Per unit and ad valorem royalties in a patent licensing game

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ABSTRACT:
In a context of product innovation, we study two-part tariff licensing between a patentee and a potential rival which compete in a differentiated product market characterized by network externalities. The latter are shown to crucially affect the relative profitability of Cournot vs. Bertrand when a per unit royalty is applied. By contrast, we find that Cournot yields higher profits than Bertrand under ad valorem royalties, regardless of the strength of network effects.

JEL CLASSIFICATION: L13, L20, D43

KEYWORDS: LICENSING, PRODUCT INNOVATION, BERTRAND VS. COURNOT, NETWORK EFFECTS

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1 Introduction

Empirical evidence suggests that firms often license to direct competitors their patented innovation (Jiang and Shi, 2018). Investments in either new technologies or in new product development allow firms to advance in economic performance, gaining a competitive advantage *via* innovation. The literature on patent licensing considers contracts that assume different forms such as either a fixed-fee or a per-unit/ad valorem royalty, as well as a two-part tariff including both a fixed fee and a royalty component, generally focusing on the optimality of a license scheme over the other.[1]

The present paper investigates, in a framework of product innovation, the optimal patent licensing by an incumbent when consumers' preferences exhibit network effects. Such consumption externalities, which are typical of markets such as telecommunications, on-line games, digital music/movies, payment systems, software and e-commerce platforms, imply that the value of a good to a consumer increases as the number of its users grows. Network effects are argued to lie behind the success of the most dynamic and impactful companies in the world such as Microsoft, PayPal. Microsoft, Facebook, Uber, Twitter and Salesforce. Most recent Industrial Organization literature points out the key role of network effects in affecting *via* expectations firms' equilibrium network size and the adoption of innovations, thus achieving a critical mass (David, 1985; Farrell and Saloner, 1985; Arthur, 1989; Choi, 1994; Economides, 1996a; Cabral et al., 1999).[2] The intensity of network effects has been also shown to impact product pricing and the strength of firms' market power (Cabral, 2011; Katz, Shapiro, 1985 and 1986), the strategic choices of product characteristics (Lambertini and Orsini, 2001; Baake and Boom, 2001; Garcia and Gabszewicz, 2007) and the determinants of market structure through firm entry (Economides, 1996b) and vertical integration (Dogan, 2009).

Licensing of new products, brands and services has become a crucial revenue source in network industries. Recent evidence suggests that licensing is a powerful value driver for Nokia, with brand and technology licensing net sales of 1.6 billion Euros in 2017 (Nokia Corporation Financial Report, 2018), and a revenue generator for Microsoft, Ericsson, IBM, Qualcomm and Texas instruments (Ludlow, 2014). Also, earnings of on-line games' developers have massively increased over the last years (State of the Developer Nation, 2018). In a lot of cases licensing occurs between firms that are direct competitors. See Microsoft that licensed mobile operating system features to Samsung and HTC (Hoffman, 2014) or Apple that obtained from Microsoft an eight-year license for Applesoft Basic that is a dialect of Microsoft Basic, adapted to the Apple II services of personal computers. Moreover General Motors (GM) licensed its OnStar service,

[1] Literature shows that the optimality of licensing schemes depends on whether the patentee is external to the market (Kamien and Tauman, 1986; Muto, 1993; Erutku, and Richelle, 2007) or rather is a producer within the market (Wang, 1998), on product differentiation (Kabiraj and Lee, 2011; Bagchi and Mukherjee, 2014), on whether firms compete with respect to quantities or price (Muto, 1993; Bagchi and Mukherjee, 2014).

[2] See Gandal (2008) for empirical studies emphasizing the role of network effects in boosting firm success.

an in-vehicle satellitebased mapping service, to other automobile manufacturers as Toyota and Honda.

Despite network effects have received wide attention in recent years both in practice and academic research, the analysis of their effects on licensing behavior in oligopolistic markets has been limited to very few studies dealing with the optimality of licensing strategies in a quantity competition framework. Wang et al. (2012) introduce network externalities in a Cournot model of process innovation, showing how they may let the patentee exploit the advantages of a larger market size achieved by favoring the competitor's production through a fixed fee, rather than charge a royalty restricting the licensees' output. The same mechanism is at work in the product innovation model of Lin and Kulatilaka (2006) who demonstrate that a pure fixed-fee license dominates a two-part tariff when the network intensity is high enough. By contrast, Zhao et al (2014) find that fixed-fee licensing never dominates royalty licensing or two-part tariff licensing when network effects interact with quality differences in a vertical product innovation model.

In the present paper we aim at investigating how the presence of network effects affects the optimal behavior of an incumbent innovator that licenses a new product technology to a potential market rival through a two-part tariff.[3] Market competition can occur under Cournot or under Bertrand, while either a per-unit and ad valorem royalty is included in the two-part licensing scheme.[4] In particular, we focus on how the strength of network externalities affects the relative profitability of Cournot vs. Bertrand, for each considered contract. The comparison on profitability between Cournot and Bertrand competition is an extensively debated issue in oligopoly theory. Following Singh and Vives (1984), much literature has found a dominance of Cournot over Bertrand with substitutes (Tanaka, 2001a; Tanaka, 2001b; Tasnádi, 2000, among others). This result, however, has been reversed in several circumstances: in mixed duopolies due to the presence of social welfare maximizing firms (Ghosh and Mitra, 2010; Matsumura and Ogawa, 2012), in vertically related industries (Correa-López and Naylor, 2004; Arya et al. 2008; Mukherjee et al., 2012; Alipranti et al., 2014), under cost and demand asymmetries (Zanchettin, 2006) and substantial quality differences (Häckner, 2000). Recently, it has been raised the question of whether the Singh and Vives (1984)'s result is robust to the presence of network effects. In this regard, Pal (2014) has shown that, when network externalities

[3] Optimal two-part licensing contracts have been studied, among the others, by Kamien and Tauman (1984), Erutku and Richelle (2007), Faulí-Oller and Sandonís (2002), Sen and Tauman (2007), Sen and Stamatopoulos (2009) and Filippini and Vergari (2017).

[4] Including a per unit or an ad valorem royalty in a licensing agreement is empirically observed (Bousquet et al., 1998; Lim and Veugelers, 2003; Trombini and Comacchio, 2012) and theoretically justified (San Martín and Saracho, 2010, 2015, 2016; Heywood et al., 2014; Colombo and Filippini, 2015a and 2015b, Fan et al., 2018). Whether a per-unit or an ad valorem royalty must be included in a licence is found to depend on the mode of competition (Colombo and Filippini, 2015a and 2015b), on demand or cost uncertainty (Bousquet et al., 1998), on product differentiation and the licensee' development cost for the new product (San Martín and Saracho, 2016), on the relative efficiency of the licensee compared to the licensor (Fan et al., 2018), on asymmetric information about the value of the patent (Heywood et al., 2014).

are strong enough, the positive effect on profits caused by tougher competition in Bertrand, which positively impacts on the equilibrium prices via consumers' expectations, dominates the standard negative effect of tougher competition on profits, leading to higher profits in Bertrand.[5]

The debate on Cournot vs. Bertrand profitability has been recently revisited by Chang et al (2017) in a patent licensing game. In this study, a product innovator charges a fixed fee plus a per unit royalty to either a potential rival or an incumbent rival. They find that the royalty rate under Bertrand competition works as a commitment of the licensor to set higher market prices, which enhance licensing revenues and let market profits to be higher under Bertrand than under Cournot, regardless of the degree of product differentiation. Our work extends Chang et al (2017) to the presence of network externalities, showing that market profitability may be reverted in favor of Cournot or Bertrand, depending on the type of the royalty payment. When a per unit royalty applies, profits may be either greater or lower under Cournot competition than under Bertrand competition, depending on the interplay between the intensity of network effects and the degree of product substitutability. Conversely, when an ad valorem royalty is included in the two part tariff, Cournot turns out to be always more profitable than Bertrand in the range of the parameters which ensures that both the fixed and the variable price are positive. These results have the following intuitive explanation. Increasing network effects, by raising end-users' utility and shifting market demand through expectations, can enhance one firm's ability to both expand its output and exploit higher consumers' willingness to pay setting higher prices. A contract with a per unit royalty, however, by restricting the licensee's output, leads higher network effects to limit the licensee's ability to set a higher price in Bertrand, while it lets higher network effects positively affect, through the channel of expectations on a larger market size, both the licensee's output and price in Cournot. As long as product substitutability is high enough, the positive effect of a higher royaly rate is still high and dominates the negative effect of network externalities on the licensee'price in Bertrand. This allows Bertrand to be still more profitable than Cournot, as in Chang et al. (2017). However, sufficiently low product substitutability, by reducing the negative effect on profits of a limited licensee's production in Cournot, yields the reversal result that Cournot is more profitable than Bertrand, which turns out to occur more likely when network effects increase. Finally, we provide the following explanation for higher profitability in Cournot relative to Bertrand under ad valorem royalty licensing. In this case, the patentee's optimal behavior is aimed to induce both a higher price and a higher output by the licensee. The latter is therefore able to exploit a larger network size through expectations both when she acts as a price-setter and as a quantity-setter, which lets market variables increase in the network externalities irrespective of the mode of competition. In such circumstances, the relative profitability between Bertrand and Cournot is determined by the

[5] The same result has been also achieved by Pal (2015) in a managerial delegation context with negative network externalities.

interplay between the positive effect on profits through consumers' expectations and the negative effect caused by more aggressive conduct, as in Pal (2014), which plays in favor of Cournot in the range of the model's parameters ensuring the positivity of the contract's components.

Our results contribute to the existing literature in several ways. First, they point out the role of network externalities in a licensing framework with price competition, which has been never investigated in previous literature. Second, the paper captures the implications of network effects on product innovation licensing, showing how the result of Chang et al. (2017) that Bertrand is more profitable than Cournot regardless of product substitutability does not hold when network effects are strong enough and product substitutability is sufficiently low. Third, it shows that the Pal (2014)'s conclusion that higher Bertrand profitability arises when the strength of network effects is high enough is reversed under the cost asymmetries induced by a contract with a per unit royalty.

The reminder of the paper is organized as follows. Section 2 develops the model, while Section 3 draws some conclusions.

2 The model

Firm 1 is an incumbent producer facing the following linear demand function:[6]

$$p_1 = a + ny_1 - q_1$$

where $a > 0$, q_1 denotes the quantity produced by the monopolist, p_1 its price, and y_1 denotes the consumers' expectation about firm 1's market size (sales), with n (with $0 \leq n < 1$) measuring the strength of network effect. For the sake of simplicity, we assume that both the variable and the fixed costs production are zero. Firm 1 has to decide whether to license its product innovation to a potential rival, firm 2, thus allowing it to produce a differentiated network good with the same technology as firm 1. In the case of licensing, market structure becomes a duopoly à la Cournot or à la Bertrand in which the two firms face the following inverse demand functions (Hoernig, 2012):

$$p_1 = a + n(y_1 + \gamma y_2) - q_1 - \gamma q_2 \tag{1}$$
$$p_2 = a + n(y_2 + \gamma y_1) - q_2 - \gamma q_1 \tag{2}$$

[6] We assume that prior licensing the patentee produces only a veriety of the good for a higher marketing or development cost that discourages her to produce an additional variety (Chang et al., 2017). This is a standard assumption in literature on product innovation (Kitagawa et al., 2014, San Martin and Saracho, 2016, Kitagawa et al., 2018) that associates the creation of new varieties with brand name, packaging, after-sales services, and switching costs (Kitagawa et al., 2014, San Martin and Saracho, 2016, Kitagawa et al., 2018). The same assumption is also used in literature on process innovation (e.g., Faulì-Oller and Sandonis, 2002; Wang, 2002; Wang and Yang, 1999; Colombo, 2015) in which firms compete on the market with imperfect substitute products under a drastic cost-reducing innovation.

q_2 and p_2 being respectively firm 2's output and price, whereas y_2 is consumers' expectations on firm 2's sales. The parameter γ in the range $[0,1]$ measures the degree of substitutability between the two varieties ($\gamma = 0$ implies that the products are unrelated, whereas $\gamma = 1$ implies that the products are perfect substitutes).

The direct demand functions can be written as:

$$q_1 = \frac{a(1-\gamma) + ny_1(1-\gamma^2) - p_1 + \gamma p_2}{(1-\gamma^2)} \tag{3}$$

$$q_2 = \frac{a(1-\gamma) + ny_2(1-\gamma^2) - p_2 + \gamma p_1}{(1-\gamma^2)} \tag{4}$$

The game timing is as follows. In the first stage, firm 1 chooses to license its new product technology or not through the payment of a two-part tariff, i.e. a lump sum payment plus either a per unit or an ad valorem royalty. In the second stage, if any, firm 2 accepts the contract offered by the rival and the two firms engage in either Cournot or Bertrand market competition. In Section 2.1 we derive the solution in a scenario with no licensing. The latter is then compared with the market outcome derived under per unit (ad valorem) royalty licensing in Section 2.2 (Section 2.3), implying a search for the subgame perfect Nash equilibrium (SPNE) either in Cournot or Bertrand.

2.1 The no licensing framework

The profit function of firm 1 is as follows:

$$\pi_1 = (a + ny_1 - q_1)\, q_1 \tag{5}$$

By maximizing (5) with respect to q_1, we obtain:

$$q_1 = \frac{a + ny_1}{2}$$

Following Katz and Shapiro (1985) and Hoernig (2012), we assume that consumers' expectations satisfy a rational expectations' condition, which implies that in equilibrium $y_1 = q_1$, thus obtaining the following output:

$$q_1 = \frac{a}{2-n} \tag{6}$$

The equilibrium price is:

$$p_1 = \frac{a}{2-n} \tag{7}$$

By substituting (6) and (7) in (5), we obtain firm 1's profits:

$$\pi_1 = \frac{a^2}{(2-n)^2} \tag{8}$$

2.2 Two-part tariff licensing with a per unit royalty

We assume that the firm 1 can license its innovation by imposing the payment of a royalty r for each unit sold by firm 2 and a fixed amount F.[7] Therefore, firms' profits are:

$$\pi_1 = p_1 q_1 + (r q_2 + F) \tag{9}$$
$$\pi_2 = (p_2 - r)\, q_2 - F \tag{10}$$

2.2.1 Cournot competition

By using the inverse demand functions (1) and (2) and maximizing profits (9) and (10) with respect to q_1 and q_1, we obtain the following reaction functions:

$$q_1 = \frac{a + n(y_1 + \gamma y_2) - \gamma q_2}{2}$$
$$q_2 = \frac{a + n(\gamma y_1 + y_2) - \gamma q_1 - r}{2}$$

Notice that each reaction function shifts outward as consumers' expectations increase, denoting that one firm's output is positively influenced by expectations on both its own sales and the rival's sales. The solution of the system of the reaction functions, under the rational expectations' conditions $y_1 = q_1$ and $y_2 = q_2$, gives the optimal quantities:

$$q_1 = \frac{a(2 - (n + \gamma(1-n))) + r\gamma(1-n)}{4 - (n(4-n) + \gamma^2(1-n)^2)} \tag{11}$$
$$q_2 = \frac{a(2 - (n + \gamma(1-n))) - r(2-n)}{4 - (n(4-n) + \gamma^2(1-n)^2)} \tag{12}$$

Notice that $\frac{\partial q_1}{\partial r} > 0$ and $\frac{\partial q_2}{\partial r} < 0$, that is, setting a higher r lets firm 1's output increase and firm 2"s output decrease.

Given the optimal quantities (11) and (12), firm 2's profit can be written as follows:

[7] This model has also been performed under the assumption of fixed fee licensing, i.e. introducing network externalities in Section 2.1 of Chapter 1. We observe that the non profitability of fixed fee licensing with respect to no licensing is less likely to occur for increasing network externalities. By shifting market demand, network effects allow the patentee to exploit the higher consumers' willingness to pay, thus benefiting from its own market profits and more consistent licensing revenues. This leads licensing to be advantageous for a higher extent of the product differentiated parameter. Consistently with the results obtained by Wang et al. (2012) and Lin and Kulatilaka (2006) respectively in a Cournot framework with process innovation and in a Cournot framework with product innovation, we show that sufficiently intense network effects make licensing advantageous in Cournot also when products are close substitutes. Moreover, by comparing the patentee's profit in Cournot and Bertrand, we show that strong enough network externalities lead Bertrand to be more profitable than Cournot. Intuitions are provided by Pal (2014) who finds the same result in a non-licensing model.

$$\pi_2 = \frac{\left(a(2-(n+\gamma(1-n))) - r(2-n)\right)^2}{\left(4-(n(4-n)+\gamma^2(1-n)^2)\right)^2} - F$$

Being firm 2's profits under no licensing equal to zero, the maximum F that the licensee accepts to pay is:

$$F = \frac{\left(a(2-(n+\gamma(1-n))) - r(2-n)\right)^2}{\left(4-(n(4-n)+\gamma^2(1-n)^2)\right)^2} \tag{13}$$

Notice that $\frac{\partial F}{\partial r} < 0$.

At the royalty setting stage, maximization of firm 1's profit in (9), calculated at the optimal quantities, gives the equilibrium royalty rate:

$$r^C = \frac{a(2-(\gamma+n(1-\gamma)))^2(\gamma(1-n)-n)}{2(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n)))} \tag{14}$$

By substituting (14) in (13), we obtain the equilibrium fixed fee:

$$F^{CU} = \frac{a^2(4-n(4-n)-\gamma(4+n^2-5n))^2}{4\left(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n))\right)^2}$$

Notice that both F and r are positive in the area above the green curve in Figure 1.

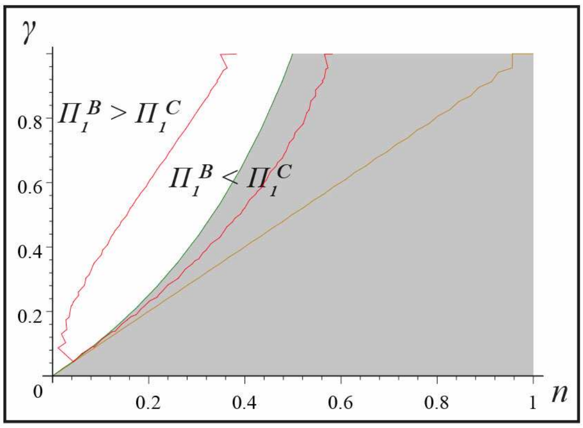

Fig. 1: Cournot vs Bertrand with per-unit royalty

The profit differential (Π_1^B-Π_1^C).

r=o in Bertrand competition.

r=o in Cournot competition.

We now calculate the market variables at the SPNE, i.e. the equilibrium quantities and prices:

$$
\begin{aligned}
q_1^{CU} &= \frac{a((2-\gamma)(2-n)-\gamma^2(1-n))}{2(4-(n(4-n)+\gamma^2(3+n^2-4n)))} \\
q_2^{CU} &= \frac{a((4+n^2)(1-\gamma)-n(4-5\gamma))}{2(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n)))}
\end{aligned}
$$

$$
\begin{aligned}
p_1^{CU} &= \frac{a((2-\gamma)(2-n)-\gamma^2(1-n))}{2(4-(n(4-n)+\gamma^2(3+n^2-4n)))} \\
p_2^{CU} &= \frac{a((2-n)^2+\gamma(1-n)(n+\gamma^2(1-n))-\gamma^2(4+n^2-5n))}{2(4-(n(4-n)+\gamma^2(3+n^2-4n)))}
\end{aligned}
$$

Firm 1's profits are as follows at the SPNE:

$$
\pi_1^{CU} = \frac{a^2(8+\gamma^2(1-n)^2-2\gamma(4+n^2-5n)-n(8-n))}{4(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n)))} \tag{15}
$$

while $\pi_2^{CU}=0$. It is worth noting that all market variables increase in n: indeed, $\frac{\partial p_1^{CU}}{\partial n}>0$, $\frac{\partial p_2^{CU}}{\partial n}>0$, $\frac{\partial q_1^{CU}}{\partial n}>0$, $\frac{\partial q_2^{CU}}{\partial n}>0$. Moreover, while the royalty rate decreases in n, the fixed fee increases in n. Finally, we find that network effects impacts positively on firm 1 profits, i.e., $\frac{\partial \pi_1^{CU}}{\partial n}>0$.

Furthermore, we find that firm 1's profits under licensing in (15) are higher than firm 1's profits under no licensing in (8) in the region of the parameters which ensure positivity of the market variables and the licensing contract components. This is stated in the following remark.

Remark 1 *By assuming quantity competition, two-part tariff licensing with a per unit royalty is always profitable for the patent holder when* $\gamma \geq \frac{n}{1-n}$ with $0 \leq n \leq \frac{1}{2}$ (i.e. above the green curve in Figure 1) *where market variables and licensing contract components are positive.*

2.2.2 Bertrand competition

By engaging in Bertrand competition, firm 1 and firm 2 face the direct demand functions (3) and (4) and maximize their own profits in (9) and (10) with respect to p_1 and p_2, respectively. The following reaction functions are then obtained:

$$
\begin{aligned}
p_1 &= \frac{a(1-\gamma)+ny_1(1-\gamma^2)+\gamma(p_2+r)}{2} \\
p_2 &= \frac{a(1-\gamma)+ny_2(1-\gamma^2)+\gamma p_1+r}{2}
\end{aligned}
$$

By solving the system of the above reaction functions, under the conditions $y_1 = \frac{a(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)(1-n)}$ and $y_2 = \frac{a(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)(1-n)}$, we obtain the following prices:

$$p_1 = \frac{a(2-(n(1-\gamma)+\gamma(1+\gamma)))+r\gamma(3-4n+n^2)}{4-(n(4-n)+\gamma^2)} \quad (16)$$

$$p_2 = \frac{a(2-(n(1-\gamma)+\gamma(1+\gamma)))+r(2-(n(3-n)-\gamma^2(1-n)))}{4-(n(4-n)+\gamma^2)} \quad (17)$$

Observe that $\frac{\partial p_1}{\partial r} > 0$ and $\frac{\partial p_2}{\partial r} > 0$, which implies that setting a higher r leads both firms to charge higher prices.

Fimr 2's profit can be written as follows:

$$\pi_2 = \frac{(a(2-(\gamma(1+\gamma)+n(1-\gamma)))-r(2-n)(1-\gamma^2))(a(2+\gamma-n)-r(2-n)(1+\gamma))}{(1+\gamma)((4-\gamma^2)-n(4-n))^2} - F$$

The maximum F that the licensee accepts to pay for the innovation is therefore:

$$F = \frac{(a(2-(\gamma(1+\gamma)+n(1-\gamma)))-r(2-n)(1-\gamma^2))(a(2+\gamma-n)-r(2-n)(1+\gamma))}{(1+\gamma)((4-\gamma^2)-n(4-n))^2} \quad (18)$$

Maximization of firm 1' profit in (9), after incorporating (18) and the optimal prices in (16) and (17), leads to the optimal royalty rate:

$$r^B = \frac{a\left(2-n+\gamma\right)^2(\gamma-n)}{2(4-(n(8+n^2-5n)-\gamma^2(5+2n^2-7n)))} \quad (19)$$

The fixed fee is then obtained by substituting (19) in (18):

$$F^{BU} = \frac{a^2\left(4-\gamma^3(2-n)+\gamma^2(2+n^2-4n)-\gamma(4+2n^2-7n)-n(4-n)\right)\left(4+\gamma^2(2-n)+n\gamma(3-n)-n(4-n)\right)}{4(1+\gamma)(4-(n(8+n^2-5n)-\gamma^2(5+2n^2-7n)))^2}$$

It is easy to check that both the variable royalty and the fixed fee are positive in the region of parameters above the brown curve in Figure 1

The market variables at the SPNE, i.e., prices and quantities, are:

$$p_1^{BU} = \frac{a(4+\gamma^2(7-3n)-\gamma(2-n^2+n)-2n)}{2(4-(n(4-n)-\gamma^2(5-2n)))}$$

$$p_2^{BU} = \frac{a(4-\gamma^3+\gamma^2(6-n)-n(4-n+\gamma))}{2(4-(n(4-n)-\gamma^2(5-2n)))}$$

$$q_1^{BU} = \frac{a\left(4+\gamma^3+\gamma^2(5-3n)+\gamma(2-n)-2n(3-n)\right)}{2\left(1-n\right)\left(1+\gamma\right)\left(4-(n(4-n)-\gamma^2(5-2n))\right)}$$

$$q_2^{BU} = \frac{a\left(4+\gamma^2(2-n)+\gamma n(3-n)-n(4-n)\right)}{2\left(1-n\right)\left(1+\gamma\right)\left(4-(n(4-n)-\gamma^2(5-2n))\right)}$$

Firm 1's profits at the SPNE are:

$$\pi_1^{BU} = \frac{a^2\left(8+\gamma^3+3\gamma^2(3-2n)+\gamma n(2+n)-n(8-n)\right)}{4(1-n)(1+\gamma)(4-(n(4-n)-\gamma^2(5-2n)))} \tag{20}$$

while $\pi_2^{BU}=0$. When n increases, we find that $\frac{\partial p_1^{BU}}{\partial n}>0$ and $\frac{\partial q_1^{BU}}{\partial n}>0$, as far as firm 1 is concerned, while $\frac{\partial p_2^{BU}}{\partial n}<0$ and $\frac{\partial q_2^{BU}}{\partial n}>0$ as regards firm 2. Also notice that the variable royalty r and the fixed fee F^{BU} respectively decreases and increases in n. Finally, network effects turn out to positively impact on firm 1 profits, i.e., $\frac{\partial \pi_1^{BU}}{\partial n}>0$.

Comparing firm 1's profits under licensing in (20) and its profits under no licensing in (8) allows us to introduce the following remark.

Remark 2 *By assuming price competition, two-part tariff licensing with a per unit royalty is always profitable for the patent holder when* $\gamma \geq n$ with $0 \leq n < 1$ (i.e. above the brown curve in Figure 1) *where market variables and licensing contract componets are positive.*

2.2.3 Cournot vs. Bertrand under a per unit royalty

Here we compare the patentee's profits under Cournot and Bertrand competition. The red curve in Figure 1 represents the profit differential $\pi_1^{CU}-\pi_1^{BU}$ which turns out to be positive below the curve, while it is negative above. By focusing on the region of the parameters in which all market variables are positive and licensing is advantageous for the patentee both in Cournot and Bertrand, that is the area above the green curve, it is easy to check that $r^B > r^C$, while $\pi_1^{CU}-\pi_1^{BU}$ may be positive or negative, according to the interplay between n and γ.

Proposition 1 follows from the above analysis.

Proposition 1 *Under two-part tariff licensing with a per unit royalty, market profitability is higher under Bertrand than under Cournot, regardless of γ when $n=0$ (Chang et al, 2017), and at sufficiently high values of γ for any given* $0<n\leq 1$. *Increasing network effects make the Bertrand profits less likely to dominate the Cournot profits.*

oppure

Proposition 1 *Under two-part tariff licensing with a per unit royalty, for a given n, market profitability is higher under Bertrand than under Cournot above a γ* limit, *while the opposite occurs below the γ limit in our region of interest* $\gamma \geq \frac{n}{1-n}$ with $0 \leq n \leq 0.5$ (i.e above the green curve of Figure 1). *When $n=0$ the patentee is better off under Bertrand competition, regardless of γ. Increasing network effects make the Bertrand profits less likely to dominate the Cournot profits.*

Proposition 1 *Under two-part tariff licensing with a per unit royalty, there always exists $\overline{\gamma}$ in the region of parameters where licensing is profitable and the contractual terms are positive, that is when $\gamma \geq \frac{n}{1-n}$* (see Remark 1), *such that market profitability under Bertrand is higher (lower) than under Cournot for $\gamma \geq \overline{\gamma}$ ($\gamma < \overline{\gamma}$), for any given $n \in \left(0, \frac{1}{2}\right)$. Since $\frac{\partial \overline{\gamma}}{\partial n} > 0$, increasing network effects make the Bertrand profits less likely to dominate the Cournot profits.*

Proof:

From (20) and (19), we obtain:
$\pi_1^{BU} - \pi_1^{CU} = \frac{\gamma(1-\gamma)\left((1-n)(2-n)^2\gamma^3 - n(1-n)(8-3n)\gamma^2 - n(2-n)\left(2+n^2-4n\right)\gamma + n^2(2-n)^2\right)}{2(1-n)(4+n^2-\gamma^2n^2+4\gamma^2n-4n-3\gamma^2)(4-4n+n^2+5\gamma^2-2\gamma^2n)(1+\gamma)}$

We get that:
$sign\left[\frac{\gamma(1-\gamma)\left((1-n)(2-n)^2\gamma^3 - n(1-n)(8-3n)\gamma^2 - n(2-n)\left(2+n^2-4n\right)\gamma + n^2(2-n)^2\right)}{2(1-n)(4+n^2-\gamma^2n^2+4\gamma^2n-4n-3\gamma^2)(4-4n+n^2+5\gamma^2-2\gamma^2n)(1+\gamma)}\right] =$
$= sign\left[f(\gamma, n)\right]$

where $f(\gamma, n) = (1-n)(2-n)^2\gamma^3 - n(1-n)(8-3n)\gamma^2 - n(2-n)\left(2+n^2-4n\right)\gamma + n^2(2-n)^2$. For any given $n \in \left(0, \frac{1}{2}\right)$, there exists a γ, say $\overline{\gamma}$, such that $f(\gamma, n) \geq 0$ if $\gamma \geq \overline{\gamma}$ ($f(\gamma, n) < 0$ if $\gamma < \overline{\gamma}$).

An explanation of Proposition 1 is as follows. When $n = 0$ (Chang et al, 2017), Bertrand competition is more profitable than Cournot due to the higher royalty rate in Bertrand which is optimally chosen to keep prices relatively high, thus raising market profitability. In Bertrand, however, more intense network effects on the one hand induce a price increase by both firms through expectations on a larger network size, on the other hand weaken the patentee's incentive to keep prices high through a high royalty rate. Due to the output contraction induced by the royalty payment, the second negative effect on the licensee's price dominates the first positive effect, leading its price to decrease when n increases. For the patentee, instead, the positive effect through market size expectations always dominates the negative effect of a lower royalty rate, which lets its price increase in n. The weaker position of the licensee firm, instead, does not refrain it from enjoying a larger expected market size and expanding both its output and price following an increase of network effects. The same occurs for the patentee. Finally, it can be observed that sufficiently high product substitutability in Bertrand, by enhancing the patentee's incentive to set a relatively high royalty rate and soften tougher market competition, limits the negative impact of network effects on the licensee's price, thus improving Bertrand profitability relative to Cournot. Conversely, sufficiently low product substitutability, by reducing the licensee's loss associated with its weaker position under increasing network effects, pushes towards higher profitability of Cournot than Bertrand.[8]

[8] This contrasts with Pal (2014) where strong enough network externalities lead the indirect positive effect of Bertrand more aggressive conduct on profits to dominate the direct (standard)

2.3 Two-part tariff licensing with an ad valorem royalty

In this section we keep the above assumptions on demand and firms' costs. Moreover, we assume that firm 1 uses a two-part tariff including a royalty d which is a fraction of rival's revenues and a fixed amount F. We can write firm 1's and firm 2's profits as follows:

$$\pi_1 = p_1 q_1 + d p_2 q_2 + F \tag{21}$$

$$\pi_2 = (1-d)\, p_2 q_2 - F \tag{22}$$

2.3.1 Cournot competition

At the market stage, firm 1 and firm 2 compete *à la* Cournot facing the inverse demand function respectively in (1) and in (2). Maximization of firm 1's profits in (21) with respect to q_1 and maximization of firm 2's profits in (22) with respect to q_2 lead to:

$$\begin{aligned} q_1 &= \frac{a + n(y_1 + \gamma y_2) - \gamma q_2 (1+d)}{2} \\ q_2 &= \frac{a + n(\gamma y_1 + y_2) - \gamma q_1}{2} \end{aligned}$$

The above reaction functions yield, under the rational expectations's conditions ($y_1 = q_1$ and $y_2 = q_2$), the optimal quantities:

$$q_1 = \frac{a\,(2 - n(1-\gamma) - \gamma(1+d))}{4 - (\gamma^2((1-n)^2 + d(1-n)) + n(4-n))} \tag{23}$$

$$q_2 = \frac{a(2 - (n + \gamma(1-n)))}{4 - (\gamma^2((1-n)^2 + d(1-n)) + n(4-n))} \tag{24}$$

Subject to (23) and (24), the licensee's profit is:

$$\pi_2 = \frac{a^2\,(1-d)\,(2 - (n + \gamma(1-n)))^2}{(4 - (\gamma^2((1-n)^2 + d(1-n)) + n(4-n)))^2} - F$$

Under the constraint $(1-d)\, p_2 q_2 - F \geq 0$, which defines the maximum amount F that the licensee pays for the innovation, i.e.

$$F = \frac{a^2\,(1-d)\,(2 - (n + \gamma(1-n)))^2}{(4 - (\gamma^2((1-n)^2 + d(1-n)) + n(4-n)))^2}$$

the patentee maximizes her own profit with respect to d, thus setting:

negative effect, which leads Bertrand to be more profitable than Cournot. In our case, by contrast, sufficiently intense network effects always enhance market profitability in Cournot, while they negatively affect the Bertrand profits by limiting, through a royalty rate reduction, the licensee's ability to exploit higher consumers' willingness to pay.

$$d^C = \frac{\gamma^3(1-n^3+3n^2-3n)-\gamma^2(4-n^3+6n^2-9n)+\gamma(4+n^3-n^2-4n)-n(2-n)^2}{(4-\gamma^2(1-n)^2-\gamma(2+n^2-3n)-2n(3-n))\gamma} \tag{25}$$

The equilibrium fixed fee is as follows:

$$F^{CV} = \frac{a^2\left(\gamma^2(2-n^3+5n^2-6n)-\gamma^3(2-n^3+4n^2-5n)-n\gamma(2+n^2-3n)+n(2-n)^2\right)(4-\gamma^2(1-n)-\gamma(2-n)-2n)}{4(4-\gamma^2(3-n^3+5n^2-7n)-n(8+n^2-5n))(4-\gamma^2(3+n^2-4n)-n(4-n))\gamma}$$

The market variables, i.e., quantities and prices, at the SPNE are:

$$\begin{aligned}
q_1^{CV} &= \frac{a\left(4-(\gamma(4+n^2-5n)+n(4-n))\right)}{2\left(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n))\right)} \\
q_2^{CV} &= \frac{a\left(4-\gamma^2(1-n)-\gamma(2-n)-2n\right)}{2\left(4-(\gamma^2(3+n^2-4n)+n(4-n))\right)}
\end{aligned}$$

$$\begin{aligned}
p_1^{CV} &= \frac{a(4+\gamma^3(1-n)^2-\gamma^2(4+n^2-5n)+\gamma n(1-n)-n(4-n))}{2\left(4-\gamma^2(3+n^2-4n)-n(4-n)\right)} \\
p_2^{CV} &= \frac{a\left(4-\gamma^2(1-n)-\gamma(2-n)-2n\right)}{2\left(4-\gamma^2(3+n^2-4n)-n(4-n)\right)}
\end{aligned}$$

Firm 1's equilibrium profits are:

$$\pi_1^{CV} = \frac{a^2(8+\gamma^2(1-n)^2-2\gamma(4+n^2-5n)-n(8-n))}{4\left(4-(\gamma^2(3-n^3+5n^2-7n)+n(8+n^2-5n))\right)} \tag{26}$$

while $\pi_2^{CV} = 0$.

It can be easily checked that both the royalty rate d^C and the fixed fee F^{CV} are positive in the region above the green curve of Figure 2, with d^C decreasing in n and F^{CV} increasing in n. In the same region, all market variables are positive and increase in n (i.e., $\frac{\partial p_1^{CV}}{\partial n} > 0$, $\frac{\partial q_1^{CV}}{\partial n} > 0$, $\frac{\partial p_2^{CV}}{\partial n} > 0$ and $\frac{\partial q_2^{CV}}{\partial n} > 0$), as well as firm 1's profits (i.e, $\frac{\partial \pi_1^{CV}}{\partial n} > 0$).

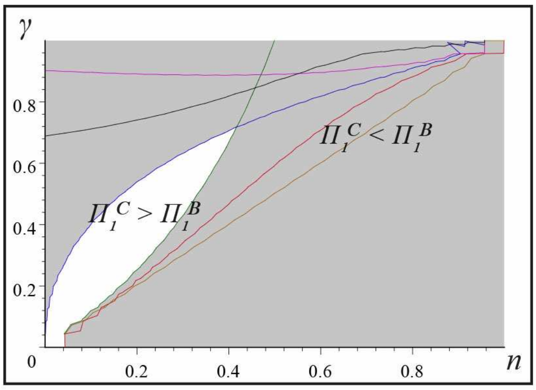

Fig. 2: Cournot vs Bertrand with ad-valorem royalty

r=0 in Cournot competition.

r=0 in Bertrand competition.

F=0 in Bertrand competition.

The profit differential (Π_1^C-Π_1^B).

q_1=0 in Bertrand competition.

The profit differential (Π_1^B-Π_1^H).

Comparing firm 1's profits under licensing in (26) with firm 1's profits under no licensing in (8), we get the result highlighted in the following remark.

Remark 3 *Under quantity competition, two-part tariff licensing with an ad valorem royalty is always profitable for the patent holder when* $\gamma \geq \frac{n}{1-n}$ with $0 \leq n \leq 0.5$ (i.e. above the green curve in Figure 2) *where market variables and licensing contract componets are positive.*

Remarkably, by comparing firm 1's profits under ad valorem licensing with profits under per unit licensing, we find that the presence of network externalities doesn't affect the Niu (2013)'s result, thus observing that:[9]

Proposition 2 *The patentee is indifferent between licensing through a per unit royalty or an ad valorem royalty*

Proof:

It follows from the identity between (15) and (26).

[9] See Niu (2013, p. 13) for an intuition of the above equivalence result.

2.3.2 Bertrand competition

Firm 1 and firm 2 compete with respect to prices facing the direct demand functions. Maximizing firm 1's and firm 2's profits with respect to p_1 and p_2, respectively, we obtain the following reaction functions:

$$\begin{aligned} p_1 &= \frac{a(1-\gamma)+ny_1(1-\gamma^2)+\gamma p_2(1+d)}{2} \\ p_2 &= \frac{a(1-\gamma)+ny_2(1-\gamma^2)+\gamma p_1}{2} \end{aligned}$$

We solve the above system under the rational expectation conditions $y_1 = \frac{a(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)(1-n)}$ and $y_2 = \frac{a(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)(1-n)}$, getting the optimal prices:

$$p_1 = \frac{a(2-\gamma^2(1+d(1-n))-n-\gamma(1-d(1-n)-n))}{4-(n(4-n)+\gamma^2(1+d(1-n)))} \tag{27}$$

$$p_2 = \frac{a(2-\gamma^2-\gamma(1-n)-n)}{4-(n(4-n)+\gamma^2(1+d(1-n)))} \tag{28}$$

At the previous stage, subject to (27) and (28), the licensee's profit can be written as follows:

$$\pi_2 = \frac{a^2(1-d)(2-n+\gamma)\left(2-\gamma^2-\gamma(1-n)-n\right)}{(4-(n(4-n)+\gamma^2(1+d(1-n))))^2(1+\gamma)} - F$$

Under the condition $(1-d)\,p_2 q_2 - F \geq 0$, where F represents the maximum amount the licensee is willing to pay for the new product technology

$$F = \frac{a^2(1-d)(2-n+\gamma)\left(2-\gamma^2-\gamma(1-n)-n\right)}{(4-(n(4-n)+\gamma^2(1+d(1-n))))^2(1+\gamma)} \tag{29}$$

the patentee maximizes her own profit, thus setting the following royalty rate:

$$d^B = \frac{\gamma^3+\gamma^2(4-3n)+\gamma(4-n(8-3n))-n(2-n)^2}{(4-\gamma^2(1-n)+\gamma(2+n^2-3n)-2n(3-n))\,\gamma} \tag{30}$$

By substituting (30) in (29), we obtain:

$$F^{BV} = \frac{a^2(1-\gamma)(4-\gamma^2+\gamma(2-n)-2n))\left(n(2-n)^2-\gamma^3(2-n)-\gamma^2(2-n^2)+n\gamma(2-n)\right)}{4(1+\gamma)(4-\gamma^2(3-2n)-n(4-n))(4-\gamma^2(3+2n^2-5n)-n(8+n^2-5n))\gamma}$$

Notice that, at equilibrium, the royalty rate d is positive above the brown curve in Figure 2, while F is positive below the blue curve.

Firms' quantities and prices at the SPNE are:

$$\begin{aligned} p_1^{BV} &= \frac{a(4-\gamma^2(4-3n)+n\gamma(1-n)-n(4-n))}{2\,(4-(\gamma^2(3-2n)+n(4-n)))} \\ p_2^{BV} &= \frac{a(4-\gamma^2+\gamma(2-n)-2n)\,(1-\gamma)}{2(4-(\gamma^2(3-2n)+n(4-n)))} \end{aligned}$$

$$q_1^{BV} = \frac{a(4-\gamma^3-\gamma^2(4-3n)-n(4-\gamma-n))}{2\left(4-\left(\gamma^2(3+2n^2-5n)+n(8+n^2-5n)\right)\right)(\gamma+1)}$$
$$q_2^{BV} = \frac{a(4-2n-\gamma^2+\gamma(2-n))}{2\left(4-\left(\gamma^2(3-2n)+n(4-n)\right)\right)(\gamma+1)}$$

with the patentee's quantity being positive below the pink curve in Figure 2. This implies that both d and F are positive in the region of parameters included between the blue curve and the brown curve, where all market variables are also positive.

Firm 1's profits at the SPNE are:

$$\pi_1^{BV} = \frac{a^2(8-\gamma^3-\gamma^2(7-6n)+n\gamma(2-n)-n(8-n))}{4\left(4-\gamma^2(3+2n^2-5n)-n(8+n^2-5n)\right)(\gamma+1)} \tag{31}$$

while $\pi_2^{BV} = 0$.

As in the previous settings, we find that the royalty rate and the fixed fee respectively decreases and increases in n. Likewise, all market variables and firm 1's profits positively depend on n.

Moreover, we find that profits under licensing in (31) are larger than firm 1's profits under no licensing in (8) below the black curve in Figure 2. We can conclude that licensing is always profitable in the region of the parameters in which both the fixed fee, the royalty rate and all market variables are positive, then getting the result included in the following remark

Remark 4 *Under price competition, two-part tariff licensing with an ad valorem royalty is profitable for the patent holder in the relevant region of the parameter space.*

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Remark 4 *Under price competition, two-part tariff licensing with an ad valorem royalty is profitable for the patent holder when* $n \leq \gamma \leq (\frac{1}{3(-2+n)}(2+\sqrt[3]{A+3B(2-n)^2}+\frac{C}{\sqrt[3]{A+3B(2-n)^2}}-n^2))$ *(i.e. the area between the blue curve and the brown curve in Figure 2) with* $0 \leq n < 1$

$A = 8 - n^6 - 18n^5 + 132n^4 - 333n^3 + 384n^2 - 180n$

$B = (\sqrt{3}\sqrt{n\,(n^6 + 11n^5 - 79n^4 + 176n^3 - 172n^2 + 71n - 8)})$

$C = (4 + n^4 + 3n^3 - 16n^2 + 12n)$

where market variables and licensing contract components are positive.

Moreover, by comparing firm 1's profits under ad valorem licensing in (31) with profits under per unit licensing in (20), we find that the presence of network externalities doesn't affect the Colombo and Filippini (2015b)'s result, thus observing that:[10]

[10] See Colombo and Filippini (2015b, p. 9) for an intuition of the above result.

Proposition 3 *Under Bertrand competition, patent licensing is more profitable when the contract includes a per unit royalty in a two-part tariff than an ad valorem royalty.*

Proof:

$\pi_1^U - \pi_1^V = \frac{a^2\gamma(2+\gamma-n)^2(n-\gamma)^2}{2(1-n)(1+\gamma)(n^2+2\gamma^2n-4n-3\gamma^2+4)(4-4n+n^2+5\gamma^2-2\gamma^2n)} > 0$

2.3.3 Cournot vs. Bertrand under an ad valorem royalty

Let us compare the patentee's profits under Cournot and Bertrand competition. The red curve in Figure 2 represents the profit differential $\pi_1^C - \pi_1^B$ which is positive above the curve, while it is negative below. By focusing on the region of the parameters in which all market variables are positive and licensing is advantageous in both Cournot and Bertrand, that is that included between the blue and the green curve, we find that $d^B > d^C$ and $\pi_1^C - \pi_1^B > 0$.

The following proposition then holds.

Proposition 4 *Licensing a new product technology to a potential rival using a two-part contract with ad valorem royalty is more profitable under Cournot competition than under Bertrand competition, regardless of the degree of product substitutability and the intensity of network effects.*

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Proposition 4 *Under two-part tariff licensing with an ad valorem royalty, regardless of the degree of product substitutability and the intensity of network effects market profitability is higher under Cournot than under Bertrand in the region* $\frac{n}{1-n} \leq \gamma \leq (\frac{1}{3(-2+n)}(\sqrt[3]{A+3B(2-n)^2} + \frac{C}{\sqrt[3]{A+3B(2-n)^2}} + 2 - n^2))$ with $0 \leq n \leq \frac{1}{2}$ *(i.e. the area between the blue curve and the green curve in Figure 2) and*

$A = 8 - n^6 - 18n^5 + 132n^4 - 333n^3 + 384n^2 - 180n$

$B = \sqrt{3}\sqrt{n\,(n^6 + 11n^5 - 79n^4 + 176n^3 - 172n^2 + 71n - 8)}$

$C = 4 + n^4 + 3n^3 - 16n^2 + 12n$

Proof:

From (26) and (31), we get $\pi_1^{CV} - \pi_1^{BV} =?????? \geq 0 \Rightarrow n \leq \overline{n}$

where $\overline{n} = \frac{4-3\gamma^3-4\gamma^2+4\gamma-\sqrt{16-32\gamma^2-3\gamma^6+20\gamma^4}}{2(2-\gamma^3-2\gamma^2+\gamma)}$. *Since the condition* $n \leq \widetilde{n}$ *(with* $\widetilde{n} = \frac{\gamma}{1+\gamma}$*) defines the region of the parameters ensuring profitability of licensing and positivity of the contractual terms (see Remark 4) and should be met and, moreover, since* $\overline{n} \leq \widetilde{n}$*, we can conclude that the Cournot profits are always higher than the Bertrand profits.*

Clearly, unlike the case of per unit royalty, market profitability is no more affected by product differentiation and the intensity of network effects in the region of parameters ensuring that both the fixed fee and the royalty rate are positive, namely when n is not too high for any given γ. Indeed, we observed that a per unit royalty, by causing an output contraction by the licensee, limits the positive effect of higher network externalities on both its price and output, with an extent that depends on the degree of product differentiation and determines higher profitability of either Cournot or Bertrand. By contrast, we find that a change of an ad valorem royalty rate does not affect directly the licensee's choices, which are distorted only through the patentee's behavior. The latter is then oriented to induce, through an appropriate choice of the ad valorem royalty, both a quantity and a price increase by the licensee,[11] which are further enhanced by increasing network effects under both Bertrand and Cournot. Under low enough network effects, which are required for positivity of both the fixed fee and the royalty rate, the negative direct effect of firms' more aggressive play in Bertrand relative to Cournot turns out dominate its positive indirect effect *via* expectations, leading to more profitable Cournot competition regardless of the degree of product differentiation and the strength of network effects.[12]

3 Concluding remarks

This paper has reconsidered the relative profitability of Cournot vs. Bertrand competition in a network market in which a patent holder licenses, through a two-part tariff, her product innovation to a potential rival.[13] We have found that the interplay between the intensity of network effects and the degree of product differentiation differently affect relative market profitability in Cournot and Bertrand under a per unit royalty. This result is driven by per unit royalties which reduces the licensee firm's output and, under increasing network effects, refrain it from fully exploiting the higher consumers' willingness to pay to set a higher price in Bertrand and exploit the higher network size in Cournot. The intensity of network effects has been shown to interact with the degree of product differentiation, determining the extent of the above effects and their impact on the licensing revenues in the two competition frameworks, causing either a dominance of Cournot or a dominance of Bertrand profits. Conversely, under ad valorem two-part licensing, the patentee's strategic incentive to behave less aggressively and favor the rival to some extent, does not limit the licensee from enjoying the benefits induced by a larger equilibrium network size, both in Cournot and Bertrand. The standard dominance of Cournot profits over

[11] See in this regard Colombo and Filippini, 2015b, p. 9.

[12] The result resembles that achieved under low enough network externalities by Pal (2014) in a standard duopoly.

[13] We have also studied the patentee's profitability on the assumption of licensing one of her varieties produced in the ex-ante monopolistic framework. In the per-unit royalty case, we observe that the absence of a 'variety effect' negatively impacts on the profitability of licensing vs. no licensing to a lesser extent in Bertrand than in Cournot. This leads the competition effect to interact with the network externalities making the dominance of Cournot over Bertrand profits more likely than one variety offered prior to licensing. When an ad valorem royalty is adopted, the conclusion that Cournot is more profitable than Bertrand under less likely licensing is then straightforward.

Bertrand then arises in the relevant range of the model's parameters, regardless of the degree of product substitutability and the strength of network effects.[14]

Our findings can provide interesting insights on profitability conditions in network markets which may give rise to antitrust concern. We leave to future research the analysis of social desirability of our findings, as well as their robustness to the assumption that either the licensee is also an incumbent in the market or the patentee endogenously chooses her R&D (quality improving) investment level.

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