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Abstract

ARCH models for the daily S&P500 log-returns are estimated, whereas the intraday prices comprise the dataset for an ARFIMAX model. Model's forecasting performance is statistically superior when the CBOE's VIX index is incorporated as an explanatory variable.

Keywords: ARFIMAX, HYGARCH, VIX Index, Volatility Forecasting.
JEL Classification Codes: C22, C32, C53, G15.

1. Introduction

In 1993, the Chicago Board of Options Exchange (CBOE) created the implied volatility index (VIX) which is considered by the market participants as the world's premier benchmark of stock market volatility. VIX index measures market expectations of the next 30 calendar days volatility conveyed by stock index option prices. On September 22nd of 2003, the CBOE announced a new computation of the volatility index, renaming the original VIX to VXO. The VXO is calculated from the Black and Scholes option pricing formula and uses eight at-the-money options on the SP100 index. The VIX index is based on S&P500 index options, uses nearly all of the available S&P500 index options, and its calculation is independent of any model. Although the VIX index was introduced in 2003, its daily prices date back to 1986. In the CBOE's website[1] details about the construction of the VIX index are available.

Blair et al. (2001) considered the VXO index and the realized volatility as explanatory variables in a Threshold ARCH (TARCH) model and concluded that the VXO index provides more accurate forecasts than either daily or intraday SP100 returns. According to Koopman et al. (2005), in a GARCH model the inclusion of realized or implied volatility as explanatory variable produces more accurate volatility forecasts. However, the models with the realized volatility as dependent variable outperform the models with the VXO index as an explanatory variable when forecasting the volatility in the SP100 index series.

Admittedly, implied volatility[2] is an informative variable for forecasting next day's volatility. The present paper tries to answer the question: does an implied volatility index provide any statistically

[1] http://www.cboe.com/micro/vix/vixwhite.pdf
[2] The listed implied volatility indices eliminate the biases and the mis-specification problems that characterized the implied volatility measures.

significant incremental predictive ability in forecasting volatility, or its predictability contains qualitatively similar information with the one that is provided by the interday or the intraday returns? The implied volatility index provides incremental information in forecasting next trading day's volatility. Evidence may be provided by using either an interday based ARCH specification or an intraday based Fractionally Integrated ARMAX (ARFIMAX) model. Therefore, the VIX index contains information about next day's realised volatility, which is not available in the past values of either daily conditional volatility or intraday realized volatility.

In the next two sections, the framework of the estimated models is presented. We investigate the possible incremental information incorporated in the VIX index in an ARCH framework, where an interday dataset is considered, as well as in an ARFIMAX framework, where the realized volatility (computed from an intraday dataset) is the dependent variable. The fourth section presents our findings and the fifth section concludes.

2. ARCH Models – Interday Data

The most common way to model the process of daily log-returns, $y_t = \ln(SP500_t) - \ln(SP500_{t-1})$, is to assume that it can be decomposed into two parts, the predictable, μ_t, and unpredictable, ε_t, component. The unpredictable component can be presented as an ARCH process:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= \mu(\theta \mid I_{t-1}) \\ \varepsilon_t &= \sigma_t z_t \\ \sigma_t &= g(\theta \mid I_{t-1}) \\ z_t &\overset{i.i.d.}{\sim} f_{z_t}(w;0,1), \end{aligned} \tag{1}$$

where $\mu(.)$ and $g(.)$ are functions of the information set I_{t-1} available in time $t-1$ depending on parameter vector θ, $f(.)$ is the density function of z_t, w is the vector of the parameters of f_{z_t} to be estimated and σ_t is the conditional standard deviation of y_t.

For the purpose of the study, the conditional mean is modelled as a first order autoregressive process in order to account for the non-synchronous trading effect. We base our analysis on three specifications of conditional variance: Bollerslev's (1986) GARCH, Glosten's et al. (1993) TARCH and Davidson's (2004) Hyperbolic GARCH (HYGARCH) models. GARCH and TARCH are typical volatility specifications of simple interday models, whereas HYGARCH is a representative example of complex ARCH models[3].

The GARCH volatility specification takes into account the volatility clustering effect:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta VIX_{t-1}^2. \tag{2}$$

The TARCH model expands the GARCH one in capturing the asymmetric relationship between the conditional volatility and the unexpected returns (parameter γ):

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta VIX_{t-1}^2, \tag{3}$$

where $d_t = 1$ if $\varepsilon_t > 0$ and $d_t = 0$ otherwise. Finally, the HYGARCH model captures not only the volatility clustering and the asymmetric effect but also the long-run dependencies in the conditional variance:

$$\sigma_t^2 = \omega + \left(1 - \beta L - (1 - aL)\left(1 + \zeta\left((1-L)^d - 1\right)\right)\right)\varepsilon_t^2 + \beta\sigma_{t-1}^2 + \delta VIX_{t-1}^2. \tag{4}$$

[3] The lag orders both for conditional mean and variance are not selected according to a model selection criterion, such as the Akaike Information Criterion or the Schwarz Bayesian Criterion, as a good in-sample performance of a model is not a prerequisite for its good out-of-sample precision.

L is the lag operator, d is the fractional integration parameter, which accounts for the slow change of the conditional variance over time. In order to model asymmetric and leptokurtic standardized innovations, z_t, we assume that they are skewed Student-t distributed:

$$f_{z_t}(v,g;0,1)=\frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}}\left(\frac{2s}{g+g^{-1}}\right)\left(1+\frac{sz_t+m}{v-2}g^{-d_t}\right)^{-\frac{v+1}{2}}, \quad (5)$$

where g is the asymmetry parameter, v denotes the degrees of freedom of the distribution, $\Gamma(.)$ is the gamma function, $d_t=1$ if $z_t \geq -m/s$, and $d_t=-1$ otherwise, $m=\Gamma((v-1)/2)\sqrt{(v-2)}\left(\Gamma(v/2)\sqrt{\pi}\right)^{-1}\left(g-g^{-1}\right)$ and $s=\sqrt{g^2+g^{-2}-m^2-1}$. The skewed Student-t density function was introduced by Fernandez and Steel (1998).

3. ARFIMAX Model – Intraday Data

The realized intraday volatility at day t is computed as in Martens (2002) and Koopman et al. (2005):

$$h_t^2=\frac{\sigma_{oc}^2+\sigma_{co}^2}{\sigma_{oc}^2}\sum_{j=1}^{m-1}100\left(\ln\left(SP500_{(j+1/m),t}\right)-\ln\left(SP500_{(j/m),t}\right)\right)^2, \quad (6)$$

where $SP500_{(m),t}$ are the five-minute linearly interpolated S&P500 prices at day t with $m=79$ observations per day, $\sigma_{oc}^2=T^{-1}\sum_{t=1}^{T}\left(\ln\left(P_{(1),t}\right)-\ln\left(P_{(1/m),t}\right)\right)^2$ is the open to close sample variance and $\sigma_{co}^2=T^{-1}\sum_{t=1}^{T}\left(\ln\left(P_{(1/m),t}\right)-\ln\left(P_{(1),t-1}\right)\right)^2$ is the close to open sample variance. The factor $\sigma_{oc}^{-2}\left(\sigma_{oc}^2+\sigma_{co}^2\right)$ accounts for overnight returns without inserting the *noisy* effect of daily returns. For a more detailed discussion on the derivation of the volatility measure the reader is referred to Hansen and Lunde (2005).

The long memory property of the realized variance has been extensively modeled in an ARFIMAX framework. We base our analysis in the following specification:

$$(1-a'L)(1-L)^{d'}\left(\ln h_t^2-w_0'-w_1'y_{t-1}-\gamma'd_{t-1}'y_{t-1}-\delta VIX_{t-1}^2\right)=(1+b'L)u_t, \quad (7)$$

where $u_t \sim N\left(0,\sigma_u^2\right)$, $d_t'=1$ when $y_t>0$ and $d_t'=0$ otherwise. Parameter γ' accounts for the asymmetric relationship between past log-returns and realized log-variance.

4. Results

The S&P500 index was obtained from Datastream, whereas the VIX index, $VIX_{close,t}$, is available from the CBOE website. The time span includes the period from January 1990 to December 2003, a total of 3516 trading days[4]. The explanatory variable VIX_t^2 is computed as $VIX_t^2=VIX_{close,t}^2/252$. The intraday dataset, which was obtained from Olsen and Associates, consists of five-minute linearly interpolated S&P500 prices, for 1748 trading days, in the period from January 1997 to December 2003[5].

We are based on eight model specifications, the three ARCH processes and the ARFIMAX model, with and without the VIX index as exogenous variable. Based on a rolling sample of constant size equal to 3017, we estimated the parameters of the ARCH models every trading day. The parameters of the ARFIMAX models were also re-estimated at each trading day, using a rolling sample of constant size equal to 1249. Thus, for both ARCH and ARFIMAX processes, we generate 499 one-day-ahead volatility forecasts.

The one-day-ahead forecast of the GARCH model is:

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4 For the interday dataset, $T=3516$, $\breve{T}=3017$ and $\tilde{T}=499$.

5 For the intraday dataset, $T=1748$, $\breve{T}=1249$ and $\tilde{T}=499$.

$$\sigma^2_{t+1|t} = \omega^{(t)} + \alpha^{(t)}\varepsilon^2_{t|t} + \beta^{(t)}\sigma^2_{t|t} + \delta^{(t)}VIX^2_t \,. \tag{8}$$

Respectively, next day's conditional variance forecasts of the TARCH and HYGARCH models are computed as:

$$\sigma^2_{t+1|t} = \omega^{(t)} + \alpha^{(t)}\varepsilon^2_{t|t} + \gamma^{(t)}d_t\varepsilon^2_{t|t} + \beta^{(t)}\sigma^2_{t|t} + \delta^{(t)}VIX^2_t \,, \tag{9}$$

and

$$\sigma^2_{t+1|t} = \omega^{(t)} + \left(a^{(t)} - \beta^{(t)}\right)\varepsilon^2_{t|t} + \sum_{i=1}^{\infty}\left(\frac{d^{(t)}\Gamma\left(i-d^{(t)}\right)}{\Gamma\left(1-d^{(t)}\right)\Gamma(i+1)}L^i\zeta^{(t)}\left(\varepsilon^2_{t+1|t+i} - a^{(t)}\varepsilon^2_{t|t+i}\right)\right) + \beta^{(t)}\sigma^2_{t|t} + \delta^{(t)}VIX^2_t \,, \tag{10}$$

where $\sum_{i=1}^{\infty} d\left(\frac{\Gamma(i-d)}{\Gamma(1-d)\Gamma(i+1)}L^i\right) = (1!)^{-1}dL + (2!)^{-1}d(1-d)L^2 + (3!)^{-1}d(1-d)(2-d)L^3 + \dots .$

The next trading day's realized variance forecast is:

$$\sigma^2_{t+1|t} = \exp\left(\ln h^2_{t+1|t} + 0.5\sigma^2_u\right), \tag{11}$$

for $\ln h^2_{t+1|t}$ denoting the one-step-ahead realized log-volatility[6].

We evaluate the forecasting accuracy of each model based on the predictive mean squared error criterion:

$$L^{(i)} = \tilde{T}^{-1}\sum_{t=1}^{\tilde{T}} L^{(i)}_t \,. \tag{12}$$

where $L^{(i)}_t = \left(h^2_{t+1} - \sigma^{2(i)}_{t+1|t}\right)^2$, for $i = 1,...,8$ models. Previous studies, such as Pagan and Schwert (1990), Bollerslev and Ghysels (1996) and Koopman et al. (2005), claimed that symmetric loss functions produce unreliable results and suggested the use of asymmetric loss functions that take into account the non-linear character of volatility. However, Hansen and Lunde (2006) analyzed how the substitution of proxy for the latent measure of volatility affects the ranking of a set of volatility forecasting models. They provided evidence that if an evaluation is based on asymmetric loss functions, the substitution of a noisy proxy for the true but unobservable conditional variance can result in an inferior model being chosen as best. Therefore, the paper measures the squared distance between conditional volatility forecast and the proxy for the true volatility. Such a loss function ensures a consistent ranking of the models[7].

In order to compare the forecasting performance of model i against its $i^* = 1,...,M$ competitors, we compute the statistic $T^{SPA} = \max_{i^*=1,...,M}\left(\sqrt{M}\,\overline{X}_{i^*}\left(\sqrt{Var\left(\sqrt{M}\,\overline{X}_{i^*}\right)}\right)^{-1}\right)$, where $\overline{X}_{i^*} = \tilde{T}^{-1}\sum_{t=1}^{\tilde{T}} L^{(i)}_t - L^{(i^*)}_t$. The p-values of the SPA criterion are obtained by using the stationary bootstrap method of Politis and Romano (1994). According to Table 1, the introduction of the VIX index decreases the value of the loss function in each case. The p-values of Hansen's (2005) superior predictive ability (SPA) hypothesis are presented, in order to carry out formal prediction superiority tests. The null hypothesis that the model with the VIX index as exogenous variable has statistically higher predictive ability than the model without the VIX index is not rejected in any case. However, the hypothesis that the ARFIMAX model with the VIX index has statistically higher forecasting performance relative to the rest of the models is rejected with a p-value of 0.811. The rejection of the hypothesis constitutes evidence that the combination of intraday and implied volatility produces the most accurate one-day-ahead volatility forecast. Figure 1, which presents, indicatively, the one-step-ahead volatility forecasts of HYGARCH and ARFIMAX models, gives a clear view of the incremental information that VIX

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6 Since $u_t \sim N\left(0,\sigma^2_u\right)$, the $\exp(u_t)$ is log-normally distributed.

7 However, we evaluated the performance of the models based on asymmetric loss functions, i.e. the heteroskedasticity adjusted squared error of Bollerslev and Ghysels (1996) and the logarithmic error of Pagan and Schwert (1990), and we reached to the same conclusion.

index provides. The models with the VIX index produce next day's standard deviation forecasts that are closer to the realized volatility[8].

Table 1: The values of the predictive mean squared error loss function and the p-values of the SPA test for the null hypothesis that that the model with the VIX index as exogenous variable has statistically

5. Conclusion

The present study has investigated whether the implied volatility index of S&P500 provides incremental predictive ability in forecasting next day's realized volatility. It was shown that when the VIX index is incorporated as exogenous variable either in an interday or in an intraday model specification, it provides incremental predictive ability. An interesting issue for future research is the possible mechanism why VIX can help to forecast the next day volatility and in which ways, this result can help us understand the microstructure of the market and the volatility predictability.

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