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Empirical Evidence of Systemic Tail Risk Premium in the Johannesburg Stock Exchange

Abstract

This paper assesses the impact of systematic tail risk of stocks, defined as a stock's exposure to market tail events, on the cross section returns of an emerging stock exchange, especially the Johannesburg Stock Exchange (JSE) from January 2002 through June 2018. If stock market investors are crash-averse, then holding stocks that experience high exposure to market tail events should be rewarded with a premium. The paper therefore sets out to determine whether high exposure to market tail events translates into higher returns of stocks traded on the JSE. To achieve this objective, the study extends on the work of Chabi-Yo, Ruenzi and Weigert (2015) based on extreme value theory (EVT) and copula models as well as the traditional asset pricing tools of portfolio formation and cross-sectional regressions. The results of the empirical analysis support the existence of a systematic tail risk premium in the JSE. Interestingly, the effect of systematic tail risk on the cross section of JSE returns is time-varying and independent from that of risk measures such as beta and downside beta and firm characteristics such as book-to-market (BTM) ratio, size and past returns. In addition, the study provides evidence on the impact of financial crises on crash aversion.

1. INTRODUCTION

The increasing frequency and severity of market tail events have highlighted the need to challenge the traditional view of risk in financial markets. Although a great deal of effort has been directed to the adequate quantification of the risk of tail events in stock markets, few studies so far seek to understand the pricing implications of this type of risk for the cross section of returns in equity markets. Furthermore, the few studies that link tail risk to the cross section of stock returns mostly focus on developed markets and fail to address the case of emerging markets (see Chabi-Yo et al., 2015). However, studies have shown the recurrence of extreme events in emerging markets, mostly due their vunerability to shocks from developed economies (see Bonga-Bonga, 2017). Emerging markets are known to experience higher tail risk than developed markets. If systematic tail risk is rewarded in the cross section of stock returns in developed markets, then stock market investors could expect a higher premium for bearing such risk in emerging markets. However, While empirical studies indeed suggest that systematic tail risk is rewarded in developed markets (Cholette and Lu, 2011; Gabaix, 2012, Kelly and Jiang, 2014; Chabi-Yo, Ruenzi and Weigert, 2015), there are evidences that emerging stock markets often do not follow asset pricing paradigms (see Adu, Alagidede and Karimu, 2015). Thus, it becomes a matter of empirical analaysis to assess whether systematic tail risk is priced in the cross section of returns in emerging equity markets. Such an assessment is important for investors and asset managers to have an insight on how these market tail events affect the pricing of stocks in emerging markets. It is in that context that this paper attempts to fill the gap in the literature by setting out to provide evidence on the pricing of systematic tail risk in the cross section of stock returns in emerging markets in Africa, with a particular attention to the Johannesburg Stock Exchange (JSE).

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This study therefore explores the extent to which systematic tail risk explains the cross section of stock returns in the Johannesburg Stock Exchange (JSE). The JSE is not only the largest stock market on the African continent. It is also the most liquid and the most efficient. The focus on this stock exchange stems from two reasons. On the one hand, African stock markets continue to receive an increasing attention from investors as they offer an opportunity for diversification through their low correlation with developed markets. On the other hand, very few studies consider a downside risk framework in asset pricing studies focusing on South Africa (Limberis, 2012; Okyere-Boakye and O'Malley, 2016). Aside Limberis (2012) who considers Value-at-Risk

(VaR), studies on the South African market do not consider the risk of low probability events. Assessing the impact of systematic tail risk on the cross section of JSE stock returns will therefore help fill this gap.

Previous empirical studies assessing the impact of systematic tail risk on the cross section of returns have suggested a number of measures to capture this particular kind of risk. Harris, Nguyen and Stoja (2016) propose using VaR estimates of stock market indices when estimating a stock's systematic tail risk. Kelly and Jiang (2014) propose computing systematic tail risk using a panel estimation approach. Similarly, Chollete and Lu (2011) use a panel estimation approach and proxy systematic tail risk with tail exponent/index. Bali, Cakici and Whitelaw (2014) estimate systematic tail risk by extending Bawa and Linderberg's (1977) framework to extreme losses. van Oordt and Zhou (2016) also proxy systematic tail risk using a beta-like sensitivity measure between stock and market extreme negative returns. Chabi-Yo, Ruenzi and Weigert (2015) rely on the concept of copulas and gauge systematic tail risk using the lower tail dependence coefficient between stock and market returns. Though innovative, their measure fails to account for crash severity (Harris et al., 2016).

The present study extends the work of Chabi-Yo et al. (2015) and proposes a new measure of systematic tail risk. Unlike Chabi-Yo et al. (2015), the study focuses on extreme returns and combines the concepts of Extreme Value Theory (EVT) and copula to estimate a stock's exposure to market tail events. In particular, the study first characterizes market and stocks tail events under the Block model and subsequently proxies a stock's systematic tail risk with the parameter estimate of an extreme value copula fitted to the bivariate Generalized Extreme Value (GEV) distribution of these tail events. Based on data for JSE All Share Index companies, results of the traditional asset pricing portfolios formation and cross-sectional regressions show that the extreme value copula parameter adequately captures a stock's systematic tail risk. More importantly, the results support the existence of a systematic tail risk premium in the JSE. Interestingly, the impact of systematic tail risk on the cross section of returns is time-varying and independent from that of risk measures such as beta and downside beta and firm characteristics such as book-to-market (BTM) ratio, size and past returns. In addition, the results provide evidence on the negative impact of the 2008 Global Financial Crisis on crash aversion in the JSE.

The remainder of this paper is as follows. Section 2 outlines the estimation of systematic tail risk. Section 3 covers the methodology used to uncover the systematic tail risk premium. Specifically, it highlights univariate and bivariate portfolio sorts as well as cross-sectional regressions. Section 4 gives a description of the data and provides the empirical results of the study, including some robustness checks. Section 5 concludes with a summary of the research findings and recommendations for future research areas.

2. ESTIMATION OF SYSTEMATIC TAIL RISK

The estimation of systematic tail risk relies on the identification of market and stock tail events and the modelling of their dependence structure. This section introduces the statistical concepts of extreme value theory (EVT) and copula used to achieve this task. While EVT provides a framework to select market and stock tail events, copulas help estimate systematic tail risk by gauging the dependence between these extremes.

2.1. Extreme Value Theory

2.1.1. The Block Model

In this study, the characterization of tail events is based on the Block model, one of two EVT frameworks. The Block model relies on the Fisher-Tippett Theorem which is analogous to the well-known Central Limit Theorem (CLT). While the CLT is concerned with the limiting distribution of sample means, the Extremal Types Theorem approximates the limiting distribution of block extrema (maxima and minima).

Suppose X_1, $X_2, \dots, X_n$ is a series of n i.i.d. random variables with a common distribution $F(x)$ and $M_n = \max(X_1, \dots, X_n)$.

$$\begin{aligned} P(M_n \leq x) &= P(X_1 \leq x, \dots, X_n \leq x) \\ &= P(X_1 \leq x) * \dots * P(X_n \leq x) \\ &= [F(x)]^n \end{aligned} \tag{1}$$

When n approaches infinity, the limiting distribution is given by the degenerate distribution in Equation 2

$$\lim_{n\to\infty} Pr\,(M_n \leq x) = \lim_{n\to\infty} [F(x)]^n = \begin{cases} 0, & if\ F(x) < 1 \\ 1, & if\ F(x) = 1 \end{cases} \tag{2}$$

To obtain a non-degenerate distribution, Fisher and Tippett (1928) suggest using the limiting distributions of M_n^*, a linear transformation of M_n given by:

$$M_n^* = \frac{M_n - a_n}{b_n} \tag{3}$$

where $a_n$and $b_n > 0$ are two sequences of constant.

Theorem 1 (Extremal types theorem)

Let $\{X_n, n \geq 2\}$ be a sequence of i.i.d.r.v.'s and $M_n = \max(X_1, \dots, X_n)$. If there exists sequences of norming constant $\{a_n > 0\}, \{b_n\}$, and a non-degenerate distribution function H such that,

$$\lim_{n\to\infty} P\left\{\frac{M_n - a_n}{b_n} \leq x\right\} = H(x), \qquad x \in \mathbb{R} \tag{4}$$

then, H is one the following three distributions:

$$\text{Gumbel:} \quad \Lambda(x) = exp\left\{-\exp[-(\tfrac{x-\lambda}{\alpha})\right\}, \; x \in \mathbb{R} \tag{5}$$

$$\text{Frechet:} \quad \phi(x) = \begin{cases} 0, & x < \lambda \\ exp\left\{-(\frac{\alpha}{x-\lambda})^{\beta}\right\}, & x \geq \lambda \; and \; \beta > 0 \end{cases} \tag{6}$$

$$\text{Weibull:} \quad \psi(x) = \begin{cases} exp\left\{-[-\left(\frac{x-\lambda}{\alpha}\right)]^{\beta}\right\}, & x \leq \lambda \; and \; \beta > 0 \\ 1, & otherwise \end{cases} \tag{7}$$

These functions are the limiting distributions of maxima and can be summarized into one single parameterization known as the Generalized Extreme Value (GEV) distribution given in equation 2.8 (von Mises, 1936; Jenkinson, 1955).

$$H_{\xi}(x) = \begin{cases} exp\left\{-\left[1 + \xi\left(\frac{x-\lambda}{\alpha}\right)\right]^{-\frac{1}{\xi}}\right\}, & if \; 1 + \xi\left(\frac{x-\lambda}{\alpha}\right) \geq 0 \text{ and } \xi \neq 0 \\ \exp\left[-\exp\left(\frac{\lambda - x}{\alpha}\right)\right], & -\infty < x < \infty \quad and \; \xi = 0 \end{cases} \tag{8}$$

where $\xi = \frac{1}{\alpha}$ is the shape parameter and $\alpha > 0$ the tail index. The Gumbel, Weibul and the Fréchet distributions are respectively identified by $\xi = 0, \xi < 0 \; and \; \xi > 0$.

Similarly, the limiting distribution of minima is given by Equation 9.

$$H_\xi(x) = \begin{cases} 1 - exp\left\{ -\left[1 - \xi\left(\frac{x-\lambda}{\alpha} \right) \right]^{-\frac{1}{\xi}} \right\}, & 1 - \xi\left(\frac{x-\lambda}{\alpha} \right) > 0 \; and \; \xi \neq 0 \\ 1 - \exp\left[-\exp\left(\frac{x-\lambda}{\alpha} \right) \right], & -\infty < x < \infty \; and \; \xi = 0 \end{cases} \quad (9)$$

2.1.2. Maximum Likelihood Estimation of the Generalised Extreme Value distribution parameters

Estimates of the GEV parameters can be obtained using the Maximum Likelihood Estimation (MLE) method. Essentially, the MLE method seeks to find parameter estimates that maximise the likelihood of a sequence $\{x_1, x_2, \dots, x_m\}$ being sampled from the GEV distribution. When $\xi \neq 0$ and $1 - \xi\left(\frac{x_i - \lambda}{\alpha}\right) > 0$, the likelihood function L is given by Equation 10:

$$L(\lambda, \alpha, \xi) = \prod_{i=1}^{m} \frac{1}{\alpha} [1 + \xi\left(\frac{x_i - \lambda}{\alpha} \right)]^{-1-\frac{1}{\xi}} \; exp\left\{ -[1 + \xi\left(\frac{x-\lambda}{\alpha} \right)]^{-1/\xi} \right\} \quad (10)$$

And when $\xi = 0$, the likelihood function is given by equation 2.11:

$$L(\lambda, \alpha) = \prod_{i=1}^{m} \frac{1}{\alpha} \exp\left(-\frac{x_i - \lambda}{\alpha} \right) . \exp[-\exp\left(-\frac{x_i - \lambda}{\alpha} \right)] \quad (11)$$

To avoid the complexity of working with Equations 10 and 11, the optimisation problem is easily achieved using their respective log-likelihood functions in Equations 13 and 14:

$$\ell = \log L(\lambda, \alpha, \xi) = m \log \alpha - (1 + \tfrac{1}{\xi}) \textstyle\sum_{i=1}^{m} \log\left[1 + \xi\left(\frac{x_i - \lambda}{\alpha} \right) \right] - \sum_{i=1}^{m} \left[1 + \xi\left(\frac{x_i - \lambda}{\alpha} \right) \right]^{-\frac{1}{\xi}} \quad (13)$$

$$\ell = \log L(\lambda, \alpha) = -m \; log\theta - \sum_{i=1}^{m} \left(\frac{x_i - \lambda}{\alpha} \right) - \sum_{i=1}^{m} \exp\left(-\frac{x_i - \lambda}{\alpha} \right) \quad (14)$$

The GEV parameter estimates are then obtained by solving the following system of equations:

$$\begin{cases} \frac{\partial l(\lambda, \alpha, \xi)}{\partial \lambda} = 0 \\ \frac{\partial l(\lambda, \alpha, \xi)}{\partial \alpha} = 0 \\ \frac{\partial l(\lambda, \alpha, \xi)}{\partial \xi} = 0 \end{cases} \quad (15)$$

2.2. Copulas

2.2.1. Definition

Copula functions provide a powerful tool to describe the dependence structure of random variables.

Definition 3.1 A 2-dimensional copula is a uniformly continuous function $C: [0,1]^2 \rightarrow [0,1]$ with the following properties (Nelsen, 1999):

1. For every $u \in [0,1]$

$$C(0, u) = C(u, 0) = 0. \tag{16}$$

2. For every $u \in [0,1]$

$$C(u, 1) = u \text{ and } C(1, u) = u \tag{17}$$

3. For every (u_1, v_1), $(u_2, v_2) \in [0,1] \times [0,1]$ with $u_1 \leq u_2$ and $v_1 \leq v_2$

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \tag{18}$$

A function that satisfies property 1 is said to be grounded. Property 3 is a two-dimensional analogue of a non-decreasing one-dimensional function. A function satisfying this property is called a 2-increasing function.

Theorem 3.2 (Sklar 1959): *Let F be a bivariate distribution with margins F_1 and F_2. Then there exists a copula C: $[0,1]^2 \rightarrow [0,1]$ such that, for all x_1, x_2 in $\mathbb{R} = [-\infty; \infty]$,*

$$F(x_1, x_2) = C\big(F_1(x_1), F_2(x_2)\big) \tag{19}$$

If the margins are continuous, then C is unique. Conversely, if C is a copula and F_1 and F_2 are univariate functions, then the function F defined in Equation 19 is a bivariate distribution function with margins F_1 and F_2.

2.2.2. Extreme Value Copulas

The dependence structure between series of extremes can be modelled using extreme value copulas. This class of copulas is governed by two conditions. First, extreme value copulas characterise the dependence structure of multivariate distributions with GEV marginals. Second, they satisfy the relationship in Equation 20 for all $\tau > 0$.

$$C(u_1^\tau, \ldots, u_d^\tau) = C^\tau(u_1, \ldots, u_d) \quad (20)$$

The most prominent extreme value copulas include copulas from the Galambos family, the Husler-Reiss family and the Gumbel family. Table 1 presents expressions of these copulas. θ is the dependence parameter; and for the Galambos family, θ is greater or equal to 0 with $\theta = 0$ representing the case of independence. For the Husler-Reiss family, the independence and full dependence cases are reached when the parameter θ is equal 0 and ∞, respectively. For the Gumbel family, the dependence parameter $\theta \in [1, \infty)$ and independence is reached when $\theta = 1$.

2.2.3. Maximum Likelihood Estimation of the Copula Parameter

To fit a copula function to the bivariate distribution of extremes, the study relies on the MLE method. For a copula function C defined such that:

$$F(x_1, x_2) = C\big(\theta; F_1(x_1), F_2(x_2)\big) \quad (21)$$

Table 1 Families of Extreme Value Copulas

Copula Family	**Function**	**Domain**	$\boldsymbol{\lambda_L}$	$\boldsymbol{\lambda_U}$
Galambos	$u.v.\exp\{[(-\ln u)^{-\theta} + (-\ln v)^{-\theta}]^{-1/\theta}\}$	$\theta \geq 0$	0	$2^{-1/\theta}$
Gumbel	$\exp\{-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]\}^{1/\theta}$	$\theta \geq 1$	0	$2-2^{1/\theta}$
Husler-Reiss	$\exp\{(lnu)\, \emptyset\left[\frac{1}{\theta} + \frac{\theta}{2}\ln\left(\frac{lnu}{lnv}\right)\right] + (\ln v)\, \emptyset\left[\frac{1}{\theta} + \frac{\theta}{2}\ln\left(\frac{lnv}{lnu}\right)\right]\}$	$\theta \geq 0$	0	$2 - 2\emptyset(1/\theta)$

Note: $u, v\ \epsilon\ [0,1], \theta$ represents the copula dependence parameter and ϕ is the univariate standard Normal distribution.

where F represents a bivariate distribution, θ the copula parameter and F_1 and F_2 are the respective marginal cumulative density functions of the random variables X_1 and X_2, the likelihood function L and the related log-likelihood function l are given by:

$$L\left(\theta;\ \hat{F}_1(x_1^i), \hat{F}_2(x_2^i)\right) = \prod_{i=1}^{n} c\left(\theta;\ \hat{F}_1(x_1^i), \hat{F}_2(x_2^i)\right) \tag{22}$$

$$l\left(\theta;\ \hat{F}_1(x_1^i), \hat{F}_2(x_2^i)\right) = \sum_{i=1}^{n} \log c\left(\theta;\ \hat{F}_1(x_1^i), \hat{F}_2(x_2^i)\right) \tag{23}$$

respectively, with c being the copula density function and $\hat{F}_1(x_1^i)$ and $\widehat{F}_2(x_2^i)$ the respective estimates of the marginal CDFs $F_1(x_1)$ and $F_2(x_2)$. The copula parameter is then obtained by maximizing the likelihood function in Equation 23.

2.3. Measuring Systematic Tail Risk

To measure a stock's systematic tail risk, the study combines extreme value theory and extreme value copulas. Following the framework of the Block Model, the study first characterises tail events and consequently uses the GEV distribution to model the univariate behaviour of these extremes. Specifically, the study defines a tail event as the minimum or the worst return realisation over a one-month period. Although the selection of monthly extremes is likely to violate the IID assumption of the block model, Leadbetter, Lindgren and Rootzen (1983) provide grounds for using the GEV distribution in the presence of short-term dependence. In addition, the choice of monthly block intervals remains practical in the case of data limitation. Moreover, the study focuses solely on minima and therefore assumes that stock market participants hold long positions. Though restrictive, this assumption is unlikely to significantly affect the results of this study.

In this study, systematic tail risk estimates are computed every semester for each stock. To do so, series of market and stock minima are first obtained over overlapping five-year periods, then transformed into series of maxima before being fitted to the univariate GEV distributions. As Gudendorf and Segers (2009) show, extreme value copulas mostly have independent lower tails. The transformation of minima into maxima therefore becomes necessary to exploit the upper tail dependence coefficients of extreme value copulas. As Table 1 reveals, there exists a relationship between the upper tail dependence coefficients and the parameters of the extreme value copulas. For each of the three extreme value copulas, it can be shown that the upper tail dependence coefficient is an increasing function of the copula parameter. In other words, the higher the copula parameter is, the higher the tail dependence coefficient, and the higher the systematic tail risk. It is therefore possible to proxy systematic tail risk with either the upper tail dependence or the parameter of the extreme value copula. This study uses the latter and arbitrarily fits the Galambos copula to the bivariate GEV of market and stock extrema to obtain the copula parameter.

Although the systematic tail risk estimates are able to reveal the dependence structure between stock and market extremes or tail events, they do not directly inform on a stock's exposure to the risk of tail events in the market during a particular semester. Recall that every semester, the study estimates a stock's systematic tail risk using overlapping five-year periods. In other words, each systematic tail risk estimate derived in this study informs on a stock's exposure to market tail events over a period of five years. So, to determine a stock's exposure to such risk in a particular semester, the study relies on changes or innovations in subsequent systematic tail risk estimates (STRIs). While a positive STRI represents an increasing exposure to market tail events, a negative STRI suggests a decrease in exposure. Stocks with a positive (negative) STRI in a particular semester therefore experienced further (reduced) exposure to tail events in the market.

3. ASSESSING THE SYSTEMATIC TAIL RISK PREMIUM

3.1. Portfolio Formation

Portfolio formations and cross sectional regressions have consistently been used in studies focusing on the cross-section of stock returns as they provide an intuitive way to assess the relationship between risk factors and stock returns. This section therefore describes the method of portfolio formation and cross-sectional regressions used to investigate the existence of a systematic tail risk premium in the cross section of stock returns in the JSE.

3.1.1 Univariate Portfolio Sorts

The study first conducts a univariate analysis where portfolios are primarily formed based on a single risk factor. Every semester, stocks are classified into quintile portfolios according to their STRI estimates with portfolios 1 and 5 comprising stocks with the lowest and highest STRI, respectively. Next, portfolio average returns are computed over the risk measurement period. The aim is to determine whether higher levels of STRI are, on average, contemporaneously compensated with higher returns.

3.1.2 Bivariate Portfolio Sorts

Although the univariate analysis might establish a relationship between a risk factor and expected returns, it still has a shortcoming. Due to the high correlation that risk measures often exhibit, it is cautious to extend the analysis to a bivariate setting. The study therefore further uses bivariate portfolio formations to isolate the effects of other risk measures such as the beta and Downside Beta (Ang, Chen and Xing, 2006). First, quintile portfolios are formed every semester based on a risk factor other than STRI. For each of the five portfolios, a second sort is conducted. Stocks within each of the quintile portfolios are further arranged into five new portfolios according to their STRI estimates. In total, twenty five double sort portfolios are formed for each pair of risk factors. These double sorts are performed subsequently using stocks beta, downside beta and firm size. The analysis then proceeds to evaluate the relationship between STRI and expected returns within each of the first quintile portfolios.

3.2. Cross-Sectional Regressions

Cross-sectional regressions are complementary alternatives to portfolio formations. Unlike portfolio sorts, they help differentiate simultaneously the impact of a wider range of risk factors. In general, cross sectional regressions in asset pricing studies assume a linear relationship between risk factors and stock returns which is can be represented as given in the Equation 24.

$$R_{i,t} = \delta_{0,t} + \delta_{1,t}\beta_{i,F_1} + \delta_{2,t}\beta_{i,F_2} + \cdots + \delta_{k,t}\beta_{i,F_k} + \varepsilon_{i,t}, \qquad \text{for } i = 1, 2, \dots, n \qquad (24)$$

where $R_{i,t}$ is stock i excess return at a specific time t, and β_{i,F_k} and δ_k are stock i loading on factor k and the premium on factor k, respectively.

To avoid the consequences of the error-in-variables problem, some studies advocate the use of portfolios of stocks for the computation of these factor loadings (Blume 1970, Black, Jensen and Scholes, 1972, Fama and McBeth, 1973). However, Ang et al. (2017) argue that using portfolios do not actually result in smaller standard errors of cross-sectional parameter estimates. Instead, the authors suggest that the use of portfolios in cross-sectional tests destroys information by reducing the spread of factor loadings. In this study, the cross-sectional regressions are therefore conducted using firm level data instead of portfolios.

To estimate the parameters of equation 24, the study relies on Ordinary Least Squares estimators. More importantly, the estimation follows a method similar to Fama and McBeth (1973) two-step regression. In particular, stock excess returns are repeatedly regressed on risk factors loadings and firm characteristics at each point in time. The result of this series of regressions is a series of estimates for each regression coefficient.

Overall parameter estimates $\hat{\delta}_k$ are then computed by averaging the series of estimates. Assuming no autocorrelation, the variance of this estimate is provided by $\sigma^2(\hat{\delta}_k)$.

$$\hat{\delta}_k = \frac{1}{T}\sum_{t=1}^{T} \hat{\delta}_{k,t} \tag{25}$$

$$\sigma^2(\hat{\delta}_k) = \frac{1}{T^2}\sum_{t=1}^{T}(\hat{\delta}_{k,t} - \hat{\delta}_k)^2 \tag{26}$$

4. DATA AND EMPIRICAL RESULTS

The following sections present the data used in this study as well as the empirical results. Section 4.1 presents data sources. Section 4.2 provides some descriptive statistics of the data and the results of a preliminary analysis. Sections 4.3 and 4.4 subsequently show the results of the portfolio formations and the Fama and McBeth (1973) cross-sectional regressions, respectively, followed by some robustness checks in section 4.5. Lastly, section 4.6 provides the findings on the temporal stability of the impact of the GFC on investors' crash aversion.

4.1. Data

4.1.1. Sources

This study uses data provided by the JSE to analyse the relationship between STRI and stocks expected returns. The sample consists of daily price data for stocks included in the JSE All Share Index over the period of July 1997 to June 2018. These price data were freely provided by the JSE. In total, 396 stocks are included in this study. In addition, the study uses an equally weighted index and the South African three-month Treasury bill rates to proxy the market and the risk-free rate, respectively. While the former is compiled using returns of the JSE All Share constituents, the latter is obtained from the INET MacGregor BFA database.

4.1.2. Equally-Weighted Market Index

To proxy overall market movements in the JSE, the study uses an equally-weighted index. The choice of the equally-weighted index over the readily available value-weighted JSE All Share index mostly stems from two reasons. First, the need to capture tail events in the market requires the use of an index that is not mostly affected by movements in large market-cap stocks. In their composition, value-weighted indices proportionally assign weights to firms according to their market capitalization and consequently give greater (smaller) weights to large (small) market cap firms. On the contrary, equally-weighted indices are do not have this bias as they aggregate return in the market by assigning an equal weight to all firms. Secondly, relying on an equally-weighted index helps isolate the size effect from the systematic tail risk proxy.

To obtain the equally weighted market index returns, daily returns for each of the JSE All share index constituents are computed using Equation 26 and then averaged daily on an equal weighting basis according to Equation 27.

$$r_{i_{t+1}} = \ln\left(\frac{P_{t+1}}{P_t}\right) \tag{26}$$

$$r_{index_{t+1}} = \frac{1}{n}\sum_{i=1}^{n} r_{i_{t+1}} \tag{27}$$

where r_i is the return on a JSE All share index constituent and r_{index}, the return on the equally-weighted index.

Figure 1 plots the returns for both the equally weighted index and the JSE All Share Index and allow some degree of comparison between these two market indices. Although the equally-weighted index returns are, in general, smaller than the JSE All Share index returns, both indices are able to mirror the dynamics of stock movements in the JSE. In particular, the observed spikes in the periods between years 1997 and 1999 and years 2007 and 2009 show that both indices capture periods of high tail risk in the JSE. Interestingly, the equally-weighted index is also able to capture some effects of the crash of the Chinese stock market of 2015.

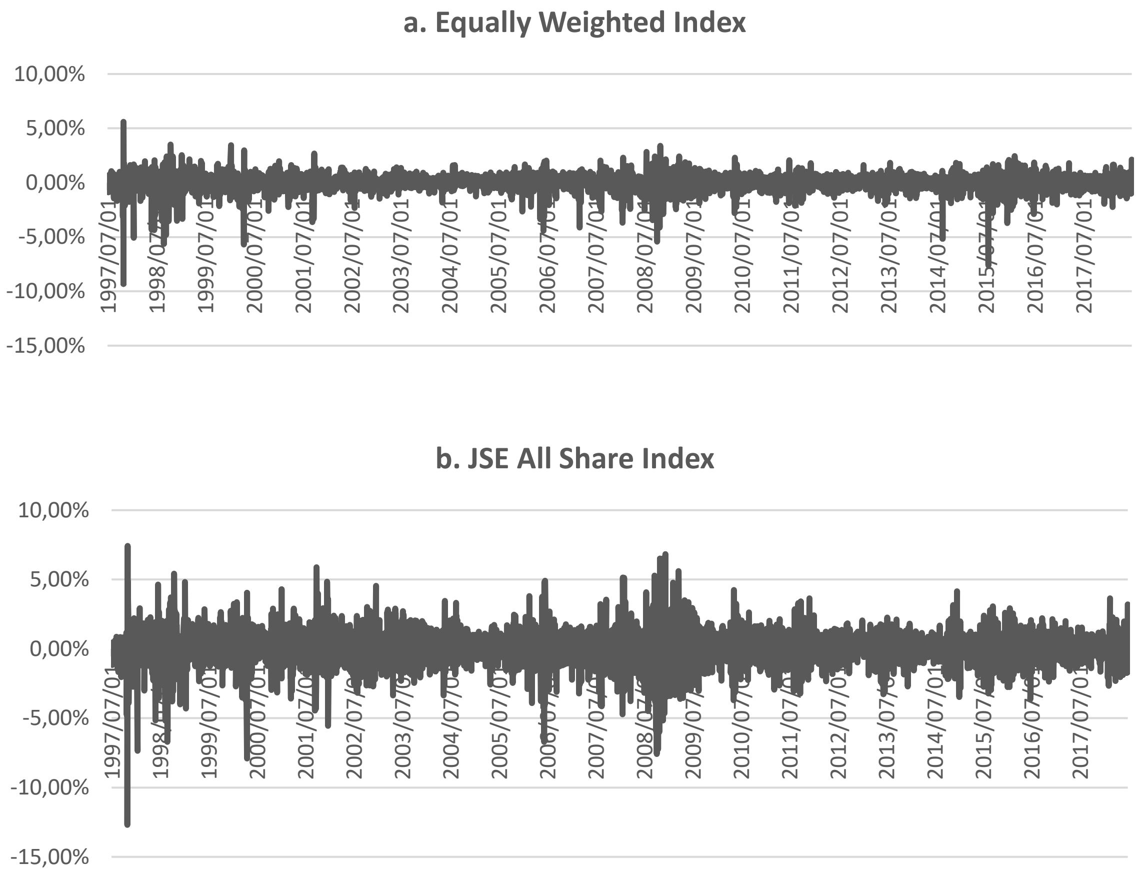

Figure 1 Daily Returns on the Equally-weighted Index and the JSE All Share Index

4.2. Systematic Tail Risk over Time

The following sections primarily assess the ability of the extreme value copula parameter to gauge systematic tail risk. First, section 4.2.1 provides some descriptive statistics on systematic tail risk estimated using overlapping five-year periods. Next, section 4.2.2 evaluates STRIs as proxy for stock's exposure to tail events during a particular semester.

4.2.1. Systematic Tail Risk Estimates Using Five-Year Overlapping Periods

Table 2 gives a summary of the five-year systematic tail risk estimates proxied by the copula parameter, and a summary of other computed statistics. As expected, estimates of the Galambos copula dependence parameter are all positive. For all stocks in the sample, the Galambos copula parameter ranges between 0 and 1.76. On average, the parameter is equal to 0.59 with a standard deviation of 0.25 and the first, second and third quartiles equalling 0.68, 1.04 and 1.43 respectively.

Table 2 Descriptive statistics

	Average	st. dev.	Quartile 1	Median	Quartile 3	min	max
Returns	0.004%	0.239%	-0.081%	0.026%	0.129%	-2.832%	1.570%
Five-year STR	0.59	0.25	0.41	0.57	0.74	0.00	1.76
Beta	1.08	0.58	0.68	1.04	1.43	-0.44	3.60
Downside Beta	1.04	0.53	0.69	0.97	1.28	-0.62	4.31
ln(Size)	23.54	1.44	22.56	23.43	24.43	18.62	28.32
Book-to-Market Proxy	0.00	0.01	0.00	0.00	0.01	-0.11	0.07
Past Returns	0.01%	0.23%	-0.07%	0.03%	0.14%	-2.83%	1.57%

If estimates of the extreme value copula parameter are able to proxy STR, then they should be able to capture periods of high tail risk in the market. Hence, to confirm the ability of the dependence parameter to represent STR, dependence parameter estimates for all stocks are averaged every semester and plotted in Figure 2.

With a minimum of 0.36 and a maximum of 0.81, these averages are relatively low over the first part of the sample period and rapidly increase to high levels from the first semester of 2007 up to the second semester of 2008, a period associated to the US Housing Bubble and the global financial meltdown. This observation not only confirms the ability of the extreme value copula parameter to capture periods of increasing tail risk in the JSE, but also captures the time-varying nature of systematic tail risk. Most importantly, the plot reveals the increasing correlation between stock extreme losses over the period leading to the global financial crisis of 2008.

Figure 3 shows semester averages of STRI for all stocks in the sample, along with a four-semester moving average unveiling the trends in STRI. As expected, STRI averages fluctuate between positive and negative regions, recording their two highest levels during periods coinciding with the GFC of 2008 and the Chinese stock market crash of 2015. Interestingly, the upward trend in STRI averages reveals that tail risk did not only increase in the JSE ahead of the crash of 2008. It also increased at faster rates.

Likewise, the downward trend in STRIs between 2009 and 2013 shows the reduction in tail risk in the JSE in the aftermath of the crash of 2008. Similar but shorter patterns of upward and downward trends in STRIs are also observed after 2015, with tail risk in the JSE reaching another peak level in the second semester of 2015.

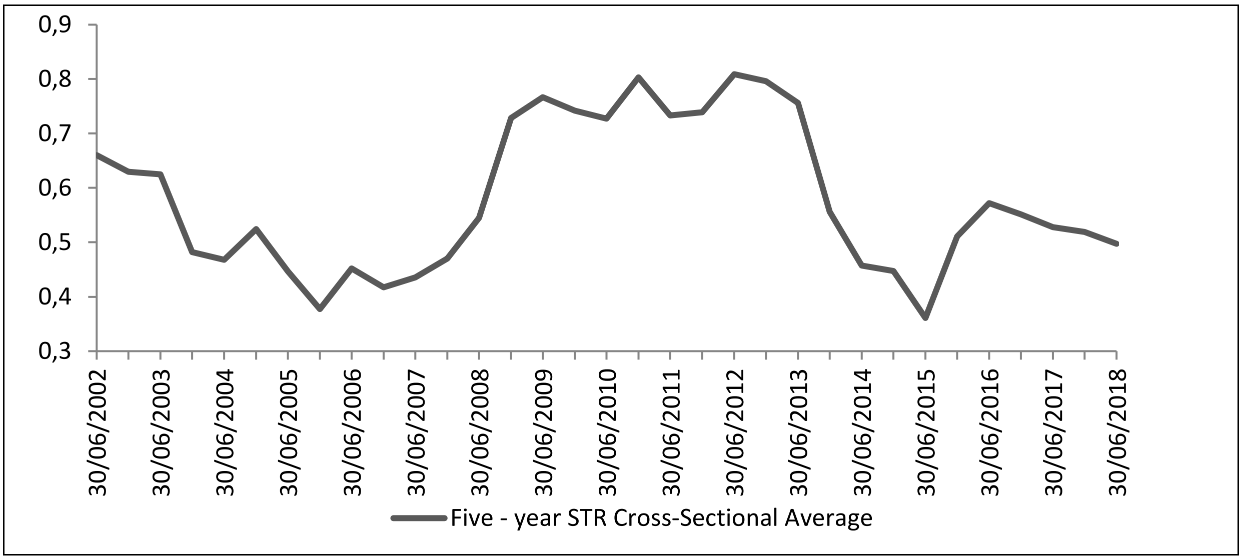

Figure 2. Systematic Tail Risk Innovations

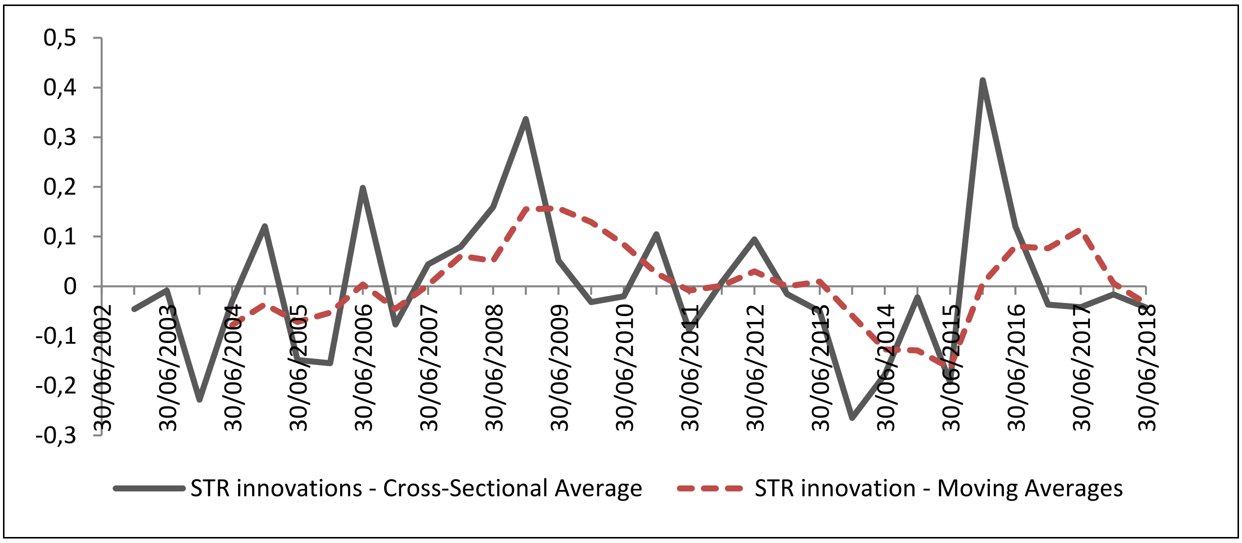

Figure 3. Systematic Tail Risk Innovations

STRIs undeniably provide information about the path of and the acceleration in tail risk in the JSE over the sample period, highlighting periods of increasing and decreasing tail risk. But STRIs, more importantly, provide a remarkable insight about the pre-GFC period. Figure 4 reveals that the likelihood of a systemic extreme downside move had been increasing despite the high levels of returns recorded during the pre-GFC period. Had market participants relied on measures of tail risk, it would have been apparent that the high level of returns was paired with a high risk of extreme downside moves.

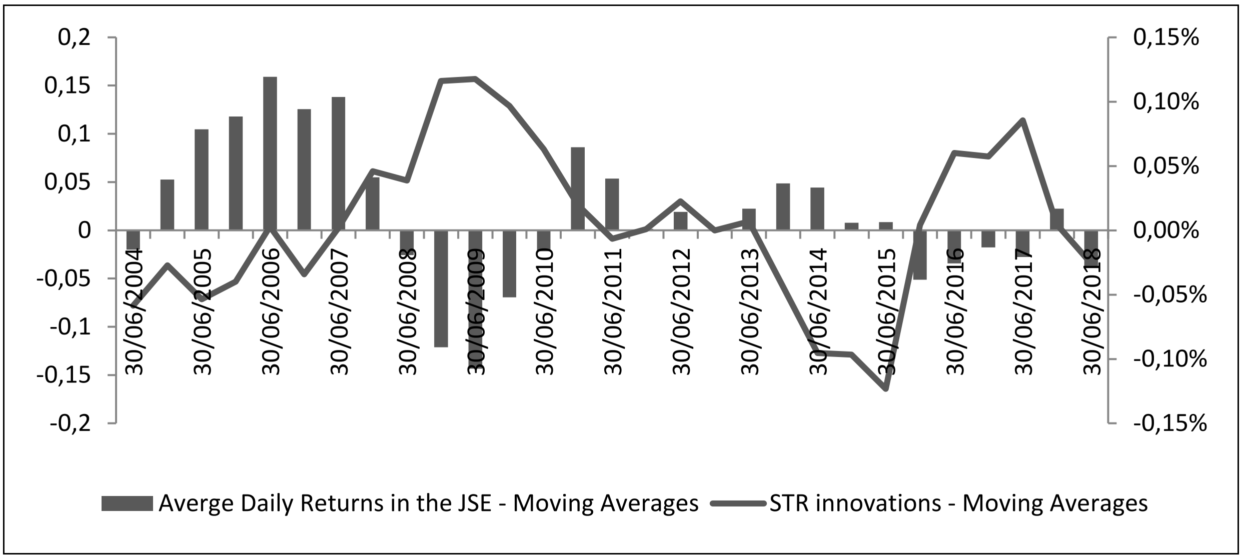

Figure 4. STRIs and Returns – Moving Averages

4.3. Portfolio Sorts

The following section assesses whether stocks with high STRIs contemporaneously receive high returns. This question is addressed using portfolio sorts as well as Fama-McBeth cross-sectional regressions. The results are presented in the sections below.

4.3.1. Univariate Sorts

Table 3 reports the results of the univariate sorts based on STRIs. Column 1 shows the average STRIs of the quintile portfolios. As expected, these averages increase from portfolios 1 to 5 and range between -11.71 and 36.74 percent. Column 2 shows the contemporaneous value-weighted portfolio returns. Mostly, portfolios with high STRIs earn higher returns on average, with the low and high STRI portfolios recording -1.97 and 2.01 percent, respectively. Notably, the relationship between STRIs and portfolio returns is close to being monotonic.

Table 3. Univariate Portfolio Sorts

STRI Portfolios	STRI (%)	VW Ret (%)	EqW Ret (%)	Beta	Downside Beta	Size
Low STRI	-11.71	-1.97	-10.07	1.18	1.12	23.23
2	-3.93	1.18	6.68	1.21	1.14	23.64
3	-0.10	1.07	5.94	1.30	1.25	23.77
4	6.07	1.64	8.93	1.30	1.22	23.87
High STRI	36.74	2.01	10.31	1.25	1.18	23.63
H-L		3.97**	20.39**			
		(2.45)	(2.33)			

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Figure 5 graphically depicts the positive relationship. The remaining columns of Table 3 show equally-weighted portfolio returns as well as averages of other risk factors. For each risk factor, these averages increase monotonically from the low to high STRI portfolio.

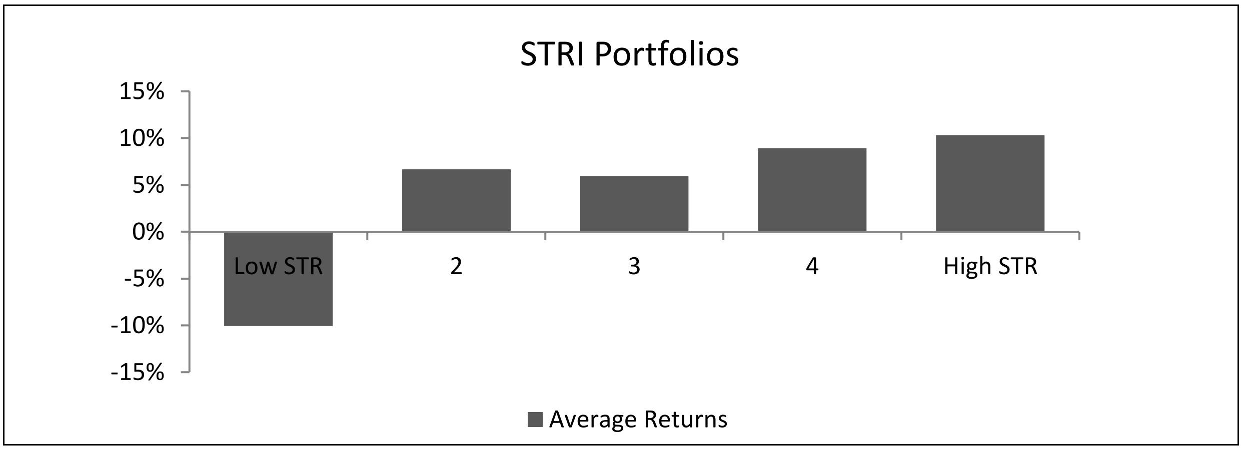

Figure 5. Average Returns of STRI Portfolios over the Period 2002 to 2018

To ensure that the high average return of the portfolio with high STRI is not due to outliers, return spreads between high and low STRI portfolios are computed each semester and presented in Figure 6. Satisfyingly, the plot reveals no outliers. Instead, the positive spreads recorded in 24 semesters, against 9 semesters with negative spread, attest that a stock's increasing exposure to tail events in the JSE is associated with higher returns. Interestingly, figure 4.6 shows that the second semester of the year 2008 recorded the lowest spread.

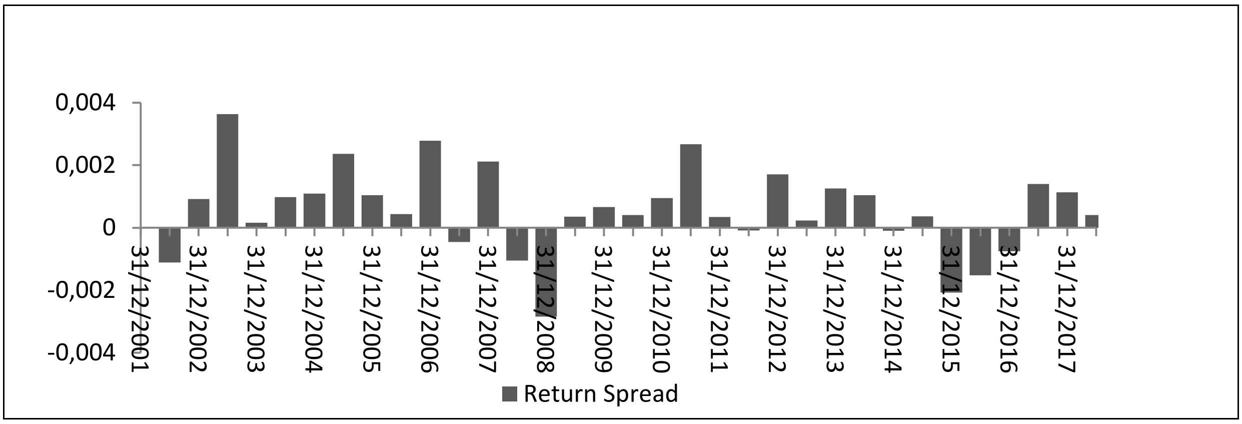

Figure 6. Return Spread between High and Low risk Portfolios

The univariate sort, so far, suggests a positive relationship between STRIs and average returns. However, since high (low) STRI stocks have high (low) betas, it is possible that the observed positive relationship between STRI and stock returns is driven by systematic risk (beta). Moreover, low (high) STRI portfolios have on average low (high) downside betas. Hence, the positive relationship between STRI quintiles and their average returns could also be the result of the downside beta as documented by Ang et al (2006). To investigate these hypotheses, the study proceeds with bivariate sorts to isolate the effects of risk factors other than STRI.

4.3.2. Bivariate sorts

This section outlines the results of the bivariate sorts. Table 4 reports the results of the bivariate sorts, with STRI as secondary risk factor. Panel A shows the results of the Beta/STRI double sort. Overall, the results show that returns increase with STRIs. On average, there is a positive and significant spread of 4.67 percent per year between the high and low STRI quintiles.

Panel B summarises the outcome of the downside beta/ STRI double sort. Again, the results mostly show that average returns rise from the portfolio with low STRIs to the one with high STRIs. Overall, returns increase monotonically from -2.33 to 1.74 percent per year on average. In addition, the spread between the high and low STRI portfolio average returns is about 4.06 percent per year and is statistically significant.

Panel C reports the results of the size/STRI double sort. Just as in the two previous cases, STRI and average returns are mostly positively related, with returns increasing from -2.37 percent to 2.52 percent. The spread return between the high and low STRI portfolios is statistically significant.

In general, the bivariate sorts confirm the positive relationship observed in the univariate sorts. The overall results for both the univariate and bivariate portfolio sorts reveal that high (low) STRI portfolios earn on average high (low) returns. These findings therefore suggest that STRI is to some extent priced in the JSE. Furthermore, the results of the bivariate sorts also suggest that the impact of STRI is independent from those of beta, downside beta and size.

Table 4. Bivariate Sorts - Value-Weighted Portfolio Returns

Panel A: Beta and Systematic Tail Risk (STR)						
	1 Low Beta	**2**	**3**	**4**	**5 High Beta**	**Average**
Low STR	2.05%	1.44%	-1.22%	0.48%	-13.32%	-2.29%
2	3.40%	1.66%	1.25%	2.24%	1.18%	1.94%
3	1.09%	3.26%	0.71%	1.60%	-0.65%	1.19%
4	2.26%	2.22%	2.42%	1.61%	-0.58%	1.58%
High STR	3.85%	1.85%	3.76%	2.08%	0.39%	2.38%
H-L	1.80%	0.42%	4.98%**	1.60%	13.71%***	4.67%***
	(0.98)	(0.21)	(2.00)	(0.78)	(2.90)	(2.70)
Panel B: Downside Beta and Systematic Tail Risk (STR)						
	1 Low Beta	**2**	**3**	**4**	**5 High Beta**	**Average**
Low STR	1.27%	0.11%	2.27%	-1.28%	-13.14%	-2.33%
2	1.59%	2.11%	1.57%	0.30%	0.56%	1.22%
3	0.80%	3.26%	2.72%	1.20%	-0.12%	1.57%
4	1.78%	1.86%	3.74%	2.77%	1.29%	2.28%
High STR	1.84%	2.65%	3.20%	2.32%	-1.27%	1.74%
H-L	0.57%	2.54%	0.93%	3.61%	11.87%	4.06%
	(0.27)	(1.11)	(0.40)	(1.44)*	(2.41)**	(2.14)**
Panel C: Size and Systematic Tail Risk (STR)						
	1 Low Beta	**2**	**3**	**4**	**5 High Beta**	**Average**
Low STR	-6.77%	-1.90%	-1.22%	-2.58%	0.80%	-2.37%
2	-3.85%	0.95%	1.08%	1.76%	2.35%	0.43%
3	-0.77%	1.70%	2.56%	4.05%	2.28%	1.95%
4	-2.93%	3.70%	3.15%	2.73%	0.47%	1.39%
High STR	1.00%	1.75%	3.94%	2.73%	3.19%	2.52%
H-L	7.77%	3.65%	5.16%	5.31%	2.39%	4.88%***
	(2.17)**	(1.04)	(1.62)*	(1.86)**	(1.20)	(2.65)

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

4.4. Cross-Sectional Regressions

This section presents the results of the cross-sectional regressions based on the Fama-McBeth procedure (Fama and McBeth, 1973). To confirm the nature of the impact of STRI on expected returns, excess returns are first regressed on the contemporaneous STRI estimates and other statistical risk measures. Next, the cross-sectional regression analysis adds firm characteristics that have long been identified as factors influencing the cross section of stock returns. Considering these variables should help distinguish their impact from that of STRI. These additional variables include firm size, past returns and change in the market value of equity as a proxy for firm BTM ratio (Gerakos and Linnainmaa, 2015).

Table 5. Multivariate Regressions

	1	2	3	4	5	6	7
	return	return	return	return	return	return	return
STRI	0.0002*			0.0002*	0.0002*	0.0001**	0.0001**
	(1.85)			(1.89)	(1.86)	(2.18)	(2.23)
Beta		-0.0003**		-0.0003**		-0.0002**	
		(-2.48)		(-2.67)		(-2.15)	
Downside Beta			-0.0003***		-0.0004***		-0.0002**
			(-2.81)		(-2.88)		(-2.26)
Size						9.8E-05	7.1E-05
						(1.68)	(1.27)
Past Returns						-0.0007***	-0.0007***
						(-6.98)	(-7.13)
Book to Market						0.0016***	0.0015***
						(13.12)	(13.77)
Constant	0.000061	0.000061	0.000061	0.000061	0.000061	0.000061	0.000061
	(0.39)	(0.37)	(0.37)	(0.37)	(0.37)	(0.38)	(0.38)
R Square	0.0365	0.0648	0.0813	0.1058	0.1207	0.5122	0.5208
F Stat	3.42	6.15	7.87	4.28	5.59	43.66	46.64

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Table 5 records the results of the regression analysis. Column 1 shows the results of regression 1, which reassesses the findings of the univariate sort by including STRI. Similar to the results of the univariate sorts, the contemporaneous relationship between excess returns and STRI is positive. One standard deviation increases in STRI results in an additional annual return of 5.5 percent, on average. However, the coefficient is only statistically significant at 0.1 level.

For comparison purposes, the study relates excess returns solely to stock betas in regression 2 and to downside betas in regression 3. The results show a negative and insignificant relationship between stocks excess returns and betas. This finding is in line with the results of the sorts and those of previous studies that find a negative relationship between stock returns and betas on the JSE (Ward and Muller, 2012). The contemporaneous relationship between excess returns and downside betas is also found to be negative and statistically insignificant. This negative relationship is in contrast with the findings of Ang, et al. (2006), in which instead finds a positive association is found between stock returns and downside betas.

Regression 4 adds beta to regression 1. Including both STRI and beta in the regression not only preserves the signs of the coefficients but also improves their statistical significance. Both STRI and beta coefficients are now significant at 0.01 level of significance. Regression 5 includes both STRI and downside beta. As before, STRI remains positively related to average returns. Downside beta also conserves its negative relationship with average returns.

Regressions 1 to 5 confirm the results of the univariate and bivariate sorts. In general, STRI not only has a positive relationship with stocks average returns, but its effect on the cross section of returns is also independent from that of other risk factors such beta and downside beta. Moreover, STRI cannot solely explain the cross section of returns. The reported regressions using the F statistic and R square confirm that the variation in the cross section of returns is better explained by a combination of risk factors.

Firm characteristics are introduced in the following series of regressions. Regression 6 simultaneously includes STRI, beta, size, a BTM proxy as well as past returns. Regression 7 runs stock returns on the set of variables in regression 6 but replaces beta with downside beta. In accord with findings of previous studies on the JSE, the coefficient of the BTM proxy is positive and statistically significant in both regressions (Auret and Sinclaire, 2006). In this study, past returns are found to be a significant explanatory of the cross section of stock returns (van Heerden and van Rensburg, 2015). The coefficient of the size factor is also positive but statistically insignificant in both regressions. In presence of all of these variables, the coefficient of the STRI remains positive and statistically significant at 0.05 level in both regressions 6 and 7.

4.5. Robustness Checks

4.5.1. Alternative Dependence Structures

To assess the extent to which the choice of the dependence structure between market and stock extremes affects the above findings, this study subsequently substitutes the Galambos copula with the Gumbel copula, the Husler-Reiss copula and the traditional Pearson's correlation coefficient in the estimation of systematic tail risk. For each alternative dependence structure, cross sectional regressions 1 to 7 of section 4.4 are performed. Table 6 shows the results relative to the Gumbel copula.

Table 6. Multivariate regressions using alternative dependence structures – Gumbel Copula

	Full Period				
	1	4	5	6	7
	return	return	return	return	return
STRI	0.0002*	0.0002**	0.0002**	0.0001*	0.0001*
	(2.01)	(2.25)	(2.35)	(1.9)	(1.92)
Beta		-0.0002**		-0.0002**	
		(-2.31)		(-2.13)	
Downside Beta			-0.0003**		-0.0002**
			(-2.57)		(-2.12)
Size				0.0001	0.0001
				(1.6)	(1.32)
Past Returns				-0.0007***	-0.0007***
				(-6.88)	(-6.96)
Book to Market				0.0015***	0.0015***
				(13.88)	(14.51)
Constant	0.0001	0.0001	0.0001	0.0001	0.0001
	(0.67)	(0.65)	(0.65)	(0.65)	(0.65)
R Square	0.0641	0.1208	0.1349	0.5187	0.5278
F Stat	5.72	4.3	5.93	45.74	47.42

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

The relationship between STRI and stock returns is positive and statistically significant in all regressions. The results also show that the impact of STRI is independent from that of the control variables.

Similarly, Table 7 shows the results relative to the Gumbel copula. Again, the relationship between STRI and stock returns is positive and statistically significant in all regressions. Just as previously, the impact of STRI is also independent from that of the control variables.

Table 7. Multivariate regressions using alternative dependence structures - Husler Reiss Copula

	Full Period				
	1	4	5	6	7
	return	return	return	return	return
STRI	0.0002**	0.0002**	0.0002**	0.0002**	0.0002**
	(2.44)	(2.29)	(2.15)	(2.69)	(2.58)
Beta		-0.0003**		-0.0002**	
		(-2.6)		(-2.15)	
Downside Beta			-0.0004***		-0.0002**
			(-2.74)		(-2.26)
Size				0.0001	0.0001
				(1.64)	(1.24)
Past Returns				-0.0007***	-0.0007***
				(-6.98)	(-7.3)
Book to Market				0.0015***	0.0015***
				(13.46)	(14.15)
Constant	0.0001	0.0001	0.0001	0.0001	0.0001
	(0.68)	(0.66)	(0.66)	(0.67)	(0.67)
R Square	0.034	0.0989	0.1158	0.5145	0.5251
F Stat	5.96	4.71	5.58	43.75	45.83

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Table 8 reports the results relative to the Pearson's correlation. Interestingly, the results differ from those based on copula dependence structures. The relationship between STRI and stock returns is mostly positive in all regressions, except regression 6. However, the size of the coefficients are smaller and the observed relationship is statistically insignificant in all regressions, perhaps suggesting the inadequacy of the Pearson's correlation coefficient to model the dependence structure of extremes.

Table 8. Multivariate regressions using alternative dependence structures – Tail Beta

	Full Period				
	1	4	5	6	7
	return	return	return	return	return
STRI	0.0001	0.0001	0.0002	-5.61E-06	7.60E-06
	(1.07)	(1.33)	(1.53)	(-0.06)	(0.09)
Beta		-0.0003**		-0.0002*	
		(-2.61)		(-2)	
Downside Beta			-0.0004***		-0.0002*
			(-2.9)		(-2.14)
Size				0.0001*	0.0001
				(1.86)	(1.56)
Past Returns				-0.0007***	-0.0007***
				(-7.23)	(-7.41)
Book to Market				0.0015***	0.0015***
				(13.27)	(13.88)
Constant	0.0001	0.0001	0.0001	0.0001	0.0001
	(0.68)	(0.67)	(0.67)	(0.67)	(0.67)
R Square	0.0582	0.1233	0.1375	0.5246	0.534
F Stat	1.15	4.02	5.32	38.57	40.84

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

These results are in line with the findings of Chabi-Yo et al. (2015) and van Oordt and Zhou (2016). Essentially, they highlight the importance of the choice of dependence structure when estimating systematic tail risk measures.

4.5.2. Alternative Regression Estimation Methods

All regressions have so far followed the (modified) Fama-McBeth approach. To ensure that the results are not specific to using that approach, further regressions are carried out in this section using other estimation methods. In particular, regressions 6 and 7 in section 4.4 are repeated using the pooled OLS estimation, firm fixed effect panel estimation and random effect panel estimation.

Table 9 reports the results. In general, STRI remains positively related to stock returns and the relationship is still statistically significant.

Table 9. Alternative Regression Methods

	Pooled OLS		Fixed Effect		Random Effect	
	6	7	6	7	6	7
	return	return	return	return	return	return
STRI	0.0001**	0.0001**	0.0001*	0.0001*	0.0001**	0.0001**
	(2.2)	(2.16)	(1.82)	(1.8)	(2.31)	(2.28)
Beta	-0.0002***		-0.0003***		-0.0002***	
	(-5.89)		(-2.99)		(-5.41)	
Downside Beta		-0.0003***		-0.0004***		-0.0003***
		(-6.88)		(-2.99)		(-6.68)
Size	0.0002***	0.0002***	-0.0002	-0.0001	0.0002***	0.0002***
	(5.13)	(4.71)	(-1.02)	(-0.87)	(3.83)	(3.52)
Past Returns	-0.0007***	-0.0007***	-0.0008***	-0.0008***	-0.0007***	-0.0007***
	(-15.39)	(-15.33)	(-9.26)	(-9.42)	(-16.06)	(-16.08)
Book to Market	0.0015	0.0015***	0.0015***	0.0015***	0.0015***	0.0015***
	(34.02)	(34)	(14.52)	(14.46)	(33.88)	(33.76)
Constant	0.0001	0.0001	0.00001	0.0001	0.0000	0.0000
	(1.41)	(1.41)	(927.77)***	(933.2)***	(0.37)	(0.39)
R Square	0.3028	0.3057				
Within			0.2807	0.2844	0.2752	0.2791
Between			0.4317	0.4521	0.5989	0.5887
Overall			0.2696	0.2786	0.3037	0.3064
F Stat	264.66	268.28	47.65	47.62		
Wald					1279.42	1300.19

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

4.6. Temporal Stability

This section compares the crash aversion of JSE investors before and after the GFC. If JSE investors were more crash averse after the crisis, then exposure to tail events in the JSE should translate into a significantly higher premium during that period. To evaluate this assertion, the study repeats univariate and bivariate sorts as well as the cross sectional regressions for the periods stretching from 1 January 2002 to 31 December 2006 and from 1 July 2009 to 30 June 2014. These periods represent the five years preceding and the five years following the 2007- 08 GFC.

Table 10 gives the results of the univariate sort. In both periods, the relationship between STRI and average returns is positive. The spreads between high and low STRI portfolios are positive and significant, with the pre-crisis and the post-crisis periods recording spreads of 8.81 percent and 6.41 percent, respectively. These spreads suggest that pre-crisis period, relative to the post-crisis period, is associated with higher returns. However, they do not inform about Investors' crash aversion. Only risk-adjusted spreads can provide such information. With risk-adjusted spreads of 11.12 and 17.15 percent for the pre-crisis and post-crisis periods respectively, the results suggest that JSE investors became more crash inverse following the 2007-08 Global Financial Crisis. In addition, the relationship between STRI and returns is strictly increasing over the post-crisis period.

Table 10. Temporal Stability - Univariate Sort

	Returns (%)		STRI	
Portfolio	Jan-2002 - Dec-2006	Jul-2009 - Jun-2014	Jan-2002 - Dec-2006	Jul-2009 - Jun-2014
Low STR	-1.62	-2.09	0.69	0.80
2	3.58	1.40	0.88	0.90
3	5.29	3.50	0.95	0.96
4	4.63	4.64	1.04	1.02
High STR	7.19	4.31	1.48	1.17
H-L	8.81	6.41	0.79	0.37
	2.93	3.74	4.15	8.22
Risk Adjusted Returns	11.12	17.15		

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Table 11 offers similar conclusion for the bivariate sorts. For all bivariate sorts, the relationship between STRI and returns is positive. However, the relationship is monotonic over the post-crisis period only. Similar to the results of the univariate sort, the risk-adjusted spreads in the bivariate sorts are also higher over the post-crisis period.

Table 11 Temporal Stability - Bivariate Sorts

Beta/STRI Sort

Portfolio	Returns (%) Jan-2002 - Dec-2006	Returns (%) Jul-2009 - Jun-2014	STRI Jan-2002 - Dec-2006	STRI Jul-2009 - Jun-2014
Low STR	-1.79	-2.75	-0.27	-0.19
2	5.11	2.67	-0.11	-0.09
3	4.23	3.57	-0.04	-0.04
4	5.28	4.51	0.03	0.01
High STR	8.51	5.36	0.41	0.16
H-L	10.30	8.11	0.68	0.35
	3.77	4.46	4.19	8.36
Risk Adjusted Returns	15.16	23.31		

Downside Beta/STRI Sort

Portfolio	Returns (%) Jan-2002 - Dec-2006	Returns (%) Jul-2009 - Jun-2014	STRI Jan-2002 - Dec-2006	STRI Jul-2009 - Jun-2014
Low STR	-2.01	-2.80	-0.27	-0.19
2	3.44	2.20	-0.11	-0.09
3	5.99	3.76	-0.04	-0.04
4	5.77	4.21	0.03	0.01
High STR	7.66	5.72	0.41	0.15
H-L	9.66	8.53	0.68	0.34
	2.95***	5.46***	4.08***	9.08***
Risk Adjusted Returns	14.22	24.76		

Size/STRI Sort

Portfolio	Returns (%) Jan-2002 - Dec-2006	Returns (%) Jul-2009 - Jun-2014	STRI Jan-2002 - Dec-2006	STRI Jul-2009 - Jun-2014
Low STR	-1.42	-2.54	-0.27	-0.18
2	3.25	1.56	-0.11	-0.10
3	6.43	3.61	-0.04	-0.05
4	3.93	4.15	0.03	0.01
High STR	9.00	5.31	0.41	0.16
H-L	10.42	7.85	0.68	0.34
	3.40***	4.85***	4.15***	8.11***
Risk Adjusted Returns	15.42	23.21		

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Table 12 reports the results of cross-sectional regressions carried out for the pre and post-crisis periods. The results show that the relationship between STRI and returns remains positive for both periods. Interestingly, the effect of STRI appears more significant over the post-crisis period. The results of the portfolio sorts and cross-sectional regressions provide evidence that JSE investors were crash-inverse before and after the GFC. More importantly, the crash aversion was more pronounced in the period following the crisis.

Table 12. Multivariate Regression Models

	Jan 2002 - Dec 2006				
	1	4	5	6	7
	return	return	return	return	return
STRI	0.000397	0.000428	0.000432	0.000337	0.000337
	2.07	2.08	2.36	1.83	2.05
Beta		-0.00027		-0.00032	
		-1.16		-1.33	
Downside Beta			-0.00032		-0.00033
			-1.44		-1.45
Size				-3E-05	-6.9E-05
				-0.26	-0.53
Past Returns				-0.00085	-0.00097
				-4.41	-5.3
Book to Market				0.001506	0.001547
				8.06	8.62
Constant	0.000558	0.000554	0.000554	0.000554	0.000554
	0.097	1.84	1.84	1.84	1.84
R Square	0.0699	0.1334	0.1292	0.4531	0.4621
F Stat	4.28	2.2	2.83	21.16	18.96

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

Table 12. Multivariate Regression Models (Continued)

July 2009 - June 2014					
	1	2	3	4	5
	return	return	return	return	return
STRI	0.000259	0.000302	0.000295	0.000161	0.000161
	2.77	3.23	2.99	2.35	2.21
Beta		-0.00032		-0.00024	
		-2.51		-1.73	
Downside Beta			-0.00033		-0.00023
			-2.02		-1.48
Size				0.000219	0.000174
				1.8	1.68
Past Returns				-0.0003	-0.00029
				-2.44	-2.26
Book to Market				0.001125	0.001122
				6.5	6.4
Constant	0.000326	0.000325	0.000325	0.000326	0.000326
	1.4	1.39	1.39	1.4	1.4
R Square	0.031	0.11	0.1264	0.4798	0.4836
F Stat	7.69	5.75	4.51	59.62	68.08

**means the coefficient is significant at 10 %, **means a coefficient is significant at 5 %, ***means a coefficient is significant at 1 %, the number in brackets represent standard errors*

4. CONCLUSION

This study sought to provide further evidence on the role of market tail events in the cross section of stock returns. While earlier studies mostly focused on US stock markets, the present work extends the scope to emerging markets. In particular, the study investigates the existence of a systematic tail risk premium in the JSE. To this end, the study follows the works of Chabi-Yo et al. (2015) and estimates a stock's STRI by combining the statistical concepts of EVT and copula.

Using data on the JSE All Share Index constituents, the results reveal a positive and significant relationship between STRI and contemporaneous stock returns over the period of January 2002 to June 2018. Notably, this relationship remains significant after controlling for other risk factors and firm characteristics such as beta, downside beta, firm size and BTM ratio. These findings suggest that an increasing exposure to tail events in the JSE is compensated with higher returns.

The relationship between STRI and stock returns in the JSE is robust to different regression estimation methods. These methods include the Fama - McBeth approach and panel estimations. Similarly, the results are also robust to different copula dependence structures considered for the computation of systematic tail risk estimates.

The study also provides evidence on the impact of financial crises on crash aversion. The results show that JSE investors were crash averse throughout the investigation period. However, they reveal a more pronounced crash aversion in the aftermath of the financial crisis of 2008.

Overall, the findings suggest that the impact of systematic tail risk on the cross section of returns in the JSE is statistically and economically significant. Investors are therefore rewarded for their exposure to tail risk in the JSE. However, the premium associated to such risk is time-varying and more prevalent in periods subsequent to market turmoil.

The evidences presented here are in accord with earlier studies that find a significant impact of tail risk on the cross section of stock returns in US stock markets (Chabi-Yo et al., 2015; Cholette and Lu, 2011; Kelly and Hao, 2014). The systematic tail risk premium is therefore not unique to developed markets. The evidence in the JSE suggests that systematic tail risk is also priced in emerging markets.

The practical relevance of these results is of an utmost importance to both academics and finance professionals. The findings implicitly provide support for downside risk framework as a legitimate perspective on investors' perception of risk in equity markets. There is therefore a need to reconsider disfavoured portfolio theories such as the safety-first criterion in asset pricing endeavours. However, despite the apparent validity of the downside risk framework, reliance on Pearson correlation- based downside risk measures could be extremely misleading.

Similar to the work of van Oordt and Zhou (2016), the results of this study show no significant relationship between Pearson correlation-based STRI and the cross section of stock returns. When comparing results based on Pearson correlation coefficient and copula dependence structures, the study finds the latter to be adequate. While this result provides additional evidence of the limitations of the Pearson correlation coefficient, it also reveals the central importance of the choice of dependence structure when estimating risk measures. Interestingly, this finding has implications for further asset pricing endeavours. On one hand, it becomes essential for subsequent asset pricing studies to consider relying on copulas when constructing dependence based risk measures. On the other hand, this finding presents new avenues for future research. In particular, future asset pricing endeavours could further investigate the extent to which dependence measures affect asset pricing results.

References

Adu, G., Alagidede, P. and Karimu, A. (2015). Stock return distribution in the brics. *Review of Development Finance*, 5(2), pp.98–109.

Ang, A., Chen, J. and Xing, Y. (2006). Downside risk. *Review of Financial Studies*, 19(4), pp.1191-1239.

Ang, A., Chen, J. and Xing, Y. (2002). Downside correlation and expected stock returns. Working Paper, Columbia University.

Ang, A., Liu, J. and Schwarz, K. (2017). Using stocks or portfolios in tests of factor models. Working Paper, Columbia University.

Auret, C. J. and Sinclaire, R. A. (2006). Book-to-market ratio and returns on the JSE. *Investment Analysts Journal*, 63, pp.31-38.

Bali, T. G., Cakici, N. and Whitelaw, R., F. (2014). Hybrid tail risk and expected stock returns: When does the tail wag the dog? *Review of Asset Pricing Studies*, 4, pp.206-246.

Bawa, V. and Lindenberg, E. (1977). Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics*, 5, pp.189-200.

Black, F. Jensen, M. C. and Scholes, M. S. (1972). The capital asset pricing model: Some Empirical Test. In Michael C. Jensen (Ed.), *Studies in the Theory of Capital Markets*, pp.79-121. New York: Praeger.

Blume, M. (1970). Portfolio theory: A step towards its practical applications. *Journal of Business*, 43 (2), pp.152-174.

Bonga-Bonga, L. (2017). "Assessing the Readiness of the BRICS Grouping for Mutually Beneficial Financial Integration", Review of Development Economics, 21(4),pp. e204-e219.

Chabi-Yo, F., Ruenzi, S. and Weigert, F. (2015). Crash sensitivity and the cross-section of expected stock returns. Unpublished Working Paper, Ohio State University, University of Mannheim, and University of St. Gallen.

Cholette, L. and Lu, C.-C. (2011). The market premium for dynamic tail risk. Working paper, University of Stavanger and National Chengchi University.

Fama, E, F. and McBeth, J. D. (1973). Risk, return, and equilibrium: empirical tests. *The Journal of Political Economy*, 81(3), pp.607-636.

Gerakos, J. and Linnainmaa, J. (2015). Decomposing value. Working Paper. University of Chicago.

Gudendorf, G. and Segers, J. (2010). Extreme-value copulas. In W. H. P. Jaworski, F. Durante and T. Rychlik (Eds.), Proceedings of the Workshop on Copula Theory and its Applications, pp.127-146. Springer.
Harris, R.D.F., Nguyen, L.H. and Stoja, E. (2016). STR. Bank of England Staff Working Paper No.547.
Jenkinson, A. F. (1955). Statistics of extremes, in *Estimation of Maximum Floods*, WMO 233, TP 126, Tech. Note 98, chap. 5, pp.183-228, World Meteorol. Off., Geneva, Switzerland, 1969.
Kelly, B. and Jiang, H. (2014). Tail risk and asset prices. *Review of Financial Studies*, 27, pp.2841-2871.
Leadbetter, M. R., Lindgren, G. and Rootzen, H. (1983). *Extremes and Related Properties of Random Sequences and Processes*. New York: Spring Verlag.
Limberis, A. (2012). An empirical investigation of the conditional risk-return trade-off in South Africa. Unpublished M-Com (Accountancy) thesis. Johannesburg: University of Witwatersrand.
Okyere-Boakye, K. and O'Malley, B., (2016). Downside CAPM: The case of South Africa. *Journal of Economic and Financial Sciences*, 9(2), pp.578-608.
Post, T. and Van Vliet, P. (2006). Downside risk and asset pricing. *Journal of Banking & Finance*, 30, pp.823-849.
Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3), pp.425-442.
Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris*, 8, pp.229-231.

van Heerden, J. D. and van Rensburg, P. (2015). The cross-section of Johannesburg Securities Exchange listed equity returns (1994-2011). *Studies in Economics and Finance*, 32(4), pp.422-444.
van Oordt, M. R. C. and Zhou, C. (2016). STR. *Journal of Financial and Quantitative Analysis*, 51 (2), pp.685-705.
von Mises, R. (1936). La distribution de la plus grande de n valeurs, *Rev., Math, Union Interbalcanique*, Vol. 1, pp.141-160, Reproduced, Selected papers of von Mises, R. (1964) *American Mathematical Society*, 2, pp.271-294.
Ward, M. and Muller, C. (2012). Empirical testing of the CAPM on the JSE. *Investment Anlaysts Journal*, 76, pp.1-12.