the typical modeling strategy assumes no uncertainty in the economy.[1] Departing from this convention, a relatively small number of contributions show that this process can have effects that are observable at the business cycle frequencies. Da-Rocha and Restuccia (2006) use a model with agriculture and non-agriculture sectors to show that the size of the employment share in agriculture can account for a large fraction of the differences in the magnitude of aggregate output volatility across countries. Similarly, Moro (2012) models an economy displaying structural transformation between manufacturing and services and finds that in the calibrated model the rise of the value added share of services can account for 28% of the decline in aggregate output volatility observed in the U.S. after 1980, while Moro (2015), in the context of a similar model, finds that structural transformation can account for at least 83% of the larger output volatility in middle-income relative to high-income economies. Carvalho and Gabaix (2013) perform a volatility accounting exercise for the U.S. and show that aggregate output volatility can be traced back to the change in size of the various sectors in the economy that display heterogeneous volatilities. More recently, Yao and Zhu (2018) use a two-sector model to show that the absence of employment-output correlation in China is due to the large size of the agricultural sector in that country, while Storesletten, Zhao, and Zilibotti (2019) provide a stochastic model displaying an acceleration of structural change in booms and a deceleration in recessions.

Ideally, to analyze the short run properties of a growth model with structural change and stochastic elements one needs to solve for the entire *stochastic growth path* and compute the statistics generated by the model along such path. However, due to structural transformation, computing it presents a technical challenge. The issue arises because, under general conditions, growth in the model is unbalanced.[2] This implies that at any point in time the model is in transitional dynamics during the relevant period of analysis, meaning that we

[1]See for instance Kongsamut, Rebelo, and Xie (2001), Ngai and Pissarides (2007), Kylymnyuk, Maliar, and Maliar (2007), Duarte and Restuccia (2010), Herrendorf, Rogerson, and Valentinyi (2014), Boppart (2014), Comin, Lashkari, and Mestieri (2015), Duernecker and Herrendorf (2016), among many others.

[2]Some papers in the literature discuss how some structural change models may display balanced growth when measured in terms of a numeraire good, and unbalanced growth when measured in line with NIPA conventions (see Duernecker, Herrendorf, and Valentinyi (2017) and Leon-Ledesma and Moro (2017)). Here we refer to theoretical unbalanced growth in terms of the numeraire good.

cannot focus our study on a steady state or a balanced growth path (BGP) . When adding uncertainty, this prevents the use of standard local solutions methods, as they involve approximations around a steady state or a BGP. Due to this technical difficulty, the existing literature employing growth models displaying structural change in a stochastic environment typically resorts to simplifying assumptions to find numerical solutions using local methods.[3]

The main contribution of this paper is to propose a tractable algorithm that allows to solve for the entire stochastic growth path with structural transformation and capital accumulation. In addition to the theory, we provide a Matlab toolbox that can be easily implemented to solve a large class of stochastic structural change models.[4] Our method relies on the technique developed by Den Haan and Marcet (1990) to compute dynamic systems, the Parameterized Expectations Approximation (PEA). The algorithm approximates the conditional expectation that typically appears in the first order conditions of a stochastic model as a function of the state variables. This procedure does not rely on the economy being in steady state or on a BGP.

We apply the algorithm to a stochastic version of the multi-sector growth model presented in Herrendorf, Rogerson, and Valentinyi (2014). While it is well known that this model fits successfully the long run properties of structural transformation in the post-war U.S., its ability to replicate the short run properties along the growth path is unexplored until now.[5] We calibrate the model to fit long run structural transformation, aggregate consumption growth and volatility and relative volatility of sectoral prices, finding that it can match long-run characteristics of the data well.

However, our analysis reveals a poor performance of the model in the short-run. The model produces a counterfactually high volatility of services and conterfactually low volatilies of agriculture and manufacturing, even when the volatility of service TFP is substantially lower than the other two sectors. We argue that the poor performance of the model in terms of volatility of the individual consumption components is due to the non-homothetic terms on services and agriculture in the utility function. Given the evolution of sectoral prices and aggregate consumption expenditure as measured in the data, the model can only account for the structural change in the U.S. economy when the income elasticity of services is larger than one and that of agriculture is smaller than one. However, these elasticities also determine the short-run behavior of the economy, by implying a high responsiveness of services consumption and a low one of manufacturing and agriculture consumption to both

[3]An exception is Storesletten, Zhao, and Zilibotti (2019), as we explain below.

[4]To download the toolbox go to https://unh.box.com/s/x6joe5ry2gds1yaqlu2490s1hkn9cki9.

[5]Buera and Kaboski (2009) note that by considering a time span starting from the late 1800's, the ability of the model to reproduce long-run structural change is substantially reduced. However, our focus here is on the post-war period.

income and price shocks. As a result, the calibrated parameters imply a counterfactually high volatility of services consumption and a counterfactually low volatility of agriculture consumption. When we set non-homothetic parameters to zero, thus imposing a common income elasticity equal to one in the three sectors, the performance of the model in terms of volatility of individual consumption components improves, while the ability of the model to fit long run structural change is dramatically reduced. The punchline is that structural change models embed a trade-off between their short- and long-run properties. To improve on the short-run fit of the model, one has to reduce the long-run fit.

To better illustrate the benefits of having an algorithm to compute the entire stochastic stochastic growth path, consider the two solution methods adopted in Moro (2012) and Moro (2015). The working tool in both works is a two-sector growth model with non-homothetic preferences and exogenous stochastic TFP in the two sectors. However, the two papers resort to different solution methods to analyze the business-cycle effects of structural transformation. Moro (2012) performs standard RBC analysis around two steady states differing in the *level* of TFP but not in the stochastic process for TFP shocks in the two sectors. Due to non-homothetic preferences, a larger TFP level endogenously creates a larger share of services in steady-state. In turn, the different size of the share of services affects the response of endogenous variables to shocks. Thus, in this case the Euler equation is made stationary by removing the growth component of TFP. This amounts to imposing no future structural change in the expectation term. Moro (2015), instead, focuses on both growth and volatility together so that, to find a solution, it simplifies the model by dropping capital accumulation and assuming an exogenous growth rate for TFP in the two sectors. In this model TFP is given by a deterministic growth component and a stochastic cyclical component, and the growth path becomes a sequence of static equilibria. In this case there is no forward looking component in the household problem (i.e. no expectation term and no Euler equation) and the effect on investment on the cyclical properties of the economy is excluded. The method presented here allows to avoid such simplifications and to study the volatility properties of the multi-sector model along the entire growth path.

A third type of method used in the literature is that proposed in Storesletten, Zhao, and Zilibotti (2019), which involves a global solution. It relies on the fact that the model converges to a one sector economy when time goes to infinity. Their algorithm assumes that this state is reached in finite time, and from then on the model can be computed with standard RBC techniques. This provides an end-point, and the model can be solved using backward induction all the way to period one. For the backward induction to work, grids for the endogenous and exogenous state variables are needed. Our algorithm is more general, not requiring the economy to converge to a one sector economy (in fact, our economy converges to a three sector economy), and does not require the use of grids for the state variables, which makes it highly tractable.

Note that these challenges do not apply forstructural change models displaying a BGP, like the ones in Kongsamut, Rebelo, and Xie (2001), Ngai and Pissarides (2007), Boppart (2014) and Comin, Lashkari, and Mestieri (2015). These models display, under certain conditions, a time invariant Euler equation so that standard techniques can be used to analyze the short-run properties of the economy. However, the existence of balanced growth relies on a specific set of assumptions in each model. Slight modifications of these assumptions imply that the balanced growth path does not exist anymore, making the Euler equation time-varying and preventing the use of standard techniques. The typical example is given by the model resulting by merging Kongsamut, Rebelo, and Xie (2001) and Ngai and Pissarides (2007), which are two models displaying a BGP. As discussed in Herrendorf, Rogerson, and Valentinyi (2014), the combination of the two models *fails* to display a BGP.[6] Thus, under general conditions, one cannot rely on the existence of a BGP to carry out high frequency analysis in a multi-sector model, and the algorithm presented in this paper appears as the only available tool that avoids simplifications as the ones used in Moro (2012) and Moro (2015).

The remainder of the paper is as follows. Section 2 describes the stochastic version of the multi-sector growth model in Herrendorf, Rogerson, and Valentinyi (2014); section 3 presents the Parameterized Expectations Algorithm; sections 4 and 5 describe the data and the calibration, respectively, while section 6 presents the main results of the model. Section 7 concludes.

2 Model

The model builds on Herrendorf, Rogerson, and Valentinyi (2014), expanded with stochastic productivity shocks on the production functions. Time is discrete and there is a representative agent with preferences defined over the consumption of three goods: agriculture (a), manufacturing (m) and services (s). The utility function is

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \log(C_t),$$

[6] Also, a common assumption of multi-sector models displaying a BGP is the same constant returns to scale Cobb-Douglas technology in each sector, which makes the relative price of two goods independent of the quantities produced of the two goods. Dropping the assumption of equal Cobb-Douglas exponent in the two sectors would imply the non-existence of a BGP in most canonical models.

where

$$C_t = \left[\sum_{j=a,m,s} \omega_j^{1/\mu} (c_{jt} + \bar{c}_j)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \tag{1}$$

and c_{jt} is period t consumption of good $j = a, m, s$. The weights $\omega_j > 0$ denote the relative importance of sector $j = a, m, s$ in the aggregate consumption index C_t and $\mu > 0$ governs the elasticity of substitution. The terms $\bar{c}_j$ introduce non-homotheticities to the problem.

Each good can be produced with a technology that inputs capital and labor. These technologies are

$$Y_{jt} = e^{z_{jt}} K_{jt}^{\alpha} L_{jt}^{1-\alpha}, \ 0 \leq \alpha \leq 1, \ j = a, m, s$$

The shock z_{jt} is the sum of a first order autoregressive process, plus a deterministic component that grows with time. More specifically,

$$z_{jt} = \hat{z}_{jt} + g_j t + A_j, \qquad \hat{z}_{jt} = \rho_j \hat{z}_{j,t-1} + \varepsilon_{jt}, \ \varepsilon_{jt} \sim \mathbb{N}(0, \sigma_j^2), \ j = a, m, s,$$

The representative agent owns the stock of capital and rents it out to the firms. Capital evolves according to

$$K_{t+1} = (1-\delta) K_t + X_t,$$

where $\delta \in [0, 1]$ is the rate of depreciation and X_t is investment. Denoting by p_{jt} the price of each good, w_t the wage rate and r_t the rental rate of capital, the period t budget constraint is

$$\sum_{j=a,m,s} p_{jt} c_{jt} + p_{mt} X_t = w_t + r_t K_t.$$

Feasibility requires the following market clearing conditions for all t:

$$Y_{at} = c_{at}, \quad Y_{mt} = c_{mt} + X_t, \quad Y_{st} = c_{st}, \quad K_t = \sum_{j=a,m,s} K_{jt}, \quad 1 = \sum_{j=a,m,s} L_{jt},$$

where we normalized labor supply to 1.

2.1 Equilibrium

A competitive equilibrium for this economy is a list of allocations $\{c_{jt}\}$ and prices $\{p_{jt}, r_t, w_t\}$ for $j = a, m, s$ and $t = 0, 1, \ldots, \infty$, such that the representative consumer maximizes utility subject to the budget constraint, firms maximize profits, and all markets clear.

We normalize the price of the manufacturing good p_{mt} to 1 each period. Profit maximization implies that equilibrium prices must satisfy

$$p_{at} = e^{z_{mt}-z_{at}},\ p_{st} = e^{z_{mt}-z_{st}}.$$

The firm first order conditions guarantee that the capital labor ratios equalize across sectors, which simultaneously implies that, since $L = 1$, for all t,

$$\frac{K_{jt}}{L_{jt}} = K_t,\ j = a, m, s.$$

This pins down the wage rate and the rental rate as a function of the stock of capital:

$$w_t = e^{z_{mt}}(1-\alpha)K_t^{\alpha}, \tag{2}$$

and

$$r_t = e^{z_{mt}}\alpha K_t^{\alpha-1}. \tag{3}$$

As shown in Herrendorf, Rogerson, and Valentinyi (2014), the dynamic problem of the consumer can be split into two: an intertemporal problem, in which the consumer makes a saving/consumption decision, that determines aggregate consumption expenditures in each period t; and a static problem, in which at each date t, by taking as given consumption expenditure from the intertemporal problem, the agent chooses the consumption level of each individual good in the consumption index. Let E_t denote consumption expenditure in period t. The static problem is

$$\max_{c_a,c_m,c_s} \log\left[\sum_{j=a,m,s} \omega_j^{1/\mu}(c_{jt}+\bar{c}_j)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \ s.t. \sum_{j=a,m,s} p_{jt}c_{jt} = E_t,$$

with solution

$$c_{mt} + \bar{c}_m = \frac{E_t + \sum_{j=a,m,s} p_{jt}\bar{c}_j}{P_t^{1-\mu}}, \tag{4}$$

$$c_{at} + \bar{c}_a = p_{at}^{-\mu}\frac{\omega_a}{\omega_m}(c_{mt}+\bar{c}_m), \tag{5}$$

$$c_{st} + \bar{c}_s = p_{st}^{-\mu}\frac{\omega_s}{\omega_m}(c_{mt}+\bar{c}_m), \tag{6}$$

where

$$P_t = \left(\sum_{j=a,m,s} \omega_j p_{jt}^{1-\mu}\right)^{\frac{1}{1-\mu}}, \tag{7}$$

is the aggregate price index.

The intertemporal problem of the consumer determines the expenditure E_t each period. Investment and consumption expenditures are linked through the feasibility condition, such that

$$E_t + X_t = r_t K_t + w_t,$$

To derive the Euler equation, we exploit the fact that

$$P_t C_t - \sum_{j=a,m,s} p_j \bar{c}_j = E_t, \tag{8}$$

as proved in Herrendorf, Rogerson, and Valentinyi (2014). Also, inserting equations (5) and (6) into (1) we obtain

$$C_t = \frac{c_{mt} + \bar{c}_m}{\omega_m} \left[\sum_{j=a,m,s} \omega_j p_j^{1-\mu} \right]^{\frac{\mu}{\mu-1}} = \frac{c_{mt} + \bar{c}_m}{\omega_m} P_t^{-\mu}. \tag{9}$$

Equation (9) shows the mapping between C_t and c_{mt}, while equations (5) and (6) show the mapping between c_{mt} and c_{at} and c_{st}. These imply that we can set C_t as the choice variable and solve the consumer problem as

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \log(C_t)$$
$$s.t.$$

$$P_t C_t - \sum_{j=a,m,s} p_{jt} \bar{c}_j + K_{t+1} - (1-\delta) K_t = w_t + r_t K_t, \tag{10}$$

where we used (8) to drop individual consumption components from the intertemporal budget constraint.

The Euler equation can then be written as

$$1 = \beta \mathbb{E} \left[\frac{P_t C_t}{P_{t+1} C_{t+1}} (r_{t+1} + 1 - \delta) \right]. \tag{11}$$

In stationary models (either with a steady state or with a BGP and detrended) this problem is solved by approximating the expectation function with some variant of a Taylor expansion around a deterministic steady state. However, in a context of structural change, growth is unbalanced so neither a steady state nor a BGP exists, and local solution methods cannot be used.

To see this why growth is unbalanced in the deterministic model, suppose that P_tC_t grows on average at a constant rate and consider set $\sigma_j = 0$ for $j = a, m, s$. The Euler equation becomes:

$$\frac{P_{t+1}C_{t+1}}{P_tC_t} = 1 + g_{c,t} = \beta(1 - \delta + r_{t+1}).$$

For $g_{c,t}$ to be time invariant, r_t must be also constant over time. From equation (3) this can only happen when K_t grows at the rate $g_k = \frac{g_m}{1-\alpha}$, which implies, by equation (2), that w_t also grows at the rate g_k. It follows that the term $w_t + r_tK_t - K_{t+1} - (1 - \delta)K_t$ in the budget constraint grows at the rate g_k. Using the intertemporal budget constraint in two subsequent period, we then find that P_tC_t grows at the rate

$$g_{c,t} = \frac{(1 + g_k)(w_t + r_tK_t - K_{t+1} - (1 - \delta)K_t) + \sum_{j=a,m,s} p_{jt+1}\bar{c}_j}{w_t + r_tK_t - K_{t+1} - (1 - \delta)K_t + \sum_{j=a,m,s} p_{jt}\bar{c}_j} - 1.$$

This rate can only be constant over time when $\sum_{j=a,m,s} p_{jt}\bar{c}_j$ grows at the rate g_k.[7] However, this is never the case. To see this, note that $\sum_{j=a,m,s} p_{jt+1}\bar{c}_j = \sum_{j=a,m,s} e^{z_{mt}-z_{jt}}\bar{c}_j$ so that prices have a well defined growth rate which is a function of exogenous parameters, and so there is nothing that guarantees that the whole term grows at rate g_k. In addition, note that when $g_{c,t}$ is time-varying, it has to be that also r_{t+1} is time varying, such that the Euler equation is satisfied in each period. Finally, if r_{t+1} is time varying, from (3) we have that the capital stock does not grow at a constant rate. It follows that, in general, the conditions for a balanced growth path do not hold.[8]

The absence of a steady state or a BGP in the deterministic model prevents the use of standard local solution techniques in the stochastic version. We turn to the PEA method developed by Den Haan and Marcet (1990) to address this issue, which we describe next.

3 The Parameterized Expectations Approximation

The PEA is an algorithm that is well suited to work in stochastic models no stationarity. This method has been successfully applied in stationary environments where the cycles are deviations from a constant trend, as in Maliar and Maliar (2003b), Marcet and Marshall (1994), and Marcet and Lorenzoni (1998). In this paper we apply it to a growth environment

[7]Alternatively, it must be that $\sum_{j=a,m,s} p_{jt+1}\bar{c}_j = 0$ for all t, which is the condition in Kongsamut, Rebelo, and Xie (2001) for a balanced growth path. However, even with this condition fulfilled, there must be homogeneous TFP growth across sectors in the model to have a balanced growth path. As this condition is at odds with the data we disregard this particular case here.

[8]For the conditions under which a BGP exists in the context of the model presented here see Herrendorf, Rogerson, and Valentinyi (2014).

without a BGP. As such, we are aware of no other method that can address this type of issues.

The PEA exploits the facts that (i) the value of the right hand side of equation (11) must depend on the state variables k, z_a, z_m and z_s; and that (ii) any function can be approximated by a polynomial of sufficiently high order.

The approximation we choose is as follows:[9]

$$\log\left[\frac{1}{P_{t+1}C_{t+1}}(r_{t+1}+1-\delta)\right] \approx \Phi(k_t, z_{at}, z_{mt}, z_{st}), \tag{12}$$

where

$$\Phi(K_t, z_{at}, z_{mt}, z_{st}) = \eta_0 + \eta_1 f(K_t) + \eta_2 z_{at} + \eta_3 z_{mt} + \eta_4 z_{st} + \eta_5 f(K_t)^2 + \eta_6 z_{at}^2 + \eta_7 z_{mt}^2 + \eta_8 z_{st}^2.$$

To understand the intuition behind the algorithm, assume to know the equilibrium sequence $\{C_t, P_t, r_t\}$. Then the value of the vector $\eta = [\eta_0\ \eta_1\ \eta_2\ \eta_3\ \eta_4\ \eta_5\ \eta_6\ \eta_7\ \eta_8]$ can be obtained via an OLS regression, where

$$\log\left[\frac{1}{P_{t+1}C_{t+1}}(r_{t+1}+1-\delta)\right] = S_t\eta' + \epsilon_t,$$

$S_t = [1, f(K_t), z_{at}, z_{mt}, z_{st}, f(K_t)^2, z_{at}^2, z_{mt}^2, z_{st}^2]$ and ϵ_t is an error term with mean 0 and variance σ_ϵ^2. Alternatively, if we had the right values for η, one could compute the right hand side of equation 11 as

$$\mathbb{E}\left[\frac{1}{P_{t+1}C_{t+1}}(r_{t+1}+1-\delta)\right] = \mathbb{E}\left[e^{\Phi}\right] = \mathbb{E}\left[e^{\eta\times\mathbf{S_t}}e^{\epsilon_t}\right] = e^{\eta\times\mathbf{S_t}}\mathbb{E}\left[e^{\epsilon_t}\right] = e^{\eta\times\mathbf{S_t}}e^{\frac{\sigma_\epsilon^2}{2}}$$

The variables in S_t are known in period t, and the exponential of the error term follows a log-normal distribution where the mean of the error term is 0 and the variance is σ_ϵ^2.

The problem is that neither the sequence $\{C_t, P_t, r_t\}$ nor the true value for the vector η are known. We then use the following algorithm, based on Den Haan and Marcet (1990), to obtain the latter.

1. Guess initial values for the vector η, and an initial guess for σ_ϵ^2.

[9] We set up the function Φ so that we can compute the expectation in a closed form solution. Alternative specifications might require the computation of numerical expectations. For such cases, Judd, Maliar, Maliar, and Tsener (2017) describe how to write these expectations efficiently.

2. Given these guesses and the value of the state variables at time 0, compute C_0 from equation (11). Compute P_0 from the realizations of the shocks.

3. Obtain prices w_0 and r_0 from equations (2 and (3) respectively, and K_1 from equation (10).

4. Given the new draw of shocks, plugging K_1 and the shocks into equation (12) gives C_1 and K_2.

5. Continue to obtain the entire sequence of $\{C_t, K_t, r_t, w_t\}$ in a similar way.

6. Given the sequences, compute the left hand side of equation (12) and regress via OLS on the polynomial Φ. This will generate new estimates $\hat{\eta}$ and $\hat{\sigma_\epsilon}$.

7. Compare the values of $\hat{\eta}$ and η and the values of $\hat{\sigma_\epsilon}$ and σ_ϵ. If these are close, then we can conclude that we have the right values of η.[10] Otherwise, go to step 8.

8. Use these to update the guesses in step 1 as follows: $\eta' = \lambda\eta + (1-\lambda)\hat{\eta}$ and $\hat{\sigma}_\epsilon = \sigma_\epsilon$ for some $\lambda \in [0, 1)$.

9. Use the values of η' and σ'_ϵ as the new guesses in step 1, and go to step 2.

Notice that the polynomial includes a function of capital, $f(k)$. In its simplest form, we may have $f(k) = k$ or $f(k) = \log(k)$. However, these are not our preferred specifications. This is because of the presence of non-homotheticities in the model. As we discuss in section 5, we estimate $\bar{c}_a < 0$ and $\bar{c}_s > 0$. As a consequence, some terms in the capital sequence may turn negative (in off equilibrium iterations), which makes the algorithm less likely to converge to a feasible solution. For example, if k_t is relatively small and $\bar{c}_a$ is large in absolute value (but negative in sign) then by equation (10) k_{t+1} may become negative, which is not feasible.

A way around this is to assume that the capital stock has a lower bound, equal to a small positive number.[11] However, this presents a problem when the bound is active for most elements in the capital sequence, since this reduces the rank of the matrix S making it non-invertible. A solution to this problem arises from rearranging the budget constraint as follows:

$$K_{t+1} - \sum_{j=a,m,s} p_{jt}\bar{c}_j = r_t K_t + w_t + (1-\delta)K_t - P_t C_t$$

[10] The measure of distance between η, σ_ϵ and $\hat{\eta}, \hat{\sigma}_\epsilon$ that we use is $\sum_{i=1}^{9}(\eta_i - \hat{\eta}_i)^2 + (\sigma_\epsilon - \hat{\sigma}_\epsilon)^2$ and the tolerance level is 5×10^{-7}.

[11] This lower bound should only be active in iterations other than the final one.

This suggests the use of the function

$$f(K_t) = \log\left(K_t - \sum_{j=a,m,s} p_{jt-1}\bar{c}_j\right)$$

We find that this simple modification reduces the problem of encountering negative elements in the series for capital and speeds up computing times greatly.[12]

To improve the efficiency of the algorithm we adopt the upper and lower bounds discussed in Maliar and Maliar (2003a). We start with an upper bound that is 10% larger than our initial guess for η and a lower bound that is 10% below. This modifies Step 8 as follows:

$$\eta' = \max(lb, \min(ub, \lambda\eta + (1-\lambda)\hat{\eta}))$$

where lb stands for lower bound and ub for upper bound. Initially, $lb = \eta - 0.1|\eta|$ and $ub = \eta + 0.1|\eta|$, and we relax these bounds each iteration by a constant equal to 0.5. We choose to make these increases as a constant amount because when the estimate is close to zero, a proportional relaxation of the bound will have little impact. These bounds prevent the system from changing too much and so make the algorithm faster and more likely to converge.

Setting the initial guess for η is challenging, since the wrong guess can prevent the algorithm from converging. We find that setting the initial guess as $\eta = [1, -1, 0, 0, 0, 0, 0, 0]$ works reliably. This follows the suggestion in Marcet and Marshall (1994) and Marcet and Lorenzoni (1998), in that the function Φ must be invertible with respect to capital in the initial guess.

Finally, a potential concern is represented by the value to choose for λ. A smaller λ speeds up the algorithm, but makes it less likely to converge to a solution. To address this issue we start with a relatively low value of λ (equal to 0.5), and then keep the simulations that successfully converged to a (calibrated) solution. To address the remaining ones, we increase λ successively until all simulations converge.

4 Data

Our data comes from the National Income and Product Accounts (NIPA) published by the Bureau of Economic Analysis (BEA), from the year 1947 until 2010, at an annual level. Using

[12]The algorithm using $\log(k)$ without non-homotheticity terms also converges, but in general requires a larger value of λ, increasing computing time, with many iterations involving matrices with less than full rank, although final iterations always have full rank matrices. The results under this alternative are practically identical.

these data Herrendorf, Rogerson, and Valentinyi (2014) construct series for consumption and prices of each type of good (agriculture, manufacturing and services), both in terms of value added and final expenditures. We use these constructed data in our calibration. In particular, we use final expenditure because it requires less manipulation than value added measures, and most of it is readily available from the BEA. While we refer to their paper for the detail of the procedure for building the data, here we mention how each sector in the model maps to the data.

Most data comes from NIPA Tables 2.4.3 "Real Personal Consumption Expenditures by Type of Product, Quantity Indexes" and Table 2.4.5 "Personal Consumption Expenditures by Type of Product". Within these tables, agriculture is "food and beverages purchased for off-premises consumption", manufacturing includes "durable goods", and "non-durable goods" excluding "food and beverages purchased for off-premises consumption". Services includes "services" and "government consumption expenditures".

5 Calibration and Simulations

As is standard in the real business cycle literature, we set the model period to be one quarter. Note, however, that our data is annual, as we use the dataset in Herrendorf, Rogerson, and Valentinyi (2013). Thus, when comparing data and model, we annualize the results from the model. Also, the calibration requires knowledge of relative prices and consumption expenditure in the first and last quarter of the sample period. Our data, being annual, means that we do not have these for the first and last quarters. To address this we assume that the prices and expenditures in these quarters are the annual prices and expenditures in the years 1947 and 2010 respectively.

Several parameters are calibrated according to the existing literature. Since one period in the model corresponds to one quarter, we set $\beta = 0.99$, while we impose α =0.3 to match a capital share of 30%. We set the depreciation of capital δ equal to 0.015, which implies an annual depreciation rate of about 6%.

The main parameters to calibrate pertain to the utility function, and we follow Herrendorf, Rogerson, and Valentinyi (2013) in calibrating them. The expenditure shares can be expressed as

$$s_{jt} = \frac{p_{jt}c_{jt}}{E_t} = \frac{\omega_j p_{jt}^{1-\mu}}{\sum_{i=a,m,s} \omega_i p_{it}^{1-\mu}} \left(1 + \sum_{i=a,m,s} \frac{p_{it}\bar{c}_i}{E_t}\right) - \frac{p_{jt}\bar{c}_j}{E_t}.$$

Given the actual expenditures on each sector, the parameters $\omega_j, \bar{c}_j$ for $j = a, m, s$ and μ can be estimated via non-linear least squares. Notice that we can at most use the shares of two sectors, not three, because the three add up to 1 and make the system linearly dependent.

We further impose that $\omega_j \geq 0$ for all $j = a, m, s$, $\sum_{j=a,m,s} \omega_j = 1$ and $\mu > 0$. To estimate the parameters without these constraints, we make the following transformations

$$\mu = e^{b_0}, \omega_a = \frac{1}{1 + e^{b_1} + e^{b_2}}, \omega_m = \frac{e^{b_1}}{1 + e^{b_1} + e^{b_2}}, \omega_s = \frac{e^{b_2}}{1 + e^{b_1} + e^{b_2}},$$

and estimate the parameters b_0, b_1, b_2. Furthermore, we follow Kongsamut, Rebelo, and Xie (2001) in imposing $\bar{c}_m = 0$.[13] We obtain slightly different estimates than Herrendorf, Rogerson, and Valentinyi (2013), because we normalize total consumption expenditures to be equal to 1 in the starting period, 1947. This normalization only affects the calibrated values of the A_j's and $\bar{c}_j$'s, for $j = a, m, s$.

The shock parameters to calibrate are g_j, A_j, ρ_j and σ_j^2. Because of a lack of suitable targets, we set $\rho_j = \rho = 0.9$ for $j = a, m, s$. We experiment with different assumptions on ρ_j finding that the results barely change. We calibrate σ_j to match the volatility of the relative (to manufacturing) prices of agriculture and services, and the volatility of aggregate consumption. We obtain these volatilities in the data by detrending each series using an HP filter with smoothing parameter 100, and compute the standard deviation of the residual.

We perform several simulations of the model to derive our results. The next set of parameters vary for each simulation. We set A_a, A_s, g_a and g_s such that, given the realizations of the shocks $\hat{z}_{jt}$ for all j and t, the relative prices of agriculture and services match the data both in 1947 and in 2010.

To see how this works, recall that $p_{at} = e^{z_{mt} - z_{at}}$ and $p_{st} = e^{z_{mt} - z_{st}}$. Taking logs

$$\log p_{jt} = z_{mt} - z_{jt} = \hat{z}_{mt} - \hat{z}_{jt} + (g_{mt} - g_{jt})t + A_m - A_j, \quad j = a, s$$

Given the relative prices for 1947 and 2010 and the values of $\hat{z}_{at}, \hat{z}_{mt}, \hat{z}_{st}, g_{mt}$ and A_m we obtain the values of g_{at}, g_{st}, A_a and A_s. These are $p_{a,1947} = 0.62, p_{s,1947} = 0.32, p_{a,2010} = 1.11, p_{s,2010} = 1.12$. We set A_m and g_m so that the consumption expenditure matches the data in 1947 and 2010 (recall we normalize consumption expenditure to be 1 in 1947). To do this, we measure the consumption expenditure using chain weighted prices, as in the data.[14] Specifically, we proceed as follows. We compute the growth rate in chain weighted total consumption expenditures between 1947 and 2010. This gives a growth factor of 8.106 over the entire period. Given a value for A_m, we compute the value for g_m that would deliver this growth. We do this via a bisect method, with bounds between 0.0020 and 0.0035. Next, we set the value of A_m so that the consumption expenditure in the first quarter of 1947 is 1,

[13] Herrendorf, Rogerson, and Valentinyi (2013) find that ignoring this constraint hardly changes the goodness of fit. See their footnote 7.

[14] That is, in the model we compute real consumption expenditure using a chain-weighted Fisher index, in the same fashion the corresponding variable in the data is constructed.

according to our normalization. Again, we do this via a bisect method, with bounds between -1.95 and -1.60.

It is noteworthy that the bisect methods work very accurately. This highlights an important feature of the PEA itself. Numerous studies starting from Den Haan and Marcet (1990) and furthered by Marcet and Lorenzoni (1998) show how parsimonious this method is when applied to stationary series. The success of the bisect method in this paper shows that this parsimony extends to non-stationary processes as well. To see this, we highlight that, in general, bisect methods only work well under strong continuity and monotonicity assumptions. However, there is no assumption that guarantees either continuity or monotonicity in this case. To see this, notice how the calibration algorithm works:

1. Start with guesses A_{m0} and g_{m0} and set $A_m = A_{m0}$ and $g_m = g_{m0}$. Set A_a, A_s, g_a and g_s as described above.

2. Run Steps 1 through 9 in section 3.

3. Check growth of consumption expenditures. If this is larger than the data, reduce g_m. If it is lower, increase g_m. Otherwise, leave g_m as it is and focus on A_m by moving on to Step 4.

4. If consumption expenditures in period 1 are larger than 1, reduce A_m, and if they are less than in the data, increase it. Once they are equal, set $A_{m0} = A_m$ and go back to Step 1.

5. Once both the expenditure in period 1 and the growth of expenditure match the data, the calibration algorithm ends.

This calibration algorithm is only guaranteed to work when Step 2 (which is the step running the PEA) is monotonic and continuous, meaning that the PEA estimates of the vector η do not change substantially when either A_m or g_m change marginally. There is no theory that guarantees that this is the case here. In practice however, we find that the calibration algorithm runs efficiently in calibrating all simulations under different specifications.

We should mention that this success is not guaranteed regardless of the choice of certain variables. Mainly, the choices of the tolerance level for how "close" the estimates η and $\hat{\eta}$ and σ_ϵ and $\hat{\sigma}_\epsilon$ are, matter. The tighter this tolerance, the likelier the success of the calibration. The downside is that a very tight tolerance level reduces speed. We find that a tolerance level of 5×10^{-7} works well. The second key element that determines the success is λ, the weight on the old guesses for the $\eta's$ in Step 8. The larger the λ, the likelier the success. Again, the cost is time. Our solution is to start with a relatively low value for λ. This successfully calibrates a large number of simulations. For the remaining ones, we increase λ. We continue this way until all simulations are successfully calibrated. The Matlab Toolbox provided does this automatically.[15]

We perform 1,000 simulations of the model, and calibrate a specific g_m and A_m for each one. The relevant period of analysis is 1947 through 2010, which amounts to 256 quarters. To minimize dependency on initial conditions (i.e., the initial capital stock k_0) we simulate the economies for 400 periods and drop the initial 144 periods to have to 256 periods (quarters) as in the time span considered. We set $k_0 = 0.4$, which determines a relatively smooth and increasing series of capital. Starting from a different initial level does not change the analysis when we drop the initial 144 periods.[16] The calibrated values are very similar across simulations. On average, $g_a = 0.001, g_m = 0.0024$ and $g_s = -0.0025$, with standard deviations across simulations of $0.00015, 0.00007$ and 0.00015, respectively. The mean values for the A's are $A_a = -1.01, A_m = -1.18$ and $A_s = 0.03$, with standard deviations $0.047, 0.021$ and 0.045, respectively.

Table 1 shows the calibrated parameters. The values for g_j and A_j reported are averages across simulations.

We set λ, the weight on the estimates for the polynomial approximating the right hand side of the Euler equation, initially to 0.5. Given this value, the algorithm was able to successfully calibrate and 984 simulations. For the remaining 16 we increased λ to 0.75, which added 12 more successful simulations. We then increased it to $\lambda = 0.875$ which included 2 more simulations, $\lambda = 0.9375$ included one more, and $\lambda = 0.96875$ added the last simulation. Appendix A shows details of the estimation results in the PEA algorithm.

6 Results

The model performs well in terms of replicating structural change in the data. Figure 1 shows the evolution of value added shares of the three sectors for one of the 1,000 simulations (dashed red lines), together with the corresponding figures in the data (solid blue lines). All simulations perform very similarly in terms of structural change.[17] The good performance of the model in replicating long run-structural change is well know from the work of Herrendorf, Rogerson, and Valentinyi (2013). Here we confirm that a *stochastic version* of the model

[15]Specifically, the codes start off with a value of λ determined by the user (we use 0.5) and updates this value by adding halfway through 1. Thus, starting with 0.5, the next iteration has 0.75, then 0.875, then 0.9375, and so on.

[16]However, as we find when working with no non-homotheticicities below, a value of k_0 very far from this may prevent a successful calibration.

[17]In Figure 1 we report the first of the 1,000 simulations.

Parameter	Target	Value
α	Labor share of 70%	0.3000
β	Annual Interest rate of 4%	0.9900
δ	Annual depreciation rate of 6%	0.1500
μ	Non-linear least squares estimation	0.8478
ω_a	Non-linear least squares estimation	0.0210
ω_m	Non-linear least squares estimation	0.1690
ω_s	Non-linear least squares estimation	0.8100
$\bar{c}_a$	Non-linear least squares estimation	-0.2730
$\bar{c}_m$	Normalization	0.0000
$\bar{c}_s$	Non-linear least squares estimation	2.2714
ρ	Normalization	0.9000
g_a	Match agriculture relative price in 2010	0.0001
g_m	Match consumption expenditure in 2010	0.0024
g_s	Match services relative price in 2010	-0.0025
A_a	Match agriculture relative price in 1947	-1.0137
A_m	Match consumption expenditure in 1947	-1.8271
A_s	Match services relative price in 1947	0.0348
σ_a	Match volatility of relative price of agriculture	0.0035
σ_m	Match volatility of total consumption	0.0116
σ_s	Match volatility of relative price of services	1×10^{-5}

Table 1: Calibration

still fits the structural change data closely once calibrated to post-war U.S. data.

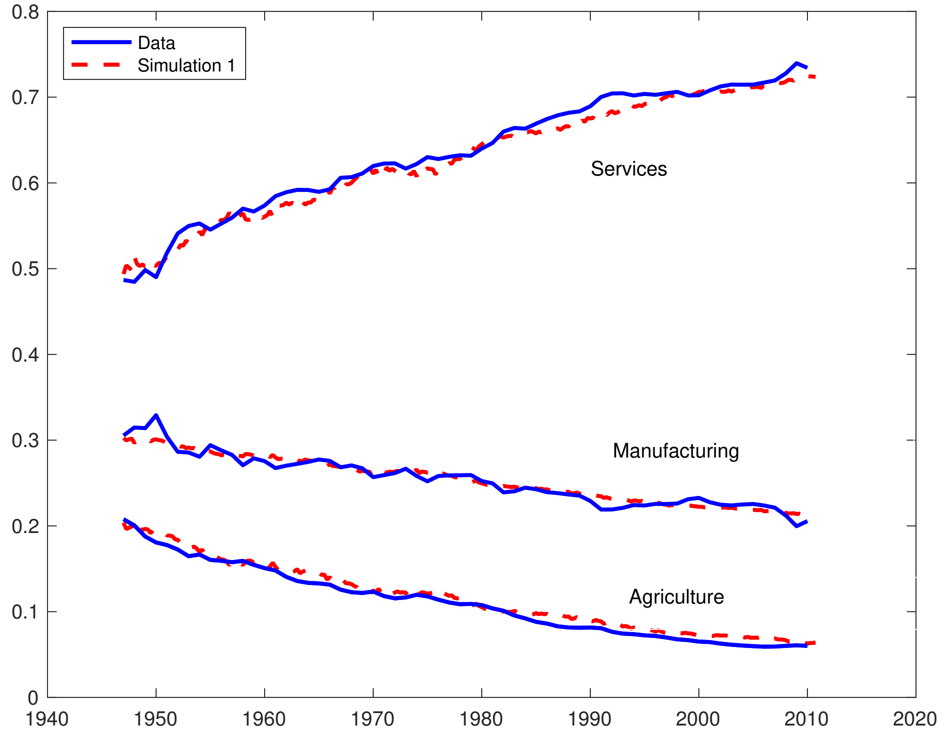

Figure 1: Structural Change in the Model and the Data.

We now turn to the business cycle performance of the model, which can be analyzed for the first time along the growth path in this class of models thanks to the PEA presented in this paper. Recall that to calibrate the stochastic components of the model we target the volatility in prices (relative to manufacturing) and in real aggregate consumption. These volatilities are reported in the first column of Table 2, while the second column reports the ability of the model in replicating these targets. While the model cannot exactly match the three volatilities, it accounts for a large fraction of them. In the less performing case of the three targets, the relative price of services, the model accounts for 84% of the volatility observed in the data. Figure 2 shows graphically the departures from trend of per-capita real consumption and relative prices in the data, along with the corresponding results of the simulation.

The last three lines of Table 3 report the volatility of individual consumption components, which are not used as targets in the calibration, in the data and in the model. Figure 3 compares these volatilities graphically. Along this metric, the model displays a remarkably

Standard deviation of	Data	Model
Real Consumption per-capita	0.0180	0.0190
Relative Price of Agriculture	0.0198	0.0192
Relative Price of Services	0.0154	0.0184
Agricultural Consumption	0.9827	0.2529
Manufacturing Consumption	2.0604	0.4856
Services Consumption	1.1876	1.5128

Table 2: Model vs Data

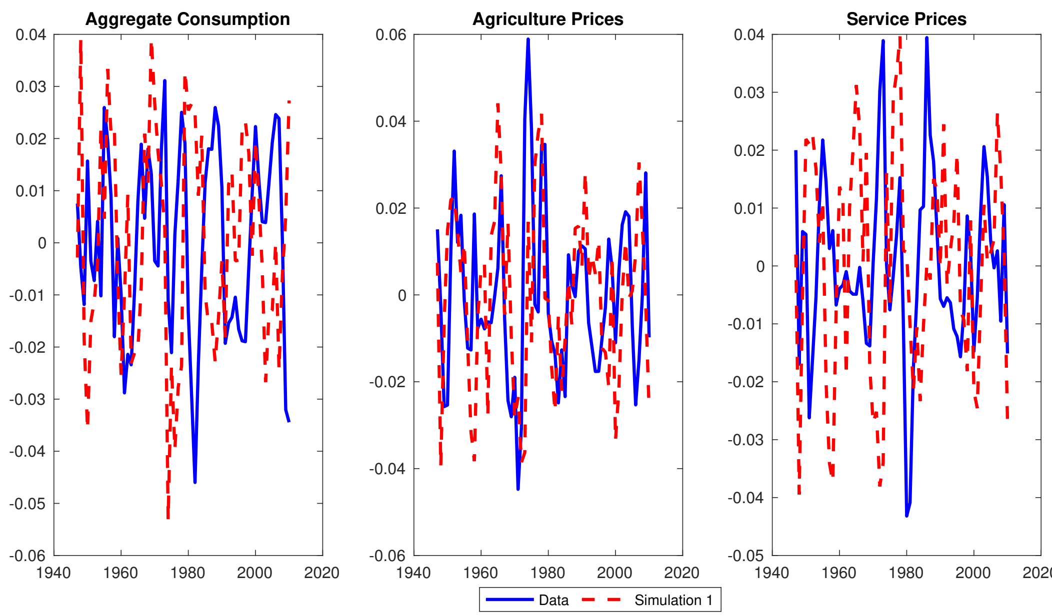

Figure 2: Volatility of Aggregate Consumption Expenditures and Relative Prices

poor performance. The volatility of agriculture is 0.98 in the data, compared to 0.26 in the model (a 27% account), that of manufacturing is 2.06 in the data compared to a 0.51 in the model (a 25% account), while that of services is 1.19 in the data versus a 1.56 in the model (a 131% account). Thus, the model predicts a large volatility of services and a small volatility of agriculture and manufacturing when compared to the data.

The poor performance of the model in terms of sectoral volatility is due to the non-homothetic terms $\bar{c}_j$, $j = a, m, s$. To see this, use equations (4) through (8) to write the demand function for good j as

$$c_{jt} = \omega_j \left(\frac{p_{jt}}{P_t} \right)^{-\mu} C_t - \bar{c}_j, \; j = a, m, s.$$

Approximating income by aggregate consumption C_t, the income elasticity is

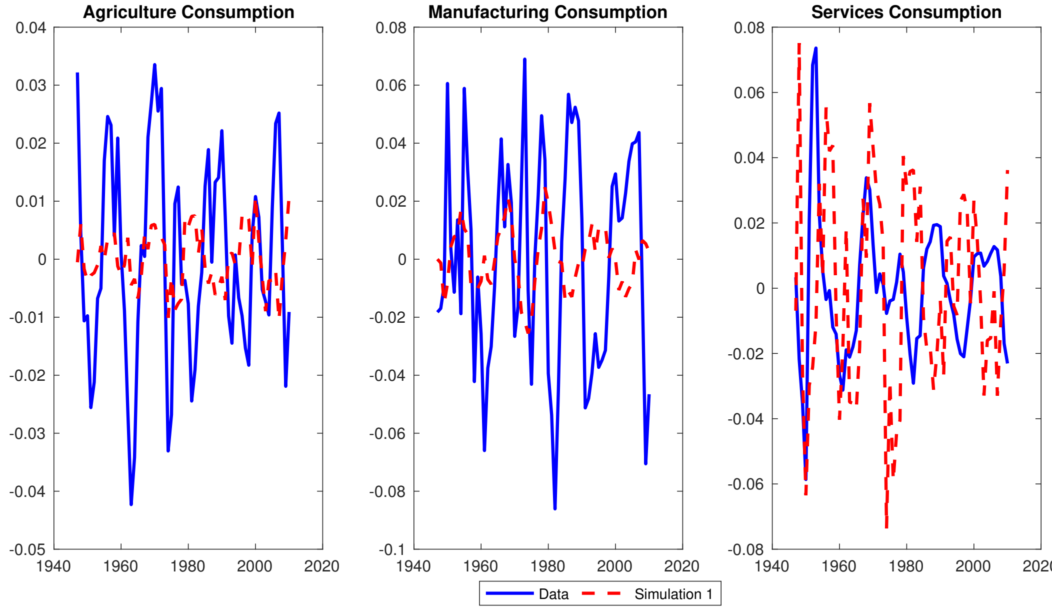

Figure 3: Volatility of Sectoral Consumption

$$\frac{\partial c_{jt}}{\partial C_t}\frac{C_t}{c_{jt}} = \frac{\omega_j \left(\frac{p_{jt}}{P_t}\right)^{-\mu} C_t}{\omega_j \left(\frac{p_{jt}}{P_t}\right)^{-\mu} C_t - \bar{c}_j}, \tag{13}$$

and the elasticity of substitution between c_{jt} and C_t is,

$$\frac{\partial (c_{jt}/C_t)}{\partial (p_{jt}/P_t)}\frac{p_{jt}/P_t}{c_{jt}/C_t} = \frac{-\mu\omega_j \left(\frac{p_{jt}}{P_t}\right)^{-\mu}}{\omega_j \left(\frac{p_{jt}}{P_t}\right)^{-\mu} - \bar{c}_j/C_t}. \tag{14}$$

These equations show that for the manufacturing sector, with $\bar{c}_m = 0$, the income elasticity is 1 and the elasticity of substitution is $-\mu$. They also show that a large $\bar{c}_j$ increases both the income and the substitution elasticities. The long-run features of the economy require a large income elasticity for the service sector (making services a luxury good), and a small one for the agricultural sector (making them necessity goods). This is achieved by setting $\bar{c}_a < 0$ and $\bar{c}_s > 0$. This implies a small elasticity of substitution of the agricultural sector, and a large one for the service sector. Thus, even when service prices are less volatile than prices in agriculture relative to the aggregate price index, the large elasticity of substitution implies much larger changes in the consumption of services than of agriculture relative to aggregate consumption.

Income elasticities also play a role in shaping the volatility of consumption components.

As income of the representative household increases with TFP growth in the long-run, these non-homothetic terms induce, given prices, an increase in the share of services and a decline in the shares of manufacturing and agriculture. However, in the short-run, these income elasticities also imply that services consumption responds to income shocks more than manufacturing consumption which, in turn, responds more than agriculture consumption, creating a volatility of individual components of consumption that is at odds with the data.

The above result suggests that structural change models embed a tension between their long- and short-run properties. To put it differently, to improve the short-run performance of the model, one should reduce the magnitude of non-homothetic components. However, in doing this, and given the evolution of prices in the data, one reduces the performance of the model in the long-run. To see this, we run a version of the model in which we set the non-homothetic terms to zero, i.e. $\bar{c}_a = \bar{c}_m = \bar{c}_s = 0$, while leaving all the other parameters values as in the calibration. Figure 4 reports the performance of the model in reproducing long-run structural change, while Table 3 reports the volatility statistics.[18] The fit of the model is now poor in terms of long-run consumption shares. The rise of services goes from 78% to 81% compared to 49% to 73% in the data. However, in terms of volatility of individual components of consumption, the model displays a better performance. The volatility of agriculture is 0.98 in the data, compared to 1.22 in the model (a 124% account), while that of services is a remarkably similar 1.19 in the data versus a 1.17 in the model. Note that the volatility of aggregate consumption is now substantially smaller than in the data (0.14 vs 0.18), suggesting that the non-homothetic terms create overall more volatility in the economy, although this is not distributed appropriately in equilibrium among the various sectors.[19]

7 Conclusions

In this paper we address a problem that has stalled the literature on structural transformation when it comes to stochastic settings. Standard real business cycle solution techniques rely on the existence of a stationary process for consumption, and use this stationarity to approximate a stochastic Euler equation. The problem arises because in a large class of models of structural change, the process for consumption is not stationary. This paper proposes

[18] To make the exercise as clean as possible we only change the value of the parameters $\bar{c}_a$ and $\bar{c}_s$. The fit would slightly improve by re-estimating the parameters μ and $\omega_j, j = a, m, s$. In doing this we obtain a slightly better match of average shares, but their evolution over time is similar to that reported in Figure 4. The results are available upon request.

[19] When setting non-homotheticities to zero in the counterfactual, we also lower k_0, the level of capital 144 periods before the first period in our sample, from 0.4 to 0.05. Starting with $k_0 = 0.4$ does not change the results, but produces a U-shape series for the capital stock during the periods we discard.

Standard deviation of	Data	Model
Real Consumption per-capita	0.0180	0.0139
Relative Price of Agriculture	0.0198	0.0192
Relative Price of Services	0.0154	0.0184
Agricultural Consumption	0.9827	1.2190
Manufacturing Consumption	2.0604	0.6103
Services Consumption	1.1876	1.1684

Table 3: Model without non-homotheticity vs Data

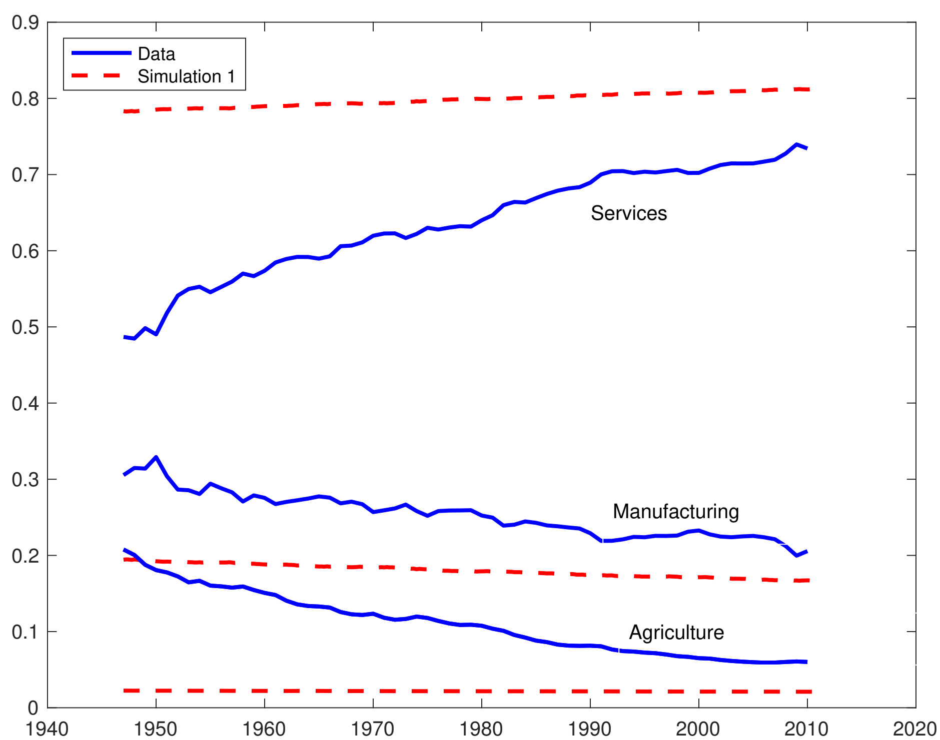

Figure 4: Structural Change in the Model with No Non-Homotheticities

an algorithm based on the PEA method by Den Haan and Marcet (1990) as an alternative. This method does not linearize around a constant, so does not require the process to be stationary. It relies on the knowledge that state variables determine the expected growth rates, and as such finds a function that will successfully map these state variables into the expectations in the Euler equation. In other words, the procedure finds the *time-invariant* approximation of the expectation function, even if the stochastic variable inside that function is non-stationary and as such is *time-varying*. We show that the algorithm works very smoothly and so it appears as a robust tool to solve stochastic structural change models. To complement the theory, we provide a Matlab Toolbox than can be used to compute these equilibria in similar settings.

We use the solution of the model calibrated to post-war U.S. data to show that structural change models embed a tension between their long- and short-run properties. To fit long-run consumption shares, these models require a value of the non-homothetic parameters that imply a counterfactually high volatility of services and counterfactually low volatilies of manufacturing and agriculture in the short-run. This result suggest that more theoretical work is needed to have structural change models that can fit concurrently short- and long-run properties of an economy along its growth path.

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Appendix

A Details - Estimation Results in the PEA Algorithm

The polynomial $\Phi(K, z_a, z_m, z_s)$ approximates closely the right hand side of equation (11). The R^2 is 1 in all simulations when approximating to 4 decimal points, with a standard deviation of 4×10^{-6}. The mean and standard deviations of estimates of the PEA algorithm across simulations are reported in Table 4. These estimates are significant in most simulations. The last column of Table 4 shows the percentage of simulations displaying significant estimates at the 95% confidence level.

Estimate	Mean of estimates across simulations	S.D. of estimates across simulations	% significant at 95% level
Constant	0.5656	4.8075	65%
$f(k)$	-0.2793	0.0133	100%
z_a	0.2544	9.6322	66%
z_m	-0.3227	1.7727	86%
z_s	1.2679	0.2178	100%
$f(k)^2$	-0.0518	0.0048	100%
z_a^2	0.1497	4.9162	64%
z_m^2	-0.0212	0.9114	63%
z_s^2	0.5422	0.0899	100%
σ_ϵ^2	3×10^{-10}	1×10^{-10}	–
R^2	1.0000	4×10^{-6}	–

Table 4: Mean and Standard Deviations across simulations of the PEA Algorithm estimates

Focusing on the standard deviations of the PEA estimates, the model shows relatively large standard deviations for the constant and the shocks, but relatively small ones for the functions of capital. This is not surprising, since large differences in shocks across simulations imply substantially different effects of those shocks on the expectations term. It is encouraging to obtain consistent estimates for capital, the endogenous state variable in the model, featuring very low standard deviations, and always significant coefficients.