Departure Time Choice and Bottleneck Congestion with Automated Vehicles: Role of On-board Activities

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1 October 2019

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Abstract

The enhanced possibility to perform non-driving activities in automated vehicles (AVs) may not only decrease the disutility of travel, but also change the AV users' departure time preferences, thereby affecting traffic congestion. Depending on the AV interior, travellers may be able to perform in the vehicle activities that they would otherwise perform at home or at work. These possibilities might make them depart at different times compared to situations, when they are not able to engage in any activities during travel or when the possible activities do not substitute any out-of-vehicle activities. This paper formalises the on-board activity and substitution effects using new scheduling preferences in the morning commute context. The new scheduling preferences are used (1) to analyse the optimal departure times when there is no congestion, and (2) to obtain the equilibrium congestion patterns in a bottleneck setting. If there is no congestion, it is predicted that AV users would choose to depart earlier (later), if the on-board environment is better suited for their home (work) activities. If there is congestion, more AV users departing earlier or later would skew the congestion in the corresponding direction. Given the minimalistic bottleneck setting, it is found that congestion with AVs is more severe than with conventional vehicles. If AVs were specialised to support only home, only work, or both home and work activities, and would do so to a similar extent, then 'Work AVs' would increase the congestion the least.

Keywords: Automated vehicles, On-board activities, Scheduling preferences, Departure time choice, Bottleneck model, Traffic congestion

1. Introduction

Among the core expected benefits of automated vehicles[1] (AVs) is their promise to let their users perform new non-driving activities, or engage more efficiently in current non-driving activities, while being on the way. It is commonly anticipated that this would make travel more pleasant, thus reducing the 'penalty' associated with travel time (see Soteropoulos et al., 2019, for a recent review of modelling studies). The reduced penalty in turn is expected to lead to acceptance of longer travel times, thereby increasing traffic congestion, which may be (partly) offset by shorter headways and increased throughput expected from AVs. The possible net congestion effects of AVs have been extensively studied in literature (e.g., van den Berg & Verhoef, 2016; Wadud et al., 2016; Auld et al., 2017; Milakis et al., 2017; Simoni et al., 2019).

However, a thought experiment can demonstrate that *the substance of on-board activities* may directly influence the timing preferences for a trip, and in so doing affect congestion patterns in ways that would not be predicted using travel time penalty. For example, an AV user may consider shifting or extending the pre- or post-travel activities into the trip. In the context of the morning commute, an individual may choose to perform in the AV 'home activities', such as getting ready, preparing and

[1] This paper considers primarily the so-called level 5 or fully automated vehicles, according to SAE International (2016) standards.

eating breakfast, getting a little more sleep, or 'work activities', such as replying emails, planning the day, adjusting meeting schedule. This shift might reduce the aversion to longer travel and encourage AV users to depart at peak times. At the same time, it might result in a desire to depart from the origin earlier, while shifting origin-type activities to the trip, or to depart and arrive at the destination later, while shifting destination-type activities to the trip. That on-board activities may have varied influence on the preferred departure times, may also be expected knowing that on-board activities differ with regard to their influence on the travel time penalty and satisfaction (Ettema & Verschuren, 2007; Susilo et al., 2012; Rasouli & Timmermans, 2014; Frei et al., 2015; Correia et al., 2019), as well as their effect on pre- and post-trip activities and daily time-use (Banerjee & Kanafani, 2008; Pawlak et al., 2015, 2017; Das et al., 2017; Krueger et al., 2019; Pudāne et al., 2018, 2019).

Yet, most current models do not differentiate among on-board activities, when considering their impact on preferred departure times and congestion (e.g., Correia & van Arem, 2016; Lamotte et al., 2017; Simoni et al. 2019). The possibility to model such scenarios is largely lost whenever the effects of a multitude of possible activities are condensed into a single travel time penalty (such as value of travel time). This treatment implies that the travel behaviour effects of various on-board activities are the same and indistinguishable from increased comfort of travel (such as more comfortable seats).

This study proposes a more flexible modelling approach, which lets to investigate, first, how various on-board activities may influence departure times of AV users and, second, how they may affect traffic congestion. Thereby, this study contributes to two streams of literature: the study of the potential travel behaviour impacts of AVs and the rich tradition of using the bottleneck model to analyse the impact of behaviour changes on congestion. It starts by formulating new scheduling preferences that let the analyst specify how suitable the travel environment is for home and work activities. Then, it uses the new scheduling preferences to analyse the optimal departure times for users of different AVs. Finally, it obtains equilibrium congestion patterns in a minimalistic bottleneck setting, where a number of travellers with the same scheduling preferences move from a single origin to a single destination on a single route.

The bottleneck model is particularly useful for studying the possible congestion effects of on-board activities in AV. Since its conception by Vickrey (1969) and Arnott et al. (1990, 1993), this model has been instrumental in investigating various factors influencing congestion (see the reviews by de Palma & Fosgerau, 2011, and Small, 2015), and notably, it often allows to obtain analytic as opposed to simulated results. Related to the present work, the bottleneck model was used to study the effects of teleworking on congestion (Gubins & Verhoef, 2011), incorporated in a whole-day activity pattern (Zhang et al., 2005; Li et al., 2014), used to predict congestion patterns when AVs and conventional vehicles use different roads (Lamotte et al., 2017), and the congestion impacts of AVs being able to park themselves (Liu, 2018; Zhang et al., 2019). In particular, this work furthers the study of van den Berg and Verhoef (2016), who investigated the effects of AVs on a congestion in a bottleneck, while assuming that any on-board activities contribute to a decreasing travel penalty. They concluded that AV users would concentrate in the middle of the peak congestion. The same conclusion was reached also by Fosgerau (2018). In another way, this study builds on the work of Pawlak et al. (2015, 2017), which uses a scenario with two out-of-vehicle activities connected by a trip, during which two in-vehicle activities may be performed. Pawlak et al. analysed a multidimensional choice in this setting: choice of activity types, departure times, duration and switching times between on-board activities, mode, route and use of ICT.

Before explaining the organisation of the paper, I would like to highlight the relation between the present paper and other on-going work that looks into departure time effects of on-board activities: Yu et al. (2019) and Abegaz and Fosgerau (personal communication, September 2019). The three approaches start from different assumptions about the utility impacts of activities on board AVs. This paper conceives that the utility of on-board activities could be obtained by multiplying the utility of home- and work-activities with an on-board efficiency factor and analyses the departure times using general scheduling preferences and congestion patterns using $\alpha - \beta - \gamma$ preferences. In contrast, Yu et al. (2019) considers an additive effect using $\alpha - \beta - \gamma$ preferences, while Abegaz and Fosgerau include a separate class of mobile activities in a general scheduling preference framework. These variations in the set-up lead to different results. Furthermore, the three works deepen the analysis in different directions: the present work focuses on optimal departure times and congestion patterns with different vehicles, while Yu et al. (2019) analyse market and AV-provision effects, and Abegaz and Fosgerau have special interest in possible changes in value of time and reliability.

The remainder of the paper is structured as follows. Section 2 introduces the scheduling preferences that capture the possibility to shift home or work activities to the trip. It also introduces three types of AVs that are used further in the paper. Sections 3 analyses the departure times for a single traveller or multiple travellers that do not create a congestion. Section 4 analyses congestion changes with AVs in a bottleneck setting. Section 5 compares the current approach with travel time penalty method, discusses its validity and applicability to other transport modes, and recommends directions for further research. Section 6 concludes and discusses the implications of this study for AV-related transport policy.

2. Model set-up

2.1. Scheduling preferences considering on-board activities

The most general form of scheduling functions (based on Vickrey, 1973) assumes that marginal home and work utilities $h[x]$ and $w[x]$ are positive and monotonously decreasing and increasing functions, respectively, of the clock-time x in a morning time interval $[0, \Omega]$. Conventionally, it is assumed that the individual cannot participate in any home or work activities during travel and therefore does not gain any home or work utility at that time. In the context of AVs however, I assume that individuals may continue with their home activities during travel or start to perform their work activities while on the way to work, but the utility of these activities would be reduced, reflecting some inconvenience of performing these activities in the vehicle. I model this reduction with multiplicative efficiency factors $e_h, e_w \in [0,1]$ for home and work activities, respectively.

Figure 1 illustrates the model set-up. It shows the marginal utilities of home and work activities (y-axis), which depend on time (x-axis) in a morning time interval. As can be seen from the distance between the solid and dashed lines, this figure illustrates a situation where home activities are better facilitated on board than work activities: $e_h > e_w$. Shaded areas represent the total utility gained from activities at home, at work and during travel.

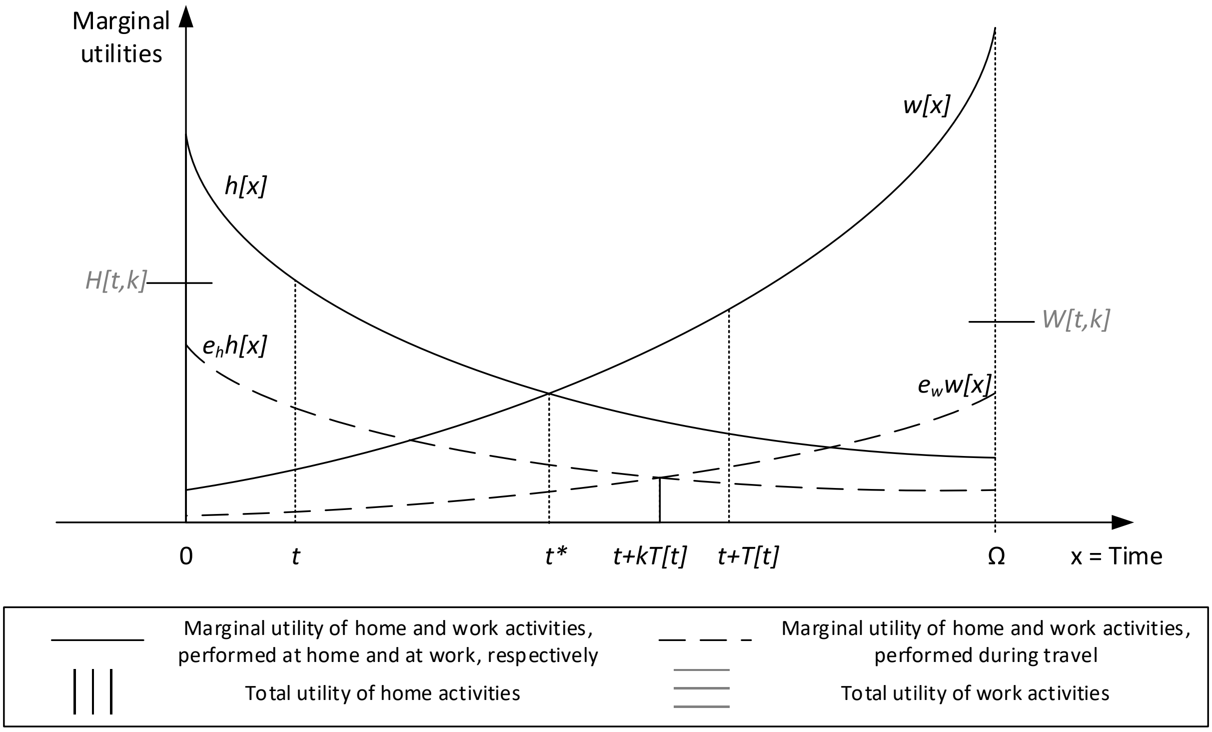

Figure 1 Scheduling preferences including the utility obtained from home and work activities on board: general scheduling preferences

The individual engages in home activity during travel at time x if $e_h h[x] > e_w w[x]$ (utility from on-board home activities is higher than utility of on-board work activities) and in work activity if $e_h h[x] < e_w w[x]$. Therefore, knowing that $e_h h[x]$ and $e_w w[x]$ are monotonously decreasing and increasing with $x \in [0, \Omega]$, respectively, the optimal time for on-board home activity (if any) is at the start of the trip, and similarly the optimal time for the on-board work activity (if any) is at the end of the trip.

Furthermore, since marginal home and work utilities $h[x]$ and $w[x]$ are assumed to be positive for $x \in [0, \Omega]$, the individual would want to continually engage in on-board activities, if they are at least slightly facilitated (i.e., if $e_h, e_w > 0$, then utilities $e_h h[x], e_w w[x] > 0$). This setting yields a single optimal switching point between the home and work activities, which can be expressed as a share of the trip duration $k \in [0,1]$. Hence, a traveller that departs from home at time t and arrives at work at time $t + T[t]$ engages in home activity on board during the time interval $[t, t + kT[t]]$ and in work activity on board during the time interval $[t + kT[t], t + T[t]]$. Travel time $T[t]$ is assumed to depend on the departure time t, which enables to model the effects of congestion. The boundary cases, where $k = 0$ or $k = 1$, correspond to individual engaging only at work or home activity on board. If travel took no time at all (the individual would be able to 'teleport' from home to work), then the optimal departure and arrival time would be t^*.

Total home utility $H[t, k]$, total work utility $W[t, k]$ and total utility $V[t, k]$ are defined as follows:

$$H[t,k] = \int_0^t h[x]dx + e_h \int_t^{t+kT[t]} h[x]dx, \tag{1}$$

$$W[t,k] = \int_{t+T[t]}^{\Omega} w[x]dx + e_w \int_{t+kT[t]}^{t+T[t]} w[x]dx, \tag{2}$$

$$V[t,k] = H[t,k] + W[t,k]. \quad (3)$$

Every traveller tries to maximise the total utility $V[t,k]$ by choosing the departure time t and the switching point between the on-board activities k. This defines the scheduling preferences that determine the optimal departure times given a broad class of home and work marginal utility functions $h[x]$ and $w[x]$. From here on, I call these 'general scheduling preferences'. While it is possible to use them to analyse the optimal departure times in case of no congestion (section 3), the analysis of equilibrium congestion patterns (section 4) requires that specific forms of $h[x]$ and $w[x]$ are used. For this purpose, I select the most widely used scheduling preferences, the $\alpha - \beta - \gamma$ model (Vickrey 1969; Small, 1982; reasons for this selection are explained at the start of section 4), which can be specified by inserting the following as the home and work utility functions in (1)-(3):

$$h[x] = \alpha, \quad (4)$$

$$w[x] = \begin{cases} \alpha - \beta, if\ x \le t^* \\ \alpha + \gamma, if\ x > t^*, \end{cases} \quad (5)$$

where α, β, γ are positive constants and α and β are assumed to have the relationship $\beta < \alpha$; t^* is the preferred arrival time at work. Parameter α is the utility of spending time at home; β and γ are the utility differences between home utility and work utility, if work is performed before or after the preferred arrival time, respectively. Figure 2 illustrates the model set-up, using the $\alpha - \beta - \gamma$ scheduling preferences. The illustrated efficiency factors e_h and e_w are such that until the time t^* it would be optimal for the individual to engage in home activities, but after time t^* it would be optimal to switch to performing work activities during travel: $e_h h[x] > e_w w[x]$ for $x \le t^*$ and $e_h h[x] < e_w w[x]$ for $x > t^*$. The figure shows a situation where traveller arrives late at work ($t + T[t] > t^*$). As before, the shaded areas represent the total utility $V[t,k]$ gained from activities at home, at work and during travel.[2]

[2] Note that the set-up (1)-(3) permits scenarios where the utility of on-board activity is higher than utility of home or work activities just before or after the trip. For example, in the context of $\alpha - \beta - \gamma$ preferences, home activity during travel may be more valuable than work activity before the preferred arrival time t^*: $e_h\alpha > \alpha - \beta$. In such cases, it is assumed that the individual would still leave the AV once it has arrived, rather than continuing with the home activity in a parked vehicle.

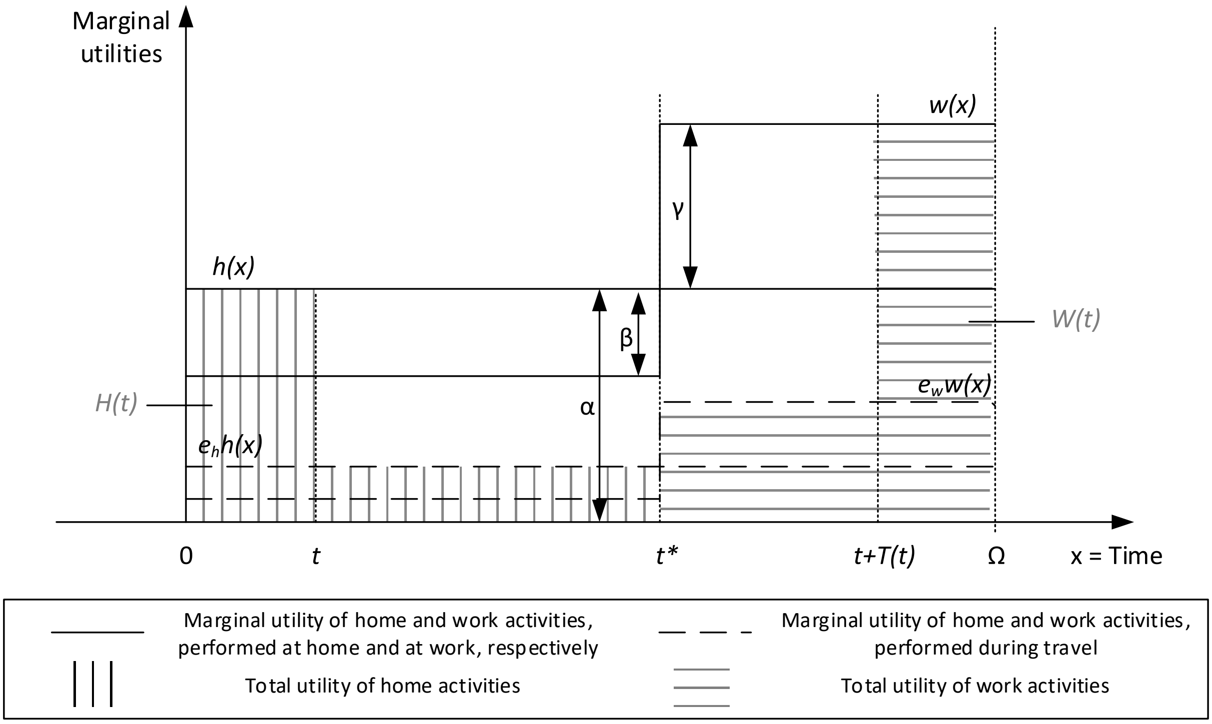

Figure 2 Scheduling preferences including the utility obtained from home and work activities on board: $\alpha - \beta - \gamma$ scheduling preferences

Note that the $\alpha - \beta - \gamma$ model does not belong to the class of general scheduling preferences. For the general scheduling preferences, the home and work utility functions are strictly decreasing and increasing, respectively, but in the $\alpha - \beta - \gamma$ model they are constant and piecewise constant, respectively.

2.2. Three types of automated vehicles

The set-up introduced in section 2.1 allows us to imagine scenarios where AVs are specialised (e.g., via interior design and equipment) to suit the needs of (1) home activities, (2) work activities, and (3) both home and work activities. In the following sections, these AV-types are called 'Home AV', 'Work AV', and 'Universal AV', respectively. However, the precise classification differs between sections 3 and 4. In section 3 with general scheduling preferences (as in Figure 1), the three types are defined using only the efficiency factors: $e_h > e_w$ characterises the Home AV, $e_h < e_w$ represents the Work AV, and $e_h = e_w$ corresponds to the Universal AV.

In section 4, which use the $\alpha - \beta - \gamma$ scheduling preferences (as in Figure 2), the definitions involve the parameters of the home and work utility functions (α, β, γ). The resulting definition of Universal AV is such that it would be optimal for the users of this AV to engage in home activities before time t^* and in work activities after t^* (as in Figure 2). The Home AV and Work AV facilitate one of the two activities much better than the other, such that, independently of the departure time t, it is optimal to engage in home activities in Home AV and work activities in Work AV during the entire trip. The parameter combinations that define each AV type in the context of $\alpha - \beta - \gamma$ preferences are shown in Figure 3. If, for example, αe_h (the utility of on-board home activity) is smaller than $(\alpha - \beta)e_w$ (the utility of on-board work activity before t^*), then these parameter values correspond to a Work AV.

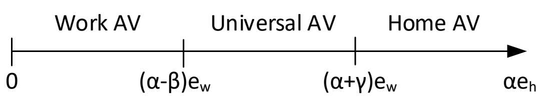

Figure 3 Definition of Home, Universal, Work AVs using $\alpha - \beta\ - \gamma$ scheduling preferences

In addition, it could be possible to distinguish a fourth type of AV that only increases the comfort of travel or facilitates such activities on board that do not substitute activities out-of-vehicle (e.g., on-board entertainment). Such an AV could be defined by replacing the home and work functions in the second integrals of equations (1) and (2) with constants (or other time-independent functions). This would define an AV that is modelled by reduced travel time penalty approach. This AV type is discussed as a point of reference in section 5.1.

3. Case of no congestion

3.1. Optimal departure times with general scheduling preferences

Having introduced the scheduling preferences, we can analyse the optimal departure time of a single traveller. The derivation would be the same in a hypothetical situation when multiple identical travellers do not create congestion, that is, when the bottleneck capacity exceeds the number of travellers. Formally, this situation can be represented as travel time being independent from the departure time and constant: $T[t] = T$. Using the general scheduling preferences, finding the optimal departure time is a 2-variable constrained optimisation problem: choose departure time t and switching point k between home- and work-type activities that maximises the total utility $V[t,k]$ from (3). The optimisation problem is constrained, because switching between activities needs to occur during the trip time ($0 \leq k \leq 1$). These conditions result in the following model:

$$\max V[t,k], \tag{6}$$

subject to:

$$g_1[k] = -k \leq 0, \tag{7}$$

$$g_2[k] = k - 1 \leq 0. \tag{8}$$

Using the definition of $V[t,k]$ from (3), the Karush–Kuhn–Tucker conditions[3] for this problem are as follows:

$$\begin{aligned}\frac{\partial}{\partial t}\left(V[t,k] - \sum_{i=1,2} \lambda_i g_i[k]\right) \\ = h[t_0] + (h[t_0 + k_0 T] - h[t_0])e_h - w[t_0 + T] \\ + (w[t_0 + T] - w[t_0 + k_0 T])e_w = 0,\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial}{\partial k}\left(V[t,k] - \sum_{i=1,2} \lambda_i g_i[k]\right) \\ = h[t_0 + k_0 T]e_h T - w[t_0 + k_0 T]e_w T + \lambda_1 - \lambda_2 = 0,\end{aligned} \tag{10}$$

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[3] The Karush-Kahn-Tucker conditions can be applied for this problem, because it fulfils the linear independence constraint qualification (Nocedal & Wright, 2006, p. 320). Since at most one of the constraints g_1 and g_2 is active for any k value, the independence is trivial.

$$g_i[k_0] \leq 0 \qquad i = 1,2, \qquad (11)$$

$$\lambda_i g_i[k_0] = 0 \qquad i = 1,2, \qquad (12)$$

$$\lambda_i \geq 0 \qquad i = 1,2, \qquad (13)$$

where the solution is denoted (t_0, k_0), and λ_i, $i = 1,2$ are the Karush–Kuhn–Tucker multipliers. The stationary points $(t_0; k_0)$ determined by (9)-(13) are the global maximum points of the utility $V[t,k]$, because the utility $V[t,k]$ is concave with respect to t and k, as shown next. The second order conditions are

$$\frac{\partial^2}{\partial t^2} V[t,k] = \frac{\partial}{\partial t}(h[t](1-e_h) + h[t+kT]e_h - w[t+T](1-e_w) - w[t+kT]e_w) < 0 \qquad (14)$$

and

$$\frac{\partial^2}{\partial k^2} V[t,k] = \frac{\partial}{\partial k}(h[t+kT]e_h T - w[t+kT]e_w T) < 0. \qquad (15)$$

The negativity of the second order conditions can be confirmed by recalling that the marginal utilities $h[x]$ and $w[x]$ are monotonically decreasing and increasing, respectively. Therefore, $\partial/\partial x\, h[x] < 0$ and $\partial/\partial x\, w[x] > 0$. Further, parameters t and k enter the marginal utilities $h[x]$ and $w[x]$ in (14) and (15) positively, therefore the derivatives of $h[x]$ and $w[x]$ with respect to t and k maintain their signs. Finally, notice that $h[x]$ and $w[x]$ enter the second order conditions with positive and negative signs, respectively. From here follows that all additive terms in (14) and (15) are negative, making both second order derivatives negative. Hence, the utility is concave with respect to t and k.

Knowing that (9)-(13) yield the global maximum points, we can analyse the optimal departure times for Home, Universal, and Work AVs. Although these equations do not reveal the optimal points in a closed form, they are nevertheless sufficient to analyse their relationships. To proceed with that, we need to separately consider the non-binding and binding cases of constraints (11).

If (11) are non-binding, then $\lambda_1 = \lambda_2 = 0$ due to the complementary slackness conditions (12), and the traveller switches from performing home to work activities during the trip. Then (10) can be rewritten as

$$\frac{\partial}{\partial k}\left(V[t,k] - \sum_{i=1,2} \lambda_i g_i[k]\right) = h[t_0 + k_0 T]e_h T - w[t_0 + k_0 T]e_w T = 0. \qquad (16)$$

Using this equality, we can simplify the first stationarity condition (9) for the non-binding case. Being an equation with a single unknown, (17) determines the optimal departure time in the non-binding case:

$$\frac{\partial}{\partial t}\left(V[t,k] - \sum_{i=1,2} \lambda_i g_i[k]\right) = h[t_0](1-e_h) - w[t_0 + T](1-e_w) = 0. \qquad (17)$$

If one of the constraints (11) is binding, then the traveller spends the entire trip performing either home or work activity. Such a situation would arise, when one of the efficiency factors e_h and e_w is much higher than the other, as well as when only one of the factors equals zero or one. In the latter case, we can observe that the non-binding condition (16) would not yield a feasible solution if one of e_h or e_w equals zero, and the non-binding condition (17) would not yield a feasible solution if one of e_h or e_w equals one. The binding cases also necessarily correspond to Home AV ($k = 1$, optimal departure time denoted t_1) or Work AV ($k = 0$, optimal departure time denoted t_3), except when $e_h = e_w = 1$ (which would correspond to a Universal AV, optimal departure time denoted t_2). We can derive the optimal departure times for the binding cases by inserting the binding k values in (9):

$$t_1, when\ k_0 = 1: h[t_1](1 - e_h) + h[t_1 + T]e_h = w[t_1 + T], \tag{18}$$

$$t_3, when\ k_0 = 0: h[t_3] = w[t_3]e_w + w[t_3 + T](1 - e_w). \tag{19}$$

Next, we can use the obtained conditions (17)-(19) to analyse the relationship between departure times of Home, Universal, and Work AV users. The results are shown in Table 1. Parameter t^* is defined such that $h[t^*] = w[t^*]$.

Table 1 Optimal departure times in case of no congestion

	t_1 – Home AV	t_2 – Universal AV	t_3 – Work AV
$e_h < 1; e_w < 1$	$t^* - T < t_1 < t_2 < t_3 < t^*$		
$e_h = 1; e_w < 1$	$t_1 = t^* - T$	-	-
$e_h = 1; e_w = 1$	-	$t_2 = [t^* - T; t^*]$	-
$e_h < 1; e_w = 1$	-	-	$t_3 = t^*$

The relationship between optimal departure times $t_1 < t_2 < t_3$ in the first line of Table 1 follows from the non-binding solution in equation (17) as well as from binding solutions (18) and (19). In the non-binding case, the definitions of the three AV types: $e_h > e_w$ for Home AVs, $e_h = e_w$ for Universal AVs, and $e_w < e_h$ for Work AVs should be inserted in (17). In the binding case, it can be noticed that both (18) and (19) contain weighted averages on one side of the equality. The following (in-)equalities arise:

$$t_1 \text{ s.t. } h[t_1] > w[t_1 + T], \tag{20}$$

$$t_2 \text{ s.t. } h[t_2] = w[t_2 + T], \tag{21}$$

$$t_3 \text{ s.t. } h[t_3] < w[t_3 + T]. \tag{22}$$

Recalling that home and work marginal utilities are decreasing and increasing, respectively, it follows that $t_1 < t_2 < t_3$. Note that equation (21) holds for any e_h and e_w values that are smaller than 1. Hence, they include the conventional vehicle, for which $e_h = e_w = 0$. This leads to the conclusion that the users of the Home AV would depart earlier and the users of the Work AV would depart later than the conventional vehicle users.

The earliest and latest optimal departure times $t_1 = t^* - T$ and $t_3 = t^*$ for Home and Work AVs follow from inserting $e_h = 1$ and $e_w = 1$ in the binding cases (18) and (19), respectively. It can be seen that for any efficiency factors lower than 1, these end-points are not reached, leading to the inequalities $t^* - T < t_1$ and $t_3 < t^*$ in the first row. Finally, if both home and work activities are perfectly facilitated in the AV ($e_h = e_w = 1$), then the traveller would experience zero disutility in such a Universal AV and would be indifferent between any departure times in the interval $[t^* - T, t^*]$, which is determined by conditions (16) and (17) and $0 \leq k \leq 1$ (constraints (11)).

Hereby, this section has obtained that, in case of general scheduling preferences, travellers whose home activities are better facilitated on board than work activities, would depart earlier than conventional vehicle users. Similarly, travellers whose work activities are better facilitated on board than home activities, would depart later than conventional vehicle users. This result holds even if there is no congestion. The implication of this finding is that a hypothetical traveller population with identical general scheduling preferences would, upon replacing their conventional vehicles with a mixture of Home, Universal and Work AVs, disperse with respect to their departure times. All departures would however still fit in the interval $[t^* - T, t^*]$.

3.2. Optimal departure times with $\alpha - \beta - \gamma$ scheduling preferences

It is useful to note that the departure time sequence $t_1 < t_2 < t_3$ does not hold for the $\alpha - \beta - \gamma$ scheduling preferences. Due to the discontinuity of $w[x]$, we cannot follow the same derivation as in the case of the general scheduling preferences. However, it is intuitive from Figure 2 that the optimal departure time generally equals $\tilde{t} = t^* - T$. Formally, it can be shown that only in two cases, the optimal departure time would be t^* instead of $\tilde{t}$: when $e_w > \gamma/(\beta + \gamma)$ for Work AV and when $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$ for Universal AV. Further, it can be demonstrated that the latter result would never occur, if it is (conservatively) assumed that e_w does not exceed 0.5 and that $\beta < \alpha < \gamma$ (as is conventional). The proofs of these results are in Appendix A.

4. Case of congestion

In order to analytically study the changes in congestion patterns, we need to assume that travellers have certain shape of departure time preferences. The previous section showed that, while general scheduling preferences lead to changing optimal departure times even if there is no congestion, the $\alpha - \beta - \gamma$ preferences lead to the same optimal departure time, unless work activity is very well facilitated on board. This makes the $\alpha - \beta - \gamma$ preferences an interesting case to be studied in the congestion setting: it would provide a conservative prediction for changes in congestion patterns, which can serve as a good starting point. Furthermore, $\alpha - \beta - \gamma$ preferences have a well-known closed form-solution for the equilibrium flow rate in a bottleneck setting – the number of travellers departing at every time moment, obtained by Arnott et al. (1990) –, which has contributed to their continuing popularity for congestion modelling. For these reasons, I adopt this form of scheduling preferences from now on.

The following derivations assume the most minimalistic bottleneck setting, where a number of individual with the same scheduling preferences travel from a single origin to a single destination on a single route. Free flow travel time is assumed to be zero, such that the total travel time equals the queueing time at the bottleneck.

4.1. Congestion with conventional vehicles

Before proceeding to compute the equilibrium congestion patterns for AVs, it is useful to recap how this is done for conventional vehicles (as per Arnott et al., 1990). As introduced in equations (4)-(5), the $\alpha-\beta-\gamma$ preferences contain a preferred arrival time t^*, when the individual starts to value being at work higher than being at home. Because everyone would like to arrive at work at exactly t^* (assuming homogeneous preferences), congestion arises – travel time is longer for trips that end around t^*. The departure time that leads to arrival at exactly t^* is denoted $\tilde{t}$ and called the 'undelayed departure time'. Eventually, it is assumed that the disutility caused by schedule delay and travel time at all departure times is perfectly balanced. This condition corresponds to the Nash equilibrium. In other words, as anyone would consider departing at another time, the gained and lost utility from so doing would cancel each other out.

Figure 4 illustrates a case where a traveller would consider postponing his departure by one time unit. The gained utility from home activity is α, whereas the lost utility from work activity is $\alpha-\beta$, if traveller arrives early, and $\alpha+\gamma$, if he arrives late. Because the travel times may differ at both considered departure times, the utility loss at the destination should be multiplied with the arrival time difference between the two considered departure times. This arrival time difference time is $1+\dot{D}/s$, where $\dot{D}$ is the change in queue length at time t: $\dot{D}=r[t]-s$. Here, $r[t]$ is the number of individuals departing at time t, and s is the number of travellers that can pass through the bottleneck (i.e. the bottleneck capacity).

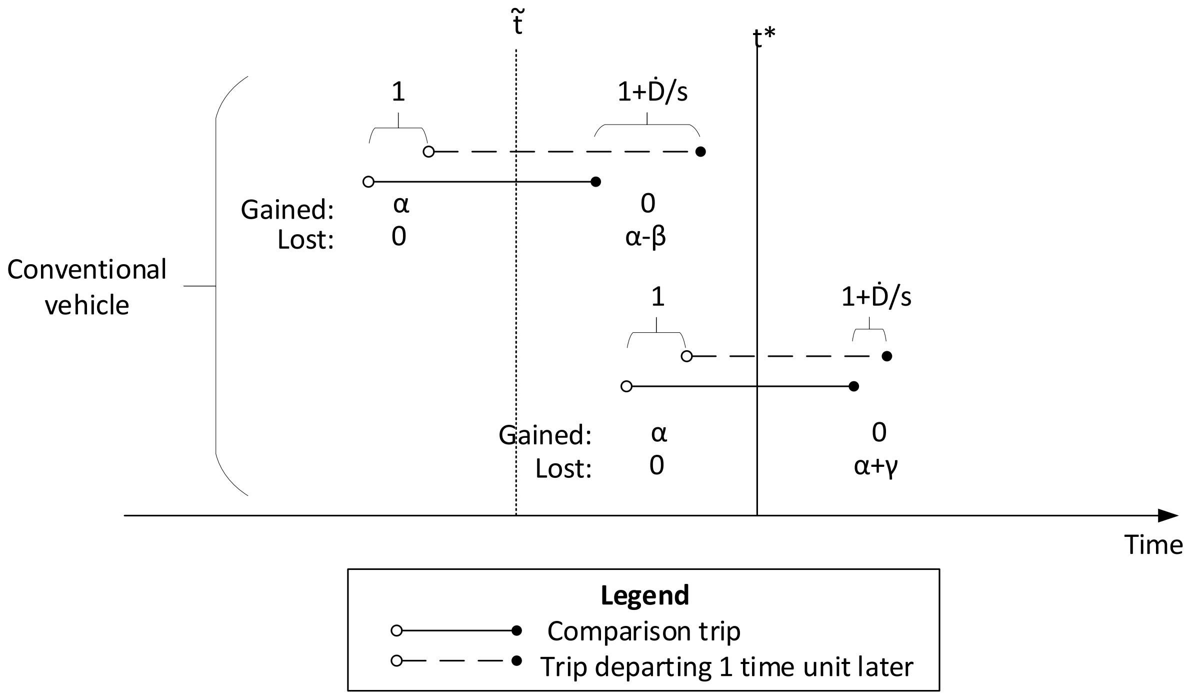

Figure 4 Utility components for computing equilibrium flow rate with conventional vehicles (CV)

By equalling the gained and lost utilities (as illustrated in Figure 4), Arnott et al. (1990) obtained the flow rates $r[t]$:

$$r[t]=\begin{cases}\dfrac{\alpha s}{\alpha-\beta}, if\ t\in[t_q,\tilde{t}) \\ \dfrac{\alpha s}{\alpha+\gamma}, if\ t\in(\tilde{t},t_{q'}],\end{cases} \tag{23}$$

where t_q and $t_{q'}$ are times at which congestion begins and ends. Arnott et al. (1990) further derived the three times characterising congestion:

$$t_q = t^* - \frac{\gamma}{\beta + \gamma}\frac{N}{s}, \tag{24}$$

$$t_{q'} = t^* + \frac{\beta}{\beta + \gamma}\frac{N}{s}, \tag{25}$$

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(\beta + \gamma)}\frac{N}{s}, \tag{26}$$

where N is the number of travellers.

4.2. Congestion with automated vehicles

The most intuitive approach when studying the congestion effects of any changes in scheduling preferences would be to consider, whether the changes can be expressed as a transformation of the parameters α, β, γ. If such transformation could be found, we could use the results (23)-(26), while only modifying the parameters therein. For the Home AV such a transformation is intuitive. Replacing α with $\alpha(1 - e_h)$ leads to the desired result (and replicates the result of van den Berg & Verhoef, 2016). In case of Universal and Work AVs however, it is not immediately clear what transformation of the α, β, γ parameters would capture the AV impact on travel costs (see Figure 2). Therefore, it is necessary to follow the path of Arnott et al. (1990) to obtain the equilibrium flow rates for these AVs. As the on-board activities lead to more complex forms for the equilibrium flow rates, Figure 5 is helpful in the derivations. Similarly to Figure 4 for conventional vehicles, Figure 5 shows all the utility components needed to compute the flow rates for AVs. Compared to the Home AV, it can be seen that the Universal and Work AV results contain an additional line for computing the equilibrium flow rate. This is needed because the utility of time spent in the AV changes depending on the clock time. Before t^*, the utility during travel is obtained from home activity carried out in a Universal AV or early work activity carried out in a Work AV. After t^*, the utility is obtained from late work activity carried out in either the Universal or Work AV.

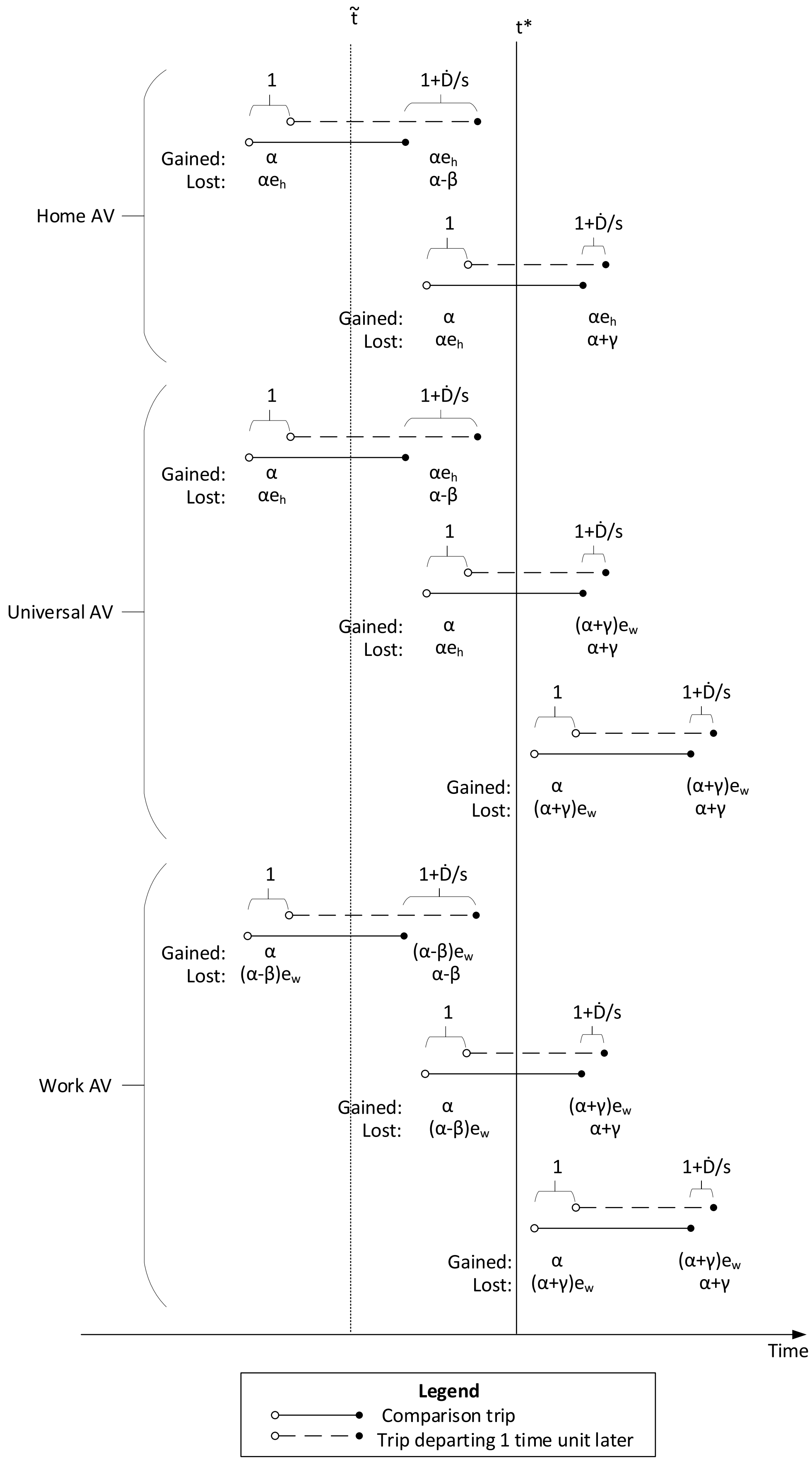

Figure 5 Utility components for computing equilibrium flow rate with AVs

Table 2 summarises the parameters needed to fully describe congestion patterns. The equilibrium flow rates can be derived from Figure 5 by balancing the utility components in each line. The congestion start, end and undelayed times are derived in Appendix B. It is found that the start and end times of congestion are the same for conventional vehicles and all AVs, while the undelayed departure time is earlier for all AV-types than for the conventional vehicles, and even earlier, if the AV facilitates home activities. The last row in Table 2 indicates that the results are valid only for the specified relationships between efficiency factors e_h, e_w and the parameters α, β, and γ. These conditions follow from the definitions of the three AV types and from a requirement that the flow rates are positive. It can be shown that these conditions are stronger than the sufficient condition for the optimal departure time to be $\tilde{t}$ in the no-congestion case (i.e., $e_w < 0.5$, as derived Appendix A). As an example, for common values in the literature $\alpha = 2$, $\beta = 1$, $\gamma = 4$ (Small, 1982, 2015), the highest possible e_h that satisfies the conditions in Table 2 would be 0.5, and the highest possible e_w would be 0.33. These values are used also for the further illustrations of the congestion patterns.[4]

Table 2 Flow rates, congestion start, end times, undelayed times for homogeneous vehicle population

	Home AV	**Universal AV**	**Work AV**
Optimal activity before t^*	Home	Home	Work
Optimal activity after t^*	Home	Work	Work
Equilibrium flow rate $r[t]$	In departure time interval $t \in [t_q, \tilde{t}]$:		
	$\frac{\alpha(1-e_h)}{\alpha(1-e_h)-\beta}s$	$\frac{\alpha(1-e_h)}{\alpha(1-e_h)-\beta}s$	$\frac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha-\beta)e_w-\beta}s$
	In departure time interval $t \in [\tilde{t}, t^*]$:		
	$\frac{\alpha(1-e_h)}{\alpha(1-e_h)+\gamma}s$	$\frac{\alpha(1-e_h)}{\alpha-(\alpha+\gamma)e_w+\gamma}s$	$\frac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s$
	In departure time interval $t \in [t^*, t_{q'}]$:		
	$\frac{\alpha(1-e_h)}{\alpha(1-e_h)+\gamma}s$	$\frac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s$	$\frac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s$
Congestion start time t_q	$t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s}$		
Undelayed departure time $\tilde{t}$	$t^* - \frac{\beta\gamma}{\alpha(1-e_h)(\beta+\gamma)}\frac{N}{s}$	$t^* - \frac{\beta\gamma}{\alpha(1-e_h)(\beta+\gamma)}\frac{N}{s}$	$t^* - \frac{\beta\gamma}{(\alpha-(\alpha-\beta)e_w)(\beta+\gamma)}\frac{N}{s}$
Congestion end time $t_{q'}$	$t^* + \frac{\beta}{\beta+\gamma}\frac{N}{s}$		

[4] Note that the absolute values of the parameters do not matter, only their ratios.

Conditions	$e_h < \frac{\alpha-\beta}{\alpha}$ $e_w < \frac{\alpha}{\alpha+\gamma} e_h$	$\frac{\alpha-\beta}{\alpha} e_w < e_h < \frac{\alpha-\beta}{\alpha}$ $\frac{\alpha}{\alpha+\gamma} e_h < e_w < \frac{\alpha}{\alpha+\gamma}$	$e_h < \frac{\alpha-\beta}{\alpha} e_w$ $e_w < \frac{\alpha}{\alpha+\gamma}$

The resulting congestion shapes for all AV-types and the base conventional vehicle are illustrated in Figure 6. This and later figures use queueing time as an indicator for the severity of the congestion. The queueing time $T[t]$ is a function of the departure rate $r[u]$:

$$T[t] = \int_0^t \frac{r[u] - s}{s} du. \tag{27}$$

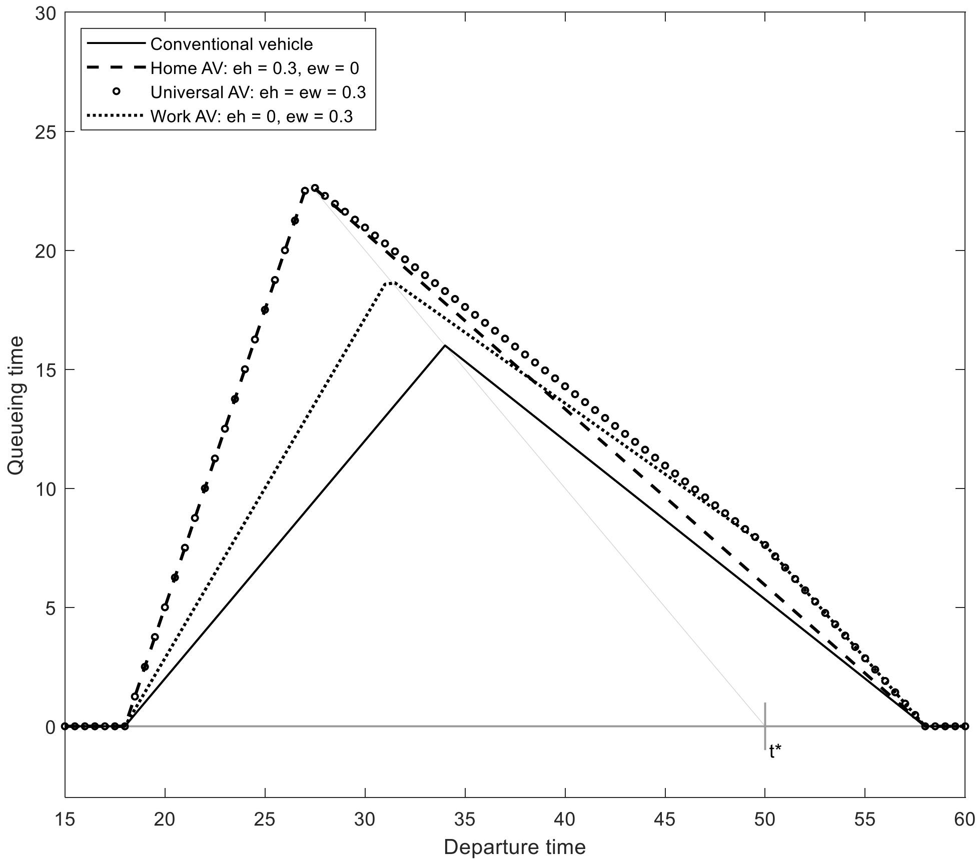

Figure 6 Development of queueing times for conventional vehicles and AVs. $N = 200$, $s = 5$, $(\alpha, \beta, \gamma) = (2,1,4)$, $t^* = 50$.

Four properties of AV congestion can be observed from Figure 6. First, congestion is more severe with AVs compared to conventional vehicles. Second, congestion is more skewed to earlier times for the Home AVs and to later times for Work AVs. Universal AVs partially overlap with both Home and Work AV graphs, thereby being skewed in both directions. It is noteworthy that this result follows from the $\alpha - \beta - \gamma$ preferences, which do not lead to any changes in optimal departure times in the no congestion case (see section 3.2). Intuitively, an even stronger skew in congestion could be expected, if general scheduling preferences were used. Third, Home and Universal AVs lead to longer maximum queueing times than Work AVs. Fourth, congestion starts and ends at the same time for all vehicles.

This leads to a conclusion that, although congestion levels are increasing, the experienced costs of congestion do not change.

It can be shown that the first, second and fourth properties apply for all parameter values, not only those used in Figure 6. However, the third property applies for such parameter values that fulfil the condition $\alpha e_h^H > (\alpha - \beta) e_w^W$, where e_h^H is the efficiency of home activities in the Home or Universal AV, and e_w^W is the efficiency of work activities in the Work AV. This condition states that before the preferred arrival time, the utility from on-board activities is higher in Home (or Universal) AVs than in Work AVs. If that was not the case, then the Home AV would be inferior to Work AV in terms of the on-board activity facilitation, and the queueing times of Home AVs would be shorter than of Work AVs at all departure times. The proofs of these four properties are in Appendix C .

4.3. Congestion with mixed vehicles

Given that all AV types intensify the congestion, but possibly in different directions, it is useful to see the net congestion effect of having different AVs in the population. Arnott et al. (1994) demonstrated how this can be done using the so-called Travel Equilibrium Frontier (TEF). The equilibrium queueing times of different travellers (as illustrated in Figure 6) can be interpreted as the cost (in terms of travel time) which they are willing to pay to depart at every time moment. Having several groups of travellers share the road, the ones who pay most occupy the respective departure time slot, which is represented by the TEF. To obtain the TEF with a specified number of travellers in each group, the graphs need to be 'scaled down or up', such that all travellers of each group depart during the time intervals, when their graph lies above other graphs.

Three combinations of vehicles are used to demonstrate the net congestion patterns in our case: Home AVs and conventional vehicles (Figure 7), Work AVs and conventional vehicles (Figure 8), and Home and Work AVs (Figure 9). Given every combination, half of the individuals use each vehicle. The necessary adjustment of one of the graphs in each case is illustrated by the move from 'original' to 'modified' graphs in Figure 7 to Figure 9.[5] Usually, the graph with the higher peak needs to be scaled down, because it would lie above the other graph at most of the departure times. This means that the longest queueing times would in general be reduced with mixed vehicles as compared to the case when all individuals use the AV with the highest peak.

The resulting graphs in Figure 7 and Figure 8 show, in line with van den Berg and Verhoef (2016), that having a mixture of AVs and conventional vehicles leads to the AVs occupying the central departure time interval, and the conventional vehicles departing as the first and last in the congestion. This illustrates how both types of on-board activities reduce the travel time costs of AV users and make them less averse to long travel times. In addition to departing at the middle of the peak however, Work AVs also tend to depart later than Home AVs and conventional vehicles (Figure 8 and Figure 9). If the Work and Home AVs offer comparable on-board activity experience (in the sense explained at the end of section 4.2), the graph of Home AV is scaled down (Figure 9). In this case, having more travellers use Work AVs reduces the congestion levels. However, if Home AVs offer inferior on-board experience compared to Work AVs, the converse is true: having more travellers use Home AVs would be beneficial. The effect would then resemble the combination of Work AVs and conventional vehicles in Figure 8.

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[5] The MATLAB code used to create figures can be found in https://gitlab.com/BaibaP/congestion-graphs-with-avs/

As a side note, the travellers whose congestion graph is scaled down benefit from mixed vehicles in the population. Therefore, if conventional vehicle users are mixed with any AV users, then the AV users would benefit (Figure 7 and Figure 8). If Home and Work AVs are mixed, then the users of Home AVs would benefit (Figure 9), unless the Home AVs are inferior to Work AVs, in which case Work AVs would benefit from the mixture. In general, the higher the efficiency of on-board activities is offered by AVs, the more likely are the users of these AVs to cause severe congestion, and the more likely they are to benefit from sharing a road with travellers whose activities are less well facilitated.

Hereby, this section has demonstrated that, given the $\alpha-\beta-\gamma$ scheduling preferences and bottleneck setting, travellers whose home activities are better facilitated on board than work activities, would prefer to depart earlier than conventional vehicle users and increase the severity of congestion in its early to middle part. Similarly, travellers whose work activities are better facilitated on board than home activities, would prefer to depart later than conventional vehicle users and increase the congestion mostly in its middle to late part. Given similar levels of activity facilitation on-board, the increase in queueing times due to Work AVs is smaller than due to Home AVs. Thereby, Work AVs have a moderating effect on the increasing congestion levels.

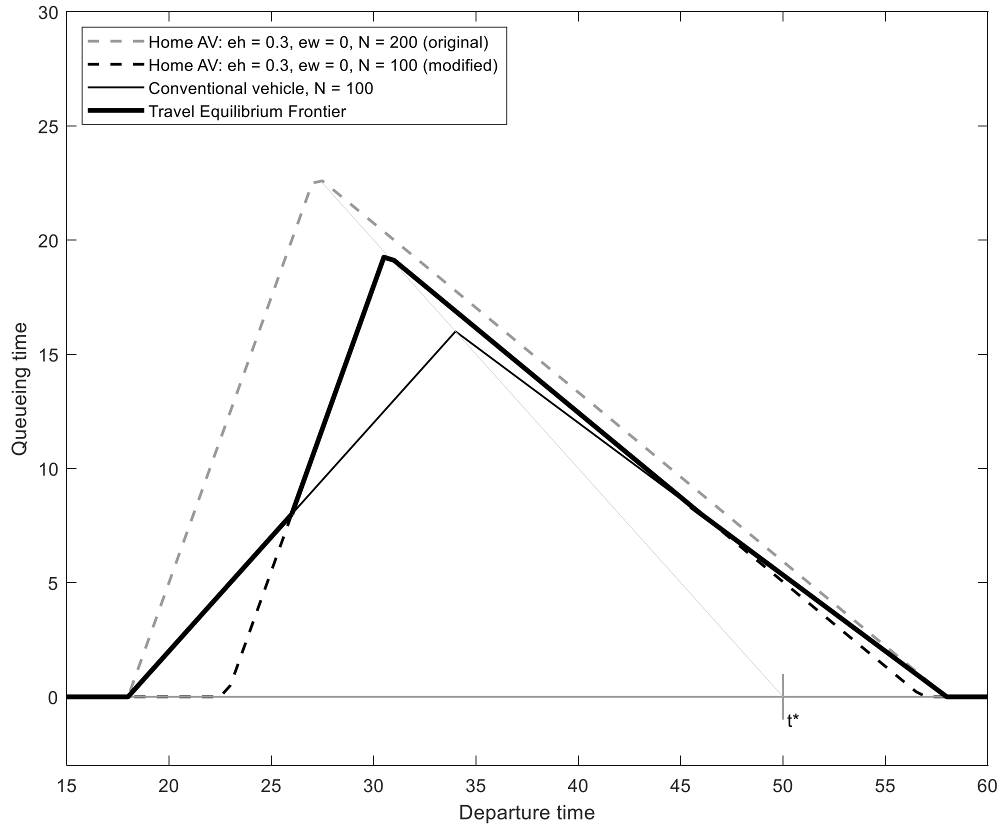

Figure 7 Development of queueing times with half-half split between conventional vehicles and home- facilitating AVs. $N_{CV} = N_{Home\ AV} = 100$, $s = 5$, $(\alpha, \beta, \gamma) = (2,1,4)$, $t^* = 50$.

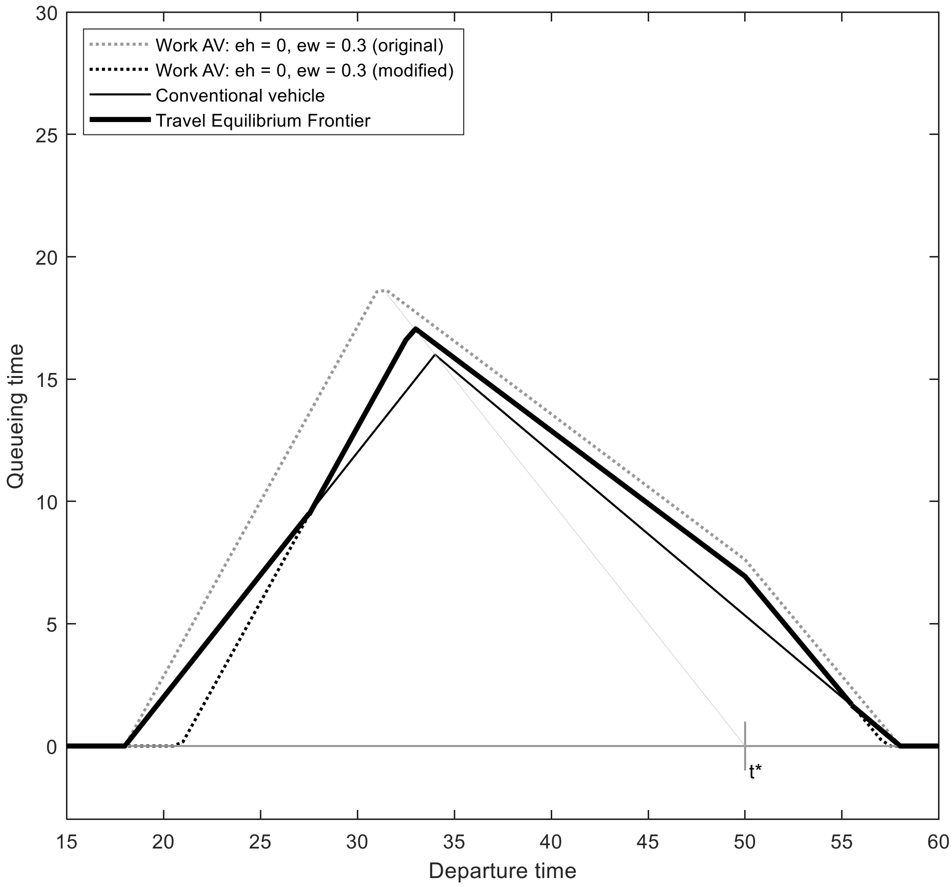

Figure 8 Development of queueing times with half-half split between conventional vehicles and work- facilitating AVs. $N_{CV} = N_{Work\ AV} = 100$, $s = 5$, $(\alpha, \beta, \gamma) = (2,1,4)$, $t^* = 50$.

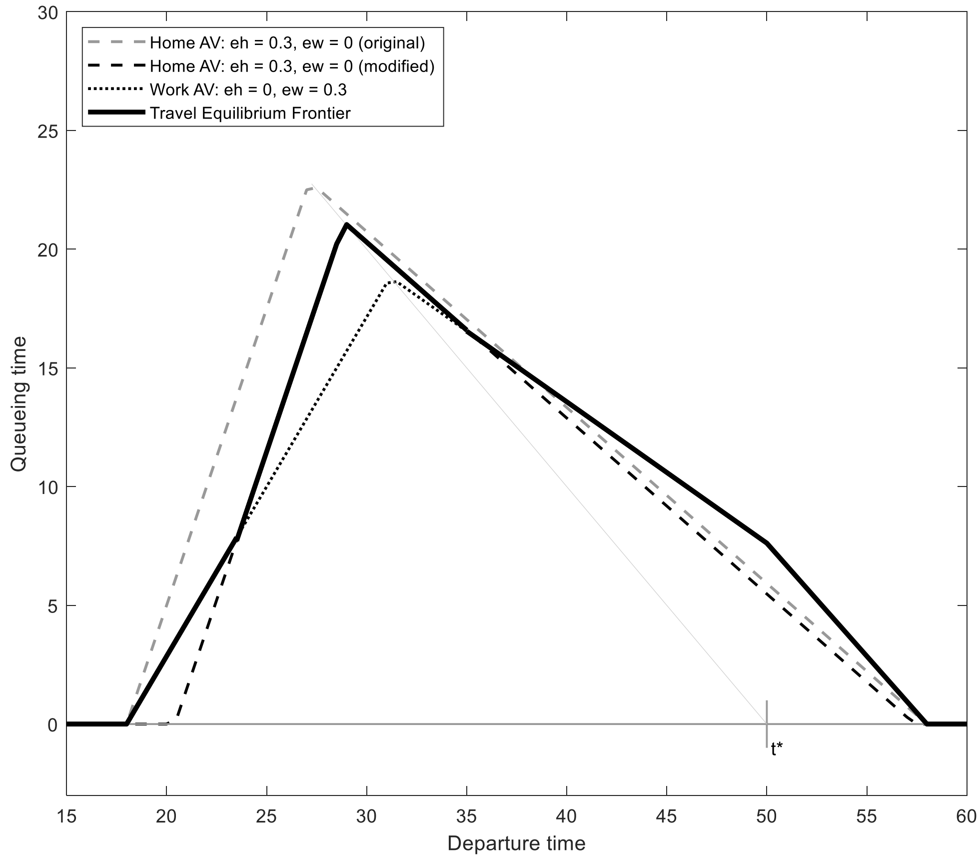

Figure 9 Development of queueing times with half-half split between home- and work- facilitating AVs. $N_{Home\ AV} = N_{Work\ AV} = 100$, $s = 5$, $(\alpha, \beta, \gamma) = (2,1,4)$, $t^* = 50$.

5. Discussion and suggestions for further research

5.1. Comparison with the travel time penalty approach

This work started from a proposition that it is important to differentiate among on-board activities when modelling departure time choice and congestion patterns. It was assumed that, similarly to out-of-vehicle activities, the utility of different on-board activities varies with clock-time. In contrast, the travel time penalty approach assumes that the utility of on-board activities is time-independent. Now we are in a position to ask: has the approach taken in this paper yielded qualitatively different results than the travel time penalty approach would have?

In the case of no congestion and general scheduling preferences, the answer is 'yes'. If the utility of on-board activities did not vary with time, the on-board activities would not influence the departure time preference, and the optimal departure time of conventional vehicles would be maintained. Formally, the second integrals of the total home and work utility functions (1) and (2) would not depend on t, and hence would disappear when the total utility (3) is differentiated with respect to t.

In the case of congestion and the $\alpha - \beta - \gamma$ preferences, the answer is 'yes, but with an exception'. Different congestion patterns were obtained for Home-, Work-, and Universal AVs. However, because the $\alpha - \beta - \gamma$ preferences assume constant home utility, the results of Home AV exactly replicate the travel time penalty approach (as derived in van den Berg & Verhoef, 2016). Since it is furthermore known that a constant home utility is a rough approximation (Tseng & Verhoef, 2008), this correspondence is not desirable. The only way to avoid this situation would be to adapt other scheduling preferences. The literature offers good alternatives for this endeavour: the so-called slope model (Fosgerau and Engelson, 2011), where the marginal utilities of out-of-vehicle activities are linear functions of time, or exponential scheduling preferences (Hjorth et al., 2015). A closed-form departure rate function for the slope model has recently been derived (Xiao et al., 2017) and would be useful for such study.

It can be expected that replacing the $\alpha - \beta - \gamma$ model with any type of general scheduling preferences (such as slope model or exponential preferences) would lead to larger congestion differences between conventional vehicles and AVs and among different AVs. Because of this consequence however, the weakness of the $\alpha - \beta - \gamma$ model is also its strength: the current approach provides conservative results – a lower bound of the possible influence of on-board activities on congestion patterns, which would apply even in contexts with a strong preference for a single work-start time. Nevertheless, exploring the effects of various scheduling functions on the congestion changes with AVs, while differentiating between home and work activities performed on board, is a highly recommended direction for further research.

In addition, it would be worthwhile to explore other ways of specifying the effect of on-board activities on home and work utility functions. For example, some travellers may be unable to perform work activity on board after the preferred arrival time, but they may engage in preparatory work tasks during travel. In this case, the current multiplicative function should be replaced with two efficiency factors, which would multiply the work utility before and after the preferred arrival time, respectively.

5.2. Validity and applicability to public transport and shared automated vehicles

As with all travel behaviour models, an important aspect is their validation and estimation. While there are not yet sufficient number of AVs on the roads, studies have occasionally turned to public transport to gain insights into possible effects of on-board activities (e.g., Pawlak et al., 2015; Malokin et al., 2019). Hence, a relevant question to the present study is: would the devised models apply and could they be validated using public transport data? Unfortunately, there are several important obstacles to such an application. First, future AVs could be expected to perform significantly better in facilitating on-board activities compared to current public transport. The difference may be even larger when considering on-board activities that substitute out-of-vehicle activities: recall the examples of morning home activities - getting ready, preparing and eating breakfast, getting a little more sleep - or work activities - replying emails, planning the day, adjusting meeting schedule. Several of these may require privacy, space, silence, continuity (absence of transfers), comfort and facilities that may be available in AVs, but not in public transport. Second, trade-offs involved in departure time choices are fundamentally different for car and public transport users: while car drivers trade off on-time arrival with travel time, public transport users balance on-time arrival with crowding levels and to a lesser extent, travel time and reliability. Third, public transport users face constraints (which the car drivers do not) when choosing departure time: they must choose from a set of scheduled departure times or predicted departure times according to public transport frequency. These characteristics would make the departure time choice model for a public transport user, who is able to engage in on-board activities during travel, fundamentally different from the model presented in this paper. Therefore, other sources of travel behaviour and departure time data could be more useful for estimation and validation of the current models: naturalistic experiments (Harb et al., 2018) or surveys (for example, stated choice experiments), which have been shown to provide trustworthy results in AV contexts (Wadud & Huda, 2019). This is an important direction for further research.

Nevertheless, even before having access to data supporting the current models, it is possible to argue for their face-validity. It was seen that the analytical results correspond to intuition: the possibility to substitute home or work activities with their on-board counterparts lead to departure time adjustments towards the most desirable time for these activities. Furthermore, the current work builds on established microeconomic models of scheduling preferences (Vickrey, 1969, 1973; Small, 1982), which have stood the test of time to predict departure time choice and resulting congestion patterns in a variety of contexts.

Another often asked and important question is: how would travel experience and behaviour differ between users of privately owned and shared AVs (including both car sharing and ride sharing), and would the same models be valid for these modes? Considering the current departure time choice model, two differences could be anticipated. First, the on-board activities may be facilitated to a different extent in shared AVs. The activities may be impaired by the reduced privacy and storage, personalisation possibilities, which would be available in privately owned AVs. At the same time, the facilitation may be increased, if fleet owners customise the AVs to suit various on-board activity needs. For example, some cars may be equipped with business and conference facilities, while others may be suited for resting and leisure. The net effect of sharing on the efficiency of on-board activities is an interesting question for future research. Second, clients of car and ride sharing may have less flexibility of choosing their departure time as compared to owners of vehicles: they may need to book the car in advance or coordinate with other users. Hence, the departure time choice and congestion models for future AV owners and users of shared AVs may differ somewhat; yet, the present model can provide a good starting point for modelling these scenarios.

5.3. Suggestions for further research

This work has presented the first steps in a detailed analysis of the impact of different on-board activities on congestion patterns. Nevertheless, and as importantly, it opens up a new field of study into the AV-effect on future mobility – and invites further work to investigate whether the proposed peak-skewing, increasing and moderating effects are also observed in more complex contexts. Previous sections mentioned the need to explore other scheduling preferences (section 5.1), as well as to obtain data to estimate and validate the current models (section 5.2). In addition, following are few other suggestions for further research.

1. A natural extension of the present work would be to simulate the effects of the proposed scheduling preferences in artificial and real city networks, as done by Correia and van Arem (2016), while incorporating heterogeneity in scheduling parameters. An extended simulation would also include other types of choices, such as mode- and route-choices, trip making and destination choice, to balance the effects of departure time changes with other anticipated AV effects, such as induced travel. Increased road capacity in high AV penetration scenarios would also need to be considered.
2. The setting where on-board activities may replace out-of-vehicle activities is also relevant for the study of value of travel time reliability (Fosgerau, & Karlström, 2010; Fosgerau & Engelson, 2011;

Xiao et al., 2017). This work showed that, although different on-board activities influence the optimal departure times differently, in general AV users tend to depart in the middle of the congestion, while prioritising on-time arrival. However, reliability is linked primarily to the costs of the varying arrival time. Therefore, the relative importance of travel time reliability (the so-called 'reliability ratio') might be expected to increase. At the same time, the absolute importance might decrease, because the costs of arriving late are reduced, when the destination activity is performed on board.

3. An important extension would be to account for various on-board activities when modelling the full day of a commuter and account for the flexibility of work hours, as is done in the activity-based bottleneck analyses (Zhang et al., 2005; Li et al., 2014; Zhang et al., 2019) and studies of departure time choice (e.g., Thorhauge et al., 2016). Some flexibility in activity schedules in general and work start times in particular is a prerequisite for the congestion shifting and moderating effects observed in this work.
4. This work emphasises the importance of understanding and empirically uncovering the sources of decreasing travel time disutility (Singleton, 2019). Note that the peak-mitigating effect would come into play only if on-board activities constitute a significant portion of the AV-benefits. If instead the travellers mainly appreciate the reduced burden and increased comfort when using AVs or even experience some disadvantages of converting resting time into busy activity time (Shaw et al., 2019; Pudāne et al., 2019), they would constitute a more homogeneous group, and hence, be more prone to intense congestion.
5. Finally, it would be important to incorporate potential endogeneity effects in the model. Travellers whose work or home activities could be performed on board may self-select to obtain access to (certain type of) AVs. Heterogeneity in the scheduling preferences (Koster & Koster, 2015) could also affect the choice among different types of AVs.

6. Conclusions and policy implications

The arrival of automated vehicles (AVs) is expected to increase the feasibility and role of on-board activities in people's daily schedules. This paper argued that the current ways of modelling the departure time choice and congestion impacts of the improved on-board activities, based mostly on the idea of a reduced travel time penalty, are not sufficient. While travel time penalty condenses effects of all on-board activities into a single indicator, different activities may in reality have varied impacts on travel behaviour. This intuition was supported in the present paper. A classical microeconomic approach – modelling departure time choices and their congestion impacts using scheduling functions – was extended to consider effects of different on-board activities in AVs. It was obtained that, if travellers are able to perform home activities on board (in Home AVs), they prefer to depart earlier than if they are able to perform work activities (in Work AVs). Results obtained in a minimalistic bottleneck setting indicate that more severe congestion could be expected in the AV era – on-board activities decrease people's aversion to longer travel times, thereby prioritising on-time arrival and increasing congestion. However, if several AV types are available that facilitate home and/or work activities to a similar extent, then Work AVs increase the congestion levels the least.

The model developed and results obtained in this paper can provide input for one of the key AV-related policy questions: will AVs lead to higher congestion levels and, if yes, how to avoid or mitigate that effect? While congestion can be expected to increase, travellers who are able to work during travel seem to mitigate that effect. This offers a valuable tool for policy makers: although some work tasks may be easily transferred to AVs, the mobile work possibilities could be further encouraged by allowing flexible working hours and, perhaps even, making work-equipped AVs available for a broader range of professions. Such measures should be tested using models that account for possibly diverging effects of different on-board activities. If their effects are significant, these measures could help to ensure that the celebrated benefits of AVs – such as allowing individuals to re-allocate their travel time for other activities – are maintained, while its potential downsides are reduced.

Acknowledgement

This work is funded by the Netherlands Organisation for Scientific Research (NWO) as part of the project 'Spatial and Transport impacts of Automated Driving' (STAD), project number 438-15-161. I would like to thank Caspar Chorus and Sander van Cranenburgh for the valuable discussions and support throughout this research. Furthermore, I am grateful to Mogens Fosgerau and Yousef Maknoon for the keen advice and feedback on different parts of this work, and to Erik Verhoef for helpful discussions during the hEART 2019 conference. All responsibility remains with the author.

Appendix A Proofs for the optimal departure times with $\alpha - \beta - \gamma$ scheduling preferences

Proposition A1. If the optimal departure time without congestion using $\alpha - \beta - \gamma$ preferences is not $\tilde{t} = t^* - T$, then it must be t^*.

Proof.

Departure time $\tilde{t}$ is better than any departure time $t < \tilde{t}$. The earlier departure times t would incur the same costs during travel as departing at $\tilde{t}$ (being the lost utility due to on-board activity being less efficient than home activity). However, the early departure would also incur costs due to arriving early.

Departure time t^* is better than any departure time $t > t^*$. The later departure times t would incur the same costs during travel as departing at t^* (being the lost utility due to on-board activity being less efficient than work activity). However, the later departure would also incur costs due to performing home instead of work activity after t^*.

Departure times between $\tilde{t}$ and t^* have either monotonously increasing or decreasing utility, which depends on whether a travel time unit costs more before or after t^*, see Figure 2. Therefore, the optimal departure time is either $\tilde{t}$ or t^*.

Proposition A2. Optimal departure time is t^* in two cases only: when $e_w > \gamma/(\beta + \gamma)$ for Work AV or when $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$.

Proof.

The necessary and sufficient condition for t^* to be the optimal departure time is that unit costs of travel before t^* is higher than after t^*.

For Home AVs, the condition equals $\alpha(1 - e_h) > \alpha(1 - e_h) + \gamma$, which is never true.

For Universal AVs, the condition leads to $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$.

For Work AVs, the condition leads to $e_w > \gamma/(\beta + \gamma)$.

Proposition A3. Optimal departure time is never t^* if $e_w < 0.5$ and if $\beta < \alpha < \gamma$.

Proof.

For Universal AVs, the condition from Proposition 2 requires that $(1-e_h)/(1-e_w) > 1+\gamma/\alpha$. Since it is assumed that $\gamma > \alpha$, then the strongest form of the condition is $(1-e_h)/(1-e_w) > 2$. If $e_w < 0.5$, then that will never occur, and optimal departure time for Universal AVs will never be t^*.

For Work AVs, the condition from Proposition 2 requires that $e_w > \gamma/(\beta+\gamma)$. Since it is assumed that $\gamma > \beta$, then the strongest form of the condition is $e_w > 0.5$. Hence, if $e_w < 0.5$, then optimal departure time for Work AVs is never t^*.

Appendix B Start, end and undelayed times of AV congestion

Three conditions determine the start, end and undelayed times of a congestion:

1. Total number of travellers departing equals N;
2. Duration of the congestion is $\frac{N}{s}$, where s is the bottleneck capacity;
3. Departing at the undelayed departure time leads to the arrival at the preferred arrival time t^*.

Congestion start, end, undelayed times for Universal AVs

Conditions 1 and 2:

$$\frac{\alpha(1-e_h)}{\alpha(1-e_h)-\beta}s(\tilde{t}-t_q)+\frac{\alpha(1-e_h)}{\alpha-(\alpha+\gamma)e_w+\gamma}s(t^*-\tilde{t}) + \frac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s(t_{q'}-t^*)=N \tag{B1}$$

$$t_{q'}-t_q=\frac{N}{s} \tag{B2}$$

Insert condition 2 into condition 1, and obtain t_q as a function of $\tilde{t}$:

$$t_q=\frac{\frac{\gamma\frac{N}{s}-t^*((\alpha+\gamma)e_w-\alpha e_h)}{\alpha-(\alpha+\gamma)e_w+\gamma}-\left(\frac{\alpha(1-e_h)}{\alpha(1-e_h)-\beta}-\frac{\alpha(1-e_h)}{\alpha-(\alpha+\gamma)e_w+\gamma}\right)\tilde{t}}{\frac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}-\frac{\alpha(1-e_h)}{\alpha(1-e_h)-\beta}}. \tag{B3}$$

Condition 3:

$$\tilde{t}=t^*-\frac{D(\tilde{t})}{s}=t^*-\frac{\int_{t_q}^{\tilde{t}}r(u)du-s(\tilde{t}-t_q)}{s}=t^*-\frac{\beta}{\alpha(1-e_h)-\beta}(\tilde{t}-t_q) \tag{B4}$$

Obtain $\tilde{t}$ as a function of t_q from condition 3:

$$\tilde{t}=\frac{(\alpha(1-e_h)-\beta)t^*+\beta t_q}{\alpha(1-e_h)}. \tag{B5}$$

Insert (B5) into (B3) to obtain t_q, which, after simplification, coincides with the t_q for the conventional vehicle case:

$$t_q = t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s}. \tag{B6}$$

Using (B2), the end of congestion $t_{q'}$ is

$$t_{q'} = t^* + \frac{\gamma}{\beta+\gamma}\frac{N}{s}. \tag{B7}$$

Inserting (B6) into (B5), we can obtain the undelayed departure time:

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(1-e_h)(\beta+\gamma)}\frac{N}{s}. \tag{B8}$$

Congestion start, end, undelayed times for Work AVs

Conditions 1 and 2:

$$\begin{aligned}\frac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha-\beta)e_w-\beta}s\left(\tilde{t}-t_q\right)+\frac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s(t^*-\tilde{t})\\ +\frac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}s\left(t_{q'}-t^*\right)=N\end{aligned} \tag{B9}$$

$$t_{q'} - t_q = \frac{N}{s} \tag{B10}$$

Insert condition 2 into condition 1, and obtain t_q as a function of $\tilde{t}$:

$$t_q = \frac{\dfrac{\gamma\frac{N}{s} - t^*(\beta+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma} - \left(\dfrac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha-\beta)e_w-\beta} - \dfrac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma}\right)\tilde{t}}{\dfrac{\alpha-(\alpha+\gamma)e_w}{\alpha-(\alpha+\gamma)e_w+\gamma} - \dfrac{\alpha-(\alpha-\beta)e_w}{\alpha-(\alpha-\beta)e_w-\beta}}. \tag{B11}$$

Condition 3:

$$\tilde{t} = t^* - \frac{\beta}{\alpha-(\alpha-\beta)e_w-\beta}\left(\tilde{t}-t_q\right) \tag{B12}$$

Obtain $\tilde{t}$ as a function of t_q from condition 3:

$$\tilde{t} = \frac{(\alpha-(\alpha-\beta)e_w-\beta)t^* + \beta t_q}{\alpha-(\alpha-\beta)e_w}. \tag{B13}$$

Insert (B13) into (B11) to obtain t_q. Congestion start and end times turn out to be the same for all vehicles. Insert t_q into (B13) to obtain the undelayed departure time for Work AV:

$$\tilde{t} = t^* - \frac{\beta\gamma}{(\alpha - (\alpha - \beta)e_w)(\beta + \gamma)} \frac{N}{s}. \tag{B14}$$

Appendix C Proofs for the properties of AV congestion

Proposition C1. The queueing times are longer with AVs compared to conventional vehicles.

Proof.

It is sufficient to show that the inflection points of AV graphs at $\tilde{t}$ are higher and lie earlier for the AV graphs than the inflection point of the conventional vehicle graph, and that the inflection point at t^* for Universal and Work AVs also lies above the conventional vehicle graph.

The highest peak at $\tilde{t}$ is as high as it is far from the preferred arrival time t^*. This follows from the definition of $\tilde{t}$ as the departure time that leads to on-time arrival. Knowing this, it can be seen from Table 2 that $t^* - \tilde{t}$ increases with e_h and e_w for all AV types. Therefore, the inflection point at $\tilde{t}$ is higher and earlier for the AV graphs than for the conventional vehicle graph, for which $e_h = e_w = 0$.

The peak at t^* for Universal and Work AVs lies above the conventional vehicle graph, because the Work AV graph in segment $[\tilde{t}, t^*]$ is flatter than the conventional vehicle graph. This is because the departure rate (from Table 2) is higher for Work AV in that interval: it can be verified that $(\alpha - (\alpha - \beta)e_w)/(\alpha - (\alpha + \gamma)e_w + \gamma)\ s > \alpha/(\alpha + \gamma)$ is always true. Since Work and Universal AV graphs overlap from t^* onward, the inflection point of Universal AVs is also necessarily above the conventional vehicle graph.

Proposition C2. Congestion is more skewed to earlier times for the Home AVs and to later times for Work AVs. Congestion with Universal AVs is skewed in both directions.

Proof.

To prove this property, we need to select an indicator that describes the skew well. I propose the following indicator, which captures the difference between the relative increase of congestion at times $\tilde{t}$ and t^*, while taking the congestion with conventional vehicles as a reference point:

$$S^{AV} = \frac{Q_{\tilde{t}}^{AV}}{Q_{\tilde{t}}^{CV}} - \frac{Q_{t^*}^{AV}}{Q_{t^*}^{CV}}, \tag{C1}$$

where $Q_{\tilde{t}}^{AV}$ and $Q_{\tilde{t}}^{CV}$ are queuing times at the undelayed departure time $\tilde{t}$ with AV and conventional vehicle (CV), respectively; $Q_{t^*}^{AV}$ and $Q_{t^*}^{CV}$ are the corresponding queueing times at t^*. If S^{AV} is positive, then the congestion is skewed towards earlier times as compared to the congestion with conventional vehicles; if it is negative, then congestion is skewed to later times.

The skew indicators for the Home AV (S^{AV_1}), Universal AV (S^{AV_2}) and Work AV (S^{AV_3}) are the following:

$$S^{AV_1} = \frac{1}{1 - e_h} - \frac{\alpha + \gamma}{\alpha(1 - e_h) + \gamma} = \frac{\gamma e_h}{(\alpha(1 - e_h) + \gamma)(1 - e_h)} > 0, \tag{C2}$$

$$S^{AV_2} = \frac{1}{1 - e_h} - \frac{1}{1 - e_w}, \tag{C3}$$

$$S^{AV_3} = \frac{\alpha}{\alpha - (\alpha - \beta)e_w} - \frac{1}{1 - e_w} = -\frac{\beta e_w}{(\alpha - (\alpha - \beta)e_w)(1 - e_w)} < 0. \qquad \text{(C4)}$$

This indicator shows that, indeed, Home AVs skew the congestion to earlier times; Work AVs skew it to later times. The indicator is zero for Universal AVs, if $e_h = e_w = 0$, and positive (negative), if e_h is larger (smaller) than e_w.

Proposition C3. Longer queueing times are reached with Home and Universal AVs compared to Work AVs.

Proof.

Having a congestion with any vehicle, the longest queueing time occurs at the undelayed departure time $\tilde{t}$. Following the definition of $\tilde{t}$, this queueing time equals $t^* - \tilde{t}$. Comparing the distance $t^* - \tilde{t}$ for Home (or Universal), and Work AVs, it can be obtained that $t^* - \tilde{t}$ is larger for Home and Universal AVs, whenever $\alpha e_h^H > (\alpha - \beta)e_w^W$, where e_h^H is the efficiency of home activities in the Home and Universal AV, and e_w^W is the efficiency of work activities in the Work AV. This condition determines that home activities would yield higher utility in Home AV than early work activities (before t^*) yield in Work AV. If this condition is not fulfilled, then Home AVs are inferior to Work AVs in terms of the quality of on-board activities, and Home AVs would lead to shorter queueing times than Work AVs (the congestion pattern would be only slightly altered from the conventional vehicle case).

However, if AVs are specialised to support only home, only work, or both home and work activities, and do so to a similar extent (such that none of AVs is inferior to another at all clock-times), then Work AVs would result in a smaller congestion increase than other AV types.

Proposition C4. Congestion costs with AVs are the same as with conventional vehicles.

Proof.

The start and end times of congestion are the same for conventional vehicles (24) and (25) and AVs (Table 2). At these times, the travel time is zero, and the individual experiences only the costs of being at work too early or too late. Since these costs are not influenced by AVs, the equilibrium costs of all congestion patterns in Figure 6 are the same and equal $(\beta\gamma/(\beta + \gamma)) * (N/s)$.

Appendix D Code used to create Figures 6-9

Code can be found in https://gitlab.com/BaibaP/congestion-graphs-with-avs/. Code was created in MATLAB R2018b.

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