Multiple-days-ahead value-at-risk and expected shortfall forecasting for stock indices, commodities and exchange rates: inter-day versus intra-day data

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Abstract

In order to provide reliable Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts, this paper attempts to investigate whether an inter-day or an intra-day model provides accurate predictions. We investigate the performance of inter-day and intra-day volatility models by estimating the AR(1)-GARCH(1,1)-skT and the AR(1)-HAR-RV-skT frameworks, respectively. This paper is based on the recommendations of the Basel Committee on Banking Supervision. Regarding the forecasting performances, the exploitation of intra-day information does not appear to improve the accuracy of the VaR and ES forecasts for the 10-steps-ahead and 20-steps-ahead for the 95%, 97.5% and 99% significance levels. On the contrary, the GARCH specification, based on the inter-day information set, is the superior model for forecasting the multiple-days-ahead VaR and ES measurements. The intra-day volatility model is not as appropriate as it was expected to be for each of the different asset classes; stock indices, commodities and exchange rates.

The multi-period VaR and ES forecasts are estimated for a range of datasets (stock indices, commodities, foreign exchange rates) in order to provide risk managers and financial institutions with information relating the performance of the inter-day and intra-day volatility models across various markets. The inter-day specification predicts VaR and ES measures adequately at a 95% confidence level. Regarding the 97.5% confidence level that has been recently proposed in the revised 2013 version of Basel III, the GARCH-skT specification provides accurate forecasts of the risk measures for stock indices and exchange rates, but not for commodities (i.e. Silver and Gold). In the case of the 99% confidence level, we do not achieve sufficiently accurate VaR and ES forecasts for all the assets.

Keywords: Basel II, Basel III, Value-at-Risk, Expected Shortfall, volatility forecasting, intra-day data, multi-period-ahead, forecasting accuracy, risk modelling.

JEL Classifications: G17; G15; C15; C32; C53.

1. Introduction and review of the literature

Risk management has now become a standard prerequisite for all financial institutions. Value-at-Risk (*VaR*) is the main risk management tool used to compute the risk of financial assets accurately. More specifically, *VaR* refers to the worst outcome of a portfolio that is likely to occur at a given confidence level over a specified period, and it focuses on market risk; see Angelidis and Degiannakis (2007). There are three methods of calculating *VaR*; the first category refers to the major representatives of *parametric family*, which are the Autoregressive Conditional Heteroskedasticity (ARCH) models. The second category, *non-parametric modeling*, relies on actual prices without assuming any specific distribution, and the main representative of this category is the *historical simulation*. The last category is the *semi-parametric* family that combines the two aforementioned frameworks. With regard to the appropriate methods of model evaluation, there are two main ones: the evaluation of the statistical properties of *VaR* forecasts, and the construction of a loss function that measures the distance between the predicted *VaR* and the actual portfolio outcome.

It is also significant that the *VaR* measurement has been adopted by bank regulators. Specifically, according to the Basel Committee on Banking Supervision (1995a,1995b, 2009), the *VaR* methodology can be used by financial institutions to calculate capital charges in accordance with their financial risk. These institutions could determine their daily capital charge by following the three prerequisites: a) The 99% confidence level must be used to make sure that institutions hold enough capital to ensure a safe and efficient market able to withstand any foreseeable problems. b) The minimum holding period must be set to 10 trading days, so that investors are able to liquidate their positions due to price changes. c) Banks could calculate *VaR* by implementing internal models.

In general, the Basel's II *VaR* quantitative requirements include: a) daily-basis estimation, b) confidence level set of 99%, c) minimum sample extension of a one year with quarterly or more frequent updates, d) no specific models prescribed, for instance, banks are free to adopt their own schemes, e) regular backtesting program for validation purposes.

The financial crisis of 2007 led to a significant number of banks becoming undercapitalized, revealing the shortfalls of the *VaR* measure as it has been defined by Basel II. The Basel Committee on Banking Supervision (2010) revised the proposed guidelines creating the Basel III. As Kinateder (2016) noted, the major disadvantage of the 2010 version of Basel III was the increased overestimation of the minimum capital requirement in extremely volatile periods (i.e. financial crises), mainly due to the introduction of the stressed *VaR* and the requirement of risk modelling at a 99% confidence level. As a result, the Basel Committee on Banking Supervision (2013) followed, which suggested the application of Expected Shortfall (*ES*) and risk modelling at a confidence level of 97.5%.

One of the most important issues in finance is the choice of one benchmark volatility model to forecast the risk that an investor faces. Since Engle's (1982) seminal paper, many other researchers have tried to find the most appropriate risk model that predicts future variability of asset returns by employing various specifications based on ARCH models, using data of different financial markets. Hence, their results are confusing and conflicting, because there is no model that is deemed adequate for all financial datasets, distributions, sample frequencies and applications. In addition, most of the empirical works are based on daily returns. Some of the most quintessential studies in the literature are presented in the following paragraphs.

Giot and Laurent (2003a), who proposed the asymmetric power of ARCH with skewed Student-*t* distributed innovations or the APARCH-skT model, estimated the daily *VaR* for stock portfolios. The findings conducted from this research performed better results for the skewed Student-*t* distribution than the pure, symmetric one. Although Giot and Laurent (2003b) kept the same distributional assumption, they chose another dataset, that of six commodities. They also claimed that more complex models (e.g. APARCH) performed better overall. Brooks and Persand (2003) concluded that the models which do not allow for asymmetries underestimate the true *VaR*.

Degiannakis (2004) suggested the fractionally integrated APARCH (FIAPARCH) model and stated that the FIAPARCH with skewed Student-*t* distributed innovations produces the most accurate one-day-ahead *VaR* predictions among three European stock indices (CAC40, DAX30 and FTSE100). Degiannakis *et al.* (2013) investigated a set of 20 stock indices worldwide and provided evidence that the fractional integration in the conditional variance model does not improve the accuracy of the *VaR* forecasts relative to the short memory GARCH specification. Additionally, other researchers, such as Angelidis *et al.* (2004) and McMillan and Kambouroudis (2009) proposed different volatility structures to estimate the daily *VaR*, but yet again without reaching a consensus and a common conclusion. They argued that the choice of the best performing model depends on the equity index. Hansen and Lunde (2005a) investigated DM-$ exchange rates and IBM stock returns and concluded that there is no evidence that the GARCH(1,1) model is outperformed by other models when the models are evaluated using the exchange rate data. This cannot be explained by the SPA test, as the ARCH(1) model is clearly rejected and found to be inferior to other models. In the analysis of IBM stock returns they found conclusive evidence that the GARCH(1,1) is inferior, and suggested that out-of-sample performance requires a specification that can accommodate a leverage effect.

Taking into consideration all the above, there is no clear agreement in the literature on which is the most adequate volatility specification. Consequently, the availability of high frequency datasets has rekindled the interest of academics for further research in forecasting risk. However, using ultra-high frequency data, researchers explore ways to extract more information that may enable them to forecast *VaR* accurately. To be more precise, Giot and

Laurent (2004) compared the APARCH-skT model with an ARFIMAX specification, in their attempt to capture *VaR* for stock indices and exchange rates as well. They conclude that the use of an intra-day dataset did not improve the performance of the inter-day *VaR* model, a fact analyzed in depth in this paper, looking not only at stocks as is the norm, but also at an extended dataset consisting of stock indices, commodities and exchange rates. Another important study that strengthens the results of the present study is that of Giot (2005), who estimated *VaR* at intra-day time horizons of 15 and 30 minutes. He proposed that the GARCH model with skewed Student-*t* distributed innovations had the best overall performance and that there were no significant differences between daily and intra-day *VaR* models once the intra-day seasonality in volatility was taken into account.

Although there is a plethora of forecasting models, the financial institutions have to abide by the recommendations of the Basel Committee on Banking Supervision. We have chosen the AR(1)-GARCH(1,1) model, a short memory model, which we compare to the Heterogeneous Autoregressive Realized Volatility, AR(1)-HAR-RV, model based on intra-day high frequency data. The distribution of the two models is the skewed Student-*t* (skT). Regarding the frequency of these forecasts, we have used 10-days-ahead and 20-days-ahead VaR and ES forecasts[1] for both models, at the 95%, 97.5% and 99% confidence level. Therefore, we will be able to study the *VaR* and *ES* forecasts at the confidence level of 97.5%, as was recently proposed in Basel III, revised in 2013, and compare them with the risk measures at the confidence level of 99%, as it has been proposed in Basel II. In support of our choice of GARCH(1,1), many researchers including Bollerslev (1986), Engle (2004), Giot and Laurent (2004) have pointed out that it has been shown to produce accurate *VaR* forecasts, among all the inter-day models, across a variety of markets and under different distributional assumptions. Some of the studies also concluded that the use of a skewed instead of a symmetrical distribution at a GARCH specification for the standardized residuals produces superior *VaR* forecasts, i.e. Giot and Laurent (2003a), Angelidis *et al.* (2004) and Degiannakis *et al.* (2014).

The HAR-RV model offers many advantages. First of all, the model retains a structure that enables the realized volatility estimates to be aggregated on different scales in order to find the realized volatility measures of the integrated volatility over different periods: daily, weekly and monthly. This is a strong advantage and the reason is simple. Typically, a financial market is comprised of participants with a large spectrum of dealing frequency. On one end of the dealing spectrum there are dealers, market makers and intraday speculators who are interested in forecasting intraday frequency data, on a daily or weekly basis. On the other end, there are central banks, commercial organization and pension fund investors, who, in their attempt to employ currency hedging, need to forecast high frequency data in a long-run period of at least a

[1] As the Basel Committee on Banking Supervision has set the minimum holding period to 10 trading days.

month. Each such participant has a different reaction to the news related to his/her investment horizon. The basic idea is that agents with different time horizons perceive, react and cause different types of volatility components, as Corsi (2002) has mentioned. Simplifying a little, the model of HAR-RV can easily identify three primary volatility components: the short-term with daily or higher trading frequency, the medium-term typically made up of portfolio managers who rebalance their positions weekly, and the long-term with a characteristic time of one month or more. Moreover, HAR can be estimated easily, as it is a multiple regression model. Surprisingly, although it does not formally belong to the class of long-memory models, the HAR-RV model is able to reproduce the same memory persistence observed in volatility (see Corsi, 2009).

The purpose of this research is to investigate the predictive ability of these models in multi-period forecasting horizons (being in line with the Basel Committee suggestion for a minimum period of 10 trading days) across a variety of markets; stocks, commodities and exchange rates. There is not an extensive literature on VaR and ES forecasting based on intra-day data[2]. Moreover, the majority of the studies dealing with intra-day volatility measures focus either on one-day-ahead VaR forecasts[3], or on the analysis of a limited dataset[4].

To summarize, for 10-days-ahead and 20-days-ahead forecasts of risk measures, the inter-day GARCH model is superior. On the other hand, the intra-day model suffers from excessive VaR violations, implying an underestimation of market risk. Undoubtedly, the inter-day GARCH specification is a safe model to adequately predict market risk measures at a 95% confidence level. The multi period-ahead VaR and ES forecasts are more accurate at the confidence level of 97.5% (as suggested in the revised version of Basel III in 2013) than at the confidence level of 99% (proposed in Basel II). Finally, a new innovative inference has emerged; the choice of the GARCH-skT has been shown to produce reasonable multiple-days-ahead VaR and ES forecasts under the skewed Student-t distribution, and most importantly, across a variety of asset classes.

The structure of the paper is as follows: Section 2 describes the VaR and ES forecasting frameworks through an ARCH process (inter-day modeling). Section 3 presents the construction of the VaR and ES multiple-days-ahead forecasts under a HAR specification (intra-day modeling). Section 4 describes the evaluation methods of VaR and ES forecasts. Section 5 gives a description of the daily log-returns and the intra-day based realized volatility measures. Section 6 investigates the empirical results of the analysis. Section 7 concludes the paper, providing the final outcomes of this research.

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[2] The most representative studies of intra-day based VaR modelling are Giot and Laurent (2004), Koopman *et al.* (2005), Beltratti and Morana (2005), Angelidis and Degiannakis (2008), Martens *et al.* (2009), Louzis *et al.*, (2013).

[3] I.e. Krzemienowski and Szymczyk (2016), Nadarajah et al. (2016), Su (2015), Watanabe (2012).

[4] I.e. Koopman *et al.* (2005), Huang and Lee (2013), Martens *et al.* (2009), Louzis *et al.* (2013).

2. Multiple-days-ahead VaR and ES forecasts under an ARCH specification (inter-day modelling)

As part of the literature on risk management and forecasting, ARCH models are used to characterize and model the financial time series. In 2003, Robert F. Engle was awarded the Nobel Prize for his pioneering work on ARCH volatility modeling. Let $\{y_t\}_{t=0}^{T} = \{\log(p_t/p_{t-1})\}_{t=1}^{T}$ refer to the continuously compounded return series, where p_t is the closing price of the trading day *t*. The return series follows the stochastic process:

$$\begin{gathered} y_t = \mu_t + \varepsilon_t \\ \varepsilon_t = z_t \sigma_t \\ z_t \sim f[E(z_t) = 0, V(z_t) = 1; \theta] \\ \sigma_t^2 = g(I_{t-1}; \theta), \end{gathered} \tag{1}$$

where $E(y_t | I_{t-1}) \equiv \mu_t$ denotes the conditional mean, given the information available, I_{t-1}, $\{\varepsilon_t\}_{t=0}^{T}$ is the ARCH process with unconditional variance $V(\varepsilon_t) = \sigma^2$ and conditional variance $V(\varepsilon_t | I_{t-1}) \equiv \sigma_t^2$, $f(.)$ is the density function of $\{z_t\}_{t=0}^{T}$, $g(.)$ is a positive measurable functional form (i.e. the ARCH volatility dynamic structure) and θ is the vector of the unknown parameters.

Because the distribution of asset returns is not symmetric, parametric *VaR* models faced difficulties in correctly modeling the tails of the distribution of returns. As a result, Angelidis and Degiannakis (2005), Giot and Laurent (2003a) and Lambert and Laurent (2001), among others, proposed the use of the skewed Student-*t* Distribution, so as to take into account the fat and platykurtic tails of the log-returns.

The τ-days-ahead $VaR_{t+\tau}^{(1-p)}$ for a long trading position at $(1-p)$ level of confidence is expressed as:

$$p = Pr\left(y_{t+\tau} \leq VaR_{t+\tau}^{(1-p)}\right) \tag{2}$$

Although the *VaR* gives important information about potential loss, it does not indicate information about expected loss. Thus, Artzner *et.al.* (1997), Artzner *et al.* (1999) and Delbaen (2002) introduced the *ES* risk measure.

ES is the expected value of loss, given that a *VaR* violation occurs, or in other words the conditional expectation of loss that takes into account losses beyond the *VaR* level.

The τ-days-ahead $ES_{t+\tau}^{(1-p)}$ for a long trading position is expressed as:

$$ES_{t+\tau}^{(1-p)} = E\left(y_{t+\tau} \middle| \left(y_{t+\tau} \leq VaR_{t+\tau}^{(1-p)}\right)\right). \tag{3}$$

Moreover, *ES* is a coherent risk measure, which satisfies the properties of sub-additivity, homogeneity, monotonicity and risk-free condition.

The empirical success of the GARCH(1,1) model has been widely spotlighted by many researchers in order to model daily volatility and calculate VaR and ES measures. The conditional mean is specified as a 1st order autoregressive process in order to allow for the non-synchronous trading effect[5] (see Degiannakis *et al.*, 2013, Lo and MacKinlay, 1990). Furthermore, we utilize the density function of skewed Student-*t* in order to take into account the fat tails and the asymmetry of the returns. The skewed Student-*t* distribution was extended to the GARCH framework by Lambert and Laurent (2000, 2001), who based their work on that of Hansen (1994). Consequently, a Monte Carlo algorithm for computing $VaR_{t+\tau|t}^{(1-p)}$ and $ES_{t+\tau|t}^{(1-p)}$ under the AR(1)-GARCH(1,1)-skT model is presented, based on Xekalaki and Degiannakis (2010) and Christoffersen (2003):

$$y_t = c_0(1-c_1) + c_1 y_{t-1} + \varepsilon_t \tag{4}$$

$$\varepsilon_t = \sigma_t z_t$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$$

$$z_t \sim skT(0,1;\nu,g)$$

$$f_{(skT)}(z_t;g,\nu) = \begin{cases} \frac{2s}{g+g^{-1}} f(g(sz_t+m);\nu) \;\; if\; z_t < -\frac{m}{s} \\ \frac{2s}{g+g^{-1}} f\left(\frac{sz_t+m}{g};\nu\right) \;\; if\; z_t \geq -\frac{m}{s} \end{cases}$$

where g and ν are the asymmetry and tail parameters of the distribution, $m = \Gamma[(\nu-1)/2]\sqrt{(\nu-2)}\left[\Gamma(\nu/2)\sqrt{\pi}\right]^{-1}(g-g^{-1})$ and $s^2 = (g^2+g^{-2}-1)-m^2$. The $f(.;\nu)$ denotes the density function of the Student-*t* distribution[6].

The Monte Carlo simulation algorithm for computing the τ-days-ahead $VaR_{t+\tau|t}^{(1-p)}$ and $ES_{t+\tau|t}^{(1-p)}$ forecasts based on the AR(1)-GARCH(1,1)-skT model is obtained as following:

One – day – ahead

Step 1: Compute the one-day-ahead conditional standard deviation:

$$\breve{\sigma}_{t+1|t} = \sqrt{a_0^{(t)} + a_1^{(t)} \varepsilon_{t|t}^2 + b_1^{(t)} \sigma_{t|t}^2}\,. \tag{5}$$

Step 2: Generate random numbers, $\{\breve{z}_{i,1}\}_{i=1}^{MC}$ from the skewed Student-*t* distribution, where MC=5000 denotes the number of draws.

Step 3: Simulate the one-trading day ahead log-returns in accordance to the AR(1) progress:

$$\breve{y}_{i,t+1} = c_0^{(t)}\left(1-c_1^{(t)}\right) + c_1^{(t)} y_t + \breve{\sigma}_{t+1|t}\,\breve{z}_{i,1}, \text{ for } i = 1,\ldots,MC. \tag{6}$$

τ-day-ahead[7]

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[5] The non-synchronous trading effect, first analyzed by Fisher (1966), expresses the autocorrelation presented in financial time series due to the fact that the values have been recorded at time intervals of one length but were recorded at time intervals of another, not necessarily regular, length.

[6] Thus, for example, $f(g(sz_t+m);\nu) = \frac{\Gamma[(\nu+1)/2]}{\Gamma(\nu/2)\sqrt{\pi(\nu-2)}}\left(1+\frac{(g(sz_t+m))^2}{\nu-2}\right)^{-\frac{\nu+1}{2}}$. For more details, the reader is referred to Giot and Laurent (2003a).

Step τ.1: Generate random numbers, $\{\breve{z}_{i,\tau}\}_{i=1}^{MC}$ from the skewed Student-*t* distribution.

Step τ.2: Create the forecast standard deviation of trading day $t+\tau$:

$$\breve{\sigma}_{i,t+\tau} = \sqrt{a_0^{(t)} + a_1^{(t)}\left(\breve{\sigma}_{i,t+\tau-1}\breve{z}_{i,\tau-1}\right)^2 + b_1^{(t)}\sigma_{i,\tau+t-1}^2} \quad . \tag{7}$$

Step τ.3: Simulate the unpredictable component: $\breve{\varepsilon}_{i,\tau} = \breve{\sigma}_{i,t+\tau}\breve{z}_{i,\tau}$.

Step τ.4: Create the hypothetical returns of time $t+\tau$, as:

$$\breve{y}_{i,t+\tau} = c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+\tau-1} + \breve{\sigma}_{i,t+\tau}\breve{z}_{i,\tau}\text{ , for } i = 1, \dots, MC. \tag{8}$$

Step τ: Calculate the τ-days-ahead $VaR_{t+\tau|t}^{(1-p)}$ and $ES_{t+\tau|t}^{(1-p)}$ as:

$$VaR_{t+\tau|t}^{(1-p)} = F_p\left(\theta^{(t)}; \{\breve{y}_{i,t+\tau}\}_{i=1}^{MC}\right)\text{ , and} \tag{9}$$

$$ES_{t+\tau|t}^{(1-p)} = \tilde{k}^{-1}\sum_{i=1}^{\tilde{k}}\left(VaR_{t+\tau|t}^{\left(1-p+ip(\tilde{k}+1)^{-1}\right)}\right) \tag{10}$$

3. Multiple-days-ahead VaR and ES forecasts under a HAR specification (intra-day modeling)

The availability of ultra-high frequency data rekindled the interest of many researchers in risk forecasting. This is illustrated by the fact that the squared daily returns are an unbiased but noisy estimator of volatility. Many researchers employ ultra-high frequency data in order to extract more information, which enables them to forecast daily *VaR* accurately. Andersen and Bollerslev (1998) showed that daily realized volatility may be constructed simply by summing up intra-day squared log-returns. Additionally, the contribution of Corsi (2009), who introduced the Heterogeneous Autoregressive for Realized Volatility (HAR-RV) model, is depicted as one of the quintessential processes. The HAR-RV model is an autoregressive structure of the realized volatilities over different time intervals. The HAR-RV model for the logarithmic transformation of the annualized realized volatility $\sqrt{252\sigma_t^{2(RV)}}$, is defined as:

$$\log\sqrt{252\sigma_t^{2(RV)}} = w_0 + w_1\log\sqrt{252\sigma_{t-1}^{2(RV)}} + w_2\log\sqrt{252\sigma_{t-5:t-1}^{2(RV)}} + w_3\log\sqrt{252\sigma_{t-22:t-1}^{2(RV)}} + u_t, \tag{11}$$

where $u_t \sim N(0,1)$. The $\sigma_{t-1}^{2(RV)}$ accounts for the volatility perception from inter-day and intra-day traders, whereas the $\sigma_{t-5:t-1}^{2(RV)}$ accounts for medium term trading strategies. Moreover, the $\sigma_{t-22:t-1}^{2(RV)}$ encompasses the perception of volatility for investment strategies with monthly or even longer time horizons. The heterogeneity is the reason of the volatility variations through different time intervals.

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[7] For Steps τ.1 to τ.4, the process is repeated. I.e. for predicting $VaR_{t+10|t}^{(1-p)}$ and $ES_{t+10|t}^{(1-p)}$, the steps τ.1 to τ.4 are computed iterated for τ=2, …, 10.

The AR(1)-HAR-RV-skT model is defined as an AR(1) process for the daily log-returns, $y_t = c_0^{(t)} - c_1^{(t)} y_{t-1} + \varepsilon_t$. The unpredictable component ε_t, is designed to follow the skewed Student-*t* distribution conditional on the most recently available information set, or $\varepsilon_t = \left(y_t - c_0^{(t)} - c_1^{(t)} y_{t-1}\right) | I_{t-1} \sim skT(0,1; g, \nu)$. Moreover, the unpredictable component is decomposed as $\varepsilon_t = z_t \sigma_t^{(RV)}$. Hence, the AR(1)-HAR-RV-skT model is defined as:

$$y_t = c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} y_{t-1} + z_t \sigma_{t|t-1}^{(RV)} \tag{12}$$

$$\sigma_{t|t-1}^{(RV)} = exp\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t-1}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t-5:t-1}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-22:t-1}^{2(RV)}}\right)/\sqrt{252} \tag{13}$$

$$z_t \sim skT(0,1; \nu, g)$$

$$f_{(skT)}(z_t; g, \nu) = \begin{cases} \frac{2s}{g+g^{-1}} f(g(sz_t + m); \nu) & if\ z_t < -\frac{m}{s} \\ \frac{2s}{g+g^{-1}} f\left(\frac{sz_t+m}{g}; \nu\right) & if\ z_t \geq -\frac{m}{s} \end{cases} \tag{14}$$

The Monte Carlo simulation algorithm for computing the $VaR_{t+\tau|t}^{(1-p)}$ and $ES_{t+\tau|t}^{(1-p)}$ forecasts based on the AR(1)-HAR-RV-skT model is illustrated:

One – day – ahead

Step 1: Compute the one-day-ahead realized volatility $\sigma_{t|t-1}^{(RV)}$ according to equation (13). Note that $\sigma_{t-5:t-1}^{2(RV)}$ denotes the average of i) actual values for points in time prior to *t* and ii) predicted values for points in time subsequent time *t*. The same case holds for $\sigma_{t-22:t-1}^{2(RV)}$.

Step 2: Generate MC=5000 random numbers, $\left\{\breve{z}_{i,1}\right\}_{i=1}^{MC}$, from the skewed Student-*t* distribution.

Step 3: The value of the unpredictable component is $\breve{\varepsilon}_{i,t+1} = \sigma_{t+1|t}^{(RV)} \breve{z}_{i,1}$.

Step 4: Simulate the one-trading day ahead log-returns in accordance to the AR(1) progress:

$$\breve{y}_{i,t+1} = c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t} + \breve{\varepsilon}_{i,t+1}, \text{for } i = 1, \dots, MC. \tag{15}$$

τ – days – ahead[8]

Step τ.1: Compute the τ-day-ahead realized volatility as:

$$\sigma_{t+\tau|t}^{(RV)} = exp\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+\tau-1|t}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t+\tau-5:t+\tau-1}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t+\tau-22:t+\tau-}^{2(RV)}}\right. \tag{16}$$

Step τ.2: Generate $\left\{\breve{z}_{i,\tau}\right\}_{i=1}^{MC}$ from the skewed Student-*t* distribution.

Step τ.3: Simulate the τ-trading days ahead log-returns:

$$\breve{y}_{i,t+\tau} = c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t+\tau-1} + \sigma_{t+\tau|t}^{(RV)} \breve{z}_{i,\tau}, \text{for } i = 1, \dots, MC. \tag{17}$$

Step τ: Calculate the τ-days-ahead $VaR_{t+\tau|t}^{(1-p)}$ and $ES_{t+\tau|t}^{(1-p)}$ as:

[8] For example, in the case of computing the $VaR_{t+10|t}^{(1-p)}$ and $ES_{t+10|t}^{(1-p)}$, the steps τ.1 to τ.3 are computed iterated for τ=2, …, 10.

$$VaR_{t+\tau|t}^{(1-p)} = F_p\left(\theta^{(t)}; \{\breve{y}_{i,t+\tau}\}_{i=1}^{MC}\right) \quad (18)$$

$$ES_{t+\tau|t}^{(1-p)} = \tilde{k}^{-1} \sum_{i=1}^{\tilde{k}} \left(VaR_{t+\tau|t}^{\left(1-p+ip(\tilde{k}+1)^{-1}\right)} \right). \quad (19)$$

4. Evaluate multiple-days-ahead VaR and ES Forecasts

The *VaR* measure must neither overestimate nor underestimate the expected loss, as in both cases the financial institution allocates the wrong amount of capital. The simplest method to measure the accuracy of the risk models is to record the total number of violations. However, there are statistical techniques for evaluating *VaR* models. The quintessential ones are the methods of Kupiec (1995) and Christoffersen (1998), called backtesting procedures.

4.1. First Stage Evaluation

The test most widely used was developed by Kupiec (1995). It examines the hypothesis of whether the average number of violations is statistically equal to the excepted one. The appropriate likelihood ratio statistic is:

$$LR_{UC} = 2\log\left(\left(1-\frac{N}{\tilde{T}}\right)^{\tilde{T}-N}\left(\frac{N}{\tilde{T}}\right)^{N}\right) - 2\log\left((1-\rho)^{\tilde{T}-N}\rho^{N}\right) \sim X_1^2, \quad (20)$$

where $N = \sum_{t=1}^{\tilde{T}} \tilde{I}_t$ is the number of days over a period $\tilde{T}$ that a violation occurred and as a result the portfolio loss was larger than the *VaR* estimate[9], and ρ is the expected ratio of violations. The risk model will be rejected if it generates too many or too few violations:

$$\tilde{I}_{t+\tau} = \begin{cases} 1, & if \;\; y_{t+\tau} < VaR_{t+\tau|t}^{(1-p)} \\ 0, & if \;\; y_{t+\tau} \geq VaR_{t+\tau|t}^{(1-p)} \end{cases} \quad (21)$$

According to Kupiec (1995), the number of violations follows a binominal distribution $N \sim B(\tilde{T}, p)$ and the hypotheses tested are:

$$H_0: N/\tilde{T} = p, \qquad H_1: N/\tilde{T} \neq p. \quad (22)$$

Christoffersen (1998) examined concurrently if the *VaR* failure process is independently distributed or not. The hypotheses presented on the second backtesting criterion are defined as:

$$H_0: \pi_{01} = \pi_{11}, \qquad H_1: \pi_{01} \neq \pi_{11}. \quad (23)$$

The $\pi_{ij} = n_{ij}/\sum_j n_{ij}$ is the corresponding probability and *i*, *j*=1 denotes that a violation has occurred, whereas *i*, *j*=0 indicates the opposite. The likelihood ratio statistics for the independence is described in the following:

$$LR_{IN} = 2(\log((1-\pi_{01})^{n00}\pi_{01}^{n01}(1-\pi_{11})^{n10}\pi_{11}^{n11}) - \log((1-\pi_0)^{n00+n10}\pi_0^{n01+n11})) \sim X_1^2. \quad (24)$$

The main advantage of using the above two backtesting tests is the fact that the managers could easily reject a *VaR* model that generates too many or too few clustered

[9] We evaluate the accuracy of risk models for long trading positions. Alternatively, for short trading positions $\tilde{I}_{t+1} = 0$ if $y_{t+1} \geq VaR_{t+1|t}^{(1-p)}$ and 1 otherwise.

violations. However, their drawback is that these two backtesting procedures cannot classify the models based only on the *p*-values of these tests.

4.2. Second Stage Evaluation

The limitation of backtesting tests leads to the excessive need of the second stage of evaluation forecasting. Lopez (1999) proposed a forecast evaluation framework which is focused on a loss function measuring the accuracy of *VaR* forecasts on the basis of the distance between the observed returns and the forecasted *VaR* values, given that a violation occurred. Through the Lopez (1999) approach, a *VaR* model is penalized when an exception takes place. Nevertheless, as Angelidis and Degiannakis (2007) noted, the returns should better be compared with *ES* instead of *VaR*, since *VaR* does not imply indications concerning the size of the expected loss. So we employ a loss function that measures the squared distance between actual daily returns and the $ES_{t+\tau|t}$ forecasts as:

$$\psi_{t+\tau} = \begin{cases} 1 + \left(ES_{t+\tau|t} - y_{t+\tau}\right)^2 & \text{if a violation has occured} \\ 0 & \text{otherwise}\,. \end{cases} \tag{25}$$

The preferable model is the one that minimizes the average loss, $\Psi = \tilde{T}^{-1}\sum_{t=1}^{\tilde{T}}\psi_t$, called the Mean Predictive Squared Error (MPSE).

Predictive accuracy is further explored with the Diebold and Mariano (1995) test[10]. We investigate whether the loss functions of the two models are statistically different. Let us define $\psi_t^{(G)}$ and $\psi_t^{(HAR)}$ as the loss functions from the AR(1)-GARCH(1,1)-skT and AR(1)-HAR-RV-skT models, respectively. The null hypothesis that these two models are of equivalent predictive ability is tested against the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability:

$$\begin{aligned} &H_0\colon E\left(\psi_t^{(G)} - \psi_t^{(HAR)}\right) = 0 \\ &H_1\colon E\left(\psi_t^{(G)} - \psi_t^{(HAR)}\right) < 0. \end{aligned} \tag{26}$$

The Diebold and Mariano statistic is computed as the test statistic of the constant coefficient from regressing $\left(\psi_t^{(G)} - \psi_t^{(HAR)}\right)$ on a constant with heteroskedastic and auto correlated consistent standard errors, or $\frac{\tilde{T}^{-1}\sum_{t=1}^{\tilde{T}}\left(\psi_t^{(G)}-\psi_t^{(HAR)}\right)}{\sqrt{V\left(\tilde{T}^{-1}\sum_{t=1}^{\tilde{T}}\left(\psi_t^{(G)}-\psi_t^{(HAR)}\right)\right)}}$.

5. Inter-Day and Intra-Day Data

In this paper, we use three types of financial asset classes; stock indices, commodities and foreign exchange rates. The 3 stock indices are the Standard and Poor's 500 from the US stock market (S&P500) with 3901 observations, the Europe Stock 50 (EurostoXX50) with 3949

[10] The Diebold-Mariano statistic has been selected for a pairwise model comparison. If we had to compare more than two model frameworks then we would have chosen the Superior Predictive Ability test of Hansen (2005) or the Model Confidence Set of Hansen et al. (2011).

observations, and the Financial Times Stock Exchange 100 from London stock market (FTSE100) with 3912 observations. The 3 commodities are Copper (HG) with 3897 observations, Silver (SV) with 3897 observations and Gold (GC), also with 3897 observations. The 3 foreign exchange rates are the Euro Exchange Rate based on the US Dollar (EUR/USD) with 3898 observations, the British Pound Exchange Rate based on the US Dollar (GBP/USD) with 3899 observations and finally, the Canadian Dollar Exchange Rate based on the US Dollar (CAD/USD) with 3899 observations[11].

The data from the nine asset prices cover a range of fifteen years, spanning the period from 3 January, 2000 to 5 August, 2015 and were conditioned to remove any non-trading days. To avoid outliers that would result from half trading days, we removed days that stock markets were not active for more than six and a half hours between 9:30 a.m. and 4:00 p.m. Furthermore, inactive trading days were excluded when stock markets were closed for the whole day, such as weekends and public or local holidays; for instance, the day after Thanksgiving and days around Christmas.

Descriptive statistics for the daily log-returns for the selected stock indices, commodities and exchange rates are presented in Table 1. All of the returns distributions are platykurtic, due to the fact that the kurtosis is a large positive value for all the nine assets. Figure 1 plots the distribution histograms of log-returns. All the asset classes are negatively skewed. The Jarque-Bera results indicate that none of the log-returns series follow a Gaussian distribution. It is clear that in almost all the graphs depicted in Figure 2 the same periods of intense volatility clustering are found. The major cluster of volatility encompasses the observations around the year of 2008 in which Lehman Brothers collapsed.

{INSERT TABLE 1}

{INSERT FIGURES 1-2}

Let us define as p_{t_j} the intra-day asset price on trading day t which has been partitioned in m equidistance points within the trading day. The realized volatility $\sigma_t^{2(RV)}$ is computed according to Hansen and Lunde (2005b) in order to scale the intra-day realized volatility with the volatility during the time that the market is closed:

$$\sigma_t^{2(RV)} = \omega_1 RV_t^{(m)} + \omega_2 \left(log(p_{t_1}) - log(p_{t-1_m})\right)^2. \quad (27)$$

[11] The dataset consists of tick-by-tick intraday prices of front-month futures contracts. The availability of reliable intraday data for a period of fifteen years and the market liquidity motivate us to choose the specific dataset. The S&P500, EurostoXX50 and FTSE100 represent, in terms of capitalization and liquidity, three of the most important stock markets worldwide. Regarding commodities, Copper, Silver and Gold belong to the most liquid commodity futures contracts. As far as currencies are concerned, the Dollar, Euro, Pound and Canadian Dollar belong to the six most traded currencies on the planet.

The $RV_t^{(m)} = \sum_{j=1}^{m-1}\left(\log\left(p_{t_j}\right) - \log\left(p_{t_{j-1}}\right)\right)^2$ term is the intra-day realized volatility, whereas the $\left(log(p_{t_1}) - log(p_{t-1_m})\right)^2$ term takes into consideration the overnight volatility. The intra-day sampling frequency is selected based on the criterion of minimizing the intra-day auto-covariance[12] that approximates the measurement errors due to microstructure frictions. The parameters ω_1 and ω_2 are estimated such as $\min_{(\omega_1,\omega_2)} V\left(\sigma_t^{2(RV)}\right)$, as $\text{argmin}_{(\omega_1,\omega_2)} E\left(\sigma_t^{2(RV)} - IV_t\right) = \text{argmin}_{(\omega_1,\omega_2)} V\left(\sigma_t^{2(RV)}\right)$, where IV_t is the actual but unobservable volatility; the integrated volatility[13]. Figure 3 plots the annualized realized standard deviations, $\sqrt{252\sigma_t^{2(RV)}}$, for the stock indices, commodities and exchange rates, whereas the descriptive statistics of $\sqrt{252\sigma_t^{2(RV)}}$ are presented in the Table 2. The kurtosis is highly positive for all the asset classes referring to leptokurtic distributions. Moreover, all the $\sqrt{252\sigma_t^{2(RV)}}$ are positively skewed. The descriptive statistics of the stock indices are qualitatively similar to those presented in the literature, i.e. Degiannakis and Floros (2016) have illustrated the descriptive information for 17 European and USA stock indices. Compared to stock indices, commodities are characterized by higher values of volatility, whereas exchange rates by much lower values of volatility. For example, the average daily annualized volatility of silver is 27.8% with a standard deviation of 15.9%, which is higher than the 22.7% average $\sqrt{252\sigma_t^{2(RV)}}$ of EurostoXX50 with a standard deviation of 12.8%. On the other hand, the EUR/USD exchange rate has an average daily annualized volatility of 9.6% with a standard deviation of 4.1%.

{INSERT FIGURE 3}

{INSERT TABLE 2}

6. Empirical Analysis

The predictive accuracy of the AR(1)-GARCH(1,1)-skT and the AR(1)-HAR-RV-skT models is investigated, concerning the 95%, 97.5% and the 99% confidence levels. Based on the total number of T observations (trading days), the rolling window approach with a fixed window length of $\breve{T} = 1000$ trading days is utilized. Hence, the models are re-estimated every trading day t, for $\tilde{T} = T - \breve{T}$ days. The results for the 10-trading-days-ahead forecasts at the 95%

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[12] The expected value of intra-day auto covariance equals to zero; see i.e. Andersen *et al.* (2006). The auto-covariance is computed as $\sum_{j=1}^{m-1}\sum_{i=j+1}^{m} y_{t_i}y_{t_{i-j}}$, for $y_{t_i} = log(p_{t_i}) - log(p_{t_{i-1}})$ denoting the intra-day log-returns.

[13] Hansen and Lunde (2005b)provided a Lemma according to which: for Y denoting a real random variable and for X_ω, for $\omega\epsilon\Omega$, being a class of real random variables, if $E(X_\omega \setminus Y) = Y$, for $\forall\ \omega\epsilon\Omega$, then: $\text{argmin}_{(\omega)} E(X_\omega - Y)^2 = \text{argmin}_{(\omega)} V(X_\omega)$.

confidence level are presented in Table 3, across the 3 asset classes. Table 3 presents the average values of $VaR_{t+10|t}^{(95\%)}$ and $ES_{t+10|t}^{(95\%)}$, the mean predicted squared error for $ES_{t+10|t}^{(95\%)}$, the observed exception rate and the *p*-values of Kupiec and Christoffersen backtesting tests. Figure 4 illustrates, indicatively, the log-returns and the $VaR_{t+10|t}^{(95\%)}$ for EurostoXX50 and FTSE100. The relative graphs for the other assets are available from the authors on request.

For the 10-trading-days-ahead forecasting horizon, the HAR-RV-skT model does not outperform the GARCH-skT specification. The GARCH-skT model framework is slightly preferable since the observed exception rates are much closer to the expected ones. The *p*-values of the Kupiec test are highly acceptable in all the cases except for Gold (for both models) and Silver (for the HAR-RV-skT model). Additionally, the independence test does not reject the hypothesis that the violations are independently distributed for both model frameworks at any reasonable level of significance for all the assets except for the Gold.

Moreover, the MPSE loss function (for the predicted $ES_{t+10|t}^{(95\%)}$) of the GARCH-skT model is lower than that of the HAR-RV-skT model in 7 out of 9 cases. However, the statistical comparison of the predictive accuracy according to the Diebold Mariano test indicates that only in the case of EurostoXX50 and Silver the $ES_{t+10|t}^{(95\%)}$ forecasts of the GARCH-skT model are statistically more accurate than those of the HAR-RV-skT model[14]. Overall, less accurate risk forecasts are estimated for Gold.

{INSERT TABLE 3}

{INSERT FIGURE 4}

For the longer time horizon of 20-days-ahead, at the 95% confidence level, the results are presented in Table 4. The forecasting performance of the GARCH-skT model has not deteriorated compared to the case of 10-days-ahead predictions. On the contrary, the HAR-RV-skT model seems to provide less accurate forecasts in this time horizon, as there are more rejections of the backtesting test of Kupiec. The Kupiec statistic for the GARCH-skT model suggests that the observed exception rate is statistically equal to the expected failure rate for all the assets. However, the Kupiec test for the HAR-RV-skT model rejects the null hypothesis at a 5% level of significance in four cases; specifically, for the EurostoXX50, FTSE100, Silver and Gold. Additionally, the independence test does not reject the hypothesis that the violations are independently distributed for both model frameworks at any reasonable level of significance for all the assets. The only exception is that of the EurostoXX50, for the risk forecasts provided by the intra-day realized volatility model.

[14] Due to space limitations, the Diebold Mariano test statistics are not presented, but they are available upon request.

Turning to the estimates for the quadratic loss function that measures the squared distance between actual returns and expected loss in the event of a $VaR_{t+20|t}^{(95\%)}$ violation (MPSE loss function for $ES_{t+20|t}^{(95\%)}$ forecast), the GARCH-skT model produces lower values in 6 out of 9 cases. The Diebold Mariano test provides evidence that only in the case of Silver, the $ES_{t+20|t}^{(95\%)}$ forecasts of the GARCH-skT model are statistically more accurate compared to those of the HAR-RV-skT model. Hence, the GARCH-skT specification seems to be preferable to that of the HAR-RV-skT, as the former satisfies most of the prerequisites concerning the $VaR_{t+20|t}^{(95\%)}$ and $ES_{t+20|t}^{(95\%)}$ forecasting. Figure 5 illustrates, indicatively, the log-returns and the $VaR_{t+20|t}^{(95\%)}$ for Copper and Gold[15].

{INSERT TABLE 4}

{INSERT FIGURE 5}

The results for the $VaR_{t+10|t}^{(97.5\%)}$ and $ES_{t+10|t}^{(97.5\%)}$ measures are similar to the 95% results and they are presented in Table 5. Overall, the daily conditional volatility model outperforms the intra-day realized volatility model. The GARCH-skT model provides accurate $VaR_{t+10|t}^{(97.5\%)}$ forecasts (as the *p*-values of the unconditional coverage test are higher than the 0.05 value) for all the indices except for the Silver and the Gold commodities. On the other hand, the HAR-RV-skT model produces more $VaR_{t+10|t}^{(97.5\%)}$ violations than expected, not only for Silver and Gold, but also for the FTSE100 index. Turning to the estimates of the MPSE loss function for the $ES_{t+10|t}^{(97.5\%)}$, the GARCH-skT model has a lower MPSE loss function compared to that of the HAR-RV-skT model in 7 out of 9 cases. Concerning the FTSE100, Silver and Gold, the $ES_{t+10|t}^{(97.5\%)}$ forecasts from the GARCH-skT model are statistically more accurate compared to those from the HAR-RV-skT model. Table 6 illustrates the information regarding the $VaR_{t+20|t}^{(97.5\%)}$ and $ES_{t+20|t}^{(97.5\%)}$ forecasts. Both models provide accurate $VaR_{t+20|t}^{(97.5\%)}$ forecasts for all the indices except for Silver and Gold, and the GARCH-skT model has a lower MPSE loss function for the $ES_{t+20|t}^{(97.5\%)}$ compared to that of the HAR-RV-skT model in 7 out of 9 cases (although for all the assets, the $ES_{t+20|t}^{(97.5\%)}$ forecasts from both models are statistically equal).

{INSERT TABLES 5-6}

The results for the $VaR_{t+10|t}^{(99\%)}$ and $ES_{t+10|t}^{(99\%)}$ measures are presented in Table 7. Overall, the daily conditional volatility model outperforms the intra-day realized volatility model. But, the GARCH-skT model provides accurate $VaR_{t+10|t}^{(99\%)}$ forecasts (as the *p*-values of the unconditional coverage test are higher than the 0.05 value) in only 5 cases. On the other hand,

[15] The relative graphs for the other assets are available from the authors upon request.

the HAR-RV-skT model produces more $VaR_{t+10|t}^{(99\%)}$ violations than expected in 7 out of 9 cases. Turning to the estimates of the MPSE loss function for the $ES_{t+10|t}^{(99\%)}$, the GARCH-skT model has a lower MPSE loss function compared to that of the HAR-RV-skT model in 7 out of 9 cases. Finally, according to Table 8, qualitatively similar findings are provided for the $VaR_{t+20|t}^{(99\%)}$ and $ES_{t+20|t}^{(99\%)}$ forecasts as in the case of the 10-days-ahead predictions.

{INSERT TABLES 7-8}

Even nowadays, the majority of the studies, i.e. Krzemienowski and Szymczyk (2016), Nadarajah *et al.* (2016), Su (2015), Watanabe (2012) investigate the one-day-ahead forecasting performance, but as it was mentioned before, the minimum holding period must be set to 10 trading days. To conclude, after checking the 10-steps-ahead and 20-steps-ahead forecasts of risk measures from GARCH-skT and HAR-RV-skT models, we can infer that the results for the risk models are not very clear across different asset classes. Hence, it is difficult for risk modelers to propose a clear-cut conclusion, concerning which model is the most accurate and reliable to adequately forecast the losses of a specific portfolio. After a careful examination, we observe that for the 95% confidence level, the daily conditional volatility model provides adequate VaR and ES forecasts for medium-term and long-term periods, such as 10-days and 20-days-ahead[16]. The only exception is the case of Gold. Turning to the 97.5% confidence level, the GARCH-skT model provides adequate VaR and ES forecasts for 7 of the cases but not for two of the commodities; Silver and Gold. The picture is more complicated in the case of the 99% confidence level, which is much more difficult to forecast accurately. At the 99% confidence level, although the GARCH-skT model outperforms the intra-day realized volatility model, we do not achieve sufficiently accurate forecasts of risk measures for all the assets.

Hence, our empirical findings recommend risk modelling at a confidence level of 97.5%. Among the changes in the regulatory treatment of financial institutions' trading book positions, the Basel Committee has proposed the replacement of 99% VaR by 97.5% ES. Kellner and Rösch (2016) provide evidence that under correctly specified models (i.e. models allowing for skewness and heavy tails) the level of capitalization would be higher when using 97.5% ES instead of the 99% VaR.

The satisfactory forecasting performance of the skewed Student-*t* distribution is in line with the findings of the literature. Previous studies, i.e. Giot and Laurent (2003a), Angelidis *et al.* (2004) and Degiannakis *et al.* (2014), have also provided strong empirical evidence of the successful application of the skewed Student-*t* distribution in forecasting risk measures.

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[16] The Füss *et al.* (2016) stated that the GARCH-type *VaR* outperforms the other *VaR's* for most of the hedge-fund-style indices. On the contrary, Christoffersen and Diebold (2000) noted that while *VaR* measures based on ARCH class specifications perform rather well for one-day time horizons, it is known that their performance is not as good for long time periods.

Recently, Braione and Scholtes (2016) showed the importance of allowing for heavy-tails and skewness in the distributional assumption with the skew Student-t outperforming the others across all tests and confidence levels.

The underperformance of the intra-day volatility model is in line with the findings of Angelidis and Degiannakis (2008), Giot and Laurent (2004). On the other hand, Huang and Lee (2013) noted that that the high-frequency intraday information has excellent forecasting performance when compared to low-frequency daily information, but their analysis is limited to S&P500 and for one-day forecasting horizon. Louzis *et al.* (2013) found that intra-day based volatility measures can produce statistically accurate multi-step VaR forecasts, but they have limited their analysis to S&P500 stock index as well.

7. Conclusions

A common question that has triggered a lot of interest in the financial literature concerns which model is most appropriate to forecast the asset returns volatility, particularly as the forecasting time horizon extends. It is well-known that investors are mainly interested in calculating VaR and forecasting volatility. In this direction, the issue of choosing one superior model among all the potential models for all cases is complicated enough, because research results are confusing and conflicting. This is due to the fact that there is no specific model that is deemed adequate for all financial datasets, sample frequencies and applications.

This paper examined whether an intra-day or an inter-day model generates the most accurate risk forecasts for different datasets, among the 3 different asset categories; stock indices (S&P500, EurostoXX50, FTSE100), commodities (Copper, Silver, Gold) and foreign exchange rates of dollar (EUR/USD, GBP/USD, CAD/USD). We employed the GARCH-skT and the HAR-RV-skT models, both under the skewed Student-t distribution. The data used capture a time horizon from January 2000 to August 2015.

The results suggest that the framework to forecast daily volatility based on intra-day volatility measures does not seem to outperform the VaR measure estimated by an inter-day model for both 10-steps-ahead and 20-steps-ahead forecasting horizons. In other words, the GARCH-skT model predicts more accurately and more effectively the losses of a portfolio when the time horizon of the estimation increases. This fact is in line with the literature, as a number of papers indicate that using intra-day data does not help when the criteria are based on daily frequency; see Angelidis and Degiannakis (2008). The empirical results of this study provide evidence that the HAR-RV-skT model suffers from excessive VaR violations, implying an underestimation of market risk for most of the asset categories. The stock indices and the commodities were quite problematic, when an attempt is made to use a realized volatility model in order to forecast the VaR measures. This is illustrated by the fact that using the HAR-RV-skT model, there were many rejections of the null hypotheses of the Kupiec's and Christoffersen's tests as well as less accurate ES forecasts.

To summarize, the results indicate firstly that investors should be extremely careful when they use one model for all cases; there is not a unique risk model for all the cases. Secondly, from the empirical analysis, a new innovative inference has emerged; the choice of the GARCH-skT has been shown to produce reasonable multiple-days-ahead VaR and ES forecasts under the skewed Student-t distribution, and most importantly, across a variety of markets; stocks, commodities and exchange rates. Finally, as the literature indicates, the use of a skewed instead of a symmetrical distribution for the standardized residuals produces accurate VaR and ES forecasts. As a consequence, the effect of the intra-day noise in the daily basis datasets is still an open area of study and requires further investigation. Undoubtedly, the GARCH-skT specification is a safe model that predicts VaR and ES adequately at a 95% confidence level. The conditional volatility model provides adequate VaR and ES forecasts for medium-term and long-term periods. Regarding the 97.5% confidence level, suggested in the revised 2013 version of Basel III, the GARCH-skT specification provides accurate forecasts of the risk measures for stock indices and exchange rates, but not for commodities (i.e. Silver and Gold). In the case of a 99% confidence level, although the GARCH-skT model outperforms the HAR-RV-skT model, we do not achieve sufficiently accurate VaR forecasts for all the assets. Hence, the multi period-ahead *VaR* and *ES* forecasts are more accurate at the confidence level of 97.5% (as suggested in the revised version of Basel III in 2013) than at the confidence level of 99% (proposed in Basel II).

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Tables and Figures

Table 1: Descriptive statistics of the daily log returns.

Table 1: Descriptive Statistics								
Index	**Obs.**	**Mean**	**Median**	**Std. dev**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Probability**
Stock Indices								
S&P500	3901	0,021517	0,078196	1,238977	-0,049957	17,79049	26443,65	0,000000
EurostocXX50	3949	0,010520	0,065985	1,537287	-0,094972	10,26722	6493,768	0,000000
FTSE100	3912	0,013624	0,059112	1,299864	-0,125363	16,04128	20643,38	0,000000
Commodities								
HG (Copper COMEX)	3897	0,025732	0,04955	1,958289	-0,191981	6,414826	1425,380	0,000000
SV (Silver COMEX)	3897	0,028808	0,151172	2,221797	-1,041196	9,544504	5693,436	0,000000
GC (Gold COMEX)	3897	0,033317	0,045465	1,237483	-0,359605	8,295233	3447,038	0,000000
Foreign Exchange Rates								
EUR/USD (EC)	3898	-0,005012	0,007898	0,643137	-0,022133	4,655992	331,3706	0,000000
GBP/USD (BP)	3899	-0,005398	0,000000	0,60121	-0,594169	7,621704	2750,703	0,000000
CAD/USD (CD)	3899	-0,000644	0,010132	0,633168	-0,146147	5,666509	869,1815	0,000000

-The last column presents the *p*-values of the Jarque-Bera statistic for testing the null hypothesis that the log-returns series are normally distributed.

Table 2: Descriptive statistics of the annualized realized volatility $\sqrt{252\sigma_t^{2(RV)}}$.

Table 2: Descriptive Statistics						
Index	**Obs.**	**Mean**	**Median**	**Std. dev**	**Skewness**	**Kurtosis**
Stock Indices						
S&P500	3901	17.0	14.1	11.2	3.4	22.6
EurostocXX50	3949	22.7	19.3	12.8	2.8	16.4
FTSE100	3912	17.6	15.0	10.7	3.1	20.3
Commodities						
HG (Copper COMEX)	3897	25.0	21.9	13.6	2.4	12.7
SV (Silver COMEX)	3897	27.8	24.6	15.9	2.7	17.6
GC (Gold COMEX)	3897	16.6	14.8	8.6	2.6	15.5
Foreign Exchange Rates						
EUR/USD (EC)	3898	9.6	8.9	4.1	2.1	13.6
GBP/USD (BP)	3899	8.5	7.7	4.2	2.5	13.3
CAD/USD (CD)	3899	8.7	8.0	4.3	1.9	10.2

Table 3: The 10-trading-days-ahead $VaR_{t+10|t}^{(95\%)}$ and $ES_{t+10|t}^{(95\%)}$ modeling results.

Part A. AR(1)-GARCH-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p* -value**	**Independence *p* -value**
Stock Indices	**GARCH-skT**						
S&P500	290	-1.787855	-2.361605	**0.068746**	5.17%	0.894479	0.199893
EurostoXX50	294	-2.427851	-3.161031	**0.209316***	5.10%	0.945827	0.788741
FTSE100	291	-1.846542	-2.440472	**0.178061**	5.84%	0.522282	0.145521
Commodities	**GARCH-skT**						
Copper	289	-2.917354	-3.722252	**0.130557**	4.50%	0.682967	0.267495
Silver	289	-3.594869	-4.574125	**0.414*** **	6.57%	0.244433	0.506262
Gold	289	-1.952748	-2.484206	**0.097983**	8.30%	**0.018587**	**0.036584**
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	289	-1.017000	-1.285351	**0.004900**	4.50%	0.682967	0.119736
GBP/USD	289	-0.937435	-1.185187	0.007209	4.50%	0.681997	0.267495
CAD/USD	289	-0.967634	-1.230028	0.012442	5.19%	0.892335	0.801553
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p* -value**	**Independence *p* -value**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	290	-1.512188	-1.905477	0.155025	7.24%	0.100002	0.246049
EurostoXX50	294	-2.045223	-2.562995	0.261478	6.80%	0.181864	0.726299
FTSE100	291	-1.640489	-2.063329	0.193879	7.22%	0.103331	0.630244
Commodities	**AR(1)-HAR-RV**						
Copper	289	-2.655036	-3.339026	0.171435	4.50%	0.682967	0.604540
Silver	289	-3.029985	-3.807393	0.536055	8.65%	**0.009832**	0.898248
Gold	289	-1.724372	-2.173198	0.105001	9.00%	**0.005005**	**0.022996**
Foreign Exchange Rates	**AR(1)-HAR-RV**						
EUR/USD	289	-0.938610	-1.179771	0.007220	5.19%	0.891263	0.214108
GBP/USD	289	-0.886966	-1.111539	**0.003910**	4.15%	0.488874	0.306938
CAD/USD	289	-0.952570	-1.195246	**0.009237**	5.19%	0.892335	0.801553

-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Table 4: The 20-trading-days-ahead $VaR_{t+20|t}^{(95\%)}$ and $ES_{t+20|t}^{(95\%)}$ modeling results.

Index	Number of 20-step-ahead VaR forecasts	Average VaR	Average ES	MPSE of ES	Observed Exception Rate	Un.Cover. *p*-value	Independence *p*-value
Part A. AR(1)-GARCH-skT							
Stock Indices	**GARCH-skT**						
S&P500	145	-1.829135	-2.478850	**0.259767**	5.52%	0.77922	0.439753
EurostoXX50	147	-2.518926	-3.344469	**0.367677**	5.44%	0.81496	0.433228
FTSE100	145	-1.899795	-2.582968	0.181323	6.90%	0.32645	0.155397
Commodities	**GARCH-skT**						
Copper	144	-2.916934	-3.756540	**0.294519**	4.17%	0.625218	0.225305
Silver	144	-3.626941	-4.649445	**0.0552*** **	8.33%	0.096048	0.137871
Gold	144	-1.971987	-2.530160	**0.198717**	8.33%	0.096048	0.993925
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	144	-1.024463	-1.299905	**0.002834**	6.94%	0.319503	0.219876
GBP/USD	144	-0.941968	-1.196488	0.017590	4.17%	0.624566	0.468414
CAD/USD	144	-0.980165	-1.256920	0.028583	5.56%	0.777733	0.330049
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	145	-1.504066	-1.895696	0.378001	6.21%	0.520418	0.569563
EurostoXX50	147	-2.034986	-2.556575	0.516498	7.48%	**0.003450**	**0.003450**
FTSE100	145	-1.629913	-2.046791	**0.138419**	8.97%	**0.049006**	0.437259
Commodities	**AR(1)-HAR-RV**						
Copper	144	-2.632820	-3.308422	0.392020	6.25%	0.518486	0.573709
Silver	144	-3.016907	-3.788454	0.183810	10.42%	**0.009253**	0.060497
Gold	144	-1.712957	-2.159939	0.205898	10.42%	**0.009253**	0.589252
Foreign Exchange Rates	**AR(1)-HAR-RV**						
EUR/USD	144	-0.943720	-1.186161	0.002974	6.25%	0.518486	0.271359
GBP/USD	144	-0.884976	-1.110776	**0.013882**	4.17%	0.624566	0.468414
CAD/USD	144	-0.949854	-1.193339	**0.019583**	6.25%	0.519129	0.271359

-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Table 5: The 10-trading-days-ahead $VaR_{t+10|t}^{(97.5\%)}$ and $ES_{t+10|t}^{(97.5\%)}$ modeling results.

Part A. AR(1)-GARCH-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**GARCH-skT**						
S&P500	290	-2.192824	-2.753454	**0.036799**	3.45%	0.328057	0.397130
EurostoXX50	294	-2.951333	-3.658311	**0.170825**	3.40%	0.352163	0.400488
FTSE100	291	-2.261826	-2.830068	**0.1299****	4.12%	0.104629	0.308685
Commodities	**GARCH-skT**						
Copper	289	-3.527295	-4.258437	**0.104294**	3.11%	0.524335	0.446020
Silver	289	-4.339416	-5.249662	**0.3006****	5.19%	**0.010532**	0.801553
Gold	289	-2.356182	-2.845725	**0.05781***	4.84%	**0.023891**	0.231556
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	289	-1.21994	-1.464450	**0.002374**	3.80%	0.188799	0.058193
GBP/USD	289	-1.122077	-1.344835	0.002675	3.11%	0.524973	0.446020
CAD/USD	289	-1.167093	-1.408604	0.008090	3.11%	0.524973	0.268707
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	290	-1.811620	-2.168892	0.122851	4.14%	0.102218	0.307813
EurostoXX50	294	-2.437388	-2.907500	0.220939	4.42%	0.057680	0.271813
FTSE100	291	-1.965353	-2.344358	0.161455	5.15%	**0.011037**	0.796385
Commodities	**AR(1)-HAR-RV-skT**						
Copper	289	-3.180947	-3.793101	0.136560	3.11%	0.524335	0.446020
Silver	289	-3.632417	-4.332523	0.411749	6.23%	**0.000634**	0.898276
Gold	289	-2.064945	-2.469107	0.072253	6.92%	**0.000074**	0.083891
Foreign Exchange Rates	**AR(1)-HAR-RV-skT**						
EUR/USD	289	-1.123763	-1.337883	0.003950	4.15%	0.101568	0.085180
GBP/USD	289	-1.056135	-1.258188	**0.001848**	3.47%	0.327071	0.396282
CAD/USD	289	-1.135762	-1.355835	**0.005627**	3.81%	0.189142	0.423481

-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Table 6: The 20-trading-days-ahead $VaR_{t+20|t}^{(97.5\%)}$ and $ES_{t+20|t}^{(97.5\%)}$ modeling results.

Part A. AR(1)-GARCH-skT							
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**GARCH-skT**						
S&P500	145	-2.291955	-2.948917	**0.170892**	2.07%	0.731489	0.036982
EurostoXX50	147	-3.098111	-3.895092	**0.294993**	4.76%	0.119209	0.318466
FTSE100	145	-2.364697	-3.054988	**0.070721**	5.12%	0.065107	0.331836
Commodities	**GARCH-skT**						
Copper	144	-3.534591	-4.313155	**0.221611**	3.47%	0.487875	0.143264
Silver	144	-4.386254	-5.332723	**0.039495**	7.64%	**0.001500**	0.175484
Gold	144	-2.374719	-2.887545	**0.148162**	5.55%	**0.043868**	0.443083
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	144	-1.235764	-1.485197	**0.000358**	4.15%	0.246967	0.468414
GBP/USD	144	-1.132676	-1.367890	0.009845	3.48%	0.488318	0.547177
CAD/USD	144	-1.184546	-1.445543	0.023636	3.47%	0.488318	0.547177
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	145	-1.805158	-2.156589	0.322902	4.82%	0.110883	0.323405
EurostoXX50	147	-2.426085	-2.896581	0.460567	4.76%	0.119209	0.029425
FTSE100	145	-1.948956	-2.323501	0.098009	6.90%	**0.005279**	0.221608
Commodities	**AR(1)-HAR-RV-skT**						
Copper	144	-3.152608	-3.766134	0.313851	3.47%	0.487875	0.143264
Silver	144	-3.608807	-4.303347	0.093733	6.94%	**0.005081**	0.219876
Gold	144	-2.053109	-2.461961	0.155016	6.26%	**0.015690**	0.573709
Foreign Exchange Rates	**AR(1)-HAR-RV-skT**						
EUR/USD	144	-1.125901	-1.341671	0.001138	4.17%	0.246967	0.468414
GBP/USD	144	-1.056633	-1.260081	**0.007564**	2.78%	0.843881	0.631340
CAD/USD	144	-1.133243	-1.351256	**0.014412**	2.78%	0.843881	0.631340

-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Table 7: The 10-trading-days-ahead $VaR_{t+10|t}^{(99\%)}$ and $ES_{t+10|t}^{(99\%)}$ modeling results.

Part A. AR(1)-GARCH-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**GARCH-skT**						
S&P500	290	-2,713784	-3,263684	**0,015066**	1,03%	0,953674	0,801911
EurostoXX50	294	-3,609985	-4,295991	**0,132889**	1,02%	0,976256	0,803254
FTSE100	291	-2,799076	-3,361837	**0,107582**	3,09%	**0,004043**	0,447644
Commodities	**GARCH-skT**						
Copper	289	-4,239563	-4,914170	**0,071091**	1,73%	0,260592	0,674235
Silver	289	-5,193212	-6,033968	**0,212725**	3,46%	**0,001043**	0,396282
Gold	289	-2,825399	-3,266032	**0,035635**	3,08%	**0,000255**	0,349885
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	289	-1,459795	-1,675273	**0,000814**	2,77%	**0,013261**	0,498933
GBP/USD	289	-1,342207	-1,544417	0,002941	2,08%	0,109574	0,613341
CAD/USD	289	-1,399029	-1,617769	0,005508	1,73%	0,260869	0,064120
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p*-value**	**Independence *p*-value**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	290	-2,158911	-2,471835	0,089942	3,10%	**0,003940**	0,446834
EurostoXX50	294	-2,908882	-3,316504	0,188173	3,06%	**0,004407**	0,450053
FTSE100	291	-2,342712	-2,677232	0,127544	3,44%	**0,001086**	0,397975
Commodities	**AR(1)-HAR-RV**						
Copper	289	-3,791532	-4,334332	0,107054	2,42%	**0,040391**	0,554799
Silver	289	-4,311206	-4,928599	0,319464	4,84%	**0,000002**	0,701767
Gold	289	-2,460248	-2,812279	0,045025	4,50%	**0,000012**	0,267495
Foreign Exchange Rates	**AR(1)-HAR-RV**						
EUR/USD	289	-1,333491	-1,522888	0,002357	3,46%	**0,001046**	0,342256
GBP/USD	289	-1,258021	-1,436047	**0,000618**	2,08%	0,109574	0,613341
CAD/USD	289	-1,353754	-1,544206	**0,003655**	1,38%	0,538860	0,737114

-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Table 8: The 20-trading-days-ahead $VaR_{t+20|t}^{(99\%)}$ and $ES_{t+20|t}^{(99\%)}$ modeling results.

Part A. AR(1)-GARCH-skT							
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p* -value**	**Independence *p* -value**
Stock Indices	**GARCH-skT**						
S&P500	145	-2.884930	-3.542076	**0.092655**	2.07%	0.25827	**0.036982**
EurostoXX50	147	-3.817402	-4.624643	**0.211576**	1.36%	0.67993	0.813661
FTSE100	145	-2.993936	-3.685127	**0.036307**	2.76%	0.08114	0.632562
Commodities	**GARCH-skT**						
Copper	144	-4.282625	-5.025230	**0.149057**	3.47%	**0.020459**	0.143264
Silver	144	-5.265473	-6.177768	**0.01014****	3.47%	**0.020430**	0.547177
Gold	144	-2.880967	-3.354845	**0.097697**	3.47%	**0.020430**	0.143264
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD	144	-1.481211	-1.708530	0.000427	2.78%	0.079897	0.631340
GBP/USD	144	-1.363087	-1.575497	0.005683	2.08%	0.257866	0.719908
CAD/USD	144	-1.429585	-1.666978	0.018167	1.39%	0.663893	0.811726
Part B. AR(1)-HAR-RV							
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MPSE of ES**	**Observed Exception Rate**	**Un.Cover. *p* -value**	**Independence *p* -value**
Stock Indices	**AR(1)-HAR-RV**						
S&P500	145	-2.147815	-2.460266	0.275627	2.76%	0.080162	0.080050
EurostoXX50	147	-2.883045	-3.295188	0.398223	4.08%	**0.004857**	0.220042
FTSE100	145	-2.313073	-2.640589	0.068693	4.14%	**0.004551**	0.470031
Commodities	**AR(1)-HAR-RV**						
Copper	144	-3.750312	-4.279910	0.241818	3.47%	**0.020459**	0.143264
Silver	144	-4.293261	-4.895013	0.061314	6.25%	**0.000020**	0.271359
Gold	144	-2.445660	-2.797862	0.098816	4.17%	**0.004432**	0.225305
Foreign Exchange Rates	**AR(1)-HAR-RV**						
EUR/USD	144	-1.342047	-1.532244	**0.000273**	3.47%	**0.020459**	0.547177
GBP/USD	144	-1.255806	-1.429632	**0.003698**	1.39%	0.663893	0.811726
CAD/USD	144	-1.351755	-1.544205	**0.010931**	2.08%	0.257866	0.719908

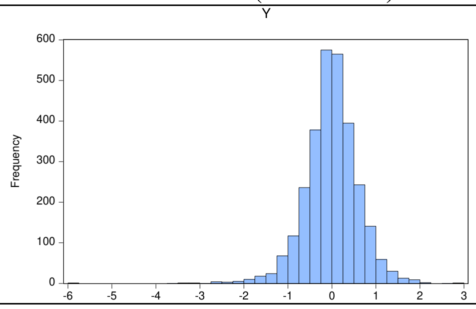
-The bold fonts of MPSE of ES loss function denote the lowest value between the two model frameworks.

- The asterisks (*,**,***) in MPSE of *ES* values indicates that according to the Diebold and Mariano statistic the alternative hypothesis that the AR(1)-GARCH(1,1)-skT model is of superior predictive ability is accepted at 1%,5% and 10% significance level, respectively.

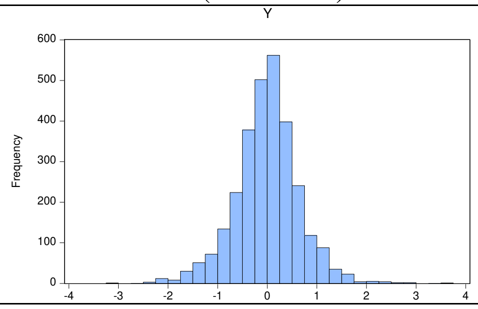
-The bold fonts in unconditional coverage *p*-value and independence *p*-value indicate the rejection of the null hypothesis that the model provides adequate *VaR* forecasts for 5% significance level.

Figure 1: The empirical distribution histograms of log-returns.

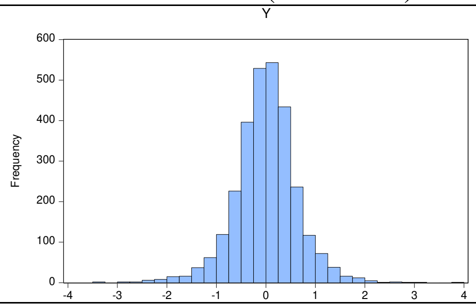
British Pound (GBP/USD)



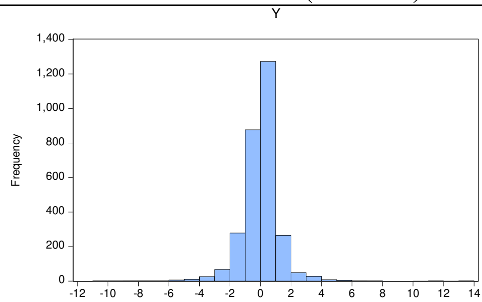
Euro (EUR/USD)



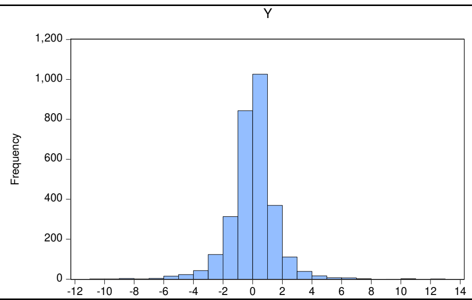
Canadian Dollar (CAD/USD)



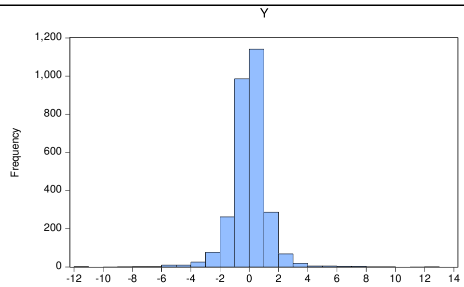
Standard and Poors (S&P500)



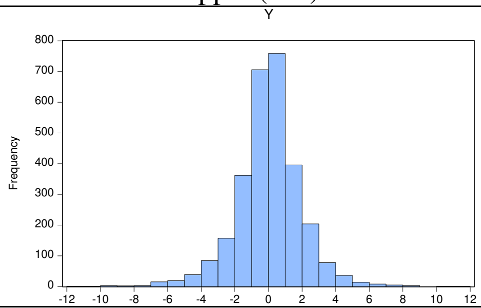
EurostoXX50



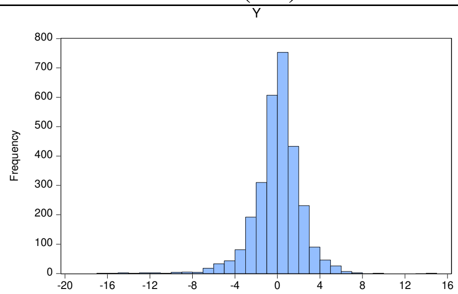
FTSE100



Copper (HG)



Silver (SV)



Gold (GC)

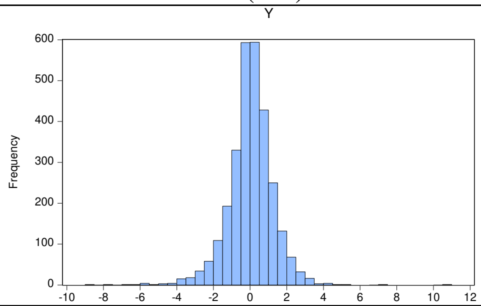
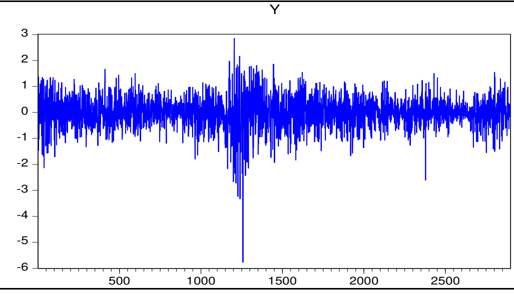
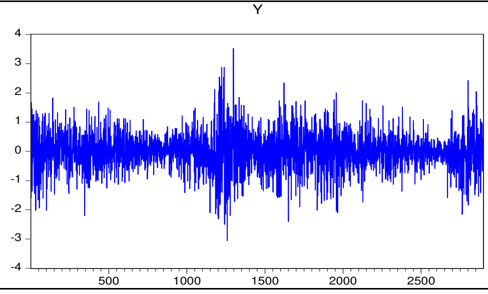


Figure 2: The daily log-returns.

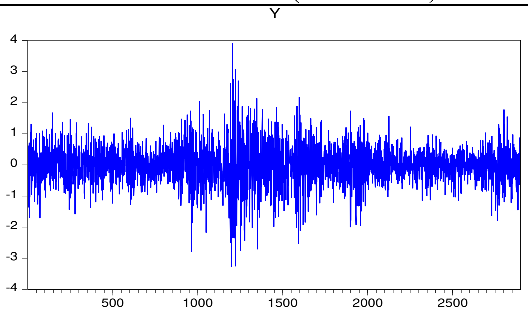
British Pound (GBP/USD)



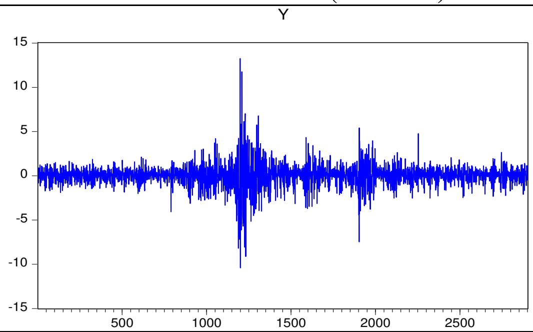
Euro (EUR/USD)



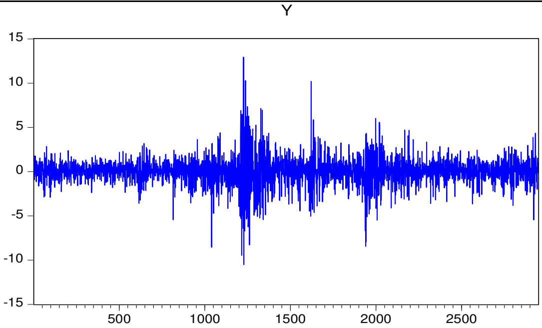
Canadian Dollar (CAD/USD)



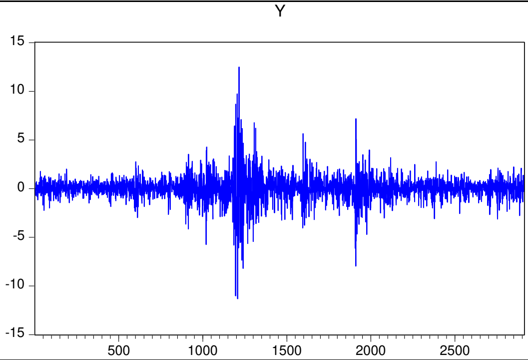
Standard and Poors (S&P500)



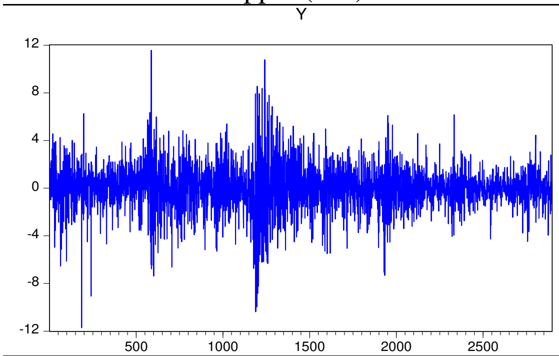
EurostoXX50



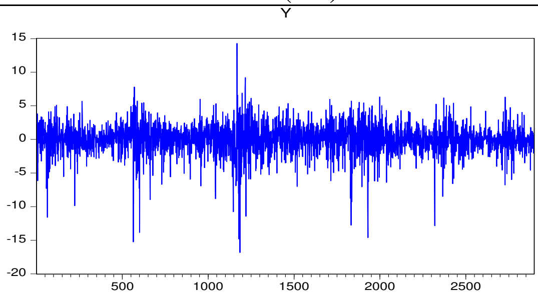
FTSE100



Copper (HG)



Silver (SV)



Gold (GC)

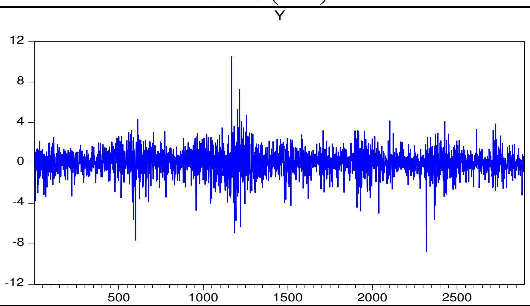
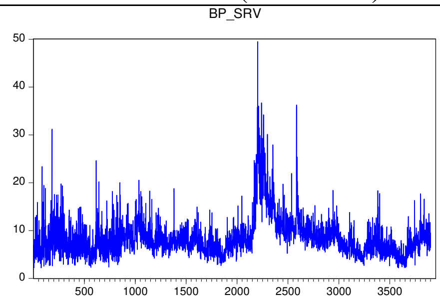
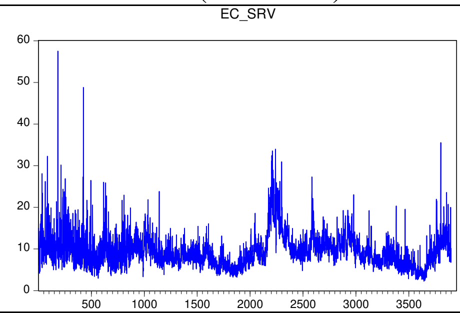


Figure 3: The daily annualized realized standard deviations, $\sqrt{252\sigma_t^{2(RV)}}$.

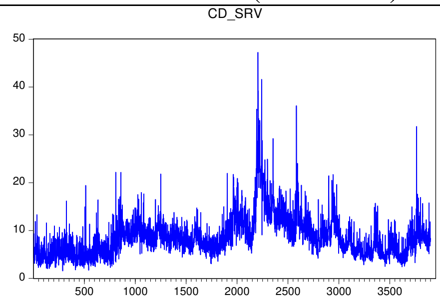
British Pound (GBP/USD)



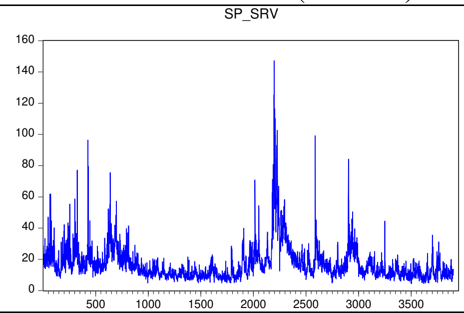
Euro (EUR/USD)



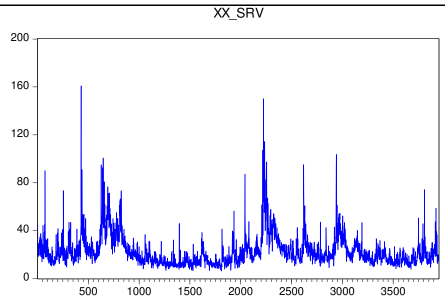
Canadian Dollar (CAD/USD)



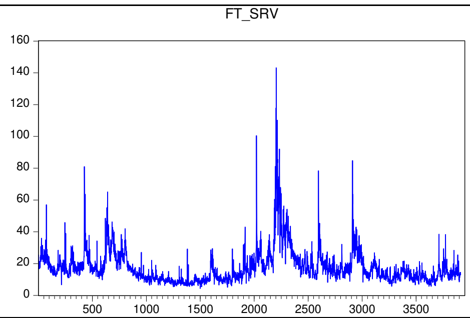
Standard and Poors (S&P500)



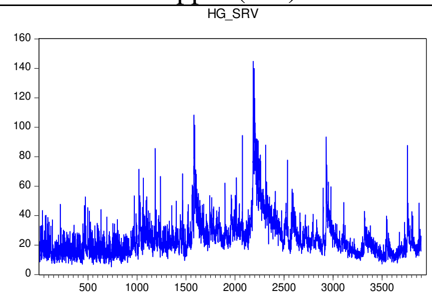
EurostoXX50



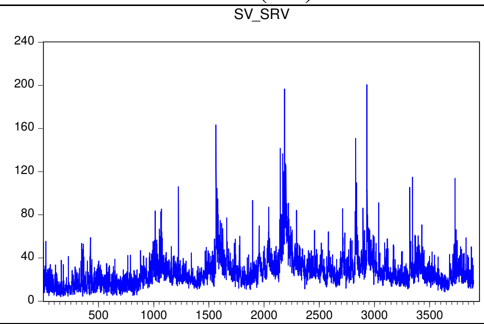
FTSE100



Copper (HG)



Silver (SV)



Gold (GC)

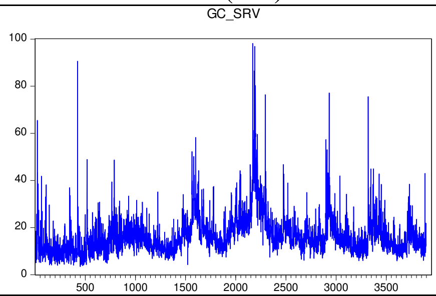


Figure 4: The 10-trading-days-ahead $VaR_{t+10|t}^{(95\%)}$ for EurostoXX50 (GARCH) and FTSE100 (HAR-RV).

EurostoXX50XX50 (GARCH)	FTSE100 (HAR-RV)

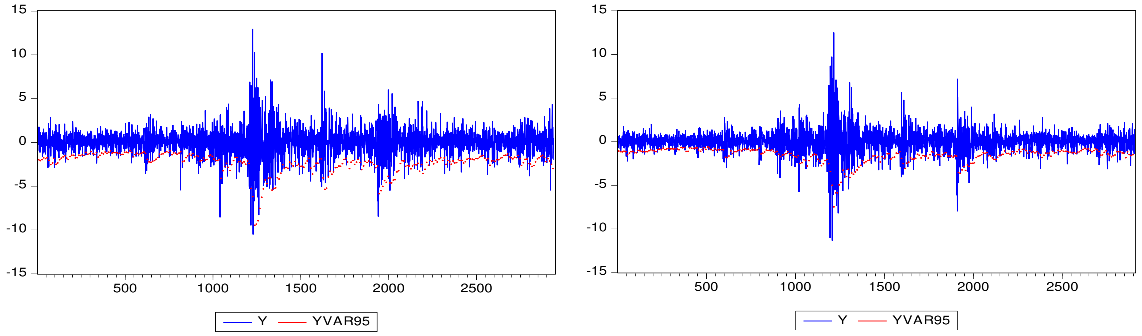


Figure 5: The 20-trading-days-ahead $VaR_{t+20|t}^{(95\%)}$ for Copper (GARCH) and Gold (HAR-RV).

Copper (GARCH)	Gold (HAR-RV)