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Abstract

This study considers how present-biased preferences influence public debt policy when a violation of debt rules is possible. To address this issue, the study extends the framework of Bisin, Lizzeri, and Yariv (American Economic Review 105, (2015), 1711–1737) by allowing for rule breaking with extra costs, and we show that rule breaking occurs when a country exhibits a strong present bias. We further extend the model by introducing a political process for determining the debt rule, and we show that a polarization of debt rules emerges between countries with high and low degrees of present bias.

Key words: Debt ceilings; Present bias; Public debt.

JEL Classification: D72, D78, H62, H63

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1 Introduction

In the last decade, many developed countries have experienced large budget deficits and rapidly growing public debt. In 2007, the average general government gross debt in Organisation for Economic Co-operation and Development (OECD) member countries, as a percentage of GDP, was 53.34%, and it increased to 85.58% in 2015. In particular, the ratio increased by more than 60 points in Greece, Japan, Portugal, Spain, and the United Kingdom.[1] This raises concerns about the sustainability of public finances and highlights the need for fiscal rules in achieving fiscal consolidation (IMF, 2009).

Fiscal rules are expected to constrain the behavior of governments, but the enforceability of these rules is questionable, as indicated by Alesina and Passalacqua (2016). As reported in Wyplosz (2013), the United Kingdom adopted two fiscal rules in 1997: (1) the budget deficit may only finance public investment and (2) the debt-to-GDP ratio may not exceed 40 percent. However, while the rule was met for a few years, slippage set in after 2002. Wyplosz (2013) also reports that in the euro area, the Maastricht treaty specifies that budget deficits cannot exceed 3 percent, but this rule has been satisfied only 60 percent of time over the thirteen years that the euro has existed. The evidence suggests that, in practice, the conditions required for fiscal institutions are rarely met.

To investigate why fiscal rule violations occur so frequently, the present study focuses on time-inconsistent, present-biased preferences. When agents are endowed with such preferences, they change their ex-ante consumption plans, choosing to consume more in the present and less in the future (Laibson, 1997). In particular, they are incentivized to support, via voting, a large public debt issue; this enables them to obtain a great deal of resources for consumption today through transfers financed by the debt issuance. Bisin, Lizzeri, and Yariv (2015) is the first study to present a model of public debt that includes such an incentive mechanism.

In the framework of Bisin, Lizzeri, and Yariv (2015), the government, representing the present-biased agents, is assumed to stick to a given debt rule; the rule breaking is

[1]Source: OECD.stat. https://stats.oecd.org/ (Accessed on June 12, 2019).

abstracted away from their analysis. However, if the rule breaking is available through the payment of extra costs, the agents may find it optimal to support the issue of public debt above the debt ceiling. Such rule breaking depends on the degree of present bias, but this degree differs among countries, as reported in Wang, Rieger, and Hens (2016). Thus, the following questions arise naturally: (a) how present-biased preferences influence the choice of public debt issuance when rule breaking is possible and (b) what kind of a debt rule must be put in place in response to the degree of present bias. The purpose of this study is to address these questions.

For analysis, we use the simple three-period model developed by Bisin, Lizzeri, and Yariv (2015). Agents are endowed with goods in period 1 and make savings and portfolio decisions; they receive utility from consumption in periods 2 and 3. Period-2 selves are endowed with present-biased preferences, so they are tempted to increase consumption in period 2 at the cost of reduced consumption in period 3. Period-1 selves use illiquid assets to constrain the consumption plans of their future selves. However, the government, representing the period-2 selves, is induced to issue public debt to respond to the desire of the period-2 selves to undo the commitment in period 1. This gives sophisticated agents an incentive to rebalance their portfolios in period 1 to reestablish their commitment consumption sequence. This in turn creates demand for further debt accumulation.

Bisin, Lizzeri, and Yariv (2015) control the behavior of present-biased agents by imposing a debt ceiling. Our model differs from theirs in that debt issue beyond the ceiling is available by incurring some additional costs. Within this extended framework, we show that the benefits of rule breaking outweigh the costs, and, thus, rule breaking occurs if the present bias is extremely strong and the debt ceiling is fairly low. The result could be viewed as providing one possible key to understanding the phenomenon of fiscal rule breaking often observed in developed countries.

The assumption of a fixed debt ceiling follows that in Bisin, Lizzeri, and Yariv (2015). This assumption is reasonable in the short run, but in the long run there must be tendency toward the revision of fiscal rules. For instance, according to the US Department of the Treasury, the US debt ceiling has been raised 78 times since 1960.[2] Another example involves Japan, which has the highest debt-to-GDP ratio among all developed countries. Despite the urgent need for fiscal consolidation, Japan pushed back the target year for achieving a primary balance surplus from 2020 to 2025 given its slow recovery from the recession.[3] The evidence suggests that fiscal rules are not necessarily rigorous requirements, but can be easily revised in accordance with economic and political conditions.

Motivated by this evidence, we extend the analysis by introducing the endogenous determination of the debt ceiling via voting. In particular, we consider a situation where period-1 selves set the debt ceiling, taking into account the response of the period-2 selves. Under this setting, we investigate how the present-biased preferences affect the design of debt ceiling, and show the following result. When the present bias is weak, period-2 selves have little incentive to change their consumption plans and thus to issue public debt. They follow this debt rule even if the ceiling is set at the lowest level, that is, even if the debt issue is prohibited. Therefore, the debt ceiling is set at zero, which is optimal from the viewpoint of the period-1 selves' utility maximization.

When the present bias is strong, the marginal benefits of rule breaking outweigh the costs for the period-2 selves. Thus, the period-1 selves find it impossible to make the period-2 selves follow the rule of no debt issue. To avoid the costs of rule breaking, period-1 selves set the ceiling at a maximum level such that the period-2 selves never break it. The result, thus far, suggests that there is a threshold level for the present bias such that the optimal ceiling for the period-1 selves varies in a discontinuous manner around the threshold. This result could be viewed as providing one possible explanation for the cross-county difference in debt rules and the resulting levels of public debt among developed countries.

As mentioned above, the present study is closely related to Bisin, Lizzeri, and Yariv (2015), who demonstrate the role of present-biased preferences in fiscal policy making. The

[2] Source: https://home.treasury.gov/policy-issues/financial-markets-financial-institutions-and-fiscal-service/debt-limit (accessed on September 27, 2019)

[3] Source: https://www5.cao.go.jp/keizai-shimon/kaigi/cabinet/2018/2018_basicpolicies_en.pdf (accessed on September 27, 2019)

study is also related to Halac and Yared (2018), who analyze the formation of fiscal rules in the presence of present-biased preferences in a multi-country economy. In particular, they compare coordinated rules, chosen jointly by a group of countries, to uncoordinated rules, chosen independently by each country, and show that the coordinated rules are slacker when the present bias is large. However, rule breaking is abstracted away from their analysis. The present study, in contrast, allows rule breaking, and it derives the optimal rules under the possibility of rule breaking by present-biased voters.

The present study also contributes to the literature on the political economy of public debt, such as Cukierman and Meltzer (1989); Song, Storesletten, and Zilibotti (2012); Azzimonti, Battaglini, and Coate (2016); Barseghyan and Battaglini (2016); and Arai, Naito, and Ono (2018). In all of these studies, it is assumed that agents are not present biased, and fiscal rules are taken as given. The present work advances the previous studies by relaxing these assumptions, and it shows how public debt accumulation and the determination of fiscal rules are affected by the present-biased preferences of voters.

This paper is organized as follows: The next section lays out the model. The third section demonstrates the agents' saving decisions and the government's fiscal policy decision. The fourth section characterizes the equilibrium allocation. The fifth section extends the model to the endogenous determination of debt rules, and the last section concludes. Proofs for the propositions are in the appendix.

2 The Model

The model is based on the one developed by Bisin, Lizzeri, and Yariv (2015). It measures identical agents who live for three periods, 1, 2, and 3. They are endowed with k units of goods in period 1 and nothing in periods 2 and 3. In period 1, agents only make savings and portfolio decisions; they receive utility from consumption in periods 2 and 3.

Agents (hereafter interchangeably called individuals, selves, and voters) have time-inconsistent, present biased preferences (Laibson, 1997). In particular, the agents' preferences over consumption in periods 2 and 3, c_2 and c_3, are given by the following utility functions:

$$U_1(c_2, c_3) = \beta \left[u(c_2) + u(c_3) \right],$$

$$U_2(c_2, c_3) = u(c_2) + \beta u(c_3),$$

where U_t $(t = 1, 2)$ is the assessed utility at time t, u is a continuous and strictly concave utility function, and $\beta \in (0, 1)$ is a parameter representing the degree of present bias; a lower β implies that period-2 agents are biased toward more period-2 consumption. Agents are assumed to be sophisticated; they are fully aware of their self-control problems.

Agents choose to invest their wealth, k, in liquid or illiquid assets in period 1. It is assumed that all liquid and illiquid assets have the same exogenous interest rate of zero. Liquid assets are one-period securities that are sold in period t $(t = 1, 2)$ and redeemed in period $t + 1$. Illiquid assets are two-period securities that are sold in period 1 and redeemed in period 3; they cannot be sold in period 2. Savings in one- and two-period securities in period 1 are denoted by, s_{12} and s_{13}, respectively; the subscript ij means the time of saving, i, and redemption, j. In period 2, agents can save the return from s_{12} in one-period securities; this saving is denoted by s_{23}.

Agents displaying present-biased preferences suffer from self-control problems. In particular, period-2 selves are tempted to increase consumption in period 2 at the cost of reduced consumption in period 3. Period-1 selves use illiquid assets to constrain the consumption plans of their future selves. However, the government, representing period-2 selves, is induced to issue public debt in the international market to respond to period-2 selves' desire to undo the commitment made in period 1. This gives sophisticated agents an incentive to rebalance their portfolios in period 1 to reestablish their consumption sequence commitment. This, in turn, creates demand for further debt accumulation. The debt issue, denoted by d, is assumed to be costly and constrained by the constitutionally imposed borrowing limits denoted by $\bar{d}$, but debt issue beyond the limit is available by incurring some additional costs, as specified below.

The timing of events and the optimization problem at each stage are as follows. In period 1, an agent, who predicts an equilibrium per capita public debt level of d and period 2 savings, s_{23}, chooses period 1 savings intended for period 2, s_{12}, and for period 3, s_{13}, to maximize the assessed utility in period 1, U_1. Since the debt level is determined through voting, each agent takes it as given when making his or her saving decision. The problem of the agent in period 1 is:

$$\begin{aligned}
\max_{s_{12},s_{13}} \quad & U_1 = \beta \left[u\left(c_2\right) + u\left(c_3\right) \right] \\
\text{s.t.} \quad & s_{12} + s_{13} \leq k, \\
& c_2 \leq s_{12} + d^e - s_{23}, \\
& c_3 \leq s_{13} + s_{23} - G(d^e), \\
& s_{12}, s_{13}, s_{23} \geq 0, \\
& \text{given } s_{23} \text{ and } d^e,
\end{aligned}$$

where d^e denotes the expected level of debt issue in period 2.

The first, second, and third constraints are the period-1, -2, and -3 budget constraints. Following Bisin, Lizzeri, and Yariv (2015), we assume that (i) private borrowing is not allowed, as expressed in the fourth constraint, and (ii) debt is financed by foreign lenders at an interest rate of zero, but can be directly distortionary.[4] The term $G(d)$, representing the costs of debt repayment, is specified as follows:

$$G(d) = \begin{cases} (1+\eta)\, d & \text{when } d \leq \bar{d}, \\ (1+\eta)\, \bar{d} + (1+\eta+\gamma)\left(d - \bar{d}\right) & \text{when } d > \bar{d}, \end{cases}$$

where $\eta > 0$ and $\gamma > 0$. The term η represents the marginal costs of debt issue, such as labor supply distortions induced by increased tax burdens for debt repayment. The term γ represents the marginal cost of rule breaking: the government is required to pay extra costs for each debt issue beyond the debt ceiling. An example of such costs is the

[4] The assumption of a zero interest rate is for simplicity of exposition.

administrative costs incurred by the government in passing a bill that allows it to issue debt above the ceiling.

In period 2, an agent chooses the savings intended for period 3, s_{23}, taking d^e as given, to maximize the assessed utility in period 2, U_2. The problem of the period-2 agents is:

$$\begin{aligned}\max_{s_{23}} \quad & U_2 = u(c_2) + \beta u(c_3) \\ \text{s.t.} \quad & c_2 \le s_{12} + d^e - s_{23}, \\ & c_3 \le s_{13} + s_{23} - G(d^e), \\ & s_{23} \ge 0, \\ & \text{given } s_{12}, s_{13}, \text{ and } d^e,\end{aligned}$$

where s_{12} and s_{13} satisfy $s_{12} + s_{13} \le k$ and solve the period-1 problem. The government, representing period-2 selves, chooses public debt issue, d, to maximize the utility of period-2 agents, subject to a non-negativity constraint, $d \ge 0$, and a constitutionally imposed debt ceiling, $\bar{d}$, given s_{12}, s_{13}, and s_{23}.[5]

For our analysis, we make the following assumptions. First, the utility function is specified as

$$u(c) = \frac{(c)^{1-\sigma} - 1}{1-\sigma},$$

where σ (> 0) is an inverse of the inter-temporal elasticity of substitution. This assumption enables us to solve the model analytically. Second, the borrowing must be below the natural debt limit, k/η, to prevent the government from defaulting. In addition, to define the debt ceiling, it is assumed that the ceiling is below the natural debt limit, as in the following assumption:

Assumption 1: $\bar{d} < k/\eta$.

[5] Lending in the international market, $d < 0$, is abstracted away from the analysis since our focus is on borrowing, $d > 0$. Allowing for $d < 0$ does not qualitatively alter the following result.

3 Decisions of Agents and Government

As mentioned above, agents are assumed to be sophisticated. Thus, we solve the model through backward induction; that is, we first solve the government's problem in period 2, then the agents' problem in period 2, and finally the agents' problem in period 1. Our result would not change if the timing within period 2 is reversed because the period-2 selves and the government share the same objective.

3.1 Government's Period-2 Decision

The problem of the government, representing the period-2 selves, is

$$\begin{aligned}
\max_{d\geq 0} \quad & V_g\left(d; s_{12}, s_{23}\right) \equiv \frac{\left(s_{12}+d-s_{23}\right)^{1-\sigma}-1}{1-\sigma} + \beta \cdot \frac{\left(k-s_{12}+s_{23}-G(d)\right)^{1-\sigma}-1}{1-\sigma} \\
\text{s.t.} \quad & G(d) = \begin{cases} (1+\eta)d & \text{when } d \leq \bar{d}, \\ (1+\eta+\gamma)d-\gamma\bar{d} & \text{when } d > \bar{d}, \end{cases} \\
& \text{given } s_{12} \text{ and } s_{23},
\end{aligned}$$

where V_g denotes the period-2 government's objective function. The first derivatives, with respect to d when $d \leq \bar{d}$ and $d > \bar{d}$, are, respectively,

$$\left.\frac{\partial V_g\left(d; s_{12}, s_{23}\right)}{\partial d}\right|_{d\leq\bar{d}} = \left(s_{12}+d-s_{23}\right)^{-\sigma} - \beta\left(1+\eta\right)\cdot\left(k-s_{12}+s_{23}-(1+\eta)d\right)^{-\sigma}, \tag{1}$$

$$\left.\frac{\partial V_g\left(d; s_{12}, s_{23}\right)}{\partial d}\right|_{d>\bar{d}} = \left(s_{12}+d-s_{23}\right)^{-\sigma} - \beta\left(1+\eta+\gamma\right)\cdot\left(k-s_{12}+s_{23}-(1+\eta+\gamma)d+\gamma\bar{d}\right)^{-\sigma}. \tag{2}$$

Let d^u and d^c denote interior solutions when $d \leq \bar{d}$ and $d > \bar{d}$, respectively. They are given by

$$d^u(s_{12}, s_{23}) = \frac{k - \left[1 + \{\beta(1+\eta)\}^{1/\sigma}\right](s_{12} - s_{23})}{(1+\eta) + \{\beta(1+\eta)\}^{1/\sigma}}, \tag{3}$$

$$d^c\left(s_{12}, s_{23}, \bar{d}\right) = \frac{k + \gamma\bar{d} - \left[1 + \{\beta(1+\eta+\gamma)\}^{1/\sigma}\right](s_{12} - s_{23})}{(1+\eta+\gamma) + \{\beta(1+\eta+\gamma)\}^{1/\sigma}}, \tag{4}$$

where $d^u(s_{12}, s_{23})$ and $d^c\left(s_{12}, s_{23}, \bar{d}\right)$ satisfy

$$d^c\left(s_{12}, s_{23}, \bar{d}\right) \gtreqless \bar{d} \Leftrightarrow A(s_{12}, s_{23}) \equiv \frac{k - \left[1 + \{\beta(1+\eta+\gamma)\}^{1/\sigma}\right](s_{12} - s_{23})}{(1+\eta) + \{\beta(1+\eta+\gamma)\}^{1/\sigma}} \gtreqless \bar{d},$$

and[6]

$$A(s_{12}, s_{23}) \leq d^u(s_{12}, s_{23}).$$

The condition of $A(s_{12}, s_{23}) \leq d^u(s_{12}, s_{23})$ implies that there are four possible cases, classified according to the relative magnitude among $d^u(s_{12}, s_{23})$, $A(s_{12}, s_{23})$, and $\bar{d}$, as illustrated in Figure 1: $d^u(s_{12}, s_{23}) \leq 0 \leq \bar{d}$ (Panel (a)), $0 < d^u(s_{12}, s_{23}) < \bar{d}$ (Panel (b)), $A(s_{12}, s_{23}) \leq \bar{d} \leq d^u(s_{12}, s_{23})$ (Panel (c)), and $\bar{d} < A(s_{12}, s_{23})$ (Panel (d)). From the figure, we can find that the solution d for the government problem, denoted by $d(s_{12}, s_{23})$, as follows:

$$d(s_{12}, s_{23}) = \begin{cases} 0 & \text{when } d^u(s_{12}, s_{23}) \leq 0, \\ d^u(s_{12}, s_{23}) & \text{when } 0 < d^u(s_{12}, s_{23}) < \bar{d}, \\ \bar{d} & \text{when } A(s_{12}, s_{23}) \leq \bar{d} \leq d^u(s_{12}, s_{23}), \\ d^c\left(s_{12}, s_{23}, \bar{d}\right) & \text{when } \bar{d} < A(s_{12}, s_{23}). \end{cases} \tag{5}$$

[Figure 1 here.]

Consider first $d^u(s_{12}, s_{23})$, which represents the optimal level of public debt when it

[6]Proof of $A(s_{12}, s_{23}) \leq d^u(s_{12}, s_{23})$ is as follows. Suppose, to the contrary, that $A(s_{12}, s_{23}) > d^u(s_{12}, s_{23})$, i.e., $0 > k/\eta + s_{12} - s_{23}$ holds. The period-2 budget constraint leads to $c_2 \leq s_{12} + d - s_{23} < s_{12} + k/\eta - s_{23}$, where the second inequality comes from $d \leq \bar{d} < k/\eta$. Given $c_2 > 0$, this implies that $0 < s_{12} + k/\eta - s_{23}$, which is a contradiction.

satisfies the debt ceiling. Eq. (3) indicates that $d^u(s_{12}, s_{23})$ increases as $(s_{12} - s_{23})$ and β decrease. The term $(s_{12} - s_{23})$, representing the period-2 consumption when there is no debt issue, implies that the marginal utility of the period-2 consumption increases as $(s_{12} - s_{23})$ decreases. The term β, representing the present bias, implies that the period-2 agents attach a larger weight on the period-2 consumption relative to the period-3 consumption as β decreases. Thus, the period-2 selves' preferences for debt financing increase as $(s_{12} - s_{23})$ and β decrease.

More precisely, suppose first that $(s_{12} - s_{23})$ and β are high, such that $d^u(s_{12}, s_{23}) \leq 0$ holds, then the optimal level of the public debt is below zero. In other words, the government prefers to lend rather than borrow in the international market. However, lending is not allowed in the present framework. Thus, the government's choice is constrained by the non-negativity constraint; the optimal level of public debt becomes $d = 0$, as illustrated in Panel (a) of Figure 1. When $(s_{12} - s_{23})$ and β are at moderate levels, such that $0 < d^u(s_{12}, s_{23}) < \bar{d}$, the government is not constrained by the non-negativity constraint or the debt ceiling. Thus, its choice is $d = d^u(s_{12}, s_{23})$, as illustrated in Panel (b) of Figure 1.

Finally, when $(s_{12} - s_{23})$ and β are low, such that $\bar{d} \leq d^u(s_{12}, s_{23})$ holds, the government may borrow over the debt ceiling. In particular, its decision depends on the relative magnitude between $A(s_{12}, s_{23})$ and $\bar{d}$. Since $A(s_{12}, s_{23})$ is decreasing in γ, which represents the costs of rule breaking, the government finds it is optimal to follow the rule and issues debt up to the limit, $d = \bar{d}$, when γ is large, such that $A(s_{12}, s_{23}) \leq \bar{d}$, as illustrated in Panel (c) of Figure 1. However, rule breaking occurs when γ is low, such that $\bar{d} < A(s_{12}, s_{23})$, as illustrated in Panel (d) of Figure 1.

3.2 Agents' Period-2 Decision

Next, we consider the period-2 agents' decision regarding one-period securities, s_{23}. The problem is

$$\begin{aligned}
\max_{s_{23}} \quad & V_2\left(s_{23}; s_{12}, d^e\right) \equiv \frac{\left(s_{12}+d^e-s_{23}\right)^{1-\sigma}-1}{1-\sigma}+\beta \cdot \frac{\left[k-s_{12}+s_{23}-G\left(d^e\right)\right]^{1-\sigma}-1}{1-\sigma}, \\
\text{s.t.} \quad & G(d^e)=\begin{cases} (1+\eta)d^e & \text{when } d^e \leq \bar{d}, \\ (1+\eta+\gamma)d^e-\gamma\bar{d} & \text{when } d^e > \bar{d}, \end{cases} \\
& s_{23} \geq 0, \\
& s_{12} \text{ and } d^e,
\end{aligned}$$

where V_2 denotes the period-2 agent's objective function. The first-order condition with respect to s_{23} leads to:

$$s_{23}=s_{23}^u\left(s_{12}, d^e\right) \equiv s_{12}-\frac{k-\left[(1+\eta)+(\beta)^{1/\sigma}\right] d^e}{1+(\beta)^{1/\sigma}} \text{ when } d^e \leq \bar{d}, \tag{6}$$

$$s_{23}=s_{23}^c\left(s_{12}, d^e, \bar{d}\right) \equiv s_{12}-\frac{k+\gamma\bar{d}-\left[(1+\eta+\gamma)+(\beta)^{1/\sigma}\right] d^e}{1+(\beta)^{1/\sigma}} \text{ when } d^e > \bar{d}. \tag{7}$$

With the private borrowing constraint, $s_{23} \geq 0$, and the expectation of $d=d^e$, an optimal level of s_{23}, denoted by $s_{23,d=d^e}^{opt}$, is given by

$$s_{23,d=d^e}^{opt}=\begin{cases} 0 & \text{when } d^e \leq \bar{d} \text{ and } s_{12} \leq S^u\left(d^e\right), \\ s_{23}^u\left(s_{12}, d^e\right) & \text{when } d^e \leq \bar{d} \text{ and } s_{12} > S^u\left(d^e\right), \\ 0 & \text{when } d^e > \bar{d} \text{ and } s_{12} \leq S^c\left(d^e, \bar{d}\right), \\ s_{23}^c\left(s_{12}, d^e, \bar{d}\right) & \text{when } d^e > \bar{d} \text{ and } s_{12} > S^c\left(d^e, \bar{d}\right), \end{cases} \tag{8}$$

where $S^u(d^e)$ and $S^c\left(d^e, \bar{d}\right)$ are defined as follows:

$$S^u(d^e) \equiv \frac{k - \left[(1+\eta) + (\beta)^{1/\sigma}\right] d^e}{1 + (\beta)^{1/\sigma}}, \tag{9}$$

$$S^c\left(d^e, \bar{d}\right) \equiv \frac{k + \gamma\bar{d} - \left[(1+\eta+\gamma) + (\beta)^{1/\sigma}\right] d^e}{1 + (\beta)^{1/\sigma}}. \tag{10}$$

The period-2 selves attach a larger weight to period-2 consumption than the period-1 selves. This implies that the former selves are induced to increase their period-2 consumption by lowering their saving in s_{23}. In particular, the period-2 selves find it optimal to save nothing in s_{23} when the expectation of d^e is low and/or when the saving in one-period securities, s_{12}, by the period-1 selves is low, such that either $s_{12} \leq S^u(d^e)$ or $s_{12} \leq S^c\left(d^e, \bar{d}\right)$ holds. If this were not the case, the period-2 selves could afford to save a portion of the return from one-period securities, s_{12}, in s_{23}.

3.3 Agents' Period-1 Decision

Consider the period-1 agent's decision regarding one-period securities, s_{12}. The problem is:

$$\begin{aligned}
\max_{s_{12}} \quad & V_1(s_{12}, d^e) \equiv \frac{\left(s_{12} + d^e - s^{opt}_{23,d^e}\right)^{1-\sigma} - 1}{1-\sigma} + \frac{\left[k - s_{12} + s^{opt}_{23,d^e} - G(d^e)\right]^{1-\sigma} - 1}{1-\sigma} \\
\text{s.t.} \quad & G(d^e) = \begin{cases} (1+\eta)d^e & \text{when } d^e \leq \bar{d}, \\ (1+\eta+\gamma)d^e - \gamma\bar{d} & \text{when } d^e > \bar{d}, \end{cases} \\
& 0 \leq s_{12} \leq k,
\end{aligned}$$

where V_1 denotes the period-1 agent's objective function. It is assumed that period-1 and -2 selves have the same expectation for d. Given the expectation of $d = d^e$ and s_{23}, the period-1 agent chooses s_{12} to maximize his/her objective. Let s^{opt}_{12,d^e} denote the solution to the problem. The solution satisfies the following first-order condition:

$$\frac{\partial V_1\left(s_{12}, d^e\right)}{\partial s_{12}} = \left(1 - \frac{\partial s_{23,d^e}^{opt}}{\partial s_{12}}\right) \cdot \left(s_{12} + d^e - s_{23,d^e}^{opt}\right)^{-\sigma}$$
$$- \left(1 - \frac{\partial s_{23,d^e}^{opt}}{\partial s_{12}}\right) \cdot \left[k - s_{12} + s_{23,d^e}^{opt} - G\left(d^e\right)\right]^{-\sigma} \leq 0, \qquad (11)$$

where a strict inequality holds if $s_{12} = 0$.

4 Equilibrium

Having described the behavior of agents and the government, we define an equilibrium in the present framework as follows.

Definition 1: A *rational expectations equilibrium* is an allocation $(s_{12}, s_{13}, s_{23}, c_2, c_3, d)$, such that (i) $s_{12} = s_{12,d^e}^{opt}$ solves the period-1 agent's problem given s_{23} and $d = d^e$; (ii) $s_{23} = s_{23,d^e}^{opt}$ solves the period-2 agent's problem given s_{12} and $d = d^e$; (iii) rational expectations hold, that is, the solution to the period-2 government's problem, d, satisfies $d\left(s_{12,d^e}^{opt}, s_{23,d^e}^{opt}\right) = d^e$; and (iv) given $s_{12} = s_{12,d^e}^{opt}$, $s_{23} = s_{23,d^e}^{opt}$, and $d = d\left(s_{12,d}^{opt}, s_{23,d}^{opt}\right)$, the allocation (s_{13}, c_2, c_3) is determined by the period-1, -2, and -3 budget constraints.

To characterize the equilibrium allocation, we proceed with the analysis in the following manner. First, we assume that period-1 and -2 selves have expectations of $d^e = 0$, $d^u \in \left(0, \bar{d}\right)$, $\bar{d}$, or $d^c (> \bar{d})$, where d^u and d^c denote the expectations of agents that the debt issuance is below or above the ceiling, $\bar{d}$, respectively. Given the expectation of the debt issuance, we solve for one-period securities, $s_{12} = s_{12,d^e}^{opt}$ and $s_{23} = s_{23,d^e}^{opt}$. Then we substitute these into the solution $d = d(s_{12}, s_{23})$ for the government problem, and identify the condition in which the expectations are rational. The following proposition presents the equilibrium level of public debt. The corresponding allocation of savings and consumption is presented in the Appendices A and B.

Proposition 1. (Equilibrium Public Debt)

(i) *If present bias is weak, such that* $1/(1+\eta) \leq \beta$ *holds, then the equilibrium debt is below the ceiling,* $d < \bar{d}$.

(ii) *If present bias is mild, such that* $1/(1+\eta+\gamma) \leq \beta \leq 1/(1+\eta)$*, then the equilibrium debt is* (a) *below the ceiling,* $d < \bar{d}$*, if* $k/\left[(1+\eta) + (\beta(1+\eta))^{1/\sigma}\right] < \bar{d}$*, and* (b) *up to the ceiling,* $d = \bar{d}$*, otherwise.*

(iii) *If present bias is strong, such that* $\beta \leq 1/(1+\eta+\gamma)$ *holds, then equilibrium debt is* (a) *below the ceiling,* $d < \bar{d}$*, if* $k/\left[(1+\eta) + (\beta(1+\eta))^{1/\sigma}\right] < \bar{d}$*;* (b) *up to the ceiling,* $d = \bar{d}$*, if* $k/\left[(1+\eta) + (\beta(1+\eta+\gamma))^{1/\sigma}\right] \leq \bar{d} \leq k/\left[(1+\eta) + (\beta(1+\eta))^{1/\sigma}\right]$*; and* (c) *beyond the ceiling,* $d > \bar{d}$*, otherwise.*

Proof. See Appendix A.1.

[Figure 2 here.]

Figure 2 illustrates the result in Proposition 1. In the current framework, present-biased agents are incentivized in period 2 to rebalance their portfolios by issuing public debt. When the bias is weak and the costs of debt issue are large, such that $\beta \geq 1/(1+\eta)$ holds, agents find it optimal to issue no debt because the benefit of public debt is outweighed by its costs. However, they prefer public debt issue when the bias is strong and the costs of debt issue are small, such that $\beta < 1/(1+\eta)$ holds. The equilibrium debt level falls below the ceiling when the debt limit is loose, $k/\left[(1+\eta) + (\beta(1+\eta))^{1/\sigma}\right] < \bar{d}$; otherwise, the debt level reaches the ceiling. In particular, the agents prefer to issue debt above the ceiling when overissue is allowed by incurring additional costs. Such a case occurs if the present bias is extremely strong and the debt ceiling is fairly low, such that $\beta \leq 1/(1+\eta+\gamma)$ and $\bar{d} < k/\left[(1+\eta) + (\beta(1+\eta+\gamma))^{1/\sigma}\right]$ hold. This result could be viewed as a possible key to understanding why the conditions required for fiscal institutions are rarely met in practice, as reported in the Introduction.

5 Vote on Debt Rule

The analysis, thus far, has assumed that the government takes the fiscal rule, represented by $\bar{d}$, as given. This assumption, which follows Bisin, Lizzeri, and Yariv (2015), is reasonable in the short run, but in the long run there must be a tendency toward revising fiscal rules, as described in the Introduction. This section extends the analysis in the previous sections by introducing endogenous determination of the debt rule, and it investigates how the present bias affects, via voting, the design of the debt rules.

For analysis, we assume that the debt rule is determined before the period-1 selves' decision on saving, s_{12} and s_{13}. Thus, the debt rule is set to maximize the period-1 selves' indirect utility. Within this setting, we consider two cases, $\beta \geq 1/(1+\eta+\gamma)$, which produces no rule breaking, and $\beta < 1/(1+\eta+\gamma)$, which involves the possibility of rule breaking when $\bar{d}$ is given, and we obtain the following result.

Proposition 2. *The optimal debt ceiling for the period-1 selves, $\bar{d}^*$, is*

$$\bar{d}^* = \begin{cases} 0 & \text{if } \beta \geq \frac{1}{1+\eta+\gamma}, \\ \dfrac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}} & \text{if } \beta < \frac{1}{1+\eta+\gamma}. \end{cases}$$

Proof. See Appendix A.2.

[Figure 3 here.]

Figure 3 illustrates the result in Proposition 2. When the present bias is weak, such that $\beta \geq 1/(1+\eta+\gamma)$ holds, period-2 selves, as voters, have little incentive to issue public debt. This implies that they follow the debt rule, even if the ceiling is set at the lowest level, $\bar{d} = 0$; that is, the issuance of public debt is prohibited. Therefore, the period-1 selves find it optimal to set $\bar{d} = 0$, which is optimal from the perspective of their utility maximization. The resulting allocation is, thus, the first-best solution to the period-1 selves' problem.

When the present bias is strong, such that $\beta < 1/(1+\eta+\gamma)$, the period-1 selves are unable to set $\bar{d} = 0$. Instead, they set the debt ceiling at a level that induces the period-2 selves to issue public debt up to the ceiling. The mechanism behind this result is as follows. As described above, period-1 selves prefer no public debt issue; thus, from the viewpoint of their utility maximization they want to prohibit debt issue in period 2. However, given such a strict constraint, period-2 selves with strongly present-biased preferences are incentivized to issue public debt beyond the ceiling because the marginal benefit of this rule breaking outweighs its cost for the period-2 selves. Given this response by the period-2 selves, the period-1 selves find it impossible to maintain the prohibition of public debt issuance. To avoid the costs of rule breaking, they set the ceiling at a maximum level, such that the period-2 selves never break it.

The result in Proposition 2 indicates that the optimal debt ceiling for the period-1 selves varies at the threshold level, $\beta = 1/(1+\eta+\gamma)$, in a discontinuous manner, as depicted in Figure 3. In other words, a slight difference in the present bias around the threshold produces large differences in the debt rules and resulting levels of public debt between countries. This result could be viewed as providing a possible explanation for the differences in debt rules and resulting levels of public debt among countries sharing similar economic backgrounds but different preferences.

6 Conclusion

This paper presented a theoretical framework to examine the political process determining public debt policy when voters are endowed with present-biased preferences. Specifically, we consider a situation in which debt issue is distortionary and constrained by the debt ceiling, but rule breaking, that is, debt issuance above the ceiling, is available through the incurrence of additional costs. Within this framework, we established that violations of fiscal rules, which are often observed in developed countries, occur when the present bias is strong and the debt ceiling is fairly low. We also studied the endogenous determination of the debt ceiling through voting and showed the debt rule polarization across countries:

the debt ceiling is set at a zero when the bias is weak, whereas it is set at some positive level when the bias is strong.

The result provides several policy implications for the international coordination of fiscal policies, such as that observed within European Union member states. The first result implies that states are more likely to deviate from international coordination, such as the Maastricht criteria, as they become more present biased. The second result implies that international agreements on strict debt rules can be formed and followed only by states endowed with weak present-biased preferences. These implications should be viewed with caution because they are derived using a simple analytical framework. However, they could provide one possible explanation for the success and failure of international agreements on fiscal rules.

A Proofs

A.1 Proof of Proposition 1 and the Equilibrium Allocation

This appendix provides the proof of Proposition 1, focusing on the case of $d > \bar{d}$ in Proposition 1(iii). Because of space limitations, the proofs of the other parts of Proposition 1 are left to Appendix B. Derivation of the consumption allocation is omitted in the following because it is immediate as a result of substituting saving and public debt into the budget constraints.

A.1.1 Equilibrium with $d > \bar{d}$

Suppose that period-1 and -2 selves expect that $d^e = d^c \left(> \bar{d}\right)$ holds. Eq. (8) leads to the savings of the period-2 Selves, when $d^e = d^c \left(> \bar{d}\right)$ as follows:

$$s_{23,d^e=d^c}^{opt} = \begin{cases} 0 & \text{when } s_{12} \leq S^c\left(d^c, \bar{d}\right) \\ s_{23}^c\left(s_{12}, d^c, \bar{d}\right) & \text{when } s_{12} > S^c\left(d^c, \bar{d}\right) \end{cases} \tag{A.1}$$

where $s_{23}^c\left(s_{12}, d^c, \bar{d}\right)$ and $S^c\left(d^c, \bar{d}\right)$ are defined by

$$s_{23}^c\left(s_{12}, d^c, \bar{d}\right) \equiv s_{12} - \frac{k + \gamma\bar{d} - \left[(1+\eta+\gamma) + (\beta)^{1/\sigma}\right] d^c}{1 + (\beta)^{1/\sigma}},$$
$$S^c\left(d^c, \bar{d}\right) \equiv \frac{k + \gamma\bar{d} - \left[(1+\eta+\gamma) + (\beta)^{1/\sigma}\right] d^c}{1 + (\beta)^{1/\sigma}}.$$

Figure A.1 illustrates $V_1\left(s_{12}, d^e = d^c\right)$. When $s_{12} \leq S^c\left(d^c, \bar{d}\right)$ holds, the first-order Condition, with respect to s_{12} in (11), is rewritten as follows:

$$\frac{\partial V_1\left(s_{12}, d^e = d^c\right)}{\partial s_{12}} = \left(s_{12} + d^c\right)^{-\sigma} - \left[k - s_{12} - (1+\eta+\gamma)\, d^c + \gamma\bar{d}\right]^{-\sigma} \leq 0,$$

where an interior solution is given by

$$s_{12} = \frac{k + \gamma \bar{d} - (2 + \eta + \gamma) d^c}{2}.$$

[Figure A.1 here.]

Alternatively, when $s_{12} > S^c\left(d^c, \bar{d}\right)$ holds, the first-order condition, with respect to s_{12} in (11), becomes

$$\frac{\partial V_1 \left(s_{12}, d^e = d^c\right)}{\partial s_{12}} = 0,$$

suggesting that V_1 is independent of s_{12} as long as $s_{12} > S^c\left(d^c, \bar{d}\right)$ holds. Notice that $V_1 \left(s_{12}, d^e = d^c\right)$ is continuous at $s_{12} = S^c\left(d^c, \bar{d}\right)$, as illustrated in Figure A.1.

The interior solution of s_{12} and threshold value $S^c\left(d^c, \bar{d}\right)$ are compared as follows:

$$S^c\left(d^c, \bar{d}\right) \gtreqless \frac{k + \gamma \bar{d} - (2 + \eta + \gamma) d^c}{2} \Leftrightarrow d^c \lesseqgtr \frac{k + \gamma \bar{d}}{\eta + \gamma}.$$

In addition, the following conditions hold:

$$\frac{k + \gamma \bar{d} - (2 + \eta + \gamma) d^c}{2} \gtreqless 0 \Leftrightarrow d^c \lesseqgtr \frac{k + \gamma \bar{d}}{2 + \eta + \gamma},$$
$$S^c\left(d^c, \bar{d}\right) \gtreqless 0 \Leftrightarrow d^c \lesseqgtr \frac{k + \gamma \bar{d}}{(1 + \eta + \gamma) + (\beta)^{1/\sigma}}.$$

Furthermore, the three threshold values of d^c are ranked as

$$\frac{k + \gamma \bar{d}}{2 + \eta + \gamma} < \frac{k + \gamma \bar{d}}{(1 + \eta + \gamma) + (\beta)^{1/\sigma}} < \frac{k + \gamma \bar{d}}{\eta + \gamma}.$$

The analysis thus far suggests that the allocation of (s_{12}, s_{23}) is given by

$$\begin{array}{ll} s_{12} > 0, s_{23} = 0 & \text{if } d^c < \frac{k+\gamma\bar{d}}{2+\eta+\gamma} < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}} < \frac{k+\gamma\bar{d}}{\eta+\gamma}, \\ s_{12} = 0, s_{23} = 0 & \text{if } \frac{k+\gamma\bar{d}}{2+\eta+\gamma} \le d^c < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}} < \frac{k+\gamma\bar{d}}{\eta+\gamma}, \\ s_{12} \in [0, k], s_{23} = s_{23}^c\left(s_{12}, d^c, \bar{d}\right) & \text{if } \frac{k+\gamma\bar{d}}{2+\eta+\gamma} < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}} \le d^c < \frac{k+\gamma\bar{d}}{\eta+\gamma}, \\ s_{12} \in [0, k], s_{23} = s_{23}^c\left(s_{12}, d^c, \bar{d}\right) & \text{if } \frac{k+\gamma\bar{d}}{2+\eta+\gamma} < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}} < \frac{k+\gamma\bar{d}}{\eta+\gamma} \le d^c. \end{array}$$

Based on this classification, the optimal levels of s_{12} and s_{23}, when $d^e = d^c (> \bar{d})$, are given as follows:

$$\begin{array}{llll} \text{(i)} & s^{opt}_{12,d^e=d^c} = \frac{k+\gamma\bar{d}-(2+\eta+\gamma)d^c}{2}, s^{opt}_{23,d^e=d^c} = 0 & \text{when } d^c < \frac{k+\gamma\bar{d}}{2+\eta+\gamma}, \\ \text{(ii)} & s^{opt}_{12,d^e=d^c} = 0, s^{opt}_{23,d^e=d^c} = 0 & \text{when } \frac{k+\gamma\bar{d}}{2+\eta+\gamma} \leq d^c < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}}, \\ \text{(iii)} & s^{opt}_{12,d^e=d^c} \in [0,k], s^{opt}_{23,d^e=d^c} = s_{12} - \frac{k+\gamma\bar{d}-\left[(1+\eta+\gamma)+(\beta)^{1/\sigma}\right]\bar{d}}{1+(\beta)^{1/\sigma}} & \text{when } \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}} \leq d^c. \end{array} \tag{A.2}$$

In what follows, we determine the conditions, such that the expectation of $d^e = d^c (> \bar{d})$ is rational, for the three cases in (A.2).

Case of $d^c < \left(k + \gamma\bar{d}\right) / (2 + \eta + \gamma)$

From (4) and (A.2), the expectation of $d^e = d^c$ is rational if the following conditions hold:

$$d^c = \frac{k + \gamma\bar{d} - \left[1 + (\beta(1+\eta+\gamma))^{1/\sigma}\right] \frac{k+\gamma\bar{d}-(2+\eta+\gamma)d^c}{2}}{(1+\eta+\gamma) + (\beta(1+\eta+\gamma))^{1/\sigma}} \text{ and } d^c < \frac{k+\gamma\bar{d}}{2+\eta+\gamma},$$

or

$$\left[1 - (\beta(1+\eta+\gamma))^{1/\sigma}\right] \cdot \left[\left(k+\gamma\bar{d}\right) - (\eta+\gamma)d^c\right] = 0 \text{ and } d^c < \frac{k+\gamma\bar{d}}{2+\eta+\gamma}. \tag{A.3}$$

The first condition in (A.3) indicates that the rational expectation of public debt is given by

$$d^c \begin{cases} \in \left(\bar{d}, \frac{k}{\eta}\right) & \text{if } \beta = \frac{1}{1+\eta+\gamma}, \\ = \frac{k+\gamma\bar{d}}{\eta+\gamma} & \text{if } \beta \neq \frac{1}{1+\eta+\gamma}. \end{cases}$$

When $\beta \neq 1/(1+\eta+\gamma)$, $d^c = \left(k+\gamma\bar{d}\right)/(\eta+\gamma)$ must satisfy the second condition in (A.3):

$$d^c = \frac{k+\gamma\bar{d}}{\eta+\gamma} < \frac{k+\gamma\bar{d}}{2+\eta+\gamma},$$

but this inequality condition fails to hold. Alternatively, when $\beta = 1/(1+\eta+\gamma)$, $d^c \in \left(\bar{d}, k/\eta\right)$ with the second condition in (A.3) gives the equilibrium level for rational expectation of public debt as

$$d \in \left(\bar{d}, \frac{k+\gamma\bar{d}}{2+\eta+\gamma}\right),$$

where the set is nonempty if $\bar{d} < k/(2+\eta)$.

Proposition A.1. *Suppose that the following conditions hold:*

$$\beta = \frac{1}{1+\eta+\gamma} \text{ and } \bar{d} < \frac{k}{2+\eta}.$$

There is a rational expectations equilibrium with $d \in \left(\bar{d}, \left(k+\gamma\bar{d}\right)/\left(2+\eta+\gamma\right)\right)$ *and*

$$(c_2, c_3, s_{12}, s_{13}, s_{23}) = \left(\frac{k+\gamma\bar{d}-(\eta+\gamma)\,d}{2}, \frac{k+\gamma\bar{d}-\eta d}{2}, \right.$$
$$\left. \frac{k+\gamma\bar{d}-(2+\eta+\gamma)d}{2}, \frac{k-\gamma\bar{d}+(2+\eta+\gamma)d}{2}, 0\right).$$

Case of $\left(k+\gamma\bar{d}\right)/\left(2+\eta+\gamma\right) \leq d^c < \left(k+\gamma\bar{d}\right)/\left(1+\eta+\gamma+(\beta)^{1/\sigma}\right)$

From (4) and (A.2), the expectation of $d^e = d^c$ is rational if the following conditions hold:

$$d^c = \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta\,(1+\eta+\gamma))^{1/\sigma}} \text{ and } \frac{k+\gamma\bar{d}}{2+\eta+\gamma}/ \leq d^c < \frac{k+\gamma\bar{d}}{1+\eta+\gamma+(\beta)^{1/\sigma}}. \quad \text{(A.4)}$$

This level of public debt is above the limit, $\bar{d}$, if

$$\bar{d} < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta\,(1+\eta+\gamma))^{1/\sigma}} \Leftrightarrow \bar{d} < \frac{k}{(1+\eta)+(\beta\,(1+\eta+\gamma))^{1/\sigma}}.$$

In addition, d^c must satisfy the second condition in (A.4):

$$\frac{k+\gamma\bar{d}}{2+\eta+\gamma} \leq d^c = \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta\,(1+\eta+\gamma))^{1/\sigma}} < \frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta)^{1/\sigma}}.$$

The first inequality holds if and only if $\beta \leq 1/(1+\eta+\gamma)$, and the second inequality always holds.

Proposition A.2. *Suppose that the following conditions hold:*

$$\beta \leq \frac{1}{1+\eta+\gamma} \text{ and } \bar{d} < \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}.$$

There is a rational expectations equilibrium with

$$d = \frac{\left(k+\gamma\bar{d}\right)}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}} \in \left(\bar{d}, \frac{k}{\eta}\right),$$

and

$$\begin{aligned}(c_2, c_3, s_{12}, s_{13}, s_{23}) = \Bigg(& \frac{\left(k+\gamma\bar{d}\right)}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}, \\ & \frac{(\beta(1+\eta+\gamma))^{1/\sigma}\left(k+\gamma\bar{d}\right)}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}, 0, k, 0\Bigg).\end{aligned}$$

Case of $\left(k+\gamma\bar{d}\right)/\left(1+\eta+\gamma+(\beta)^{1/\sigma}\right) \leq d^c$

From (4) and (A.2), the expectation of $d^e = d^c$ is rational if the following condition holds:

$$d^c = \frac{\left(k+\gamma\bar{d}\right) - \left[1+(\beta(1+\eta+\gamma))^{1/\sigma}\right]\frac{k+\gamma\bar{d}-\left[(1+\eta+\gamma)+(\beta)^{1/\sigma}\right]d^c}{1+(\beta)^{1/\sigma}}}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}$$

and $\left(k+\gamma\bar{d}\right)/\left(1+\eta+\gamma+(\beta)^{1/\sigma}\right) \leq d^c$, or

$$\frac{k+\gamma\bar{d}}{1+\eta+\gamma+(\beta)^{1/\sigma}} \leq d^c = \frac{k+\gamma\bar{d}}{\eta+\gamma}.$$

The associated level of s_{23} is

$$s^{opt}_{23,d^e=d^c} = s_{12} + \frac{k+\gamma\bar{d}}{\eta+\gamma},$$

and the corresponding consumption levels are $c_2 = c_3 = 0$, which contradicts the first-order conditions with respect to c_2 and c_3. Thus, there is no rational expectations equilibrium in this case.

$\blacksquare$

A.2 Proof of Proposition 2

When $\beta \geq 1/(1+\eta+\gamma)$, the allocation of consumption for a given $\bar{d}$ is

$$(c_2, c_3) = \begin{cases} \left(\frac{k+\gamma\bar{d}-(\eta+\gamma)d}{2}, \frac{k+\gamma\bar{d}-\eta d}{2}\right) \text{ with } d \in \left(\bar{d}, \frac{k+\gamma\bar{d}}{2+\eta+\gamma}\right) & \text{when } \beta = \frac{1}{1+\eta+\gamma} \text{ and } \bar{d} < \frac{k}{2+\eta}, \\ \left(\frac{k-\eta\bar{d}}{2}, \frac{k-\eta\bar{d}}{2}\right) & \text{when } \frac{1}{1+\eta+\gamma} \leq \beta \leq \frac{1}{1+\eta} \text{ and } \bar{d} < \frac{k}{2+\eta}, \\ \left(\frac{k-\eta d}{2}, \frac{k-\eta d}{2}\right) \text{ with } d \in \left(0, \min\left\{\frac{k}{2+\eta}, \bar{d}\right\}\right) & \text{when } \beta = \frac{1}{1+\eta}, \\ \left(\frac{k}{2}, \frac{k}{2}\right) & \text{when } \beta \geq \frac{1}{1+\eta}, \end{cases}$$

where the first, second, third, and fourth allocations come from Propositions A.1, B.4, B.2, and B.1, respectively. For each allocation, c_2 and c_3 are set at $c_2 = c_3 = k/2$ by choosing $\bar{d} = 0$. This allocation of consumption is consistent with the solution to the following period-1 selves' utility maximization problem:

$$\max \frac{(c_2)^{1-\sigma}}{1-\sigma} + \frac{(c_3)^{1-\sigma}}{1-\sigma} \text{ s.t. } c_2 + c_3 \leq k.$$

Thus, the optimal level of $\bar{d}$, when $\beta \geq 1/(1+\eta+\gamma)$, is $\bar{d} = 0$.

When $\beta < 1/(1+\eta+\gamma)$, the equilibrium allocation of consumption for a given $\bar{d}$ is

$$(c_2, c_3) = \begin{cases} \left(\frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}, \frac{(\beta(1+\eta+\gamma))^{1/\sigma}\left(k+\gamma\bar{d}\right)}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}\right) & \text{when } 0 \leq \bar{d} < \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}, \\ \left(\bar{d}, k-(1+\eta)\bar{d}\right) & \text{when } \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}} \leq \bar{d} < \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}, \\ \left(\frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}, \frac{(\beta(1+\eta))^{1/\sigma}k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}\right) & \text{when } \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} \leq \bar{d}, \end{cases}$$

where the first, second, and third allocations come from Propositions A.2, B.5, and B.3, respectively. Thus, the period-1 selves' indirect utility, V_1, becomes:

$$V_1 = \begin{cases} V_{1A} \equiv \frac{1}{1-\sigma}\left\{\left[\frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}\right]^{1-\sigma}\left[1+(\beta(1+\eta+\gamma))^{(1-\sigma)/\sigma}\right]-2\right\} \\ \qquad \text{when } 0 \leq \bar{d} < \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}, \\ V_{1B} \equiv \frac{1}{1-\sigma}\left\{(d)^{1-\sigma}+[k-(1+\eta)d]^{1-\sigma}-2\right\} \\ \qquad \text{when } \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}} \leq \bar{d} < \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}, \\ V_{1C} \equiv \left\{\left[\frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}\right]^{1-\sigma}\left[1+(\beta(1+\eta))^{(1-\sigma)/\sigma}\right]-2\right\} \\ \qquad \text{when } \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} \leq \bar{d}. \end{cases}$$

The function V_1 is continuous for $\bar{d} \in (0, \infty)$ because the following properties hold:

$$\lim_{\bar{d} \to \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}} V_{1A} = V_{1B}\left(\frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}\right)$$

and

$$\lim_{\bar{d} \to \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}} V_{1B} = V_{1C}\left(\frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}\right).$$

In addition, the differentiation of $V_{1i} (i = A, B, C)$ with respect to $\bar{d}$ leads to

$$\begin{aligned} \frac{\partial V_{1A}}{\partial \bar{d}} &= \left[\frac{k+\gamma\bar{d}}{(1+\eta+\gamma)+(\beta(1+\eta+\gamma))^{1/\sigma}}\right]^{-\sigma}\gamma\left[1+(\beta(1+\eta+\gamma))^{(1-\sigma)/\sigma}\right] > 0, \\ \frac{\partial V_{1B}}{\partial \bar{d}} &= (d)^{-\sigma}-(1+\eta)\left[k-(1+\eta)d\right]^{-\sigma}, \\ \frac{\partial V_{1C}}{\partial \bar{d}} &= 0, \end{aligned}$$

where the following condition holds:

$$\frac{\partial V_{1B}}{\partial \bar{d}} \gtreqless 0 \Leftrightarrow \bar{d} \lesseqgtr \frac{k}{(1+\eta)+(1+\eta)^{1/\sigma}}.$$

Given the assumption of $\beta < 1/(1+\eta+\gamma)$, we have

$$\frac{k}{(1+\eta)+(1+\eta)^{1/\sigma}} < \frac{k}{(1+\eta)+(\beta\,(1+\eta+\gamma))^{1/\sigma}},$$

implying that V_{1B} is decreasing in $\bar{d}$ for the range of $\left[k/[(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}],\right.$ $\left.k/[(1+\eta)+(\beta(1+\eta))^{1/\sigma}]\right)$. Thus, the optimal $\bar{d}$ becomes $\bar{d} = k/[(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}]$, and the corresponding allocation of saving, consumption, and public debt is given by

$$(s_{12}, s_{13,} c_2, c_3) = \left(0, 0, \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}, \frac{(\beta(1+\eta+\gamma))^{1/\sigma}\,k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}\right)$$

and

$$\bar{d} = \frac{k}{(1+\eta)+(\beta(1+\eta+\gamma))^{1/\sigma}}.$$

■

B Supplementary Materials (Not for Publication)

This appendix provides the proof of Proposition 1 for cases of $d \leq \bar{d}$.

B.1 Equilibrium with $d < \bar{d}$

In this subsection, we first show the equilibrium with $d = 0$, and then show the equilibrium with $d \in \left(0, \bar{d}\right)$.

B.1.1 Equilibrium with $d = 0$

Suppose that period-1 and -2 selves expect that $d^e = 0$ holds. Eq. (8) leads to saving by period-2 selves when $d^e = 0$ as follows:

$$s^{opt}_{23,d^e=0} = \begin{cases} 0 & \text{when } s_{12} \leq S^u(0) \equiv \frac{k}{1+(\beta)^{1/\sigma}}, \\ s^u_{23}\left(s_{12}, 0\right) \equiv s_{12} - \frac{k}{1+(\beta)^{1/\sigma}} & \text{when } s_{12} > S^u(0) \equiv \frac{k}{1+(\beta)^{1/\sigma}}. \end{cases}$$

Panel (a) of Figure B.1 illustrates $V_1\left(s_{12}, d^e = 0\right)$. When $s_{12} \leq S^u(0)$ holds, the first-order condition, with respect to s_{12} in (11), is rewritten as follows:

$$\frac{\partial V_1\left(s_{12}, d^e = 0\right)}{\partial s_{12}} = \left(s_{12}\right)^{-\sigma} - \left(k - s_{12}\right)^{-\sigma} \leq 0. \tag{B.1}$$

An interior solution, given by $s_{12} = k/2$, is feasible because it holds that $s_{12} = k/2 < S^u(0) \equiv k/\left[1 + (\beta)^{1/\sigma}\right]$. Thus, an optimal level of s_{12} is $s_{12} = k/2$ when $s_{12} \leq S^u(0)$, as illustrated in Panel (a) of Figure B.1.

[Figure B.1 here.]

Alternatively, when $s_{12} > S^u(0)$ holds, the first-order condition, with respect to s_{12} in (11), becomes

$$\frac{\partial V_1\left(s_{12}, d^e = 0\right)}{\partial s_{12}} = 0,$$

suggesting that V_1 is independent of s_{12} as long as $s_{12} > S^u(0)$ (see Panel (a) of Figure B.1). Notice that V_1 is continuous at $s_{12} = S^u(0)$.

Given the expectation of $d^e = 0$, the optimal level of s_{12} becomes

$$s_{12,d^e=0}^{opt} = \frac{k}{2},$$

and the corresponding level of s_{23} is $s_{23,d^e=0}^{opt} = 0$. From (5), the expectation of $d^e = 0$ is rational if the following condition holds:

$$d^u \left(s_{12,d^e=0}^{opt} = \frac{k}{2}, s_{23,d^e=0}^{opt} = 0 \right) \leq 0 \Leftrightarrow \beta \geq \frac{1}{1+\eta}.$$

Proposition B.1. *Suppose that $\beta \geq 1/(1+\eta)$ holds. There is a rational expectations equilibrium with $d = 0$ and*

$$(c_2, c_3, s_{12}, s_{13}, s_{23}) = \left(\frac{k}{2}, \frac{k}{2}, \frac{k}{2}, \frac{k}{2}, 0 \right).$$

B.1.2 Equilibrium with $d \in \left(0, \bar{d}\right)$

Suppose that period-1 and -2 selves expect that $d^e = d^u \in \left(0, \bar{d}\right)$ holds. Eq. (8) leads to saving by period-2 selves when $d^e = d^u \in \left(0, \bar{d}\right)$ as follows:

$$s_{23,d^e=d^u}^{opt} = \begin{cases} 0 & \text{when } s_{12} \leq S^u(d^u), \\ s_{23}^u(s_{12}, d^u) & \text{when } s_{12} > S^u(d^u), \end{cases} \tag{B.2}$$

where $s_{23}^u(s_{12}, d^u)$ and $S^u(d^u)$ are defined as

$$s_{23}^u(s_{12}, d^u) \equiv s_{12} - \frac{k - \left[(1+\eta) + (\beta)^{1/\sigma}\right] d^u}{1 + (\beta)^{1/\sigma}},$$

$$S^u(d^u) \equiv \frac{k - \left[(1+\eta) + (\beta)^{1/\sigma}\right] d^u}{1 + (\beta)^{1/\sigma}}.$$

Panel (b) of Figure B.1 illustrates $V_1(s_{12}, d^e = d^u)$. When $s_{12} \leq S^u(d^u)$ holds, the first-order condition, with respect to s_{12} in (11), is rewritten as follows:

$$\frac{\partial V_1\left(s_{12}, d^e = d^u\right)}{\partial s_{12}} = \left(s_{12} + d^u\right)^{-\sigma} - \left[k - s_{12} - (1+\eta)\, d^u\right]^{-\sigma} \leq 0,$$

where an interior solution, given by $s_{12} = \left[k - (2+\eta)\, d^u\right]/2$, is feasible if it satisfies the following:

$$\frac{k - (2+\eta)\, d^u}{2} < S^u(d^u) \Leftrightarrow d^u < \frac{k}{\eta}.$$

This condition is satisfied under Assumption 1, $\bar{d} < k/\eta$, and the definition of $d^u (< \bar{d})$.

Alternatively, when $s_{12} > S^u(d^u)$ holds, the first-order condition, with respect to s_{12} in (11), becomes:

$$\frac{\partial V_1\left(s_{12}, d^e = d^u\right)}{\partial s_{12}} = 0,$$

suggesting that V_1 is independent of s_{12} as long as $s_{12} > S^u(d^u)$ holds. Notice that V_1 is continuous at $s_{12} = S^u(d^u)$, and that

$$\frac{k - (2+\eta)\, d^u}{2} > 0 \Leftrightarrow d^u < \frac{k}{2+\eta},$$
$$S^u(d^u) > 0 \Leftrightarrow d^u < \frac{k}{(1+\eta) + (\beta)^{1/\sigma}}.$$

Given these properties, we can conclude that the optimal levels of s_{12} and s_{23}, when $d^e = d^u$, become

$$\begin{array}{llr} \text{(i)} & s^{opt}_{12,d^e=d^u} = \frac{k-(2+\eta)d^u}{2},\, s^{opt}_{23,d^e=d^u} = 0 & \text{when } d^u < \frac{k}{2+\eta}, \\ \text{(ii)} & s^{opt}_{12,d^e=d^u} = 0,\, s^{opt}_{23,d^e=d^u} = 0 & \text{when } \frac{k}{2+\eta} \leq d^u < \frac{k}{(1+\eta)+(\beta)^{1/\sigma}}, \\ \text{(iii)} & s^{opt}_{12,d^e=d^u} \in [0,k]\,,\, s^{opt}_{23,d^e=d^u} = s_{12} - \frac{k-\left[(1+\eta)+(\beta)^{1/\sigma}\right]d^u}{1+(\beta)^{1/\sigma}} & \text{when } \frac{k}{(1+\eta)+(\beta)^{1/\sigma}} \leq d^u. \end{array} \quad \text{(B.3)}$$

In what follows, we determine the conditions such that the expectation of $d^e = d^u$ is rational for the three cases in (B.3).

Case of $d^u < k/(2+\eta)$

From (3) and (B.3), the expectation of $d^e = d^u$ is rational if the following conditions hold:

$$d^u = \frac{k - \left[1 + (\beta(1+\eta))^{1/\sigma}\right] \cdot \left(s^{opt}_{12,d^e=d^u} - s^{opt}_{23,d^e=d^u}\right)}{(1+\eta) + (\beta(1+\eta))^{1/\sigma}} \text{ and } d^u < \frac{k}{2+\eta},$$

or

$$\left[1 - (\beta(1+\eta))^{1/\sigma}\right] \cdot \eta d^u = \left[1 - (\beta(1+\eta))^{1/\sigma}\right] \cdot k \text{ and } d^u < \frac{k}{2+\eta}. \tag{B.4}$$

The first condition in (B.4) implies that the rational expectations level of d^u is given by

$$d^u \begin{cases} \in \left(0, \bar{d}\right) & \text{when } \beta(1+\eta) = 1, \\ = k/\eta & \text{when } \beta(1+\eta) \neq 1. \end{cases}$$

When $\beta(1+\eta) \neq 1$, the candidate for a solution is $d^u = k/\eta$. This candidate is not suitable for the solution because a focus on the case of $d < \bar{d}$ and $\bar{d} < k/\eta$ is assumed in Assumption 1. When $\beta(1+\eta) = 1$, any level of $d^u \in \left(0, \bar{d}\right)$ with $d^u < k/(2+\eta)$ is rational. Thus, the equilibrium level of public debt becomes

$$d \in \left(0, \min\left\{\frac{k}{2+\eta}, \bar{d}\right\}\right).$$

Proposition B.2. *Suppose that $\beta(1+\eta) = 1$ holds. There is a rational expectations equilibrium with $d \in \left(0, \min\left\{k/(2+\eta), \bar{d}\right\}\right)$ and*

$$(c_2, c_3, s_{12}, s_{13}, s_{23}) = \left(\frac{k-\eta d}{2}, \frac{k-\eta d}{2}, \frac{k-(2+\eta)d}{2}, \frac{k+(2+\eta)d}{2}, 0\right).$$

Case of $k/(2+\eta) \leq d^u < k/\left[(1+\eta)+(\beta)^{1/\sigma}\right]$

From (3) and (B.3), the expectation of $d^e = d^u$ is rational if the following conditions hold:

$$d^u = \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} \text{ and } \frac{k}{2+\eta} \leq d^u < \frac{k}{(1+\eta)+(\beta)^{1/\sigma}},$$

or

$$\frac{k}{2+\eta} \leq \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} < \frac{k}{(1+\eta)+(\beta)^{1/\sigma}}.$$

The first inequality holds if and only if $\beta(1+\eta) < 1$; the second inequality always holds. In addition, d^u must satisfy $d^u < \bar{d}$, that is,

$$d^u = \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} < \bar{d}.$$

Proposition B.3. *Suppose that the following conditions hold:*

$$\beta < \frac{1}{1+\eta} \text{ and } \frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} < \bar{d}.$$

There is a rational expectations equilibrium with $d = k/\left[(1+\eta)+(\beta(1+\eta))^{1/\sigma}\right]$ *and*

$$(c_2, c_3, s_{12}, s_{13}, s_{23}) = \left(\frac{k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}, \frac{(\beta(1+\eta))^{1/\sigma}k}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}}, 0, k, 0\right).$$

Case of $k/\left[(1+\eta)+(\beta)^{1/\sigma}\right] \leq d^u$

From (3) and (B.3), the expectation of $d^e = d^u$ is rational if the following conditions hold:

$$d^u = \frac{k - \left[1+(\beta(1+\eta))^{1/\sigma}\right]\cdot\left(s^{opt}_{12,d^e=d^u} - s^{opt}_{23,d^e=d^u}\right)}{(1+\eta)+(\beta(1+\eta))^{1/\sigma}} \text{ and } \frac{k}{(1+\eta)+(\beta)^{1/\sigma}} \leq d^u,$$

or

$$d^u = \frac{k - \left[1 + (\beta (1+\eta))^{1/\sigma}\right] \cdot \frac{k - \left[(1+\eta)+(\beta)^{1/\sigma}\right]\cdot d^u}{1+(\beta)^{1/\sigma}}}{(1+\eta) + (\beta(1+\eta))^{1/\sigma}} \text{ and } \frac{k}{(1+\eta)+(\beta)^{1/\sigma}} \leq d^u. \tag{B.5}$$

Solving the first condition in (B.5) for d^u leads to $d^u = k/\eta$. Following the same reasoning as the previous case, this candidate is not suitable for the solution. Thus, there is no rational expectations equilibrium with $k/\left[(1+\eta)+(\beta)^{1/\sigma}\right] \leq d$.

B.2 Equilibrium with $d = \bar{d}$

Suppose that period-1 and -2 selves expect that $d^e = \bar{d}$ holds. Equation (8) leads to the period-2 selves' saving when $d^e = \bar{d}$ as follows:

$$s^{opt}_{23,d^e=\bar{d}} = \begin{cases} 0 & \text{when } s_{12} \leq S^u\left(\bar{d}\right), \\ s_{12} - \frac{k - \left[(1+\eta)+(\beta)^{1/\sigma}\right]\cdot \bar{d}}{1+(\beta)^{1/\sigma}} & \text{when } s_{12} > S^u\left(\bar{d}\right), \end{cases}$$

where $S^u\left(\bar{d}\right)$ is defined as

$$S^u\left(\bar{d}\right) \equiv \frac{k - \left[(1+\eta)+(\beta)^{1/\sigma}\right]\cdot \bar{d}}{1+(\beta)^{1/\sigma}}.$$

Panel (c) of Figure B.1 illustrates $V_1\left(s_{12}, d^e = \bar{d}\right)$. When $s_{12} \leq S^u\left(\bar{d}\right)$ holds, the first-order Condition, with respect to s_{12} in (11), is rewritten as follows:

$$\frac{\partial V_1\left(s_{12}, d^e = \bar{d}\right)}{\partial s_{12}} = \left(s_{12} + \bar{d}\right)^{-\sigma} - \left[k - s_{12} - (1+\eta)\bar{d}\right]^{-\sigma} \leq 0,$$

where an interior solution, given by $s_{12} = \left[k - (2+\eta)\,\bar{d}\right]/2$, is feasible because it satisfies $S^u\left(\bar{d}\right) > \left[k - (2+\eta)\,\bar{d}\right]/2 \Leftrightarrow k/\eta > \bar{d}$. Thus, the optimal level of s_{12} is given by $s_{12} = \left[k - (2+\eta)\,\bar{d}\right]/2$ when $s_{12} \leq S^u\left(\bar{d}\right)$.

Alternatively, when $s_{12} > S^u\left(\bar{d}\right)$ holds, the first-order condition, with respect to s_{12}

in (11), becomes

$$\frac{\partial V_1\left(s_{12}, d^e = \bar{d}\right)}{\partial s_{12}} = 0,$$

suggesting that V_1 is independent of s_{12} as long as $s_{12} > S^u\left(\bar{d}\right)$. Notice that V_1 is continuous at $s_{12} = S^u\left(\bar{d}\right)$, and that

$$\frac{k - (2+\eta)\,\bar{d}}{2} > 0 \Leftrightarrow \bar{d} < \frac{k}{2+\eta},$$
$$S^u\left(\bar{d}\right) > 0 \Leftrightarrow \bar{d} < \frac{k}{(1+\eta) + (\beta)^{1/\sigma}}.$$

Given these properties, we can conclude that the optimal levels of s_{12} and s_{23}, when $d^e = \bar{d}$, become:

$$\begin{array}{lll} \text{(i)} & s^{opt}_{12,d^e=\bar{d}} = \frac{k-(2+\eta)\bar{d}}{2},\, s^{opt}_{23,d^e=\bar{d}} = 0 & \text{when } \bar{d} < \frac{k}{2+\eta}, \\ \text{(ii)} & s^{opt}_{12,d^e=\bar{d}} = 0,\, s^{opt}_{23,d^e=\bar{d}} = 0 & \text{when } \frac{k}{2+\eta} \leq \bar{d} < \frac{k}{(1+\eta)+(\beta)^{1/\sigma}}, \\ \text{(iii)} & s^{opt}_{12,d^e=\bar{d}} \in [0,k]\,,\, s^{opt}_{23,d^e=\bar{d}} = s_{12} - \frac{k-\left[(1+\eta)+(\beta)^{1/\sigma}\right]\bar{d}}{1+(\beta)^{1/\sigma}} & \text{when } \frac{k}{(1+\eta)+(\beta)^{1/\sigma}} \leq \bar{d}. \end{array} \tag{B.6}$$

In what follows, we determine the conditions, such that the expectation of $d^e = \bar{d}$ is rational, for the three cases in (B.6).

Case of $\bar{d} < k/(2+\eta)$

From (5) and (B.6), the expectation of $d^e = \bar{d}$ is rational if the following conditions hold:

$$\begin{aligned} A&\left(s^{opt}_{12,d^e=\bar{d}} = \frac{k-(2+\eta)\,\bar{d}}{2}, s^{opt}_{23,d^e=\bar{d}} = 0\right) \leq \bar{d} \\ &\leq d^u\left(s^{opt}_{12,d^e=\bar{d}} = \frac{k-(2+\eta)\,\bar{d}}{2}, s^{opt}_{23,d^e=\bar{d}} = 0\right) \text{ and } \bar{d} < \frac{k}{2+\eta}. \end{aligned} \tag{B.7}$$

The first condition in (B.7) is reformulated as follows:

$$\frac{k - \left[1 + \{\beta\,(1+\eta+\gamma)\}^{1/\sigma}\right]\frac{k-(2+\eta)\bar{d}}{2}}{(1+\eta) + \{\beta\,(1+\eta+\gamma)\}^{1/\sigma}} \leq \bar{d} \Leftrightarrow \frac{1}{1+\eta+\gamma} \leq \beta,$$

and

$$\bar{d} \leq \frac{k - \left[1 + \{\beta(1+\eta)\}^{1/\sigma}\right] \frac{k-(2+\eta)\bar{d}}{2}}{(1+\eta) + \{\beta(1+\eta)\}^{1/\sigma}} \Leftrightarrow \beta \leq \frac{1}{1+\eta}.$$

The equilibrium conditions are summarized in the following proposition.

Proposition B.4. *Suppose that the following conditions hold:*

$$\frac{1}{1+\eta+\gamma} \leq \beta \leq \frac{1}{1+\eta} \text{ and } \bar{d} < \frac{k}{2+\eta}.$$

There is a rational expectations equilibrium with $d = \bar{d}$ and

$$(c_2, c_3, s_{12}, s_{13}, s_{23}) = \left(\frac{k - \eta\bar{d}}{2}, \frac{k - \eta\bar{d}}{2}, \frac{k-(2+\eta)\bar{d}}{2}, \frac{k+(2+\eta)\bar{d}}{2}, 0\right).$$

B.2.1 Case of $k/(2+\eta) \leq \bar{d} < k/\left[(1+\eta) + (\beta)^{1/\sigma}\right]$

From (5) and (B.6), the expectation of $d^e = \bar{d}$ is rational if the following condition holds:

$$A\left(s^{opt}_{12,d^e=\bar{d}} = 0, s^{opt}_{23,d^e=\bar{d}} = 0\right) \leq \bar{d} \leq d^u\left(s^{opt}_{12,d^e=\bar{d}} = 0, s^{opt}_{23,d^e=\bar{d}} = 0\right) \text{ and}$$
$$\frac{k}{2+\eta} \leq \bar{d} < \frac{k}{(1+\eta) + (\beta)^{1/\sigma}},$$

that is, if

$$\frac{k}{(1+\eta) + \{\beta(1+\eta+\gamma)\}^{1/\sigma}} \leq \bar{d} \leq \frac{k}{(1+\eta) + \{\beta(1+\eta)\}^{1/\sigma}} \text{ and}$$
$$\frac{k}{2+\eta} \leq \bar{d} < \frac{k}{(1+\eta) + (\beta)^{1/\sigma}}.$$

These are summarized as in the following propositions.

Proposition B.5. *Suppose that the following condition holds:*

$$\max\left\{\frac{k}{2+\eta}, \frac{k}{(1+\eta) + \{\beta(1+\eta+\gamma)\}^{1/\sigma}}\right\} \leq \bar{d} \leq \frac{k}{(1+\eta) + \{\beta(1+\eta)\}^{1/\sigma}}.$$

There is a rational expectations equilibrium with $d = \bar{d}$ and

$$\left(c_2, c_3, s_{12}, s_{13}, s_{23}\right) = \left(\bar{d}, k - (1+\eta)\,\bar{d}, 0, k, 0\right).$$

Case of $k/\left[(1+\eta) + (\beta)^{1/\sigma}\right] \leq \bar{d}$

From (5) and (B.6), the expectation of $d^e = \bar{d}$ is rational if the following condition holds:

$$\begin{aligned} A&\left(s^{opt}_{12,d^e=\bar{d}} \in [0,k]\,, s^{opt}_{23,d^e=\bar{d}} = s_{12} - \frac{k - \left[(1+\eta) + (\beta)^{1/\sigma}\right]\bar{d}}{1+(\beta)^{1/\sigma}}\right) \leq \bar{d} \\ &\leq d^u\left(s^{opt}_{12,d^e=\bar{d}} \in [0,k]\,, s^{opt}_{23,d^e=\bar{d}} = s_{12} - \frac{k - \left[(1+\eta) + (\beta)^{1/\sigma}\right]\bar{d}}{1+(\beta)^{1/\sigma}}\right) \quad \text{and} \\ &\frac{k}{(1+\eta) + (\beta)^{1/\sigma}} \leq \bar{d}. \end{aligned}$$

The inequality $\bar{d} \leq d^u(\cdot,\cdot)$ is rewritten as

$$\bar{d} \leq \frac{k - \left[1 + \left\{\beta\,(1+\eta)\right\}^{1/\sigma}\right]\frac{k - \left[(1+\eta)+(\beta)^{1/\sigma}\right]\bar{d}}{1+(\beta)^{1/\sigma}}}{(1+\eta) + \left\{\beta\,(1+\eta)\right\}^{1/\sigma}} \Leftrightarrow \eta \leq 0,$$

which fails to hold for any $\eta > 0$. Thus, there is no rational expectations equilibrium with $d = \bar{d}$ when $k/\left[(1+\eta) + (\beta)^{1/\sigma}\right] \leq \bar{d}$.

■

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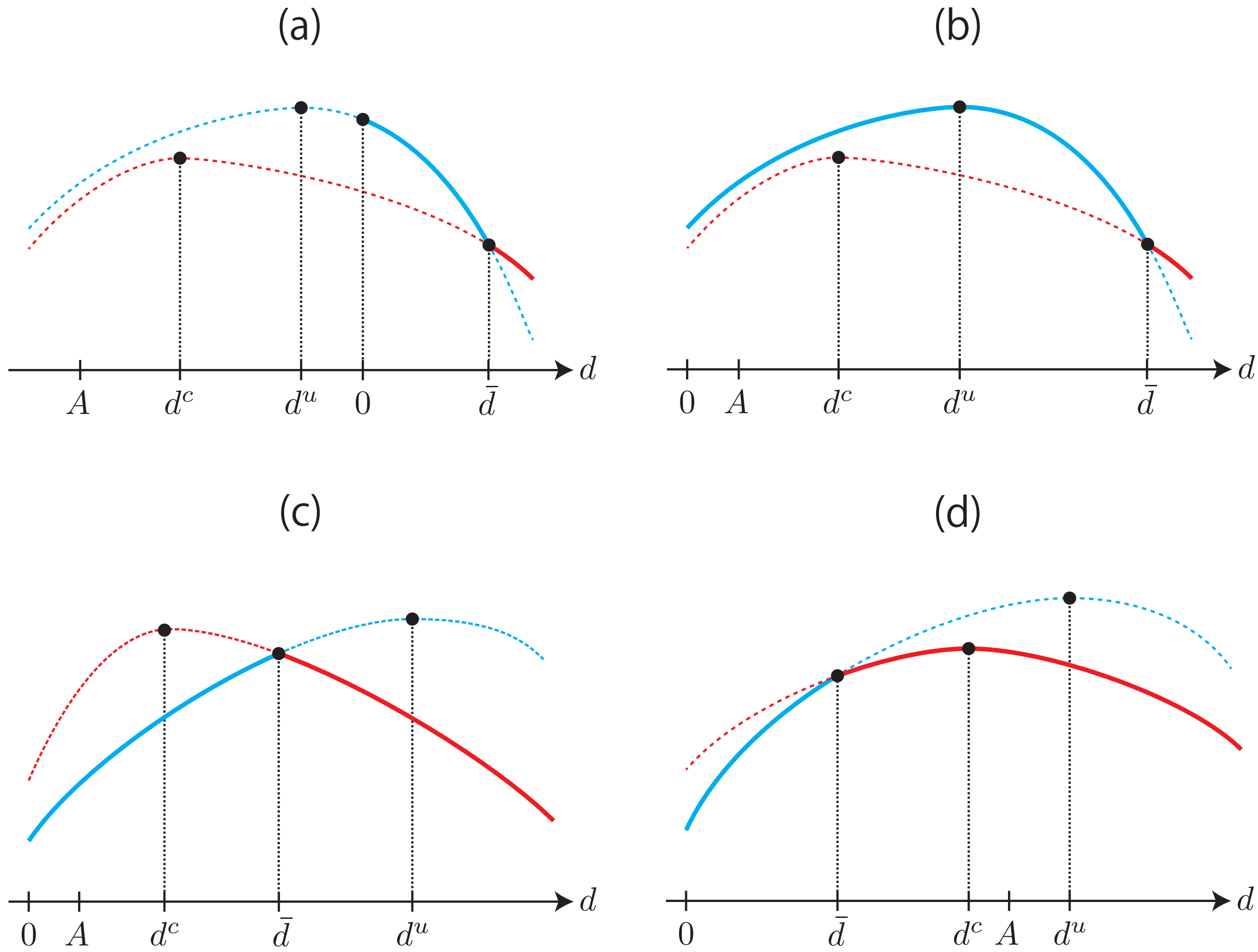

Figure 1: Illustration of the period-2 government's objective function when $d^u(s_{12}, s_{23}) \leq 0 \leq \bar{d}$ (Panel (a)), $0 < d^u(s_{12}, s_{23}) < \bar{d}$ (Panel (b)), $A(s_{12}, s_{23}) \leq \bar{d} \leq d^u(s_{12}, s_{23})$ (Panel (c)), and $\bar{d} < A(s_{12}, s_{23})$ (Panel (d)).

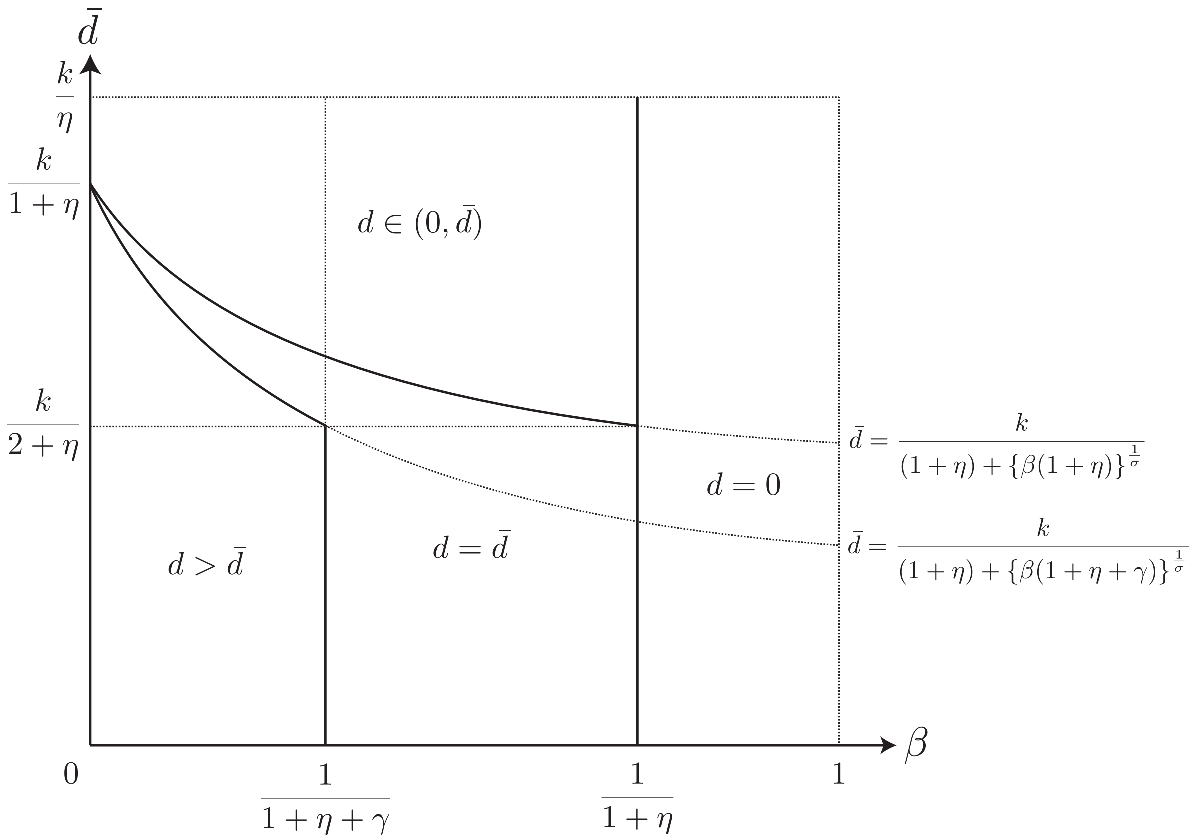

Figure 2: Classification of the equilibrium states according to the level of public debt. The horizontal axis takes β; the vertical axis takes $\bar{d}$.

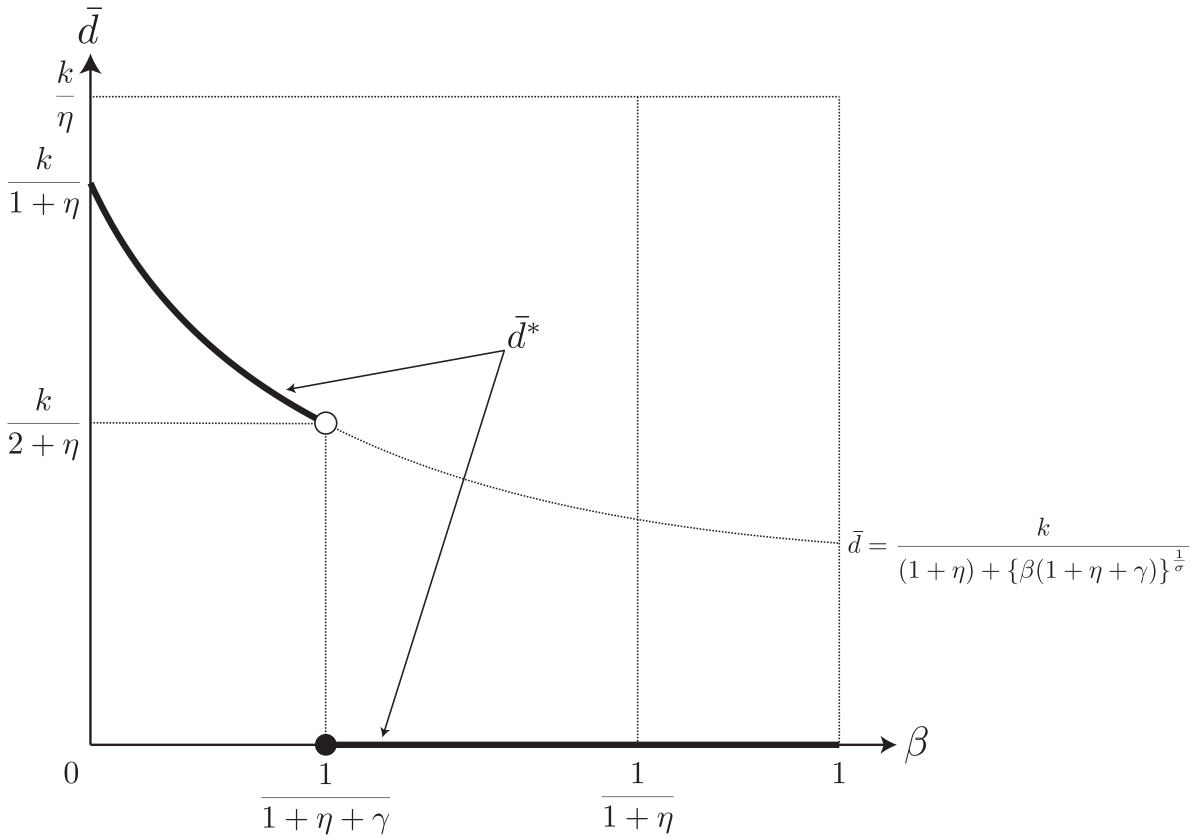

Figure 3: The optimal debt ceiling for the period-1 selves according to β.

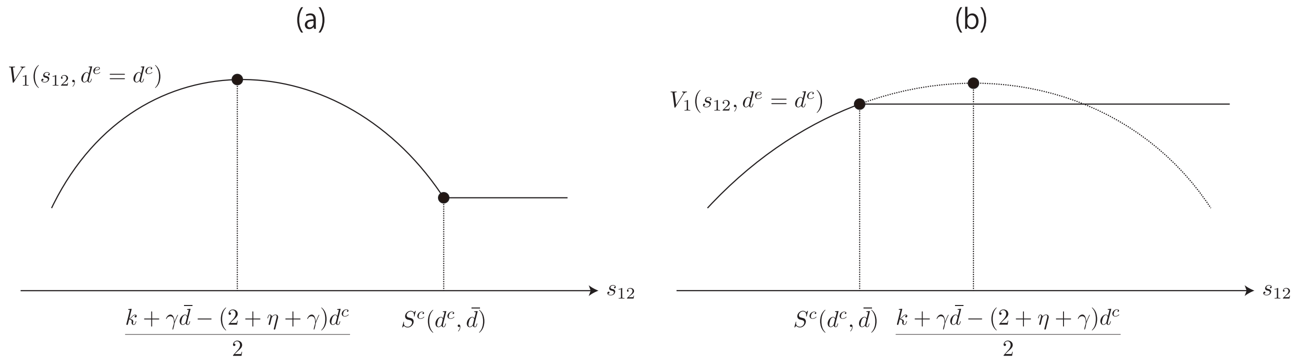

Figure A.1: Illustration of the period-1 selves' utility, $V_1(s_{12}, d^e = d^c)$, when $\left[k + \gamma\bar{d} - (2 + \eta + \gamma)d^c\right]/2 \leq S^c\left(d^c, \bar{d}\right)$ (Panel (a)) and $\left[k + \gamma\bar{d} - (2 + \eta + \gamma)d^c\right]/2 > S^c\left(d^c, \bar{d}\right)$ (Panel (b)).

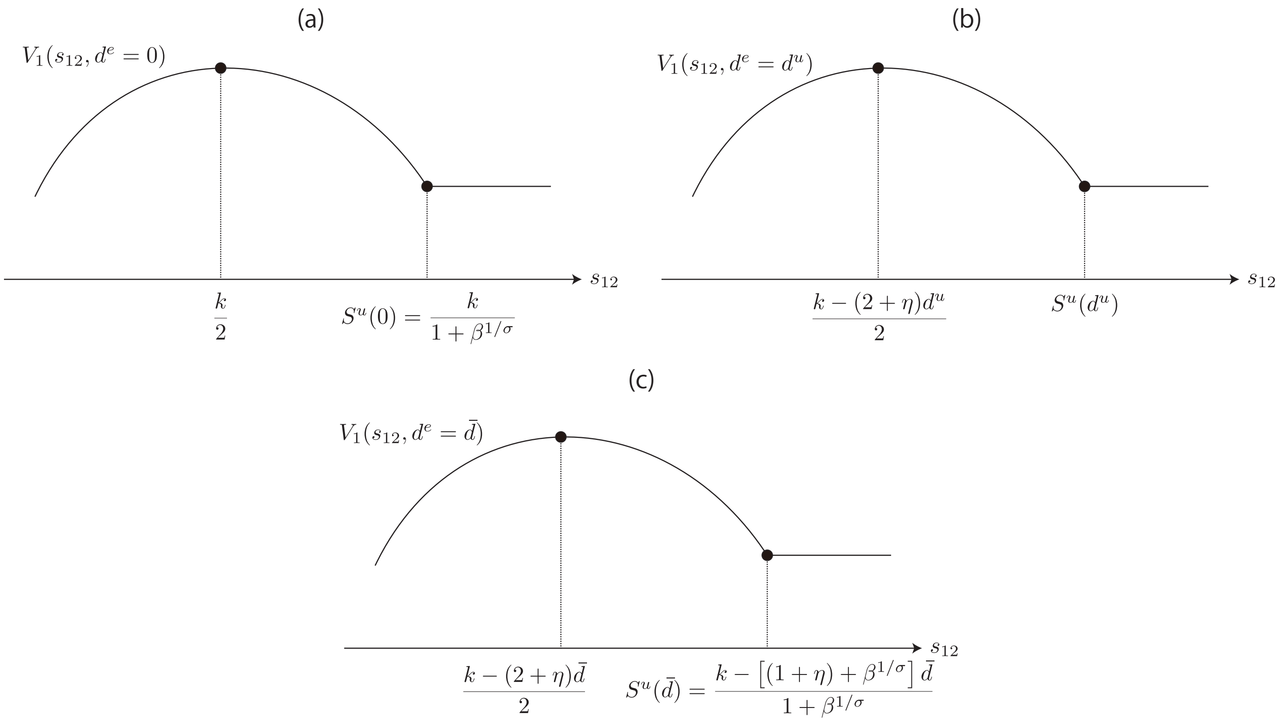

Figure B.1: Illustration of the period-1 selves' utility when $d^e = 0$ (Panel (a)), $d^e = d^u$ (Panel (b)), and $d^e = \bar{d}$ (Panel (c)).