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Optimal paid job-protected leave policy*

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Abstract

Although many countries give workers the right to return to their previous workplace after a temporary leave period ends, little is known about the details of such policies. This study characterizes the optimal paid job-protected leave policy using a model in which a worker has to leave his or her job with a certain probability and chooses whether to return to work after the leave period ends. An optimal policy, consisting of the consumption of a worker, that of a worker on leave, and the length of the leave, maximizes social welfare subject to the resource feasibility constraint and incentive constraint under which a worker on leave voluntarily chooses to return to work after the leave period ends. The present study finds that when the incentive constraint does not bind, the income risk caused by the leave should be perfectly shared among workers and workers on leave and that the leave period balances the marginal welfare gain and loss from a slight increase in the leave period. When the incentive constraint binds, the income risk caused by the leave is not perfectly shared among workers and workers on leave and that workers consume more than workers on leave because the constrained-optimal allocation has to give an incentive to workers on leave to return to work. By lengthening the leave period, another feasible allocation improves social welfare, which implies that the length of the leave period at the constrained-optimal allocation is too short. In addition, the study compares two economies, one that experiences a high discount factor and the other that experiences a low discount factor. In the former economy, the incentive constraint does not bind, whereas it does bind in the latter economy. Comparing the constrained-optimal allocations in these two economies, I find that workers consume more in the latter economy than in the former economy and that *total* consumption during the leave period in the former economy is larger than that in the latter economy. Moreover, as an application of the theory, the study focuses on the paid parental leave policies adopted by most OECD countries. Using a numerical simulation, I conclude that the negative relationship between the replacement rate, namely the ratio of cash benefits during the leave to wages while working, and length of the leave period could result from constrained-optimal allocations.

Keywords: Paid job-protected leave policy, lack of commitment, incentive constraint, constrained-optimal allocation, paid parental leave policy

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1 Introduction

Workers face several types of shocks that force them to leave their jobs temporarily such as pregnancy, sickness, and caring for old parents. For these workers, many countries provide compensation during such leave and give them the right to return to their previous workplace after the leave period ends. However, although this policy is recognized as necessary for workers, little is known about its details. How much should workers on leave be compensated? How long should the leave period be? To answer these questions, this study constructs a model in which a worker stochastically changes his/her status and theoretically characterizes an optimal paid job-protected leave policy that maximizes social welfare under certain constraints.

This study analyzes the following model. In each period, a risk-averse worker produces output and derives utility from consumption. At the end of each period, a shock hits a worker with a certain probability and this worker then has to leave his/her job. If such a shock does not hit a worker, s/he continues to work. A worker on leave derives utility from consumption and leisure. At the end of each period, another shock hits a worker on leave with a certain probability. If this shock hits a worker on leave, then s/he has to decide whether to return to work. In this model, the inverse of this probability is considered as the length of the parental leave. If a worker on leave decides to return to work, then s/he will work in the next period. If not, s/he becomes unemployed. An unemployed worker derives utility from leisure and disutility from searching for a new job. With a certain probability, an unemployed worker finds a new job and will start working from the next period. In this model, I focus on a steady state in which a worker on leave decides to return to work if a shock hits him/her and define social welfare as the sum of all workers' values in that steady state. An *allocation* is defined by the consumption of a worker, the consumption of a worker on leave, and the probability that a shock hits a worker on leave. I characterize a *constrained-optimal* allocation that maximizes social welfare subject to the resource feasibility constraint and *incentive constraint* in this model. A worker on leave is not usually obliged to return to work after the leave period ends.[1] Thus, an optimal allocation must give a worker on leave an incentive to return to work.

Constrained-optimal allocations depend on whether the incentive constraint binds. When the incentive constraint does not bind, the consumption of a worker and that of a worker on leave should be equal. Since a worker is risk-averse, consumption should be unchanged based on a worker's status. As for the length of the leave, the optimal length should equalize the marginal welfare gain and marginal welfare loss caused by the additional extension of the leave period. An increase in the leave period decreases the number of workers in the steady state, which lowers aggregate output and consumption. Thus, social welfare marginally decreases. At the same time, an increase in the leave period gives a worker on leave additional utility from leisure, which increases social welfare. The optimal length of the leave thus equalizes the marginal welfare loss and marginal welfare gain.

When the incentive constraint binds, the above results no longer hold. The consumption of a worker is greater than that of a worker on leave. Hence, to give a worker on leave an incentive to return to

[1] According to data published by the Ministry of Health, Labour and Welfare in Japan, around 10% of female workers who take parental leave do not return to work even after the leave period ends. For institutional background of parental leave policy in Japan, see, for instance, Yamaguchi (2019).

work after the leave period ends, constrained-optimal allocations have to guarantee higher utility when a worker on leave chooses to return to work than that when s/he does not. The efficient way to achieve this goal is to increase the consumption of a worker and decrease the consumption of a worker on leave. This makes the consumption of a worker and that of a worker on leave different, which implies that the constrained-optimal allocation offers imperfect income insurance. In addition, the present study shows that a worker's consumption when the incentive constraint does not bind is larger than that when the incentive constraint binds.

As for the length of parental leave, the leave period at the constrained-optimum is too short in the sense that we can find another feasible allocation whose leave period is slightly longer than the constrained-optimal one and it raises social welfare. To give a worker on leave an incentive to return to work, the consumption of a worker is too large. Social welfare is the sum of all the statuses of workers' utilities in a steady state. Since the number of workers in the steady state depends on the length of the leave, a short leave period increases the number of workers, which can raise social welfare. At this constrained-optimal allocation, another feasible allocation may raise social welfare above that achieved by the constrained-optimal allocation even though the incentive constraint is not satisfied.

In addition, the study compares the constrained-optimal allocations in two economies, say A and B. In economy A, the discount factor is sufficiently high that the incentive constraint does not bind at the optimum. In economy B, the discount factor is so low that the incentive constraint binds at the constrained optimum. It is shown that the consumption of a worker in economy B is larger than that in economy A and that the *total* consumption of a worker on leave during the leave period in economy B is smaller than that in economy A. Note that the *periodic* consumption of a worker on leave in economy B is not necessarily smaller than that in economy A. This result contrasts with that in literature of providing insurance with limited commitment (see, for instance, Chapter 20 in Ljungqvist and Sargent (2012).) This difference comes from the model setting that the length of the leave is an endogenous variable in this study.

As an application of the analytical exercise, I consider the paid parental leave policies adopted mandatorily by most OECD countries. Figure 1 shows that the leave benefits in most OECD countries cover only a proportion of previous earnings and that leave benefits and the duration of the leave have a negative relationship.[2] This study shows that this feature may result from the constrained-optimal allocations using a numerical simulation.

A paid job-protected leave policy can be considered as a type of unemployment insurance since payment provides the main source of income for the period when a worker cannot work. Indeed, unemployment insurance does not fully cover previous earnings, although the policy can be constrained optimal. In optimal unemployment insurance literature, for instance, Hopenhayn and Nicolini (1997) showed that since an unemployed worker's effort to search for a job is unobservable, to prevent the moral hazard problem, unemployment insurance partially covers previous earnings. Thus, they claimed that observed unemployment insurance can be constrained optimal. However, this study's important friction is not a worker's hidden action as in Hopenhayn and Nicolini (1997), but rather his/her incentive constraint under which a worker on leave voluntarily returns to work after the leave period ends.[3]

[2] The average payment rate refers to the proportion of previous earnings replaced by the benefit over the length of the paid leave entitlement for a person earning 100% of average national (2015) earnings. Length in weeks is the sum of the weeks of paternity leave and those of parental leave available to mothers.

[3] In an extended model of Hopenhayn and Nicolini (1997) such as Mitchell and Zhang (2010), the same result on optimal consumption holds.

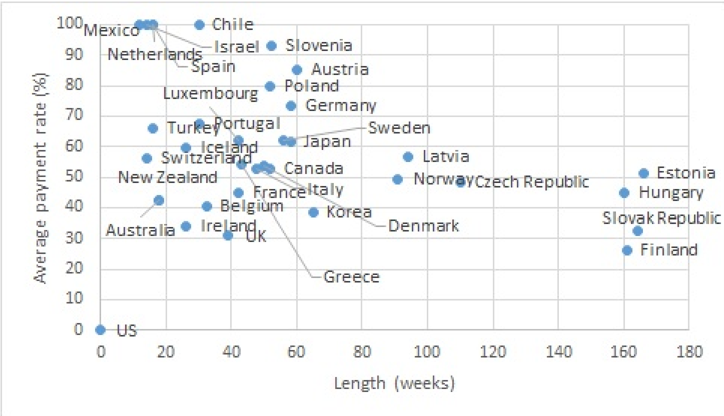

Figure 1: Paid leave entitlements available to mothers in April 2016 (Data source: OECD)

In this study, an agent's lack of commitment prevents workers from sharing the income risk perfectly. The mechanism is similar to that in Kocherlakota (1996), who considered a pure exchange economy in which two infinitely lived agents face an income risk in every period. From an ex-ante welfare point of view, it is socially optimal that both agents consume the same amount in all periods. That is, in every period, a rich agent transfers some amount to a poor agent. However, since agents cannot commit to future cooperative behavior, an agent may defect if s/he wants. To avoid such defection by agents, an allocation has to give a rich agent an incentive not to defect. Then, the resulting allocation does not perfectly share the income risk between two agents even though the allocation is constrained-efficient. This study applies a similar logic to a different setting.

Most analyses of parental leave policies have been conducted by empirical work, whereas theoretical analyses are scarce except Erosa et al. (2010) and Del-Rey et al. (2017). One of the closest works to this study is Erosa et al. (2010). They investigated the effect of paid maternity leave policies on fertility, leave-taking behavior, employment, and welfare using a labor search model. Although the model is rich, it is hard to solve analytically. Thus, the study calibrated the model consistent with U.S. data, finding that parental leave policies lead to an aggregate welfare loss even though female welfare actually increases. However, they focused on the redistribution role of parental leave policies, whereas this study sheds light on their insurance role. Del-Rey et al. (2017) considered a similar model with that in Erosa et al. (2010) and showed that the effects of the length of the leave on wages and unemployment are ambiguous. Again, Del-Rey et al. (2017) did not focus on the insurance role of parental leave policies that this study focuses on.

The remainder of the paper is organized as follows. Section 2 describes the economic model and Section 3 characterizes the constrained-optimal allocations. In Section 4, as an application of the analysis so far, a paid parental leave policy is investigated. Section 5 concludes.

2 Model

The economy is populated by a continuum of workers, distributed uniformly on the unit interval. Time is discrete and continues forever; $t = 1, 2, \ldots$.

In every period, a worker produces $y > 0$, which is constant over time and is given exogenously. A worker consumes c_t and derives utility $u(c_t)$. A periodic utility function, $u : \mathbb{R}_+ \to \mathbb{R}_+$, is assumed to be strictly increasing, strictly concave, and continuously differentiable, and $u(0) = 0$. Assume that the good is perishable and there is no storage technology. At the end of each period, a worker leaves his/her job with probability $\sigma \in (0, 1]$, which is an exogenous variable.

A worker who leaves his or her job consumes d_t and enjoys leisure in period t. Thus, s/he derives periodic utility $u(d_t) + \nu$. At the end of the period, with probability $\gamma \in \Gamma := [\underline{\gamma}, 1]$, where $\underline{\gamma} > 0$ can take any arbitrarily small number, the leave period ends. The assumption of a lower bound of γ is only a technical assumption that guarantees the existence of a solution to the optimization problem defined below. The inverse of γ, $\frac{1}{\gamma}$, is the expected duration of leave in this model. As shown below, γ is one part of the paid job-protected leave policy. If the leave period ends, then a worker on leave has to decide whether to return to work. If s/he chooses to return to work, then s/he will start working in the next period. If not, then s/he is unemployed.

An unemployed worker consumes nothing and enjoys leisure in each period. S/he searches for a job at search cost $e \geq 0$, and at the end of the period, with probability $\delta \in (0, 1)$, s/he finds a new job.

All the shocks in this economy are assumed to be i.i.d. across workers and over time.

For each status, a worker makes his/her decision to maximize his/her continuation value. Let W_t, L_t, and Q_t be the values of a worker, a worker on leave, and an unemployed worker in period t, respectively. Then, the Bellman equations for each status of a worker are given by

$$W_t = u(c_t) + \beta[\sigma Q_{t+1} + (1 - \sigma)W_{t+1}], \tag{1}$$

$$L_t = u(d_t) + \nu + \beta[\gamma \max\{W_{t+1}, Q_{t+1}\} + (1 - \gamma)L_{t+1}], \tag{2}$$

and

$$Q_t = \nu - e + \beta\left[\delta W_{t+1} + (1 - \delta)Q_{t+1}\right]. \tag{3}$$

Hereafter, I focus on a *steady state* in which workers on leave choose to return to work.[4] An *allocation* is defined by (c, d, γ). Focusing on the situation in which a worker on leave chooses to return to work once the leave period ends, an allocation, (c, d, γ), is *feasible* if

$$\mathbb{W} \times y = \mathbb{W} \times c + \mathbb{L} \times d,$$

[4] A *steady state* is a state in which the number of workers and that of workers on leave are constant over time and consumption only depends on the current status of a worker.

where[5]

$$\mathbb{W} = \frac{\gamma}{\sigma+\gamma} \quad \text{and} \quad \mathbb{L} = \frac{\sigma}{\sigma+\gamma}.$$

Given an allocation, (c,d,γ), Equations (1), (2), and (3) in a steady state are

$$W = u(c) + \beta[\sigma L + (1-\sigma)W], \tag{4}$$

$$L = u(d) + \nu + \beta[\gamma \max\{W,Q\} + (1-\gamma)L], \tag{5}$$

and

$$Q = \nu - e + \beta\left[\delta W + (1-\delta)Q\right]. \tag{6}$$

Since a worker cannot commit to his/her future action, s/he can choose not to return to work even after the leave period ends, as presented by $\max\{W,Q\}$ in Equation (5). An *incentive constraint* is defined by

$$W \geq Q. \tag{7}$$

I assume that when $W = Q$, a worker on leave chooses to return to work. A feasible allocation, (c,d,γ), is *incentive-feasible* if it satisfies the incentive constraint. From Equation (6), Equation (7) can be rewritten as

$$W \geq \frac{\nu - e}{1-\beta}. \tag{8}$$

Given an incentive-feasible allocation, (c,d,γ), from Equations (4) and (5),

$$L = \frac{u(d) + \nu + \beta\gamma W}{1-\beta(1-\gamma)}. \tag{9}$$

Plugging Equation (9) into Equation (4),

$$W = \frac{[1-\beta(1-\gamma)]u(c) + \beta\sigma[u(d)+\nu]}{(1-\beta)(1-\beta+\sigma\beta+\gamma\beta)}. \tag{10}$$

Applying Equation (10) to Equation (8), Equation (8) is rewritten as

$$u(c) + \frac{\beta\sigma}{1-\beta(1-\gamma)}\left[u(d)+e\right] \geq \nu. \tag{11}$$

[5]Letting $\mathbb{W}$ and $\mathbb{L}$ be the number of workers and workers on leave, $\mathbb{W}$ and $\mathbb{L}$ satisfy

$$\mathbb{W} = (1-\sigma)\mathbb{W} + \gamma\mathbb{L},$$

and

$$\mathbb{W} + \mathbb{L} = 1.$$

Solving these equations gives $\mathbb{W}$ and $\mathbb{L}$.

Social welfare is defined by the sum of all workers' values in a steady state. Formally, given an incentive-feasible allocation, (c,d,γ), *social welfare* is measured by

$$\frac{\gamma}{\sigma+\gamma}W+\frac{\sigma}{\sigma+\gamma}L=\frac{\gamma}{\sigma+\gamma}\frac{u(c)}{1-\beta}+\frac{\sigma}{\sigma+\gamma}\frac{u(d)+v}{1-\beta}.$$

An incentive-feasible allocation, (c,d,γ), is *constrained optimal* if (c,d,γ) is a solution to the problem:

$$\begin{array}{cc} \max_{(c,d,\gamma)\in F} & \frac{\gamma}{\sigma+\gamma}\frac{u(c)}{1-\beta}+\frac{\sigma}{\sigma+\gamma}\frac{u(d)+v}{1-\beta} \\ \text{s.t.} & \text{Equation (11),} \end{array}$$

where $F:=\{(c,d,\gamma)\in[0,y]^2\times\Gamma:(c,d,\gamma)\text{ is feasible}\}$ is a set of feasible allocations. To avoid the non-existence of the solution to the above problem, define $\overline{v}$ by

$$\overline{v}:=\max_{(c,d,\gamma)\in F}\left\{u(c)+\frac{\beta\sigma}{1-\beta(1-\gamma)}[u(d)+e]\right\}.$$

Since the objective function is continuous in (c,d,γ) and F is non-empty and compact, from the Weierstrass theorem, $\overline{v}$ is well defined. Hereafter, I assume that $v\in(0,\overline{v}]$.

Lemma 2.1. *A constrained-optimal allocation exists.*

Proof. Since the objective function is continuous in (c,d,γ) and the constraint set is non-empty and compact, from the Weierstrass theorem, a solution exists. *Q.E.D.*

3 Characterization of the constrained-optimal allocations

To characterize the constrained-optimal allocations, consider the following problem:

$$\begin{array}{cl} \max_{(c,d,\gamma)} & \frac{\gamma}{\sigma+\gamma}\frac{u(c)}{1-\beta}+\frac{\sigma}{\sigma+\gamma}\frac{u(d)+v}{1-\beta} \\ \text{s.t.} & \frac{\gamma}{\sigma+\gamma}y=\frac{\gamma}{\sigma+\gamma}c+\frac{\sigma}{\sigma+\gamma}d, \qquad (12) \\ & u(c)+\frac{\beta\sigma}{1-\beta(1-\gamma)}[u(d)+e]\geq v. \qquad (13) \end{array}$$

Letting λ and μ be Lagrange multipliers to Equations (12) and (13), respectively, set the Lagrangean as

$$\mathscr{L}:=\frac{\gamma}{\sigma+\gamma}\frac{u(c)}{1-\beta}+\frac{\sigma}{\sigma+\gamma}\frac{u(d)+v}{1-\beta}+\lambda\left\{\frac{\gamma}{\sigma+\gamma}(y-c)-\frac{\sigma}{\sigma+\gamma}d\right\}+\mu\left\{u(c)+\frac{\beta\sigma}{1-\beta(1-\gamma)}[u(d)+e]-v\right\}.$$

The first-order necessary conditions for an interior solution are

$$\frac{\gamma}{\sigma+\gamma}\frac{u'(c)}{1-\beta}-\lambda\frac{\gamma}{\sigma+\gamma}+\mu u'(c)=0, \tag{14}$$

$$\frac{\sigma}{\sigma+\gamma}\frac{u'(d)}{1-\beta}-\lambda\frac{\sigma}{\sigma+\gamma}+\mu\frac{\beta\sigma}{1-\beta(1-\gamma)}u'(d)=0, \tag{15}$$

$$\frac{\sigma}{(\sigma+\gamma)^2}\frac{u(c)}{1-\beta}-\frac{\sigma}{(\sigma+\gamma)^2}\frac{u(d)+v}{1-\beta}+\lambda\frac{\sigma}{(\sigma+\gamma)^2}(y-c+d)$$

$$-\mu\frac{\beta^2\sigma}{[1-\beta(1-\gamma)]^2}[u(d)+e]=0, \tag{16}$$

$$\frac{\gamma}{\sigma+\gamma}(y-c)-\frac{\sigma}{\sigma+\gamma}d=0, \tag{17}$$

$$\mu\left\{u(c)+\frac{\beta\sigma}{1-\beta(1-\gamma)}[u(d)+e]-v\right\}=0.$$

3.1 When the incentive constraint does not bind

First, consider the case in which an interior solution satisfies the incentive constraint with strict inequality. The solution in this case is considered as that if a worker could commit to returning to work after the leave period ends.

Since the incentive constraint holds with strict inequality, in the first-order conditions, $\mu = 0$. Let $(c^*, d^*, \gamma^*, \lambda^*, \mu^*)$ be a solution in this case. Then, Equations (14) and (15) imply

$$\lambda^* = \frac{u'(c^*)}{1-\beta} = \frac{u'(d^*)}{1-\beta} > 0.$$

Since u is strictly concave, $c^* = d^*$. Plugging $c^* = d^*$ into Equation (17),

$$c^* = d^* = \frac{\gamma^*}{\sigma+\gamma^*}y.$$

This implies that when the incentive constraint does not bind, at the optimum, an income risk caused by a shock should be fully insured and that consumption should be smoothed across the different statuses of a worker.

From Equation (14), $\lambda^* = \frac{u'(c^*)}{1-\beta}$. Plugging this into Equation (16), γ^* satisfies

$$u'(\frac{\gamma^*}{\sigma+\gamma^*}y)y = v. \tag{18}$$

Since u is strictly concave, γ^* is uniquely determined by this equation. An increase in γ raises the number of workers in the economy, which increases output and consumption. At the same time, an increase in γ shortens the leave period, which loses the opportunity of enjoying leisure. At the optimum, the marginal welfare gain from an increase in γ and the marginal welfare loss from that are equalized, as implied by Equation (18).

Proposition 3.1. *Suppose that* (c^*, d^*, γ^*) *is an interior, constrained-optimal allocation that satisfies the incentive constraint with strict inequality. Then,* (c^*, d^*, γ^*) *satisfies*

$$c^* = d^* = \frac{\gamma^*}{\sigma + \gamma^*} y,$$

and

$$u'(\frac{\gamma^*}{\sigma + \gamma^*} y) y = \nu.$$

Under $(c^*, d^* \gamma^*)$, the incentive constraint, Equation (13), is

$$\left[1 + \frac{\sigma\beta}{1 - \beta(1 - \gamma^*)}\right] u(c^*) + \frac{\beta\sigma}{1 - \beta(1 - \gamma^*)} e \geq \nu. \tag{19}$$

Note that c^* and γ^* are independent of β. Since $\frac{\sigma\beta}{1-\beta(1-\gamma^*)}$ is increasing in β, one case in which the incentive constraint does not bind is when β is sufficiently high (i.e., a worker is sufficiently patient).

3.2 When the incentive constraint binds

Next, characterize a constrained-optimal allocation when the incentive constraint binds. Let $(c^{**}, d^{**}, \lambda^{**}, \mu^{**})$ denote an interior, constrained-optimal allocation in this case. Since the incentive constraint does not bind, $\mu^{**} > 0$. From Equations (14) and (15),

$$\lambda^{**} = u'(c^{**}) \left[\frac{1}{1-\beta} + \mu^{**} \frac{\sigma + \gamma^{**}}{\gamma^{**}}\right] = u'(d^{**}) \left[\frac{1}{1-\beta} + \mu^{**} \frac{\sigma + \gamma^{**}}{\sigma} \frac{\beta\sigma}{1 - \beta(1 - \gamma^{**})}\right] > 0. \tag{20}$$

Since $\beta \in (0,1)$ implies $\frac{1}{\gamma^*} > \frac{\beta}{1-\beta(1-\gamma^{**})}$,

$$\frac{1}{1-\beta} + \mu^{**} \frac{\sigma + \gamma^{**}}{\gamma^{**}} > \frac{1}{1-\beta} + \mu^{**} \frac{\sigma + \gamma^{**}}{\sigma} \frac{\beta\sigma}{1 - \beta(1 - \gamma^{**})}.$$

For Equation (20) holding,

$$u'(c^{**}) < u'(d^{**})$$

must hold. Since u is strictly concave, $c^{**} > d^{**}$ holds.

From Equation (15), Equation (16) can be rewritten as

$$\begin{aligned}
&\frac{\sigma}{(\sigma + \gamma^{**})^2} \frac{u(c^{**})}{1-\beta} - \frac{\sigma}{(\sigma + \gamma^{**})^2} \frac{u(d^{**}) + \nu}{1-\beta} + \frac{u'(d^{**})}{1-\beta} \frac{\sigma}{(\sigma + \gamma^{**})^2} (y - c^{**} + d^{**}) \\
&+ \mu^{**} \frac{\beta\sigma}{1 - \beta(1 - \gamma^{**})} \left\{ \frac{u'(d^{**})}{\sigma + \gamma^{**}} (y - c^{**} + d^{**}) - \frac{\beta}{1 - \beta(1 - \gamma^{**})} [u(d^{**}) + e] \right\} = 0.
\end{aligned}$$

From this equation, the following lemma holds.

Lemma 3.1.

$$\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta} < \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+\nu}{1-\beta} - \frac{u'(d^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) \tag{21}$$

holds at the constrained optimum.

Proof. See the Appendix. *Q.E.D.*

An implication of Equation (21) is that by lengthening the leave period, another feasible allocation improves social welfare. In this sense, γ^{**} is too large.[6] The explanation is as follows. Suppose γ slightly decreases from γ^{**}. This decreases the number of workers and increases the number of workers on leave. The first term on the left-hand side of Equation (21) is the marginal welfare loss when the number of workers decreases, while the term on the right-hand side of Equation (21) is the marginal welfare gain when the number of workers on leave increases. A change in γ also changes output and consumption. A decrease in the number of workers lowers total output by y and the total consumption of workers by c^{**}, whereas an increase in the number of workers on leave increases the total consumption of workers on leave, d^{**}. Overall, a decrease in γ reduces total resources by $\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**})$. To satisfy the feasibility constraint, some workers have to consume less than before. The second term on the right-hand side of Equation (21) represents the marginal welfare loss if the consumption of workers on leave uniformly decreases by $\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**})$. This implies that (c^{**},d,γ), where γ is slightly smaller than γ^{**} and d is set to satisfy the feasibility constraint, raises social welfare above that achieved by $(c^{**},d^{**},\gamma^{**})$.

A summary of the findings so far is as follows.

Proposition 3.2. *Suppose that $(c^{**},d^{**},\gamma^{**})$ is an interior, constrained-optimal allocation that satisfies the incentive constraint with equality. Then,*

1. $c^{**} > d^{**}$.
2. *Ignoring the incentive constraint, social welfare achieved by $(c^{**},d^{**},\gamma^{**})$ can be improved by another feasible allocation, (c,d,γ), that satisfies $\gamma < \gamma^{**}$.*

Regarding the implementation of constrained-optimal allocations in the economy, for instance, set the lump-sum tax levied on workers to $\tau := y-c$ and the length of the leave to $\frac{1}{\gamma}$, where c and γ are the constrained-optimal consumption of workers and length of the leave characterized in this subsection.

3.3 Comparison with the constrained-optimal allocations

In this subsection, I compare (c^*,d^*,γ^*) with $(c^{**},d^{**},\gamma^{**})$. To do this, consider two economies in which all the exogenous variables besides β are the same. The discount factor in one economy, say economy A, is so high that Equation (19) holds, while the discount factor in the other economy, say economy B, is too low to satisfy Equation (19). Thus, the constrained-optimal allocation in economy A is characterized by (c^*,d^*,γ^*) and that in economy B is characterized by $(c^{**},d^{**}\gamma^{**})$.

[6] Of course, this another feasible allocation breaks the incentive constraint.

First, consider c^* and c^{**}. Since $u'(c^{**}) < u'(d^{**})$, Equation (21) implies

$$\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta}+\frac{u'(c^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) < \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+\nu}{1-\beta}. \tag{22}$$

The strict concavity of u and $c^{**} > d^{**}$ implies

$$u(c^{**})-u(d^{**})-u'(c^{**})(c^{**}-d^{**}) > 0. \tag{23}$$

Applying Equation (23) to Equation (22), I obtain

$$u'(c^{**})y < \nu.$$

Since $u'(c^*)y = \nu$ from Equation (18),

$$u'(c^{**}) < u'(c^*)$$

holds. The strict concavity of u implies $c^{**} > c^*$.

Next, consider (d^*, γ^*) and (d^{**}, γ^{**}). From Equation (17), the constrained-optimal allocations satisfy

$$y-c=\frac{\sigma}{\gamma}d.$$

Thus, $c^{**} > c^*$ implies

$$\frac{\sigma}{\gamma^*}d^* = y-c^* > y-c^{**} = \frac{\sigma}{\gamma^{**}}d^{**}. \tag{24}$$

Since $\sigma > 0$, Equation (24) is equivalent to

$$\frac{d^*}{\gamma^*} > \frac{d^{**}}{\gamma^{**}}.$$

What does this equation mean? Since $\frac{1}{\gamma}$ is the expected duration of the leave and d is the consumption of workers on leave in each period, $\frac{d}{\gamma}$ is interpreted as the total consumption of workers on leave during the leave period. Which of d^* and d^{**} is larger is inconclusive. However, if total consumption during the leave period is considered, that when the incentive constraint does not bind is larger than that when the incentive constraint binds.

A summary of the findings in this subsection is as follows.

Proposition 3.3. *Suppose there are two identical economies except β and that the discount factor in one economy is so high that Equation (19) holds, whereas that in the other economy is too low to satisfy Equation (19). Then,*

1. $c^{**} > c^*$.
2. $\frac{d^*}{\gamma^*} > \frac{d^{**}}{\gamma^{**}}$.

This result does not necessarily imply that $d^* > d^{**}$. Consider the following numerical example. The periodic utility function is $u(x) = \frac{x^{0.9}}{0.9}$, $\sigma = 0.1$, $y = 2$, $\nu = 2$, and $e = 0$. β can take a value between 0.95 and 0.99. Figure 2 illustrates c and d with different β values, showing that d^{**} for low β is larger than $d^* = 1$. From Proposition 3.3 (2), this implies $\gamma^* < \gamma^{**}$.

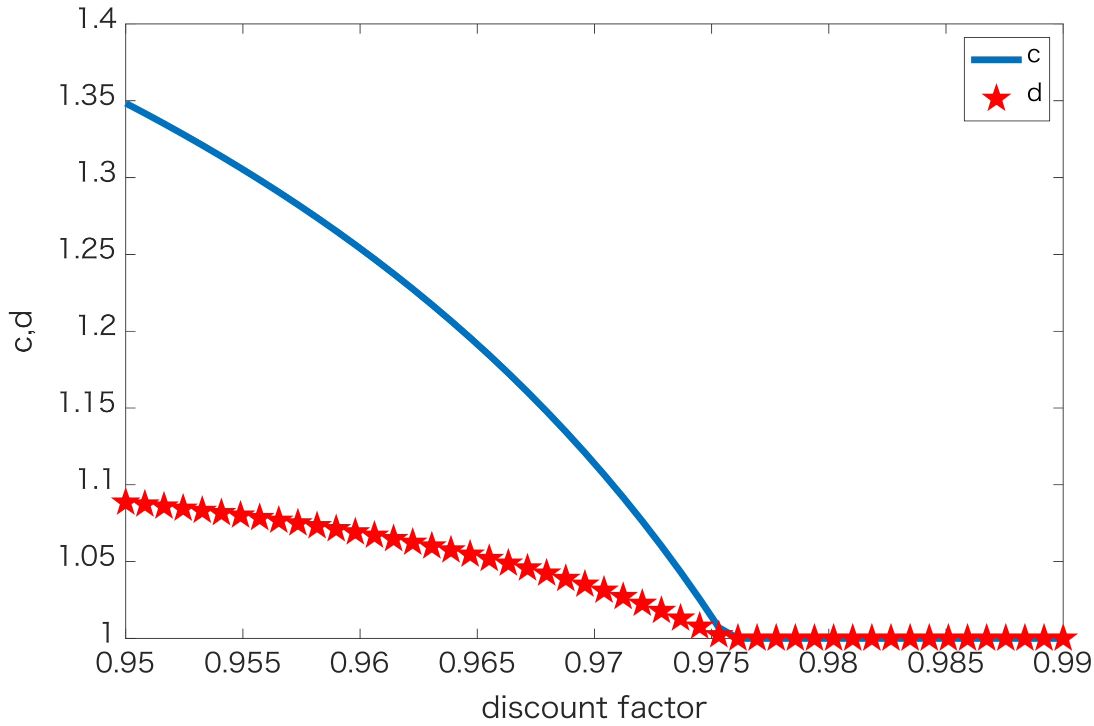

Figure 2: Simulation result of the length of the leave and replacement rate

4 Application to paid parental leave policies

In this section, as an application of the previous theoretical work, I consider paid parental leave policies.

4.1 Constrained-optimal allocations

Mandatory paid parental leave policies have been introduced in most OECD countries. Cash benefits can mitigate the financial hardship associated with foregone earnings during leave. In some countries such as those in Europe and Japan, these cash benefits are financed by the tax revenue collected from workers. Thus, from an ex-ante point of view, cash benefits serve as unemployment insurance. Job-protected leave can save job search costs even though workers temporarily leave their job. This is an important role in the labor market in an economy in which a worker who quits his/her job cannot find a new job quickly.

The cash benefits and leave period vary across OECD countries. In Figure 3, which is the same figure as Figure 1, the horizontal axis stands for the length of the leave and the vertical axis stands for the replacement rate, which is the ratio of cash benefits during the leave to wages while working. Each (red) * expresses an OECD country and the (blue) line is the approximated line.[7] The data show a negative relationship between the length of the leave and replacement rate. Why do they show negative

[7] The equation of the line is $y = -0.1609x + 67.845$.

relationship?

This observation may result from the implementation of constrained-optimal policies, and the key parameter is σ. To show this, consider the following numerical example. The periodic utility function is $u(x) = \frac{x^{0.9}}{0.9}$, $\beta = 0.99$, $y = 2$, $\nu = 2$, and $e = 0$. σ can take a value between 0.025 and 0.05. The values of σ and β are taken from Erosa et al. (2010),[8] and the other values are chosen arbitrarily. The replacement rate is measured by $\frac{d}{c}$ and the length of the leave is measured by $\frac{1}{\gamma}$ in the model. Figures 4 and 5 plot the simulation result.

Figure 4 shows the extent to which c and d vary as σ changes. In this case, $c^* = d^* = 1$, and $d^{**} > d^*$ holds when the incentive constraint binds. In Figure 5, as σ rises, a point moves from the bottom right to the top left. As σ increases, the incentive constraint does not bind. Thus, once the replacement rate reaches one, only the length of the leave shortens.

Assuming OECD countries typically share similar points, this simulation result suggests that the negative relationship between the average payment rate and length of the leave could result from constrained-optimal allocations and that the key parameter across countries is the probability that a worker leaves his/her job temporarily, σ, as discussed next.

For countries whose average payment rate is one, the length of the leave also varies. To derive this result, from Equation (18), different σ values lead to different γ^* values, while d^*/c^* is constant at one. More precisely, as σ rises, so does γ^*. When σ is too low, the incentive constraint, that is, Equation (11), is more likely to bind. In the social welfare function, more weight is placed on workers' welfare than the welfare for workers on leave. Therefore, as σ falls, the gap between c^{**} and d^{**} increases and thus d^{**}/c^{**} lowers. In Equation (11), a smaller γ raises the left-hand side of Equation (11). Thus, γ^{**} becomes smaller, or $\frac{1}{\gamma^{**}}$ becomes larger as σ becomes smaller. This explains why σ is my focus here.

4.2 Do workers voluntarily choose to leave their job?

Even if a shock that makes a worker leave his or her job hits, a worker may choose not to leave his or her job voluntarily in some cases. For instance, when a worker has the opportunity to have a child, s/he may not choose to leave because the value of a worker is higher than that of a worker on leave. In this subsection, I show that at the constrained optimum, a worker voluntarily chooses to leave his/her job when a shock hits.

Proposition 4.1. *Suppose an interior constrained-optimal allocation. Then, a worker voluntarily chooses to leave his or her job at the constrained optimum, that is, $L \geq W$ at the constrained optimum.*

Proof. See the Appendix. *Q.E.D.*

5 Conclusion

This study examines a paid job-protected leave policy using a simple dynamic model in which a key friction is that a worker on leave cannot commit to returning to work after the leave period ends. Therefore, the optimal policy has to give a worker on leave an incentive to return to work. Optimal paid job-protected leave policies are characterized by whether the incentive constraint binds.

[8] To be precise, the lower value of σ is $\sigma(4)$ and the higher value of σ is $\sigma(1)$.

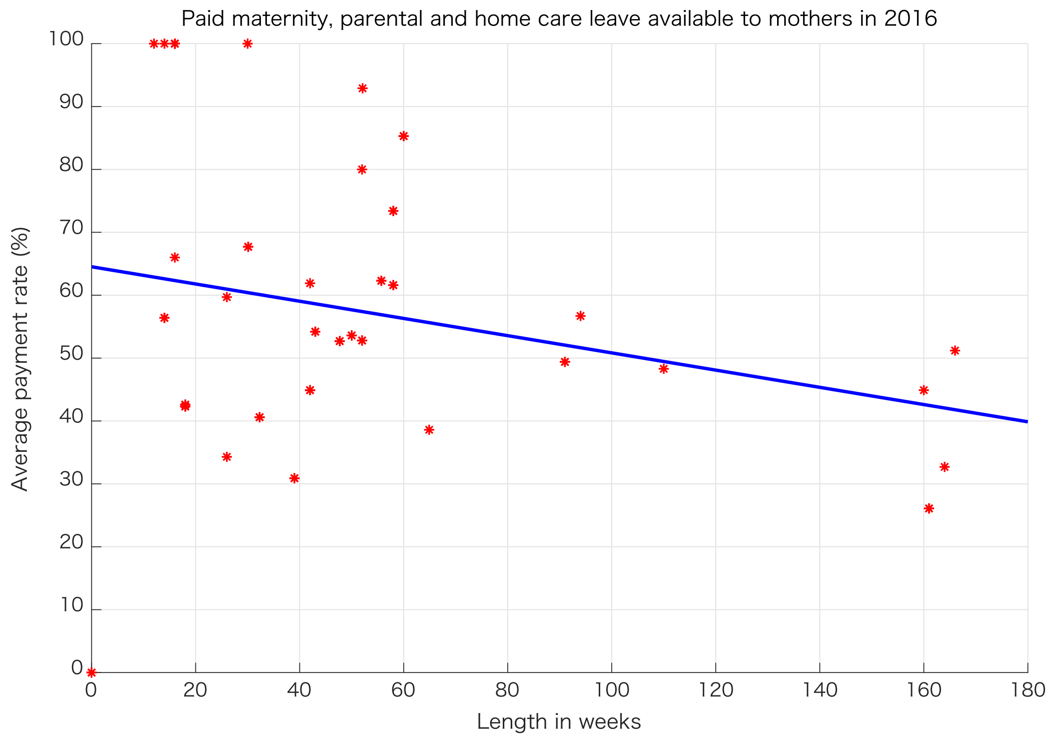

Figure 3: Paid leave entitlements available to mothers in April 2016 (Data source: OECD)

When the incentive constraint does not bind, the consumption of a worker and that of a worker on leave should be equal; that is, the income risk caused by the leave should be shared among workers. The length of the leave is set to equalize the marginal welfare gain and marginal welfare loss from a marginal increase in the length of the leave. When the incentive constraint does not bind, the consumption of a worker is greater than that of a worker on leave to satisfy the incentive constraint. Thus, the imperfect risk sharing caused by the leave might be constrained optimal. As for the length of the leave, at the constrained optimum, it is too short in the sense that social welfare can be improved by lengthening the leave period, although this allocation does not satisfy the incentive constraint.

Furthermore, the present study compares constrained-optimal allocations when the discount factor is sufficiently high to satisfy the incentive constraint strictly with those when the discount factor is too low to satisfy the incentive constraint. I show that the consumption of a worker when the incentive constraint does not bind is larger than that when the incentive constraint binds. Regarding the consumption of workers on leave and length of the leave, it is shown that total consumption during the leave period when the incentive constraint does not bind is larger than that when the incentive constraint binds. This, however, does not necessarily imply that the periodic consumption of a worker on leave when the incentive constraint does not bind is larger than that when the incentive constraint binds.

Based on the theoretical analysis, I examined whether the paid parental leave policies in OECD countries are constrained optimal. Using a numerical simulation, I showed that the negative relationship

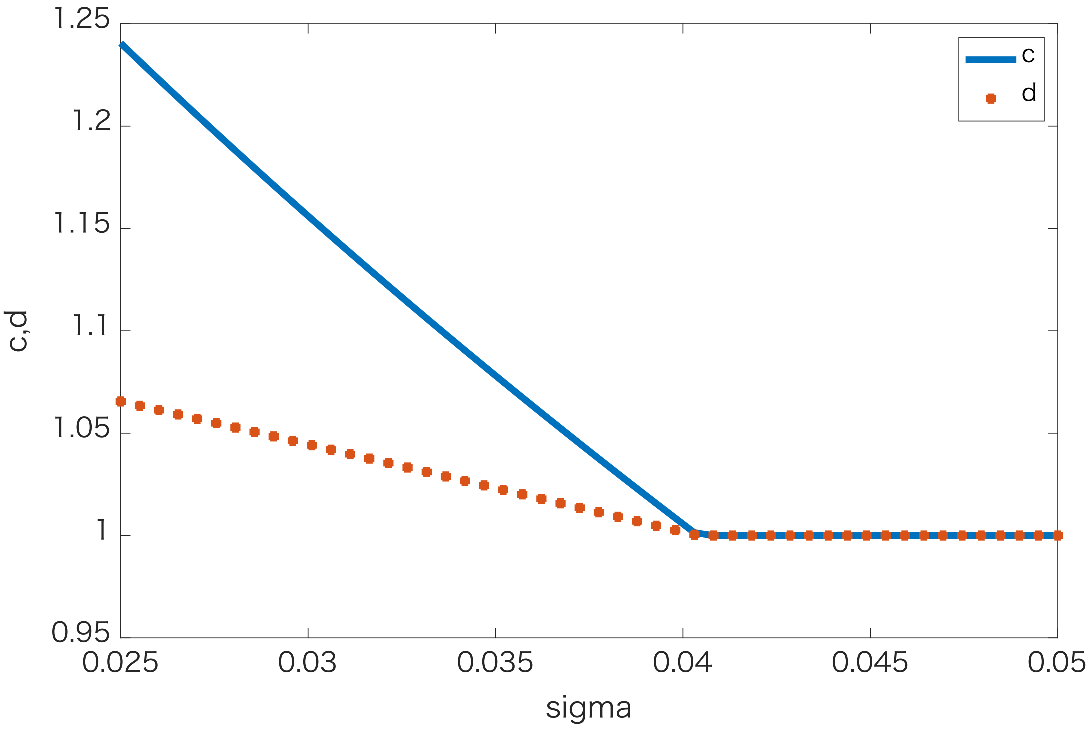

Figure 4: Simulation results of c and d

between the length of the leave and replacement rate could result from constrained-optimal paid parental leave policies and that the key parameter is the probability that a worker takes parental leave.

Since the model used in this study is simple, it can be extended in many directions. One interesting extension of the model could be to add a worker's savings as Ábrahám and Laczó (2018) extended Kocherlakota (1996) and Mitchell and Zhang (2010) extended Hopenhayn and Nicolini (1997).

A Appendix

A.1 Proof of Lemma 3.1

Proof. To claim $\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+v}{1-\beta} + \frac{u'(d^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) < 0$, suppose, by way of a contradiction, that

$$\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta} - \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+v}{1-\beta} + \frac{u'(d^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) \geq 0.$$

Since $\mu^{**} > 0$,

$$\frac{u'(d^{**})}{\sigma+\gamma^{**}}(y-c^{**}+d^{**}) - \frac{\beta}{1-\beta(1-\gamma^{**})}[u(d^{**})+e] \leq 0. \tag{25}$$

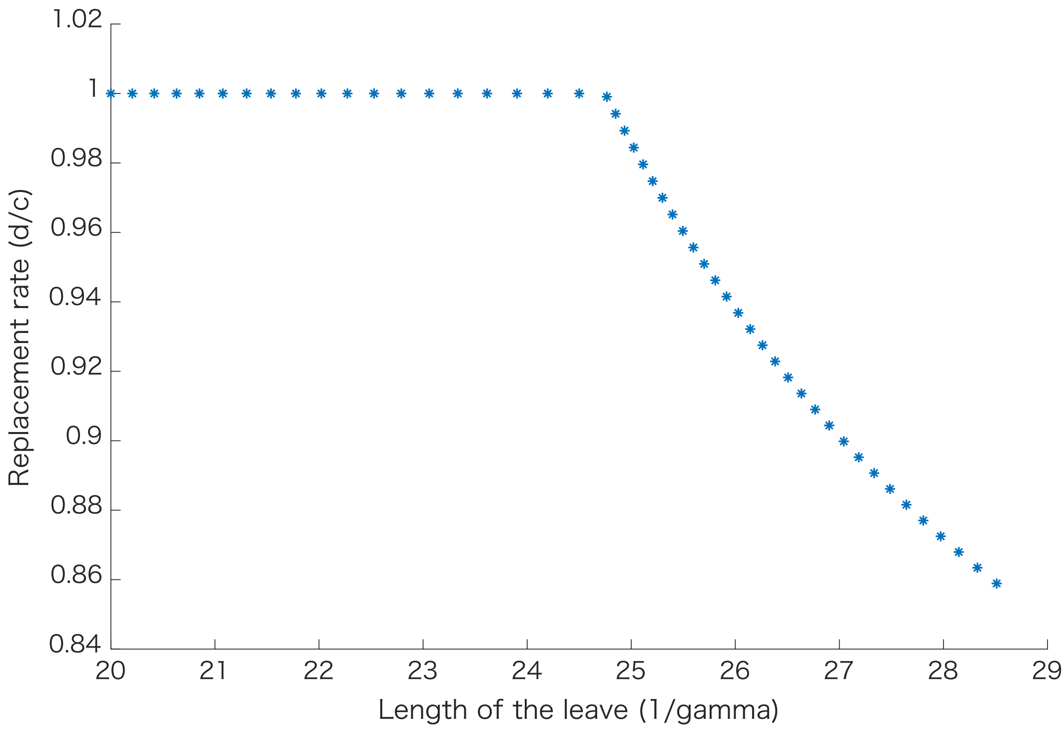

Figure 5: Simulation results of the length of the leave and replacement rate

Since the incentive constraint binds,

$$u(c^{**})+\frac{\beta\sigma}{1-\beta(1-\gamma^{**})}[u(d^{**})+e]=\nu\geq u(c^{**})+\frac{\sigma}{\sigma+\gamma^{**}}u'(d^{**})(y-c^{**}+d^{**}),$$

where Equation (25) is applied to the second inequality. From this equation

$$\begin{aligned}
& \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta}-\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+\nu}{1-\beta}+\frac{u'(d^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) \\
\leq\ & \frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(c^{**})}{1-\beta}-\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{u(d^{**})+u(c^{**})+\frac{\sigma}{\sigma+\gamma^{**}}u'(d^{**})(y-c^{**}+d^{**})}{1-\beta} \\
& +\frac{u'(d^{**})}{1-\beta}\frac{\sigma}{(\sigma+\gamma^{**})^2}(y-c^{**}+d^{**}) \\
=\ & -\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{1}{1-\beta}\left[u(d^{**})+\frac{\sigma}{\sigma+\gamma^{**}}u'(d^{**})(y-c^{**}+d^{**})-u'(d^{**})(y-c^{**}+d^{**})\right] \\
=\ & -\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{1}{1-\beta}\left[u(d^{**})-\frac{\gamma^{**}}{\sigma+\gamma^{**}}u'(d^{**})(y-c^{**}+d^{**})\right] \\
=\ & -\frac{\sigma}{(\sigma+\gamma^{**})^2}\frac{1}{1-\beta}\left[u(d^{**})-\frac{\gamma^{**}}{\sigma+\gamma^{**}}u'(d^{**})\frac{\sigma+\gamma^{**}}{\gamma^{**}}d^{**}\right]<0,
\end{aligned}$$

where the feasibility condition is applied to the fourth equality and the strict concavity of u implies the last inequality. This is a contradiction. Thus, Equation (21) holds. *Q.E.D.*

A.2 Proof of Proposition 4.1

Proof. Using Equations (9) and (10), $L \geq W$ is equivalent to

$$u(d) + \nu \geq u(c). \tag{26}$$

First, consider the case in which the incentive constraint does not bind. From Proposition 3.1, $c^* = d^*$ holds. Since $\nu > 0$, Equation (26) holds with strict inequality.

Suppose the incentive constraint binds. Thus,

$$u(c^{**}) + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}\left[u(d^{**}) + e\right] = \nu$$

holds. Since $u(\cdot) \geq 0$, $\frac{\beta\sigma}{1-\beta(1-\gamma^{**})} > 0$, and $e \geq 0$,

$$u(c^{**}) + \frac{\beta\sigma}{1-\beta(1-\gamma^{**})}\left[u(d^{**}) + e\right] = \nu \geq u(c^{**})$$

holds. Since $u(d^{**}) > 0$,

$$u(d^{**}) + \nu > u(c^{**})$$

holds, which completes the proof. *Q.E.D.*

References

Á. Ábrahám, S. Laczó, 2018. Efficient risk sharing with limited commitment and storage, *The Review of Economic Studies* 85:1389–1424.

E. Del-Rey, M. Racionero, J.I. Silva, 2017. On the effect of parental leave duration on unemployment and wages, *Economics Letters* 158:14–17.

A. Erosa, L. Fuster, D. Restuccia, 2010. A general equilibrium analysis of parental leave policies, *Review of Economic Dynamics* 13:742–758.

H. Hopenhayn, J. P. Nicolini, 1997. Optimal unemployment insurance, *Journal of Political Economy* 105:412–438.

N. Kocherlakota, 1996. Implications of efficient risk sharing without commitment, *Review of Economic Studies* 63:595–610.

L. Ljungqvist, T. Sargent, 2012. *Recursive Macroeconomic Theory* 3rd edition, Cambridge: The MIT Press.

M. Mitchell, Y. Zhang, 2010. Unemployment insurance with hidden savings, *Journal of Economic Theory* 145: 2078–2107.

OECD Family database, 2016. http://www.oecd.org/els/soc/PF2_1_Parental_leave_systems.pdf

S. Yamaguchi, 2019. Effects of parental leave policies on female career and fertility choices, *Quantitative Economics* 10:1195–1232.