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Misleading Advertising in Mixed Markets: Non-profit orientation and welfare outcomes

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Abstract

In this paper, we analyse misleading advertising competition between private firms (profit oriented) and consumer-oriented firms (concerned about consumer welfare) in the context of mixed markets. The nature of advertising in this paper is assumed to be non-rival in nature and is beneficial to all the firms in the market. We find that, both private and consumer-oriented firms incur positive expenditure on misleading advertising. Further, the profit of consumer-oriented firms is higher than that of private firms. Moreover, irrespective of whether firms are concerned about consumer welfare or not, the level of misleading advertising is socially excessive.

Keywords: Misleading advertising, Non-rival advertising, Consumer-oriented firm, Mixed markets, Cournot competition

JEL Classification: D21, L10, L29, L30

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1. INTRODUCTION

This paper analyses the misleading advertising competition between private firms (profit oriented) and consumer-oriented firms, followed by product market competition (i.e. Cournot or quantity setting). The motivation for this study is to extend and validate the findings of Matsumura and Sunada (2013) and Pi et al. (2017), who analyse misleading advertising in a mixed market with public and private firms. We extend this analysis by substituting a public firm with a firm which is consumer-oriented (measured by consumer welfare) but differs from a public firm in that it is also concerned about its competitors’ profit and consumer surplus. Examples of such entities include some of the public sector firms in developed and developing countries, firms in the health sector (trust hospitals), co-operative firms such as AMUL in India and so on. Further, the objective function of a firm concerned about consumer surplus could also be used to analyse the case of firms with a focus on both profit as well as consumer social responsibility either for strategic or non-strategic reasons (Sharma 2018; Bian et al., 2016). The primary objective for this study was to analyse whether consumer oriented firms would not only stay away from misleading advertising but also deter other firms in the market from engaging in such activities. In contrast to our expectations and in line with earlier studies in the context of mixed markets (Matsumura and Sunada, 2013; Hattori and Higashida, 2012) we find that a consumer-oriented firm also engages in aggressive advertising competition.

This paper deviates from the traditional literature on advertising competition and its welfare outcomes in two ways. First, we assume that firm advertising is non-rival in nature. This means that advertising is of a more generic nature. In other words, positive advertising done by one firm affects the market outcomes in terms of increased consumer valuation of the products sold by all the firms as well as positively affecting demand (Crespi and Marette, 2002). So advertising done by any firm is beneficial to all. There are a handful examples of such advertising in Tobacco Industry (Pierce et al. 1998), Airline Industry (i.e. destination advertising) and so forth². Further, we assume that advertising provides some information that is persuasive but can be either misleading or cautionary. Thus, it does not increase the utility of consumers but creates a perception of increased value for the product (Dixit and Norman, 1978).

²For more discussion on this, please refer to Glaeser and Ujhelyi (2010).
Second, we consider advertising competition between private (profit oriented) and consumer-oriented (consumer welfare oriented) firms, which to the best of our knowledge has not been analysed so far in the theoretical literature on advertising. In the mixed market, the orientation of firms lies between the extremes of own profit to social welfare comprising of total profit of all firms (i.e. producer surplus) as well as consumer surplus of all consumers. The orientation of the consumer-oriented firm remains to its own profit along with the welfare of the consumers. Thus these firms are different from public firms in the sense that they are not concerned about the rival firm's profit and different from private firms in the sense that their objective is own profit as well as consumer surplus optimization. These firms are also termed consumer cooperatives (Kopel and Marini, 2014), CSR oriented firms (Bian, Li and Guo, 2016) and consumer-oriented commercial firms (Goering, 2008) in the literature. There is empirical evidence to suggest that these firms do compete with private firms in some sectors of the economy (Goerke, 2003).

The empirical evidence of advertising competition between private and consumer-oriented firm have been widespread across different sectors. One example is National Cancer Institute and Kellogg Company's All Bran Cereal Campaign in 1984 (Freimuth, Hammond and Stein, 1988). The campaign had a positive impact on not only consumer's perception and dietary habits but also on the profit of Kellogg. The entry of consumer-oriented firms in advertising or in a broad sense in marketing is termed as Social Marketing in the relevant literature.

In this paper, we attempt to analyse the market outcomes in the wake of the interaction of these two aspects of firms and nature of product markets i.e. nature of advertising competition and orientation of firms, as discussed above. Firms compete in a mixed market by choosing the level of advertising expenditure and quantity produced/sold in a simultaneous move product market competition (Cournot game).

The main results are as follows. First, we show that non-rival misleading advertising is socially excessive in case of mixed market competition between profit-oriented and consumer-oriented firms. This finding is in line with Matsumara and Sunada (2013), who consider the case of a public and a private firm. Thus, we highlight that a relatively less strict nature of social orientation of the firm does not change its spending on excessive misleading advertising.

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3See Andreasen (1994) for a detailed discussion on this.

4Though, it is not always the case that in a mixed market with socially concerned firm or public firm, there would be positive and excessive advertising. Pi et al. (2017) show that in a Stackelberg setting, public firms actually have negative level of advertising (corrective instead of misleading) to counter the misleading information from the private firms in the market.
Second, we show that there seems to be a case of free riding in the spending on advertising. We show that with increased consumer-orientation of firm 2, there is an increase in the advertising level of firm 2 but firm 1 decreases its level of advertising. This is a new finding. Lastly, we show that as firm 2 increases its level of consumer-orientation, it is likely to be more aggressive in the market with higher profit than firm 1. However, beyond a threshold the profits of firm 2 become negative. The reason is excessive production and excessive advertising expenditure incurred.

The rest of the paper is organized as follows. In the next section, we outline the basic model. In section 3, we solve the game using backward induction method and discuss the implications of the findings. Section 4 focuses on the socially optimal level of advertising and whether it differs from the market outcome or not. Section 5 concludes.

2. BASIC MODEL

We consider a mixed market duopoly where a private (profit oriented) firm competes with a consumer-oriented (own profit and consumer welfare oriented) firm. Let’s say, Firm 1 is a private firm and Firm 2 is a consumer-oriented firm. While the private firm maximises its own profit i.e. \( \pi_1 c \); the consumer-oriented firm maximises the weighted sum of its own profit, \( \pi_{2c} \) and consumer surplus \( CS \). We consider a two stage game where at the first stage, both the firms choose the level of advertisement, \( e_i \in (−\infty, \infty) \) simultaneously with advertising cost of quadratic nature i.e. \( \frac{e_i^2}{2} \), where \( i = 1, 2 \). We assume that the level of advertising can be either negative or positive. Since the advertising is of non-rival nature, it affects the overall demand function by the level, \( m = \sum_{i=1}^{2} e_i = e_1 + e_2 \). At the second stage, after observing the level of advertisement in the market i.e. \( m \), each firm decides the level of output to be produced/sold, \( q_i \in [0, \infty) \) of a homogenous good. The inverse demand function is given by \( p = a + m - \sum_{i=1}^{2} q_i \). We assume that firms have a symmetric constant marginal cost, \( c \) and without any loss

5 By negative advertising, we mean advertising that negatively affects the value of the product purchased by the consumer. Such advertising is not an unheard of phenomenon. Some firms as well as government use this type of advertising to counter misleading adverts.

6 The underlying utility function of the consumer for the inverse demand function is, \( U = q_0 + (a + m)q_1 + (a + m)q_2 - \frac{(q_1^2 + 2q_1q_2 + q_2^2)}{2} \), with the income constraint being, \( y = q_0 + pq_1 + pq_2 \). Please refer to
of generality, we consider it to be equal to zero. Further, market size is large enough than marginal cost i.e. $a > c = 0$.

The objective functions of both the firms are as follows:

Firm 1 (private firm): $\max q_1, e_1 F_1 = \pi_{1C} = pq_1 - \frac{e_1^2}{2}$ (1)

Firm 2 (consumer-oriented firm): $\max q_2, e_2 F_2 = \pi_{2C} + \beta \text{CS} = \left[ pq_2 - \frac{e_2^2}{2} \right] + \beta \left[ \frac{1}{2} (q_1 + q_2)^2 \right]$ (2)

Here, $\beta \in [0, 1]$ is the relative weight assigned to the consumer surplus in the objective function of consumer-oriented firm. If $\beta$ takes the value one, then we say that equal weight is assigned to both own profit and consumer surplus by the consumer-oriented firm. If $\beta$ takes the value zero then it corresponds to the objective of the private firm. In this model, to ensure that the stability and second order conditions are satisfied, we should have $\beta < 3$, which is satisfied based on our assumption about $\beta$.

Lastly, before moving on to solve the stages of this game, we discuss about the impact of misleading advertising and how it is treated in the model. As discussed in the last section, we assume that advertising is of persuasive nature but does not create actual increase in the consumer surplus for the buyers of the product. Dixit and Norman (1978) argue that to analyse the welfare implications of such advertising, we should analyse the consumer surplus both ex-ante and ex-post. This provides us with two measures of consumer surplus which we term as actual and perceived. While the actual consumer surplus ignores the changes in the product valuation due to misleading non-rival advertising; the perceived consumer surplus accounts for the impact of misleading advertising (Hattori and Higashida, 2012). The equations for actual and perceived consumer surplus are as follows:

Singh and Vives (1984) for a detailed discussion on the properties of such utility and corresponding demand function.

7 Please note that we have a lower and upper value restriction on consumer-orientation for the purpose of this study, i.e. to analyse the behaviour of firms that value the consumer surplus or in other words, have some social responsibility towards their consumers. The upper restriction on the value of $\beta < 1$ is under the assumption that though the firms are sensitive towards consumer surplus but do not value it more than their own profit i.e. still the commercial nature of the firms is equal or dominant characteristic visible from the objective function. In a more general context, the value of $\beta$ can take any value either positive or negative based on the varying orientations of firms. This aspect remains beyond the scope of this paper but can be considered for further work.
$$CS_{\text{Perceived}} = \frac{1}{2}(q_1 + q_2)^2$$ (3)

$$CS_{\text{Actual}} = \frac{1}{2}(q_1 + q_2 - 2m)(q_1 + q_2)$$ (4)

Further, we also discuss about the socially optimal level of advertising and compare the consumer surplus changes in these conditions to provide a public policy and consumer oriented analysis of misleading advertising.

3. EQUILIBRIUM AND COMPARATIVE STATICS

In this section, we solve the Cournot game with product market and advertising competition using the backward induction method. We first solve the second stage i.e. product market competition assuming the level of advertisement as given and, then solve the first stage for the level of advertising.

3.1 Quantity Competition

Firm 1 (private) and Firm 2 (consumer-oriented) maximise their objective functions given in equation (1) and (2) respectively, assuming the level of advertising (own and rival firm's) and quantity produced by other firm as given. From the first order conditions of the constrained optimization, we get the reaction functions (i.e. first order condition) of both the firms as follows:

**Reaction function of firm 1** : 
$$q_1 = \frac{1}{2}(a + m - q_2)$$ (5)

**Reaction function of firm 2** : 
$$q_2 = \frac{a + m - (1-\beta)q_1}{(2-\beta)}$$ (6)

A closer look at the reaction functions reveals some interesting effects of the joint level of advertising denoted by \(m\) and consumer-orientation indicated by \(\beta\). We will discuss their role in determining the equilibrium level of output for each firm subsequently.

As advertising level of a firm is non-rival in nature, it affects both the reaction functions equally. For the sake of simplicity, if we assume that \(\beta = 0\), then the reaction functions of both the firms become symmetric i.e. \(R1: q_1 = \frac{1}{2}(a + m - q_2)\) and \(R2: q_2 = \frac{1}{2}(a + m - q_1)\). In this case the total level of advertising, \(m\), shifts both the reaction curves outwards (inwards) if the level of advertising is positive (negative) leading to increase (decrease) in the level of output being sold.
in the product market. The reason being that consumers’ valuation of the product increases therefore they are willing to pay higher price and firms would be willing to sell more\(^8\).

Next, looking at the role of consumer-orientation \((\beta > 0)\) of firm 2 in quantity competition, we observe that it only affects the reaction curve of the consumer-oriented firm (2) and the effect on private firm is channelized through the change in the output level of firm 2. We observe that both the intercept as well as slope of the reaction function changes due to consumer-orientation of firm 2. This leads to an outward shift of firm 2’s reaction function affecting the output of firm 1 negatively and output of firm 2 positively. This is in technical terms similar to Stackelberg type (sequential choice with leader and follower firms) equilibrium in product market but the reasoning remains altogether different. Interestingly, the value of \(\beta = 1/3\) provides a Stackelberg outcome in the game with firm 2 being the leader with half of the total possible output produced (monopoly level) and firm 1 produces follower’s output, i.e. one fourth of total market size (after factoring in the cost). This is one example, where in a simultaneous Cournot game, Stackelberg outcome is the equilibrium\(^9\).

Lastly, solving the equation (5) and (6) for \(q_1\) and \(q_2\), we get the product market equilibrium as follows:

\[
p = \frac{(1-\beta)(a+m)}{3-\beta}; \quad q_1 = \frac{(1-\beta)(a+m)}{3-\beta}, \quad \text{and} \quad \pi_{1C} = \frac{(a+m)^2(1-\beta)^2}{(3-\beta)^2} - \frac{e_1^2}{2}; \quad q_2 = \frac{(1+\beta)(a+m)}{3-\beta} \quad \text{and} \quad \pi_{2C} = \frac{(a+m)^2(1-\beta^2)}{(3-\beta^2)} - \frac{e_2^2}{2}.
\]

\(7\)

**Proposition 1:**

i) The positive (negative) non-rival misleading advertising affects output and prices of both the firms as well as their profit positively (negatively).

ii) The consumer-orientation \((\beta > 0)\) of firm 2 in duopoly quantity competition, leads to increase (decrease) in the output of consumer-oriented (private) firm and the equilibrium price in the market decreases.

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\(^8\) As we will see subsequently in the quantity competition outcome, that both price charged and quantity produced/sold by each of the firms would increase due to level of advertising, keeping the consumer-orientation being zero i.e. \(\beta = 0\)

\(^9\) See Basu (1995) and Sharma (2018) for such cases in the context of managerial (incentives) delegation by the firms in product market competition.
iii) With higher value of $\beta$ (higher consumer-orientation), the profit of firm 1 (private firm) decreases whereas profit of firm 2 increases up to a critical value of $\beta \leq \frac{1}{3}$ and then starts decreasing.

**Lemma 1:** At the optimal value of $\beta$ (i.e. 1/3) for profit maximization by consumer-oriented firm, the consumer-oriented firm behaves like a Stackelberg leader and the private firm acts like a follower firm in product market.

**Proof:** For $\beta = 1/3$, we observe that $p = \frac{(a+m)}{4}$; $q_1 = \frac{(a+m)}{4}$ and $q_2 = \frac{(a+m)}{2}$. This outcome corresponds to the Stackelberg equilibrium in the product market with market size being, $a + m$.

Next, we solve the first stage of the game i.e. advertising competition.

3.2 Advertising Competition

In the first stage advertising competition, by substituting (7) in equations, (1) and (2) along with $m = \sum_{i=1}^{2} e_i = e_1 + e_2$, we get:

Firm 1 (private firm): $\text{Max}_{e_1} F_1 = \pi_1C = \frac{(a+m)^2(1-\beta)^2}{(3-\beta)^2} - \frac{e_1^2}{2}$ (8)

Firm 2 (consumer oriented firm): $\text{Max}_{e_2} F_2 = \pi_2C + \beta CS = \frac{(a+m)^2(1+(2-\beta)\beta)}{(3-\beta)^2} - \frac{e_2^2}{2}$ (9)

By differentiating with the level of advertising for each firm, the first order conditions i.e. advertising reaction functions (ARF) of firms are as follows:

$ARF_1$: $e_1 = \frac{2(1-\beta)^2(a+\epsilon_2)}{7-\beta(2+\beta)}$ (10)

$ARF_2$: $e_2 = \frac{2(1+(2-\beta)\beta)(a+\epsilon_1)}{(1-\beta)(7-3\beta)}$ (11)

It is clear from equations (10) and (11), that level of advertising by both the firms is strategic substitute to each other i.e. $\frac{de_2}{de_1} > 0$. Further, a positive increase in the value of $\beta$ i.e., consumer orientation, leads to decrease in the level of advertising for private firm and increase in the level of advertising for consumer-oriented firm.

The equilibrium outcome of the game after advertising competition stage is as follows:
Price in the market: $p = a - \frac{2a}{(5-\beta)}$ \hspace{1cm} (12)

For firm 1: $q_1 = a - \frac{2a}{(5-\beta)}$, $\pi_1 = \frac{a^2(7-\beta(2+\beta))}{(5-\beta)^2}$, $e_1 = \frac{2a(1-\beta)}{5-\beta}$ \hspace{1cm} (13)

For Firm 2: $q_2 = \frac{a(3-\beta)(1+\beta)}{(5-\beta)(1-\beta)}$, $\pi_2 = \frac{a^2(7-\beta(14+\beta(12+\beta(-14+3\beta))))}{(5-6\beta+\beta^2)^2}$, $e_2 = \frac{2a(1+2\beta-\beta^2)}{5-6\beta+\beta^2}$ \hspace{1cm} (14)

The second order and stability conditions are satisfied if $\beta < 3$.

First, from (12), it is clear that price is affected negatively ($\frac{\partial p}{\partial \beta} < 0$) by the consumer-orientation of firm 2. Therefore, an introduction of consumer-orientation parameter in this game leads to reduction in the equilibrium price in the mixed market. The reasoning for this is as follows. A higher value of $\beta$ emphasizes that consumer surplus be given more importance in the decision making which is positively dependent on the total output being sold in the market. This leads to reduction in the price being set in the market.

Second, the quantity sold by firm 2 always remains more than firm 1 for any positive value of $\beta$, which is the implication of the consumer-orientation of firm 2. The marginal effect of increase in $\beta$ parameter is negative (positive) on output of firm 1 (firm 2). We can say that consumer-orientation leads to higher output and lesser prices in the market.

Third, in the equilibrium, profit of firm 2 is more than firm 1 if $0 < \beta < 0.097$, otherwise for any larger value of $\beta$, there is a role reversal in terms of profit. We also observe that profit of firm 1 always remains positive irrespective of any non-negative value of $\beta$, but this not the case for firm 2. The profit of the firm 2 remains positive if $\beta < 0.42$, beyond which profit become negative.

[Insert Figure 1 Here]

**Lemma 2:**

i) Consumer-oriented firm has a higher profit than private firm below a threshold value of $0 < \beta < 0.097$.

ii) Consumer-oriented firm has a lower profit than private firm for value of $\beta$ between 0.097 and 0.42.

iii) Consumer-oriented firm has a negative profit beyond the threshold value of $\beta = 0.42$. 

9
Lastly, coming to the equilibrium level of advertising, we observe that both the firms have positive level of advertising irrespective of the consumer-orientation of firm 2. It means that for $\beta = 0$, since both the firms have profit maximizing objective they both spend positively on advertising i.e. $e_1 = e_2 = \frac{2a}{5}$, in a homogenous product market. One should note that even though the advertising is non-rival in nature i.e. having perfect positive externality for the other firm, both firm have positive level of advertising, indicating that there is a unilateral incentive for either of the firms to spend on advertising. Now, when $\beta > 0$, we observe that level of advertising by firm 2 ($e_2$) is always more than firm 1 ($e_1$).

Further, we have $\frac{\partial e_2}{\partial \beta} > 0$ and $\frac{\partial e_1}{\partial \beta} < 0$. This indicates that consumer-orientation leads to increase in the misleading advertising for firm 2 and decrease for firm 1. Interestingly, the total amount spent on advertising in the market i.e. $m = \sum_{i=1}^{2} e_i = e_1 + e_2$, has first increasing and second decreasing component. Jointly, the effect of consumer-orientation ($\beta$) on total level of misleading advertising is positive.

[Insert Figure 2 Here]

As is evident from figure 2, consumer-oriented firm is more aggressive than the private firm in the advertising competition. This is a new result.

**Proposition 2:**

i) In the advertising competition, private and consumer-oriented firms have positive level of misleading and non-rival advertising nature.

ii) Consumer-oriented firm remains more aggressive than private firm in the level of advertising.

**Proof:** The explanation for this is as follows. A closer look at the effect of both the strategic variables on the objective functions of both the firms reveals that, if $a + m > 0$ (i.e. there is a market for the product) and when $0 \leq \beta \leq 1/3$ then,

\[
\text{Firm 1: } \frac{\partial^2 F_1}{\partial m \partial \beta} = \frac{\partial^2 \pi_{1C}}{\partial m \partial \beta} - ve
\]

\[
\text{Firm 2: } \frac{\partial^2 F_2}{\partial m \partial \beta} = \frac{\partial^2 \pi_{2C}}{\partial m \partial \beta} + ve + \frac{\partial^2 CS_{perceived}}{\partial m \partial \beta} + ve
\]

whereas, if $\frac{1}{3} < \beta \leq 1$, then
Firm 1: \( \frac{\partial^2 F_1}{\partial m \, \partial \beta} = \frac{\partial^2 \pi_{1C}}{\partial m \, \partial \beta} \)

Firm 2: \( \frac{\partial^2 F_2}{\partial m \, \partial \beta} = \frac{\partial^2 \pi_{2C}}{\partial m \, \partial \beta} + \frac{\partial^2 CS_{\text{perceived}}}{\partial m \, \partial \beta} \)

Therefore, under any condition, a positive change in \( \beta \) leads to negative (positive) change in the marginal effect of advertising on the profit of firm 1 (firm 2). It means that firm 2 always has an incentive to increase the level of advertising whereas firm 1 does not. That is why, firm 2 is more aggressive than firm 1. Moreover firm 1 decides to reduce the level of advertising with any positive change in \( \beta \) i.e. consumer-orientation of the firm 2.

In the next section, we discuss the welfare aspects of advertising and implications of its non-rival and misleading nature.

4. WELFARE ASPECTS OF MISLEADING ADVERTISING

As suggested in Dixit and Norman (1978), to understand the welfare aspects of persuasive advertising, one should analyse the difference in the consumer surplus, pre and post change in the consumers’ tastes and preferences, due to advertising competition among the firms.

In this section, we examine the nature of consumer surplus and social welfare in the equilibrium, followed by the socially optimal level of advertising while considering the consumer surplus i.e. either actual (without considering the distorted tastes and preferences) or perceived (i.e. after accounting for change in consumer taste and preferences). We also analyse whether the level of advertising in the mixed market is socially excessive or not.

In the equilibrium of advertising competition, we observe that consumer surplus is as follows:

\[
CS_{\text{Perceived}} = \frac{1}{2} (q_1 + q_2)^2 = \frac{2a^2(3 - \beta)^2}{(5 - 6\beta + \beta^2)^2}
\]

\[
CS_{\text{Actual}} = \frac{1}{2} (q_1 + q_2 - 2m)(q_1 + q_2) = \frac{2a^2(-3 + \beta)(1 + \beta)}{(5 - 6\beta + \beta^2)^2}
\]

As discussed earlier, the difference between actual and perceived consumer surplus is the adjustment for change in taste and preference of consumers (which we assume to be misleading in nature).
Looking at these values of consumer surplus, we can say that due to the persuasive advertising consumer believe that they get positive surplus irrespective of the fact that they are paying higher price for the good. On the other hand, after adjusting for the misleading level of advertising, we observe that actual consumer surplus becomes negative.

This is an interesting result and holds irrespective of whether any of the firms have consumer-orientation or not. We will come back to the role of consumer-orientation in affecting consumer surplus in the next subsection.

Next, if we think from a social planner or a regulator’s perspective, what should be the optimal level of advertising for the mixed market? In other words if the government is able to regulate the level of advertising what should be the first best choice?

To answer this question, the social planner will optimally choose social welfare maximizing level of advertising. The problem of social planner is as follows: \( \text{Max}_e \ SW(e) \), where \( e \) is the level of advertising for both the firms. In this problem, social welfare would include profits of both the firms as well as consumer surplus. The consumer surplus to be considered here is the perceived consumer surplus.

\[
\text{Social Planner: } \text{Max}_{e=e_1=e_2} \ SW_{\text{Perceived}} = \pi_{1C} + \pi_{2C} + CS_{\text{Perceived}}
\]

\[
= \frac{4(a + m)^2(2 - \beta) - (3 - \beta)^2 e_1^2 - (3 - \beta)^2 e_2^2}{2(-3 + \beta)^2}
\]

In the equilibrium, first best socially optimal level of advertising with perceived consumer surplus is:

\[
e_{SWP} = -\frac{4a(2-\beta)}{7-\beta(2+\beta)}
\]

On the other side, if the social planner would have considered actual consumer surplus instead of perceived consumer surplus, then the optimization problem would be as follows:

\[
\text{Social Planner: } \text{Max}_{e=e_1=e_2} \ SW_{\text{Actual}} = \pi_{1C} + \pi_{2C} + CS_{\text{Actual}}
\]

\[
= \frac{4(a + m)(a(2 - \beta) - m) - (3 - \beta)^2 e_1^2 - (3 - \beta)^2 e_2^2}{2(3 - \beta)^2}
\]

In the equilibrium the first best socially optimal level of advertising with actual consumer surplus is:

\[
e_{SWA} = \frac{2a(1 - \beta)}{17 - (6 - \beta)\beta}
\]
Comparing the socially optimal level of advertising with the actual level of advertising done by the firms in mixed market, we get the following result:

**Proposition 3:** Both private and consumer-oriented firms engage in socially excessive level of misleading advertising in the mixed markets, irrespective of whether we consider actual or perceived consumer surplus in social welfare.

**Proof:** In our model, $e_{SWP} < e_{SWA} < e_1$ and $e_{SWP} < e_{SWA} < e_2$, irrespective of the level of consumer-orientation ($\beta$). This means that firms are spending socially excessive amount on the misleading advertising. QED

5. CONCLUSION

This paper focuses on the impact of non-rival misleading advertising by firms on product market competition and equilibrium outcomes in a mixed market. We contribute to the literature in the following manner. First, we show that consumer-orientation leads to higher (lower) output for the consumer-oriented (private) firm. For a value of $\beta$ (consumer-orientation) below a threshold, a consumer-oriented firm can actually earn more profit than a private firm. This corresponds to a Stackelberg outcome with the consumer-oriented firm being the leader and the private firm behaving like follower. Second, we show that both firms spend excessively on non-rival misleading advertising and, interestingly, the consumer-oriented firm spends more than the private firm on advertising. Lastly, we demonstrate that both firms engage in a socially excessive level of advertising competition.

This paper suggests that misleading advertising is not only restricted to private profit oriented firms but also observed in a mixed market. Moreover, even though a consumer-oriented firm might care about consumers surplus, it still engages in a race-to-the-top in advertising competition.

There are several potential extensions to the analysis in this study. First, future research might explore the mechanisms (market and government interventions) by which the excessive level of misleading advertising in the market could be restricted. A second extension would be to analyse the differences in Cournot and Bertrand outcomes in the context of non-rival misleading advertising.
REFERENCES


Figure 1: Profits and Perceived Consumer Surplus (X-axis: $\beta$ and Y-axis: Profits and Consumer Surplus, we assume $a=1$ for this graphical depiction)
Figure 2: Level of advertising (Y-axis) and Non-profit orientation of firm 2 (X-axis)

(We assume $a=1$ for this graphical depiction)