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## Abstract

Performance assessment in the presence of undesirable outputs, such as pollutant emissions, is usually modelled within the framework of data envelopment analysis (DEA). In this paper we propose a new approach to measuring eco-efficiency in generalized input-output (gIO) models which may be used as a supplementary method to traditional DEA. Unlike DEA this approach takes into account detailed data on intersectoral flows in supply- and demand-driven gIO models. We focus on cases of traditional and sector-size-adjusted measures of interindustry linkages in gIO models and in each case we suggest respective indices of eco-efficiency and prove their usefulness in policymaking.

In order to illustrate possible applications of the new approach we conduct an empirical analysis aimed at identifying the eco-efficient sectors based on the 1995 and 2009 national input-output tables and environmental accounts for Poland which are provided by the World Input Output Data (WIOD) database.

**Keywords:** generalized input-output models, intersectoral linkages, eco-efficiency, nonlinear optimization.

JEL Classification: C5; O1; Q3

## 1. Introduction

In recent years the issue of seeking ways to increase 'eco-efficiency', understood as a management philosophy that aims at minimizing ecological damage while maximizing the efficiency of a firm's production processes, has become a topic of considerable interest for both researchers and politicians.[1] Due to the increasing environmental burdens caused by dramatic economic expansion, the issue of examining the ecological impact of economic activities has turned out to be of special importance for developing and transition economies.[2]

From an operational perspective measuring the efficiency of economic activity with respect to its environmental impact usually requires several steps. A crucial stage involves defining the input and output variables used to build a model that could help to measure eco-efficiency. At the next stage a set of specialized quantitative tools is used to establish the eco-efficiency levels of the decision making units (DMUs) analysed, i.e. firms, managers, sectors of an economy, etc. In this context data envelopment analysis (DEA)[3] has grown into an increasingly popular non-parametric tool used for performance evaluation. The main advantage of DEA lies in the fact that it accomplishes the task of measuring performance merely on the basis of the mathematical optimization approach without the need of subjective weight assignment for inputs and outputs. In other words, DEA allows any number of DMUs to be evaluated, with any number of inputs and outputs. The method requires the inputs and outputs for each DMU to be specified and defines the efficiency of each DMU as the weighted sum of outputs (i.e. the total output) divided by the weighted sum of inputs (i.e. total input). Moreover, all measures of efficiency lie between 0% and 100% while the numerical values of the weights are chosen in a way that maximises the efficiency of the particular DMU. When it comes to the general advantages of DEA one should also mention the fact that this approach allows a simultaneous analysis of outputs and inputs to be conducted and does not require an a priori definition of the form of the efficiency frontier. On the other hand, DEA ignores the effect of exogenous variables, ignores statistical errors and does not provide practical recommendations on how to improve efficiency (Odeck and Alkadi, 2001; Avkiran and Rowlands, 2008; Pestana and Peypoch, 2010; Jordá et al. 2012).

[1] For recent reviews of eco-efficiency-related topics see Merli et al. (2018), Cheng et al. (2018) and Pham et al. (2019), among others.
[2] See Muller and Yan (2018), Aklin et al. (2018), Kim and Park (2018), Kounetas (2018), Xing et al. (2018), Andrić et al. (2019), Brunel and Johnson (2019) and Freire-González and Puig-Ventosa (2019), among others.
[3] This concept was theorized and developed by Charnes et al. (1978).

In recent years, DEA has been mainly used in the context of measuring the eco-efficiency of sectors that operate in an economy. An updated discussion about DEA models that involve undesirable outputs, with an emphasis on economic-environmental context analysis may be found in Scheel (2001) and Gomes and Lins (2008), among others. Despite all the undisputable general advantages of this approach, one must underline that this concept fails to fully satisfactorily address important features of economic systems in at least three particular aspects.

First, as stressed by Dyckhoff and Allen (2001), the DEA approach must only be used when the decision maker has no doubts about the technical relations between undesirable outputs and certain inputs and outputs. At the same time, a DEA-based analysis of the eco-efficiency of multiple (say, $n$) sectors operating in an economy is solely based on an analysis of aggregated sector-specific inputs and outputs (usually just a few) while crucial information on intersectoral linkages (recall that for $n$ sectors one has $n \times n$ such interrelations) is not taken into account. As a consequence, this kind of detailed information is not used when establishing the levels of sectoral eco-efficiency, although it is clear that the structure of intersectoral input-output relations is an important factor that influences the levels of aggregated sectoral pollutant emission and output. This oversimplification seems to significantly reduce the possible range of policy recommendations as the conclusions following from DEA-based analyses usually take the form of listing the sectors which should reduce the amount of undesirable outputs. At the same time, no information is provided on possible changes in the underlying input-output relations with the remaining sectors of an economy that could also influence the levels of eco-efficiency.

Secondly, the DEA-based approach to measuring eco-efficiency does not allow supply- and demand-driven production processes that take place in economic systems to be analysed separately and in detail. The demand-driven effect occurs when production in sector $j$ expands which, in turn, implies that there will be higher demands from sector $j$ for goods and services produced by other sectors, which are required as intermediate inputs in sector $j$'s production. If one assumes that there are no supply limitations, the latter implies that other industries will react to the increased intermediate demands originating from sector $j$ and will expand their own production. When the origin of an impact (shock) comes from a sector as a purchaser of intermediate inputs, Leontief's demand-driven input-output model is used in order to quantify its consequences on the entire economy, as this model fully captures the demand-side impact and sectoral interlinkages (Temurshoev, 2016). Alternatively, if sector $j$

experiences expansion of output, the latter implies that increased amounts of products of sector *j* become available on the market, and, in particular, this surplus output could be used as intermediate inputs in production in other sectors of the economy. In this case, supplies from sector *j* (which plays the role of a seller of intermediate goods) for all the industries that use products or services produced by sector *j* in their production will increase. In such a context Ghosh's supply-driven input-output model is used in order to quantify its influences on the entire economy as this model fully captures the supply-side impact and sectoral interlinkages.

Finally, as underlined by Tarancón et al. (2008) and Gurgul and Lach (2018c), in practical applications one may face complex scenarios in which coefficients and benchmark variables are subjected to a set of restrictions, or even a group of benchmark variables. In such cases, the relevance of DEA-based measures of efficiency is not only determined by the elasticity of the benchmark variable, but also by any type of limitations on free changes in the coefficients.[4] It must be underlined that although the DEA approach is doubtlessly a useful tool for verifying performance, a purely mathematical approach to measuring efficiency may not be able to fully satisfactorily take into account many economic mechanisms like production technologies (especially the question of what is technically feasible when changing input coefficients), competitive advantages, international trade relations and the structure of global value chains. One should consider all these issues when interpreting DEA-based results, which unfortunately was not always the case in the majority of previous empirical studies.

Taking into account the three general problems listed above, in this paper we propose a new approach to measuring eco-efficiency in generalized input-output (gIO) models. This method is not intended to replace DEA as neither approach can be straightforwardly considered a substitute for the other, rather they can be considered complementary methods. The approach proposed in this paper builds upon a theory of intersectoral linkages and thus it looks at economic processes from a perspective other than that of DEA-based models. The latter makes the approach presented in this paper a supplementary proposal to the mainstream approach. In contrast to DEA we suggest that detailed data on intersectoral flows should be taken into account, which are available in demand- and supply-oriented gIO models. We focus on cases of traditional and sector-size-adjusted measures of interindustry linkages in

[4] Such restrictions could be budgetary, political, or technological (for instance, the balance between certain inputs) in nature (Gurgul and Lach, 2018c).

gIO models and in each case we propose respective indices of eco-efficiency and prove their general usefulness in policymaking. In order to illustrate possible applications of the new approach, the empirical analysis aimed at identifying the eco-efficient sectors is based on the 1995 and 2009 national input-output tables and environmental accounts for Poland, which are provided by the WIOD database. Importantly, unlike previous studies that focus on intersectoral flows and output-oriented key sector analyses in post-communist CEE economies, we avoid the negative effects of double-counting by focusing on two particular fundamental policy target variables – income per gross output and $CO_2$ emissions per gross output. To the best of our knowledge, both the theoretical and empirical parts of our paper are novel proposals in the economic literature.

The structure of this paper is as follows. Section 2 contains a brief overview of the literature on measuring eco-efficiency (also within the framework of input-output models) with particular attention paid to previous empirical analyses conducted for Poland. In Section 3 the outline of a new methodology for measuring eco-efficiency in supply- and demand-driven gIO models is described. Section 4 presents the dataset and the main results of an illustrative empirical example conducted for the Polish economy. Finally, in Section 5 we summarize the major findings and suggest some directions for future research.

## 2. Literature review

There is a long list of studies particularly aimed at presenting the state of the art in DEA models in the presence of undesirable outputs.[5] One of the most comprehensive reviews of the theoretical models and methods of DEA is the handbook edited by Zhu (2015). As the editor is one of the most prominent researchers in the field and the authors of particular chapters are major contributors to DEA theory, the book may be treated as a source of the current state of the art in the theory of DEA research. The range of topics covers distance functions and their value duals, cross-efficiency measures in DEA, integer DEA, weight restrictions and production trade-offs, facet analysis in DEA, scale elasticity, benchmarking and context-dependent DEA, fuzzy DEA, non-homogenous units, partial input-output relations, super efficiency, treatment of undesirable measures, translation invariance, stochastic nonparametric envelopment of data, and global frontier index. In a subsequent work Zhu (2016) provides an

[5] Comp. Färe et al. (1989, 2000), Yaisawarng and Klein (1994), Lovell et al. (1995), Färe and Grosskopf (2003, 2004), Thanassoulis (1995), Rheinhard et al. (1999, 2000), Scheel (2001), Hailu and Veeman (2001), Zofío and Prieto (2001), Dyckhoff and Allen (2001), Sun (2002), Seiford and Zhu (2002), Korhonen and Luptáčik (2003), Gomes et al. (2003), Gomes and Lins (2008) and more recently Tajbakhsh and Hassini (2018), Sueyoshi and Wang (2018), Allevi et al. (2018), Wu et al. (2018), Ma et al. (2018), Wei et al. (2019), and Galindro et al. (2019), among others.

extensive compilation of state-of-the-art empirical studies and applications for DEA. It includes a collection of 18 chapters written by world DEA experts.[6]

Given the quality of existing comprehensive reviews of theoretical variants of DEA along with reviews of their practical implementations, including the two excellent joint publications edited by Zhu (2015, 2016) mentioned above, in subsequent parts of this section we will focus solely on selected papers that form the backdrop to the outline of the new methodology presented in Section 3.

### 2.1.General approaches to modelling undesirable outputs in a DEA context

As the field of uses of DEA has grown progressively, one distinctive research current has focused on employing this technique to address the environmental consequences of production processes. Nowadays DEA-based models not only handle conventional outputs and inputs, but also allow bad or environmentally undesirable outputs, i.e., waste and polluting effluents obtained as by-products of commercial outputs and inputs, to be included in an analysis (Picazo-Tadeoa et al., 2011).

Dyckhoff and Allen (2001) and Gomes and Lins (2008) discuss the advantages and disadvantages of three main approaches to modelling undesirable outputs in a DEA context, which differ slightly in terms of the frequency of uses in empirical studies. The first one uses the reciprocal of the undesirable output as DEA output, that is, the numerical variable representing the undesirable output is first inverted and then modelled as a usual desirable output. This approach is employed, for example, by Lovell et al. (1995) and is called 'reciprocal multiplicative' (Golany and Roll, 1989; Scheel, 2001). The second method (Rheinhard et al., 1999) considers DEA to be a multi-criteria approach in which the

[6] The range of empirical topics analysed covers the performance of CEOs of U.S. banks and thrifts, the network operational structure of transportation organizations and the relative network data envelopment analysis model, examples of using different types of DEA models to compute total-factor energy efficiency scores with an application to energy efficiency, the exploration of the impact of incorporating customers' willingness to pay for service quality in benchmarking models on the cost efficiency of distribution networks, a brief review of previous applications of DEA in the professional baseball industry, a DEA-based examination of the efficiency and productivity of U.S. property-liability insurers, a two-stage network DEA model that decomposes the overall efficiency of a DMU into two components, a review of the literature of DEA models for the performance assessment of mutual funds, a discussion on the management strategies formulation of the international tourist hotel industry in Taiwan, a novel use of the two-stage network DEA to evaluate sustainable product design performances, a description of the limitations of some DEA environmental efficiency models, reviews of the use of DEA in secondary and tertiary education, a review of measures of the relative performance of New York State school districts in the academic year 2011-2012, a detailed description of uses for DEA in marketing, a description of the decomposition of a new total factor productivity index that satisfies all economically-relevant axioms from index theory with an application to U.S. agriculture and a unique study that conducts a DEA research front analysis, using a network clustering method to group the DEA literature over the period 2000 to 2014.

undesirable output is modelled as an input. In this case, both the CCR and BCC[7] variants of the DEA models can be used, depending on the operational scale of the DMUs. As mentioned by Scheel (2001), when considering undesirable outputs to be inputs, one creates the same set of production variants as in the case of considering the undesirable outputs to be desirable ones by using a reciprocal additive transformation. The third approach is based on values translation (Ali and Seiford, 1990), which technically requires the reciprocal additive transformation of the undesirable output to be increased by adding a positive scalar big enough to ensure that the modified values are positive for each DMU.[8]

### 2.2. IO-related modifications

The initial DEA-based approach to modelling the undesirable approach has been modified in many ways. A few modifications that turned out to be especially influential for empirical analysis can be found in works by Färe et al. (1996) and Kuosmanen (2005), who formulate an alternative approach to modelling undesirable emissions as outputs, assuming that these undesirable outputs are 'weakly disposable', which generally means that it is possible to abate emissions by decreasing the level of production activity. Korhonen and Luptáčik (2004) also describe several alternative models for assessing eco-efficiency and test their ability to provide similar efficiency scores for a sample of European power plants. They expound two different approaches used to incorporate eco-efficiency in DEA models. The first requires separate evaluations of technical and ecological efficiency to be computed via a DEA-based approach and then these efficiency figures should be used as output variables in a new DEA model. The second model consists in building up a ratio which simultaneously takes into account both desirable and undesirable outputs, and leads to a wide variety of alternative models, depending on how the undesirable outputs are treated.

From the perspective of the goal of this paper special attention must be paid to those studies that connect DEA-based concepts of measuring the eco-efficiency of economies with methods of exploring detailed information on interindustry relations contained in input-output data. At this point it seems necessary to refer to the pioneering works of Luptáčik and Böhm (2010) and Mahlberg and Luptáčik (2014). Both these studies are concerned with efficiency analysis applied to a single economy represented by Leontief's input-output model extended by primary factor constraints. This methodological approach requires an efficiency frontier to be

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[7] These abbreviations refer to the names of the creators of the models, i.e. Charnes, Cooper and Rhodes for the CCR (Charnes et al., 1978) and Banker, Charnes and Cooper for the BCC (Banker et al., 1984).

[8] Gomes and Lins (2008) stress that this approach is only valid for BCC and additive DEA models (Charnes et al., 1985), since CCR is not translation invariant (Cooper et al., 2000).

generated on the basis of a multi-objective optimization model instead of having to use data from different DMUs. The solutions to the multi-objective optimization problems define efficient virtual DMUs and the efficiency of a given economy is next defined as the difference between the potential of an economy and its actual performance and can be obtained as a solution to a DEA model. At the second stage the approach is extended to Leontief's augmented model, including emissions of pollutants and abatement activities, which in turn provides an IO-based tool for the analysis of eco-efficiency of an economy.

The most serious drawback of the proposals of Luptáčik and Böhm (2010) and Mahlberg and Luptáčik (2014) is the lack of statistical data required to construct the augmented Leontief model. The models of Luptáčik and Böhm (2010) and Mahlberg and Luptáčik (2014) require data on the matrix of primary input coefficients for abatement activities. As these data are not available from any data source they are approximated by the authors using the perpetual inventory method based on the time series of gross fixed capital formation for environmental protection. Although the perpetual inventory method is a popular tool used for approximating the unknown matrices of parameters in augmented and dynamic IO models (comp. Gurgul and Lach, 2016, 2018d, among others), its accuracy in practical uses is still a source of serious concern among empirical IO practitioners.[9]

An alternative approach to traditional DEA-based efficiency analysis in input-output models is proposed by Prieto and Zofío (2007). They undertake a network efficiency analysis within an input-output model that allows them to assess potential technical efficiency gains by comparing technologies corresponding to different economies. As a result they outline an algorithm for optimizing primary input allocation, intermediate production and final demand production by way of non-parametric DEA techniques. Moreover, this model optimizes the underlying multi-stage technologies that the input-output system comprises, identifying best practice economies. However, there are two crucial shortcomings of the approach of Prieto and Zofío (2007). First, the final outcome of the algorithm proposed takes the form of a matrix of optimal changes to be implemented in an original input matrix. However, as emphasized by Tarancón et al. (2008) and Gurgul and Lach (2018c), proper modelling of changes in IO coefficients should take into account budgetary, political, or technological limitations to free changes in the coefficients examined. Secondly, even if each individual

[9] Recently Gurgul and Lach (2019) went beyond the limitations of previous studies and instead of setting the crucial parameters of the dynamic endogenous input–output model with layers of techniques on an arbitrary basis they proposed a new optimization-based approach to approximating the elements of capital matrices (which are not available in statistical bureaus) on the basis of available historical data.

element of the matrix of optimal changes obtained lies at a reasonable (from an economic point of view) interval one cannot say that the whole set of changes suggested may take place in an economy at the same time (this type of general recommendation is the main policy implication of this approach). Taking this issue into account seems to be a promising direction for future extensions and generalizations of the approach.[10]

### 2.3. Measuring eco-efficiency in the Polish economy

There are several reasons for choosing Poland as a case study for the empirical part of this paper, in which focus is placed on two particular fundamental policy target variables – income per gross output and $CO_2$ emissions per gross output. First, Poland has the sixth-highest GDP (measured in PPP standards) in the EU and is an increasingly important player in the world economy. At the same time Poland has the fifth-highest level of greenhouse gas emissions in the EU. According to recent data published by WHO, 36 out of the 50 most polluted European cities are located in Poland. As a consequence, particulate pollution from fossil fuel combustion causes almost 50,000 deaths each year in Poland.

Secondly, in recent years Poland has not embraced tough emissions reductions targets, while most European countries have implemented various environment-friendly policies.[11] The lack of enthusiasm for climate policy in Poland can be explained to a great extent by the strong impact of trade union leaders in the coal industry on politicians. Since mining and burning coal has a long tradition in Poland, many citizens (especially miners and their families living in Śląskie province) cannot imagine their future without the existence of coal mines. In addition, a significant number of politicians believe that coal mines in Poland are necessary to provide an energy supply that will guarantee Poland's independence from Russia in terms of energy, Russia being the main supplier of natural gas and oil to Poland. However, since coal has long been on a downward trajectory in Poland, as foreign countries with easily-accessible coal and more modern mining technology have driven down global coal prices, Poland has switched to importing millions of tons of coal per year. Ironically, most of this coal comes from Russia.

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[10] Gurgul and Lach (2018c) propose a new algorithm for tracing value-added-redistribution-important coefficients (VARDI coefficients, in short) in a global IO model that deals with the issue of ranking the importance of individual coefficients belonging to an optimal solution, i.e. the set of VARDI coefficients.

[11] For example, in 2015, Poland prevented the 28-member EU from ratifying the Kyoto Protocol on $CO_2$ emissions as a bloc by voting for an amendment to the treaty. In 2016, Poland passed a law that hinders the development of wind energy (the new regulations included a requirement that turbines be located at a distance of at least 10 times the turbine's height from any buildings or forest, and a requirement that allowed for extended shutdowns for inspections; moreover a fourfold increase in taxes on wind farms was authorized). In 2018, Poland's climate negotiators officially admitted that pushing for stronger national pledges to reduce pollution is not one of the highest priorities of the authorities.

Taking all the these facts into account it seems obvious why in recent years one has noticed a growing number of studies aimed at analysing eco-efficiency in the Polish economy that were usually based on the use of some variants of the DEA approach.[12] Rączka (2001) identifies factors influencing the technical efficiency of heat plants in the Wielkopolska Region of Poland. Technical efficiency scores were first obtained from a DEA model for a cross-section sample of 41 heat plants. Next, a tobit model was used to explain the variability of the efficiency index. Rączka (2001) showed that government intervention in the household segment of the market decreased the efficiency of heat generation. At the same time coal quality and capital utilization were found to increase the technical efficiency. Finally, the results provided a basis to claim that public heat plants perform on average better than municipal and industrial ones.

Czaplicka-Kolarz et al. (2015) suggest a new method for assessing the eco-efficiency of mining production processes in hard coal mines in Poland, which enables the results of evaluating both the environmental and economic aspects to be integrated. This method uses the life cycle assessment (LCA) approach to assess the environmental efficiency and the results of operating activities to assess economic efficiency. A comprehensive method for assessing mining production processes is proposed as the key performance indicator in hard coal mines in Poland to be used to support decision making in mining companies.

Masternak-Janus and Rybaczewska-Błażejowska (2017) examine the possibility of using the concept of eco-efficiency at a regional level to promote the sustainable transformation of the regions of Poland. They decide to base their respective calculations on the input-oriented CCR DEA method as this approach is highly capable of measuring regional eco-efficiency. The results of the study reveal that the provinces of Lubuskie, Mazowieckie, Śląskie, Warmińsko-Mazurskie, and Wielkopolskie are relatively eco-efficient, whereas the remaining regions use too many environmental resources in relation to the value of goods and services produced. Six of the eleven eco-inefficient regions in Poland have increasing returns to scale, that is, the usage of natural resources connected with the negative impact upon the environment is rising more slowly than the values of goods and services.

In a subsequent study Rybaczewska-Błażejowska and Masternak-Janus (2018) once again focus on the case of Polish regions and the combined use of LCA and the input-oriented BCC

[12] In general, in recent years one could notice that a broad set of energy-and-climate-related topics in economic literature have been deeply analysed based on Polish data (comp. e.g. Dobrowolski et al. 2006; Boratyński, 2015; Gurgul and Lach, 2011, 2016b; Lach, 2011, among others). However, given the scope of this paper, we will focus mainly on recent DEA-related empirical studies which used Polish energy economics data.

DEA model. The ultimate goal of this approach is to support the strategic decision-making process. Firstly, it is shown that four of the sixteen Polish regions were found to be relatively eco-efficient, with agriculture and services making the greatest contribution to GDP. The study proves that the region with the most detrimental impact on the environment in all areas of protection, i.e. Śląskie province, is also the most eco-inefficient one. Among the fundamental sources of eco-inefficiency the authors list cumulative airborne emissions (primarily $CO_2$) and the excessive consumption of fuels, energy and heat in relation to the value of the goods and services produced.

To summarize, the analysis of the current state of the art in the eco-efficiency literature reveals an important fact. When it comes to the methodological details of the mainstream approach to measuring the eco-efficiency of economies one can see that DEA-based tools occupy a dominant position. This is also true in the case of studies that aim at examining the eco-efficiency of Poland. Unlike the dominant approach, in this paper we propose a new approach to measuring eco-efficiency in gIO models, which may be used as a supplementary method to traditional DEA.

## 3. Methodology

The main advantage of the proposed method is the fact that it takes into account detailed data on intersectoral flows in supply- and demand-oriented gIO models. Thus, in order to formulate respective eco-efficiency indices and prove their usefulness in policymaking we start this section with a brief overview of the most important measures of linkages which we shall then use to construct eco-efficiency indices. In general, our approach to measuring eco-efficiency builds upon the fundamentals of so-called 'key sector analysis'. In general, the purpose of key sector analysis is to identify those sectors which have the greatest effects on the rest of the economy (Gurgul and Lach, 2015).

### 3.1. Key sector measures - overview

The specific branch of literature that focuses solely on the analysis of sectoral input-output (IO) linkages and key sectors occupies a central place in the field of input-output analysis. In this context one should agree with Temurshoev (2016), who emphasizes that the analysis of IO linkages provides all the necessary tools required to better understand the importance of intersectoral interrelations in an economy. More precisely, such analyses may focus on addressing and/or evaluating various economic development policies that target specific industries. In particular, the direct and indirect strength of sectoral interdependencies can be quantified, which sheds light on the significance of individual sectors in the functioning of the entire economy (Temurshoev and Oosterhaven, 2014). Thus, it is not surprising that for many years the identification of key sectors in an economy has been one of the most important research topics in input–output analysis.[13] In any economy the identification and classification of its most influential branches can provide the basis for a taxonomy of the economy and can contribute to a better understanding of growth and development problems.

As stressed by Miller and Blair (2009), in the framework of input-output models, the process of production that takes place in a particular sector *j* implies two kinds of economic effects on other sectors of the economy:

- Assume that sector *j* increases its output. The latter implies that demands from sector *j* (which acts as a purchaser) on those sectors whose goods are used as production inputs in sector *j* will increase. This direction of a causal relationship is typical for a usual demand-side input-output model. In the input-output literature, this kind of interconnection between a particular sector and those ('upstream') sectors from which the latter purchases inputs is referred to as a so-called 'backward linkage'.
- Alternatively, an increase in output in sector *j* implies that additional amounts of products from sector *j* become available to be used as inputs to other sectors for their own production. In other words, an increase in supplies from sector *j* (acting as a seller) for the sectors that use the products of sector *j* in their production processes takes place. This in turn is the direction of causality in the usual supply-side IO model, and the term 'forward linkage' is used to indicate this kind of interconnection between a particular sector and those ('downstream') sectors to which it sells its output.

Following Temurshoev's (2016) review of the literature on the methods of key sector analysis, throughout this study two general types of measures of interindustry linkages will be studied within the framework of gIO models. These measures are related to two general concepts of measuring interindustry linkages: traditional mathematical measures of backward and forward linkages, and sector-size-adjusted interindustry linkages.[14]

[13] Since the release of the pioneering work of Hirschman (1958), the concept of the use of linkages in measuring interindustry relationships in an economy has attracted considerable attention among theoreticians and empiricists (Chenery and Watanabe, 1958; Hewings and Romanos, 1981; Hewings, 1982; Defourny and Thorbecke, 1984; Gurgul and Lach, 2016, 2018a, 2018b, 2018c, 2019; Zheng et al., 2018).

[14] In the IO literature there are several main types of interindustry linkages (Lahr, 2001). However, as pointed out by Temurshoev (2016), all the main types of sector-size-independent linkage may be easily derived on the basis of traditional linkages. Similarly, from a mathematical point of view the main types of sector-size-adjusted linkage are also functions of traditional linkages. Thus, throughout this study we restrict our analysis to the two variants of linkage measures.

### 3.2. Traditional linkages

Before providing respective formulas for calculating traditional interindustry linkages let us start by presenting some notational remarks which will make reading subsequent parts of the study easier. Since the aim of the empirical part of this paper is to analyse the efficiency of income generation with respect to the corresponding $CO_2$ emission of sectors operating in Polish economy, let us assume that the economy being studied consists of $n$ sectors. Next, let $\pi_{i,t}^{INC}$ stand for sector $i's$ direct coefficient indicating the generation of income per unit of gross output and let $\pi_{i,t}^{CO_2}$ denote sector $i's$ direct coefficient of $CO_2$ emissions per unit of gross output, both given at time point $t$. Next, let $\mathbf{x}_t = [x_i^t, i = 1, \dots, n]$ stand for the $n-$element vector of output, $\mathbf{f}_t = [f_i^t, i = 1, \dots, n]$ stand for the $n-$element vector of final demand, $\mathbf{v}_t = [v_i^t, i = 1, \dots, n]$ stand for the row of sectoral value added, $\mathbf{L}_t = (\mathbf{I} - \mathbf{A}_t)^{-1} = [l_{ij}^t, i, j = 1, \dots, n]$ denote $n \times n$ Leontief inverse and $\mathbf{G}_t = (\mathbf{I} - \mathbf{C}_t)^{-1} = [g_{ij}^t, i, j = 1, \dots, n]$ stand for $n \times n$ Ghosh inverse, where $\mathbf{A}_t = [a_{ij}^t, i, j = 1, \dots, n]$ and $\mathbf{C}_t = [c_{ij}^t, i, j = 1, \dots, n]$ denote input and output matrices, respectively.[15]

In general, key sector measures were predominantly used in studies that focused on gross output, especially in the early input-output research (Gurgul and Lach, 2015). However, the output-oriented measures of intersectoral dependencies obtained in basic Leontief/Ghosh models have a serious drawback. Namely, in order to be relevant to actual policymaking, key sector measures should be defined in a way that could reflect not only the gross-output-related processes but also the other main policy goals, including income generation, job creation, or reduction of greenhouse gas emission (Oosterhaven, 1981; Lenzen, 2003; Garrett-Peltier, 2017). Moreover, gross output reflects double-counting as it includes both the sales of intermediate and final products (thus, it is also often referred to as 'gross duplicated output'). Therefore, in practical applications one uses generalized IO models to conduct policy-oriented multiplier analysis (Gurgul and Lach, 2018a). Since the level of factor production/use in any sector $i$ (henceforth we will denote this as $e_i^t$) can be easily computed as $e_i^t = \pi_i^t x_i^t$, where $\pi_i^t$ stands for analyzed policy goal variable,[16] the so-called 'generalized demand-driven IO model' may be defined using the following formula (Miller and Blair, 2009):

$$\mathbf{e}_t = \hat{\boldsymbol{\pi}}_t \mathbf{L}_t \mathbf{f}_t, \tag{1}$$

[15] Following the usual notation in the IO literature, throughout this paper matrices are indicated by bold capitals, vectors by bold lowercases and scalars by italic capitals and lowercases. In this paper the symbols $\hat{\mathbf{x}}$ and $diag(\mathbf{x})$ will be used interchangeably to denote a diagonal matrix with elements of vector $\mathbf{x}$ on the main diagonal.

[16] In the case of the empirical example analysed in this paper $\pi_i^t = \pi_{i,t}^{INC}$ or $\pi_i^t = \pi_{i,t}^{CO_2}$.

where $\mathbf{e}_t = [e_i^t, i = 1, \dots, n]$, and $\widehat{\boldsymbol{\pi}}_t$ stands for a diagonal matrix with elements $\pi_i^t$ on the main diagonal. Similarly, one may define the so-called 'generalized supply-driven IO model', which links sectoral primary inputs to factor production/use by means of the following formula (Miller and Blair, 2009; Temurshoev, 2016):

$$\mathbf{e'}_t = \mathbf{v}_t \mathbf{G}_t \widehat{\boldsymbol{\pi}}_t. \quad (2)$$

As pointed out by Temurshoev (2016) and Gurgul and Lach (2018a), in input-output linkage analysis it is now widely accepted and strongly advocated that any backward linkage indicator, which measures the economy-wide degree of the complex (direct and indirect) interrelatedness of sectors in their role as intermediate purchasers, should be based on the demand-driven input-output model. Similarly, measuring the overall extent of the complex interconnectedness of industries in their role as intermediate suppliers, should be based on the supply-driven input-output model.[17] Thus, forward linkage indicators should be based on an application of Ghosh input-output model. Taking both these facts into account, in later parts of this paper we will use the following definition of traditional backward input-output linkage for sector $i$ and policy goal variable $\boldsymbol{\pi}_t$:

$$B_{i,t}(\boldsymbol{\pi}) = \sum_{k=1}^{n} \pi_k^t l_{ki}^t, \quad (3)$$

and the following definition of traditional forward linkage:

$$F_{i,t}(\boldsymbol{\pi}) = \sum_{k=1}^{n} g_{ik}^t \pi_k^t. \quad (4)$$

The backward input-output linkage defined in (3) reflects the demand-pull effects in an economy. $B_{i,t}(\boldsymbol{\pi})$ measures the total (i.e. direct and indirect) intermediates' purchase-related linkages/importance of sector $i$ which are associated with its unit final demand. In other words, this indicator is a measure of the quantitative significance of the chains of sector $i$'s demands for intermediate inputs from all sectors of the economy. The forward input-output linkage defined in (4) reflects the cost-push effects in the economy; i.e., it is assumed that the input and output prices may change, but their quantities will remain fixed. $F_{i,t}(\boldsymbol{\pi})$ refers to the total (i.e. direct and indirect) intermediates' sales-related linkages/importance of sector $i$ (in the sense of the quantitative significance of the chains of sector $i$'s supplies of its intermediate inputs to all sectors of the economy) which are associated with its primary inputs equal to one unit.

[17] From a historical point of view one should also mention 'direct' input-output linkages, which were defined as column sums of input matrix $\mathbf{A}_t$ (backward linkages; see e.g. Chenery and Watanabe (1958) for an early empirical application) and row sums of output matrix $\mathbf{C}_t$ (forward linkages). However, as stressed in Temurshoev (2016) these measures are nowadays rarely used in practical applications.

### 3.3. Sector-size-adjusted linkages

It should be stressed that the heterogeneity of industries in terms of their size should also be explicitly taken into account in empirical applications, which is not always the case in the existing key sector studies. Temurshoev (2016) and Gurgul and Lach (2018a) show that if the effect of sector size is not corrected for, one would very often obtain the expected outcome that big (small) industries have a big (small) impact on the whole economy, which will further disregard the greater cost of stimulating a large industry. Therefore, in practical applications it is also important to consider the total economy-wide impact of sectors per unit of their direct size/contribution. For this purpose, along with the traditional input-output linkages their sector-size-adjusted variants are also used in this study. This adjustment is simply based on taking into account the relevant size or direct impact of the sectors:

$$\bar{B}_{i,t}(\boldsymbol{\pi}) = \frac{B_{i,t}(\boldsymbol{\pi})}{\pi_i^t}, \tag{5}$$

$$\bar{F}_{i,t}(\boldsymbol{\pi}) = \frac{F_{i,t}(\boldsymbol{\pi})}{\pi_i^t}. \tag{6}$$

The sector-size-adjusted linkage defined in (5) is a dimensionless indicator that expresses the relevant traditional backward IO linkage of a particular sector per unit of its size given in terms of the policy goal variable. From a supply-side perspective, one may analogously define the sector-size-adjusted forward IO linkage using the formula (6). Like its traditional counterpart, the sector-size-adjusted forward input-output linkage also reflects the cost-push effects in the whole economy.

It is worth emphasizing that in comparison to the traditional linkages given in (3) and (4), the sector-size-adjusted linkages in (5) and (6) treat all country-sectors similarly irrespective of their size in generating the policy goal variable (Miller and Blair, 2009). Therefore, the sector-size-adjusted IO linkages are more effective indicators of the indirect economy-wide impact of the sectors relative to their own direct contribution. In this sense, sector-size-adjusted IO linkages are somewhat superior to sector-size-independent measures as they are free of biases resulting from the size of the sector (Temurshoev, 2016; Gurgul and Lach, 2018a).

A convenient way of interpreting the linkages in (3)-(6) is based on re-calculating their values relative to the relevant economy-wide average. Henceforth, let $\|\cdot\|$ denote the normalizing operator which for the vector of nonnegative numbers $\mathbf{x} = (x_i)_{i=1,\dots,n}$ is defined as follows:

$$\|\mathbf{x}\| = (\|x_i\|)_{i=1,\dots,n} = \left(\frac{nx_i}{\sum_{s=1}^{n} x_s}\right)_{i=1,\dots,n}. \tag{7}$$

Under such a notation the normalized linkages in (3)-(6) are simply defined as $\|B_{i,t}(\boldsymbol{\pi})\|$, $\|\bar{B}_{i,t}(\boldsymbol{\pi})\|$, $\|F_{i,t}(\boldsymbol{\pi})\|$ and $\|\bar{F}_{i,t}(\boldsymbol{\pi})\|$, respectively. If the backward (forward) linkage of sector $i$ is greater than the economy-wide average of the corresponding backward (forward) linkages of all sectors then the normalized backward (forward) linkage of this sector is greater than unity.

### 3.4. Measuring eco-efficiency in generalized IO models

We suggest that eco-efficiency should be measured in gIO models in a way similar to the general DEA-based approach, i.e. to define the eco-efficiency indicator as a ratio of output effect to input stimulation. Since we distinguish between backward and forward linkages in an economy, one should define separately the respective measures of the 'backward eco-efficiency' and 'forward eco-efficiency'. In the case of the two-dimensional (i.e., single output and single input) case, examined in the empirical part of this paper, we propose the following formulas to define the linkage-based measures of the eco-efficiency of sector $i$ at time point $t$:

$$ECOEFF_{i,t}^{BACK} = \frac{\|BACK_{i,t}(\boldsymbol{\pi}^{OUT,t})\|}{\|BACK_{i,t}(\boldsymbol{\pi}^{IN,t})\|}, \tag{8}$$

$$ECOEFF_{i,t}^{FORW} = \frac{\|FORW_{i,t}(\boldsymbol{\pi}^{OUT,t})\|}{\|FORW_{i,t}(\boldsymbol{\pi}^{IN,t})\|}, \tag{9}$$

where:

- $ECOEFF_{i,t}^{BACK}$ ($ECOEFF_{i,t}^{FORW}$) stands for the backward (forward) eco-efficiency measure,
- $BACK_{i,t}(\cdot)$ ($FORW_{i,t}(\cdot)$) stands for the chosen type (i.e. traditional or sector-size-adjusted) of backward (forward) linkage measure, i.e. $BACK_{i,t}(\cdot) = B_{i,t}(\cdot)$ or $BACK_{i,t}(\cdot) = \bar{B}_{i,t}(\cdot)$ ($FORW_{i,t}(\cdot) = F_{i,t}(\cdot)$ or $FORW_{i,t}(\cdot) = \bar{F}_{i,t}(\cdot)$),
- $\boldsymbol{\pi}^{OUT,t} = [\pi_{i,t}^{OUT}, i = 1, \dots, n]$ ($\boldsymbol{\pi}^{IN,t} = [\pi_{i,t}^{IN}, i = 1, \dots, n]$) stands for the output (input) policy goal variable,[18]
- $\|\cdot\|$ denotes the normalizing operator defined in (7).

The efficiency indexes defined in (8) and (9) can be interpreted intuitively and in a straightforward way. For example, if for a chosen backward linkage measure, say

[18] In the case of the empirical example analysed in this paper $\pi_i^{OUT,t} = \pi_{i,t}^{INC}$ and $\pi_i^{IN,t} = \pi_{i,t}^{CO_2}$.

$BACK_{i,t}(\boldsymbol{\pi}^t) = B_{i,t}(\boldsymbol{\pi}^t)$, and for the chosen input and output policy goal variables, say $\pi_{i,t}^{OUT} = \pi_{i,t}^{INC}$ and $\pi_{i,t}^{IN} = \pi_{i,t}^{CO_2}$, one obtains $ECOEFF_{i,t}^{BACK} > 1$, it implies that at time point $t$ the direct and indirect backward effect of a unitary rise in final demand in sector $i$ on economy-wide income generation was larger than the corresponding backward effect on economy-wide $CO_2$ emission. In other words, if sector $i$ increases its output then the demands from sector $i$ (which acts as a purchaser) on those sectors whose goods are used as production inputs in sector $i$ will impose a stronger positive effect on economy-wide income generation than on economy-wide $CO_2$ emission.

**Table 1.** Sectoral classification based on backward and forward linkages.

**Backward linkages**

| Sector type | Income generation (Leontief gIO $\mathbf{e}_t = \hat{\boldsymbol{\pi}}_t^{INC}\mathbf{L}_t\mathbf{f}_t$) | $CO_2$ emission (Leontief gIO $\mathbf{e}_t = \hat{\boldsymbol{\pi}}_t^{CO_2}\mathbf{L}_t\mathbf{f}_t$) |
|---|---|---|
| Eco-effective (ECO-EFF) | Above economy-wide average | Below economy-wide average |
| Bi-Key sector (BI-KEY) | Above economy-wide average | Above economy-wide average |
| Bi-Weak sector (BI-W) | Below economy-wide average | Below economy-wide average |
| Eco-ineffective (ECO-INEFF) | Below economy-wide average | Above economy-wide average |

**Forward linkages**

| Sector type | Income generation (Ghosh gIO $\mathbf{e}'_t = \mathbf{v}_t\mathbf{G}_t\hat{\boldsymbol{\pi}}_t^{INC}$) | $CO_2$ emission (Ghosh gIO $\mathbf{e}'_t = \mathbf{v}_t\mathbf{G}_t\hat{\boldsymbol{\pi}}_t^{CO_2}$) |
|---|---|---|
| Eco-effective (ECO-EFF) | Above economy-wide average | Below economy-wide average |
| Bi-Key sector (BI-KEY) | Above economy-wide average | Above economy-wide average |
| Bi-Weak sector (BI-W) | Below economy-wide average | Below economy-wide average |
| Eco-ineffective (ECO-INEFF) | Below economy-wide average | Above economy-wide average |

**Source:** Own elaboration.
**Note:** Linkages are given relative to their relevant economy-wide average values. Symbols in parentheses in the first column represent abbreviated names of sector types.

In addition to the simple measures of eco-efficiency defined in (8) and (9) this methodology allows one to classify sectors of an economy according to the links between income generation and pollutant emissions. Taking into account the possible values of the linkages in (3)-(6) we propose the eco-efficiency-focused sectoral classification presented in Table 1.

As can be seen from Table 1 the sectoral classification proposed is based on comparing the levels of normalized linkages. To summarize, the approach presented in this paper offers a two-dimensional eco-efficiency measure that takes into account both the eco-efficiency indexes defined in (8) and (9) as well as the classification scheme presented in Table 1. As a consequence, we propose that two-prong notation should be used, according to the scheme "abbreviated name of sector's type + (value of efficiency index)", e.g. for backward-linkage-oriented results the notation ECO-EFF(1.34) will denote a backward eco-efficient sector with the value of the efficiency index equal to 1.34.

### 3.5. Formulating policy implications

This methodology allows one to study the differences between sectoral eco-efficiency levels by looking at these quantities from a different point of view compared to the traditional DEA-based approach. For an economy with $n$ sectors the results of this type of analysis provide two $n$-element sets of linkages, both in backward- and forward-linkage-oriented cases. These values are used to calculate the efficiency indexes defined in (8) and (9) as well as to obtain the sectoral classification described in Table 1. An interesting question in the context of analysing the eco-efficiency of the economy as a whole is what the general pattern in the relationship between output (e.g. income generation) and input (e.g. $CO_2$ emission) linkages is among the sectors operating within the economy.[19] For this purpose one may study the properties of scatterplots of the values of normalized output and input linkages.

In Figure 1 we present an exemplary plot with income (i.e. the chosen output policy goal variable) linkages on the vertical axis and $CO_2$ (i.e. the chosen input policy goal variable) linkages on the horizontal axis. In practical applications this plot may be interpreted in a meaningful way. If the linkages exhibit a downward trend (upper left plot in Figure 1) it implies that the larger the $CO_2$-related linkage, the smaller the corresponding income-related linkage. In other words, the sectors operating in such an economy show a tendency to emit relatively large amounts of $CO_2$ while having relatively low potential for income generation. Alternatively, in the upper right plot in Figure 1 the trend is positive, suggesting that larger $CO_2$ emission levels are accompanied by a relatively higher potential for income generation.

[19] The latter seems especially important given the rising prices of $CO_2$ emission allowances in the EU.

However, in this particular case one still could have a significant number of eco-inefficient sectors. From the point of view of eco-oriented policymaking the most desirable relationship between the linkages would take the form of a nonlinear inverted-L-shaped relationship (henceforth we will refer to such a relationship between linkages as to the 'eco-optimal' case). Namely, if an economy reduced the number of ECO-INEFF sectors to zero the scatter plot of income-$CO_2$ linkages would look like the one presented in the bottom plot in Figure 1.[20]

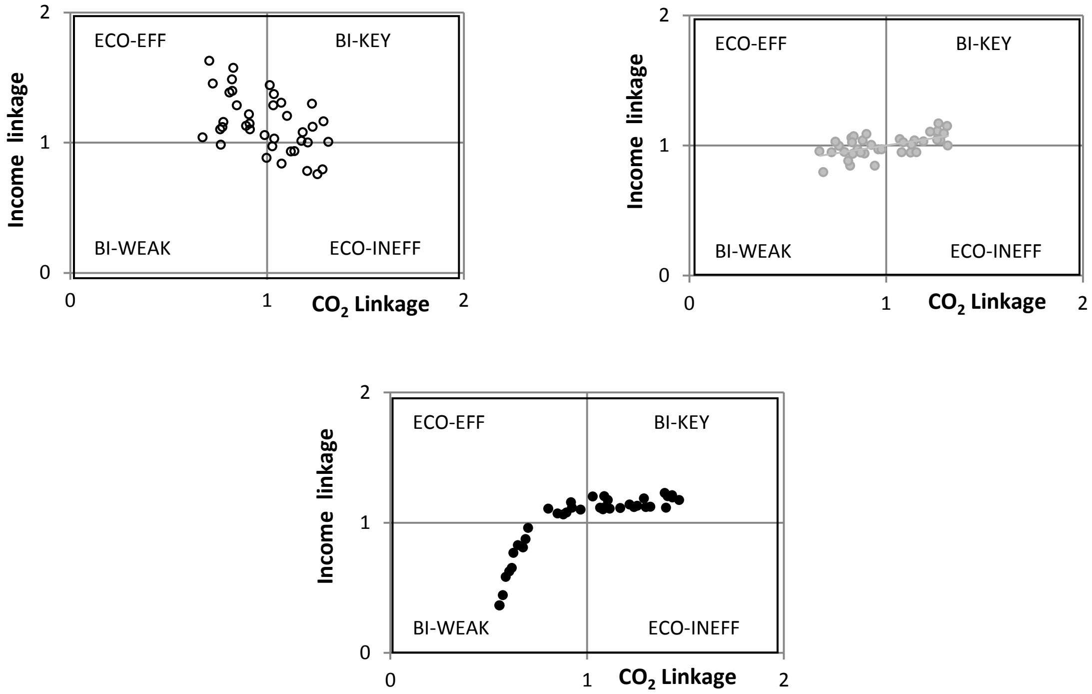


**Figure 1.** Examples of two-dimensional scatterplots of input and output linkages.
**Note:** Linkages are given relative to their relevant economy-wide averages.

To summarize, studying the properties of two-dimensional scatterplots of datasets on input and output linkages provides meaningful information about the eco-efficiency of the economy examined as a whole. Taking into account the interpretation of this type of plot (comp. the example of the linkage datasets presented in Figure 1) one may claim that from the point of view of policymaking economies for which such output/input scatterplots exhibit a downward trend might be interested in reversing this regularity, as a downward trend indicates the existence of highly eco-inefficient sectors. Therefore, in a backward-linkage-oriented[21] case one may be interested in solving the following optimization problem aimed at transforming the shape of the original scatterplot of normalized linkages towards the inverted-L-shape

[20] Note that since the averages of normalized income and $CO_2$ linkages are equal to 1, the inverted-L-shaped relationship in bottom panel of Figure 1 cannot involve only ECO-EFF and BI-KEY sectors.
[21] The respective procedure in the forward-linkage-oriented case is analogous and therefore will not be repeated here.

presented in the bottom panel of Figure 1:

**OPTIMIZATION PROBLEM NO 1**

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**Goal:** Given the data on normalized output backward linkages $\|BACK_{i,t}(\boldsymbol{\pi}^{OUT,t})\|$ for output policy target goal variable $\boldsymbol{\pi}^{OUT,t}$, and normalized input backward linkages $\|BACK_{i,t}(\boldsymbol{\pi}^{IN,t})\|$ for input policy goal variable $\boldsymbol{\pi}^{IN,t}$, where $i = 1, \dots, n$ and $t$ stands for a fixed time point, find shift vectors $\Delta_t^{OUT} = \left(d_i^{OUT,t}\right)_{i=1,\dots,n}$ and $\Delta_t^{IN} = \left(d_i^{IN,t}\right)_{i=1,\dots,n}$ that maximize the objective function:

$$ECO\_SCORE = \sum_{i=1}^{n}\left[1 + \frac{\left|\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{OUT,t}\right)+d_i^{OUT,t}\right\|-1\right|}{2\left(\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{OUT,t}\right)+d_i^{OUT,t}\right\|-1\right)} - \frac{\left|\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{IN,t}\right)+d_i^{IN,t}\right\|-1\right|}{2\left(\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{IN,t}\right)+d_i^{IN,t}\right\|-1\right)}\right],[22] \quad (10)$$

assuming that $-l_i^{t,OUT-} \leq d_i^{OUT,t} \leq l_i^{t,OUT+}$ and $-l_i^{t,IN-} \leq d_i^{IN,t} \leq l_i^{t,IN+}$ for some pairs of vectors of the upper ($0 \leq l_i^{t,OUT+}$, $0 \leq l_i^{t,IN+}$) and lower ($0 \leq l_i^{t,OUT-}$, $0 \leq l_i^{t,IN-}$) bounds and the following constraints hold true:

$$\sum_{i=1}^{n} d_i^{OUT,t} = 0, \quad (11)$$

$$\sum_{i=1}^{n} d_i^{IN,t} = 0, \quad (12)$$

$$\sum_{i=1}^{n}\left|d_i^{OUT,t}\right| \leq M_t^{OUT} \sum_{i=1}^{n} max\,(l_i^{t,OUT-}, l_i^{t,OUT+}), \quad (13)$$

$$\sum_{i=1}^{n}\left|d_i^{IN,t}\right| \leq M_t^{IN} \sum_{i=1}^{n} max\,(l_i^{t,IN-}, l_i^{t,IN+}), \quad (14)$$

where $0 \leq M_t^{OUT} \leq 1$, $0 \leq M_t^{IN} \leq 1$.

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The objective function in (10) takes the form of a three-valued pointer indicating the sector's type based on the values of the modified output linkages (vertical axis) and input linkages (horizontal axis).

Note that for eco-efficient sectors $\left|\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{OUT,t}\right) + d_i^{OUT,t}\right\| - 1\right| = \left\|BACK_{i,t}\left(\boldsymbol{\pi}^{OUT,t}\right) + d_i^{OUT,t}\right\| - 1$ and $\left|\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{IN,t}\right) + d_i^{IN,t}\right\| - 1\right| = -\left(\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{IN,t}\right) + d_i^{IN,t}\right\| - 1\right)$. Thus, for these sectors one has $ECO\_SCORE = 2$. Similarly, for sectors of type BI-KEY and BI-WEAK one has $ECO\_SCORE = 1$, while for sectors of type ECO-INEFF one has $ECO\_SCORE = 0$.[23]

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[22] $|x|$ stands for absolute value of $x$.

[23] The fact that for $ECO\_SCORE = 1$ both for BI-KEY and BI-WEAK sectors seems to stand in line with the idea of linkage-based measures of eco-efficiency defined in (8) and (9). Namely, the relative impact of BI-WEAK sectors on the economy-wide generation of income and $CO_2$ is similar to the analogous relative impact of BI-KEY sectors. If the income generating potential and the $CO_2$ emission potential of a sector are both below the respective economy-wide-averages, the net income of such a sector (i.e. the income less environmental costs) per unit of product is similar to the corresponding ratios calculated for BI-KEY sectors.

Conditions (11) and (12) ensure that the modified linkages remain normalized. To make the overall change in linkages more realistic we follow the arguments of Gurgul and Lach (2018c) and assume (comp. (13), (14)) that the overall change of input and output linkages cannot exceed a chosen threshold level (i. e. $M_t^{OUT}$ for output linkages and $M_t^{IN}$ for input linkages) of the maximal possible change (i. e. $\sum_{i=1}^{n} max\ (l_i^{t,OUT-}, l_i^{t,OUT+})$ for output linkages and $\sum_{i=1}^{n} max\ (l_i^{t,IN-}, l_i^{t,IN+})$ for input linkages). This assumption implies that not all changes in normalized linkages may reach maximal absolute values at the same time. As suggested by Gurgul and Lach (2018c), in such a case the optimal solution to **OPTIMIZATION PROBLEM NO 1** will point out only the most important linkages in terms of transforming the actual relationship between linkages towards the eco-optimal inverted-L-shape presented in the bottom plot in Figure 1.

### 3.6. Practical implementation

Another important problem is the practical implementation of the solution to **OPTIMIZATION PROBLEM NO 1.** The latter takes the form of two lists of modified linkages, i.e. $\left(\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{OUT,t}\right) + d_i^{OUT,t}\right\|\right)_{i=1,\dots,n}$ and $\left(\left\|BACK_{i,t}\left(\boldsymbol{\pi}^{IN,t}\right) + d_i^{IN,t}\right\|\right)_{i=1,\dots,n}$. Therefore, in order to translate the solution to **OPTIMIZATION PROBLEM NO 1** into a set of practical policy recommendations one must know what policies should be taken in order to influence the output and input linkages of particular sectors within an economy. In general, for each type of linkage measures two general answers may be given to this question. To illustrate the two respective policies let us focus on the case of increasing the traditional output backward linkage for a particular sector $i_0$.[24]

*Policies for changing the traditional backward linkages of sector* $i_0$

- **Strategy 1**: Modifying the policy goal variable $\boldsymbol{\pi}^{OUT,t}$

As shown in (3):

$$\left\|BACK_{i_0,t}\left(\boldsymbol{\pi}^{OUT,t}\right)\right\| = \left\|\sum_{k=1}^{n} \pi_k^{OUT,t} l_{ki_0}^t\right\|, \tag{15}$$

and

$$\left\|BACK_{i_0,t}\left(\boldsymbol{\pi}^{IN,t}\right)\right\| = \left\|\sum_{k=1}^{n} \pi_k^{IN,t} l_{ki_0}^t\right\|. \tag{16}$$

In other words, the traditional backward linkages for sector $i_0$ take the form of a scalar

[24] The respective procedure in a forward-linkage-oriented case is analogous to the backward-oriented scheme, therefore will not be repeated here.

product of the policy goal variable and the $i_0 -$ th column of the Leontief inverse. Thus, intuitively the simplest policy for increasing $\left\|\sum_{k=1}^{n} \pi_k^{OUT,t} l_{ki_0}^t\right\|$ is to increase $\pi_{i_0}^{OUT,t}$ (e.g., increase income per unit of product in sector $i_0$). The latter follows from the properties of the power series approximation of the Leontief inverse. As pointed out by Miller and Blair (2009) it is clear that all the on-diagonal elements in a Leontief inverse are greater than one. Also, it is virtually always observed in real-world Leontief inverse matrices that $l_{rs} < 1$ $(r \neq s)$ (off-diagonal elements are less than one). Thus, $\pi_{i_0}^{OUT,t} l_{i_0 i_0}^t$ is always greater than $\pi_{i_0}^{OUT,t} l_{i_0 k}^t$, for $k \neq i_0$, and as a consequence increasing $\pi_{i_0}^{OUT,t}$ will increase $BACK_{i_0,t}(\boldsymbol{\pi}^{OUT,t})$ to a greater extent than $BACK_{k,t}(\boldsymbol{\pi}^{OUT,t})$, for $k \neq i_0$, and therefore $\|BACK_{i_0,t}(\boldsymbol{\pi}^{OUT,t})\|$ will rise. Using the same logic one may easily show that the simplest policy for lowering $\|BACK_{i_0,t}(\boldsymbol{\pi}^{IN,t})\|$ is to lower $\pi_{i_0}^{IN,t}$ (e.g. lower $CO_2$ emission per unit of product in sector $i_0$).

In order to obtain precise policy recommendations (different from the simple proposals listed above) one must formulate and solve a respective linkage-oriented optimization problem. In the case of increasing the traditional output backward linkage for a particular sector $i_0$ the latter could take the following form:

**OPTIMIZATION PROBLEM NO 2**

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**Goal:** Given the data on elements of Leontief inverse, find the shift vector $\Delta_t^{OUT} = \left(d_i^{OUT,t}\right)_{i=1,\dots,n}$, where $i = 1, \dots, n$ and $t$ stands for a fixed time point, which maximize the objective function:

$$\left\|\sum_{k=1}^{n}\left(\pi_k^{OUT,t} + d_k^{OUT,t}\right) l_{ki_0}^t\right\|,$$

for output policy target goal variable $\boldsymbol{\pi}^{OUT,t}$, assuming that $-l_i^{t,OUT-} \leq d_i^{OUT,t} \leq l_i^{t,OUT+}$ for some vectors of the upper $(0 \leq l_i^{t,OUT+})$ and lower $(0 \leq l_i^{t,OUT-})$ bounds and the following constraints hold true:

$$\left|\sum_{i=1}^{n} d_i^{OUT,t}\right| \leq M_t^{OUT} \sum_{i=1}^{n} max\,(l_i^{t,OUT-}, l_i^{t,OUT+}), \qquad (17)$$

where $0 \leq M_t^{OUT} \leq 1$.

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The condition in (17) plays an analogous role to the conditions (13), (14) in **OPTIMIZATION PROBLEM NO 1**. It is important to stress that one of the main advantages of **Strategy 1** is that it may be simultaneously used to change output and input linkages, since $\boldsymbol{\pi}^{OUT,t}$ and $\boldsymbol{\pi}^{IN,t}$ are assumed to be independent.

□ **Strategy 2**: Modifying the Leontief inverse

To shed some light on the alternative approach to the practical implementation of the solution to **OPTIMIZATION PROBLEM NO 1**, let $\mathbf{L}_t^{\mathbf{M}}$ stand for a modified Leontief inverse obtained for input matrix $\mathbf{A}_t$ modified by adding elements of a shift matrix $\Delta\mathbf{A}_t$, i.e.:

$$\mathbf{L}_t^{\mathbf{M}} = [\mathbf{I} - \mathbf{A}_t - \Delta\mathbf{A}_t]^{-1}. \tag{18}$$

One may rewrite (18) in the following form:

$$\Delta\mathbf{A}_t = \mathbf{I} - \mathbf{A}_t - (\mathbf{L}_t^{\mathbf{M}})^{-1}. \tag{19}$$

Formula (19) shows how one can simply find the modification in a given input matrix (i.e., $\Delta\mathbf{A}_t$) which corresponds to a set of known changes in the corresponding Leontief matrix ($\mathbf{L}_t^{\mathbf{M}}$). At the same time formulating the conditions necessary for changing the IO coefficients stored in $\mathbf{A}_t$ is relatively simple given a technological- and terms-of-trade-based interpretation of $\mathbf{A}_t$ (Gurgul and Lach, 2018c). Thus, the only remaining issue is to find the modification of Leontief inverse ($\mathbf{L}_t^{\mathbf{M}}$) which will maximize output linkages given some policy goal variable. In the case of increasing traditional output backward linkage this problem may be solved by means of the following:[25]

**OPTIMIZATION PROBLEM NO 3**

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**Goal:** Given the data on output policy goal variable $\boldsymbol{\pi}^{OUT,t}$, find shift matrices $\Delta_t^{OUT} = \left(d_{ij}^{OUT,t}\right)_{i,j=1,\dots,n}$ (where $i,j = 1,\dots,n$, and $t$ stands for a fixed time point), which maximize the objective function:

$$\left\| \sum_{k=1}^{n} \pi_k^{OUT,t} \left( l_{ki_0}^t + d_{ki_0}^{OUT,t} \right) \right\|,$$

assuming that $-p_{ij}^{t,OUT-} \leq d_{ij}^{OUT,t} \leq p_{ij}^{t,OUT+}$, for some matrices of the upper ($0 \leq p_{ij}^{t,OUT+}$) and lower ($0 \leq p_{ij}^{t,OUT-}$) bounds, and the following constraints hold true:

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[25] The respective procedures in forward-linkage-oriented case are once again analogous to their backward-oriented counterparts, and thus will not be presented in detail. Moreover, for sector-size-adjusted linkages one could formulate analogous problems to the **OPTIMIZATION PROBLEM NO 2** and **OPTIMIZATION PROBLEM NO 3.** One could also move one step forward and consider an extended optimization problem in which both the policy goal variable and the elements of Leontief matrix are allowed to change simultaneously in some reasonable bounds.

$$\left|\sum_{i,j=1}^{n} d_{ij}^{OUT,t}\right| \leq M_t^{OUT} \sum_{i,j=1}^{n} max\ (p_{ij}^{t,OUT-}, p_{ij}^{t,OUT+}), \tag{20}$$

$$P_t \ \leq \frac{\left\|\sum_{k=1}^{n} \pi_k^{IN,t}\left(l_{ki_0}^t + d_{ki_0}^{OUT,t}\right)\right\|}{\left\|\sum_{k=1}^{n} \pi_k^{IN,t}\left(l_{ki_0}^t\right)\right\|} \leq U_t \ , \tag{21}$$

where $0 \leq M_t^{OUT} \leq 1$ and $0 \leq P_t \ \leq U_t$ .

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The practical implementation of **Strategy 2** is based on employing formula (19), which gives a shift in the initial input matrix which is necessary to obtain a given modified Leontief matrix. In our case the modified Leontief matrix is simply equal to the solution to **OPTIMIZATION PROBLEM NO 3**. It is also important to note that one of the features of **Strategy 2** is that it influences both input and output linkages at the same time since both these types of linkage depend on the same Leontief (or Ghosh) matrix. Thus, in practical applications it is necessary to control the level of change of the linkage which is not included in the objective function (e.g. control the change of the $CO_2$ linkage in the case when **Strategy 2** is intended to maximize the income linkage). Such a control mechanism is included in constraint (21) which ensures that the corresponding input backward linkage for the $i_0$-th sector must be at least equal to $P_t$% of the initial input linkage but cannot exceed $U_t$% of the initial level.[26]

### 3.7. Multiple inputs and outputs

The methodology proposed provides a useful tool for a deep analysis of eco-efficiency in both demand- and supply-oriented generalized IO models. However, in the previous subsections the problems discussed were formulated and solved in a two-dimensional case in which a thorough and meaningful graphical analysis could be conducted for a single input (e.g. $CO_2$ emission) and output (e.g. income generation) policy goal variables. At the same time in empirical studies it is especially important to be aware of carefully choosing appropriate indices in order to attain appropriate conclusions and conduct sound economic policy (Palan, 2010). Thus, in the case of analysing multiple inputs and outputs we propose a two-step modification of the two-dimensional procedure. First, as in the traditional DEA approach the multiple inputs (outputs) are transformed into a single combined input (output) using a particular type of weighting scheme. Next, the two-dimensional approach described in detail in the previous subsections is used to conduct a linkage-based analysis of eco-efficiency for

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[26] In particular, $U_t = 100\%$ ensures that the modified input linkage will not exceed the original input linkage.

the combined input and combined output variables.

In order to shed some light on the multidimensional case let us focus on the problem of measuring backward eco-efficiency based on traditional linkages.[27] Let $\widehat{\boldsymbol{\pi}}_t^{OUT,j} = diag\left(\left(\pi_{t,k}^{OUT,j}\right)_{k=1,\dots,n}\right)$, where $j = 1,..,J$ stand for a $J$-element set of output policy goal variables at time point $t$ (e.g. income, employment, etc.). If one divides both sides of the demand-driven Leontief model constructed for the $j$-th output variable:

$$\mathbf{e}_t^j = \widehat{\boldsymbol{\pi}}_t^{OUT,j}\mathbf{L}_t\mathbf{f}_t, \tag{22}$$

by a scalar equal to the average value of the $j$-th output policy goal variable the following model is obtained:

$$\bar{\bar{\mathbf{e}}}_t^j = diag\left(\left\|\boldsymbol{\pi}_t^{OUT,j}\right\|\right)\mathbf{L}_t\mathbf{f}_t, \tag{23}$$

where:

$$\bar{\bar{\mathbf{e}}}_t^j = \frac{\mathbf{e}_t^j}{\frac{1}{n}\sum_{k=1}^n \pi_{t,k}^{OUT,j}}. \tag{24}$$

What is important, nevertheless, is that the physical units of $\mathbf{e}_t^j$ the vector $\bar{\bar{\mathbf{e}}}_t^j$ in (24) are expressed in monetary units while $\left\|\boldsymbol{\pi}_t^{OUT,j}\right\|$ is a dimensionless quantity. If we now define the combined output policy goal variable as:

$$\boldsymbol{\pi}_t^{OUT} = \sum_{j=1}^J \omega_j \left\|\boldsymbol{\pi}_t^{OUT,j}\right\|, \tag{25}$$

where weights $\left(\omega_j\right)_{j=1,\dots,J}$ satisfy the condition:

$$\sum_{j=1}^J \omega_j = 1, \tag{26}$$

and define the combined output as:

$$\mathbf{e}_t^{OUT} = \sum_{j=1}^J \omega_j \bar{\bar{\mathbf{e}}}_t^j, \tag{27}$$

we may examine the following generalized demand-driven Leontief IO model:

$$\mathbf{e}_t^{OUT} = \widehat{\boldsymbol{\pi}}_t^{OUT}\mathbf{L}_t\mathbf{f}_t, \tag{28}$$

by means of the two-dimensional approach presented in the previous subsections.

Since the weight $\omega_j$ measures the importance of the $j$-th output policy goal variable $\widehat{\boldsymbol{\pi}}_t^{OUT,j}$ ($j = 1,\dots,J$) in the overall output policy goal variable $\widehat{\boldsymbol{\pi}}_t^{OUT}$, they might be chosen on an arbitrary basis by the policy decision maker. Alternatively one may use non-subjective

[27] In a forward-linkage-oriented case as well as for sector-size-adjusted data the respective procedure is analogous to the backward-oriented scheme and thus will not be repeated here.

statistical methods, e.g. the OECD's approach to setting the weights in the multi-criteria rankings of importance proposed by Nicoletti et al. (2000).[28]

## 4. Illustrative example

### 4.1. The dataset

As mentioned in the introductory section, the second general goal of this paper is to present an illustrative empirical example in which the new approach for measuring eco-efficiency in gIO models challenges real data. We create a medium-scale example by using a dataset comprising Polish input-output (IO) tables. As already mentioned, one of key issues in the upcoming decades of transition in Poland is the need for an efficient and fast decarbonizing of the economy. Poland must focus on reducing greenhouse gas emissions but the country stands at a crossroads and soon will have to make a crucial strategic decision. It must choose whether to continue supporting an unprofitable and heavily polluting coal industry, shift to natural gas (which is mainly imported from Russia), or embrace clean technology that could both improve energy security and save thousands of human lives per year. The empirical part of the paper is partly intended to provide results which could help with finding the answer to this fundamental question.

The IO data used in this study comes from the World Input-Output Database (WIOD) 2013 Release and covers two of the most distinct time points available, i.e. 1995 and 2011. The IO tables are published by the WIOD in current prices, expressed in millions of dollars. In this paper we focus on the interrelations between 34 sectors of the Polish economy, thus all the IO tables used in the calculations are in an aggregation of $34 \times 34$.[29] Along with the matrices on interindustry flows the WIOD 2013 database offers access to sectoral data on income per unit of output, i.e. $\pi_{i,t}^{INC}$, where $i = 1, \ldots, 34$ and $t = 1995, 2009$, which we choose an as output policy goal variable.

In comparison to a more recent release (i.e. the WIOD 2016 Release) the WIOD 2013 Release provides a free access to a set of detailed environmental accounts including country-sector-specific data on industry energy use, $CO_2$ emissions and other types of emissions into the air.

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[28] The construction of the weights is based on the loadings and proportion of explained variance in the factor analysis conducted for the sequence $\left(\boldsymbol{\pi}_t^{OUT,j}\right)_{j=1,\ldots,J}$. An analogous procedure may be straightforwardly conducted for the Ghosh supply-driven model, thus we will not report it here in detail.

[29] In general, the national IO tables published by the WIOD 2013 are $35 \times 35$ in size. However, in the case of Poland there were no inflows and no outflows in the case of the sector *Private Households with Employed Persons* over the period 1995-2011. Thus, we excluded this sector from the empirical analysis and focused on the tables of size $34 \times 34$.

The environmental satellites are defined such as to cover the broadest range of environmental topics (Genty et al., 2012). Given the specificity of environmental challenges in the Polish economy discussed in Section 2, in this illustrative example we focus on a particular input policy goal variable, i.e. $CO_2$ emissions[30] per unit of output, denoted as $\pi_{i,t}^{CO_2}$, where $i = 1, \dots, 34$ and $t = 1995, 2009$.[31]

### 4.2. Direct sectoral indexes of eco-efficiency

To give a brief overview of the statistical data on the two policy goal variables we abandon presenting descriptive statistics in tabular form, but instead we focus on analysing the properties of direct sectoral eco-efficiency indexes, denoted as $EI_{i,t}$, where $i = 1, \dots, 34$ and $t = 1995, 2009$. For each sector we define this coefficient as a ratio of normalized output policy goal variable (i.e. income per unit of output) to normalized input policy goal variable (i.e. $CO_2$ emission per unit of output):

$$EI_{i,t} = \frac{\|\pi_{i,t}^{INC}\|}{\|\pi_{i,t}^{CO_2}\|}, \tag{29}$$

where $i = 1, \dots, 34$ and $t = 1995, 2009$. In Figure 2 we present the plots of the top 10 and bottom 10 sectoral indexes of $EI_{i,t}$ for $t = 1995, 2009$.

As one can see in both years examined the highest values of the direct eco-efficiency index were found for the sector *Real Estate Activities*. In 1995 this index exceeded a level of 70, indicating that relative to the global-economy-average the income generating potential of this sector was 70 times larger than its relative potential to generate $CO_2$ emissions. Interestingly this index almost halved during the period 1995-2009 which most likely was caused by the fact that during transition the profitability of the sector *Real Estate Activities* dropped. On the other hand, for both years examined the lowest level on the $EI_{i,t}$ index was reported for the sector *Electricity, Gas and Water Supply*. Since the value of this index was approximately equal to 0.07 it implies that relative to the global-economy-average the $CO_2$ generating potential of this sector was approximately 14 times as large as its relative potential to generate income.

[30] In the WIOD database the levels of $CO_2$ emissions are given in gigagrams (Gg).

[31] It is worth to mention that data on $\pi_{i,t}^{CO_2}$ available in the WIOD 2013 database is obtained by applying $CO_2$ emission coefficients to emission relevant energy use and then adding process-based emissions. Such a detailed data framework (as opposed to providing only aggregate $CO_2$ emissions per sector) is important if one wants to be able to simulate the environmental impact of energy mix changes, such as for instance of a substitution of gas for coal in the power sector (Genty et al., 2012).

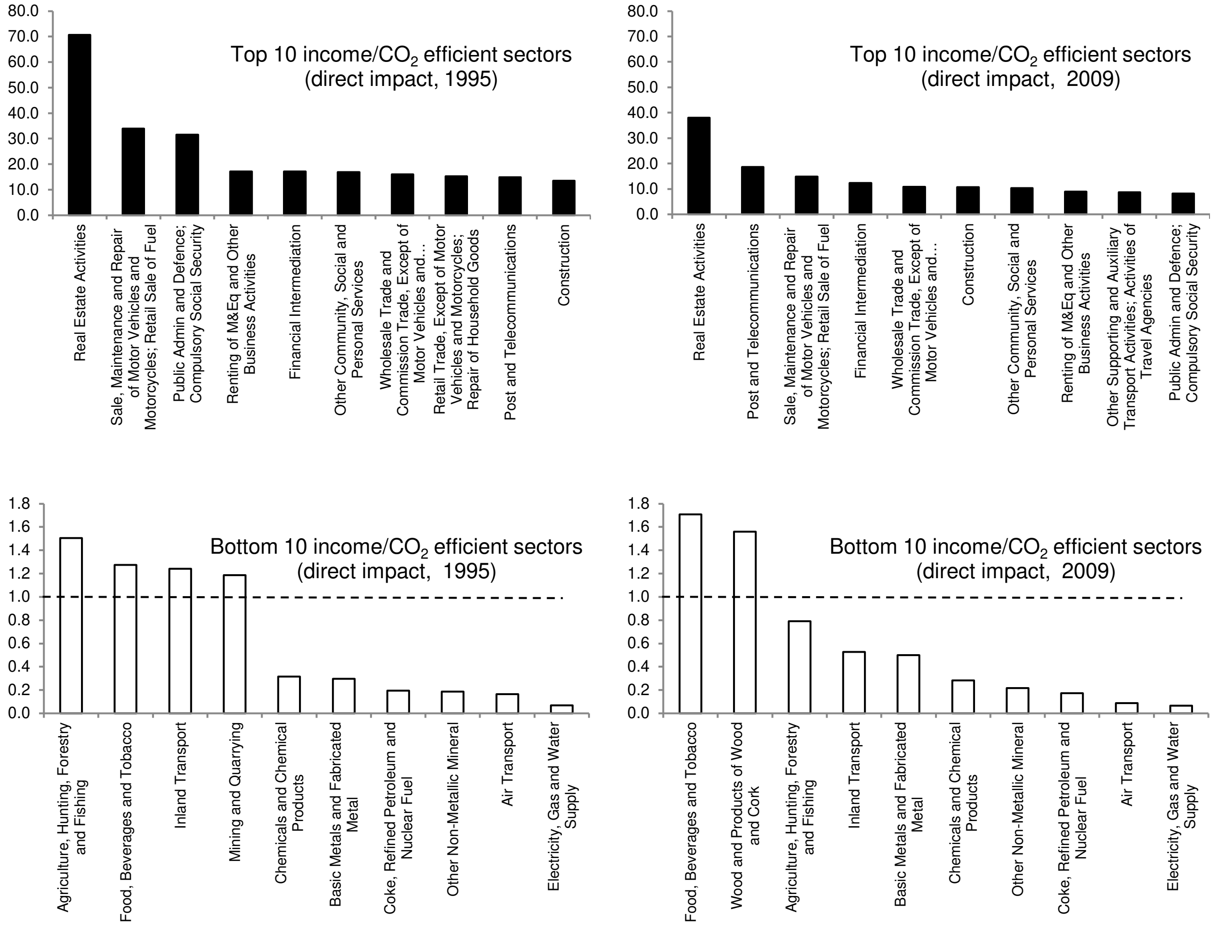


**Figure 2.** Top 10 and bottom 10 indexes of direct eco-efficiency.

**Notes:** Plots present sectoral data on direct efficiency measures defined as a ratio of normalized output policy goal variable (i.e. income per unit of output) to normalized input policy goal variable (i.e. $CO_2$ emission per unit of output).
**Source:** Own calculations based on WIOD 2013 Release.

## 4.3. Linkage-based analysis of eco-efficiency

The data presented in Figure 2 provides a basic framework for discussing the time evolution of the direct measures of the eco-efficiency of sectors operating in the Polish economy. However, the methodology presented in Section 3 provides tools which enable the sectoral eco-efficiency to be measured by taking into account crucial information on the indirect interindustry relations among the sectors. Thus, we calculated the forward and backward linkages for all sectors of the Polish economy using both the traditional as well as the sector-size-adjusted measures. Next, we calculated the linkage-based measures of eco-efficiency using (8) and (9) and classified the sectors on the basis of the scheme presented in Table 1. Table 2 presents the empirical results.

**Table 2.** Results of linkages-based analysis of eco-efficiency of sectors operating in Polish economy

| | 1995 | | | | Change between 1995 and 2009 | | | |
|---|---|---|---|---|---|---|---|---|
| | Traditional | | Sector-size-adjusted | | Traditional | | Sector-size-adjusted | |
| | Backward | Forward | Backward | Forward | Backward | Forward | Backward | Forward |
| Agriculture, Hunting, Forestry and Fishing | EFF(1.36) | EFF(1.69) | W(3.7) | EFF(3.59) | ->KEY(0.94) | ->KEY(1.09) | | |
| Mining and Quarrying | EFF(1.08) | KEY(0.54) | W(3.73) | EFF(1.45) | | | | ->KEY(0.56) |
| Food, Beverages and Tobacco | W(1.35) | W(1.53) | EFF(4.35) | W(3.83) | | | | ->EFF(2.4) |
| Textiles and Textile Products | W(1.46) | W(2.01) | W(2.81) | W(3.02) | | | | |
| Leather, Leather and Footwear | W(1.6) | W(2.37) | EFF(2.26) | W(2.61) | | | ->W(1.17) | |
| Wood and Products of Wood and Cork | EFF(1.11) | W(1.86) | EFF(2.21) | EFF(2.89) | ->W(1.16) | | | |
| Pulp, Paper, Printing and Publishing | W(1.42) | EFF(1.72) | W(3.6) | EFF(3.39) | | | ->EFF(2.4) | |
| Coke, Refined Petroleum and Nuclear Fuel | INEFF(0.46) | INEFF(0.39) | EFF(9.78) | EFF(6.43) | | | | |
| Chemicals and Chemical Products | INEFF(0.58) | INEFF(0.51) | EFF(7.58) | EFF(5.19) | | | | |
| Rubber and Plastics | W(1.34) | EFF(1.37) | EFF(1.96) | EFF(1.56) | | ->W(2.35) | ->KEY(1.05) | |
| Other Non-Metallic Mineral | INEFF(0.34) | KEY(0.38) | EFF(7.57) | EFF(6.55) | | | ->W(5.16) | |
| Basic Metals and Fabricated Metal | INEFF(0.48) | INEFF(0.49) | EFF(6.6) | EFF(5.24) | | | | |
| Machinery, Nec | W(1.11) | EFF(1.1) | W(2.22) | EFF(1.72) | | ->W(2.54) | ->KEY(0.94) | ->W(0.88) |
| Electrical and Optical Equipment | W(1.45) | W(1.65) | EFF(1.33) | EFF(1.18) | | | ->KEY(0.88) | ->W(0.83) |
| Transport Equipment | W(1.06) | W(1.53) | EFF(2.36) | W(2.64) | | | ->KEY(0.88) | |
| Manufacturing, Nec; Recycling | W(1.47) | W(2.16) | EFF(1.19) | W(1.36) | | | ->KEY(1.09) | |
| Electricity, Gas and Water Supply | INEFF(0.14) | KEY(0.15) | W(8.22) | EFF(7.2) | ->KEY(0.14) | | | |
| Construction | W(1.61) | W(3.08) | INEFF(0.49) | INEFF(0.73) | | | ->KEY(0.59) | ->KEY(0.37) |
| Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel | EFF(2.01) | EFF(2.22) | INEFF(0.24) | INEFF(0.21) | | | ->W(0.83) | |
| Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles | EFF(3.01) | EFF(2.14) | INEFF(0.78) | INEFF(0.43) | | | ->W(0.9) | |
| Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods | EFF(2.39) | EFF(2.39) | INEFF(0.64) | INEFF(0.5) | | | ->W(1.09) | |
| Hotels and Restaurants | EFF(1.76) | EFF(2.73) | INEFF(0. | W(0.97) | | | ->W(1.27) | |
| Inland Transport | EFF(1.09) | KEY(1.14) | W(3.6) | EFF(2.94) | ->INEFF(0.71) | | | |
| Water Transport | W(1.46) | W(1.88) | KEY(1.55) | W(1.55) | | | | |
| Air Transport | INEFF(0.46) | INEFF(0.26) | EFF(11.47) | W(5.04) | | | | |
| Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies | EFF(1.83) | W(1.61) | EFF(1.2) | KEY(0.82) | ->W(1.69) | | ->KEY(0.61) | |
| Post and Telecommunications | EFF(2.88) | EFF(2.8) | W(0.8) | KEY(0.6) | | | ->INEFF(0.76) | |
| Financial Intermediation | EFF(4.71) | EFF(3.38) | W(1.13) | INEFF(0.63 | | | | |
| Real Estate Activities | EFF(1.32) | EFF(2.98) | INEFF(0.08) | INEFF(0.13) | ->KEY(0.98) | | | |
| Renting of M&Eq and Other Business Activities | EFF(2.62) | EFF(2.32) | INEFF(0.63) | KEY(0.43) | | | ->W(1.33) | |
| Public Admin and Defence; Compulsory Social Security | EFF(3.92) | W(34.56) | INEFF(0.51) | W(3.51) | | ->EFF(5.73) | ->W(1.73) | |
| Education | EFF(3.65) | EFF(9.01) | W(1.69) | W(3.25) | | | | |
| Health and Social Work | EFF(2.36) | W(5.88) | W(1.68) | W(3.27) | | ->EFF(5.46) | | |
| Other Community, Social and Personal Services | EFF(1.79) | W(5.55) | INEFF(0.4 | W(1.05) | | ->EFF(2.85) | | ->INEFF(0.62) |

**Notes:** Sectoral classification is based on the definitions given in Table 1. Numbers in brackets represent the values of the linkage-based measures of eco-efficiency defined in (8) and (9). In the first four columns referring to the 1995 IO table we use shading to indicate eco-efficient sectors (abbreviation EFF, grey shading) and eco-inefficient sectors (INEFF, black). No shading was used for BI-WEAK (abbreviation W) and BI-KEY (abbreviation KEY) sectors. In the four columns referring to classification change between 1995 and 2009 we use black shading to indicate the sectors losing their status of eco-efficient, grey shading to indicate sectors gaining ECO-EFF status and black framing to indicate sectors losing their ECO-INEFF status.
**Source:** Own calculations based on WIOD 2013 Release.

As one can see, the values of the linkage-based measures of eco-efficiency defined in (8) and (9) do not duplicate the results of the sectoral classification based on the general scheme outlined in Table 1. In other words, the measures of eco-efficiency defined in (8) and (9) provide supplementary information with respect to the results of the sectoral classification. In particular, even some BI-WEAK sectors may be characterized by high values of the linkage-based measure of eco-efficiency. For example, in 1995 the sector *Public Admin and Defence; Compulsory Social Security* was classified as a BI-WEAK sector, but at the same time it was characterized by a traditional forward eco-efficiency measure equal to 34.56. Both these facts imply that in the case of this sector the traditional income forward linkage and the traditional $CO_2$ forward linkage were both below the respective economy-wide-averages (BI-WEAK class), but the income linkage was 34.56 times as close to the respective economy-wide-average as the corresponding $CO_2$ linkage.

Moreover, the results of the linkage-based analysis of the eco-efficiency of the sectors operating in the Polish economy seem to strongly depend on the chosen measure of the intersectoral linkages. This is not surprising given the arguments presented by Temurshoev (2016) and Gurgul and Lach (2018a), who stress that each of the two measures of the two types of intersectoral linkage used in this empirical example has its own merit as far as economic interpretation is concerned. And so, one can see that on the basis of the 1995 data several sectors characterized by low levels of direct eco-efficiency (comp. Figure 2) and high levels of aggregated $CO_2$ emission (e.g. *Coke, Refined Petroleum and Nuclear Fuel*; *Chemicals and Chemical Products*; *Other Non-Metallic Mineral*; *Basic Metals and Fabricated Metal*; *Electricity, Gas and Water Supply* and *Air Transport*) were found to be eco-inefficient with respect to the traditional linkages. On the contrary, sectors characterized by relatively low levels of $CO_2$ emission per unit of output (e.g. *Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel*, *Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles*; *Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods* and *Real Estate Activities*) were found to be eco-efficient with respect to at least one of the traditional linkages.

Interestingly, an almost inverted sectoral classification was obtained for sector-size-adjusted linkages based on the 1995 data. To some extent the latter follows directly from the definitions of forward and backward sector-size-adjusted linkages given in (5) and (6). The lower the level of $CO_2$ emission per unit of output the larger the sector-size-adjusted forward and backward linkages. As a consequence the sectors classified as ECO-EFF with respect to the traditional linkages were usually classified as ECO-INEFF with respect to the sector-size-adjusted linkages, and vice versa. However, these results cannot be said to be contradictory given the fundamental differences between the aims and scopes of the traditional and the sector-size-adjusted measures.

After the two first decades of economic transformation, the set of eco-efficient sectors in the Polish economy underwent a change. This change was observed for both traditional and sector-size-adjusted measures. When it comes to a comparison of the eco-efficiency-oriented sectoral classification in the Polish economy in 1995 and 2009 one notices several important facts. First, a dominating pattern of change was a situation when a sector lost its ECO-EFF status. This was mainly the case for sector-size-adjusted backward linkages. On the other hand, a significant number of trade- and service-related sectors lost the status of ECO-INEFF with respect to sector-size-adjusted backward linkages.[32]

In the context of analysing the properties of eco-efficiency measures listed in Table 2 it is interesting to study the relationship between income and $CO_2$ linkages for all four types of measure examined. Following the general discussion on the nature of such relationships presented in Section 3 (comp. e.g. Figure 1) in Figure 3 we present the respective scatterplots along with fitted linear regression lines.

The most evident regularity shown in the two upper panel plots in Figure 3 is the fact that the sector *Electricity, Gas and Water Supply* is characterized by the largest traditional backward and forward $CO_2$ linkages. As a consequence, removing the sector from the sample had a significant impact on the regression slope coefficient. Interestingly, notwithstanding the sample used to estimate the regression line, one can see that for traditional backward linkages the slope coefficients are negative (although not statistically significant at a level of 5% in any sample-selection variant). In other words, the scatterplot exhibits a pattern similar to that presented in the upper left plot in Figure 1. On the contrary, notwithstanding the sample used to estimate the regression line, one can see that for traditional forward linkages the slope coefficients are slightly positive (but also not statistically significant at a level of 5% for the samples analysed). In other words, the scatterplots for traditional forward linkages exhibit a pattern similar to that presented in the upper right plot in Figure 1.

[32] Technically, this is not surprising given the fact that the economy-wide average for each type of normalized linkage is equal to unity and at the same time the number of sectors which lost their ECO-EFF status was relatively large.

Similarly, the most evident regularity shown in the two bottom panel plots in Figure 3 is the fact that the sector *Real Estate Activities* is characterized by the largest sector-size-adjusted backward and forward $CO_2$ linkages. The respective slope coefficients are either negative (sector-size-adjusted backward linkages) or oscillate around zero (sector-size-adjusted forward linkages).

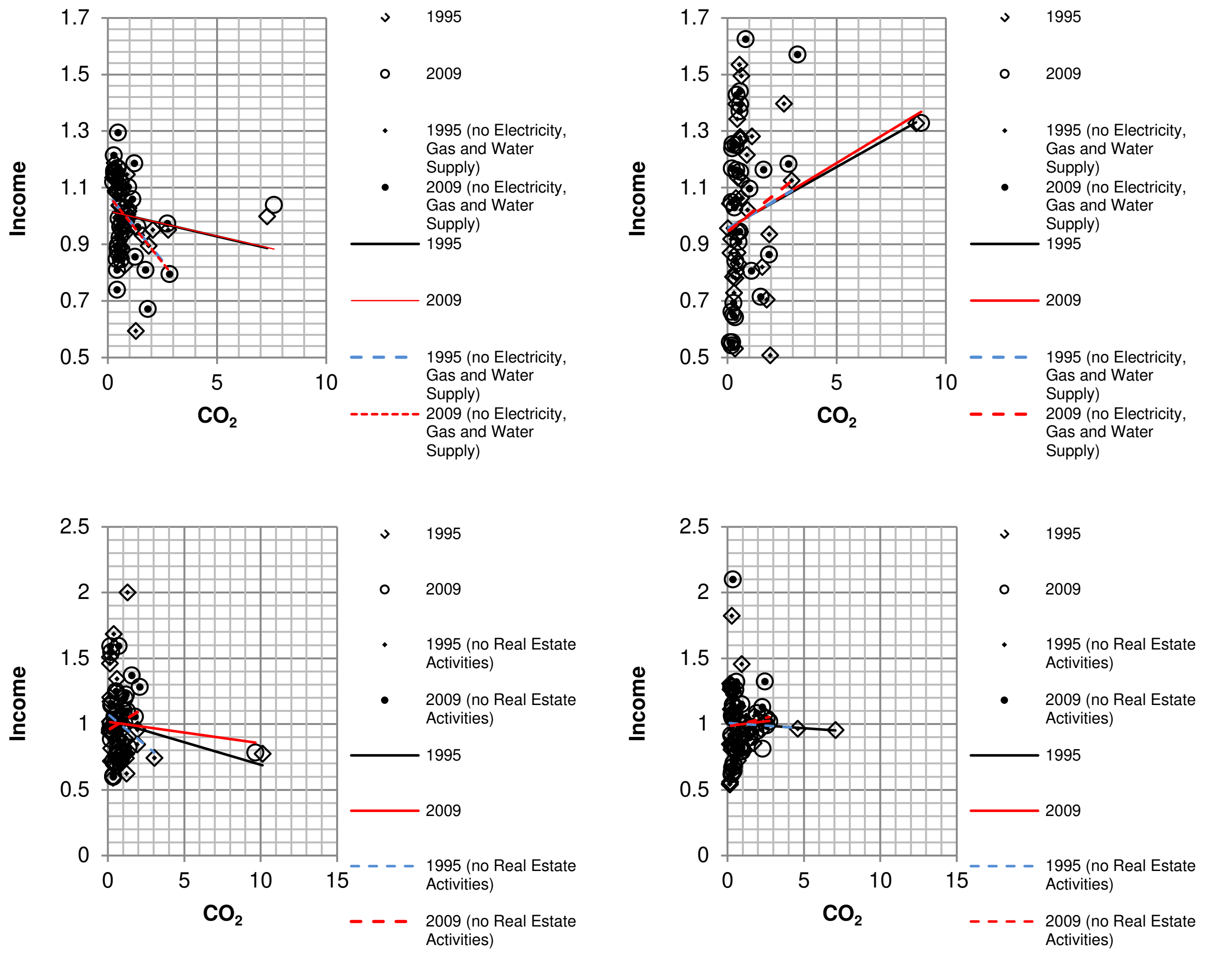


**Figure 3.** Scatterplots of normalized income and $CO_2$ linkages in Polish economy.

**Notes:** The plots present sectoral data on normalized income and $CO_2$ linkages in the Polish economy (top left corner – traditional backward linkages, top right corner – traditional forward linkages, bottom left corner – sector-size-adjusted backward linkages, bottom right corner – sector-size-adjusted forward linkages). The plots contain linear regression lines fitted to the respective datasets of the linkages (i.e. complete annual datasets (solid lines) and the datasets with the sectors with highest $CO_2$-related linkages removed from the samples (dashed lines)).
**Source:** Own calculations based on WIOD 2013 Release.

Since one does not notice any statistically significant relationship between the income and $CO_2$ linkages in scatterplots in Figure 3, this proves that in both years examined the sectors characterized by abnormal $CO_2$ emissions did not generate above-average income levels. This conclusion refers to both traditional and sector-size-adjusted linkages calculated for the demand- and supply-oriented models. In other words, even after excluding outliers from the analysis, the results suggest that the number of eco-inefficient sectors operating in the Polish economy may have risen in the period 1995-2009.

An interesting question is what changes in the linkages are required to transform the plots in Figure 3 towards the eco-optimal pattern shown in the bottom plot of Figure 1. In order to illustrate how the methodology outlined in Section 3 could help to answer this important question, in the next subsection we will focus on the case of eco-optimizing the distribution of traditional backward linkages in the Polish economy.

### 4.4. Formulating general sectoral policy implications

As mentioned in Section 3 one of the analytical methods for finding the eco-optimal distribution of linkages for a given type of intersectoral measure is to solve a respective variant of **OPTIMIZATION PROBLEM NO 1.** In this small illustrative example we focus on optimizing the distribution of traditional backward linkages in the Polish economy on the basis of the 2009 data. We focus on this particular type of linkage because of three main reasons. First, traditional linkages take the sector size into account which implies that modifying this type of measure may have a much more significant impact on the economy's overall $CO_2$ emissions than changing the sector-size-adjusted measures. Secondly, in the case of the traditional backward linkages the actual scatterplot for the 2009 dataset is characterized by a negative slope coefficient. Finally, for this type of linkage we did not report any positive shift in the slope coefficient between 1995 and 2009, which makes room for some sort of policy recommendation.

In order to solve **OPTIMIZATION PROBLEM NO 1** we used a number of General Algebraic Modeling System (GAMS) solvers dedicated to finding global solutions to non-linearly constrained discontinuous optimization problems.[33] All the computations were conducted using the NEOS Server.[34] After solving the optimization problem we obtained a maximized value of the objective function (i.e. the *ECO_SCORE* function) and a list of corresponding changes in the two sequences of linkages. In Figure 4 we present the report on the change in the objective function (*ECO_SCORE*) and detailed information on the numbers of different

[33] Following Gurgul and Lach (2018c) we assumed a ±10% interval for maximal changes in all linkages and a 70% threshold level , i.e. we assumed that $l_i^{OUT-} = l_i^{OUT+} = 10\% \times BACK_{i,t}(\boldsymbol{\pi}^{OUT,t})$, $l_i^{IN-} = l_i^{IN+} = 10\% \times BACK_{i,t}(\boldsymbol{\pi}^{IN,t})$ and $M_t^{OUT} = M_t^{IN} = 70\%$, where $t = 2009$.

[34] NEOS Server (http://www.neos-server.org) is a free internet-based service for solving numerical optimization problems. Hosted by the Wisconsin Institutes for Discovery at the University of Wisconsin in Madison, the NEOS Server provides access to more than 60 state-of-the-art solvers in more than a dozen optimization categories. The NEOS Server offers a variety of interfaces for accessing the solvers, and jobs run on distributed high-performance machines enabled by the HTCondor software. For more details see http://www.neos-server.org.

types of sector in the actual and modified set of linkages along with a scatterplot of modified linkages.

The *ECO_SCORE* function calculated on the basis of the solution to **OPTIMIZATION PROBLEM NO 1** turned out to be larger than the *ECO_SCORE* for the original 2009 dataset by 7 points. The latter follows mainly from the fact that in comparison to the sectoral classification based on the actual 2009 data the number of ECO-INEFF sectors for the modified set of linkages dropped by 6 while the number of ECO-EFF sectors grew by 5 at the same time. As can be seen in the scatterplot in Figure 4 the largest shifts in traditional backward and forward linkages were reported in the case of the sector *Electricity, Gas and Water Supply*. In Table 3 we present detailed results of solving the traditional-backward-linkage-oriented variant of **OPTIMIZATION PROBLEM NO 1** with nominal and relative changes in both types of linkages reported.

| | ECO-INEFF | BI-KEY | BI-WEAK | ECO-EFF | ECO_SCORE |
|---|---|---|---|---|---|
| 2009 (actual) | 6 | 3 | 13 | 12 | 40 |
| 2009 (modified) | 4 | 6 | 7 | 17 | 47 |

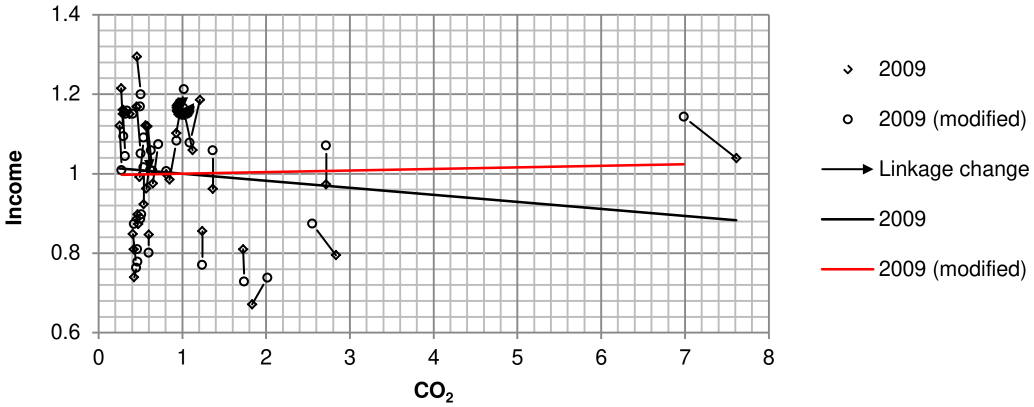


**Figure 4.** Sectoral classification and scatterplots of normalized traditional backward income-$CO_2$ linkages – original data and the results of solving **OPTIMIZATION PROBLEM NO 1**.

**Notes:** The values in the table refer to the numbers of different types of sectors in the Polish economy on the basis of actual and modified distributions of normalized traditional backward income-$CO_2$ linkages. The values of the *ECO_SCORE* function are given in the first column from the right. The plots present 2009 sectoral data on normalized traditional income-$CO_2$ backward linkages in the Polish economy along with respective modified linkages obtained after solving **OPTIMIZATION PROBLEM NO 1**. Solid lines represent linear regression models fitted to the respective datasets, while arrows indicate the magnitudes and directions of linkage change in the two-dimensional plane.
**Source:** Own calculations based on WIOD 2013 Release.

Before analysing the values presented in Table 3 one should recall an important feature of normalized linkages. Since the average value of a set of any type of normalized linkage is always equal to unity one may claim that increasing the linkages of some sectors is always accompanied by a lowering in the values of linkages of some other sectors.

**Table 3.** Results of solving the traditional-backward-linkage-oriented variant of **OPTIMIZATION PROBLEM NO 1**

| | Nominal change of $CO_2$ linkage | Nominal change of income linkage | Relative change of $CO_2$ linkage | Relative change of income linkage |
|---|---|---|---|---|
| Agriculture, Hunting, Forestry and Fishing | -0.112 | 0.106 | <-90% | >90% |
| Mining and Quarrying | 0.093 | 0.11 | >90% | >90% |
| Food, Beverages and Tobacco | 0 | 0.008 | 0.00% | 8.02% |
| Textiles and Textile Products | -0.047 | 0 | <-90% | 0.00% |
| Leather, Leather and Footwear | 0.045 | 0 | >90% | 0.00% |
| Wood and Products of Wood and Cork | 0.085 | 0.098 | >90% | >90% |
| Pulp, Paper, Printing and Publishing | 0.065 | 0.098 | >90% | >90% |
| Coke, Refined Petroleum and Nuclear Fuel | 0.183 | 0.067 | >90% | >90% |
| Chemicals and Chemical Products | 0.014 | -0.081 | 8.12% | <-90% |
| Rubber and Plastics | 0 | -0.046 | 0.00% | -54.35% |
| Other Non-Metallic Mineral | 0 | 0.097 | 0.00% | >90% |
| Basic Metals and Fabricated Metal | 0 | -0.085 | 0.00% | <-90% |
| Machinery, Nec. | 0.046 | 0 | >90% | 0.00% |
| Electrical and Optical Equipment | 0.042 | 0 | >90% | 0.00% |
| Transport Equipment | 0.042 | 0.039 | >90% | 52.77% |
| Manufacturing, Nec; Recycling | 0 | 0.092 | 0.00% | >90% |
| Electricity, Gas and Water Supply | -0.627 | 0.154 | <-90% | >90% |
| Construction | 0.049 | 0.099 | >90% | >90% |
| Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel | 0.029 | 0 | >90% | 0.00% |
| Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles | 0.037 | 0 | >90% | 0.00% |
| Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods | 0.045 | 0 | >90% | 0.00% |
| Hotels and Restaurants | 0.058 | -0.112 | >90% | <-90% |
| Inland Transport | 0 | 0.096 | 0.00% | >90% |
| Water Transport | 0.041 | -0.085 | >90% | -<-90% |
| Air Transport | -0.283 | 0.079 | <-90% | >90% |
| Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies | 0.057 | 0.096 | >90% | >90% |
| <-90%Post and Telecommunications | 0.025 | -0.112 | >90% | <-90% |
| Financial Intermediation | 0.029 | -0.116 | >90% | <-90% |
| Real Estate Activities | -0.121 | -0.108 | <-90% | <-90% |
| Renting of M&Eq and Other Business Activities | 0.031 | 0 | >90% | 0.00% |
| Public Admin and Defence; Compulsory Social Security | 0.027 | -0.121 | >90% | <-90% |
| Education | 0.046 | -0.095 | >90% | -73.42% |
| Health and Social Work | 0.046 | -0.117 | >90% | <-90% |
| Other Community, Social and Personal Services | 0.056 | -0.112 | >90% | <-90% |

**Note:** In the first two columns which refer to the nominal changes in the backward linkages we use shading to indicate the top 5 (black) and bottom 5 (grey) changes. In the two columns that refer to the percentage changes the nominal linkage changes are given relative to the maximal positive change (e.g. the value "52%" means that the change in linkage was positive and reached a level of 52% of the maximal possible change). In this case we use shading to indicate above-90% levels (black) and levels below -90% (grey) of the relative changes in linkages.
**Source:** Own calculations based on WIOD 2013 Release.

Thus, from the point of view of policymaking one should carefully interpret two particular outcomes of solving one of the three optimization problems defined in Section 3: increasing

$CO_2$ linkages and lowering income linkages. A recommendation to increase the normalized $CO_2$ linkage of a particular sector *j* does not imply the need to taking measures in order to (irrationally) increase the $CO_2$ generating potential of this sector. It only implies that there are some other sectors in an economy for which $CO_2$ linkages should be reduced first. As a consequence, the relative $CO_2$ linkage of sector *j* will rise although no actions were taken with its nominal levels of $CO_2$ generation. Similarly, lowering the relative income linkage of sector *j* impies that there are some other sectors in an economy whose relative income linkages should be improved first.

Taking these facts into account one should focus on listing the sectors for which easily-interpretable policies for increasing income linkages or lowering $CO_2$ emissions are recommended. And so the results presented in Table 3 suggest that in order to increase the *ECO_SCORE* of the Polish economy with respect to traditional backward linkages one should implement policies intended to reduce the $CO_2$ backward linkage in the case of the sectors *Electricity, Gas and Water Supply*; *Textiles and Textile Products* and *Air Transport*, among others. At the same time one may list the sectors (e.g. *Mining and Quarrying*; *Wood and Products of Wood and Cork*; *Pulp, Paper, Printing and Publishing*) in the case of which reducing the $CO_2$ backward linkage is not a priority from the point of view of increasing the *ECO_SCORE* objective. Similarly, increasing the *ECO_SCORE* of the Polish economy with respect to traditional backward linkages requires policies intended to increase the income backward linkage in the case of the sectors *Electricity, Gas and Water Supply* and *Agriculture, Hunting, Forestry and Fishing*, among others to be implemented. At the same time one may list sectors (mainly service-related, e.g. *Financial Intermediation*, *Hotels and Restaurant*) in the case of which increasing the income backward linkage is not a priority given the general goal of increasing the *ECO_SCORE* value.

### 4.5. Formulating sector-specific policy implications - a case study

As stressed in Section 3 an important problem is the practical implementation of the solution to **OPTIMIZATION PROBLEM NO 1**. In order to translate the solution to **OPTIMIZATION PROBLEM NO 1** into a set of practical policy recommendations two basic strategies were formulated. The first set of recommendations, named **Strategy 1**, was aimed at modifying the respective policy goal variable. In order to give an illustrative example of implementing the strategy, let us focus on a particular case of decreasing the traditional $CO_2$ linkage of the sector *Electricity, Gas and Water Supply* (henceforth abbreviated as *EGWS*). The motivation to focus on the traditional $CO_2$ linkage of this particular sector is threefold. First, according to the solution to **OPTIMIZATION PROBLEM NO 1** the recommended nominal decrease in traditional $CO_2$ backward linkage in the case of this sector reached the highest nominal level among all the sectors of the Polish economy. Secondly, as shown in Figure 5, in 2009 the sector *EGWS* was characterized by the largest $CO_2$-emission-to-output ratio values.

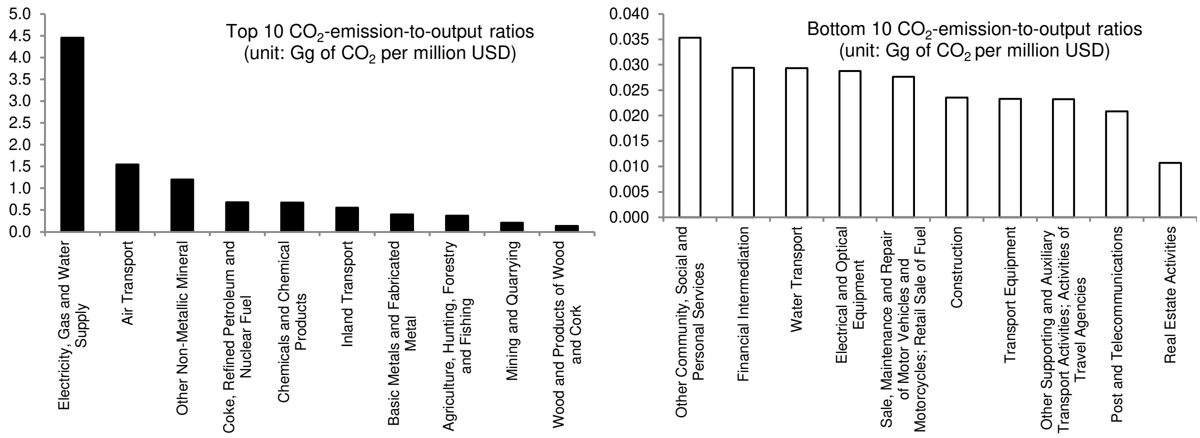


**Figure 5.** Top 10 and bottom 10 sectoral $CO_2$-emission-to-output ratios in 2009.

**Notes:** The plots present sectoral data on the top 10 and bottom 10 $CO_2$-emission-to-output ratios. The data is expressed in gigagrams (Gg) of $CO_2$ emission per million USD of output.
**Source:** Own calculations based on WIOD 2013 Release

The latter implies that this sector will play a dominant role in shaping the backward and forward linkages of the remaining sectors of the Polish economy in the gIO models (comp. the role of the coefficient $\pi_{EGWS,2009}^{CO_2}$ in formulas (3)-(6)). Finally, one cannot forget that electricity generation is responsible for the lion's share of greenhouse gas emissions in Poland, making up approximately 40% of the country's overall emissions. Thus, the power sector, in which emissions originate predominately from coal power plants, should decarbonize in as short a period as possible. The latter could be accomplished by replacing the old plants with newer coal and gas plants, and supporting the installation of solar plants.

Below we will present the results of solving the particular modified[35] variant of **OPTIMIZATION PROBLEM NO 2,** intended to minimize the traditional input backward linkage of the sector *EGWS*.[36]

[35] Unlike the original formulation of **OPTIMIZATION PROBLEM NO 2** presented in Section 3 the modified optimization problem is aimed at minimizing the traditional input linkage (i.e. the $CO_2$ linkage) of the sector $EGWS$. As a consequence, the technical parameters of the modified problem are denoted $l_{EGWS}^{2009,IN-}$, $l_{EGWS}^{2009,IN+}$ (sequences of bounds) and $M_{2009}^{IN}$ (threshold level).

[36] Following Gurgul and Lach (2018c) we assumed a 10% lower bound for maximal drops in all elements of the vector of input policy goal variable and a 50% threshold level.

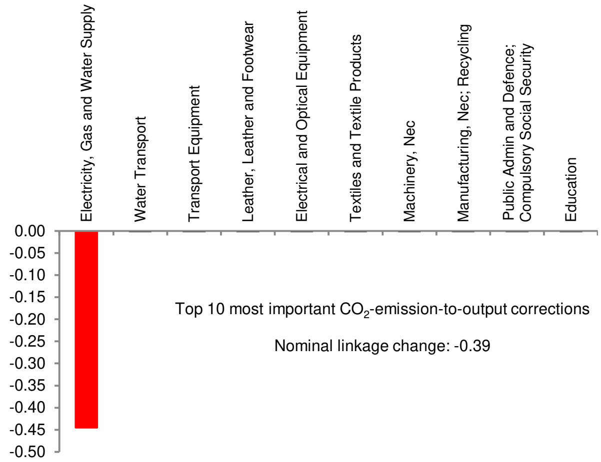


**Figure 6.** The results of implementing **Strategy 1** for the sector *Electricity, Gas and Water Supply*.

**Notes:** The plots present sectoral data on the top 10 most important changes in the $CO_2$-emission-to-output ratios obtained after solving **OPTIMIZATION PROBLEM NO 2** aimed and minimizing the traditional backward linkage of the sector *Electricity, Gas and Water Supply* (i.e. implementing **Strategy 1**) via changing the elements of the vector of the input policy goal variable. The data is expressed in gigagrams (Gg) of $CO_2$ emission per million USD of income.
**Source:** Own calculations based on WIOD 2013 Release

As one can see, implementing this particular variant of **Strategy 1** for the sector *EGWS* allowed the sectoral $CO_2$ linkage to be reduced (from a level of 7.61 to 7.22). At the same time the intuitive policy of reducing the sectoral coefficient of $CO_2$ emission per unit of output turned out to be an especially useful one, as the biggest drop of $CO_2$-emission-to-output ratio was reported precisely for the sector *EGWS.*

Next we followed a procedure for implementing the alternative strategy intended to find the modifications of the input matrix required to reduce the backward $CO_2$ linkage of the sector *EGWS*. The results of solving the respective optimization problem take the form of matrix $\Delta\mathbf{A}_t$. This way one obtains recommendations regarding which input coefficients should be respectively modified by changing the flows from source to destination sectors. The results of implementing **Strategy 2**, i.e. the solution to the particular modified[37] variant of **OPTIMIZATION PROBLEM NO 3** are summarized in Table 4.[38]

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[37] Unlike the original formulation of **OPTIMIZATION PROBLEM NO 3** presented in Section 3 the modified optimization problem is aimed at minimizing the traditional input linkage (i.e. the $CO_2$ linkage) of the sector *EGWS*. As a consequence, the technical parameters of the modified problem are denoted $p_{ij}^{2009,\mathrm{IN}-}, p_{ij}^{2009,IN+}$ (matrices od bounds), $M_{2009}^{IN}$ (threshold level) and $P_{2009}, U_{2009}$ (income linkage bounds), where $i,j = 1,\dots,34$.

[38] Following Gurgul and Lach (2018c) we assumed a ±5% interval for maximal changes in all elements of the initial Leontief inverse, and a 50% threshold level. Finally, we assumed that the modified output linkage must be at least as large as the original one. To summarize, we assumed that $p_{ij}^{2009,IN-} = p_{ij}^{2009,IN+} = 5\% \times l_{ij}^{2009,}$, $M_{2009}^{IN} = 50\%$, and $P_{2009} = 100\%$.

**Table 4.** The results of implementing **Strategy 2** aimed at reducing the $CO_2$ traditional linkage of the sector *Electricity, Gas and Water Supply*.

**Top 5 nominal decreases in IO coefficients**

| **Source sector** | **Destination sector** |
|---|---|
| *Electricity, Gas and Water Supply* | *Electricity, Gas and Water Supply* |
| *Machinery, Nec.* | *Machinery, Nec.* |
| *Leather, Leather and Footwear* | *Leather, Leather and Footwear* |
| *Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles* | *Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles* |
| *Agriculture, Hunting, Forestry and Fishing* | *Food, Beverages and Tobacco* |

**Top 5 nominal increases in IO coefficients**

| **Source sector** | **Destination sector** |
|---|---|
| *Air Transport* | *Air Transport* |
| *Inland Transport* | *Inland Transport* |
| *Construction* | *Electricity, Gas and Water Supply* |
| *Wood and Products of Wood and Cork* | *Wood and Products of Wood and Cork* |
| *Electricity, Gas and Water Supply* | *Real Estate Activities* |

**Notes:** This table presents interindustry flow data on the most important changes in input coefficients obtained after solving **OPTIMIZATION PROBLEM NO 3** aimed at minimizing the traditional backward linkage of the sector *Electricity, Gas and Water Supply* (i.e. implementing **Strategy 2**) via changing the elements on the Leontief inverse.
**Source:** Own calculations based on WIOD 2013 Release.

Implementing this particular variant of **Strategy 2** for the sector *EGWS* allowed the sector's traditional $CO_2$ linkage to be reduced from a level of 7.61 to 7.09. This significant decrease in the $CO_2$ backward linkage was mainly caused by increasing the IO intraefficiency of the sector *EGWS*.[39] However, one cannot forget that although **Strategy 2** led to a larger reduction in the $CO_2$ backward linkage compared to **Strategy 1**, it also influences the structure of the corresponding income linkage of *EGWS*. However, setting the parameter $P_{2009}$ at a level of 100% ensures that the modified output linkage cannot be smaller than the original one, thus in the modified scenario the income generating potential of the sector *EGWS* will not drop.

As the last part of this empirical analysis let us give an example of the role of the sector *Electricity, Gas and Water Supply* in shaping the overall structure of linkages in the Polish economy. And so, if we implemented **Strategy 1** (with the same set of technical parameters as in the previous case) to the sector *Leather, Leather and Footwear* we would obtain a 19% decrease in the traditional $CO_2$ linkage after taking into account corrections to the input policy goal vector listed in Figure 7.

[39] Increasing IO intraefficiency is understood here as decreasing sector *EGWS* use of goods produced in this sector. For more details on this particular issue see Gurgul and Lach (2018c).

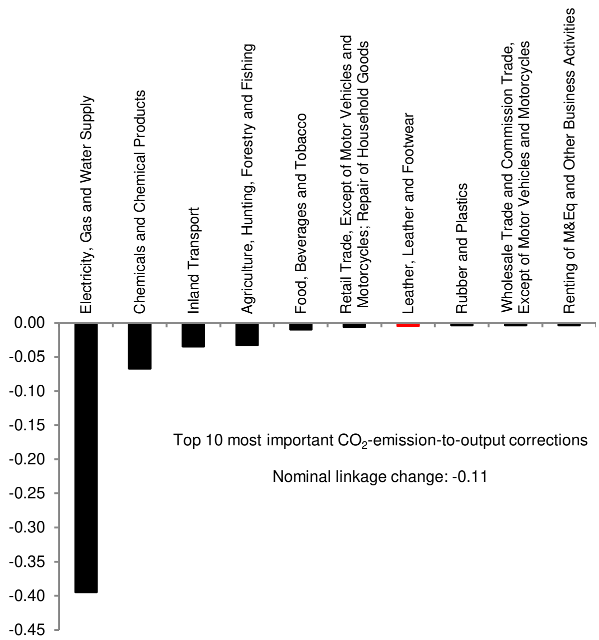


**Figure 7.** The results of implementing **Strategy 1** for the sector *Leather, Leather and Footwear*.

**Notes:** The plots present sectoral data on the top 10 most important changes in the $CO_2$-emission-to-output ratios obtained after solving the modified variant of **OPTIMIZATION PROBLEM NO 2** aimed at minimizing the traditional backward linkage of the sector *Leather, Leather and Footwear* (i.e. implementing **Strategy 1**) via changing the elements of the vector of input policy goal variable. The data is expressed in gigagrams (Gg) of $CO_2$ emission per million USD of income.
**Source:** Own calculations based on WIOD 2013 Release.

Despite the low nominal value of the absolute change in the $CO_2$-emission-to-output ratio the intuitive policy of reducing the intrasectoral coefficient of $CO_2$ emissions per unit of output also turned out to be a useful strategy in this case. The latter follows from the fact that the decrease in the $CO_2$-emission-to-output ratio for the sector *Leather, Leather and Footwear* reached the lowest possible value, i.e. 10% of the actual sectoral value of the input policy goal variable. Even more interestingly, lowering the traditional backward $CO_2$ linkage of the sector *Leather, Leather and Footwear* by implementing **Strategy 1** would require an almost 90% reduction in the $CO_2$-emision-to-output coefficient of the sector *Electricity, Gas and Water Supply*. The latter proves that lowering the $CO_2$ emissions of *Electricity, Gas and Water Supply* is of strategic importance for the eco-efficiency of many sectors of the Polish economy and should thus doubtlessly be treated as a priority by the authorities.

## 5. Conclusions

The value added of this study is twofold. First, when it comes to methodological aspects we proposed a new approach to measuring eco-efficiency in generalized input-output (gIO) models which may be used as a supplementary method to traditional DEA. Unlike DEA this approach takes into account detailed data on intersectoral flows in demand- and supply-oriented gIO models.

Secondly, we illustrated possible applications of the new approach by conducting an empirical analysis aimed at identifying eco-efficient sectors. This part of the study was based on the application of the 1995 and 2009 national input-output tables and environmental accounts for Poland, which were taken from the WIOD database.

Despite the simplicity and illustrative purpose of the real-data-based example presented in the empirical part of this paper, the analysis conducted in our study may be of some use for the future policy of Poland – a country that aims to transform into a more eco-efficient economy, which is especially important given the scope of the 2030 EU climate policy. In our opinion the methodology and initial empirical results presented in this paper may support the identification of those policies that are necessary to accelerate the transition to a clean economy while maximizing financial benefits. Given the significant role of the energy generation sector in Poland in shaping the interindustry ecologically-economic relations between sectors operating in the economy one may claim that in future years the country should focus on several policies. First, meeting renewable portfolio standards in the power sector can help increase Poland's capacity to drive more zero-carbon electricity, such as solar or wind, onto its grid. The decreasing costs of solar panels and wind turbines mean that Poland can make the transition to a low-carbon economy while saving money, as renewables have no fuel costs.[40] These policies will be particularly important, as a significant share of Poland's coal capacity will retire in the near future and need to be replaced. Poland's grid is somewhat small and inflexible, and additional measures that add to grid flexibility – such as expanded transmission, demand response, energy storage, and fast ramping supply – can help integrate the growing share of renewables. Moreover, raising energy efficiency standards, especially for the manufacturing industries, could help reduce the energy demanded by Poland's industry sector. However, one cannot forget that transformation of the energy sector in Poland is a complex problem. As concluded by Köppl and Schleicher (2018) policy strategies focusing on individual components of an energy system like shifting to renewables may, from a comprehensive perspective on more sustainable energy systems, prove even counterproductive. Thus, one may claim, that complementary policies supporting combined heat and power, waste heat recovery, and an improved design of industrial facilities may further lower energy consumption. In our opinion, increasing the price of carbon is still one of the most powerful, economy-wide incentives to undertake measures that reduce emissions of greenhouse gasses.

[40] Recently PGE, the largest power company in Poland, announced that it aims to build over 1,000 megawatts of offshore wind capacity by 2030.

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