The Correlation between Time Preference and Incomes Is Spurious: They Are Bridged by Fluid Intelligence

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Abstract

The rate of time preference (RTP) has been observed to be negatively correlated with incomes, but the mechanism behind this correlation is not yet sufficiently understood. Here, I examine it on the basis of fluid intelligence in an economy in which households behave according to the maximum degree of comfortability. I show that heterogeneity in fluid intelligences among households causes heterogeneous RTPs and incomes at the same time. This means that the negative correlation between RTP and incomes is spurious, and there is no direct causality between them. They only appear to be correlated because they are bridged by fluid intelligences.

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1 INTRODUCTION

The rate of time preference (RTP) is regarded as negatively correlated with income, and many empirical studies have supported this correlation (e.g., Fisher, 1930; Lawrance, 1991; Becker and Mulligan, 1997; Samwick, 1998; Frederick et al., 2002; Ventura, 2003). Even though the causality in this relationship is not necessarily clear, it is believed by many economists to exist (e.g., Fisher, 1930; Uzawa; 1968; Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990; Becker and Mulligan, 1997; Frederick et al., 2002), and many endogenous RTP models in which households' RTPs are assumed to be formed on the basis of household current income have been constructed and used (e.g., Uzawa, 1968; Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990). However, the mechanism behind this causality is not clearly demonstrated in these endogenous RTP models. In addition, these models have the inherent problem that they are unstable because of the assumption of this causality.

In this paper, I examine the mechanism behind the observed negative correlation between RTP and income from a different point of view on the basis of the concept of fluid intelligence in an economy under the maximum degree of comfortability (MDC)-based procedure introduced by Harashima (2019). Fluid intelligence is one of several types of human intelligence, and it is usually defined as the ability to solve novel problems by thinking logically without depending only on previously acquired knowledge. The importance of fluid intelligence has been emphasized in psychology and psychometrics (e.g., Cattell, 1963, 1971), and it is usually modeled on the basis of item response theory (e.g., Lord and Novick, 1968; van der Linden and Hambleton, 1997). Harashima (2012) showed that, as fluid intelligence increases, the probability of solving unexpected problems increases and therefore productivity and labor incomes increase (Harashima, 2009, 2016, 2017b).

Harashima (2018, 2019) presented a MDC-based procedure by which households reach a steady state. In most economic studies, it has been assumed that a household reaches a steady state by generating rational expectations on the basis of its RTP (i.e., the RTP-based procedure). However, rational expectations impose substantial demands on households in that they have to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Harashima (2018, 2019) indicated that the capital-wage ratio (CWR) at MDC under the MDC-based procedure is equivalent to RTP under the RTP-based procedure. Furthermore, it is highly likely that households do not actually use the RTP-based procedure but instead use the MDC-based procedure, because the latter is far easier to use than the former but both equally lead households to the same steady state. The MDC-based procedure is very simple—a household simply behaves on the basis of whether or not it feels most comfortable with its combination of labor income and capital (wealth).

In this paper, I show that heterogeneity in fluid intelligences among households causes heterogeneities in CWR at MDC and RTP as well as incomes. That is, the observed correlation between RTP and income is spurious and there is no direct causality between the two. They only appear to be correlated because they are bridged by fluid intelligence.

2 FLUID INTELLIGENCE

2.1 Model of fluid intelligence

In psychology and psychometrics, many types of intelligence have been considered, including fluid intelligence, crystallized intelligence, short-term memory, long-term storage and retrieval, reading and writing ability, and visual processing. Among these, the importance of the difference between fluid intelligence and crystallized intelligence has been particularly emphasized. According to Cattell (1963, 1971), fluid intelligence is the ability to solve novel problems by thinking logically without depending only on knowledge previously acquired. This is the ability to deal with unexpected situations without relying only on knowledge obtained from schooling or previous experience. With the help of fluid intelligence, people can flexibly adapt their thinking to new problems or situations. By contrast, crystallized intelligence is the capacity to acquire and use knowledge or experience. This is the ability to communicate one's knowledge and to reason by using previously learned experiences.

Fluid intelligence can be modeled on the basis of item response theory, which is widely used in psychometric studies (e.g., Lord and Novick, 1968; van der Linden and Hambleton, 1997). In particular, the item response function is used to describe the relationship between abilities and item responses (e.g., test scores or performances). A typical item response function is

$$p(\eta) = c + \frac{1-c}{1+\exp[-a(\eta-b)]} ,$$

where p is the probability of a correct response (e.g., answer) to an item (e.g., test or question), η ($\infty > \eta > -\infty$) is a parameter that indicates an individual's ability, a (> 0) is a parameter that characterizes the slope of the function, b ($\infty \geq b \geq -\infty$) is a parameter that represents the difficulty of an item, and c ($1 \geq c \geq 0$) is a parameter that indicates the probability that an item can be answered correctly by chance.

2.2 Fluid intelligence, productivity, and labor income

2.2.1 Productivity

Harashima (2009, 2012, 2016, 2017b) showed that total factor productivity is positively correlated with the probability of ordinary (typical) workers' solving unexpected problems in a unit of time, where "ordinary" means workers who are not highly educated and trained experts. In the process of production, many (although minor) unexpected problems occur, and fluid intelligence is indispensable to solve these unexpected problems.

On the basis of item response theory, the probability of a worker's solving unexpected problems in a unit of time, $\hat{p}(FI)$, can be modeled as

$$\hat{p}(FI) = \hat{\delta} + \frac{1-\hat{\delta}}{1+\exp\left[-\hat{\gamma}\left(FI-\widehat{D}\right)\right]} , \tag{1}$$

where *FI* ($\infty > FI > -\infty$) is a parameter that indicates an ordinary worker's (household's) fluid intelligence, $\hat{\gamma}$ (> 0) is a parameter that characterizes the slope of the function, $\widehat{D}$ is a parameter that indicates the average difficulty of unexpected problems that the worker has to solve, and $\hat{\delta}$ ($1 \geq \hat{\delta} \geq 0$) is the probability that unexpected problems are solved by chance. Harashima (2012) showed that productivity is positively correlated with $\hat{p}(FI)$; that is, fluid intelligence is positively correlated with productivity. Equation (1) indicates that the higher a worker's fluid intelligence (i.e., a higher value of *FI*), the greater the probability of solving unexpected problems in a unit of time.

An ordinary worker's ability to solve problems quickly with fluid intelligence affects productivity substantially. Fluid intelligence is therefore closely related to productivity, and furthermore, the causality runs from fluid intelligence to productivity. Equation (1) indicates that as a worker's fluid intelligence increases, the worker's productivity increases; that is,

$$\frac{dP_R}{dFI} > 0 ,$$

where P_R is an ordinary worker's (household's) productivity.

2.2.2 Labor incomes

Harashima (2009, 2012, 2016, 2017b) showed that labor incomes are positively proportionate to productivity, because a production function can be deduced from the basic natures of capital and labor inputs and the experience curve effect to be

$$Y = \bar{\sigma}\omega_A\omega_L A^{\alpha}K^{1-\alpha}L^{\alpha} \ , \tag{2}$$

where Y is output, A is technology, K is capital input, L is labor input, α is a parameter ($0 < \alpha < 1$) that indicates labor share, $\bar{\sigma}$ (> 0) a parameter that represents a worker's accessibility limit to capital with regard to location, and ω_A and ω_L are positive parameters. The value of $\bar{\sigma}\omega_A\omega_L A^{\alpha}$ indicates total factor productivity and $\omega_A\omega_L$ is heterogeneous among workers, but $\bar{\sigma}$ and A^{α} are common to all workers. Hence, a worker's productivity can be represented by $\omega_A\omega_L$. In essence, workers can be interpreted as households, and therefore, worker productivity can be interpreted as household productivity.

By equation (2), the labor income of a worker (household) (w) is given by

$$w = (1-\alpha)\bar{\sigma}\omega_A\omega_L A^{\alpha}k^{1-\alpha} \ , \tag{3}$$

where $k = \frac{K}{L}$. Hence, the worker's (household's) labor income is a linear increasing function of $\omega_A\omega_L$. Because productivity is caused by, and positively correlated with, FI (as shown in Section 2.2.1), a worker's (household's) labor income (w) is also caused by, and positively correlated with, its fluid intelligence.

2.2.3 Approximation

Ordinary workers are required to solve unexpected problems, but only minor ones. Solving difficult unexpected problems is basically delegated to highly educated and trained experts. Because unexpected problems with regard to $\omega_A\omega_L$ that are delegated to ordinary workers are not difficult for most of these workers, the average difficulty (i.e., $\widehat{D}$) of the problems will be far smaller than most ordinary workers' FIs. When $\widehat{D}$ is sufficiently smaller than most ordinary workers' FIs, the value of $\exp\left[-\hat{\gamma}\left(FI-\widehat{D}\right)\right]$ will be far smaller than unity, and therefore approximately

$$\frac{1}{1+\exp\left[-\hat{\gamma}\left(FI-\widehat{D}\right)\right]} \cong 1-\exp\left[-\hat{\gamma}\left(FI-\widehat{D}\right)\right] .$$

Hence, by equation (1), approximately,

$$\hat{p}(FI) \cong \hat{\delta} + \left(1-\hat{\delta}\right)\left\{1-\exp\left[-\hat{\gamma}\left(FI-\widehat{D}\right)\right]\right\} . \tag{4}$$

Taking this nature into consideration, the $\omega_A \omega_L$ determined by $\hat{p}(FI)$ as shown in Sections 2.2.1 and 2.2.2 can be approximately described as an exponentially decelerating increasing function of *FI* such that

$$\omega_A \omega_L \cong \bar{\delta}\{\hat{\delta} + (1 - \hat{\delta})[\exp(-\hat{\gamma}\widehat{D}) - \exp(-\hat{\gamma}FI)]\}$$

by equation (4), where $\bar{\delta}$ is a positive constant. Here, it is assumed for simplicity that $\hat{\delta} = 0$, and therefore

$$\omega_A \omega_L \cong \bar{\delta}[\exp(-\hat{\gamma}\widehat{D}) - \exp(-\hat{\gamma}FI)] \,. \tag{5}$$

Because the worker's (household's) labor income (w) is a linear increasing function of $\omega_A \omega_L$, as shown in Section 2.2.2, then it is also approximately an exponentially decelerating increasing function of *FI* such that

$$w \cong \check{\delta}[\exp(-\hat{\gamma}\widehat{D}) - \exp(-\hat{\gamma}FI)] \tag{6}$$

by equation (5), where $\check{\delta}$ is a positive constant.

This approximation means that, in a situation where unexpected problems with solutions that are (1) delegated to ordinary workers and (2) not difficult (i.e., $\widehat{D}$ is far smaller than most workers' *FI*s), an ordinary worker's wage does not have the same relative increase even if the worker's fluid intelligence is relatively high.

3 MDC-BASED PROCEDURE

The MDC-based procedure is explained briefly in this section. For a more detailed presentation, see Harashima (2018, 2019).

3.1 "Comfortability" of the capital-wage ratio (CWR)

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$ where $\breve{k}_t$ and $\breve{w}_t$ are the k_t and w_t of the household, respectively. Let Γ be the household's subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i (i = 1, 2, 3, … , M). The household should next assess whether it feels comfortable with its current Γ, that is, its combination of income and capital. "Comfortable" in this context means at ease, not anxious, and other similar related feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable capital-wage ratio (CWR) value, because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC), and let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ therefore indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is the Γ_i of household i at $\tilde{s}_i$.

3.2 Homogeneous population

Suppose first that all households are identical (i.e., a homogeneous population).

3.2.1 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.
Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

3.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t, and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under an RTP-based procedure, that is, one derived in a Ramsey-type growth model in which households behave by discounting utilities by θ and generating rational expectations, where θ (> 0) is household RTP. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See Harashima (2018, 2019).

Proposition 1 indicates that we can interpret that $\tilde{S}_{MDC}$ is equivalent to $\tilde{S}_{RTP}$. This means that both procedures can function equivalently and that CWR at MDC is substitutable for RTP as a guide for household behavior.

3.3 Heterogeneous population

In actuality, however, households are not identical—they are heterogeneous—and, if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker, 1980; Harashima, 2010, 2017a). However, Harashima (2010, 2017a) showed that a sustainable heterogeneity (SH) at which all optimality conditions of all heterogeneous households are simultaneously satisfied exists under the RTP-based procedure. In addition, Harashima (2018, 2019) showed that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised and a rule for the government must be added in a heterogeneous population.

Suppose that households are identical except for their CWRs at MDC (i.e., their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be the Γ_R of household i. Also let T be the net transfer that households receive from the government with regard to SH and T_i be the net transfer that household i receives ($i = 1,2,3, \ldots, M$).

3.3.1 Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.
Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, the government must act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

3.3.2 Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved, and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure. In addition, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See Harashima (2018, 2019).

Proposition 2 indicates that we can interpret that $\tilde{S}_{MDC,SH,ap}$ is equivalent to $\tilde{S}_{RTP,SH}$ and that CWR at MDC is substitutable for RTP as a guide for heterogeneous households' behavior. Furthermore, no matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are severally estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

4 CWR AT MDC AND FLUID INTELLIGENCE

4.1 The degree of freedom of choice

4.1.1 Fluid intelligence and economic activities

As discussed in Section 2.2 and shown by Harashima (2009, 2012, 2016, 2017b), total factor productivity depends not only on the level of technology (A) but also on ordinary workers' abilities to solve a large number of unexpected minor problems ($\omega_A \omega_L$), and the workers' fluid intelligence is indispensable to solving these unexpected problems (Harashima, 2012).

Nevertheless, fluid intelligence is indispensable not only after unexpected problems occur but also before they occur. The future is uncertain, and people understand that there are always risks of unexpected problems in the future. However, if the future can be more precisely foreseen, the probability of occurrence of these kinds of problems can be reduced, so that some of the problems can be anticipated or solved before they occur. As more previously unexpected problems are anticipated, higher levels of consumption can be enjoyed. Hence, reducing the number of unexpected problems *ex ante* is as important as solving them *ex post*.

Both solving and preventing or avoiding unexpected problems require fluid intelligence, and fluid intelligence is indispensable to fix unexpected problems, as discussed in Section 2 and by Harashima (2012). Both solving and reducing the number of unexpected problems are achieved in the same manner—by newly uncovering the mechanism by which a hitherto unknown problem occurs and innovating a way to fix or avert it.

Humans are endowed with reason and therefore can foresee the future and plan for their future actions. As Harashima (2018, 2019) indicates, even under the MDC-based procedure, households behave fully considering the future and choose the best plan for their futures. Hence, households make an effort to foresee the future to the extent possible even under the MDC-based procedure. Of course, foreseeing the future includes activities related to reducing the number of unexpected problems to the greatest degree possible before they occur.

What should a household foresee when making a future plan with regard to MDC? Households need to clarify choices between consumption and saving, because MDC indicates how much a household prefers to save out of its labor income. In a future plan for consumption and saving, goods and services that are to be consumed in the future should be clearly distinguished from those consumed at present. Hence, the plan should be made fully considering what kinds of goods and services can be consumed and how they can be consumed in the future while at the same time preventing or avoiding unexpected problems as much as possible with fluid intelligence. Because CWR means the combination of consumption and saving, therefore a household’s CWR at MDC reflects its future plan for consumption and saving.

4.1.2 Consumption opportunities

Let a “consumption opportunity” be a technologically and physically feasible opportunity of consuming a good or service in a future period for a household. Opportunities of consuming different goods and services in the same future period are counted as different consumption opportunities. Nevertheless, a household cannot simultaneously realize more than one consumption opportunity from among many consumption opportunities.

This means that each household has a large number of consumption opportunities in each period, but it can realize only a fraction of them in that period. Therefore, when a household makes its future plan, it has to carefully examine which consumption opportunities should be realized from among the many competing consumption opportunities.

Some of the consumption opportunities may be technologically or physically feasible, but they still may not be accessible to all households. It is highly likely that the number of consumption opportunities that each household can actually realize is heterogeneous among households, because the scopes of accessible consumption opportunities from which the households choose are highly likely to be heterogeneous among households.

When a household makes its future plan for consumption and saving, it should first ascertain which consumption opportunities it can access. Households will sort consumption opportunities into "accessible consumption opportunities" that are thought to be accessible and "inaccessible consumption opportunities" that are not. Different households sort consumption opportunities differently because households are by nature heterogeneous.

Various factors will influence household sorting choices. For example, affordability will matter because consumption requires money or other exchangeable resources. If the opportunity for consumption of an expensive good or service is not affordable because of a budget constraint, it will be thought of as inaccessible. As compared with necessities, luxuries will be thought to be inaccessible far more often than necessities. Another factor is qualification or privilege, because some consumption opportunities require a household to possess some type of qualification—for example, a high score on an entrance exam for an exclusive school. Even if a household can afford this consumption opportunity, it cannot access it if it does not possess the required qualification.

However, the most important factor is probably uncertainty. The future is uncertain because humans are imperfect, and unexpected problems inevitably will occur. The affordability and qualification factors themselves are not certain. If a household is uncertain whether some consumption opportunities are affordable because its budget constraint is uncertain, it may sort these opportunities as inaccessible. When a household makes a future plan for its consumption and saving, therefore, its perception of uncertainty about its future will be very important. If a household estimates that its future is more uncertain, it will give up a larger amount of consumption opportunities and sort them as inaccessible. Conversely, it may sort more opportunities as accessible if it perceives a more certain future. A household's amount of accessible consumption opportunities will therefore depend significantly on its perceived uncertainty about the future.

It will not matter to a household nevertheless even if it can access only a handful of consumption opportunities, because consumption opportunities include both necessities and luxury items. In the industrialized economies, most of the consumption opportunities could be considered to be luxuries in a broad sense, and consumption opportunities corresponding to civil minimums will make up a relatively small fraction of all consumption opportunities. Hence, a household will generally be able to access many consumption opportunities of necessities.

4.1.3 Timing of consumption

Households obtain utilities by consuming goods and services, but even if the same good or service is consumed, the level of utility obtained by each household will differ, depending on the timing of consumption. A household may strongly desire to consume a good or service in a specific favorable period but may never want it in other periods. If a household cannot avoid consuming a good or service in a less desirable or even unwanted consumption period, the utility it obtains will be far less than that if it can consume it in the desired period. For example, in summer, people do not want to consume a good or service that is useful only in winter. Similarly, when people are old, they do not want to consume a good or service that is useful only for young people. In addition, people may become tired of consuming a good or service if the amount is excessive, and they may want to consume only part of it in the current period.

The most desirable period or best timing of consumption will vary across goods and services as well as households. The best timing may occur only once, or it may be repeated at some interval. In any case, it may not always be easy for a household to consume a good or service exactly at the best or most desired timing. In general, a household can realize the consumption of daily necessaries at the best consumption timings, but it may not necessarily be easy to realize the consumption of some goods and services that are consumed much more infrequently at the best timing.

If a household cannot realize consumption opportunities that match the desired timings, its utilities are lower than if it can. Hence, households also have to estimate whether they can realize the consumption opportunities that match the best consumption timings in future plans for consumption and saving. If a household can clearly foresee that a consumption opportunity does not match the best timing, it will most likely exclude this consumption opportunity from consideration. In general, the less certain a household is about whether a consumption opportunity matches the best timing, the more likely it will be to exclude the opportunity from consideration. The reluctance to give up consumption opportunities because of uncertainty about the best timings will be far lower in the case of luxuries than necessities. Because most consumption opportunities are for luxuries, as discussed in Section 4.1.2, a household will generally not pursue an accessible consumption opportunity if it is uncertain whether this opportunity matches the best timing.

4.1.4 The degree of freedom of choice

4.1.4.1 Consumption accessibility

Suppose for simplicity that each household has N consumption opportunities. A household can access only a small portion of the opportunities because of the constraints discussed in Section 4.1.2. Therefore, in making its future plan for consumption and saving, it determines which consumption opportunities it can access. Here, household i estimates that it can access n_i ($\leq N$) out of N consumption opportunities. Let the ratio

$$\frac{n_i}{N} = \mu_i \tag{7}$$

be the "consumption accessibility" of household i, and evidently,

$$\frac{dn_i}{d\mu_i} = N = \text{const.} \tag{8}$$

4.1.4.2 Consumption accuracy

Suppose that, for any household, the best consumption timing of any good or service arrives within a finite period, and even if the best consumption timings are multiple and arrive repeatedly, each of the repeated best timings arrives in a finite period after the last one. Let a "best timing consumption opportunity" be a consumption opportunity that matches the best consumption timing. Household i estimates that it can realize m_i best timing consumption opportunities out of n_i accessible consumption opportunities. Let the ratio

$$\frac{m_i}{n_i} = \pi_i \tag{9}$$

be the "consumption accuracy" of household i, and evidently,

$$\frac{dm_i}{d\pi_i} = n_i = \text{const.}$$

4.1.4.3 Degree of freedom of choice in consumption

By the nature of consumption accessibility and accuracy,

$$F_i = \mu_i \pi_i = \frac{m_i}{N} \tag{10}$$

indicates how much household i has freedom of choice in consumption in the future. If F_i is higher, household i can choose a consumption opportunity in a future period from among a larger amount of best timing consumption opportunities. Let F_i be the "degree of freedom of choice" (DFC).

By the natures of μ_i and π_i, for any i,

$$\frac{\partial F_i}{\partial \mu_i} > 0 \tag{11}$$

and

$$\frac{\partial F_i}{\partial \pi_i} > 0 \ . \tag{12}$$

4.2 The DFC model

4.2.1 Fluid intelligence and DFC

4.2.1.1 Fluid intelligence and accessible consumption opportunities

As discussed in Section 4.1.2, uncertainties about the future are significantly important in a household's determination of the accessibility of its consumption opportunities. In addition, as discussed in Section 4.1.1, fluid intelligence is indispensable to reducing uncertainties about consumption opportunities. Hence, the number of accessible consumption opportunities is substantially influenced by fluid intelligence. Taking this nature into consideration, it is highly likely that if a household possesses a higher fluid intelligence it can access a larger number of consumption opportunities than households with lower fluid intelligences. That is,

$$\frac{d\mu_i}{dFI_i} > 0 \ , \tag{13}$$

where FI_i is the degree of fluid intelligence of household i.

Note that affordability, which is a factor in accessibility, directly depends on labor incomes, and labor incomes (as well as productivity) depend on fluid intelligence, as indicated in Section 2.2. Hence, the number of accessible consumption opportunities is also influenced by fluid intelligence directly through this channel. In addition, in modern societies, many qualifications, which are also a source of accessibility (as discussed in Section 4.1.2), are based on abilities in specific fields, because a "qualification" in this sense basically means some kind of superiority over other people. Because fluid intelligence is one of the most important sources of human intellectual abilities, fluid intelligence will also substantially affect the amount of accessible consumption opportunities through this channel.

4.2.1.2 Fluid intelligence and best timing of consumption

As discussed in Section 4.1.1, fluid intelligence is indispensable to reducing uncertainty as a whole, and therefore it is also reduces uncertainty about best consumption timings. If a household possesses a higher fluid intelligence, it can probably realize best timing consumption opportunities than households with lower fluid intelligences. That is,

$$\frac{d\pi_i}{dFI_i} > 0 \ . \tag{14}$$

4.2.1.3 Fluid intelligence and DFC

By the total derivative of F_i, for any i,

$$\frac{dF_i}{dFI_i} = \frac{\partial F_i}{\partial \mu_i}\frac{d\mu_i}{dFI_i} + \frac{\partial F_i}{\partial \pi_i}\frac{d\pi_i}{dFI_i} \tag{15}$$

By inequalities (11), (12), (13), and (14) and equation (15),

$$\frac{dF_i}{dFI_i} > 0 \ . \tag{16}$$

That is, DFC (F_i) is a function of fluid intelligence FI_i, and as FI_i increases, F_i increases.

4.2.2 The DFC model

Because DFC (F) is a function of fluid intelligence (FI), item response theory can be used to construct a model of DFC. That is, F_i can be modeled on the basis of equation (1) such that

$$F_i = \tilde{p}(FI_i) = \tilde{\delta} + \frac{1-\tilde{\delta}}{1+\exp\left[-\tilde{\gamma}\left(FI_i - \widetilde{D}\right)\right]} \ , \tag{17}$$

where $\tilde{\gamma}$ (> 0) is a parameter that characterizes the slope of the function, $\widetilde{D}$ is a parameter with regard to "difficulty" in the "economic environment," and $\tilde{\delta}$ ($1 \geq \tilde{\delta} \geq 0$) is the probability that a consumption opportunity is accessible and also accurately realized by chance. The "economic environment" consists of various elements such as technology, political stability, legal system, diplomatic situation, natural environment, and restrictions on usable natural resources. "Difficulty" indicates how difficult it is for a household to reduce future uncertainties (i.e., to anticipate potential problems) because of various obstacles existing in the economy. For example, a more difficult economic environment means more unstable technological progress, more frequent political turbulence, a more unpredictable legal system, more unstable international situations, and more unsure restrictions on natural resources.

Equation (17) indicates that DFC is determined by fluid intelligence (FI_i), as well as by difficulty in the economic environment ($\widetilde{D}$). FI_i is an internal factor and $\widetilde{D}$ is an external factor for a household. As is evident from equation (17), if FI_i is higher, F_i is higher, and if $\widetilde{D}$ is higher, F_i is lower. Suppose that FI_i, $\widetilde{D}$, $\tilde{\delta}$, and $\tilde{\gamma}$ are all not correlated with each other, and that $\widetilde{D}$, $\tilde{\delta}$, and $\tilde{\gamma}$ are common to all households. Hence, by equation (17),

$$\frac{dF_i}{dFI_i} > 0 . \tag{18}$$

This inequality is the same as inequality (16). Also, by equation (17),

$$\frac{dF_i}{d\widetilde{D}} < 0 . \tag{19}$$

Suppose for simplicity that $\widetilde{D}$, $\tilde{\delta}$, and $\tilde{\gamma}$ are constant.

4.2.3 Approximation

Even if households are heterogeneous, they will not be largely different from each other. Hence, it is highly likely that most F_i are contiguously located around a certain value between 0 and 1. Furthermore, because a household can realize only a small fraction of the huge number of consumption opportunities, most households' F values will be close to 0; that is, most households' FI will be sufficiently smaller than $\widetilde{D}$ in equation (17). If $\exp\left[-\tilde{\gamma}\left(F_i - \widetilde{D}\right)\right]$ is sufficiently large because FI_i is sufficiently less than $\widetilde{D}$, then approximately

$$\frac{1}{1+\exp\left[-\tilde{\gamma}\left(FI_i-\tilde{D}\right)\right]} \cong \frac{1}{\exp\left[-\tilde{\gamma}\left(FI_i-\tilde{D}\right)\right]} \ .$$

Hence, approximately, there is an exponential relation between F_i and FI_i such that

$$F_i = \tilde{p}(FI_i) \cong \tilde{\delta} + \left(1-\tilde{\delta}\right)\frac{\exp(\tilde{\gamma}FI_i)}{\exp\left(\tilde{\gamma}\tilde{D}\right)} \ .$$

Here, it is assumed for simplicity that $\tilde{\delta} = 0$, and therefore

$$F_i = \tilde{p}(FI_i) \cong \frac{\exp(\tilde{\gamma}FI_i)}{\exp\left(\tilde{\gamma}\tilde{D}\right)} \ . \tag{20}$$

That is, approximately, F_i increases exponentially as FI_i increases.

What this approximation means is that in a situation where a household can realize only a small fraction of consumption opportunities and therefore most households' F values are close to 0, if FI increases even a little, a large number of consumption opportunities becomes additionally accessible and realizable. A small increase in FI reduces the uncertainty substantially and provides a large number of additional consumption opportunities (i.e., an exponential increase in consumption opportunities).

In addition, taking the natures of μ_i and π_i discussed in Section 4.1 into consideration, equation (10) strongly implies that μ_i and π_i are affected by FI_i in the same manner. Hence, by equation (20), it is highly likely that

$$\mu_i \cong \lambda\frac{\exp(\tilde{\gamma}FI_i)}{\exp\left(\tilde{\gamma}\tilde{D}\right)} \tag{21}$$

and

$$\pi_i \cong \lambda^{-1}\frac{\exp(\tilde{\gamma}FI_i)}{\exp\left(\tilde{\gamma}\tilde{D}\right)} \ , \tag{22}$$

where λ is a positive constant.

4.3 DFC and capital

4.3.1 Motives for saving and investment

Under the MDC-based procedure, a household's CWR at MDC reflects its future plan for consumption and saving, as discussed in Section 4.1.1, particularly in the sense that the household's CWR indicates its feeling about whether the future is sufficiently secure economically. Because CWR is the ratio of labor income to capital, CWR at MDC indicates the extent to which a household eventually wants to accumulate capital by saving out of its labor income. Therefore, to examine the relation between DFC and CWR at MDC, we first should examine a household's motives to save and invest money (or other types of economic resources).

First, suppose a primitive agricultural society in which currency or money has not yet been invented. Even in this society, people should save some economic resources. For example, harvested grains are not eaten only at the time of harvest but throughout the year, so most grains are saved at the time of harvest. That is, people put aside and stock some portion of their resources for future consumption. People save resources not only for later consumption, however, but also to increase production in the future so as to consume a much larger amount than the amount saved. A portion of the current resources that are put aside can be exchanged for tools or animals that enable production to increase in the future. This is the second motive for saving—resources are saved to be invested.

Even in our modern society, these essential motives remain the same. Savings and investments are made primarily because (a) people need to spread out the consumption of production (earnings) over time, and (b) people sacrifice some portion of current resources to increase production in the future, thereby increasing possible future consumption.

Savings from motive (a) are not invested and therefore no return (i.e., increases in future production) is expected from these resources. They do, however, depend on how accurately the best consumption timings in the future can be foreseen. Savings from motive (b) are invested and returns are expected. Production (i.e., the number of accessible consumption opportunities) is expected to increase by investments. Hence, the amount of resources that should be saved from motive (b) depends on the amount of expected future increases in production.

4.3.2 Capital, accessibility, and accuracy

It is assumed for simplicity that the returns from, and risks of, any investment are identical, finite, and common knowledge for all households (e.g., interest rates in financial markets are identical and commonly known to all households). Suppose also for simplicity that the amount of money (economic resources) that is used to realize a consumption opportunity is identical for any consumption opportunity; in other words, any consumption opportunity equally consists of a unit of consumption opportunity that has the same "price."

4.3.2.1 Capital, motive (b)

A household will save and invest with motive (b) only if it estimates that some consumption opportunities are made additionally accessible in the future as a result of the investment. It will not accumulate capital infinitely, because the number of consumption opportunities is finite in a future period. A household will accumulate capital with motive (b) up to the point at which it feels that the combination of the estimated future increase in accessible consumption opportunities and the necessary present resulting decrease in consumption is most comfortable. There will be a unique combination that is felt to be most comfortable by each household.

An important point is that a household only intuitively feels whether the aforementioned combination is most comfortable. There is no predetermined objectively true and correct most comfortable combination. Nobody knows whether the combination with which a household subjectively feels most comfortable is equal to the predetermined objectively true and correct most comfortable combination. We can only say that the amount of capital eventually held by a household is the one that makes it subjectively feel most comfortable.

Because any consumption opportunity consists of a unit of consumption opportunity that has the same price as assumed above, a household's amount of capital derived from motive (b) is a linear increasing function of the number of additional consumption opportunities that are estimated to be accessible by investments. Let $\bar{n}_i + \tilde{n}_i$ be accessible consumption opportunities of household i, where $\bar{n}_i$ is the accessible consumption opportunities that are originally accessible without the capital from motive (b), and $\tilde{n}_i$ is those that are additionally made accessible because of the capital (accumulated investments) of household i from motive (b) at MDC ($k_{b,i}$). Because $k_{b,i}$ is a linear increasing function of $\tilde{n}_i$,

$$\tilde{n}_i = \chi k_{b,i} \ ,$$

where χ is a positive constant.

Taking this capital accumulation behavior into consideration, the consumption accessibility of household i indicated by equation (7) should be modified such that

$$\mu_i = \frac{\bar{n}_i + \tilde{n}_i}{N} = \frac{\bar{n}_i + \chi k_{b,i}}{N} \tag{23}$$

at MDC for household i. By total derivative of equation (23),

$$\frac{dk_{b,i}}{d\mu_i} = \chi^{-1}\left(N - \frac{d\bar{n}_i}{d\mu_i}\right) . \tag{24}$$

Here, it seems highly likely that $\frac{d\bar{n}_i}{d\mu_i}$ is constant because $\bar{n}_i$ is irrelevant to motive (b) and therefore will have the same natures as equations (7) and (8). Hence, by equation (24),

$$\frac{dk_{b,i}}{d\mu_i} = \text{const.} \tag{25}$$

In addition, it seems highly likely that $N > \frac{d\bar{n}_i}{d\mu_i}$, because N indicates the number of all consumption opportunities, and thereby, by equation (24)

$$\frac{dk_{b,i}}{d\mu_i} > 0 . \tag{26}$$

Therefore, by equation (25) and inequality (26), as μ_i increases, the amount of capital from motive (b) at MDC ($k_{b,i}$) increases linearly.

4.3.2.2 Capital, motive (a)

A household also sets aside some resources with motive (a). As discussed in Section 4.1.3, if a household feels less certain about the best timing of an accessible consumption opportunity, it will be less likely to set aside resources for this consumption opportunity and will thereby accumulate less capital with motive (a).

Let $\overline{m}_i + \widetilde{m}_i$ be best-timing consumption opportunities of household i, where $\overline{m}_i$ is the best-timing consumption opportunities that are irrelevant to capital (savings) with motive (a) and $\widetilde{m}_i$ is those that are estimated to certainly match the best timings and make household i save money with motive (a) at MDC ($k_{a,i}$). Because any consumption opportunity consists of a unit of consumption opportunity that has the same price as assumed above, a household's capital (accumulated investments) from motive (a) is a linear increasing function of the number of consumption opportunities that are set aside with motive (a); that is, $k_{a,i}$ is a linear increasing function of $\widetilde{m}_i$ such that

$$\widetilde{m}_i = \xi k_{a,i}$$

where ζ is a positive constant.

Taking this capital accumulation behavior with motive (a) into consideration, the consumption accessibility of household i, as indicated by equation (9), should be modified such that

$$\pi_i = \frac{\bar{m}_i + \tilde{m}_i}{\bar{n}_i + \tilde{n}_i} = \frac{\bar{m}_i + \xi k_{a,i}}{\bar{n}_i + \chi k_{b,i}} \tag{27}$$

at MDC for household i. By total derivative of equation (27),

$$\frac{dk_{a,i}}{d\pi_i} = \xi^{-1}\left[\bar{n}_i + \chi k_{b,i} - \frac{d\bar{m}_i}{d\pi_i} - \frac{\bar{m}_i + \xi k_{a,i}}{\bar{n}_i + \chi k_{b,i}}\left(\chi \frac{dk_{b,i}}{d\pi_i} + \frac{d\bar{n}_i}{d\pi_i}\right)\right] . \tag{28}$$

It seems highly likely that the values of $\frac{d\bar{m}_i}{d\pi_i}$, $\frac{\bar{m}_i+\xi k_{a,i}}{\bar{n}_i+\chi k_{b,i}}$, $\chi \frac{dk_{b,i}}{d\pi_i}$, and $\frac{d\bar{n}_i}{d\pi_i}$ are all far smaller than that of $\bar{n}_i + \chi k_{b,i}$ because $\bar{n}_i + \chi k_{b,i}$ indicates the number of all accessible consumption opportunities. Therefore, by equation (28), approximately,

$$\frac{dk_{a,i}}{d\pi_i} \cong \xi^{-1}(\bar{n}_i + \chi k_{b,i}) . \tag{29}$$

Here, both μ_i and π_i will be affected by FI_i in the same manner as shown in Section 4.2.3; therefore, $\bar{n}_i$ and $k_{b,i}$ will increase as π_i increases and thereby

$$\frac{d^2 k_{a,i}}{d\pi_i^2} \cong \xi^{-1}\left(\frac{d\bar{n}_i}{d\pi_i} + \chi \frac{dk_{b,i}}{d\pi_i}\right) > 0 .$$

In addition, as with the case of $\frac{d\bar{n}_i}{d\mu_i}$, it seems highly likely that $\frac{d\bar{n}_i}{d\pi_i}$ and $\frac{dk_{b,i}}{d\pi_i}$ are positive constants and thereby approximately

$$\frac{d^2 k_{a,i}}{d\pi_i^2} \cong \text{const.} \tag{30}$$

Equations (29) and (30) indicate that, as π_i increases, the amount of capital from motive (a) increases at a greater than linear rate; that is, $k_{a,i}$ is approximately a quadratic and increasing function of π_i.

As was the case with capital from motive (b), the amount of a household's capital from motive (a) is also finite, because the number of accessible consumption opportunities is finite in a future period and the best consumption timing of any consumption opportunity arrives within a finite period.

4.4 Fluid intelligence and MDC

4.4.1 Existence of a unique CWR at MDC

As shown in Section 4.3.2, capital from cases (a) and (b) has the common feature that it is finite. Hence, for a given level of labor income and, equivalently, for given levels of technology (A) and fluid intelligence (FI_i), any household has a certain finite level of capital that it feels is most comfortable. That is, a unique finite value of CWR at MDC exists for each household.

4.4.2 Fluid intelligence and capital

As equation (25) indicates, the amount of capital from motive (b) ($k_{b,i}$) is a linear increasing function of consumption accessibility (μ_i) and, as equation (21) indicates, μ_i is approximately an exponentially increasing function of FI_i. Hence, the amount of capital from motive (b) of household i ($k_{b,i}$) is approximately an exponentially increasing function of FI_i. In addition, as equation (30) indicates, the amount of capital from motive (a) ($k_{a,i}$) is approximately a quadratic and increasing function of consumption accuracy π_i and, as equation (22) indicates, π_i is approximately an exponentially increasing function of FI_i. Hence, the amount of capital from motive (a) of household i ($k_{a,i}$) is also approximately an exponentially increasing function of FI_i. Therefore, the amounts of both types of capital are approximately exponentially increasing functions of FI_i, and thereby the combined capitals ($k_{a,i} + k_{b,i}$) are also approximately an exponentially increasing function of FI_i.

As equations (17), (20), (21), and (22) indicate, a higher FI_i causes higher F_i, μ_i, and π_i, and as Section 4.3.2 indicates, increases in F_i, μ_i, and π_i cause an increase in the amount of capital. That is, an increase in FI_i causes an increase in the amount of capital. The causality clearly runs from fluid intelligence to capital.

4.4.3 Fluid intelligence and MDC

Whereas capital ($k_i = k_{a,i} + k_{b,i}$) amounts are approximately an exponentially accelerating increasing function of FI_i, as shown in Section 4.4.2 and by equation (20), labor incomes (w_i) are an exponentially decelerating increasing function of FI_i, as equation (6) indicates. Therefore, by equations (6) and (20), the CWR at MDC of household i (the ratio of w_i to k_i at MDC; i.e., $\Gamma(\tilde{s}_i)$) can be approximately described as

$$\Gamma(\tilde{s}_i) = \Phi \frac{\exp(-\hat{\gamma}\widehat{D}) - \exp(-\hat{\gamma}FI_i)}{\exp(\tilde{\gamma}FI_i)} \ , \tag{31}$$

where Φ is a positive constant. In equation (31), $\exp(-\hat{\gamma}\hat{D}) - \exp(-\hat{\gamma}FI_i)$ represents labor income (w_i) and $\exp(\tilde{\gamma}FI_i)$ represents capital (k_i).

By equation (31),

$$\frac{d\Gamma(\tilde{s}_i)}{dFI_i} = \frac{\Phi}{\exp[(\hat{\gamma}+\tilde{\gamma})FI_i]} \langle \hat{\gamma} - \tilde{\gamma}\{\exp[\hat{\gamma}(FI_i - \hat{D})] - 1\} \rangle \ . \tag{32}$$

Because $\exp[-\hat{\gamma}(FI - \hat{D})]$ is far smaller than unity, as discussed in Section 2.2.3 (i.e., $\exp[\hat{\gamma}(FI_i - \hat{D})]$ is far larger than unity) and both $\hat{\gamma}$ and $\tilde{\gamma}$ are parameters that characterize the slopes of item response functions with regard to solving unexpected problems by *FI* for the same population and therefore will take similar values, then by equation (32), generally

$$\frac{d\Gamma(\tilde{s}_i)}{dFI_i} < 0 \ . \tag{33}$$

Inequality (33) indicates that as *FI* increases, the CWR at MDC decreases. The CWR at MDC of a household with a higher *FI* is lower than that of a household with a lower *FI*.

As shown in Section 2.2, a higher fluid intelligence causes higher labor income, and as shown in Section 4.4.2, a higher fluid intelligence also causes higher capital amount. Therefore, equation (31) clearly shows that the causality runs from fluid intelligence to CWR at MDC.

4.5 Technological progress and MDC

4.5.1 Technological progress and consumption opportunities

As technologies progress, the number of technologically and physically feasible opportunities of consuming goods and services will increase because, for example, technological progress increases the varieties of goods and services available to households. However, how do consumption opportunities increase by technological progress?

Consumption opportunities will probably expand evenly or uniformly as technologies progress, whereas the composition structure of consumption opportunities remains unchanged on average because firms will keep the user interfaces with their products unchanged even as technologies progress. If the user interface is worse, sales of products and profits will decrease. Hence, if a firm recognizes that its sales and profits are lower because the user interface with its product is worse than expected, it will improve the user interface up to the level at which the firm's profits are maximized. Because the average intelligence of households will not change basically, even if technologies progress, the level of user interface that is required for a firm to maximize its profits also will not change on average, even if technologies progress. Because the average user interface is unchanged, the number of consumption opportunities (*N*) will increase at the same rate as production in the economy increases by technological progress.

4.5.2 Technological progress, DFC, and difficulty

Because fluid intelligence is highly likely given by nature, it will not change by technological progress. On the other hand, difficulty ($\widetilde{D}$) in equations (17) and (20) may change as technologies progress. However, because firms will keep the user interfaces unchanged on average, $\widetilde{D}$ will not be affected by technological progress. By equations (17) and (20), therefore, the DFC that is determined by FI and $\widetilde{D}$ will also not change by technological progress.

4.5.3 Technological progress and MDC

The accessible consumption opportunities that are originally accessible without the capital from motive (b) $(\bar{n}_i)$ and the best timing consumption opportunities that are irrelevant to capital (savings) with motive (a) $(\overline{m}_i)$ will increase at the same rate as *N* considering the natures of $\bar{n}_i$ and $\overline{m}_i$. Therefore, given a level of fluid intelligence and thereby values of μ_i and π_i, the amount of capital (k_i) will increase at the same rate as *N*, as equations (23) and (27) imply. Hence, because FI and $\widetilde{D}$ are not affected by technological progress (as indicated in Section 4.5.2) and *N* increases at the same rate as production in the economy increases with technological progress (as indicated in Section 4.5.1), the amount of capital (k_i) will increase at the same rate as production in the economy increases with technological progress. This nature is consistent with most economic growth models in that, at steady state and on a balanced growth path, the entire capital amount in an economy increases at the same rate as production in the economy increases by technological progress.

At the same time, by equations (2) and (3), a household's labor income (w_i) increases at the same rate as the production in the economy increases with technological progress at steady state and on a balanced growth path. This nature is also consistent with most economic growth models.

Therefore, the capital (k_t) and labor income (w_t) of each household increase at the same rate as the production in the economy increases with technological progress. As a result, CWR at MDC does not change with technological progress; that is, $\Gamma(\tilde{s}_i)$ is irrelevant to technological progress.

5 SPURIOUS CORRELATION

I examine the correlation between CWR at MDC (equivalently RTP) and labor income in Section 5.1, and then I examine the correlation between CWR and capital as well as capital income in Section 5.2. The case where SH (i.e., when all optimality conditions of all heterogeneous economies are simultaneously satisfied; Harashima, 2010, 2017a) is not achieved is not examined, because if SH is not achieved, then the extreme and highly unrealistic state where the household with the lowest CWR at MDC (i.e., the highest fluid intelligence) eventually monopolizes all capital will emerge as Becker (1980) and Harashima (2010, 2017a) indicate.

For this discussion, I assume for simplicity that there are many economies in a country, and each economy has a homogeneous population (households). In addition, economies are identical except for household fluid intelligence.

5.1 Correlation with labor income

The correlation with labor income is examined first in the case where economies are isolated from each other, and therefore SH does not matter. In this case, a higher fluid intelligence causes a higher productivity, as shown in Section 2.2.1, and higher labor income, as shown in Sections 2.2.2 and 2.2.3. At the same time, a higher fluid intelligence causes a lower CWR at MDC, as shown in Section 4.4.3. Hence, a household with a relatively low CWR at MDC can obtain relatively high labor incomes. As a result, CWRs at MDC can be observed to be negatively correlated with labor incomes among economies. Because CWR at MDC is substitutable with RTP, as indicated in Section 3 and by Harashima (2018, 2019), RTPs can also be observed to be negatively correlated with labor incomes.

An important point is that the observable correlation between CWRs at MDC (or RTPs) and labor incomes is spurious and does not indicate any direct causality between the two. Causalities exist only between fluid intelligence and other relevant elements. Correlations between CWRs at MDC (or RTPs) and these other elements can be observed only because all of them are being bridged by fluid intelligence.

Next, I examine the case where heterogeneous economies are fully open with each other except for labor force, and SH is maintained among them by appropriate government interventions. As noted above, a higher fluid intelligence causes a lower CWR at MDC and also causes higher productivity and labor income. These properties are unchanged even under SH, because SH affects capital accumulation, not the production function. Therefore, CWRs at MDC (or RTPs) can be still observed to be negatively correlated with labor incomes among economies. The observable correlation between CWRs at MDC (or RTPs) and labor incomes under SH is still spurious and does not indicate any direct causality between the two.

5.2 Correlation with capital and capital income

Next, the correlations with capital and capital income are examined. In the case where economies are isolated from each other and SH does not matter, CWRs at MDC (or RTPs) can also be observed to be negatively correlated with capital and capital income by the same reasoning discussed in Section 5.1, and the observable correlations are still spurious. The case where heterogeneous economies are fully open with each other except for labor force, and SH is maintained among them by appropriate government interventions, is not as simple, however, because SH substantially and differently affects capital accumulations across economies.

Because a higher fluid intelligence causes not only lower CWRs at MDC (as shown in Section 4.4.3) but also higher productivity and labor incomes as well as high amounts of capital and capital incomes (as shown in Sections 2.2 and 4.4.2), all of these factors become simultaneously heterogeneous with heterogeneous fluid intelligences. Hence, the state where only CWRs at MDC are heterogeneous does not exist.

Under SH, capital accumulations of economies with relatively low CWRs at MDC are restrained and those with relatively high CWRs at MDC are enhanced. As a result, the amounts of capital that economies own do not greatly differ. Hence, CWRs at MDC (or RTPs) may not clearly be observed to be negatively correlated with capital and capital income.

However, in reality, SH may not be “correctly” achieved (Harashima, 2010, 2017a). In this case, a government will intervene only up to the point at which the number of votes cast in elections in response to increases in the level of economic inequality is equivalent to that in response to decreases. If government intervention for SH is implemented in this manner, the amounts of capital that economies own still may differ substantially, even though SH appears to have been achieved. Therefore, CWR at MDC (or RTP) may be still clearly be observed to be negatively correlated with capital and capital income. An important point, however, is that even if negative correlations between them are observed, these correlations are still spurious and no direct causality exists.

6 DISCUSSION

6.1 RTP as a shadow of CWR at MDC

As Section 3 and Harashima (2018, 2019) indicate, RTP and CWR at MDC can be substituted for one another. The nature that fluid intelligence significantly influences CWR at MDC therefore means that it should also influence the process of RTP formation—or at least it can be interpreted as doing so.

Harashima (2004, 2014) presented a model of RTP that eliminates the serious drawback of Uzawa's (1968) RTP model, which is one of the most familiar endogenous RTP models. The key variable in Harashima's (2004, 2014) model is W, which indicates the size of the utility stream. It is defined as

$$W = \lim_{T \to \infty} E \int_0^T \rho(t) u(c_t) \exp(-\psi t) dt \tag{34}$$

on a balanced growth path on which y_t, k_t, and c_t as well as technology grow at the same constant rate (ψ), where c_t is consumption in period t, $u(\bullet)$ is the utility function, E is the expectation operator, and

$$\rho(t) = \frac{1}{T} \quad \text{if } 0 \leq t \leq T$$
$$\rho(t) = 0 \qquad \text{otherwise.}$$

Let c^* be

$$Eu(c^*) = \lim_{t \to \infty} E[u(c_t)\exp(-\psi t)] \ . \tag{35}$$

If $\psi = 0$ (i.e., no economic growth), c^* is a positive constant and indicates consumption at steady state. Even if the economy grows (i.e., $\psi > 0$), c^* is still a positive constant because c_t grows at rate ψ. In this sense, we can interpret c^* as consumption at "steady state" not only when the economy does not grow but also when it is growing. By equations (34) and (35),

$$W = E[u(c^*)] \ .$$

An essential feature of this endogenous RTP model is that RTP (θ) is sensitive to, and a function of, W such that

$$\theta = \tilde{\theta}(W) = \tilde{\theta}\{E[u(c^*)]\} \ ,$$

where the function $\tilde{\theta}(\cdot)$ is monotonically continuous and continuously differentiable and

$$\frac{d\theta}{dW} = \frac{d\theta}{dE[u(c^*)]} < 0 \ . \tag{36}$$

That is, RTP (θ) has a one-to-one correspondence with the expected utility at steady state, $E[u(c^*)]$. As equations (3) and (6) indicate, a higher FI means a larger value of $W = E[u(c^*)]$, and by inequality (36), a lower RTP. The causality clearly runs from FI to RTP. Hence, fluid intelligence also significantly influences the formation of RTP.

Because W indicates steady state consumption, a larger value means a better future in a sense, which means that an estimated brighter future lowers RTP. It seems highly likely that a higher FI causes a household to have a brighter outlook on the future, which in turn lowers its RTP.

As shown in Section 4.2, a household with a higher FI can realize a greater number of best-timing consumption opportunities. This also means that a higher FI indicates a brighter future. Because a household with a higher FI has a lower CWR at MDC, as shown in Section 4.4.3, a brighter future also means a lower CWR at MDC. The common property in determining RTP and CWR at MDC is therefore that both are governed by future prospects—particularly by an indicator that signals how optimistic a household's view of the future is—and fluid intelligence significantly influences this view.

Note that, in the above endogenous RTP model, technological progress does not affect the formation of RTP because W is not affected by it, as indicated in equation (34). As shown in Section 4.5, technological progress also does not affect CWR at MDC.

Also note that the above endogenous RTP model predicts that an increase in uncertainty about the future economy results in an increase in RTP (Harashima, 2004, 2014). An increase in uncertainty will correspond to an increase in difficulty ($\widetilde{D}$) in equations (17) and (20). An increase in $\widetilde{D}$ consequently decreases F and increases CWR at MDC by inequalities (19), (18), and (33). Hence, CWR at MDC and RTP also have this common property.

6.2 *Indicator of a brighter future and impatience*

RTP has been regarded as indicating the degree of impatience. However, the endogenous RTP model shown in Section 6.1 and by Harashima (2004, 2014) indicates that impatience may be irrelevant to RTP. Considering the nature of RTP as a "shadow" of CWR at MDC, however, we can still interpret RTP as an indicator of the degree of impatience.

As shown in Section 6.1, a higher FI leads to a brighter future and a lower CWR at MDC and RTP. It is highly likely that a household with a brighter future behaves less impatiently—in other words, it can tolerate more current displeasure. We can see this nature in the capital derived from motive (b). As discussed in Section 4.3.2.1, a household accumulates capital with motive (b) until it reaches its most comfortable combination of the additional future increase and the present decrease in accessible consumption opportunities. In this model, some resources are not consumed at present to gain future rewards, which means that a household must exercise patience. A larger additional future increase means a brighter future, but it also means a larger amount of resources that has to be given up at present, and a brighter future means a higher degree of patience (i.e., a lower degree of impatience). Hence, a lower CWR at MDC and RTP mean a lower degree of impatience. RTP as a shadow of CWR at MDC therefore still can be interpreted as an indicator of the degree of impatience. Nevertheless, the origin of impatience is not RTP (or CWR at MDC)—it is fluid intelligence, because a higher *FI* causes a lower RTP (or CWR at MDC).

Although RTP can be interpreted as an indicator of impatience, we cannot necessarily say that a household discounts its expected utilities with a constant RTP in every future period, as the RTP-based procedure indicates. Because of the nature of CWR at MDC, we can say that a household with a lower RTP (CWR at MDC) is less impatient overall, but we do not know whether its degree of impatience remains unchanged for any future period.

6.3 "True and correct" CWR at MDC

As discussed in 4.3.2.1, nobody knows whether the combination that a household subjectively feels to be most comfortable eventually is equal to the predetermined objectively true and correct most comfortable combination. If CWR at MDC is essentially determined by fluid intelligence, however, the objectively true and correct CWR at MDC itself may exist, because fluid intelligence is highly likely given by nature (i.e., exogenously). Even if the objectively true and correct value does exist, a household still cannot know its objectively true and correct values of *FI* and CWR at MDC. It can only make its best estimate at several economic values and behave on the basis of whether its CWR at MDC is most comfortable in an uncertain environment.

7 CONCLUDING REMARKS

RTP has been regarded to be negatively correlated with income, and many empirical studies have supported this correlation. A causality from income to RTP has generally been thought to exist, but the mechanism behind this causality has not been sufficiently explained theoretically.

In this paper, I examine this problem from a different point of view, one based on the concept of fluid intelligence in an economy under the MDC-based procedure (Harashima 2019). Fluid intelligence is a type of human intelligence, and its importance has been emphasized in psychology and psychometrics (e.g., Cattell, 1963, 1971). Harashima (2019) described the MDC-based procedure through which a household reaches a steady state and showed that households probably use this procedure rather than a RTP-based procedure, because the former is far easier to use than the latter and both procedures lead households to the same steady state.

I show that a higher fluid intelligence causes a lower CWR at MDC (RTP) and causes higher levels of productivity, labor income, capital income, and capital. Hence, CWRs at MDC (RTP) can be observed to be negatively correlated with labor incomes. CWRs at MDC (RTPs) may not be clearly observed to be negatively correlated with capital and capital income at SH, because SH affects capital accumulation in different ways across heterogeneous households. However, if a government intervenes up to the point at which the numbers of votes cast in elections in response to increases and decreases in the level of economic inequality are balanced, SH may not be achieved correctly. Therefore, negative correlations between the above factors may be still clearly observed.

In any case, however, the observed negative correlations are spurious and there is no direct causality between them. They appear to be correlated only because they are bridged by fluid intelligence.

References

Becker, Gary S. and Casey Mulligan (1997) “The Endogenous Determination of Time Preference,” *Quarterly Journal of Economics*, Vol. 112, No. 3, pp. 729–758.

Becker, Robert A. (1980) “On the Long-run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households,” *The Quarterly Journal of Economics*, Vol. 95, No. 2, pp. 375–382.

Cattell, Raymond Bernard. (1963) “Theory of Fluid and Crystallized Intelligence: A Critical Experiment,” *Journal of Educational Psychology*, Vo. 54, pp. 1-22.

Cattell, Raymond Bernard. (1971) *Abilities: Their structure, growth, and action*, Houghton Mifflin, Boston.

Epstein, Larry G. (1987) “A Simple Dynamic General Equilibrium Model,” *Journal of Economic theory*, Vol. 41, No. 1, pp. 68–95.

Epstein, Larry G. and J. Allan Hynes (1983) “The Rate of Time Preference and Dynamic Economic Analysis,” *Journal of Political Economy*, Vol. 91, No. 4, pp. 611-635.

Fisher, Irving (1930) *The Theory of Interest*, Macmillan, New York

Frederick, Shane, George Loewenstein and Ted O’Donoghue (2002) “Time Discounting and Time Preference: A Critical Review,” *Journal of Economic Literature*, Vol. 40, No. 2, pp.　351-401.

Harashima, Taiji (2004) “A More Realistic Endogenous Time Preference Model and the Slump in Japan,” *EconWPA Working Papers*, ewp-mac0402015.

Harashima, Taiji (2009) “A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers’ Innovations as an Essential Element,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 15508.

Harashima, Taiji (2010) “Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 24233.

Harashima, Taiji (2012) “A Theory of Intelligence and Total Factor Productivity: Value Added Reflects the Fruits of Fluid Intelligence,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 43151

Harashima, Taiji (2014) “Time Preference Shocks,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 60205.

Harashima (2016) “A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers’ Innovations as an Essential Element,” in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 50, No.1, pp. 55-80.（原嶋　耐治「全要素生産性の理論と収斂仮説：根源的要素としての一般労働者のイノベーション」『金沢星稜大学論集』第50巻第1号 55～80頁）

Harashima, Taiji (2017a) “Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population,” in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 51, No.1, pp. 31-80.（原嶋　耐治「持続可能な非均質性—均質ではない構成員からなる経済における不平等、経済成長及び社会的厚生—」『金沢星稜大学論集』第51巻第1号 31～80頁）

Harashima (2017b) “A Theory on the Economic Impacts of Immigration,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 78821.

Harashima (2018) “Do Households Actually Generate Rational Expectations? “Invisible Hand” for Steady State,” *MPRA* (*The Munich Personal RePEc Archive*) *Paper* No. 88822.

Harashima (2019) “Do Households Actually Generate Rational Expectations? ‘Invisible Hand’ for Steady State,” in Japanese, *Journal of Kanazawa Seiryo University*, Vol. 52, No.2, pp. 49-70.（原嶋　耐治「家計は実際に合理的期待を形成して行動しているのか—定常状態への「見えざる手」—」『金沢星稜大学論集』第52巻第2号 49～70頁）

Lawrance, Emily C. (1991) “Poverty and the Rate of Time Preference: Evidence from Panel Data,” *Journal of Political Economy*, Vol. 99, No. 1, pp. 54–77.

Lord, Frederic M. and Melvin R. Novick. (1968) *Statistical Theories of Mental Test Scores*, Addison-Wesley, Reading, MA.

Lucas, Robert E., Jr. and Nancy L. Stokey (1984) “Optimal Growth with Many Consumers,” *Journal of Economic Theory*, Vol. 32, pp. 139-171.

Obstfeld, Maurice (1990) “Intertemporal Dependence, Impatience, and Dynamics,” *Journal of Monetary Economics*, Vol. 26, No. 1, pp. 45-75.

Samwick, Andrew A. (1998) “Discount Rate Heterogeneity and Social Security Reform,” *Journal of Development Economics*, Vol. 57, No. 1, pp. 117–146.

Uzawa, Hirofumi (1968) “Time Preference, the Consumption Function, and Optimal Asset Holdings,” in *Value, Capital, and Growth: Papers in Honour of Sir John Hicks* (J. N. Wolfe ed.), University of Edinburgh Press, Edinburgh, Scotland.

van der Linden, Wim J. and Ronald K. Hambleton. (Eds.) (1997) *Handbook of Modern Item Response Theory*, Springer-Verlag, New York.

Ventura, Luigi. (2003) “Direct Measure of Time-preference,” *Economic and Social Review*, Vol. 34, No. 3, pp. 293–310.