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2010

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Abstract

This paper examines hedging in South African stock index futures market. The hedge ratios are estimated by six econometric techniques: the standard OLS regression, simple and vector error correction models, the ECM with generalised autoregressive heteroskedasticity (GARCH) as well as time-varying CCC-ARCH and Diag-BEKK ARCH models. The empirical results show that the ECM-GARCH model (capturing volatility clustering) provides best hedging ratios, while CCC-ARCH is superior to OLS, ECM and VECM. We conclude that there is not a unique model specification for measuring hedge ratios. For each market (emerging and mature), a model's comparative analysis must be conducted in order to extract the best performing model.

JEL Classification: G13, G15.

Keywords: Hedging, Hedge Ratio, Futures, SAFEX, OLS, ECM, VECM, GARCH.

I. INTRODUCTION AND THEORY

Hedging is the most important function of futures markets. It is concerned with the management of risk. Theory description of hedging include Working (1953), Johnson (1960), Stein (1961), Rutledge (1972), Ederington (1979) and Floros and Vougas (2004). In general, hedging is the action taken by a buyer or seller to protect his/her business or assets against a change in prices, see Floros and Vougas (2004). From the theoretical point of view, there are three goals of hedging: risk minimisation, profit maximisation, and the portfolio approach, see Rutledge (1972). Hedging is carried out to (i) eliminate risk due to adverse price fluctuation, (ii) reduce risk due to adverse price moves, (iii) profit from changes in the basis, and (iv) maximise expected return for a given risk and minimise risk for a stated return (Sutcliffe, 1993).

Stock index futures contracts can be used to hedge the risk. Hedging uses futures markets to reduce risk of a cash (spot) market position. According to Hull (2000, p. 66), when the relationship between the cash price and the price of a futures contract is very close, the hedge is more effective. However, because this relationship is usually not perfect (spot and futures positions do not move together), the hedge is a cross-hedge. In this case, the hedger should trade the right number of futures contract to control the risk. In other words, the determination of the optimal hedge ratio[1] (minimum variance hedge ratio, or MVHR) is required. MVHR is the optimal amount of futures bought or sold expressed as a proportion of the cash position. It is important for the hedger to be able to identify the number of contracts needed to hedge the portfolio. Thus, the hedge ratio (HR) will be used, so one can choose the right number of futures contracts minimising risk. The HR is the number of futures contracts bought, or sold, divided by the number of spot contracts whose risk is being hedged.

Several measures have been proposed for the HR computation. Usually, the HR is estimated from an OLS regression of cash on futures prices. The method is introduced by Ederington (1979), and Anderson and Danthine (1980). The slope coefficient of the OLS regression is the MVHR, which is constant over time. An alternative estimation of the optimal HR is based on the phenomenon that cash and futures prices display volatility clustering, and, hence, GARCH models are to be

[1] Optimal hedge ratio is derived from the optimisation of certain objective functions, such as minimising the variance of the return from a hedged portfolio or maximising a function of the portfolio return that is related to the expected utility level (Lien and Shrestha, p. 627, 2010).

preferred. These models are used for estimating heteroscedastic optimal hedge ratios, see Cecchetti *et al*. (1988) and Floros and Vougas (2004). Furthermore, several studies use error correction models (ECM) to estimate the hedge ratios, see Chou *et al*. (1996) and Lien (1996). Other papers use error correction terms with a time-varying risk structure when analysing the spot-futures relationship, see Kroner and Sultan (1993), and Lien and Tse (1999). According to Lee and Chien (2010), various econometric models give different conclusions when estimate HR. Miffre (2004) shows that the conditional OLS model outperforms the OLS and GARCH models, while Alexander and Barbosa (2007) find no evidence that time-varying conditional covariance and ECM can improve upon the OLS hedge ratio. Recently, Hsu et al. (2008) suggest that copula-based GARCH models perform more effectively than OLS, CCC-GARCH and DCC-GARCH models (for more information about the performance of various econometric models for HR estimation see Lee and Chien, 2010).

Lien and Zhang (2008) summarise theoretical and empirical research on the roles and functions of emerging derivatives markets and report mixed results. The present paper focuses on model specification and empirical comparison of several models for HR estimation using data from an emerging market; the South African futures market (FTSE/JSE 40 Index). The traditional regression model OLS, the ECM, the VECM, and the ECM-GARCH models as well as the dynamic CCC-ARCH and Diag-BEKK ARCH models are employed. According to authors' knowledge this is the only empirical investigation using data from South African futures market.

The manuscript is organised as follows: Section II shows an overview of econometric models employed for estimating hedge ratios. Section III describes the data, and Section IV presents the empirical results from the econometric models. Section V discusses a comparison between models and Section VI concludes the paper and summarises our findings.

II. METHODOLOGY

The futures hedge ratios are mainly calculating via the OLS regression model. Butterworth and Holmes (2000) estimated the (ex post) MVHR using OLS, by regressing the first order log difference in the spot prices, S_t, against the first order log difference in the futures prices, F_t, or:

$$\Delta S_t = c + b\Delta F_t + u_t$$
$$u_t \sim N(0,\sigma^2), \tag{1}$$

where $\Delta S_t = \log S_t - \log S_{t-1}$, $\Delta F_t = \log F_t - \log F_{t-1}$. The coefficient b is the hedge ratio[2].

Nevertheless, the former model specification assumes the absence of auto-correlation and heteroskedasticity in log-returns. There is substantial evidence in financial literature to suggest that financial time series do not comply with the assumption of uncorrelated and homoskedastic returns to financial instruments. Chou *et al*. (1996), following the method proposed by Engle and Granger (1987), estimated the hedge ratio, using an error correction model (ECM). Assuming the series are co-integrated, there exists an ECM of the form:

$$\Delta S_t = c + a\hat{\varepsilon}_{t-1} + b\Delta F_t + \theta_1 \Delta F_{t-1} + \phi_1 \Delta S_{t-1} + u_t,$$
$$u_t \sim N(0,\sigma^2), \tag{2}$$

where $\hat{\varepsilon}_{t-1} = S_{t-1} - \left(\hat{c}_0 + \hat{b}_0 F_{t-1}\right)$. The coefficient b is the hedge ratio.

In addition, one may use ECM with time varying terms in the variance equation (GARCH errors), or:

$$\Delta S_t = c + a\hat{\varepsilon}_{t-1} + b\Delta F_t + \theta_1 \Delta F_{t-1} + \phi_1 \Delta S_{t-1} + u_t,$$
$$u_t = \sigma_t z_t$$
$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3}$$
$$z_t \sim N(0,1)$$

An iterative procedure is used based upon the method of Marquardt algorithm. Heteroskedasticity Consistent Covariance option is used to compute quasi-maximum likelihood covariances and standard errors using the methods described by Bollerslev and Wooldridge (1992). This is normally used if the residuals are not conditionally normally distributed (for more details about GARCH models see Xekalaki and Degiannakis, 2010).

[2] For all the models, the Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors are computed, as they are consistent estimators in the presence of both heteroskedasticity and autocorrelation of unknown form.

Ghosh (1993) and Lien (1996) calculated the optimal hedge ratio using a VECM specification:

$$\Delta S_t = a_S \hat{\varepsilon}_{t-1} + \theta_{11}\Delta F_{t-1} + \phi_{11}\Delta S_{t-1} + u_{S,t}\,, \tag{4}$$

$$\Delta F_t = a_F \hat{\varepsilon}_{t-1} + \theta_{12}\Delta F_{t-1} + \phi_{12}\Delta S_{t-1} + u_{F,t}\,, \tag{5}$$

where $\begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_S^2 & \sigma_{S,F} \\ \sigma_{S,F} & \sigma_F^2 \end{pmatrix}\right)$. The hedge ratio is calculated as $\rho\sigma_S\sigma_F^{-1}$, where $\rho = \frac{\sigma_{S,F}}{\sqrt{\sigma_S^2\sigma_F^2}}$ is the correlation coefficient between $u_{S,t}$ and $u_{F,t}$, and σ_S and σ_F are the standard deviations of $u_{S,t}$ and $u_{F,t}$, respectively.

Moreover, we proceed to the estimation of the optimal hedge ratio using two bivariate ARCH specifications. An ARCH system of 2 regression equations is defined:

$$\begin{gathered} \mathbf{y}_t = \mathbf{B}'\mathbf{x}_t + \mathbf{u}_t \\ \mathbf{u}_t \mid I_{t-1} \sim N[\mathbf{0}, \mathbf{H}_t] \\ \mathbf{H}_t = g(\mathbf{H}_{t-1}, \mathbf{H}_{t-2}, \ldots, \mathbf{u}_{t-1}, \mathbf{u}_{t-2}, \ldots), \end{gathered} \tag{6}$$

where

$$\mathbf{y}_t = \mathbf{B}'\mathbf{x}_t + \mathbf{u}_t \Leftrightarrow \begin{pmatrix} \Delta S_t \\ \Delta F_t \end{pmatrix} = \begin{pmatrix} a_S & \theta_{11} & \phi_{11} \\ a_F & \theta_{12} & \phi_{12} \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix}, \tag{7}$$

I_{t-1} is the available information set, $N[0, \mathbf{H}_t]$ is the bivariate normal distribution with $E(\mathbf{u}_t) = \mathbf{0}$, conditional mean, and $V(\mathbf{u}_t) = \mathbf{H}_t$, conditional variance, respectively.

We are based on two successfully applied versions of the bivariate ARCH process; Bollerslev's (1990) constant conditional correlation, or CCC-ARCH, model, and Baba's et al. and Engle and Kroner's (1995) diagonal BEKK, or Diag-BEKK ARCH, model.

In the CCC-ARCH model the conditional variance of $\mathbf{u}_t$ is decomposed as:

$$\mathbf{H}_t = \mathbf{\Sigma}_t^{1/2}\mathbf{C}\mathbf{\Sigma}_t^{1/2}, \tag{8}$$

where $\mathbf{\Sigma}_t^{1/2} = \begin{bmatrix} \sigma_{S,t} & 0 \\ 0 & \sigma_{F,t} \end{bmatrix}$ is the diagonal matrix with the conditional standard deviations along the diagonal, and $\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ is the matrix of conditional correlations. The conditional standard deviations are computed as univariate GARCH(1,1) models:

$$\sigma_{S,t}^2 = a_{1,0} + \alpha_{1,1} u_{S,t-1}^2 + \beta_{1,1} \sigma_{S,t-1}^2 , \tag{9}$$

$$\sigma_{F,t}^2 = a_{2,0} + \alpha_{2,1} u_{F,t-1}^2 + \beta_{2,1} \sigma_{F,t-1}^2 , \tag{10}$$

and the conditional covariance is computed as:

$$\sigma_{S,F,t} = \rho \sigma_{S,t} \sigma_{F,t} . \tag{11}$$

The hedge ratio is calculated as $\frac{\sigma_{S,F,t}}{\sigma_{F,t}^2} = \rho \frac{\sigma_{S,t}}{\sigma_{F,t}}$.

In the Diag-BEKK ARCH model the conditional variance of $\mathbf{u}_t$ is decomposed as:

$$\mathbf{H}_t = \mathbf{A_0 A_0'} + \mathbf{A}_1 \mathbf{u}_{t-1} \mathbf{u}_{t-1}' \mathbf{A}_1' + \mathbf{B}_1 \mathbf{H}_{t-1} \mathbf{B}_1' = \begin{bmatrix} \sigma_{S,t}^2 & \sigma_{S,F,t} \\ \sigma_{S,F,t} & \sigma_{F,t}^2 \end{bmatrix} . \tag{12}$$

The conditional variances are computed as GARCH(1,1) models in forms follow:

$$\sigma_{S,t}^2 = a_{0,1,1} + a_{1,1,1} u_{S,t-1}^2 a_{1,1,1} + \beta_{1,1,1} \sigma_{S,t-1}^2 \beta_{1,1,1} , \tag{13}$$

$$\sigma_{F,t}^2 = a_{0,2,2} + a_{1,2,2} u_{F,t-1}^2 a_{1,2,2} + \beta_{1,2,2} \sigma_{F,t-1}^2 \beta_{1,2,2} , \tag{14}$$

and the conditional covariance is computed as:

$$\sigma_{S,F,t} = a_{0,1,2} + a_{1,1,1} u_{S,t-1} u_{F,t-1} a_{1,2,2} + \beta_{1,1,1} \sigma_{S,F,t-1} \beta_{1,2,2} . \tag{15}$$

The hedge ratio is calculated as $\frac{\sigma_{S,F,t}}{\sigma_{F,t}^2}$. For technical details about the aforementioned bivariate ARCH models the interested reader is referred to Xekalaki and Degiannakis (2010).

III. DATA DESCRIPTION

This study employs 1043 trading days on the FTSE/JSE Top 40 stock index and stock index futures contract for the period 2 January 2002 to 28 February 2006. Closing prices for the spot index were obtained from *DataStream International*, while closing futures prices were obtained from the official webpage of the South African Futures Exchange, or SAFEX (http://www.safex.co.za).

FTSE/JSE Top 40 stock index consists of the largest 40 companies ranked by full market capitalisation (value) that is before the application of any weighting in the All Share Index. The futures contract is the FTSE/JSE's Top 40 future nearest to expiration, assuming a rollover to the next contract expiration. Analysis is confined to the nearby contract because almost all trading volume is in the near month so that liquidity is much great in that contract compared with the far contract.

The futures contracts are quoted in the same units (South African Rand) as the underlying index without decimals, with the price of a futures contract or contract size being the quoted number (index level) multiplied by the contract multiplier, which is R10 for the contract. Futures expiry months are March, June, September and December. The stock index futures contract is cash-settled and marked to market on the last trading day, which is at 15:40 South African time on the third Thursday in the delivery or expiration month. The formal futures exchange was established in 1988 as well as the SAFEX clearing company. For more details about the South African market, see Motsa (2006) and Floros (2009).

Figures 1 and 2 present the plots of logarithmic FTSE/JSE Top 40 stock index and stock index futures, respectively. Figures 3 and 4 show the behaviour of returns of both indices over time, indicating volatility clustering or pooling in FTSE/JSE Top 40 spot and futures returns.

<< Figure 1 about here >>

<< Figure 2 about here >>

<< Figure 3 about here >>

<< Figure 4 about here >>

IV. EMPIRICAL RESULTS

First, we apply unit root tests for log-stock prices and log-futures prices for FTSE/JSE Top 40. Augmented Dickey and Fuller (1979), or ADF test statistic, and Phillips and Perron (1988), or PP test statistic, indicate that both series are I(1). Cointegration are used to confirm whether there exists such a cointegrating structure between spot and futures markets. Johansen's (1988, 2004) approach suggests that spot and futures are cointegrated, with one cointegration relationship[3]. Thus, there exists a linear combination of the South African spot and futures prices.

A. The Conventional Approach - OLS Regression

The optimal hedge ratio can be derived from the regression in equation (1), where the returns to holding spot asset are regressed on the returns to holding the hedging instruments. Table 1 presents the results for FTSE/JSE Top 40 index. The hedge ratio is 0.9043, and it is significantly less than unity.

[3] The results obtained from the ADF, PP and Johansen tests are available upon request.

<< Table 1 about here >>

B. An Error Correction Approach

In our case, *S* and *F* are cointegrated, and therefore, the optimal hedge ratio can be calculated from an error correction model, see equation (2). We apply an ECM to obtain alternative estimates for the hedge ratio, so we can compare them with the ones obtained from the conventional method. The results are reported in Table 2.

<< Table 2 about here >>

The results show a hedge ratio of 0.9150 for FTSE/JSE Top 40. The hedge ratio coefficient in the hedge equation (ECM) is significantly less than unity at any level of significance. Comparing estimated hedge ratio, we conclude that the hedge ratio estimated by equation (2) is greater than the one estimated by equation (1). This implies that the conventional model under-estimates the number of futures contracts needed to hedge the spot portfolio. This is not in line with Floros and Vougas (2004), who provided evidence that in the case of the Greek stock market the FTSE/ASE-20 hedge ratio estimated by the ECM is less than that obtained from the OLS method.

C. An Error Correction Approach with GARCH Errors

The ECM specification, in equation (3), is also taken into consideration under the assumption of time varying conditional variance. The coefficient *b* equals to 0.9212.

<< Table 3 about here >>

D. A Vector Error Correction Approach

If spot and futures prices are cointegrated, we can use a Vector Error Correction Model (VECM) to estimate hedge ratio. The hedge ratio is calculated as $h = \frac{\sigma_{S,F}}{\sigma_F^2}$. From equations (4) and (5), $\sigma_{S,F}$ is the covariance coefficient between the innovations $u_{S,t}$ and $u_{F,t}$, and σ_S and σ_F are the standard deviations of $u_{S,t}$ and $u_{F,t}$, respectively. Thus, the hedge ratio from VECM is calculated as $h = \frac{\sigma_{S,F}}{\sigma_F^2} = \frac{0.000128}{0.000140} = 0.9143$. The estimation of the model is presented in Table 4. The hedge ratio estimated from VECM, is close to the one obtained from ECM. In the studies of Ghosh (1993) and Floros and Vougas (2004) the hedge ratios estimated from VECM were greater than the ones obtained from OLS and ECM specifications.

<< Table 4 about here >>

E. The CCC-ARCH Model

The fifth model for estimating hedge ratio is by employing VECM model with time varying conditional variances and covariance. To incorporate both short- and long-run information of data, we model the mean equation (first moment) with an error correction model, and in addition, we take into account heteroscedastic variances and covariances (to capture volatility clustering), by modelling the conditional variance matrix with Bollerslev's (1990) constant conditional correlation framework.

Table 5 reports the results from CCC-ARCH model. Figure 5 plots the hedge ratios across time. The average hedge ratio is 0.9169 for FTSE/JSE Top 40 index.

<< Table 5 about here >>

<< Figure 5 about here >>

F. The Diag-BEKK ARCH Model

The last model for estimating hedge ratio is the DIAG-BEKK ARCH model. Table 6 reports the estimation of the Diag-BEKK ARCH model. Figure 6 plots the hedge ratios across time. The average hedge ratio is 0.9074 for FTSE/JSE Top 40 index. Hence, the hedge ratio estimated by CCC-ARCH model is greater than the one obtained from that model. So, the hedge ratio estimated by CCC-ARCH model should be more efficient in reducing risk of spot prices.

<< Table 6 about here >>

<< Figure 6 about here >>

V. MODELS COMPARISON

Table 7 shows the hedge ratios estimated from the six econometric models. The hedge ratio estimated by the ECM-GARCH model performs better in terms of hedging. It is greater than the ones obtained from OLS, ECM and VECM. Hence, hedgers need more futures contracts to reduce the market risk of their cash portfolios (their losses are going to be reduced substantially). So, the hedge ratio estimated by ECM-GARCH model should be more efficient in reducing risk of spot prices. Furthermore, this implies that all other constant models (OLS, ECM and VECM) under-estimate the number of futures contracts needed to hedge spot prices. Therefore, hedge ratio estimated by ECM-GARCH significantly improves hedging. Our results show that it is superior to hedge ratios obtained from all other constant models, and therefore this hedge ratio should provide better hedging (ECM-GARCH performs well in terms of variance reduction).

Furthermore, we find that the dynamic hedge ratios obtained from the CCC-ARCH and Diag-BEKK ARCH models have a sample mean less than unity, but greater than the constant hedge ratios obtained from the traditional OLS and ECM. In particular, the hedge ratio estimated from CCC-GARCH is greater than those from OLS, ECM and VECM, while the hedge ratio obtained from the Diag-BEKK ARCH is greater than that from the simple OLS and ECM. These estimates suggest that the naive 1:1 hedging strategy is inappropriate; this is in line with Yang and Allen (2004). We should note that the hedge ratio series obtained from CCC-ARCH and Diag-BEKK ARCH are time varying hedge ratios, which, in turn, incorporates a time-varying conditional correlation coefficient between the spot and futures prices and, hence, generates more realistic time-varying hedge ratios (Yang and Allen, 2004). Even though the sample mean hedge ratio form the Diag-BEKK ARCH model is smaller in magnitude from the ones obtained from the traditional constant models (ECM, VECM and ECM-GARCH), the average time-varying hedge ratio is just an indicative figure which doesn't represent the actual hedge ratios from all time periods. The fact that time-varying models capture time-varying hedge ratios with success, shows that these dynamic models are somewhat superior to the traditional models (in particular to the OLS, ECM and VECM). Similarly, the mean of the time-varying CCC-GARCH hedge ratio is larger than those derived from the simple OLS, the ECM and the VECM, which means that the constant hedge ratios would lead to a smaller than optimal hedging position (Sim and Zurbruegg, 2001).

<< Table 7 about here >>

VI. SUMMARY AND CONCLUSIONS

Futures contracts can be a very effective risk management instrument due to its high liquidity and low transaction cost (Lien and Shrestha, 2010). When using stock index futures for hedging (a technique to minimise risk), we require estimates of the so-called hedge ratio. Various approaches for risk minimisation lead to different estimation approaches and conclusions for the (optimal) hedge ratio. According to Ghosh (p. 751, 1993), "*Underestimating the optimal hedge ratio results in a suboptimal hedge of the cash portfolio. Improved optimal hedge ratios appear to reduce considerably the risk of the risk minimizing portfolio. This means loss from a* *suboptimal hedge is significantly reduced and helps to reduce the impact of the costs of hedging*".

In this paper, we focus on model specification and empirical comparison for (optimal) hedge ratio estimation using data from South African futures market (an emerging market). We examine the behaviour of futures prices from FTSE/JSE Top 40 index by employing six econometric methods, which include: the traditional OLS regression model, ECM, ECM-GARCH, VECM, CCC-ARCH, and Diag-BEKK ARCH models. The empirical results show that GARCH framework is superior to traditional hedging models (OLS, ECM and VECM). It is found that the traditional models underestimate the number of futures contracts, needed to hedge the spot portfolio. In other words, portfolio managers can incur significant loss by using traditional (constant) models.

In particular, we find that hedge ratio, estimated by ECM-GARCH, is greater than the hedge ratio estimated by the other methods. We show that the FTSE/JSE Top 40 index hedge ratio estimated by ECM-GARCH significantly improves hedging. It is superior to the other models implying better hedging; therefore, the hedge ratio derived from ECM-GARCH is more effective in controlling and reducing risk of the cash portfolio (Ghosh and Clayton, 1996). This indicates that a financial analyst or trader whose portfolio includes the South Africa stock market should select the optimal spot portfolio to be hedged and minimise risk exposure by estimating the ECM-GARCH model.

This hedge ratio is more efficient than those estimated by all other techniques; we also confirm that the constant hedge ratio derived from OLS is unable to recognise the trend in the spot and futures changes (Park and Switzer, 1995). The ECM-GARCH performs better than the other hedge ratios in terms of capturing conditional variances (spot and futures changes). We show that hedgers in South African stock index futures are able to estimate the number of futures contracts needed using an ECM-GARCH model to reduce losses as well as overall costs of hedging.

However, we should note that the hedging strategy using the time-varying model (CCC-ARCH) is superior to the traditional methods (OLS, ECM and VECM). This is in line with Park and Switzer (1995). In particular, the mean of the time-varying hedge ratios is larger than that derived from the simple models above, which means that the simple hedge ratios would lead to a smaller than optimal hedging position (Sim and Zurbruegg, 2001).

Hence, we reach to a different conclusion in comparison to other similar studies; e.g. the Greek emerging market, see Floros and Vougas, 2004. Therefore, there is not a unique model specification for all the markets. For each market (emerging and mature), a model's comparative analysis must be conducted in order to extract the best performing model.

Future research should evaluate the hedging effectiveness of the constant and time-varying hedge ratios, measured in terms of ex-ante and ex-post risk-return trade-off at various forecasting horizons, for several emerging markets across geographies and regions.

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TABLES AND FIGURES

Table 1. Estimated parameters of the regression model in equation (1). The coefficient to standard error ratios are reported in brackets.

$$\Delta S_t = c + b\Delta F_t + u_t = 0.00005[0.72] + 0.9043[63.44]\Delta F_t + u_t .$$

Table 2. Estimated parameters of the regression model in equation (1). The coefficient to standard error ratios are reported in brackets.

$$\Delta S_t = c + a\hat{\varepsilon}_{t-1} + b\Delta F_t + \theta_1 \Delta F_{t-1} + \phi_1 \Delta S_{t-1} + u_t =$$
$$= 0.00003[0.34] - 0.00001[-4.71]\hat{\varepsilon}_{t-1} + 0.915[74.80]\Delta F_t + 0.23[4.51]\Delta F_{t-1} - 0.21[-3.72.]\Delta S_{t-1} + u_t ,$$

where $\hat{\varepsilon}_{t-1} = S_{t-1} - \left(\hat{c}_0 + \hat{b}_0 F_{t-1}\right) = S_{t-1} - \left(-75.97[-5.30] + 1.002[784.3]F_{t-1}\right)$.

Table 3. Estimated parameters of the regression model in equation (1). The coefficient to standard error ratios are reported in brackets.

$$\Delta S_t = c + a\hat{\varepsilon}_{t-1} + b\Delta F_t + \theta_1 \Delta F_{t-1} + \phi_1 \Delta S_{t-1} + u_t =$$
$$= 0.00018[1.35] - 0.00002[-3.44]\hat{\varepsilon}_{t-1} + 0.921[98.34]\Delta F_t + 0.24[5.43]\Delta F_{t-1} - 0.25[-5.50]\Delta S_{t-1} + u_t ,$$

$$\sigma_t^2 = a_0 + a_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 = 6.5E-09[0.14] + 0.018[1.80]u_{t-1}^2 + 0.98[109.6.]\sigma_{t-1}^2$$

where $\hat{\varepsilon}_{t-1} = S_{t-1} - \left(\hat{c}_0 + \hat{b}_0 F_{t-1}\right) = S_{t-1} - \left(-75.97[-5.30] + 1.002[784.3]F_{t-1}\right)$.

Table 4. Estimated parameters of the CCC-ARCH model. The coefficient to standard error ratios are reported in brackets.

$$\mathbf{y}_t = \begin{pmatrix} \Delta S_t \\ \Delta F_t \end{pmatrix} = \begin{pmatrix} a_S & \theta_{11} & \phi_{11} \\ a_F & \theta_{12} & \phi_{12} \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \mathbf{u}_t = \begin{pmatrix} 0.000014[2.07] & 0.283[2.82] & -0.22[-2.09] \\ 0.00003[4.42] & 0.057[0.54] & -0.013[-0.12] \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix}$$

$$\begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.000128 & 0.000128 \\ 0.000128 & 0.000140 \end{pmatrix} \right)$$

Table 5. Estimated parameters of the CCC-ARCH model. The coefficient to standard error ratios are reported in brackets.

$$\mathbf{y}_t = \begin{pmatrix} \Delta S_t \\ \Delta F_t \end{pmatrix} = \begin{pmatrix} a_S & \theta_{11} & \phi_{11} \\ a_F & \theta_{12} & \phi_{12} \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \mathbf{u}_t = \begin{pmatrix} 0.00005[2.64] & 0.315[2.08] & -0.25[-1.59] \\ 0.00006[3.58] & 0.092[0.62] & -0.044[-0.28] \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix}$$

$$vech(\mathbf{H}_t) = \begin{pmatrix} \sigma^2_{S,t} \\ \sigma_{S,F,t} \\ \sigma^2_{F,t} \end{pmatrix} = \begin{pmatrix} a_{1,0} + \alpha_{1,1} u^2_{S,t-1} + \beta_{1,1} \sigma^2_{S,t-1} \\ \rho \sigma_{S,t} \sigma_{F,t} \\ a_{2,0} + \alpha_{2,1} u^2_{F,t-1} + \beta_{2,1} \sigma^2_{F,t-1} \end{pmatrix} = \begin{pmatrix} \underset{[1.22]}{9.6E-07} + \underset{[5.32]}{0.044}\; u^2_{S,t-1} + \underset{[83.6]}{0.95}\; \sigma^2_{S,t-1} \\ \underset{[171.7]}{0.957}\; \sigma_{S,t}\sigma_{F,t} \\ \underset{[1.30]}{9.1E-07} + \underset{[5.02]}{0.038}\; u^2_{F,t-1} + \underset{[98.0]}{0.96}\; \sigma^2_{F,t-1} \end{pmatrix}$$

Table 6. Estimated parameters of the Diag-BEKK ARCH model. The coefficient to standard error ratios are reported in brackets.

$$\mathbf{y}_t = \begin{pmatrix} \Delta S_t \\ \Delta F_t \end{pmatrix} = \begin{pmatrix} a_S & \theta_{11} & \phi_{11} \\ a_F & \theta_{12} & \phi_{12} \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \mathbf{u}_t = \begin{pmatrix} 0.000008[1.26] & 0.221[2.21] & -0.145[-1.39] \\ 0.00003[5.09] & -0.007[-0.07] & 0.084[0.81] \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_{t-1} \\ \Delta F_{t-1} \\ \Delta S_{t-1} \end{pmatrix} + \begin{pmatrix} u_{S,t} \\ u_{F,t} \end{pmatrix}$$

$$vech(\mathbf{H}_t) = \begin{pmatrix} \sigma^2_{S,t} \\ \sigma_{S,F,t} \\ \sigma^2_{F,t} \end{pmatrix} = \begin{pmatrix} a_{0,1,1} + a_{1,1,1} u^2_{S,t-1} a_{1,1,1} + \beta_{1,1,1} \sigma^2_{S,t-1} \beta_{1,1,1} \\ a_{0,1,2} + a_{1,1,1} u_{S,t-1} u_{F,t-1} a_{1,2,2} + \beta_{1,1,1} \sigma_{S,F,t-1} \beta_{1,2,2} \\ a_{0,2,2} + a_{1,2,2} u^2_{F,t-1} a_{1,2,2} + \beta_{1,2,2} \sigma^2_{F,t-1} \beta_{1,2,2} \end{pmatrix} =$$

$$= \begin{pmatrix} \underset{[2.57]}{2.5E-06} + \underset{[9.08]}{0.207}\; u^2_{S,t-1} 0.207 + \underset{[144.5]}{0.968}\; \sigma^2_{S,t-1} 0.968 \\ \underset{[2.33]}{2.0E-06} + 0.207 u_{S,t-1} u_{F,t-1} 0.185 + 0.968 \sigma_{S,F,t-1} 0.978 \\ \underset{[2.11]}{1.6E-06} + \underset{[6.40]}{0.185}\; u^2_{F,t-1} 0.185 + \underset{[186.8]}{0.978}\; \sigma^2_{F,t-1} 0.978 \end{pmatrix}$$

Table 7. Hedge ratios estimated from the six models.

OLS	0.9043
ECM	0.9150
ECM-GARCH	0.9212
VECM	0.9143
CCC-ARCH	0.9169
Diag-BEKK ARCH	0.9074

Figure 1: Logarithmic Stock Index

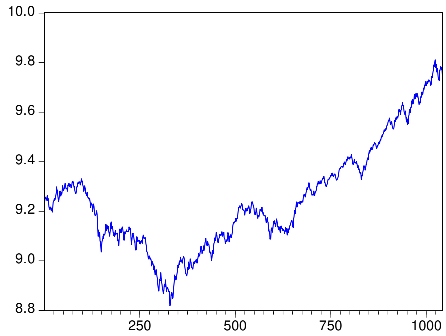

Figure 2: Logarithmic Stock Index Futures

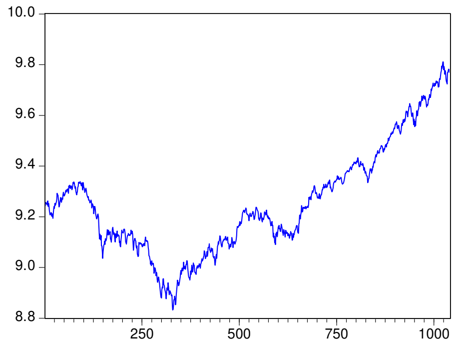

Figure 3: Stock Index Returns

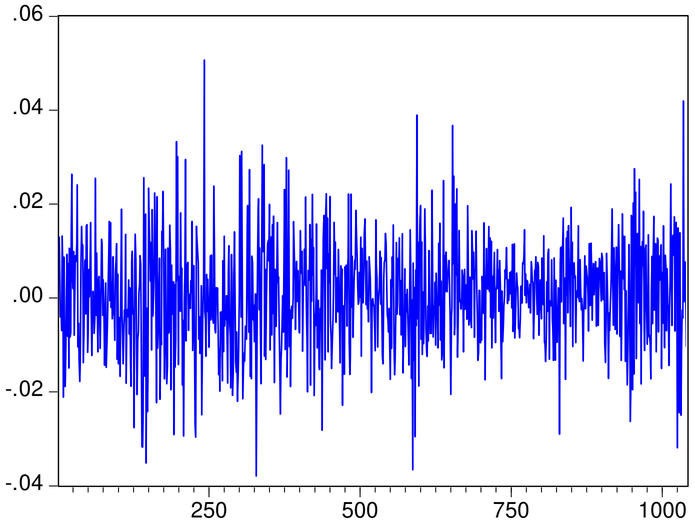

Figure 4: Stock Index Futures Returns

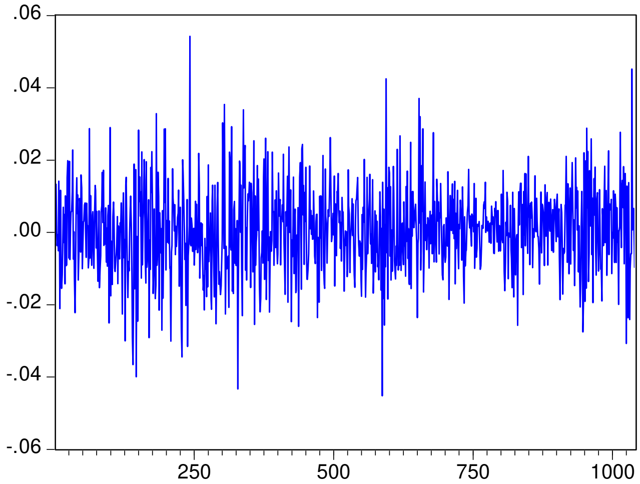

Figure 5. Hedge ratio across time from CCC-ARCH model. The hedge ratio is calculated as $\frac{\sigma_{S,F,t}}{\sigma_{F,t}^2} = \frac{\rho\sigma_{S,t}\sigma_{F,t}}{\sigma_{F,t}^2} = \frac{\rho\sqrt{a_{1,0} + \alpha_{1,1}u_{S,t-1}^2 + \beta_{1,1}\sigma_{S,t-1}^2}}{\sqrt{a_{2,0} + \alpha_{2,1}u_{F,t-1}^2 + \beta_{2,1}\sigma_{F,t-1}^2}}$.

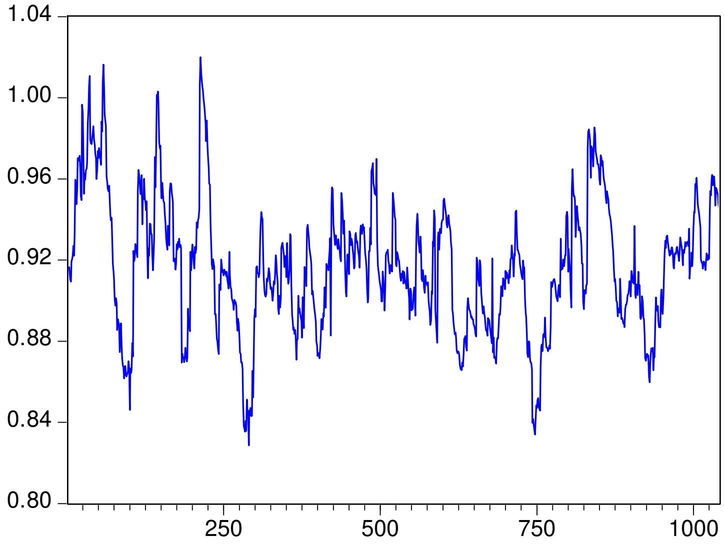

Figure 6. Hedge ratio across time from Diag-BEKK ARCH model. The hedge ratio is calculated as $\frac{\sigma_{S,F,t}}{\sigma_{F,t}^2} = \frac{a_{0,1,2} + a_{1,1,1}u_{S,t-1}u_{F,t-1}a_{1,2,2} + \beta_{1,1,1}\sigma_{S,F,t-1}\beta_{1,2,2}}{a_{0,2,2} + a_{1,2,2}u_{F,t-1}^2 a_{1,2,2} + \beta_{1,2,2}\sigma_{F,t-1}^2\beta_{1,2,2}}$.