Electricity market competition when forward contracts are pairwise efficient

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Abstract

This paper investigates competition in electricity markets when each pair of strategic firms exchanges forward obligations pairwise-efficiently. The gains from pairwise trade are specific to the counterparty, which can be horizontally- or vertically-related depending on whether it has access to flexibility in the spot market. The analysis shows that pairwise efficient forward trade rules out a bilateral oligopoly spot market where net buyers and net sellers strategically interact. Firms without flexibility close their position entirely in the forward market. Forward markets serve to absorb renewable energy shocks, even if forward contracts are unobservable and firms are risk-neutral.

Keywords: Nash-in-Nash bargaining, bilateral oligopoly, renewables

JEL classification: D43, L13, L94

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1. Introduction

Forward obligations are known to crucially determine the exercise of market power in wholesale electricity spot markets. Consequently, the question how firms sign forward contracts in equilibrium is a classic one, and is one that continues to be highly relevant. What are the interactions between the forward market and the real-time market? And how can we expect the forward market to absorb supply shocks caused by renewables; which firms take up which positions?

This paper addresses these questions by using the concept *pairwise efficiency*. This concept requires that, in any equilibrium, each pair of strategic firms should not be able to gain from additionally exchanging forward obligations bilaterally, considering how doing so would affect competition in the spot market and taking as given the other forward obligations in the market.[2] This equilibrium condition has the important advantage that it is relevant whenever there is an over-the-counter market where firms can trade secretly, which is the case for most real-world electricity markets.[3]

This approach advances on previous work studying forward trade in two important ways. First, the literature following Allaz and Vila (1993) is based on the idea that forward obligations can act as a commitment device.[4] The commitment value, however, is known to break down when firms can also sign contracts secretly, for

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[2] Similar concepts have been employed in a variety of other settings, many of them involving vertical relations. See e.g. Crémer and Riordan (1987) on contract equilibrium, Hart and Tirole (1990), McAfee and Schwartz (1994) on passive beliefs and pairwise-proof equilibrium, and Collard-Wexler et al. (2019) on Nash-in-Nash bilateral bargaining. Jeon and Lefouili (2018) use the concept of bilateral efficiency to analyze cross-licensing between horizontal competitors.

[3] Section 4 discusses possible sources of frictions that determine how well pairwise efficient forward trade holds in practice.

[4] This literature includes Newbery (1998), Green (1999), Ferreira (2003), Mahenc and Salanié (2004), Liski and Montero (2006), Bushnell (2007), Anderson and Hu (2008), Bushnell et al. (2008), Holmberg (2011), Ito and Reguant (2016), van Eijkel et al. (2016), Acemoglu et al. (2017), Brown and Eckert (2017), Miller and Podwol (2018), and Wölfing (2019).

example in an over-the-counter market.[5] This paper contributes to that literature by revealing strategic motivations for forward trading that do not rely on the heroic assumption that all contracts are perfectly observable to third parties. Second, it is well-understood—and documented—that both sellers *and* buyers can behave strategically in electricity markets.[6] For instance, it is not uncommon for producers to, after receiving unfavorable renewable generation forecasts, act as a buyer in the forward market or the spot market. Also, retailers or aggregators can behave strategically, for example by adjusting their offtakes using smart-metering technologies. This paper is first to analyze trade between strategic buyers and sellers, both in the spot market *and* in the forward market.[7]

By studying pairwise trade, this paper analyzes how a firm's incentive to sign a forward contract depends on the characteristics of the counterparty. A crucial characteristic of the counterparty is whether it has the flexibility to strategically alter its physical impact on the electricity system, either by adjusting its production or by adjusting its consumption. Firms with access to such flexibility in the spot market are denoted as *dispatchable firms*, and firms without such flexibility are labelled as *non-dispatchable firms*. Consequently, the concept pairwise efficiency captures two types of relations. When the pair consists of two dispatchable firms, the relation in the forward market is horizontal. Alternatively, trade between a dispatchable and a non-dispatchable firm constitutes a vertical relation.

Horizontal relations—The analysis of horizontal relations generates the following finding: strategic dispatchable firms are either all net buyers or all net sellers in the spot market. The intuition is as follows. If the finding would not hold, the market would be a so-called *bilateral oligopoly*, where net buyers exercise market power

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[5] For an exposition of the argument, see Bagwell (1995) who shows that small amounts of noise can eliminate a firm's first-mover advantage. Hughes and Kao (1997) study the implications of unobservability in the context of forward markets.

[6] See e.g. the studies by Wolak (2000), Mansur (2007), Bushnell et al. (2008), Hortacsu and Puller (2008), Hendricks and McAfee (2010), and Hortacsu et al. (2019).

[7] This is unlike models of forward trade in the style of Allaz and Vila (1993), where producers can trade with speculators but not with each other. In Spiegel (1993), Anderson and Hu (2008), Coutinho (2013), and Van Moer (2019), strategic firms can trade forward with each other, but the spot market consists of only strategic sellers or only strategic buyers.

by underconsuming and net sellers exercise market power by underproducing as compared to the social optimum (see Hendricks and McAfee (2010) on a theory of bilateral oligopoly). Firms would thus suffer from a deadweight loss: in equilibrium the net buyers' willingness to pay exceeds the net sellers' marginal cost.

Now consider a pairwise contract between a net buyer and a net seller. The net buyer, by purchasing an additional unit from the net seller in the forward market, reduces its volume exposed to the spot market price. Consequently, it has fewer incentives exercise buyer power. Similarly, the net seller, by selling an additional unit in the forward market, also reduces its volume exposed to the spot market price and has fewer incentives to exercise seller power.

Such a pairwise contract is always profitable for two reasons. First, cost-efficiency improves by bringing firms' markups—and hence firms' marginal costs—closer to each other. Second, bilateral oligopoly markets have the unique feature that firms, when exercising market power, impose negative externalities on the firms that have net positions of the oppose sign. In particular, a net buyer is hurt by a net seller's exercise of seller power and a net seller is hurt by a net buyer's exercise of buyer power. By taking positions of the same sign, firms rule out these negative competitive externalities. Pairwise trade in the forward market can thus be profitable as well as pro-competitive by avoiding the inefficiencies associated with bilateral oligopoly.

The analysis thus predicts that renewable energy shocks are processed through the forward market in a way such that all firms have a net position of the same sign. This indicates a role for forward trade to absorb firm-specific shocks and spread them across multiple firms. This prediction, which relies on imperfect competition, would also follow from a model of perfect competition where forward trade is motivated by risk-aversion. In those models, too, there would be incentives for firms to spread risks by aligning their net positions.[8] The strategic incentives for

[8] For seminal work on hedging and uncertainty, see Holthausen (1979) and Bessembinder and Lemmon (2002).

forward trading reported in this paper can thus complement incentives for forward trading that relate to risk hedging.

The analysis also has implications for the design of short-term electricity markets with renewables. The model fits this application as follows: firms can trade forward in the intraday timeframe and the spot market is the real-time market. Firms' renewable energy generation forecasts typically become precise only the last hours before real-time.[9] This may hinder firms to trade pairwise-efficiently, an issue illustrated in figure 1. The upper part of the diagram displays the scenario where intraday trading is no longer possible after firms learn about idiosyncratic shocks to their renewable energy portfolios. If so, firms who suffer from adverse shocks to their portfolios are no longer able to trade with others who enjoy positive shocks. In other words, unexpected shocks to firms' renewable energy portfolios can cause firms' forward contracts to violate pairwise efficiency. Consequently, the real-time market risks being a bilateral oligopoly. Extending the intraday trading timeframe closer to real-time remedies this issue by facilitating firms to trade with more accurate information, as depicted in the lower diagram. Firms' positions at the end of the intraday trading timeframe are then less likely to violate pairwise efficiency. In this way, a late intraday gate closure time reduces the inefficiencies associated with bilateral oligopoly.[10]

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[9] Wind power generation forecasts, for example, are known to become substantially more precise between six and one hour before physical delivery (see e.g. Herrero et al., 2018). Consistent with this, in Europe, ACER reports that 68% of the trade in half-hourly products occurs between 60 and 120 minutes before delivery (see https://bit.ly/2R7ppa1).

[10] In Europe, there is a trend to facilitate firms to trade electricity shorter before physical delivery, often up to only five minutes before real-time. This differs from the U.S. tradition where intraday trading is less common (see Herrero et al., 2018). This suggests that European real-time markets may be less prone to being bilateral oligopolies than their U.S. counterparts.

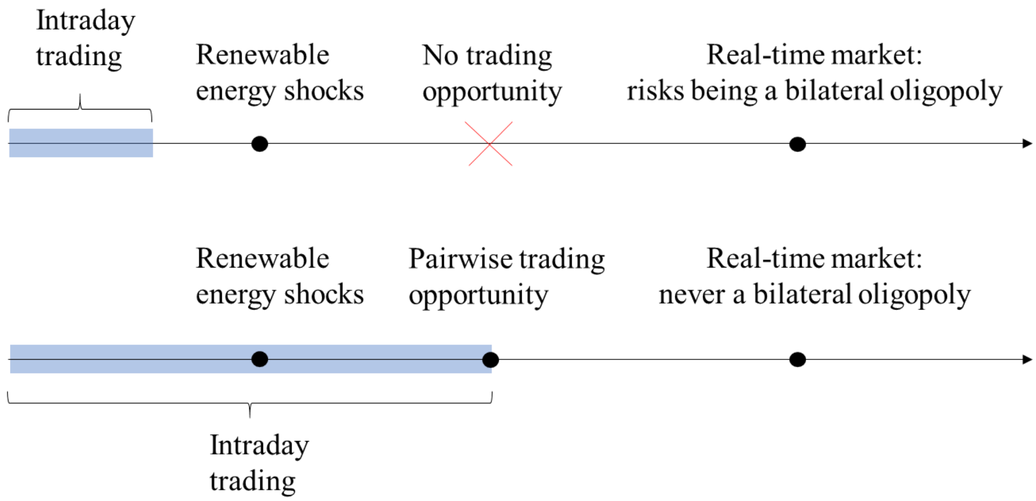

Figure 1: early vs. late intraday gate closure time.

Vertical relations—The analysis of vertical relations finds that non-dispatchable firms do not incur a shortage nor a surplus in the spot market. Pairwise efficient trade essentially mutes their role in the spot market: the pair optimally makes the dispatchable firm the residual claimant on all variable profits.

This result contributes to the debate on whether short-term electricity markets should allow *virtual bidders*—market participants without physical assets (see e.g. Birge et al. (2018) and Jha and Wolak (2019)). The usual concern by policymakers is that these market participants would reduce system reliability and predictability. This paper shows that under pairwise efficiency, firms without dispatchable physical assets, despite acting strategically, do not contribute to the system imbalance.

The result also predicts how renewable energy generators who lack flexibility in the real-time market deal with shocks to their portfolio. If they are sophisticated enough to trade pairwise-efficiently in the forward market, they trade to *not* incur imbalances in real-time. This insight can also be phrased normatively: firms without access to flexibility should attempt to close their position at the end of the intraday market timeframe.

Finally, this paper combines the analysis of horizontal and the analysis of vertical relations. The former analysis shows that strategic dispatchable firms do not transact with each other in the spot market. The latter analysis shows that strategic non-dispatchable firms incur a net position equal to zero in the spot market. Combining both results, at least one "side" of the spot market—the net buyers or the net sellers—have either foregone profitable forward trading opportunities, or have behaved sub-optimally in the spot market. There can be several explanations for such an outcome. First, there may be frictions such as uncertainty or asymmetric information that hinder firms to trade pairwise efficiently. Second, firms may fail to trade pairwise efficiently or to profit-maximize due to limited strategic ability. The latter explanation is in line with findings by Hortacsu and Puller (2008) and Hortacsu et al. (2019), who study heterogeneity in firms' strategic ability in the Texas real-time electricity market.

The remainder of this paper is phrased to fit its application to short-term electricity markets. The plan is as follows. Section 2 models the real-time market and demonstrates the bilateral oligopoly problem. Section 3 analyzes pairwise efficient forward trade and proves the main results. Section 4 provides discussion. Section 5 concludes.

2. The real-time market

This section models the real-time market for exogenous forward contract positions. We start by modelling the components of firms' real-time market trade volumes and proceed by modelling and analyzing the strategic interaction in the real-time market.

The components of firms' real-time market trade volumes

A firm's trade volume in the real time market is determined by (1) its forward contract position, (2) its non-dispatchable physical assets, and (3) its dispatchable physical assets.

1. *Forward contract position*

First, firms can have forward contracts in their portfolio.[11] Firm *i*'s net forward purchases are denoted by $\tilde{q}_i$ and its net financial transfer received in the forward market is denoted by t_i. We allow variable $\tilde{q}_i$ to be negative, in which case the firm is a net seller in the forward market. Since any commitment taken up by one firm is an asset held by another firm, the sum of all forward contract positions equals zero, or $\sum \tilde{q}_i = 0$.

2. *Non-dispatchable physical assets.*

Second, firms can have physical assets in their portfolio that are non-dispatchable. For example, some power facilities are prohibitively expensive to shut down (e.g. nuclear power plants), making them non-dispatchable in the short-run. As another example, technologies such as wind power and solar power, if they enjoy dispatch priority, are also non-dispatchable. Also, a consumption portfolio can consist of households who consume inflexibly due to the absence of smart-metering.

We denote firm i's net surplus from its non-dispatchable portfolio by $\overline{q}_i$. This surplus is defined as the difference between firm i's injections originating from its non-dispatchable production assets and firm i's offtakes originating from its non-dispatchable consumption assets. We allow $\overline{q}_i$ to be negative, in which case it should be interpreted as a net shortage.

3. *Dispatchable physical assets*

Finally, firms can have generation or consumption assets in their portfolio that are dispatchable. For example, gas-fired power plants can be called upon to produce,

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[11] Whether contracts are physical or financial does not alter the analysis.

depending on market conditions. As another example, flexibility could come from demand-side response, enabling firms to reduce consumption when needed.

Firm i 's net surplus from its dispatchable portfolio is denoted by q_i . We allow q_i to be negative, in which case it should be interpreted as a net shortage.

Firm i 's cost to attain a net surplus of q_i with its dispatchable portfolio is denoted by $C_i\left(q_i\right)$, assumed twice continuously differentiable, and characterized by strictly increasing marginal costs ($C_i^{''}\left(q_i\right) > 0$).

The increasing marginal cost curve captures that firms dispatch their dispatchable assets following their merit-order. For example, each firm activates its cheapest generation units first before using more expensive technologies. Likewise, each firm first decreases consumption from its consumption assets characterized by the lowest willingness to pay for electricity, before switching off consumption assets characterized by a higher willingness to pay. Importantly, the marginal cost curve represents dispatchable production units as well as dispatchable consumption units: flexibility can originate both from the supply side and the demand side.

We distinguish two types of firms: dispatchable firms and non-dispatchable firms.

Dispatchable firms: *firms who have dispatchable production or consumption assets in their portfolio.*

Non-dispatchable firms: *firms who do not have dispatchable production or consumption assets.*

When firm i is non-dispatchable, we write that $q_i = 0$ and $C_i\left(0\right) = 0$.

We can summarize that firm i 's net surplus in the real-time market is determined by its financial and physical assets and equals

$$Q_i \equiv \tilde{q}_i + \overline{q}_i + q_i \, .$$

Figure 2 visualizes an example marginal cost curve of a dispatchable firm.

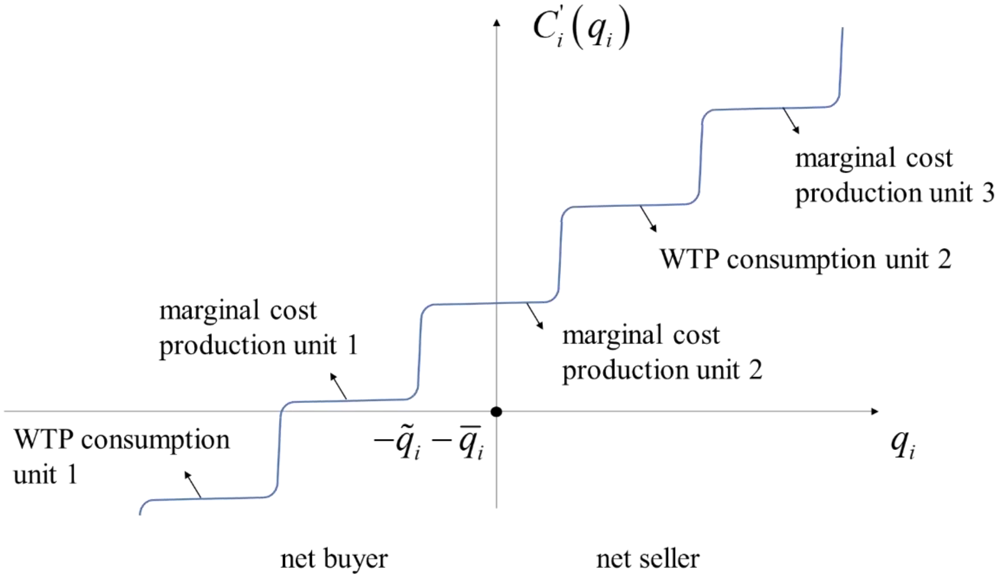

Figure 2: an example marginal cost curve of a dispatchable firm.

Several observations are useful to point out. First, the firm is a *net buyer* in the real-time market when $Q_i < 0 \Leftrightarrow q_i < -\tilde{q}_i - \overline{q}_i$, and is a *net seller* in the real-time market when $Q_i > 0 \Leftrightarrow q_i > -\tilde{q}_i - \overline{q}_i$. Second, the marginal cost at which the firm produces $q_i = -\tilde{q}_i - \overline{q}_i$, the amount needed to be balanced, equals $C_i\left(-\tilde{q}_i - \overline{q}_i\right)$. This marginal cost is larger when the firm has a lower net surplus from its non-dispatchable portfolio ($\overline{q}_i$). Third, the firm's marginal cost can originate from a production unit or a consumption unit, both when the firm is a net buyer and when the firm is a net seller in the real-time market.

Strategic interaction in the real-time market

The real-time market balances electricity injections and offtakes such that the following equation holds.

Balanced electricity system equation: *a balanced electricity system requires that the physical surplus aggregated over all firms equals zero, or*

$$\sum\left(\overline{q}_i + q_i\right) = 0.$$

There is a uniform price, denoted by P, that firms receive for their net surplus or pay for their net shortage. We will distinguish non-strategic firms from strategic firms. The non-strategic firms act as a fringe and are represented as being one firm, indexed $i = f$. The N strategic firms are indexed by $i = 1,...,N$ and interact as Cournot competitors.[12]

1. *Non-strategic fringe*

The non-strategic fringe is defined as follows.

Non-strategic fringe: *firms who, in the real-time market, bid energy from their physical assets independently from firms' forward contract positions.*

This definition includes the following two specific forms of non-strategic behavior. First, it includes the possibility that non-strategic firms behave competitively and bid marginal cost. So, the non-strategic firms could be interpreted as a competitive price-taking fringe. Second, the set of non-strategic firms can also include firms who produce and consume fixed amounts with their physical assets, regardless of market conditions.

We model the net surplus of the fringe's physical assets, $\overline{q}_f + q_f$, as a linearly increasing function of the uniform price P as follows:

$$\overline{q}_f + q_f = -\frac{a}{b} + \frac{P}{b},$$

where a and $b > 0$ are constants. We can rewrite the balanced electricity system equation to plug in $\overline{q}_f + q_f = -\sum_{i \in N} \left(\overline{q}_i + q_i \right)$. We obtain

$$P = a - b \times \sum_{i \in N} \left(\overline{q}_i + q_i \right).$$

This shows that the schedule offered by the non-strategic fringe determines the inverse demand function faced by the strategic firms.

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[12] The discussion section motivates this approach.

2. *Strategic firms*

We are now ready to characterize the behavior of the strategic firms in the real-time market. Strategic firms are defined as follows.

Strategic firms: *firms who are profit-maximizers in the real-time market.*

We subsequently analyze strategic non-dispatchable firms and strategic dispatchable firms.

a. *Strategic non-dispatchable firms*

Strategic non-dispatchable firms do not have dispatchable physical assets available for strategic use. When firm i is non-dispatchable, its net surplus in the real-time market equals $\tilde{q}_i + \overline{q}_i$.

b. *Strategic dispatchable firms*

The strategic dispatchable firms compete à la Cournot in the real-time market. Each strategic dispatchable firm simultaneously decides about the net surplus of its dispatchable portfolio. When firm i is dispatchable, its variable profits can be written as

$$\pi_i = t_i + P \times \underbrace{\left(\tilde{q}_i + \overline{q}_i + q_i \right)}_{Q_i} - C_i\left(q_i\right).$$

Since the inverse demand curve is linear and the marginal cost curve is strictly increasing, the first-order conditions for maximization with respect to q_i are necessary and sufficient. They equal

$$P - b \times \left(\tilde{q}_i + \overline{q}_i + q_i^* \right) = C_i^{'}\left(q_i^*\right).$$

This shows that dispatchable firms with a positive net surplus produce at a marginal cost *below* the market-clearing price. In contrast, dispatchable firms with a negative net surplus produce at a marginal cost *above* the market-clearing price.

Real-time market is a bilateral oligopoly: *at least one strategic dispatchable firm is a net buyer and at least one strategic dispatchable firm is a net seller.*

An important feature in electricity markets is that firms' injections and offtakes originating from non-dispatchable physical assets are subject to shocks. For example, household demand for electric heating and cooling is subject to temperatures. Also, wind power generation requires the wind to blow, and generation from solar panels depends on the amount of sunshine. Result 1 states that this establishes the potential for bilateral oligopoly.

Result 1: *If firms cannot reconsider their forward contract positions after learning about renewable energy shocks, the real-time market can be a bilateral oligopoly.*

The appendix offers a proof by example. If forward markets close early, firms can no longer reconsider their forward contract position after learning about late idiosyncratic shocks to their portfolios caused by renewables. As a result, the real-time market can be composed of strategic dispatchable firms who are net buyers due negative shocks (unexpectedly low $\overline{q}_i$) and other strategic dispatchable firms who are net sellers because of positive shocks (unexpectedly high $\overline{q}_i$).

3. Pairwise efficient forward trade

This section investigates pairwise efficient forward trade among strategic firms. Pairwise efficiency requires firms' forward contract positions to be such that each pair of strategic firms cannot gain by engaging in additional bilateral trade, taking as given the forward contract positions of other firms in the industry, and considering unilaterally optimal behavior in the real-time market. The bilateral contract is unobservable to firms that do not take part in the contract. This corresponds to the notion of private or secret contracting familiar from the vertical relations literature.

We distinguish the following types of pairs:

- ***Horizontally-related pair:*** consists of two strategic dispatchable firms.

- ***Vertically-related pair:*** consists of a strategic dispatchable and a strategic non-dispatchable firm.
- For completeness, we also investigate pairs consisting of two strategic non-dispatchable firms.

The following notation, depicted in table 1, distinguishes the different types of pairs. Firm 1 and firm 2 are representative examples of strategic dispatchable firms. Firm 3 and firm 4 are representative examples of strategic non-dispatchable firms. Firm 1 and firm 2 then form a representative horizontally-related pair. Firm 1 and firm 3 form a representative vertically-related pair. Finally, firm 3 and firm 4 form a representative pair of non-dispatchable firms.

	firm 1 dispatchable	firm 4 non-dispatchable
firm 2 dispatchable	*horizontally-related pair*	
firm 3 non-dispatchable	*vertically-related pair*	*pair of strategic non-dispatchable firms*

Table 1: representative pairs of firms

We proceed by analyzing (1) a horizontally-related pair, (2) a vertically-related pair, and (3) a pair of strategic non-dispatchable firms.

1. Horizontally-related pair

We consider pairwise trade between firm 1 and firm 2.[13] We denote firms' *candidate* net forward contract positions by an asterisk (*): $\tilde{q}_1^*$ for firm 1 and $\tilde{q}_2^*$ for firm 2. Next, we investigate whether firms have incentives to sign an additional forward contract that commits firm 1 to purchase γ additional units from firm 2. Consequently, $\tilde{q}_1 = \tilde{q}_1^* + \gamma$ and $\tilde{q}_2 = \tilde{q}_2^* - \gamma$. The choice about which firm acts as

[13] The techniques used in this subsection partly draw from Spiegel (1993)'s analysis. My model differs by studying more than two firms and, crucially, by studying the possibility of bilateral oligopoly.

seller and which firm acts as buyer is without loss of generality because γ can be positive or negative.

Two remarks are useful. First, the size of the financial transfer between the two firms is zero-sum from a bilateral point of view. Since the transfer paid by one firm is received by the other firm, the sum $t_1 + t_2$ is unaffected. Moreover, the financial transfer occurs in the forward market and is regarded as sunk when firms compete in the real-time market. Consequently, the transfer does not determine equilibrium behavior; it is not part of the first-order conditions that characterize q_1^* or q_2^*. Second, since contracts are private, the behavior of outsiders to the contract is independent of γ. We can thus write that $q_j^*(\gamma) = q_j^*$ for $j \neq 1, 2$.

Firms' variable profits are

(1) $$\pi_1 = t_1 + PQ_1 - C_1\left(q_1^*(\gamma)\right)$$

(2) $$\pi_2 = t_2 + PQ_2 - C_2\left(q_2^*(\gamma)\right),$$

where

$$P = a - b\left[\bar{q}_1 + q_1^*(\gamma) + \bar{q}_2 + q_2^*(\gamma) + \sum_{\substack{j \in N \\ j \neq 1,2}} \left(\bar{q}_j + q_j^*\right)\right]$$

$$Q_1 = \tilde{q}_1^* + \gamma + \bar{q}_1 + q_1^*(\gamma)$$

$$Q_2 = \tilde{q}_2^* - \gamma + \bar{q}_2 + q_2^*(\gamma).$$

The pair of firms maximizes profits with respect to λ. Bilateral profits equal (1) + (2). Maximizing with respect to γ, we obtain the necessary first-order condition

$$\frac{d\left(\pi_1(\gamma) + \pi_2(\gamma)\right)}{d\gamma} = 0,$$

which can be written as

$$\underbrace{\frac{\partial \pi_1}{\partial \gamma} + \frac{\partial \pi_2}{\partial \gamma}}_{=0} + \underbrace{\frac{\partial \pi_1}{\partial q_1^*}}_{=0} \frac{dq_1^*(\gamma)}{d\gamma} + \frac{\partial \pi_1}{\partial q_2^*} \frac{dq_2^*(\gamma)}{d\gamma} + \frac{\partial \pi_2}{\partial q_1^*} \frac{dq_1^*(\gamma)}{d\gamma} + \underbrace{\frac{\partial \pi_2}{\partial q_2^*}}_{=0} \frac{dq_2^*(\gamma)}{d\gamma} = 0 .$$

The first two terms represent the direct effects, which equal $\frac{\partial \pi_1}{\partial \gamma} + \frac{\partial \pi_2}{\partial \gamma} = P - P = 0$. Moreover, we know that firm 1 and 2's quantity choices in the real-time market satisfy their first-order conditions, so that $\frac{\partial \pi_1}{\partial q_1^*} = \frac{\partial \pi_2}{\partial q_2^*} = 0$ (the envelope theorem). Therefore, we obtain that pairwise efficiency requires

(3) $$\frac{\partial \pi_1}{\partial q_2^*} \frac{dq_2^*(\gamma)}{d\gamma} + \frac{\partial \pi_2}{\partial q_1^*} \frac{dq_1^*(\gamma)}{d\gamma} = 0 .$$

Firms' bilateral profits $\pi_1(\gamma) + \pi_2(\gamma)$ are not necessarily concave with respect to γ .[14] Equation (3) thus represents a necessary condition for pairwise efficiency, but does not serve as a sufficient condition. Nevertheless, the following result, proven in the appendix, is a powerful one that generally holds.

Result 2: *Under pairwise efficient forward trade, the real-time market is never a bilateral oligopoly.*

The intuition is as follows. Suppose it would be a bilateral oligopoly. Then, there must exist a pair of firms that consists of a net buyer in the real-time market and a net seller in the real-time market. Without loss of generality, denote the net buyer in the real-time market by firm 1 and denote the net seller in the real-time market by firm 2.

Firms can then always gain from letting firm 1 purchase from firm 2 in the forward market. By doing so, firm 2's net surplus in the real-time market shrinks, so that it has fewer incentives to exercise seller power in the real-time market. This effect benefits firm 1, who is a net buyer in the real-time market, and is formally

[14] See Van Moer (2019).

represented by the first term of equation (3). Also, the forward contract reduces firm 1's volume purchased in the real-time market, which reduces its incentives to exercise buyer power. This effect benefits firm 2, who is a net seller in the real-time market, and is formally represented by the second term of equation (3). So, the forward contract reduces the competitive externalities that firms inflict upon each other, which is mutually beneficial. For this reason, firms always have incentives to sign forward contracts to rule out a bilateral oligopoly real-time market.

2. Vertically-related pair

We next consider pairwise trade between firm 1 and firm 3. We apply the same techniques: we denote firms' *candidate* net forward contract positions by an asterisk, and we suppose without loss of generality that firm 1 purchases γ units from firm 3, so that $\tilde{q}_1 = \tilde{q}_1^* + \gamma$ and $\tilde{q}_3 = \tilde{q}_3^* - \gamma$. As before, since contracts are private, the real-time market behavior of outsiders to the contract is independent of γ. We can thus write that $q_j^*(\gamma) = q_j^*$ for $j \neq 1,3$.

Firms' variable profits equal

$$\pi_1 = t_1 + PQ_1 - C_1\left(q_1^*(\gamma)\right) \tag{4}$$

$$\pi_3 = t_3 + PQ_3, \tag{5}$$

where

$$P = a - b\left(\bar{q}_1 + q_1^*(\gamma) + \sum_{\substack{j \in N \\ j \neq 1}} \left(\bar{q}_j + q_j\right)\right)$$
$$Q_1 = \tilde{q}_1^* + \gamma + \bar{q}_1 + q_1^*(\gamma)$$
$$Q_3 = \tilde{q}_3^* - \gamma + \bar{q}_3.$$

The pair of firms maximizes the gains from bilateral trade with respect to λ. In other words, firms should not be able to increase bilateral profits (4) + (5) by altering γ. We obtain the necessary first-order condition

$$\underbrace{\frac{\partial \pi_1}{\partial \gamma}+\frac{\partial \pi_3}{\partial \gamma}}_{=0}+\underbrace{\frac{\partial \pi_1}{\partial q_1^*}}_{=0}\frac{dq_1^*(\gamma)}{d\gamma}+\frac{\partial \pi_3}{\partial q_1^*}\frac{dq_1^*(\gamma)}{d\gamma}=0.$$

As before, the first two terms represent the direct effects, which sum up to zero. The third term is zero because of the envelope theorem. Consequently, additional forward trade affects bilateral profits only through firm 1's quantity choice. We work out and obtain

(6) $$\left[-b\left(\tilde{q}_3^*-\gamma+\bar{q}_3\right)\right]\frac{dq_1^*(\gamma)}{d\gamma}=0.$$

We are now ready to state result 3.

Result 3: *Under pairwise efficient forward trade, the strategic non-dispatchable firms have a net surplus of zero in the real-time market.*

The proof is in the appendix. If result 3 would not hold, firm 1 would pose an externality on firm 3 because its quantity decision affects the price that firm 3 receives or pays in the real-time market. Under pairwise efficiency, firms sign the forward contract that eliminates this externality by closing firm 3's position.

3. Pair of non-dispatchable firms

Finally, for completeness, consider pairwise trade between two strategic non-dispatchable firms 3 and 4. As before, we investigate firms' incentives to trade γ additional units in the forward market, so that $\tilde{q}_3=\tilde{q}_3^*+\gamma$ and $\tilde{q}_4=\tilde{q}_4^*-\gamma$. Firms' bilateral profits equal

$$\pi_3+\pi_4=t_3+t_4+\underbrace{\left(a-b\sum_{j\in N}\left(\bar{q}_j+q_j^*\right)\right)}_{P}\underbrace{\left(\tilde{q}_3^*+\gamma+\bar{q}_3+\tilde{q}_4^*-\gamma+\bar{q}_4\right)}_{Q_3+Q_4},$$

and are unaffected by γ.

We can now combine Result 2 and Result 3. From result 2, we know that strategic dispatchable firms have incentives to trade with each in the forward market rather than in the real-time market, in order to avoid inefficiencies from bilateral oligopoly. Result 3 states that strategic non-dispatchable firms close their position entirely in the forward market. Consequently, the counterparties of strategic firms in the real-time market cannot be strategic firms themselves. This insight is stated in Result 4.

Result 4: *All net buyers or all net sellers in the real-time market have either foregone profitable pairwise forward trading opportunities or have behaved sub-optimally in the real-time market.*

When firms act strategically, pairwise efficient forward trading crowds out trading in the real-time market. Consequently, observing that firms take large net positions in the real-time market indicates frictions. These frictions can for example originate from firms failing to trade pairwise efficiently due to uncertainties or asymmetric information. Alternatively, they can relate to firms having limited strategic capacities, a feature that has been documented in electricity markets by Hortacsu and Puller (2008) and Hortacsu et al. (2019).

4. Discussion

Real-time markets in practice

In this subsection, I describe how real-time markets work in practice and motivate the model of Cournot competition.

The organizations responsible for the balance between electricity injections and offtakes on the high-voltage grid are called system operators. When the system risks being long, maintaining a balanced grid requires a production-decrease or a consumption-increase. When the system risks being short, maintaining a balanced grid requires a production-increase or a consumption-decrease. In most countries, due to unbundling, system operators are not allowed to own or operate physical production or consumption assets. They therefore procure energy by means of auctions. The bids offered by firms are stacked from cheap to expensive in a so-called merit order. As more energy is required to balance the system, the bid that is activated on the margin is a more expensive one. System operators recover the costs of activating these bids by holding market participants responsible for the surplus or shortage they contribute to the system. In most countries, there is a uniform price that firms receive for each MWh of surplus or pay for each MWh of shortage they incur, although variations of this mechanism exist in some countries. The uniform price is typically determined by the system operator's cost of activating the marginal bid.

1. Auction-based balancing

In markets with auction-based balancing, the decisions to activate and deactivate dispatchable assets are all determined by centralized auctions.[15] The relevant framework to analyze competition under this regime is supply function competition (Klemperer and Meyer, 1989). In the absence of uncertainty, models of supply function competition are known to be characterized by multiple equilibria. The Cournot outcome analyzed in this paper represents one of them and is the one most preferred by firms. The feature of the Cournot model that dispatchable firms who are net sellers "underproduce" as compared to the social optimum corresponds to them bidding "positive markups" in the auction. Likewise, "underconsumption" by net buying firms corresponds to them bidding "negative markups" in the auction.

2. Decentralized balancing

Under decentralized balancing, firms can decentrally activate or deactivate the dispatchable assets they did not commit in the auction.[16] The bids submitted in the auction then serve to resolve the residual need for balancing. Under this regime, the

[15] Examples of markets with auction-based balancing include the centralized dispatch systems in the U.S. and e.g. Spain in Europe.
[16] Several European countries, such e.g. as Belgium, the Netherlands and Germany, are characterized by decentral balancing.

Cournot model has a natural interpretation by capturing firms' choices how much to produce decentrally.[17]

Possible frictions that can hinder pairwise efficiency

In this subsection, I discuss two possible sources of frictions that determine how well pairwise efficient forward trade holds in practice.

A first source of possible frictions is uncertainty. Whether uncertainty hinders pairwise efficiency depends on whether the uncertainty reveals *before* or *after* the intraday gate closure time. In case the information updates arrive before the intraday gate closure time, firms still have opportunities to respond to them and alter their forward contract positions pairwise-efficiently. In contrast, in case the information updates arrive after the intraday gate closure time, firms can no longer alter their forward contract positions. Consequently, markets with a later intraday gate closure time are likely to be better characterized by pairwise efficient forward trading than markets with an earlier intraday gate closure time. For example, it is conceivable that most uncertainties have revealed by an intraday gate closure time at five minutes before real-time. However, an earlier intraday gate closure time would leave more uncertainties unresolved.

A second source of possible frictions that can interact with pairwise efficiency is asymmetric information. A deep analysis on this subject would need to account for the market microstructure and is beyond the scope of this study. Nevertheless, it is useful to point out several factors that can be expected to reduce information asymmetries. First, asymmetric information regarding renewable energy generation would be reduced by publicly available weather forecasts. Second, many countries have regulations that require firms to report plant outages, thereby reducing asymmetric information regarding such events. Finally, intraday market prices serve as a public signal for abundance or scarcity. Therefore, intraday markets,

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[17] Firms can prefer balancing decentrally over participating in the auction because, by doing so, they keep their rivals' residual demand functions as price-inelastic as possible. This induces their rivals to compete less aggressively (see Singh and Vives, 1984).

particularly when liquid, contribute to diffusing information across market participants quickly.

5. Conclusions

This paper has investigated the implications of pairwise efficient forward contracting in wholesale electricity markets. The analysis has distinguished different types of pairs. A horizontally-related pair consists of two firms with access to flexibility in the spot market, and a vertically-related pair consists of one firm with flexibility and another firm without flexibility in the spot market.

The analysis of horizontal relations has shown a new efficiency rationale for forward trade: firms have incentives to trade forward to avoid a so-called *bilateral oligopoly* spot market, where firms incur net positions of the opposite sign. Net buyers and net sellers, by exercising market power, impose negative externalities on each other. Forward trading to remove these externalities is always in firms' joint interest and is pro-competitive by reducing markups.

This strategic motive for forward trading is distinct from the strategic commitment mechanism emphasized in previous studies, which requires all contracts to be observable. It is also distinct from forward trading to hedge risk, the rationale traditionally emphasized in the field of finance. The analysis has found that forward trading to avoid bilateral oligopoly reduces firms' volumes exposed to the spot market price. Consequently, when firms trade for the strategic motive presented in this paper, they may also mitigate their exposure to possible price risk. Both forward trading motives can thus complement each other.

The result also sheds new light on how to design short-term electricity markets. In particular, it points out a new channel through which late intraday market gate closure times can improve efficiency in markets with renewables: when firms face idiosyncratic shocks to their portfolios, a late trading opportunity reduces the risk of a bilateral oligopoly real-time market.

The analysis of vertical relations has indicated that firms who have access to flexibility, when deciding about how competitively to act, exert externalities on the firms without flexibility. Pairwise efficient forward contracting entirely removes these externalities by closing the positions of the firms without flexibility. This insight predicts that virtual bidders, who do not have access to physical generation or consumption assets, do not cause real-time imbalances between electricity injections and offtakes.

Both results combined imply that, if pairwise forward trade is frictionless, strategic market participants have a strict preference for trading in the forward market rather than in the spot market. Consequently, non-zero net positions in the spot market indicate that some firms have either foregone profitable pairwise forward trading opportunities, did not behave profit-maximizingly in the spot market, or both. This outcome could be explained by frictions related to uncertainty, asymmetric information, or by firms having limited strategic abilities.

Finally, the results of this paper also offer advice to new market participants who are designing their short-term electricity market trading strategies: it is not profit-maximizing to sell in the real-time market when competitors are short (and the price is high) and to buy when competitors are long (and the price is low). Instead, firms should search for bilateral forward trading opportunities to avoid net positions of the opposite sign in the real-time market. Moreover, for renewable energy generators who lack flexibility, pairwise trading to minimize imbalances in the real-time market is always a solid business strategy, even absent risk management considerations.

6. References

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7. Appendix

Proof of result 1.

We construct an example that proves result 1.

The example

There are two strategic dispatchable firms, firm 1 and firm 2. The non-dispatchable firms, for simplicity, have a zero net surplus in the real-time market. The fringe's net surplus in the real-time market equals

$$Q_f = \tilde{q}_f + \overline{q}_f + q_f = \tilde{q}_f - \frac{a}{b} + \frac{P}{b}.$$

Consequently, the fringe is a (weak) net buyer if $P \le a - b \times \tilde{q}_f$ and a net seller if $P > a - b \times \tilde{q}_f$.

We construct the example so that firm 1 enjoys a positive shock to its non-dispatchable portfolio and firm 2 suffers from a negative shock to its non-dispatchable portfolio. We will argue that as a result of these shocks, firm 1 acts as a net seller and firm 2 acts as a net buyer: the real-time market is a bilateral oligopoly.

More specifically, suppose that firm 1 enjoys a positive shock to its non-dispatchable portfolio such that the marginal cost at which it would produce the amount needed to be balanced ($q_1 = -\tilde{q}_1 - \overline{q}_1$) satisfies

$$C_1^{'}\left(-\tilde{q}_1 - \overline{q}_1\right) < a - b \times \tilde{q}_f .$$

Firm 2 suffers from a negative shock to its non-dispatchable portfolio such that the marginal cost at which it produces the amount needed to be balanced ($q_2 = -\tilde{q}_2 - \overline{q}_2$) satisfies

$$C_2^{'}\left(-\tilde{q}_2 - \overline{q}_2\right) > a - b \times \tilde{q}_f .$$

Analysis

Without loss of generality, we can state that the market-clearing price satisfies one of two possibilities: $P \le a - b \times \tilde{q}_f$ or $P > a - b \times \tilde{q}_f$. In both scenarios we will show that the market is a bilateral oligopoly.

1. $P \le a - b \times \tilde{q}_f$.

Since $P \le a - b \times \tilde{q}_f$, the fringe is a (weak) net buyer in the real-time market.

We prove that firm 2 is a (strict) net buyer in the real-time market by contradiction. Suppose that firm 2 would not be a (strict) net buyer: $\tilde{q}_2 + \overline{q}_2 + q_2^* \ge 0$. Then, we would have $C_2^{'}\left(q_2^*\right) \ge C_2^{'}\left(-\tilde{q}_2 - \overline{q}_2\right) > a - b \times \tilde{q}_f$. We plug in this inequality in firm

2's first-order condition and obtain $P - b \times \left(\tilde{q}_2 + \overline{q}_2 + q_2^* \right) > a - b \times \tilde{q}_f$. Since $P \leq a - b \times \tilde{q}_f$, we get $-b \times \left(\tilde{q}_2 + \overline{q}_2 + q_2^* \right) > 0$, which contradicts $\tilde{q}_2 + \overline{q}_2 + q_2^* \geq 0$.

Therefore, the fringe and firm 2 buy a jointly positive net volume in the real-time market. Consequently, the balanced electricity system equation requires that firm 1 is a net seller in the real-time market. This shows that the real-time market is a bilateral oligopoly.

2. $P > a - b \times \tilde{q}_f$.

Since $P > a - b \times \tilde{q}_f$, the fringe is a net seller in the real-time market.

We prove that firm 1 is a (strict) net seller in the real-time market by contradiction. Suppose that firm 1 would not be a (strict) net seller: $\tilde{q}_1 + \overline{q}_1 + q_1^* \leq 0$. Then, we would have $C_1^{'}\left(q_1^* \right) \leq C'\left(-\tilde{q}_1 - \overline{q}_1 \right) < a - b \times \tilde{q}_f$. We can plug in this inequality in firm 1's first-order condition and obtain $P - b \times \left(\tilde{q}_1 + \overline{q}_1 + q_1^* \right) < a - b \times \tilde{q}_f$. Since $P > a - b \times \tilde{q}_f$, we get $-b \times \left(\tilde{q}_1 + \overline{q}_1 + q_1^* \right) < 0$, which contradicts $\tilde{q}_1 + \overline{q}_1 + q_1^* \leq 0$.

Therefore, the fringe and firm 1 sell a jointly positive net volume in the real-time market. Consequently, the balanced electricity system equation requires that firm 2 is a net buyer in the real-time market. This shows that the real-time market is a bilateral oligopoly.

Proof of result 2.

The proof is by contradiction. Suppose there is a bilateral oligopoly. Then, there must exist a pair of firms that consists of a net buyer in the real-time market and a net seller in the real-time market. Without loss of generality, we denote the net buyer in the real-time market by firm 1 and denote the net seller in the real-time market by firm 2. Formally, we can write that $\tilde{q}_1^* + \gamma + \overline{q}_1 + q_1^*\left(\gamma \right) < 0$ and that $\tilde{q}_2^* - \gamma + \overline{q}_2 + q_2^*\left(\gamma \right) > 0$.

The contradiction is proven in two steps. Step 1, presented below, shows that, the direct effects in equation (3) have the following signs

$$\text{(A1)} \qquad \underbrace{\frac{\partial \pi_1}{\partial q_2^*}}_{>0} \frac{dq_2^*(\gamma)}{d\gamma} + \underbrace{\frac{\partial \pi_2}{\partial q_1^*}}_{<0} \frac{dq_1^*(\gamma)}{d\gamma}.$$

Step 2, presented below, proves that $\frac{dq_1^*(\gamma)}{d\gamma} < 0$ and $\frac{dq_2^*(\gamma)}{d\gamma} > 0$. Expression (A1) is thus strictly positive, which contradicts that equation (3) holds.

Step 1.

We can use (1) to write

$$\frac{\partial \pi_1}{\partial q_2^*} = -b\left(\tilde{q}_1^* + \gamma + \bar{q}_1 + q_1^*(\gamma)\right).$$

Since $\tilde{q}_1^* + \gamma + \bar{q}_1 + q_1^*(\gamma) < 0$, we know that $\frac{\partial \pi_1}{\partial q_2^*} > 0$.

Also, we can use (2) to write

$$\frac{\partial \pi_2}{\partial q_1^*} = -b\left(\tilde{q}_2^* - \gamma + \bar{q}_2 + q_2^*(\gamma)\right).$$

Since $\tilde{q}_2^* - \gamma + \bar{q}_2 + q_2^*(\gamma) > 0$, we know that $\frac{\partial \pi_2}{\partial q_1^*} < 0$.

Step 2.

Firm 1's first-order condition in the real-time market equals

$$\left(a - b\left[\bar{q}_1 + q_1^*(\gamma) + \bar{q}_2 + q_2^*(\gamma) + \sum_{\substack{j \in N \\ j \neq 1,2}} \left(\bar{q}_j + q_j^*\right)\right]\right)$$
$$-b\left(\tilde{q}_1^* + \gamma + \bar{q}_1 + q_1^*(\gamma)\right) - C_1'\left(q_1^*(\gamma)\right) = 0.$$

Totally differentiating with respect to γ gives

(A2) $$-b+\frac{dq_1^*(\gamma)}{d\gamma}\left(-2b-C_1^{"}\left(q_1^*(\gamma)\right)\right)+\frac{dq_2^*(\gamma)}{d\gamma}(-b)=0\,.$$

This shows that $\frac{dq_1^*(\gamma)}{d\gamma}$ and $\frac{dq_2^*(\gamma)}{d\gamma}$ are not simultaneously zero.

Firm 2's first-order condition in the real-time market equals

$$\left(a-b\left[\overline{q}_1+q_1^*(\gamma)+\overline{q}_2+q_2^*(\gamma)+\sum_{\substack{j\in N\\ j\neq 1,2}}\left(\overline{q}_j+q_j^*\right)\right]\right)$$
$$-b\left(\tilde{q}_2^*-\gamma+\overline{q}_2+q_2^*(\gamma)\right)-C_2^{'}\left(q_2^*(\gamma)\right)=0.$$

Totally differentiating with respect to γ gives

(A3) $$b+\frac{dq_1^*(\gamma)}{d\gamma}(-b)+\frac{dq_2^*(\gamma)}{d\gamma}\left(-2b-C_2^{"}\left(q_2^*(\gamma)\right)\right)=0\,.$$

Adding up (A2) and (A3), we obtain that

(A4) $$\frac{dq_1^*(\gamma)}{d\gamma}\underbrace{\left(-3b-C_1^{"}\left(q_1^*(\gamma)\right)\right)}_{<0}+\frac{dq_2^*(\gamma)}{d\gamma}\underbrace{\left(-3b-C_2^{"}\left(q_2^*(\gamma)\right)\right)}_{<0}=0\,.$$

Since marginal costs are increasing, we can sign the second and fourth term in equation (A4).

Moreover, since we know that $\frac{dq_1^*(\gamma)}{d\gamma}$ and $\frac{dq_2^*(\gamma)}{d\gamma}$ are not simultaneously zero, from (A4), *each* of these derivatives must be non-zero. As a result, from (A4), it must be true that $\frac{dq_1^*(\gamma)}{d\gamma}$ and $\frac{dq_2^*(\gamma)}{d\gamma}$ have an opposite sign.

Finally, we show that $\frac{dq_1^*(\gamma)}{d\gamma}<0$ and $\frac{dq_2^*(\gamma)}{d\gamma}>0$ by contradiction. Suppose not. Then, $\frac{dq_1^*(\gamma)}{d\gamma}>0$ and $\frac{dq_2^*(\gamma)}{d\gamma}<0$. Writing (A2) – (A3) then gives

$$-2b+\underbrace{\frac{dq_1^*(\gamma)}{d\gamma}\left(-b-C_1''\left(q_1^*(\gamma)\right)\right)}_{<0}+\underbrace{\frac{dq_2^*(\gamma)}{d\gamma}\left(b+C_2''\left(q_2^*(\gamma)\right)\right)}_{<0}=0,$$

a contradiction.

Proof of result 3.

Step one establishes that $\frac{dq_1^*(\gamma)}{d\gamma}<0$. Step two establishes that $\gamma=\tilde{q}_3^*+\overline{q}_3$.

Step one.

We start from firm 1's first-order condition in the real-time market, which equals

$$\underbrace{\left(a-b\left(\overline{q}_1+q_1^*(\gamma)+\sum_{\substack{j\in N\\ j\neq 1}}\left(\overline{q}_j+q_j\right)\right)\right)}_{P}-b\left(\tilde{q}_1^*+\gamma+\overline{q}_1+q_1^*(\gamma)\right)-C_1'\left(q_1^*(\gamma)\right)=0.$$

Totally differentiating with respect to γ gives

$$-b+\frac{dq_1^*(\gamma)}{d\gamma}\left(-2b-C_1''\left(q_1^*(\gamma)\right)\right)=0,$$

rewritten as

$$\frac{dq_1^*(\gamma)}{d\gamma}=\frac{b}{-2b-C_1''\left(q_1^*\right)}<0.$$

Step two.

The necessary condition for pairwise efficiency (6) is satisfied if and only if

$$-b\left(\tilde{q}_3^*-\gamma+\overline{q}_3\right)=0,$$

equivalently written as

$$\gamma=\tilde{q}_3^*+\overline{q}_3.$$