Volatility Forecasting: Evidence from a Fractional Integrated Asymmetric Power ARCH Skewed-t Model

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Abstract

Predicting the one-step-ahead volatility is of great importance in measuring and managing investment risk more accurately. Taking into consideration the main characteristics of the conditional volatility of asset returns, I estimate an asymmetric Autoregressive Conditional Heteroscedasticity (ARCH) model. The model is extended to also capture i) the skewness and excess kurtosis that the asset returns exhibit and ii) the fractional integration of the conditional variance. The model, which takes into consideration both the fractional integration of the conditional variance as well as the skewed and leptokurtic conditional distribution of innovations, produces the most accurate one-day-ahead volatility forecasts. The study recommends to portfolio managers and traders that extended ARCH models generate more accurate volatility forecasts of stock returns.

Keywords: ARCH models, Fractional Integration, Intra-Day Volatility, Long Memory, Skewed-t Distribution, Value-at-Risk, Volatility Forecasting.

JEL: C32, C52, C53, G15.

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The usual disclaimer applies.

1. Introduction

I investigate the forecasting ability of a set of conditional volatility models in predicting the one-day-ahead conditional standard deviation of three stock indices. Taking into consideration the properties that characterize financial markets (non-synchronous trading, volatility clustering, integrated conditional variance, asymmetries in the response of volatility to the sign of returns, Box-Cox power transformation of the conditional volatility process and the asymmetric absolute innovations), I estimate the Generalized ARCH (GARCH), the Integrated Generalized ARCH (IGARCH) and the Asymmetric Power ARCH (APARCH) models. Moreover, I extend the APARCH model in order to capture the conditional variance response to past innovations by introducing the fractional integration of the conditional variance. Also, the used model framework is modified for explaining the skewness and excess kurtosis that the asset returns exhibit by assuming that the conditional innovations are skewed-t distributed. In total, 5 conditional volatility specifications are considered in the context of ARCH models and their forecasting ability is explored in two ways. First, I consider two loss functions that measure the distance between predicted and realized intra-day volatility. Second, the ability of the models in forecasting the one-day-ahead Value-at-Risk (VaR) measure, for both long and short trading positions, is investigated. In both cases, the statistical significance of model's predictive accuracy is tested based on Diedold and Mariano (1995) methodology.

The APARCH model that takes into consideration both the fractional integration of the conditional variance and the skewed and leptokurtic conditional distribution of innovations produces the most accurate one-day-ahead volatility forecasts. The extended ARCH model exhibits superior forecasting ability over the parsimonious ARCH models. This result appears in accord with the studies of Brooks and Persand (2003), Giot and Laurent (2003), Hansen and Lunde (2003) and Vilasuso (2002). The present study reinforces the conclusions of the previous studies, providing evidence for the accuracy of the one-day-ahead volatility forecasts as measures of the realized intra-day volatility and of the VaR estimation. In respect of one-day-ahead volatility forecasting, the extended model does not suffer from over-fitting but gives the most accurate predictions.

In the second section of this article, a short and concise description of the ARCH framework is provided. Section three contains the used dataset and illustrates the estimation method of the models. The fourth section compares the predictive ability of the estimated models. In the last section, the conclusions of this study are presented.

2. *The ARCH framework of Estimating Volatility*

For P_t denoting the price of an asset at time t, let $y_t = \ln(P_t / P_{t-1})$ denote the continuously compounded return series and $E(y_t \mid I_{t-1}) = E_{t-1}(y_t) = \mu_t$ denotes the conditional mean given the information set I_{t-1} available in time $t-1$. The innovation process for the conditional mean is then given by $\varepsilon_t = y_t - \mu_t$ with corresponding unconditional variance $V(\varepsilon_t) = \sigma^2$ and zero unconditional mean. The conditional variance is defined by $V(y_t \mid I_{t-1}) = V_{t-1}(y_t) = \sigma_t^2$. An ARCH process, $\{\varepsilon_t\}$, can be presented as:

$$
\begin{aligned}
y_t &= \mu_t + \varepsilon_t \\
\varepsilon_t &= z_t \sigma_t \\
z_t &\overset{i.i.d.}{\sim} f(0,1;w) \\
\sigma_t^2 &= g(\sigma_{t-1}, \sigma_{t-2}, \ldots; \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots; \upsilon_{t-1}, \upsilon_{t-2}, \ldots),
\end{aligned}
\tag{1}
$$

where $f(.)$ is the density function of z_t, with $E(z_t) = 0, V(z_t) = 1$, w is the vector of the parameters of f, $g(.)$ is a linear or nonlinear functional form and υ_t is a vector of predetermined variables included in I_t. The conditional mean is considered as a first order autoregressive process, $\mu_t = c_0 + c_1 y_{t-1}$, in order to account for the non-synchronous trading. According to Campbell et al. (1997), "The non-synchronous trading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular lengths".

Engle (1982) introduced the original form of $\sigma_t^2 = g(.)$ as a linear function of the past q squared innovations:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} \left(a_i \varepsilon_{t-i}^2\right), \tag{2}$$

where $\alpha_0 > 0$, $a_i \geq 0$, for $i = 1,...,q$. Bollerslev (1986) proposed a generalization of the ARCH(q) process, the GARCH(p,q) model, by allowing for past conditional variances in the current conditional variance equation:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} \left(a_i \varepsilon_{t-i}^2\right) + \sum_{j=1}^{p} \left(b_j \sigma_{t-j}^2\right), \tag{3}$$

where $\alpha_0 > 0$, $a_i \geq 0$, $i = 1,...,q$ and $b_j \geq 0$, $j = 1,..., p$. The unconditional variance is equal to $\sigma^2 = a_0 \left(1 - \sum_{i=1}^{q} a_i - \sum_{j=1}^{p} b_j\right)^{-1}$. However, in the majority of empirical applications, the estimate for $\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} b_j$ turns out to be very close to unity, providing the motivation, for the development of the so-called integrated GARCH, or IGARCH(p,q), model by Engle and Bollerslev (1986):

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i L^i \varepsilon_t^2 + \sum_{j=1}^{p} b_j L^j \sigma_t^2 \text{, for } \sum_{i=1}^{q} a_i + \sum_{j=1}^{p} b_j = 1, \tag{4}$$

where L is the lag operator. The polynomial $\sum_{i=1}^{q} a_i L^i + \sum_{j=1}^{p} b_j L^j = 1$ has $d > 0$ unit roots and $\max(p,q) - d$ roots outside the unit circle. The Exponentially Weighted Moving Average (EWMA) model, used by RiskMetrics^{TM} (1995) in their VaR methodology for daily data, is a special case of the IGARCH(1,1) model with zero intercept and $b_1 = 0.94$.

Ding et al. (1993) introduced the APARCH(p,q) model, which allows the power δ of the heteroscedasticity equation to be estimated from the data:

$$\sigma_t^{\delta} = a_0 + \sum_{i=1}^{q} a_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^{\delta} + \sum_{j=1}^{p} b_j \sigma_{t-j}^{\delta}, \tag{5}$$

where $a_0 > 0$, $\delta \geq 0$, $a_i \geq 0$ and $-1 < \gamma_i < 1$, for $i = 1,...,q$ and $b_j \geq 0$, for $j = 1,..., p$. The model introduces a Box-Cox power transformation on the conditional standard deviation process and on the asymmetric absolute innovations.

Tse (1998) based on the observation that volatility tends to change quite slowly over time, introduced the Fractionally Integrated Asymmetric Power ARCH, or FIAPARCH(p,q), model in the following form:

$$\sigma_t^{\delta} = a_0 + \left(1 - (1 - b(L))^{-1} a(L)(1-L)^{-d}\right)\left(|\varepsilon_t| - \gamma\varepsilon_t\right)^{\delta}, \tag{6}$$

where $a(L) = \sum_{i=1}^{q} a_i L^i$ and $b(L) = \sum_{j=1}^{p} b_j L^j$. The model (6) is strictly stationary and ergodic for $0 \leq d \leq 1$. In contrast to the GARCH and IGARCH models where shocks to the conditional variance either dissipate exponentially or persist indefinitely, for the FIAPARCH model the response of the conditional variance to past shocks decays at a slow hyperbolic rate.

In the original paper of Engle (1982), the density function of z_t, $f(.)$, was considered as the standard normal distribution. Bollerslev (1987) was the first who introduced non-normality for $f(.)$ in order to produce unconditional distribution with thicker tails. Lambert and Laurent (2000, 2001), based on Fernández and Steel (1998), suggested that not only the conditional distribution of innovations may be leptokurtotic, but also asymmetric and proposed the use of the skewed Student t density function:

$$f(z_t; v, g) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left(\frac{2s}{g + g^{-1}}\right)\left(1 + \frac{sz_t + m}{v-2} g^{-d_t}\right)^{-\frac{v+1}{2}}, \quad v > 2, \tag{7}$$

where g is the asymmetry parameter, v denotes the degrees of freedom of the distribution, $\Gamma(.)$ is the gamma function, $d_t = 1$ if $z_t \geq -m/s$, and $d_t = -1$ otherwise, $m = \Gamma((v-1)/2)\sqrt{(v-2)}\left(\Gamma(v/2)\sqrt{\pi}\right)^{-1}\left(g - g^{-1}\right)$ and $s = \sqrt{g^2 + g^{-2} - m^2 - 1}$.

Financial literature is full of ARCH presentations whose modelisation was motivated by the various characteristics of financial markets. A wide range of proposed ARCH processes is covered in surveys such as Bera and Higgins (1993), Bollerslev et al. (1994), Degiannakis and Xekalaki (2004), Gouriéroux (1997), Li et al. (2001) and Poon and Granger (2003).

3. Dataset and Method of Model Estimation

The data set used in the present study consists of the CAC40, DAX30 and FTSE100 stock index daily returns in the period from July 10th, 1987 to June 30th, 2003 and is obtained from DataStream. Figure 1 plots the daily returns of the three stock indices and Table 1 presents their basic statistics. There is negative skewness and excess kurtosis in the three stock index daily returns, indicating the use of an asymmetric and leptokurtic conditional distribution of innovations such as the skewed Student t distribution.

A number of studies, such as Brooks and Persand (2003), Giot and Laurent (2003) and Hansen and Lunde (2003), investigated whether more flexible models are able to beat the forecasting ability of the parsimonious GARCH(1,1) model. In the present study the GARCH(1,1) model with normally distributed innovations (GARCH(1,1)-N) and its extensions, the IGARCH(1,1)-N, the APARCH(1,1)-N, the FIAPARCH(1,1)-N and the FIAPARCH(1,1) with skewed-t conditional distributed innovations (FIAPARCH(1,1)-skT) models are estimated. The main purpose of the study is to provide evidence for the use of extended ARCH models, such as the FIAPARCH(1,1)-skT model, in predicting future volatility.

The five models are estimated using a rolling sample of constant size equal to 2000 observations, by the maximum likelihood method. The GARCH(1,1)-N model is the most parsimonious model and requires the estimation of 5 parameters (c_0, c_1, a_0, a_1, b_1). On the other hand, the most extended model is the FIAPARCH(1,1)-skT, which has 10 parameters for estimation

($c_0, c_1, a_0, a_1, b_1, \delta, d, \gamma, v, g$). Since, in estimating non-linear ARCH models, no closed form expressions are obtainable for the parameter estimators, the BHHH iterative algorithm (Berndt et al. 1974) is employed. For technical details about the model estimation, the interested reader is referred to Bollerslev et al. (1994, section 2.2.1) and Degiannakis and Xekalaki (2004, section 4.1). The parameters of the models are re-estimated every trading day, in order to incorporate the most recent information for the trading behavior. This is a major difference with other studies such as Hansen and Lunde (2003), Klaassen (2002), Vilasuso (2002), where the forecasts were calculated using the parameters of the model that had been estimated once. Vilasuso (2002) estimated the parameters of his model using all the available dataset, while Klaassen (2002) and Hansen and Lunde (2003) estimated the in-sample parameters of their models and based on them, they derived the volatility forecasts. Giot and Laurent (2003) re-estimated the model parameters every 50 trading days as they supported that there are no qualitative differences as when one updates the parameters on a daily base.

4. Evaluate the Predictive Ability of the ARCH Models

In order to investigate the predictability of the models, a two-fold evaluation procedure is followed. First, I define two statistical criteria to measure the distance between predicted and realized intra-day volatility. Second, I compute the VaR measure and investigate which model can predict the next day's financial loss more accurately.

4.1 Predicting Intra-day Volatility

Two measures of the closeness of the forecasts to the realizations are used in order to evaluate the ability of the models in forecasting one-step-ahead intra-day volatility: 1) the heteroscedasticity-adjusted squared error (HASE) and the logarithmic error (LE) loss functions. Denoting the one-step-ahead forecasting variance by $\sigma^2_{t+1|t}$, and the realized intra-day variance at time $t+1$ by h^2_{t+1}, the loss functions were considered as:

$$HASE = T^{-1}\sum_{t=1}^{T}\left(1 - h^2_{t+1}/\sigma^2_{t+1|t}\right)^2 , \tag{8}$$

$$LE = T^{-1}\sum_{t=1}^{T}\ln\left(h^2_{t+1}/\sigma^2_{t+1|t}\right)^2 , \tag{9}$$

where T is the number of the one-step-ahead volatility forecasts. The HASE function and the LE function were introduced by Andersen et al. (1999) and Pagan and Schwert (1990), respectively. The realized intra-day volatility of day t is computed as:

$$h^2_t = \sum_{j=1}^{m-1}\left(\ln\left(P_{(j+1/m),t}\right) - \ln\left(P_{(j/m),t}\right)\right)^2 , \tag{10}$$

where $P_{(m),t}$ is the discretely observed series of prices of an asset at day t with m observations per day. In order to avoid market microstructure frictions without lessening the accuracy of the continuous record asymptotics, I used five-minute linearly interpolated prices. The 5-minutes sampling frequency were also used by Andersen and Bollerslev (1998), Andersen et al. (1999), Andersen et al. (2000), Andersen et al. (2001a) and Kayahan et al. (2002) among others. For information and reference about the construction and the properties of the intra-day data, the reader is referred to Andersen et al. (2001b), Andersen et al. (2003) and Andersen et al. (2004). Olsen and Associates provided the intra-day quotation data.

In the sequel, the model with the lowest value of the loss function was tested against the other models in order to investigate whether its forecasting performance is statistically superior. The statistical significance of the volatility forecasts was investigated using the Diedold and Mariano (1995) methodology. Let $DM_t = HASE^A_t - HASE^B_t$, where $HASE^A_t$ and $HASE^B_t$ are the HASE values at time t of models A and B, respectively. The Diebold-Mariano statistic is the t-statistic derived by the regression of DM_t on a constant with Newey and West (1987) heteroscedastic and consistent (HAC) standard errors. Under the null hypothesis, the model with the lowest value of the loss function has equal predictive ability with the alternative model. Under the alternative hypothesis, the model with the lowest value of the loss function has superior predictive ability. According to Table 2, which presents the values of the HASE and LE loss functions and the relative Diebold-Mariano statistics, the FIAPARCH(1,1)-skT model either yields the lowest value of the loss functions or produces volatility forecasts whose predictive accuracy is not statistically significant to the forecasts of the model with the lowest value of the loss function. Only, in the case of the FTSE100 index and the LE loss function, the FIAPARCH(1,1)-skT model is statistically significant to the FIAPARCH(1,1)-N model, which yields the lowest value of the LE loss function. According to the 2nd Figure, which plots the realized intra-day volatility and the relative one-day-ahead volatility forecasts of the FIAPARCH(1,1)-skT model, it tracks the realized volatility very close.

4.2 Predicting VaR Measure

VaR at level of significance a, is a measure, which refers to the predicted financial loss over a specified period with a given probability $1-a$. Traders do not concentrate only on buying assets, as their portfolios may consist of both long and short trading positions. Thus, the ability of the models discussed here in forecasting VaR should be evaluated for trades that are profitable regardless of whether the asset price increases or decreases. The VaR number for the next trading day, given the information set at day t, is computed as:

$$VaR_{t+1|t} = F_a \sigma_{t+1|t} , \tag{11}$$

where F_a is the corresponding quantile of the assumed unconditional distribution of innovations. For long and short trading positions F_a is the left and right quantile at $a\%$, respectively. The adequacy of the models, in a risk management framework, is investigated by the construction of a loss function that measures the squared distance between actual daily returns and one-step-ahead VaR.[1] The model with the minimum value of (12) is considered as the most appropriate:

$$T^{-1}\sum_{t=1}^{T}\left(d_{t+1}\left(VaR_{t+1|t}-y_{t+1}\right)^{2}\right). \tag{12}$$

For long trading positions, $d_t = 1$ if $y_t < VaR_{t|t-1}$ and $d_t = 0$ otherwise, whereas, for short trading positions, $d_t = 1$ if $y_t > VaR_{t|t-1}$ and $d_t = 0$ otherwise. According to Table 3, the FIAPARCH(1,1)-skT model achieves the lowest value of loss function (12) for both long and short trading positions and the three stock indices under investigation. Under the null hypothesis of the Diebold-Mariano test, the model with the lowest value of the loss function has equal predictive ability in forecasting one-day-ahead VaR with the alternative model. The accuracy of the FIAPARCH(1,1)-skT model's VaR predictions is statistically superior in the majority of the cases.

Brooks and Persand (2003), Giot and Laurent (2003), Hansen and Lunde (2003) and Vilasuso (2002) among others have reached to the conclusion that flexible models produce accurate volatility forecasts. Brooks and Persand (2003) modeled volatility for S&P500 and Southeast Asian stock market indices. They noted that models, which allow for asymmetries either in the unconditional return distribution or in the response of volatility to the sign of returns, lead to VaR measures, which would be deemed more accurate under the Basle Committee rules. Giot and Laurent (2003) estimated daily VaR for stock index returns by using an APARCH model with skewed distribution and pointed out that it performed better than the pure symmetric one, because it reproduced the characteristics of the empirical distribution more accurately. Hansen and Lunde (2003) investigated DM-$ exchange rates and IBM stock returns and concluded that, although, the parsimonious GARCH(1,1) model was not outperformed in its forecasting ability by more sophisticated models in the case of DM-$ exchange rates, in the case of stock returns, models, which account for asymmetric effects, have produced more accurate volatility forecasts. Vilasuso (2002) showed that a FIGARCH model with normally distributed innovations generated superior out-of-sample exchange rate volatility forecasts. The present study has reached the conclusion that the FIAPARCH(1,1)-skT, an extended ARCH model, generates more accurate volatility forecasts. Usually, researchers reach to the conclusion that the extended models provide excellent in-sample performance but poor out-of-sample predictability. However, in the case of one-day-ahead volatility forecasting, the FIAPARCH(1,1)-skT model does not seem to suffer from the over-fitting symptom. On the contrary, it produces the most accurate volatility forecasts in the majority of the cases.

5. Conclusion

The ability of volatility models, under the ARCH framework, to produce accurate forecasts of i) one-day-ahead realized intra-day volatility and ii) one-day-ahead VaR was investigated. It was found that the FIAPARCH(1,1)-skT model generates improved one-day-ahead volatility predictions. It is an asymmetric ARCH model that takes into consideration the Box-Cox power transformation of the conditional standard deviation process and the asymmetric absolute innovations, the fractional integration of the conditional variance as well as the skewed and leptokurtic conditional distribution of innovations. Therefore, the use of flexible models, which account for recent developments in the area of asset returns' volatility, is important in obtaining more accurate one-step-ahead volatility forecasts. Portfolio managers should take into consideration the ability of volatility specifications, such as the FIAPARCH(1,1)-skT model, in forecasting one-day-ahead volatility more accurately.

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Table 1. Daily returns basic statistics.

	CAC40	DAX30	FTSE100
Mean	0.000183	0.000206	0.000132
Standard Deviation	0.014108	0.015186	0.011155
Skewness	-0.2671	-0.4850	-0.7660
Kurtosis	7.1050	8.8777	13.2623
Number of Observations	4001	4008	4031

Table 2. HASE and LE loss functions, as they are computed in (8) and (9), respectively, and the relative Diebold-Mariano statistics.

	CAC40		DAX30		FTSE100	
Model	HASE Loss Function	DM Statistic	HASE Loss Function	DM Statistic	HASE Loss Function	DM Statistic
GARCH(1,1)-N	9.045655	-2.52774*	0.485761	-1.94042	0.488443	-1.58756
IGARCH(1,1)-N	7.970780	-2.21427*	0.447328	--	0.479234	-1.14582
APARCH(1,1)-N	7.349019	-2.47113*	0.474691	-0.57499	0.454476	-0.64122
FIAPARCH(1,1)-N	6.341504	-1.51987	0.464788	-0.49778	0.445565	--
FIAPARCH(1,1)-skT	6.252786	--	0.452104	-0.11375	0.453901	-1.44746
Model	LE Loss Function	DM Statistic	LE Loss Function	DM Statistic	LE Loss Function	DM Statistic
GARCH(1,1)-N	0.762832	-5.67353**	1.473186	-7.59009**	1.207131	-5.60130**
IGARCH(1,1)-N	0.891378	-8.71770**	1.570528	-10.1103**	1.213684	-5.53343**
APARCH(1,1)-N	0.704857	--	1.292694	-6.26598**	1.139915	-3.81473**
FIAPARCH(1,1)-N	0.724752	-2.02689*	1.456512	-7.11328**	1.062456	--
FIAPARCH(1,1)-skT	0.719542	-1.26751	1.136389	--	1.079832	-2.71098**

*Statistically significant at 5%.

** Statistically significant at 1%.

Table 3. The loss function in (12), which measures the squared distance between actual daily returns and one-day-ahead VaR forecast, and the relative Diebold-Mariano statistics. The first and the second panel refer to the VaR at $a = 5\%$ and $a = 1\%$ levels of significance, respectively.

	CAC40		DAX30		FTSE100	
			$a = 5\%$			
			Long Positions			
Model	Loss Function	DM Statistic	Loss Function	DM Statistic	Loss Function	DM Statistic
GARCH(1,1)-N	0.065551	-3.89179**	0.087170	-4.007482**	0.041990	-3.816774**
IGARCH(1,1)-N	0.055388	-2.47118**	0.066870	-2.338562*	0.037960	-3.222050**
APARCH(1,1)-N	0.065596	-5.40206**	0.087871	-4.897153**	0.037681	-4.473745**
FIAPARCH(1,1)-N	0.063253	-5.97315**	0.086078	-4.510446**	0.037282	-5.822649**
FIAPARCH(1,1)-skT	0.042675	--	0.053523	--	0.023751	--
			Sort Positions			
Model	Loss Function	DM Statistic	Loss Function	DM Statistic	Loss Function	DM Statistic
GARCH(1,1)-N	0.044903	-3.85406**	0.038727	-3.288739**	0.017074	-2.740793**
IGARCH(1,1)-N	0.035263	-1.74338	0.029910	-0.837396	0.014234	-1.132025
APARCH(1,1)-N	0.041037	-4.98423**	0.037286	-4.913541**	0.014402	-2.031883*
FIAPARCH(1,1)-N	0.040013	-5.04504**	0.037238	-3.385501**	0.015622	-4.657467
FIAPARCH(1,1)-skT	0.029958	--	0.027336	--	0.012470	--
			$a = 1\%$			
			Long Positions			

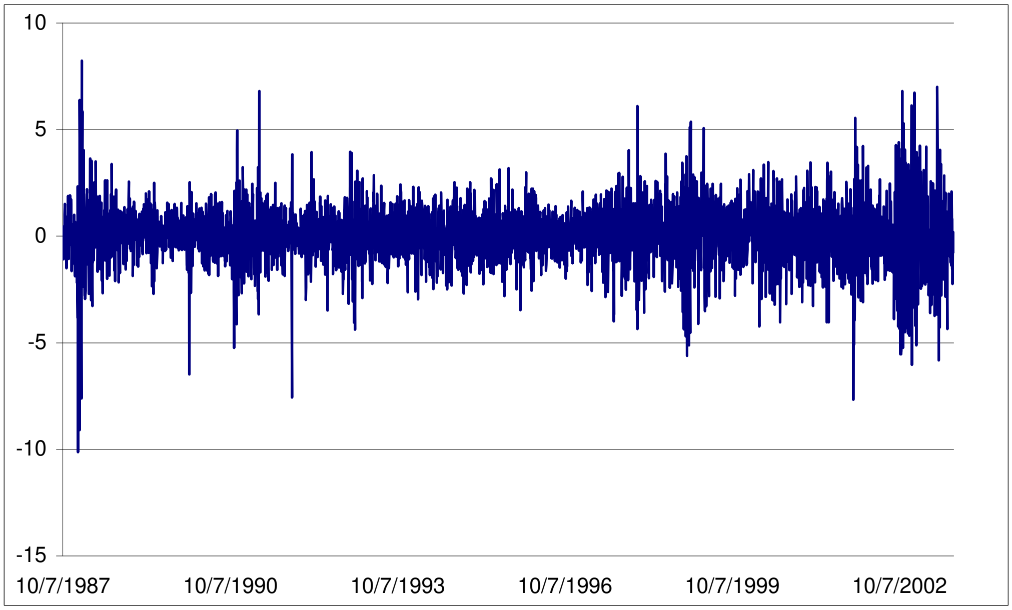
Model	Loss Function	DM Statistic	Loss Function	DM Statistic	Loss Function	DM Statistic
GARCH(1,1)-N	0.018611	-2.07918*	0.026076	-2.071137*	0.011895	-2.148599*
IGARCH(1,1)-N	0.016305	-1.81311	0.017439	-1.813110	0.010593	-1.930740
APARCH(1,1)-N	0.017799	-2.70251**	0.027427	-2.480588*	0.009448	-2.334256*
FIAPARCH(1,1)-N	0.015945	-2.72616**	0.025671	-2.344886*	0.008977	-2.815065**
FIAPARCH(1,1)-skT	0.007119	--	0.009478	--	0.003468	--
			Sort Positions			
Model	Loss Function	DM Statistic	Loss Function	DM Statistic	Loss Function	DM Statistic
GARCH(1,1)-N	0.006970	-2.42590**	0.007536	-2.356176*	0.002387	-1.785638
IGARCH(1,1)-N	0.003923	-1.68471	0.005083	-1.723925	0.001786	-1.142707
APARCH(1,1)-N	0.006059	-2.63054**	0.007156	-2.655640**	0.001846	-1.735150
FIAPARCH(1,1)-N	0.005716	-2.69535**	0.009186	-2.425682*	0.002126	-2.891783**
FIAPARCH(1,1)-skT	0.002142	--	0.003344	--	0.001103	--

*Statistically significant at 5%.

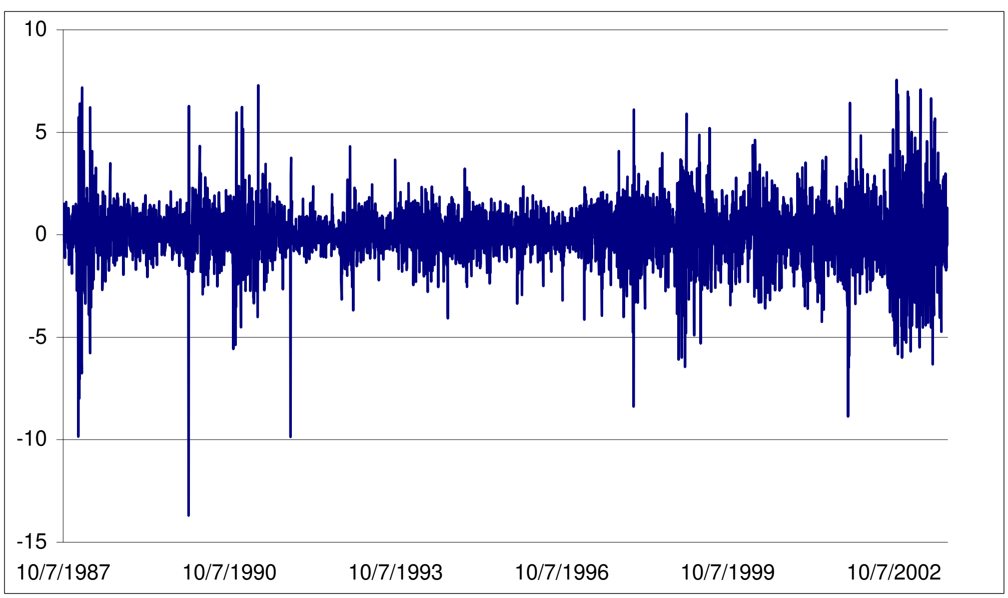
** Statistically significant at 1%.

Figure 1. CAC40, DAX30 and FTSE100 stock index daily returns in the period from July 10th, 1987 to June 30th, 2003.

CAC40



DAX30



FTSE100

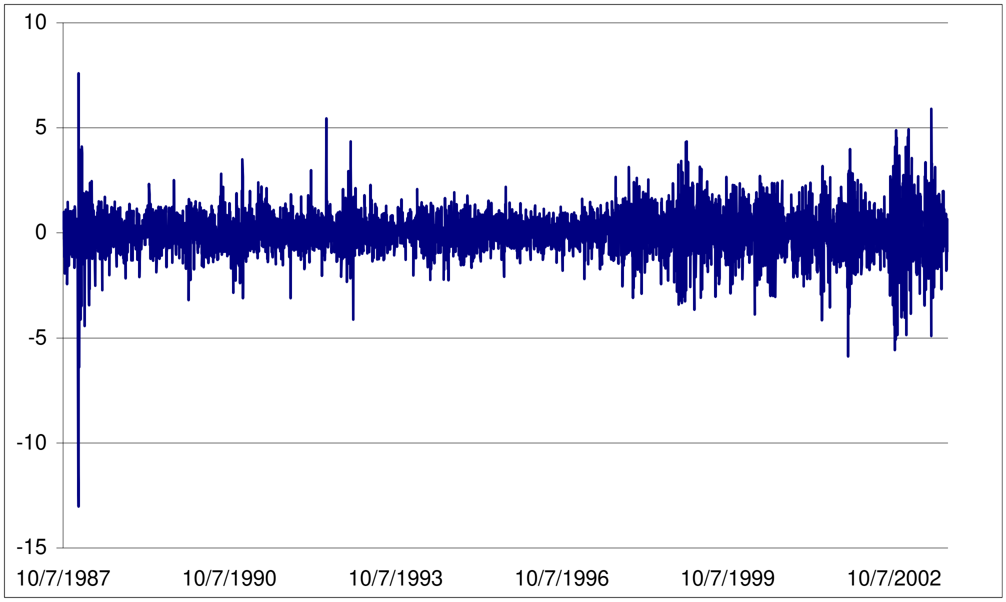
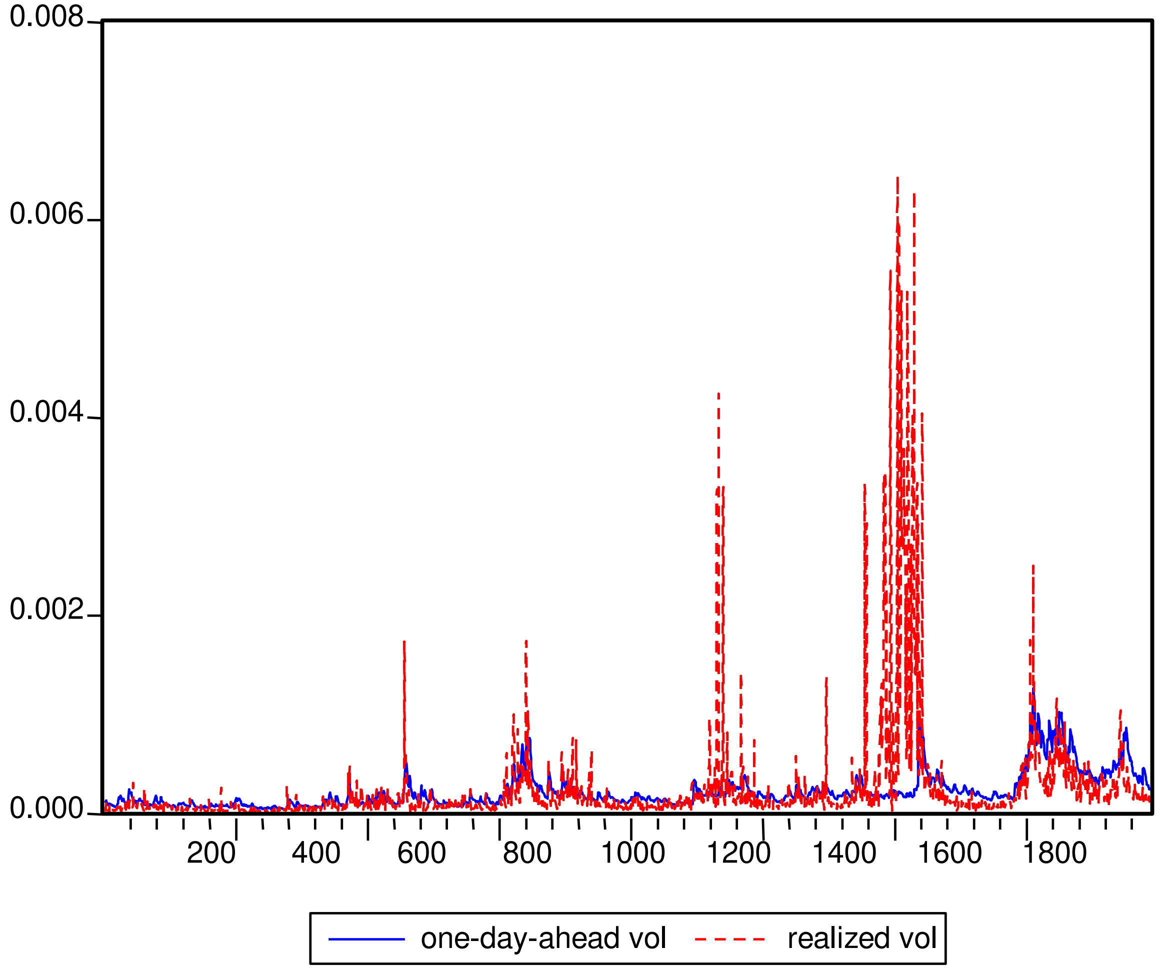
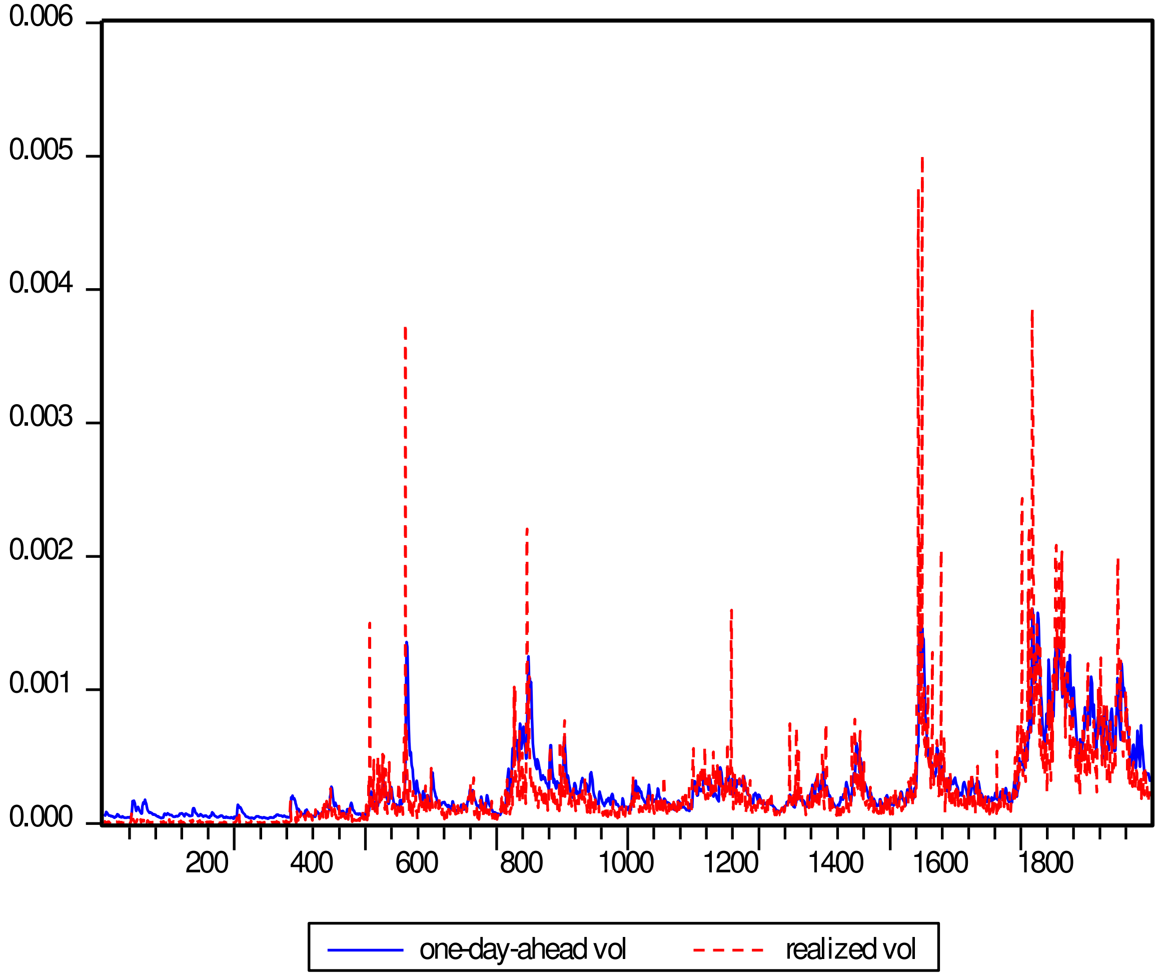

10
5
0
-5
-10
-15
10/7/1987
10/7/1990
10/7/1993
10/7/1996
10/7/1999
10/7/2002

Figure 2. The realized intra-day volatility and the relative one-day-ahead forecasts of the FIAPARCH(1,1)-skT model for the CAC40 (July 20th 1995 – June 30th 2003), DAX30 (July 11th 1995 – June 30th 2003) and FTSE100 indices (June 14th 1995 – June 30th 2003).

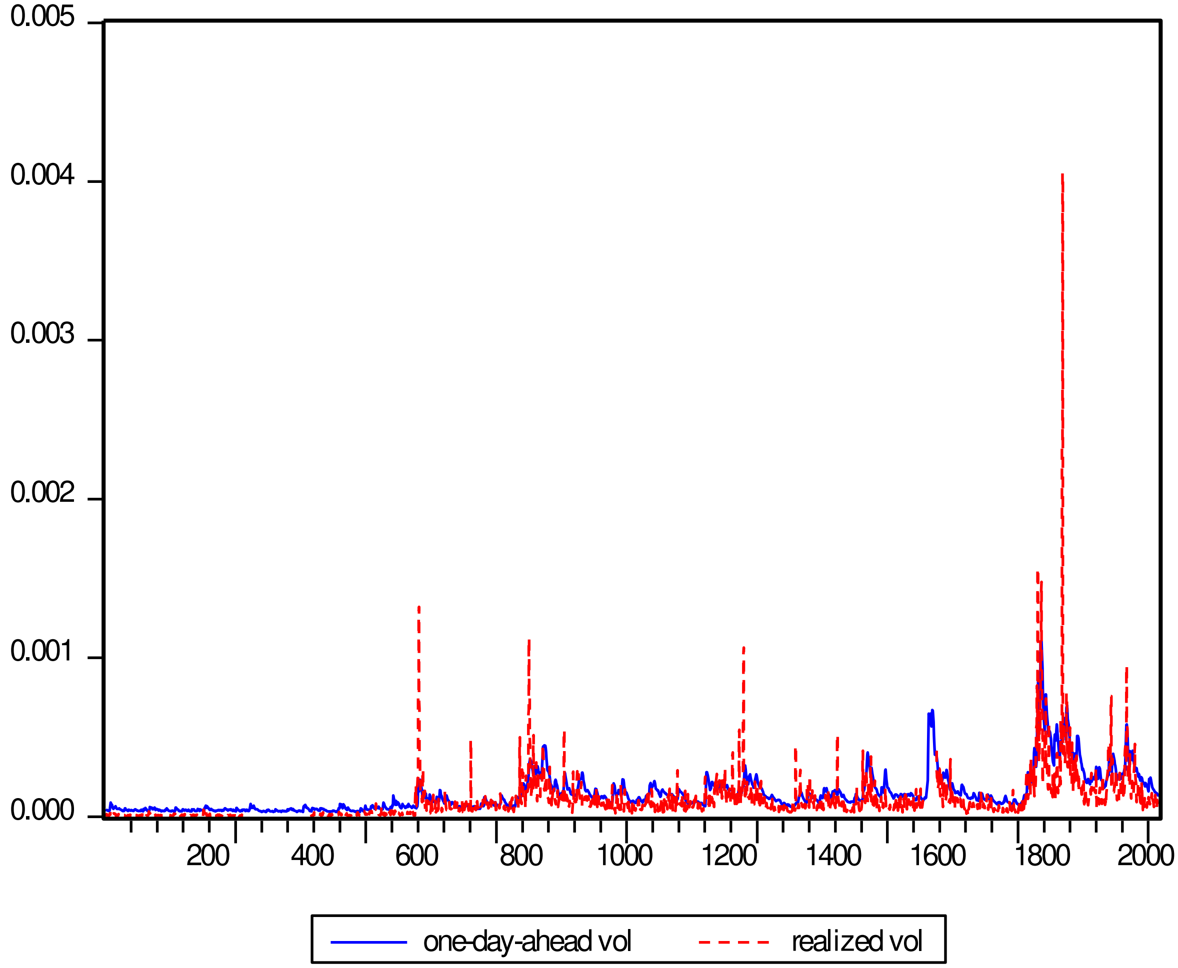
CAC40



DAX30


0.006
0.005
0.004
0.003
0.002
0.001
0.000
200
400
600
800
1000
1200
1400
1600
1800
one-day-ahead vol
realized vol

FTSE100


0.005
0.004
0.003
0.002
0.001
0.000
200
400
600
800
1000
1200
1400
1600
1800
2000
one-day-ahead vol
realized vol