Chaos in the tourism industry

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2008

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July 22, 2008

Abstract

The paper presents an application of the chaos theory to tourism, a sector in which operators' choices are particularly elaborate and complex. The dynamics of the tourist industry are, in fact, the result of close interactions between units of production, tourist flows, local authorities and natural resources. These interactions do not necessarily lead to a regular trend in the development of the tourist industry as proposed by Butler; on the contrary, irregularities of various types are very possible. The model microfounds rigorously on both the demand and the supply side. Firms and tourists operate under the hypothesis of limited rationality, the former in an oligopolistic context, the latter on the basis of mechanisms of evolutionary selection. Although not exhaustive, the model forms a theoretical platform that can be easily adapted to hypotheses and situations that differ from those originally hypothesized. As a consequence, this paper presents a series of numerical simulations. The results show the chaotic nature of a tourist flow, which limits the practicability of measures introduced to stabilise the system. In their place, measures are needed that stimulate a continuous reshaping of the system in relation to the factors that tend to change it. **JEL classification:** C73, L10, L83, Q01

Keywords: sustainable tourism, chaos, evolutionary games

1 Introduction

The theory of the life cycle of tourist destinations proposed by Butler (1980) is still at the centre of intense scientific debate today. Even though it is recognised by scholars as the reference framework for the study of the dynamics of the tourist industry, Butler's approach does not account for the frequent differences to be found between the time series of tourist flows and their theoretical forecasts. The life cycle follows a regular succession of five stages, *exploration*, *involvement*, *development*, *consolidation* and *stagnation* (Figure 1), followed by a stage of decline or rejuvenation. The first four stages imply the growth of the system, whilst the stage of stagnation occurs once the maximum sustainable tourist flow has been reached. Without economic policy measures aimed at requalifying the tourist area, stagnation will lead to inevitable decline. However, empirical evidence shows the dynamics of this situation to be much more complex and characterised, for example, by phases of uniform growth followed by more or less marked (Christaller, 1963; Plog, 1973; Lundtorp-Wanhill, 2001) or even chaotic (Russell-Faulkner, 2004) cyclical trends. The main anomalies are to be found in the last phases of the cycle, so much so that some scholars believe that the theory of the post-stagnation stages needs to be revised (Agarwal, 1997; Priestley-Mundet, 1998; Aguilò-Alegre-Sard, 2005).

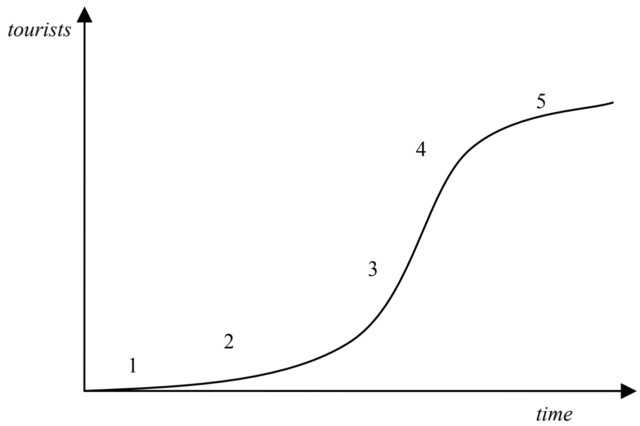

Figure 1: Butler's cycle - 1) Exploration 2) Involvement 3) Development 4)Consolidation 5) Stagnation.

The general view that emerges from the numerous empirical tests carried out is that Butler's model has to be considered primarily as descriptive rather than normative (Haywood, 1986; Cooper-Jackson, 1989; Ioannides, 1992; Opperman, 1995, Hovinen, 1981, 2002). Certainly there are some studies that reveal a significant correlation between time series and the theoretical model (see for example Meyer-Arendt, 1985; Douglas, 1997), but they are to be considered as one of the many possible manifestations of the development of a tourist locality. One of the main limitations of the model is that it does not explicitly consider the effects of factors, both external and internal to the system, on the evolutionary dynamics of the tourist industry under examination: variations in the number of firms working in the sector, the preferences of tourists, competition or the quality of the environment can generate substantial changes in the normal development of a tourist destination. For example, Lundtorp-Wanhill (2001) show how, when assessing the nature of tourist flows from the habitual to the occasional, substantial changes can be seen in the regularity of the cycle as proposed by Butler, so much so as to make the authors define it as a "*caricature*" of the real situation.

The belief that there are many more complex aspects behind the evolutionary dynamics of the tourist industry than Butler's theory would suggest has led scholars to reconsider their theoretical paradigms. Tourism begins to be seen as operating as a non deterministic and non linear system, characterised by a complex network of relations between a large number of elements, each one of which is subject to continuous changes and stimuli (McKercher, 1999). From this point of view the chaos theory can form a paradigm on which a new theory of tourist development can be built. Butterfly effects, bifurcations, strange attractors and chaos can provide valid explanations for the anomalies observed during the long life cycle of many tourist localities (Russell, Faulkner, 1999; Prideaux, 2000; Hovinen, 2002; Zahra, Ryan, 2007), but this requires a complete re-conceptualisation of the theory that integrates research on tourism with other scientific sectors, such as environmental economics, ecology and the theory of complexity (Farrel, Twining-Ward, 2004). An interesting theoretical model that has moved in this direction is the one proposed by Casagrandi-Rinaldi (2002). Their description of a tourist system is based on the interaction of three fundamental elements, tourist flow, environment and capital, described by a system of differential equations. A study of the system reveals the presence of points of bifurcation at which significant changes in the behaviour of the system can be observed with minimal variations in the reference parameters [1]. Although this model provides interesting suggestions for analysis, it nevertheless lacks a rigorous microfoundation of the equations that describe the system: in particular, it does not explain the mechanisms that regulate the decision-making processes of a tourist. The same fault can be found in the model proposed by Hernandez-Leon (2007), which is very similar to the one that has just been described. Once again the fundamental equations of the system, even though they are able to reproduce the dynamics of Butler's cycle, are not the result of a rigorous formalization of the behaviour of the agents involved.

The model presented in this paper starts from these points and attempts to describe the evolutionary dynamics of a hypothetical tourist industry through the interaction of demand (tourist flow), supply (oligopolistic firms) and natural environment, but also introduces some new aspects that can be summarised in four points:

1. the tourist industry is assumed to be made up of a certain number of local firms (destinations), each one organised in a Cournot framework of symmetrical oligopoly. The formal situation of oligopoly means that the quantity of services produced and the price charged can be calculated for each locality in each period. The choice of an oligopolistic form of the market is logical if we consider that there is a limited number of firms in a local industry, that entry is expensive and that there exists a high degree of substitutability between the tourist services offered by the firms. This last point justifies the symmetry of oligopoly. Two levels of competition are considered in the model: the first concerns firms in the same locality and therefore in competition with each other, whilst the second involves different localities competing to gain an ever larger share of the tourist population;

2. the tourist flow towards a locality is regulated by the level of surplus obtained on average by the tourist that decides to visit it: this surplus depends on both the level of the services in the industry and the quality of the environment. The mathematical law that formalises the dynamics of a tourist flow draws on the replicator theory (Taylor-Jonker, 1978; Weibull, 1998), which can often be found in evolutionary game theory. In summarising the decision-making processes of a tourist, it takes into account not only the surplus that can be obtained in the different localities, but also factors such as popularity and congestion, all in a non deterministic

[1] A similar analysis applied to the interaction between collective actions and the use of natural resources can be found in Anderies (2000).

context. Choice will also depend on the probability of acquiring information about the localities. This information is acquired through random exchanges about the experiences of single tourists. The use of the evolutionary approach makes it possible to implicitly consider a more realistic and limited rationality of tourists; in fact, in a world characterised by scarce information a tourist will not necessarily choose the destination that guarantees the greatest surplus.
Papatheodorou (2005) formalises a tourist's choice as if he were perfectly rational, informed and maximising his own utility function. Although it is a well-established approach and can be easily applied, we have preferred to adopt what, in our opinion, is a more realistic selection model, in which the tourist has little information about the different tourist destinations and limited rationality, in the sense that he can continue to prefer localities that guarantee lower levels of surplus;

3. the mathematical law that describes the dynamics of a tourist flow is unknown to firms, which therefore find that they have to optimize their profits period by period, as they cannot plan a path for the efficient development of production and tourism in advance. We believe that this approach to be truer to the real situation and in any case closer to the spirit of the model developed here, which aims to investigate the chaotic aspects of the system rather than define the deterministic laws that are valid in the long run;

4. the model has been developed in discrete time. The time unit chosen refers to the time necessary for the tourist industry to significantly change its production apparatus and therefore the quantity of services produced. The assumption is that the tourist industry changes its capacity slowly; even if individual firms can change their production of tourist services within a short period of time, we believe that the overall impact on the system is negligible and only after a fairly long period can a relevant change be observed. Furthermore, this choice of method is essential, given the fact that, unlike differential equations, difference equations obtained in a model with discrete time can generate chaotic dynamics if they take on specific mathematical forms[2].

In addition to productive activities and the movement of tourists, the model also tries to highlight the deterioration and recovery of the quality of the environment which characterises the development phases of the tourist locality, in order to assess environmental sustainability. The structure of the model provides for the tourist flow to be distributed among different localities (oligopolies), thus affecting the quality of the environment in each one, both directly through the pollution produced by each tourist and indirectly by stimulating the production of services. The levels of production and the quality of the environment, in turn, condition the dynamics of the tourist flow by changing its distribution.

As we were unable to trace the general properties of the model to test whether the model structured in this way is capable of reproducing both regular Butler-type dynamics and chaotic trajectories, and thus to explain the

[2]It is well known in the literature how the logistic equation $x_{t+1} = ax_t(1-x_t)$ can generate cyclical and chaotic dynamics according to variations in parameter a. For greater detail, see Elaydi (1999).

complexity of the evolutionary processes observed in many tourist localities, we preferred to use the method of numerical simulations and limit the study to only two competing tourist localities. We assumed the system is in an initial state of equilibrium, in which only one of the two destinations is fully developed. We then concentrated our attention on the less developed locality and studied the processes of growth in relation to changes in the parameters of the model.

Even though it is not exhaustive, the picture that emerges from the study of the bifurcation diagrams enables us to make some important observations about the complexity of the evolutionary dynamics of the tourist industry and the role of policy measures[3]. In particular, the scenarios that emerge from the simulations suggest the following observations:

1. Significant changes in the state of the system can be induced by variations that concern one or more of the following factors: elasticity of demand, tourist preferences, costs of production, number of firms and the environmental impact of the tourist industry;

2. As McKercher (1999) maintained, both linear and non linear processes are active in tourist systems; the prevalence of one or the other depends on the phase in which the system is at a certain moment. The tourist flow can therefore appear stable, or at least evolve in a regular and predictable way, for long periods and then suddenly become chaotic. The simulations have in fact reproduced this type of behaviour of tourist flows fairly consistently, thus showing how the initial Butler-type phases of development can be followed by markedly unstable, typically cyclical or chaotic dynamics;

3. The more or less stable nature of the dynamics is strongly influenced by factors that can be traced back mainly to consumer tastes, sensibility to price and, to a certain extent, unforeseeable events such as environmental disasters and political instability. The model focuses mainly on the first two factors, leaving to one side the role played by catastrophic events[4]

Although the model proposed here does not consider all the factors that play a role in the development of a tourist industry (for example, it does not consider transport costs), it nevertheless forms a valid theoretical platform. Further extensions and elaborations may be introduced, with particular reference to the mechanisms of interaction between the industrial organisation of the production system and the decision-making of the tourist.

[3] We were guided in our choice of analysis techniques by an interesting article by Currie-Kubin (2006), in which the use of bifurcation diagrams helped to highlight the chaotic nature of the dynamics generated in *Core-Periphery* models, thus seriously questioning all the basic assumptions of 'New Economic Geography'.

[4] The inclusion of catastrophic factors would have meant considering some stochastic disturbance; this approach, however, would go against the basic idea of the model which is to reproduce endogenously chaotic dynamics without the intervention of exogenous shocks. For a discussion of the management of catastrophic events in the tourist industry, see Ritchie (2004). In particular the inconstancy of tourists can produce significant changes in both the level of demand elasticity and the criteria followed in the selection of the localities to visit, thus generating strong and unforeseeable fluctuations in tourist flows;

4. Some important observations about the role of economic policy can be drawn from the inherent instability of tourist flows. Development cannot be controlled directly, because it is determined by factors that cannot be directly managed by authorities (see above). Authorities can influence the direction of growth, but the measures adopted may, as well as increasing the size of the flows of visitors, also determine the instability of the flow itself.

The paper is structured in three parts: in the first, the mathematical structure of the model is developed to obtain the basic equation that describes the dynamics of a tourist flow; the second part presents the results of the numerical simulations and the analysis of the bifurcation diagrams; the third part concludes by discussing the possible developments and extensions of the model.

2 The Model

2.1 Tourists

The tourist flow towards a given locality is generally regulated by its popularity and by the average level of satisfaction obtained by those who choose to visit it. The positive assessment attributed to the locality by a tourist will be communicated to other potential tourists who, with a certain degree of probability, will then decide to spend their holidays at that destination. The level of probability tends to increase, the greater the popularity of the tourist destination and the degree of satisfaction reached by the visitors, whereas it decreases as the level of popularity and satisfaction of other competing localities increases.

Consider a set $i \in D$ of tourist localities competing to attract ever greater shares of tourists by attracting them away from other competing localities. The potential population of tourists is exogenous and equal to M_{max}, whilst $m_{i,t}$ indicates the share of tourists that at time t chooses destination i, for a total number of tourists equal to $M_{i,t} = m_{i,t}M_{max}$. The vector $m_t = \{m_{1,t}, ..., m_{i,t}, ...\}$ describes, therefore, the state of the tourist population at time t.

A tourist compares his experience at time t with that of another tourist chosen at random from the population. The exchange of information can force him, with a certain probability, to review his preferences and choose a different locality for the following period. If both tourists have had the same experience, that is, at time t they visited the same locality, they will not obtain any additional information and so they have no incentive to change tourist destination.

The only element that distinguishes one tourist from another, therefore, is the different experiences they have acquired by visiting different localities in the same period. We rule out the possibility that the two tourists can have different opinions about the same locality they have visited [5].

Given the constancy of the tourist population, it is assumed implicitly that the tourist has an infinite life or that, once information is obtained, he is substituted by a perfect copy of himself (descendent), to whom all the information acquired will be transferred (which means, for the purposes of the results, exactly the same thing). It will be the descendent at that point to decide whether to return to the old destination or go to a new one[6]. From this moment onwards, for the sake of simplicity, we assume that the tourist lives eternally and therefore

[5]Further extensions of the model could take into account a certain heterogeneity in the assessments of tourists who have had the same experiences, by distinguishing, for example, mass tourism from ecotourism. This hypothesis, however, would require a subdivision of the tourist population into at least two subpopulations and the identification of an evolutionary mechanism that explains the prevalence of one type of tourism over the other, thus making the model much more complex.

[6]The model also allows for the fact that a tourist can acquire information from other sources, such as Internet or specialised agencies. We assume, however, that before he takes a decision he will try to get confirmation from tourists with the experience necessary to provide a more objective assessment.

there will be no need to continuously distinguish between the descendent and the parent.

It is well to point out that the experience of a tourist is limited to the knowledge of the characteristics of the two localities: the one visited at time t and the one that he decides to visit at time $t+1$. The experiences before time t have therefore been forgotten.

Let $P_t\left(V_i \rightarrow V_j \left| T_k\right.\right)$ with $i, j, k \in D$ be the probability that a tourist, visiting locality i at time t decides to visit locality j in the following period, on the basis of the information received at time t by a tourist in locality k.

Having assumed that a tourist considers the possibility of changing destination from i to j, only after having exchanged opinions with someone who has already visited the other locality, we can write that $P_t\left(V_i \rightarrow V_j \left| T_k\right.\right) = 1$ if $i = j = k$ and $P_t\left(V_i \rightarrow V_j \left| T_k\right.\right) = 0$ if $k = i \neq j$ or $k \neq i, k \neq j, i \neq j$. If the comparison was made with someone with the same experience, the tourist (or likewise his direct descendent) will return to the old destination with probability equal to 1; similarly, the probability that he decides to visit a different locality without having had information from a tourist who has already been there is zero.

In other cases in which $i \neq k, j = k$ or $i = j \neq k$, the law of conditional probability $P_t\left(V_i \rightarrow V_j \left| T_k\right.\right) = P_t\left(V_i \rightarrow V_j, T_k\right) / P_t\left(T_k\right) > 0$ stands.

If we consider locality i we can determine its net tourist flow by finding the difference between the number of tourists arriving and leaving in a certain period of time. On the basis of the assumed imitative process, the flow of tourists arriving is equal to $\sum_{k \in D, k \neq i} m_{k,t} M_{max} P_t\left(V_k \rightarrow V_i, T_i\right)$ whilst those leaving is $\sum_{k \in D, k \neq i} m_{i,t} M_{max} P_t\left(V_i \rightarrow V_k, T_k\right)$, from which we obtain the dynamic equation for the tourist flow:

$$
\begin{aligned}
m_{i,t+1} - m_{i,t} = \sum_{k \in D, k \neq i} & m_{k,t} P_t\left(V_k \rightarrow V_i \left| T_i\right.\right) P_t\left(T_i\right) + \\
& - m_{i,t} P_t\left(V_i \rightarrow V_k \left| T_k\right.\right) P_t\left(T_k\right)
\end{aligned} \tag{1}
$$

where $P_t\left(T_h\right) = m_{h,t} \; \forall h \in D$.

The exchange of information between tourists mainly concerns their level of satisfaction and the popularity of the destination they visited. And therefore it is reasonable to suppose that the conditional probability, that is, the probability of changing destination once the information has been obtained from another tourist, will depend exactly on these factors, that is:

$$
\begin{gathered}
P_t\left(V_k \rightarrow V_i \left| T_i\right.\right) = f_k\left(Sc_{k,t}, Sc_{i,t}, m_{i,t}, m_{k,t}\right), \\
P_t\left(V_k \rightarrow V_k \left| T_i\right.\right) = 1 - f_k\left(Sc_{k,t}, Sc_{i,t}, m_{i,t}, m_{k,t}\right)
\end{gathered}
$$

with $\partial f_k / \partial Sc_{k,t} < 0, \quad \partial f_k / \partial Sc_{i,t} > 0, \quad \partial f_k / \partial m_{k,t} > 0, \quad \partial f_k / \partial m_{i,t} < 0,$ $0 \leq f_k \leq 1, \quad \forall k \neq i \in D.$

Let us indicate with $Sc_{i,t}$ and $Sc_{k,t}$ the utility (surplus) obtained on average in time t from tourists in localities i and k, whose popularity is measured by the share of the tourist population, $m_{i,t}$ and $m_{k,t}$ respectively.

As suggested by Weibull (1998), it is assumed that the conditional probability to reach a locality in the following period is proportional to the popularity of that locality and that the factor of proportionality is positively correlated with the present surplus that can be obtained in the same locality. By indicating with $\omega_k(Sc_{i,t}) > 0$ the factor of proportionality that the tourist in k associates with locality i, we can write

$$\begin{array}{l} P_t(V_k \to V_i | T_i) = \frac{\omega_k(Sc_{i,t})m_{i,t}}{\omega_k(Sc_{i,t})m_{i,t}+\omega_k(Sc_{k,t})m_{k,t}} \\ P_t(V_k \to V_k | T_i) = \frac{\omega_k(Sc_{k,t})m_{k,t}}{\omega_k(Sc_{i,t})m_{i,t}+\omega_k(Sc_{k,t})m_{k,t}} \end{array}$$

with $\partial\omega_k/\partial Sc_{i,t} > 0 \quad \forall i,k \in D$.

By substituting in [1] we obtain the basic equation of the model

$$\begin{aligned} m_{i,t+1} = m_{i,t} + \sum_{k \in D, k \neq i} & m_{k,t} \frac{\omega_k(Sc_{i,t}) m_{i,t}}{\omega_k(Sc_{i,t}) m_{i,t} + \omega_k(Sc_{k,t}) m_{k,t}} m_{i,t} + \\ & - m_{i,t} \frac{\omega_i(Sc_{k,t}) m_{k,t}}{\omega_i(Sc_{i,t}) m_{i,t} + \omega_i(Sc_{k,t}) m_{k,t}} m_{k,t} \end{aligned} \quad (2)$$

from which it only remains to specify the exact form of the factors of proportionality ω and its surplus, which are given in paragraphs 3 and 2.2 respectively.

2.2 Firms

Each locality of set D represents a tourist industry which is assumed to be structured as a Cournot-type oligopoly with homogeneous firms[7]. The term 'industry' should not be understood as the whole tourist system in a country, but the set of firms in a certain locality whose organisation, geographical position and environmental resources can be considered a system in itself and which has certain characteristics that distinguish it from other tourist localities (industries) (the classic example is an island that attracts tourists during the summer period)[8]. The group of firms belonging to each of the tourist destinations is considered exogenous and is indicated by $N = \{N_d\}_{d \in D}$.

Each firm belonging to locality d chooses at time t the quantity of tourist services to be produced and supplied, in order to maximise the following profit function in that period:

$$\Pi_{d,i,t} = P\left(\sum_{j=1}^{N_d} q_{d,j,t}, M_{d,t}, E_{d,t}\right) q_{d,i,t} - c_d q_{d,i,t},$$

[7] The hypothesis that the tourist industry is organised along the lines of oligopolistic competition has been analysed by Davies (1999) and Baum-Mudambi (1995). For a review, see Davies and Dawnward (2005).

[8] It is necessary to point out that the dynamics of a tourist industry are not neutral in relation to the scale chosen for the analysis. Development trajectories can undergo marked changes if the analysis moves from a single tourist locality to the whole tourist industry in the geographical area.

with $\forall i = (1, ..., N_d), \forall d \in D$ where $P(.)$ indicates the inverse demand function that depends on the total quantity of services produced by the tourist industry $\sum_{j=1}^{N_d} q_{d,j,t}$, on the flow of tourists arriving $M_{d,t}$ and on the level of environmental quality $E_{d,t}$, whilst c_d is the same marginal cost for all the firms in the same industry [9].

The greater the flow of tourists and the better the quality of the environment, the higher the price per unit of tourist service, that is $\partial P/\partial M_{d,t} > 0$ and $\partial P/\partial E_{d,t} > 0$. If we assume that the inverse demand function is $p_t = \left(M_{d,t}(E_{d,t}+1) / \sum_{j=1}^{N_d} q_{d,j,t}\right)^{\epsilon_d}$, with $1/\epsilon_d > 1$ to indicate the (constant) price elasticity of demand, and consider the symmetrical nature of oligopoly, we obtain the quantity of tourist services produced by firm i at time t

$$q_{d,i,t}^* = q_{d,t}^* = \frac{M_{d,t}(E_{d,t}+1)}{N_d}\left(\frac{N_d - \epsilon_d}{c_d N_d}\right)^{1/\epsilon_d},$$

with $N_d > \epsilon_d \; \forall d \in D$.

If we substitute the equilibrium quantity in the inverse demand function we obtain the equilibrium price $p^* = \left(\frac{N_d - \epsilon_d}{c_d N_d}\right)^{-1}$ which depends entirely on the structural parameters of the local industry. A fixed factor, equal to 1, is added to the value of the quality of the environment in order to avoid the cessation of all productive activities, if it is annulled. It is correct, in fact, to assume that a tourist locality can continue to exist even in the presence of a very degraded natural environment, as in the case of mass tourism which is very often attracted to the wide range of entertainment facilities on offer rather than to the natural beauty of a place.

By integrating the demand function, the average surplus $Sc_{d,t}$ obtained by a tourist who visits locality d at time t is:

$$Sc_{d,t} = \frac{1}{M_{d,t}} \int_{p^*}^{\infty} \frac{M_{d,t}(E_{d,t}+1)}{p_{d,t}^{1/\epsilon_d}} dp = (E_{d,t}+1)\frac{\epsilon_d}{1-\epsilon_d}\left(\frac{N_d - \epsilon_d}{c_d N_d}\right)^{\frac{1-\epsilon_d}{\epsilon_d}}.$$

Given that a tourist flow towards a certain locality is also affected by the surplus obtained in other localities, the model presents two types of strategic interaction: one between firms in the same locality and the other between competing tourist destinations. But, whereas in the first the quantity of tourist services produced by firms in the same industry acts as an element of reciprocal conditioning, the second works through variations in the surplus and therefore in the flows of tourists that go to the different localities[10].

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[9]Firms act under the assumption of bounded rationality as they do not know the law of motion of tourist flow (2). This hypothesis, as well as being realistic, also simplifies somewhat the calculations. If this had not been so, we would, in fact, have had to solve the complex problem of inter-temporal optimization in a differential game (oligopoly). For an application of this type, see Candela-Cellini (2004).

[10]A possible extension of the model could be that the profits of firms in a certain locality were also dependent on the quantity of services produced in other competing destinations, thus introducing a further factor of strategic interaction between local tourist industries. In this case the demand function would assume the following form: $p_t = \left(M_{d,t}(E_{d,t}+1) / \left(\sum_{j=1}^{N_d} q_{d,j,t} + \sum_{g \in D, g \neq d} \sum_{i=1}^{N_g} q_{j,i,t}\right)\right)^{\epsilon_d}$.

2.3 The natural environment

The production of services and the flow of tourists have an immediate impact on the quality of the environment. By indicating the maximum possible level for the quality of the environment in locality d with $\hat{E}_d$, that is, the level that would be reached in the absence of a tourist industry, the quality of the environment at time t is expressed by the following expression:

$$E_{d,t} = max\left\{\hat{E}_d - \alpha M_{d,t} - \beta N_d q^*_{d,t} \quad ,0\right\},$$

with $\alpha > 0$ and $\beta > 0$ to indicate the impact of tourist flows and the production of tourist services on the environment respectively. Once the quality of the environment has been lost, it can only be recovered by reducing production or the number of tourists. Natural resources used by the industry are not renewable so long as the source of pollution persists in the territory: therefore, once the quality of the environment has been destroyed, it can be recovered only by reducing the number of tourists and production.

The natural environment is considered as a fixed stock of environmental resources localised in a specific territory. This can be left in its original state, by guaranteeing the natural wealth that is present, or it can be allocated to an economic use, by establishing infrastructures for tourism with a consequent reduction in its quality. By substituting in $q^*_{d,t}$ and solving the equation for $E_{d,t}$ we obtain

$$E_{d,t} = max\left\{\frac{\hat{E}_d - M_{d,t}\left[\alpha + \beta\left(\frac{N_d-\epsilon_d}{c_d N_d}\right)^{1/\epsilon_d}\right]}{1+\beta M_{d,t}\left(\frac{N_d-\epsilon_d}{c_d N_d}\right)^{1/\epsilon_d}}, 0\right\},$$

from which the maximum limit of sustainable tourism can be calculated, that is, the share of the tourist population which corresponds to quality zero of the environment:

$$\hat{m}_d = \frac{\hat{E}_d}{M_{max}\cdot\left[\alpha + \beta\left(\frac{N_d-\epsilon_d}{c_d N_d}\right)^{1/\epsilon_d}\right]},$$

with $E_{d,t} = 0 \quad \forall m_{d,t} > \hat{m}_d$.

The limit $\hat{m}_d$ is therefore a specific factor of a locality which is determined by the combination of environmental factors $(\hat{E}_d, \alpha, \beta)$, structural components of the industry (N_d, c_d) and elements connected with the psychology of the tourist (ϵ_d).

At this point a tourist industry can be defined as sustainable if it combines a positive level of environmental quality with a positive flow of visitors, that is, if it is capable of satisfying both the following conditions $m_{d,t} > 0$ and $E_{d,t} > 0$ $\forall t,\ d \in D$ at the same time. Given that the trajectories of a tourist flow can vary and converge on a stationary, cyclical or even chaotic state, it is necessary to specify the stability conditions for each of these cases:

1. if the orbit of the tourist flow converges on the fixed point m^* the industry is sustainable on condition that $0 < m^* < \hat{m}$.

2. if the orbit attractor is a P cycle in period n, $P \equiv \{m_j, m_{j+1}, \ldots, m_{j+n-1}$: $g(m_{j+n-1}) = m_j\}$, then the industry is perfectly sustainable on condition that $E_{d,t} > 0\ \forall m_{d,t} \in P$. There may also be cycles that alternate phases of total environmental exploitation with phases of a partial recovery of the quality of the environment in a general alternation between environmental sustainability and unsustainability. The same can be said about the orbits which have a chaotic character, but are limited to an interval of values (m_{min}, m_{max}) ; in this case the industry is perfectly sustainable in terms of the environment if $\hat{m}_d > m_{max}$, unsustainable if $\hat{m}_d < m_{min}$, whilst if $\hat{m}_d \in (m_{min}, m_{max})$ there will be periods of sustainability followed by phases of total exploitation of the environment.

The possible alternation between environmental sustainability and unsustainability can be explained by the fact that a single period of time, taken as the temporal unit of reference in the model, is not completed in a single tourist season, but is considered to be sufficiently long (that is, a number of tourist seasons) to allow an appropriate adaptation of both the quantity of services supplied made by the firms and the recovery of part of the natural environment that has been destroyed[11].

3 The Simulations

A simulation of the model requires an explicit formalization of the factors of proportionality used in the definition of conditional probability.

As in Weibull (1998), let us suppose that $\omega_k(Sc_{i,t}) = exp(\sigma \cdot Sc_{i,t})$ with $\sigma \geq 0$. This specification is sufficiently general, because different choice models can be adopted by the tourist according to the value of the parameter σ:

1. if $\sigma \to 0$, tourists tend to prefer the most popular tourist locality independently of the real benefits that can be obtained;

2. if $\sigma \to \infty$, tourists tend to choose the tourist locality that guarantees a higher level of surplus independently of its popularity.

As in fact

$$P_t(V_k \to V_i | T_i) = \frac{exp(\sigma \cdot Sc_{i,t}) m_{i,t}}{exp(\sigma \cdot Sc_{i,t}) m_{i,t} + exp(\sigma \cdot Sc_{k,t}) m_{k,t}},$$

we have

$$\lim_{\sigma \to 0} P_t(V_k \to V_i | T_i) = \frac{m_{i,t}}{m_{i,t} + m_{k,t}}$$

[11]On the other hand, the existence of cyclical dynamics in natural capital has been widely noted and discussed in studies of ecology and sustainable development; see the literature on dynamic models developed by Anderies (1998, 2000) and Brander-Taylor (1998).

$$\lim_{\sigma\to\infty} P_t\left(V_k \to V_i \left|T_i\right.\right) = \begin{cases} 0 & \text{if } Sc_{k,t} > Sc_{i,t} \\ 1 & \text{if } Sc_{k,t} < Sc_{i,t} \end{cases}$$

Similar specifications about the probability of a tourist's choice can be found in Papathodorou (2005) and Morley (1994).

Let us assume that there are two localities, that is $D = \{A, B\}$, in order to make the study of the model simpler without compromising the purpose of the analysis, which is to identify the chaotic behaviours of a tourist flow in a system of interacting tourist industries [12]. This does not prevent us from considering industry A as a single industry, whilst B is the set of tourist industries in competition with it, that is to say $B = \{B_1, ..., B_n\}$. On the basis of this specification the conditional probabilities are

$$\mathrm{P}_t\left(V_A \to V_B \left|T_B\right.\right) = f_A = \frac{e^{\sigma\cdot Sc_{B,t}}(1-m_{A,t})}{e^{\sigma\cdot Sc_{B,t}}(1-m_{A,t})+e^{\sigma\cdot Sc_{A,t}}m_{A,t}}$$

$$\mathrm{P}_t\left(V_B \to V_A \left|T_A\right.\right) = f_B = \frac{e^{\sigma\cdot Sc_{A,t}}m_{A,t}}{e^{\sigma\cdot Sc_{B,t}}(1-m_{A,t})+e^{\sigma\cdot Sc_{A,t}}m_{A,t}}$$

$$\mathrm{P}_t\left(V_A \to V_A \left|T_B\right.\right) = 1 - f_A$$

$$\mathrm{P}_t\left(V_B \to V_B \left|T_A\right.\right) = 1 - f_A$$

which, substituted in [2], make it possible to obtain the basic equation of the model in an explicit form in relation to the tourist flow that concerns destination A:

$$m_{A,t+1} = m_{A,t} + m_{A,t}(1-m_{A,t})\left\{\frac{e^{\sigma\cdot Sc_{A,t}}m_{A,t} - e^{\sigma\cdot Sc_{B,t}}(1-m_{A,t})}{e^{\sigma\cdot Sc_{A,t}}m_{A,t} + e^{\sigma\cdot Sc_{B,t}}(1-m_{A,t})}\right\} \tag{3}$$

where for $m_{A,t} \in [0,1]$ also $m_{A,t+1} = g(m_{A,t}) \in [0,1]$, that is $g : [0,1] \to [0,1]$.

The reduction to two tourist industries makes it possible to limit the analysis to the study of the dynamics of the tourist flow of just one industry since, the dynamics of destination B is determined by the residue, as $m_{B,t} = 1 - m_{A,t}$.

Given the initial state $m_{A,0}$, the orbit of the system is unequivocally defined. The first iteration is $m_{A,1} = g(m_{A,0})$, the second $m_{A,2} = g(m_{A,1}) = g(g(m_{A,0}))$

[12] Remember that, in addition to competition between industries which determines the distribution of a tourist flow in the different localities, there is competition between firms inside each industry which conditions the quantity of tourist services produced in each period. The two levels of competition are, however, strictly intertwined: whilst the surplus obtainable by a visiting tourist can be determined by the competition between firms in the same locality, this surplus then enters the basic equation of the model [3] as a reference parameter in the choice of the tourist, thus in turn conditioning the dynamics of the tourist flow that underlie competition between destinations.

and so on. By indicating the n-th iteration by $g^{[n]}(m_{A,0})$ we can represent the orbit by the vector $(m_{A,0}, g(m_{A,0}), g^{[2]}(m_{A,0}), g^{[3]}(m_{A,0}), \ldots, g^{[n]}(m_{A,0}), \ldots)$, which in a more compact form becomes $\left\{g^{[n]}(m_{A,0}) : n \geq 0\right\}$ with $g^{[0]}(m_{A,0}) = m_{A,0}$. If $g(m^*) = m^*$ then the system is in stationary equilibrium and m^* is a fixed point. In the same way, if $g^{[k]}(\hat{m}) = \hat{m}$ then $\hat{m}$ is a periodic point and the orbit $\left\{g^{[n]}(\hat{m}) : n \geq 0\right\}$ is periodic in period k. An orbit is defined as chaotic if it is limited, not periodic and shows a marked sensitivity to the initial conditions, in the sense that, following arbitrarily small variations in these conditions, there are significant changes in the course of the orbit itself.

The function $g(m_{A,t})$ has the following fixed points: $m_{A,t} = 0$, $m_{A,t} = 1$ and $\forall m_{A,t} \in (0,1)$ so that $e^{\sigma \cdot Sc_{A,t}} m_{A,t} = e^{\sigma \cdot Sc_{B,t}}(1 - m_{A,t})$. In general there is no guarantee that a solution $m_{A,t}$ exists that satisfies this condition. In the particular case of perfect structural homogeneity among tourist localities, which occurs when two destinations have the same parameter values but not necessarily similar levels of development, the system has at least three fixed points: $m_A^* = 0$, $m_A^* = 1$ and $m_A^* = 0.5$. Furthermore, in this case if a fixed point $\bar{m}_A \in (0,1)$ exists with $\bar{m}_A \neq 0.5$, then $1 - \bar{m}_A$ is also a fixed point in the system.

If, in fact, $e^{\sigma \cdot Sc_A(\bar{m}_A)} \bar{m}_A - e^{\sigma \cdot Sc_B(1-\bar{m}_A)}(1 - \bar{m}_A) = 0$, from the hypothesis of structural homogeneity we have that $Sc_A(1 - \bar{m}_A) = Sc_B(1 - \bar{m}_A)$ and $Sc_A(\bar{m}_A) = Sc_B(\bar{m}_A)$, and therefore also $e^{\sigma \cdot Sc_A(1-\bar{m}_A)}(1-\bar{m}_A) - e^{\sigma \cdot Sc_B(\bar{m}_A)} \bar{m}_A = 0$.

Independently of the degree of structural similarity between the tourist localities, the fixed internal points cannot be analytically determined because $m_{A,t}$ cannot be made explicit from the condition $e^{\sigma \cdot Sc_{A,t}} m_{A,t} = e^{\sigma \cdot Sc_{B,t}}(1 - m_{A,t})$. It is therefore necessary to proceed with the study of the model through numerical simulations on the computer[13]. They do not provide a detailed study of the general properties of the model, but highlight how, even in a deterministic context, significant changes in the dynamics of a tourist flow can be generated after minimal variations in the parameters. These changes make it very difficult to predict the trend of a tourist flow and limits the practicability of any policy measure aimed at stabilising the system. The emergence of irregular and unstable fluctuations is endogenous to the model and is in no way generated by the interference of stochastic shocks.

Each of the simulations starts with the system in equilibrium and with two tourist localities, which, though structurally homogeneous, are at different stages of development. More precisely, it is assumed that locality A is still in an initial stage of development, whilst destination B has already reached the stage of consolidation. If we assign the values in Table 1 to the parameters, we have a situation as described above and shown in Figure 2.

In this case the system presents three internal equilibrium points, $m_1^* \approx 0.01$, $m_2^* = 1 - m_1^* \approx 0.99$ and $m_3^* = 0.5$ as well as two fixed points $m_4^* = 0$ and $m_5^* = 1$. In view of the perfect structural homogeneity of the tourist localities and since $|g'(m_1^*)| = 0.6734 < 1$, the equilibriums m_1^* and m_2^* are locally stable. Equilibrium m_3^* is instead unstable, as $|g'(m_3^*)| = 1.44982 > 1$. At time zero, therefore, locality A is positioned at point $m_{A,0} = m_1^*$ whilst, by symmetry, locality B absorbs almost all the tourist flow available and positions itself at $1 - m_{A,0} = m_2^*$.

[13]The Mathematica software was used for the simulations.

Non - conditional parameters $\begin{cases} \varepsilon_A = \varepsilon_B = 0.5 \\ \sigma = 0.01 \end{cases}$

Conditional parameters $\begin{cases} N_A = N_B = 100 \\ c_A = c_B = 10 \\ \alpha_A = \alpha_B = 1 \\ \beta_A = \beta_B = 2 \end{cases}$

$M_{\max} = 10^5$

$\hat{E}_A = \hat{E}_B = 10^5$

Table 1: base values for the numerical simulation of the model.

Starting from the basic case with the parameter values shown in Table 1, one of the parameters will be varied each time in conditions of *ceteris paribus*. In order to assess the effect produced by economic policies, the parameters that can be conditioned by public intervention are kept distinct from those that cannot be directly influenced by policy measures.

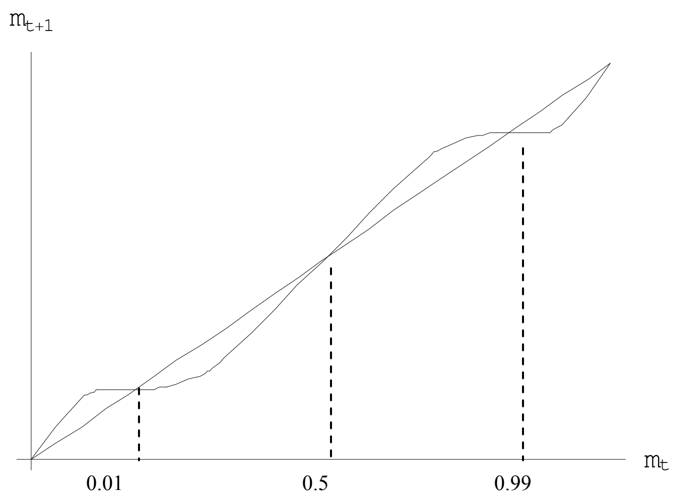


Figure 2: Initial state of the system. Locality A is in a stable equilibrium position $m \approx 0.01$, whilst locality B is in stable equilibrium $m \approx 0.99$. The point of equilibrium $m = 0.5$ is unstable.

The following parameters belong to the first category:

a. Parameters α and β for environmental impact (of tourists and firms respectively), which the policy maker can change with measures aimed at reducing emissions per tourist (for example, by encouraging recycling or limiting the use of private means of transport) and favouring use of productive processes and materials that pollute less;

b. The number of firms working in the industry (N_d), which can be modified by adopting appropriate industrial policies to favour or reduce competition;

c. Costs (c_d), which can be affected by subsidies for the production of tourist services and incentives for the use of efficient technologies and tax concessions.

The second type includes the parameters σ and ϵ_d which describe the tastes and price sensibility of tourists and therefore are beyond the direct control of public authorities. The parameter σ is an indicator of the criterion followed by a tourist in his choice of the locality to be visited: an increase means greater sensibility to the real benefits obtained by visiting a certain locality rather than its reputation; its value will depend therefore on factors linked to lifestyle and fashion. ϵ_d represents the perception that the tourist has of the service that he is offered: the closer the value is to 1 (rigid demand), the more the service is perceived as a luxury good; in the same way, a low value of ϵ_d implies high elasticity of demand and therefore an increased sensibility to price associated with mass tourism [14].

In each of the simulations, the key element is the bifurcation diagram which, given a certain initial state, describes the long term behaviour of the system in relation to variations in a parameter. The chosen parameter is made to vary within a specific interval in 500 steps; the orbit is calculated for at least 400 periods for each value of the parameter considered in the specified interval. However, in order to highlight the long term behaviour, the diagram shows only the last 150 repetitions; that is, for each value of the parameter, the bifurcation diagram represents $g^{[t]}(m_{A,0})$ for $250 \leq t \leq \tau$ with $\tau \geq 400$.

3.1 Changes in demand elasticity - ϵ

Let us analyse the case in which, starting from a situation of equilibrium as described in Figure 2, a change in demand elasticity takes place for locality A. Although the two localities under consideration are supposed to be structurally identical, it is possible that at a later stage differences in their level of development change the tourist's sensibility to price according to the locality visited. For example, given the negative effects of congestion and productive activities on the environment, the tourist may develop a greater sensibility to price with a consequent increase in demand elasticity for the destination with a larger number of visitors. On the other hand, the less developed locality is still able to offer a high level of environmental quality, thus encouraging the tourist to accept a higher price for the same service. The opposite situation may also occur, where the less developed locality ends up with a higher demand elasticity as the tourist is more sensitive to the reputation of a destination rather than its quality. These considerations persuade us that variations in parameter ϵ_A are possible in all directions. In general, a tightening of demand (increase in ϵ_A) leads to an improvement in the quality of the environment and a consequent increase in the surplus Sc_A. It is more likely, therefore, that condition

[14]We assume that the two localities can present different levels of demand elasticity. This hypothesis presumes that the tourist can perceive the nature of the service that is offered to him in a different way, depending on who is supplying it.

$e^{\sigma \cdot Sc_{A,0}} m_{A,0} > e^{\sigma \cdot Sc_{B,0}}(1 - m_{A,0})$ prevails with the tourist flow beginning to move from B to A. On the other hand, an increase in elasticity can start an inverse mechanism and lead to the decline of destination A, even before this locality begins to attract a significant share of the tourists. Figure 3 shows the bifurcation diagram where we indicate the interval of the values for ϵ_A with $I = (0, 1)$ to facilitate the reading of the diagram, before subdividing it into three subintervals $I = \{I_1, I_2, I_3\}$ to identify the different parts of the diagram which show the behaviour of the system to be basically homogeneous:

1. $\epsilon_A \in I_1 = (0, \epsilon_1 \approx 0.35)$: locality A experiences a gradual loss of visitors to the competing locality. In the long term all the tourists settle in B;

2. $\epsilon_A \in I_2 = [\epsilon_1, \epsilon_2 \approx 0.88)$: locality A manages to keep a small share of the potential tourists who continue to prefer alternative destinations;

3. $\epsilon_A \in I_3 = [\epsilon_2, 0.999)$: there is a radical change in the dynamics of the development of destination A which, given the low elasticity of demand and the consequent increase in the surplus of a tourist, is now able to attract a substantial share of tourists.

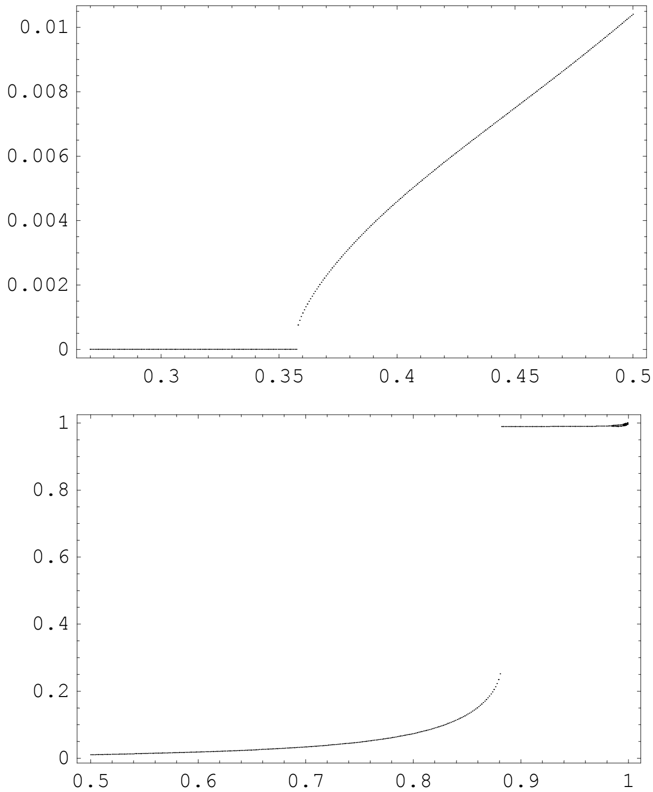
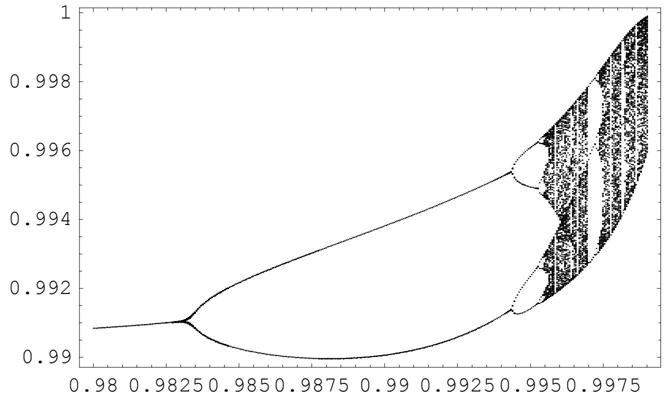


Figure 3: Bifurcation diagrams in relation to ϵ_A with the initial state $m_0 \approx 0.01$. x axis: ϵ_A; y axis: m_A.

A first reading of the diagram therefore seems to suggest general stability in the system in relation to variations in ϵ_A. A more detailed examination, however, reveals elements of a more complex nature. Figure 4 shows two details of the bifurcation diagram in which the long term dynamics do not seem to converge to a fixed value, but instead become cyclical, confirmed by the presence of bifurcations, or chaotic. The existence of cycles of any order and chaotic trajectories is confirmed by the presence of a cycle in period 3 (Li-Yorke theorem, 1975)[15], which is very evident in the central part of the detail on the right.



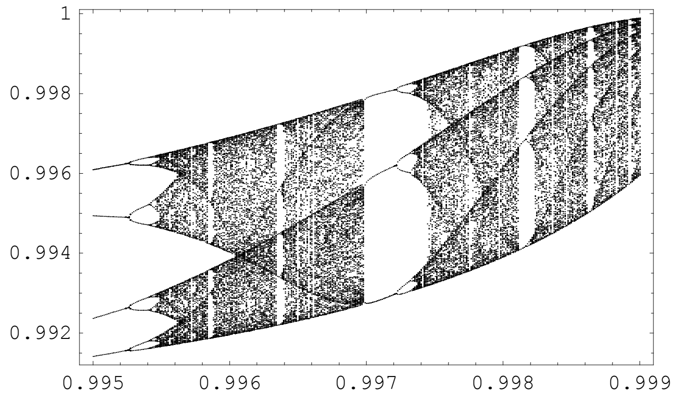


Figure 4: Details of the bifurcation diagram shown in Figure 3. For the values of ϵ_A close to 1 the dynamics in the long run take on chaotic or cyclical trends. x axis: ϵ_A; y axis: m_A.

[15] On the basis of the Li-Yorke theorem (1975):
Given a continuous function $m_{t+1} = g(m_t)$ at interval $J \rightarrow J \subset \Re$ and supposing that point $m \in J$ exists such that

$$g^{[3]}(m) \leq m < g(m) < g^{[2]}(m)$$

then

i for each $n = 1, 2, 3....$ there exists a trajectory of period n in J;

ii g is chaotic in a non-enumerable set $S \subset J$.

Even if these observations are not particularly significant as they are located in an interval with limited values[16], they do, however, show an important property of the model, namely, it generates endogenously complex dynamics in the development of a tourist locality. These complexities will become evident in later simulations. Another important aspect emerges when the trajectories of the tourist flow with specific values of ϵ_A are simulated. As the graphs in Figure 5 show, the dynamics of the tourist flow follow the type of trend theorised by Butler. The interesting point is the difference in the behaviour of the cycle in the post-consolidation phase. In the first simulated case ($\epsilon_A = 0.88$) the cycle tends to become stable following the period of consolidation with a phase of stagnation; in other cases, growth picks up again with a new Butler-type cycle ($\epsilon_A = 0.888$) and an infinite series of chaotic oscillations begins ($\epsilon_A = 0.998$).

On the basis of the values assigned to the parameters, the maximum sustainable tourist flow is positively correlated with the elasticity of demand $1/\epsilon_d$, that is $\partial \hat{m}_d / \partial \epsilon_d < 0$ and, given that in the case we analysed high values of ϵ_A are combined with a significant growth in the tourist flow towards destination A, it is possible that, at those values, there emerges a condition in the industry that was environmentally unsustainable, that is $m_{A,t} > \hat{m}_A$. For example, in the case analysed above in which $\epsilon_A = 0.998$, the maximum limit of sustainable tourism becomes $\hat{m}_A = 0.835$, well below the values of the tourist flow which, even with an irregular trend, is always more than 99% of the total tourist population, as seen in the details in Figure 5.

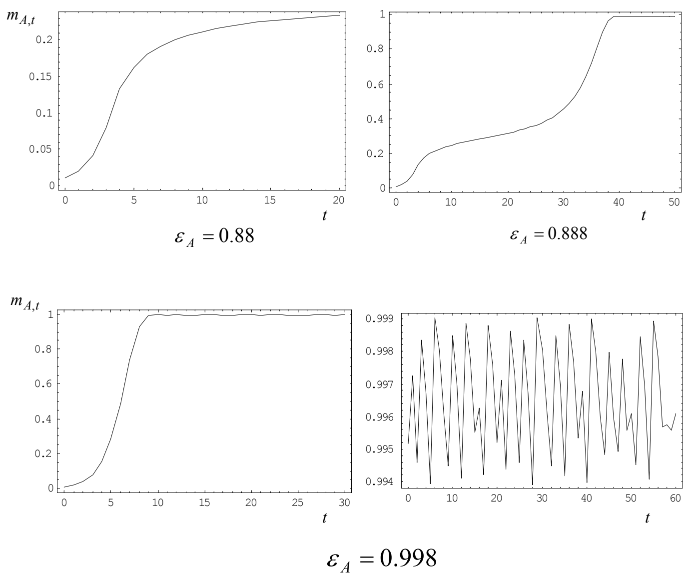


Figure 5: Time series of the tourist flow towards destination A with different values of ϵ_A. In the first two cases the flow converges towards a stationary state and in the third towards a seemingly stable phase, but it is in fact characterised by chaotic irregularities. x axis: t; y axis: $m_{A,t}$.

[16]Although it follows chaotic trajectories, the tourist flow is always close to 99.5% of the population of potential tourists.

Following the same reasoning, environmental unsustainability can be shown in the case in which $\epsilon_A = 0.888$ with $\hat{m}_A = 0.871$ and sustainability for $\epsilon_A = 0.88$ with $\hat{m}_A = 0.873$. In general, development appears sustainable so long as $\epsilon_A < \epsilon_2 \approx 0.88$ whilst for higher values the number of visitors grows so much that it exceeds the maximum sustainable limit.

3.2 Changes in the criterion for the choice of locality - σ

Parameter σ is an indicator of the criterion followed by the tourist in his choice of locality[17]: if it increases, it means less attention is being paid to the reputation of a certain tourist locality and more to the real benefit that can be obtained; if σ is low, the tourists tend to stay in B, given that the rival locality is not sufficiently attractive since it is still in the initial phases of development.

At time zero the system is in equilibrium, that is $e^{\sigma \cdot Sc_{A,0}} m_{A,0} = e^{\sigma \cdot Sc_{B,0}}(1 - m_{A,0})$ with $m_{A,0} \approx 0.01041$. This condition can be rewritten as $\sigma \cdot (Sc_{A,0} - Sc_{B,0}) = ln\left(\frac{1-m_{A,0}}{m_{A,0}}\right)$ with, given perfect structural homogeneity and as $\frac{\partial Sc_d}{\partial m_d} < 0 \ \forall d$, $Sc_{A,0} > Sc_{B,0}$ if $m_{A,0} < 0.5$. Following variations in σ, therefore, the system is going to be in disequilibrium with the result that there will be a flow of tourists towards locality A if σ increases and away from it if it decreases.

The bifurcation diagram in Figure 6 shows this trend and highlights the increasing complexity of the dynamics as σ increases.

When the choice of a tourist destination is guided by fashion and evidence of a greater number of visitors (low level of σ), the tourist flow tends to be stable in so far as it is difficult for the new tourist localities to attract tourists away from well-established destinations. If the choice is taken in a more rational way, that is, privileging those localities that guarantee a higher level of well-being, it is likely that the tourist flow will begin to move towards the less visited localities, before starting to fluctuate cyclically or chaotically from one locality to another, following the continuous oscillations in the levels of surplus that can be obtained in each one of them.

The simulation was limited to an interval with the values $\sigma \in [0, 10]$, without substantial changes in the behaviour of the system for wider intervals.

The study of the dynamics in the long term is made, as above, by dividing the interval into three specific subintervals:

1. $\sigma \in [0, \sigma_1 \approx 0.1)$: the flow of tourists tends to abandon locality A with values of σ close to zero and to reach a stationary level for $\sigma \to \sigma_1 \approx 0.1$;

2. $\sigma \in [\sigma_1, \sigma_2 \approx 0.5)$: the tourist flow tends to settle equally at both tourist destinations;

3. $\sigma \in [\sigma_2, 10)$: the long term trajectories of the tourist flow have mostly cyclical or chaotic behaviours, showing evident fluctuations in a wide interval that goes from 30% to 70% of the total tourist population.

Figure 7 shows three possible developments of the tourist flow, which correspond to the same number of values for parameter σ. Cycles, chaos and asymptotic convergence are possible future developments of a tourist destination, but

[17]Given the values in Table 1 and that σ does not directly condition tourists' surplus, the two localities remain structurally homogeneous, independently of the value of this parameter. The set of fixed points in the system therefore also includes $m_A^* = 0.5$.

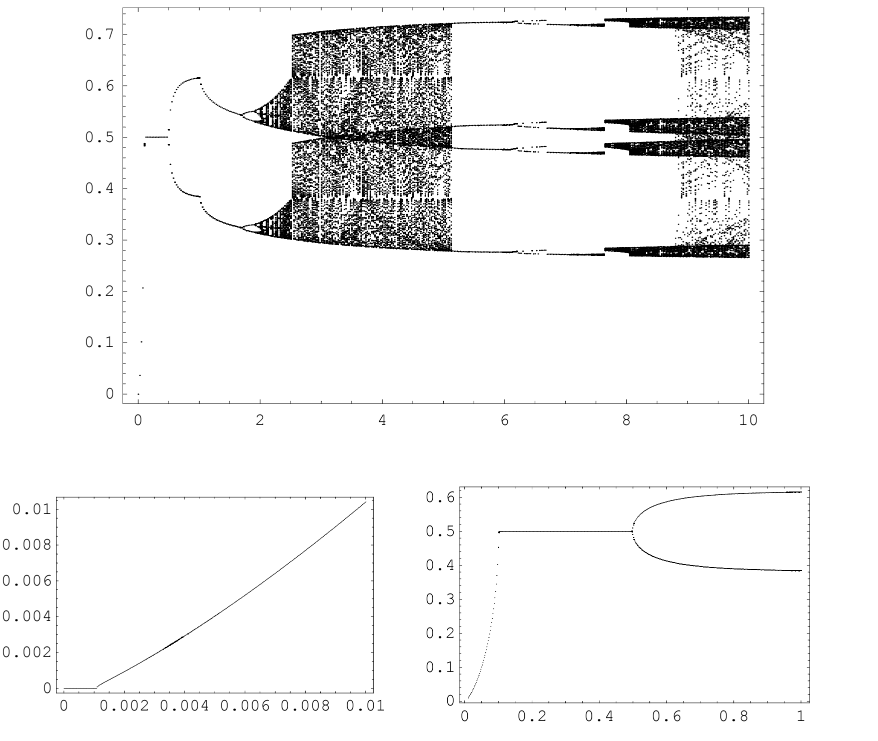


Figure 6: Bifurcation diagram with respect to parameter σ with an initial state $m_{A,0} \approx 0.01$. x axis: σ; y axis: m_A.

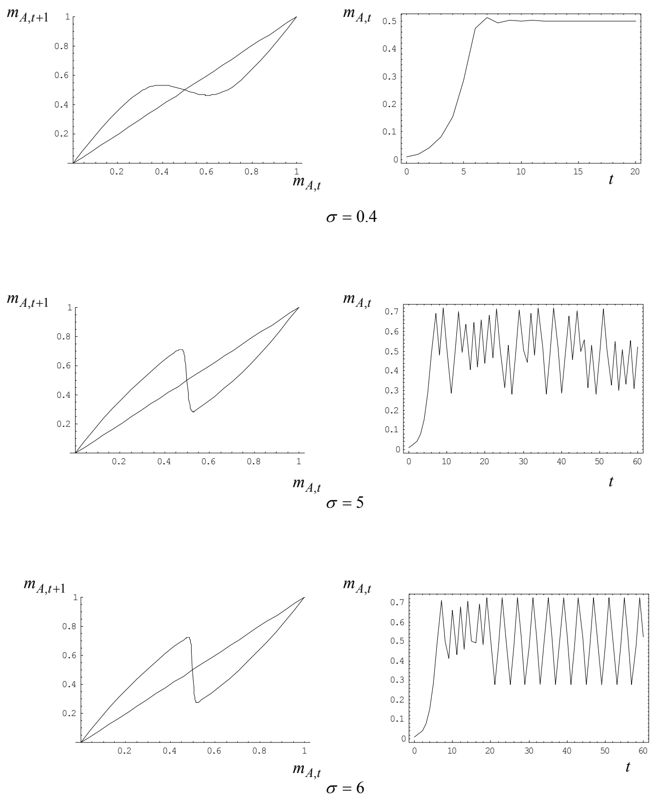


Figure 7: Time series of the tourist flow and the corresponding Staircase Diagram for the different values of σ. The tourist flow converges towards a stationary state in the first case and follows a chaotic and cyclical trend in the second and third cases respectively. x axis: t; y axis: $m_{A,t}$.

the element which is common to each one of the scenarios is the Butler-type initial growth phase. In all three cases, the stagnant, chaotic or cyclical phase emerges only after a first phase of marked development that takes more than 50% of potential tourists to locality A.

With reference to the sustainability of the environment, variations in σ do not change the maximum level of sustainable tourism, which therefore remains constant. As $\hat{m}_A = 0.98$ and observing how the share of tourist population never exceeds 80%, we can conclude that the condition of sustainability is always satisfied with each value of σ in the interval considered.

3.3 Changes in the number of firms (N) and marginal cost (c)

Considering $\Omega_d = \frac{N_d - \epsilon_d}{c_d N_d}$ we can rewrite the expression of surplus as

$$Sc_d = \frac{\hat{E}_d - \alpha M_d + 1}{1 + \beta M_d \Omega_d^{1/\epsilon_d}} \cdot \frac{\epsilon_d}{1 - \epsilon_d} \cdot \Omega_d^{\frac{1-\epsilon_d}{\epsilon_d}} \quad \text{if} \quad E_{d,t} > 0$$

$$\mathrm{Sc}_d = \frac{\epsilon_d}{1-\epsilon_d} \cdot \Omega_d^{\frac{1-\epsilon_d}{\epsilon_d}} \quad \text{if} \quad E_{d,t} \leq 0.$$

Surplus and therefore the dynamic equation of the model depend on the number of firms $N_d \in \aleph$ and production costs $c_d > 0$ by means of the factor Ω_d; we can then study the effect produced by variations in one or both parameters through variations in that factor. If $E_{A,t} > 0$ and for a given value of a tourist

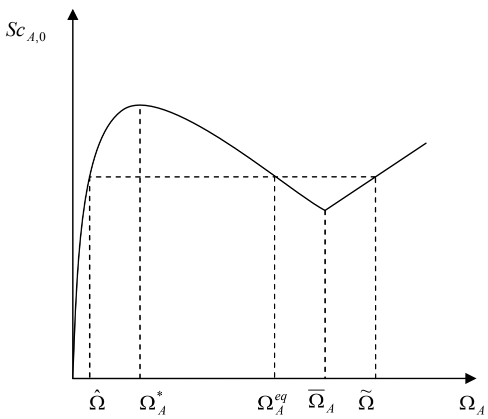


Figure 8: Surplus function of the tourist who visits locality A at time $t = 0$. For $\Omega_A < \bar{\Omega}_A$ the function takes on the form of a parabola with a maximum in Ω_A^*. If $\Omega_A > \bar{\Omega}_A$ the function is linear and growing.

flow $m_{A,t} \in (0, 1)$, surplus is first a growing function of Ω_A , before falling once the maximum point has been reached $\Omega_A^* = \left(\frac{1-\epsilon_A}{\epsilon_A \beta M_A}\right)^{\epsilon_A}$. This means that the attractiveness of a locality is not generally correlated directly with the number of firms or the efficiency of the productive processes. In fact, if these parameters become too high, the locality can undergo a sudden reduction in size because of excessive environmental costs. Nevertheless, there is also a second effect on surplus, this time connected with variations to the maximum limit of sustainable tourism $\hat{m}_A$ caused by Ω_A. This limit tends to decrease as Ω_A grows, thus there will be a $\bar{\Omega}_A$ such that the maximum limit of sustainable tourism is lower than the actual level of tourism, $\hat{m}_A < m_{A,t}$, for all $\Omega_A > \bar{\Omega}_A$ so that the tourist industry is no longer sustainable in terms of the environment ($E_{d,t} = 0$), and surplus becomes a growing monotone function of Ω_A.

Given the initial equilibrium condition ($m_{A,0} \approx 0.01041$) and the values assigned to the parameters, we obtain $\hat{\Omega} \approx 0.0048$, $\Omega_A^* \approx 0.0219$, $\bar{\Omega}_A \approx 6.89$, $\tilde{\Omega} \approx 455.5$ and $\Omega_A^{eq} = 0.0995$ where Ω_A^{eq} indicates the value of the parameter in the initial equilibrium state. The situation before the variation in Ω_A is described in Figure 8 and on that basis we could expect, in relation to the starting point, a fall in the tourist flow for $\Omega_A < \hat{\Omega}$ and $\Omega_A^{eq} < \Omega_A < \tilde{\Omega}$, an increase for $\hat{\Omega} < \Omega_A < \Omega_A^{eq}$ and $\Omega_A > \tilde{\Omega}$, with $Sc_A\left(\hat{\Omega}\right) = Sc_A\left(\Omega_A^{eq}\right) = Sc_A\left(\tilde{\Omega}\right)$, as the bifurcation diagrams confirm in Figure 9.

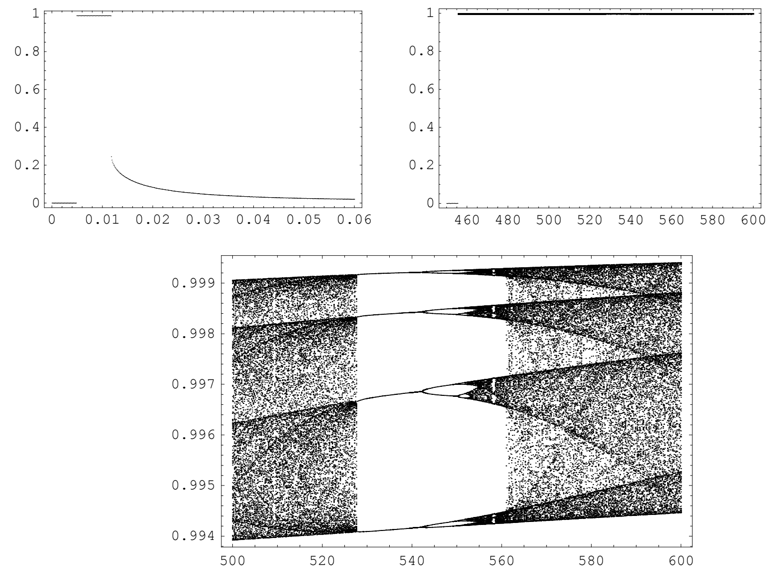


Figure 9: Bifurcation diagram with respect to Ω_A in the initial condition $m_{A,0} \approx 0.01$. x axis: Ω_A; y axis: m_A.

Peaks of tourist flows can be observed for $\Omega_A \in (\Omega_1 \approx 0.005)$, $\Omega_2 \approx 0.01173$ and $\Omega_A > \Omega_3 \approx 455.53$. In this last case, however, the detail in the diagram for interval $500 < \Omega_A < 600$, shows how the final phase of development in the tourist industry presents unstable or cyclical trends, even if they are on the whole insignificant, because they are within a very small interval of values given that the tourist flow is always higher than 99% of the total population of tourists. In Figure 10 three possible paths of development are shown for locality A. In the first case, $\Omega_A = 0.012$, development follows the typical phases of the Butler cycle before stabilizing around 20% of the total tourist population; in the second case, $\Omega_A = 0.01$, growth follows a double Butler-type cycle that remains around 99% of the total tourist population; in the third case, $\Omega_A = 520$, growth consists in a single Butler cycle culminating in an apparently stable phase, but in fact, it is marked by continuous chaotic fluctuations, as shown for the 60 periods that go from $t = 15$ to $t = 75$.

The results of the simulation lead to some important considerations. The fact that development can be made by either increasing or reducing Ω_A leaves industrial policy ample room for manoeuvre. The two types of intervention, however, present substantial differences both in terms of practicability and environmental sustainability. If tourist development is to be started by increasing Ω_A, the measures will have to be such that $\Omega_A > \Omega_3$, which requires a substantial increase in the number of firms together with a significant reduction in the costs of production, but this is not always practicable.

On the other hand, decreases in Ω_A can generate development, but only for values included in a limited interval. If the reduction is excessive, the industry will lose all chance of growth, whereas development will have a limited effect on small shares of the tourist population if the decrease is not sufficiently strong. As a consequence, policy errors can compromise the future development of tourist destinations or at least reduce its impact.

The first type of intervention therefore seems preferable, since it allows both the full development of the locality, even where Ω_A increases excessively, and a reduction in the level of prices, as $p_d^* = 1/\Omega_d$. This reasoning, however, does not give enough attention to the environmental implications of the two different industrial policies. Increases in Ω_A, in fact, reduce the maximum limit of sustainable tourism, thus making this policy unadvisable from an environmental point of view. More specifically, as $\hat{m}_A \approx 0 \ \forall \Omega_A > \Omega_3$, no value of Ω_A higher than Ω_3 is capable of guaranteeing sustainable development, which would be possible if Ω_A were reduced, so that we obtained $\Omega_A \in (\Omega_1, \Omega_2)$ given that in this case we would have $\hat{m}_A \approx 1$.

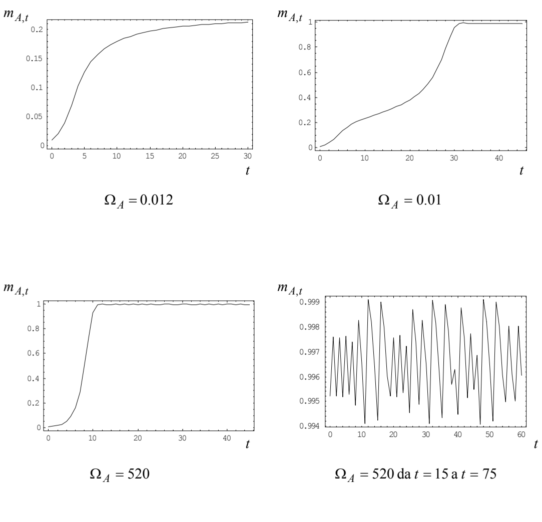


Figure 10: Time series of tourist flows in A for different values of Ω_A.

The fact that the state of chaos (or cyclicity) in the long term occurs with very high values of Ω_A but does not bring about significant fluctuations to the number of visitors depends entirely on the basic values assigned to the parameters. Therefore, a very different picture emerges if, for example, the simulation is repeated hypothesizing that at time zero parameter σ moves from 0.01 to 1.

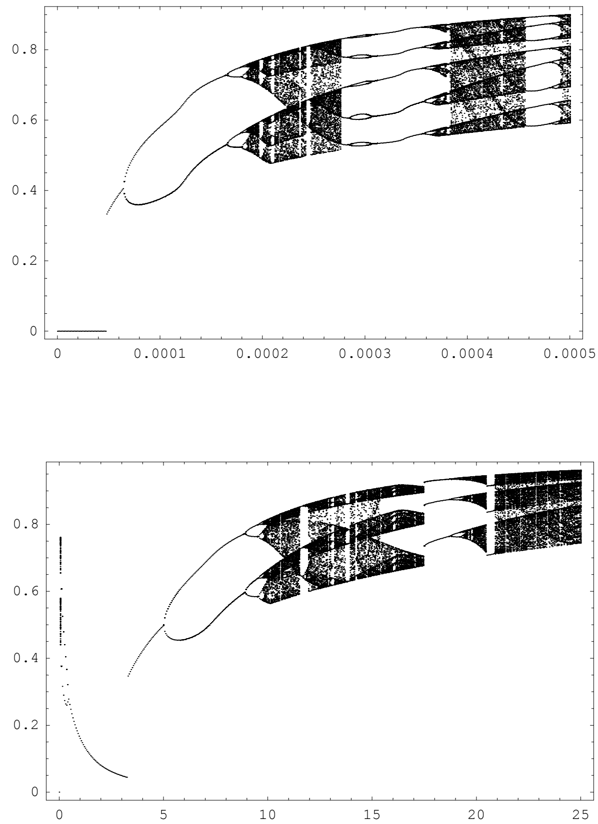


Figure 11: Bifurcation diagrams for Ω_A with $\sigma = 1$ and an initial state $m_{A,0} \approx 0.01$. x axis: Ω_A; y axis: m_A.

As Figure 6 shows, this structural variation would start a flow of tourists away from B to A, until the flow stabilized in a cycle in period 2 [18]. Faced with the prospect of growth, local institutions could decide to adopt expansionary policies to attract a greater number of firms and to reduce the price of tourist services or, at the same time, make the entry of new firms conditional upon specific investments capable of guaranteeing a high standard of services, with a consequent increase in the marginal costs of production and the equilibrium price. Given that $\partial\Omega_A/\partial N_A > 0$ and $\partial\Omega_A/\partial c_A < 0$, the final effect of this manoeuvre cannot be unequivocally determined, as it could lead to either an increase in Ω_A if the positive effect of the proliferation of firms prevailed, or a reduction, if the negative impact of production costs prevailed.

It is evident from the bifurcation diagrams in Figure 11 that the chaotic nature of the paths of development emerges for both low and higher values of Ω_A. Furthermore, unlike the preceding case, the chaotic (or cyclical) fluctuations

[18] The tourist flow would fluctuate between 40% and 60% of the total population of tourists.

are much bigger, which indicates that there can be important variations in the number of visitors from one period to another.

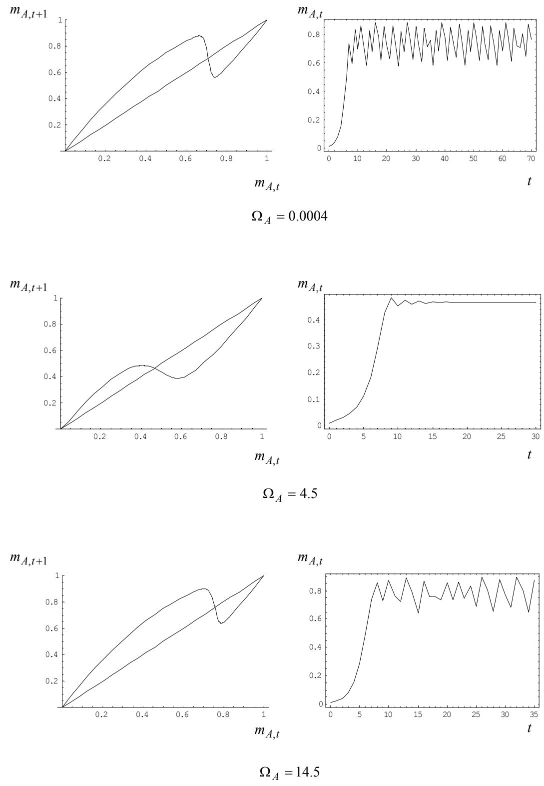


Figure 12: Time series of the tourist flow to A for different values of Ω_A. The Staircase Diagram is linked to each one of the time series that it generated starting from the initial state $m_{A,0}$, with $\sigma = 1$.

Figure 12 shows three possible trajectories for development corresponding to the same number of values of Ω_A. As in the cases analysed above, the first phases of development follow the typical trend of the Butler cycle, whilst the main differences can be seen in the post-consolidation phase; in the first and third case the dynamics become irregular and are probably chaotic, whereas in the second they converge towards a stationary state. As far as environmental sustainability is concerned, what was said above is true, as there are no variations in the limit $\hat{m}_A$ following a change in σ. As $\partial\hat{m}_A/\partial\Omega_A < 0$, development tends to absorb growing quotas of environmental resources the higher the value of Ω_A. To be more specific, environmental sustainability is present only in the first of the three cases analysed in Figure 12 where the trajectories of the tourist flow, though showing obvious fluctuations, remain lower than the maximum sustainable limit that is $\hat{m}_A \approx 1$ whereas in the other two cases the limit is exceeded by the arrival of a larger tourist flow. To be more precise, for $\Omega_A = 4.5$ the locality attracts about 50% of the tourist population when the maximum sustainable limit is $\hat{m}_A \approx 0.024$, whilst for $\Omega_A = 14.5$ the flow fluctuates chaotically between approximately 60% and 80% of the tourist population, well above the limit of environmental sustainability equal to $\hat{m}_A \approx 0.0023$.

3.4 Changes in the environmental impact of tourists (α) and firms (β)

The other parameters sensitive to economic policy measures concern the consequences of economic activities (β) and the tourist flow (α) on the environment. We will limit the analysis to the study of the effects produced by variations in the parameters of environmental impact on locality A, first with two simulations for the variations in α_A and β_A, respectively, then with a third where the two parameters are made to vary together. Certainly these three simulations do not exhaust all the possible cases, but we believe that they are sufficient to test whether, starting from a condition of stability in the long term, the economic policies that change the environmental impact of productive activities and the tourist flow have the capacity to destabilise the system[19].

Changes in α_A: the system is stable for the whole interval of values considered, $\alpha_A \in (0, 10)$. Reductions in the environmental impact of the tourist flow are not sufficient to guarantee the beginning of significant development in the tourist locality, which continues to attract little more than 1% of the potential tourist population for $\alpha < 1$ (figure 13).

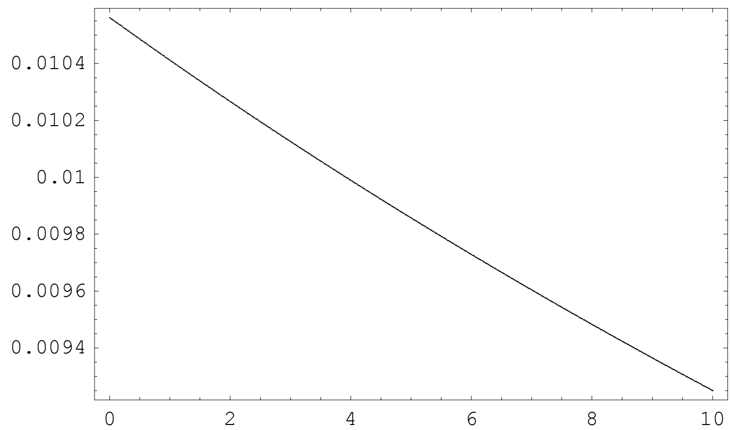


Figure 13: Bifurcation diagram for α_A with initial state $m_{A,0} \approx 0.01$. x axis: α_A; y axis: m_A.

We can define this situation as a *lock-in*, as the system is in fact blocked in its initial position, even after a significant change in the environmental impact of the tourist flow (Faulkner-Russel, 1997).

However, it is sufficient that, as in the previous simulation, an increase from 0.01 to 1 in parameter σ occurs at time zero for the behaviour of the system to change radically.

The diagrams in Figure 14 show a marked complexity in the dynamics of the tourist flow in this case. The popularity of a tourist destination plays a less

[19] Alternative scenarios could emerge if policy measures affecting the parameters of environmental impact involving both tourist destinations were considered; in this case $\alpha_A = \alpha_B$ and $\beta_A = \beta_B$ could continue to hold even after their variation.

important role in the choice of tourists and locality A can therefore hope for a greater flow of visitors.

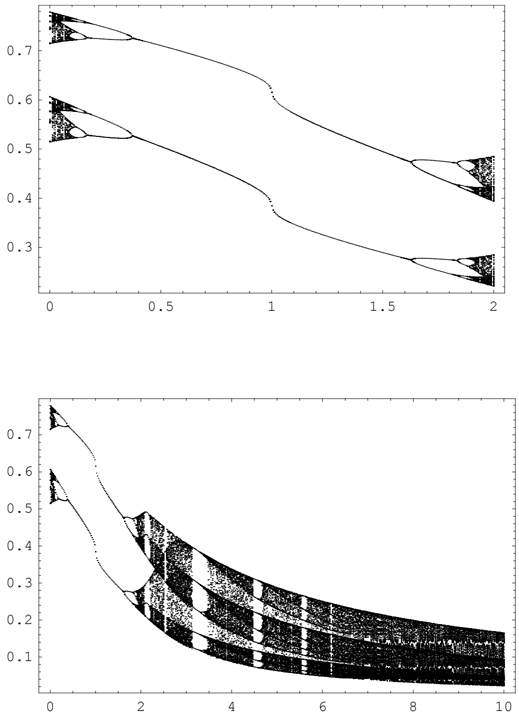


Figure 14: Bifurcation diagram for α_A with initial $m_{A,0} \approx 0.01$ and $\sigma = 1$. x axis: α_A; y axis: m_A.

Development is all the greater, the lower the value of α_A . However, if the interval of values between $\alpha_1 \approx 0.5$ and $\alpha_2 \approx 1.5$, in which the trajectories are periodic in period 2, is omitted, the tourist flow appears generally complex with big cyclical or chaotic fluctuations. This complexity is not present in the previous case with $\sigma = 0.01$. Figure 15 shows two possible developments for $\alpha_A = 0.05$ and $\alpha_A = 4$; once again, what seems to be the first phase of Butler-type development is followed by an indefinite series of apparently chaotic fluctuations[20].

If the environmental impact of tourist flows is reduced, the maximum level of sustainable tourism can be increased. It can be immediately verified that, for $\alpha_A \leq \bar{\alpha} \approx 0.98$, $\hat{m}_d \geq 1$ is obtained, so that, whatever the level of the tourist flow in a certain period, locality A will be able to grow without compromising all its natural resources. On the other hand, growth is also possible for higher values of α_A, which are higher than they were at the beginning, although in this case growth is more contained and there is a risk of using all the available environmental resources. For $\alpha_A = 4$, for example, the maximum level of sus-

[20]Enlargements of the diagram in figure 14 (not shown in the text) reveal the presence of cycles in period 3, thus confirming the hypotheses of Li-Yorke's theorem and therefore the existence of chaotic orbits.

tainable tourism becomes $\hat{m}_d \approx 0.248$ which, as shown in figure 15, is frequently exceeded by much higher levels of tourists arriving.

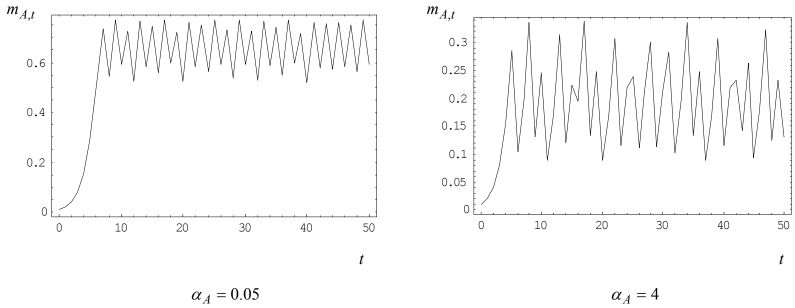


Figure 15: Time series of the tourist flow to A for different values of α_A, $\sigma = 1$.

Changes in β_A: even in the presence of a strong 'popularity effect' generated by a low value of σ, Figure 16 shows how the reductions of β_A are, at least in this case, more effective in favouring development in the tourist industry. It is, in fact, possible to attract a larger share of visitors by reducing the environmental impact of firms, even after starting with a small number of visitors, but in a context in which reputation plays a central role in attracting them.

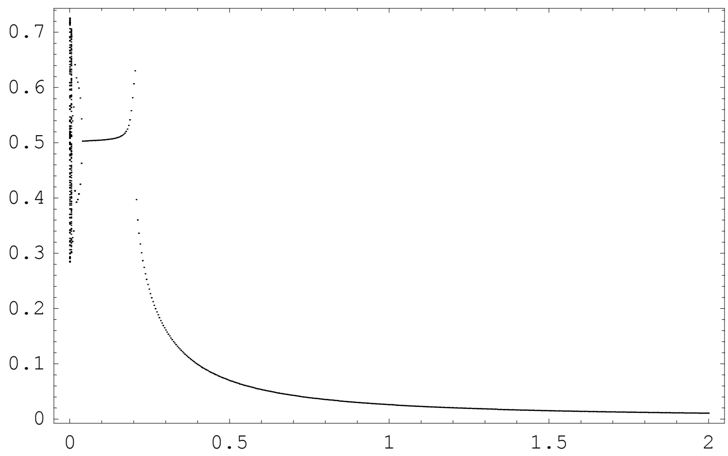


Figure 16: Bifurcation diagram for β_A with initial condition $m_{A,0} \approx 0.01$. x axis: β_A; y axis: m_A.

The details of the diagram shown in Figure 17 reveal that the trajectories of the tourist flow tend towards a stationary state for $\bar{\beta} \approx 0.038 \leq \beta_A \leq 2$ and to follow a cyclical or chaotic trend for $0 \leq \beta_A \leq \bar{\beta}$.

An example is given in Figure 18 for $\beta_A = 0.001$; in this case the first phase, which would seem to be similar to the Butler cycle, is followed by a phase with a noticeably irregular tourist flow, with values that fluctuate from about 30% to more than 70% of the total tourist population. Stable, but nevertheless significant, growth can be realized for smaller reductions in the environmental

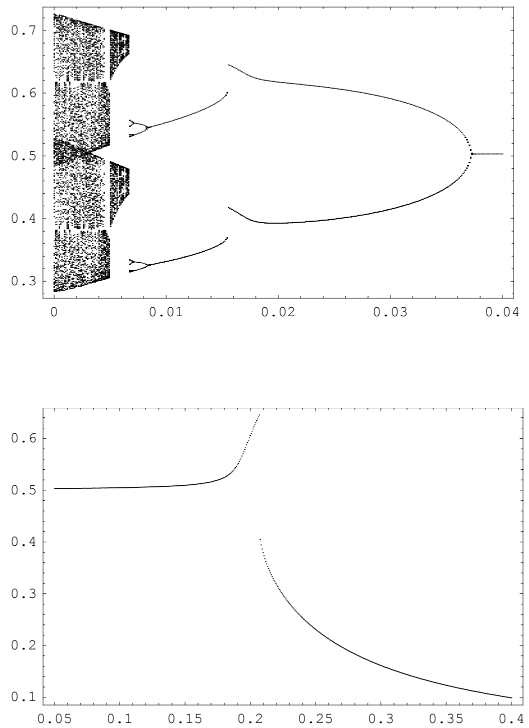


Figure 17: Details of the bifurcation diagram shown in Figure 16. x axis: β_A; y axis: m_A.

impact. Figure 18 also shows the path of development for $\beta_A = 0.16$; the flow can be seen to become stationary around 50% of the total tourist population. The differences between the previous case and this one, however, are not to be found just in the different behaviour of the flow in the post-consolidation phase, but also, and above all, in the time necessary to complete that phase. In the first case, with a very low β_A, 70% of the tourist population can be reached in just 7 periods, after which the flow begins to fluctuate irregularly, whilst in the second case growth is gradual, requiring more than 30 periods to reach half the potential tourism.

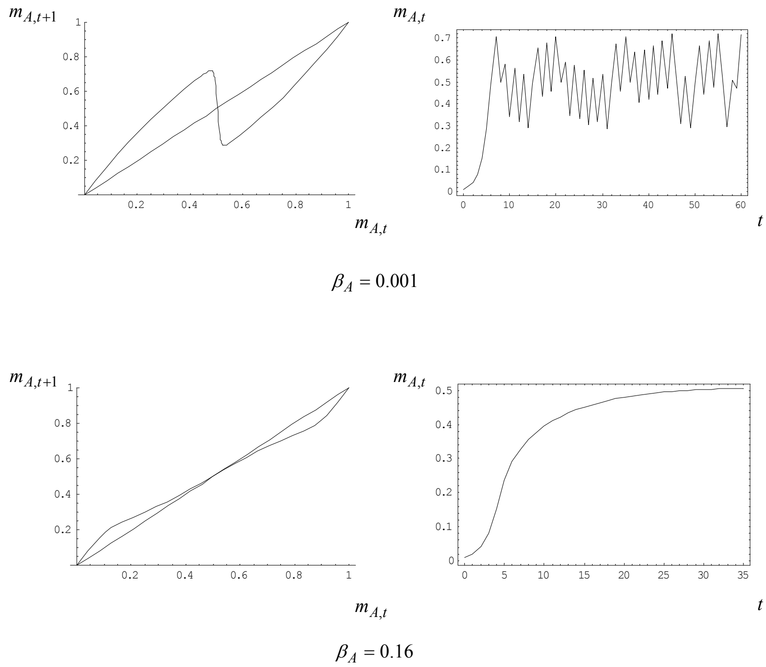


Figure 18: Time series of the tourist flow to A for different values of β_A.

The development is sustainable in environmental terms, as it is for all values of β_A below 2, as $\partial \hat{m}_A / \partial \beta_A < 0$ and $\hat{m}_A \approx 0.98$ for $\beta_A = 2$.

Changes in α_A, β_A: as the analysis allows the study of only one parameter at a time through the bifurcation diagram, we can hypothesise α_A/β_A will always be constant and $\alpha_A = (1+h)$, $\beta_A = 2\,(1+h)$ with $h \in [-1, \infty)$. In this way we can study the effect of variations in the two parameters of environmental impact simply by observing what happens by varying parameter h.

The parameters of environmental impact are reduced with $h < 0$. In this case, although the lower values of pollution substantially increase the number of visitors to locality A (figura 19), the development that will start can follow very different trends, depending on the size of the reduction. Figure 20 shows different sections of the bifurcation diagram for $h < 0$. In the first interval of values, $h \in (h_1 \approx -0.978, 0]$, the system remains stable, with trajectories converging towards a stationary state. Further reductions, with $h \in (h_2 \approx -1, h_1]$, are instead associated with an increase in the complexity of the trajectories that

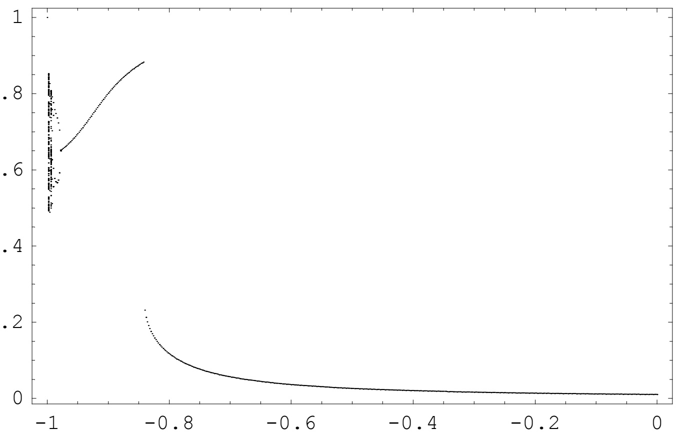


Figure 19: Bifurcation diagram for h, with initial condition $m_{A,0} \approx 0.01$. x axis: h; y axis: m_A.

have a cyclical or chaotic trend. At the limit, for $h \to -1$, the system becomes stable once again in the long term, because the tendency for the parameters of environmental impact to go to zero generates a general move of the tourist population away from B to A.

Two specific trajectories of development are shown in Figure 21. In the first case Butler-type growth culminates in an unstable phase with continuous fluctuations in the tourist flow between approximately 90% and 60% of the tourist population, whilst in the second it follows the typical trend with a final stage of stagnation. In this case locality A absorbs about 70% of the whole tourist population, taking over the leadership from locality B.

As $\partial \hat{m}_A / \partial h < 0$, $\hat{m}_A \approx 0.98$ for $h = 0$ and $\hat{m}_A = 1$ for $h \approx -0.019$, and given what is shown in the bifurcation diagram, the development of locality A can be said to be always sustainable for $h \in [-1, 0]$.

It is easy to imagine that reductions in pollution will be followed by an increase in the tourist flow, but not that these reductions can actually give rise to irregular trends in the flow, leading to marked fluctuations without any obvious explanation. The cause of these fluctuations can be found in the continuous changes in the levels of surplus resulting, firstly, from the action taken on the parameters for environmental impact and, later, from the continuous variations in the number of visitors. In fact, the reduction in one or both the parameters at time zero provokes an initial growth in the surplus, thus breaking the equilibrium and generating a flow of tourists towards locality A[21]. At that point the increase in the share of visitors once again reduces surplus because of the growing congestion, so much so that the dynamics of the flow are inverted and tourists begin to move back to locality B. The continuous fluctuations in surplus caused by the mechanism described above could either diminish, thus allowing

[21]Remember that at time zero the system is in equilibrium and that, as $m_{A,0} < 0.5$ in a situation of perfect structural homogeneity, there is also $Sc_{A,0} > Sc_{B,0}$. Therefore, if there is an increase in Sc_A, for example if the tourist industry has less impact on the environment, there will be an increase, at least at first, in the tourist flow towards locality A.

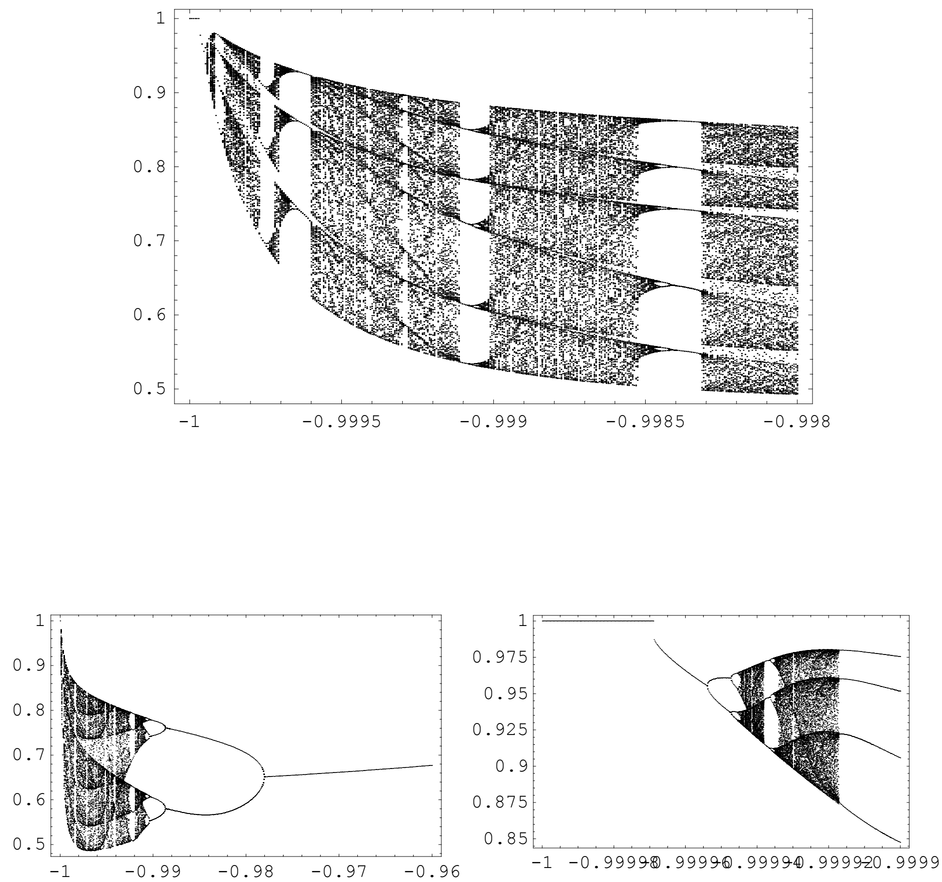


Figure 20: Details of the bifurcation diagram shown in figure 19. x axis: h; y axis: m_A.

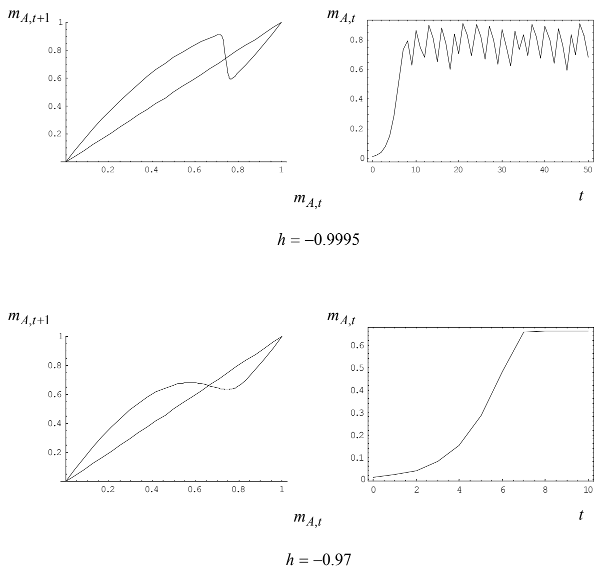


Figure 21: Time series and Staircase Diagram for different values of h.

the system to come to a stationary state, or continue indefinitely, in which case the cyclical or chaotic trends would persist.

4 Conclusions

Our model produces some important theoretical results which can be summarized as follows:

- As many scholars have pointed out (see, for example, Hovinen 2002), the chaos theory provides a useful complement to Butler's model and allows a deeper understanding of the development process in the highly complex context which results from the numerous interactions of the different agents. In literature the traditional approach is based mainly on the formalization of particular areas in the system of the tourist industry: in some cases the evolution of the tourist system is the result of intertemporal choices in social planning (Kort et.al., 2002; Giannoni-Cellini, 2004), in other cases it arises from the strategic interaction of oligopolistic firms (Candela-Cellini, 2004). However, these models lack a rigorous formalization of the dynamic mechanism that determines the choice of tourists. It is, in fact, this very mechanism, once inserted in a standard context of oligopoly, which allows our model not only to reproduce the dynamics of the Butler cycle, which must be considered as one of the possible manifestations of the growth process, but also to account for the irregularity of the tourist flow that is often observed in the post-consolidation phases of the cycle itself.

- Usually the sudden and unpredictable changes in the dynamics of a system, such as a tourist system, are explained by external factors or exogenous shocks. The application of the chaos theory means that these changes need an endogenous explanation and it provides an appropriate method to identify the factors that generate these changes. An example could be our simulations based on the variations in the elasticity parameter ϵ_A. It has been seen how, the tourist flow being constant, the tightening of demand generally generates an increase in prices and an improvement in the quality of the environment. In the case under consideration in the simulation it is shown that $\partial Sc_A/\partial\epsilon_A > 0$ for $\epsilon_A \in (0, 1)$, thus there is an increase in the surplus of the tourist as the elasticity of demand falls. Starting from a situation of perfect structural homogeneity, as hypothesized at the beginning, locality A experiences a gradual increase in the tourist flow as ϵ_A increases. The development cycle follows the path theorized by Butler up to the stage of consolidation. The nature of the dynamics of the tourist flow then takes on characteristics that vary according to the value of elasticity, as it can converge to a stationary state (stagnation) or a final stage that can be defined as "chaotic stability", in which the tourist flow fluctuates cyclically or chaotically in a limited interval of values.

- Tourists are changeable and inconstant, making the destination vulnerable to changes in their tastes and preferences. One locality may experience rapid growth and remain among the favourite destinations of tourists for a long time thanks to its popularity, but then fall into an inexorable decline if tourists change their selection criteria, preferring, for example, new localities to the more popular ones. These changes may have little to do with the effective quality and quantity of services provided. We analysed this aspect by simulating the effects produced by variations in parameter σ. As hypothesized in the model, a tourist could prefer one locality to another purely for economic reasons ($\sigma \to \infty$) or, more simply, following a fashion, almost or totally independently of the real utility that he obtains by visiting that place ($\sigma \to 0$). In this case, it can become very difficult to start the development of a tourist locality that does not have many visitors, even if it succeeds in guaranteeing high levels of services and quality of environment. On the other hand, greater attention paid by tourists to the real benefits rather than fashion might favour an increase in the tourist flow, but it can also increase the instability of the system. The mobility of tourists is greater in those cases where they are attracted by higher levels of surplus rather than the popularity of the destination. After an initial period of Butler-type development a phase characterised by marked cyclical or chaotic fluctuations can be expected, with the tourist population moving from one locality to another in pursuit of higher levels of surplus.

Our model seems to confirm some important empirical evidence. Of particular relevance in this field is Lundtorp-Wanhill's article (2001), in which the authors examine the development trajectories of two tourist localities, the Isle of Man and Bornholm, using the data of tourist flows from 1884 in the first case and 1910 in the second[22]. In both cases the first phases of growth seem to follow

[22] The scarcity of available data over a long period of time, as in the case of the growth of a tourist locality, makes it difficult to estimate the empirical basis of a theory of tourist development.

what was proposed by Butler's cycle, but the picture changes radically once the consolidation phase has been reached. In the case of the Isle of Man the tourist flow begins to fluctuate noticeably, taking on dynamics reminiscent of the chaotic fluctuations found in the simulations. In the second case, however, the cycle culminates in a stagnant phase lasting ten years, after which there is a recovery in a kind of double Butler cycle. In the absence of more recent data this case would also appear to be characterized by irregular fluctuations in the final phase of the cycle that can be traced back to chaotic dynamics. The authors explain these erratic trends by the increase in competition from rival destinations.

As far as policy is concerned, the unpredictability of the tourist flow and its innate tendency to follow irregular trends implies the need for a significant change in the role of intervention. The simulations of the model confirm the hypotheses already proposed by some scholars of the impossibility of directly controlling the size of a tourist flow. Intervention on the structure of costs, the size of the industry and environmental impact, even if it can guide the growth of the industry, is not capable of guaranteeing either the desired levels or the stability of the tourist flow, which instead proves to be very sensitive to factors that cannot be directly influenced. The simulations of the model based on variations in the parameters that can be controlled by authorities $(\Omega_A, \alpha, \beta)$ have produced the following results:

- *Parameter* Ω_A): One type of intervention widely adopted by local institutions which aim to start the development of a tourist destination is to guarantee firms free entry or to allow the lowering of production costs in order to favour competition. Very often, however, it is not realized that the same levels of development can be reached with diametrically opposed policies, as, for example, those that limit the proliferation of firms and aim at the protection of the environment. By simulating the effect produced by variations in Ω_A it can be observed how a significant increase in the tourist flow can be obtained even by reducing the number of firms (as $\partial\Omega_A/\partial N_A > 0$) or by increasing production costs (as $\partial\Omega_A/\partial c_A < 0$). The preference often shown for measures that can compromise the quality of the environment can be explained by the fact that they are easier to realize. In the model there is a very high probability that mistakes will be made whilst trying to encourage development by reducing Ω_A as it is possible to reach this result only in a limited interval of values. An extreme reduction, as seen above, would lead to the definitive abandonment of the locality by tourists or growth below forecasts. There will also be an obvious and significant destabilization in the system in the presence of a higher value of parameter σ; the nature of the preferences made by a tourist thus plays a central role in conditioning the final effects of the economic policies.

- *Parameters* α, β): the general tendency is to increase the tourist flow as the impact of the industry on the environment decreases. The nature of the dynamics caused by these variations is, however, anything but regular. These manoeuvres can in fact be ineffective if, for example, popularity carries weight in attracting tourists, or they can destabilize the system, thus giving rise to chaotic fluctuations.

A tourist industry is capable of developing even in the presence of a spoilt natural environment. Once the environment has been destroyed, the supply of services can continue to attract tourism. This result matches perfectly with what has been observed in the development of some important tourist destinations. The Balearic islands are an example of how a tourist industry with high environmental costs can continue to prosper, contrary to all the forecasts of scholars that foresaw a gradual decline (Aguilò-Alegre-Sard, 2005).

As mentioned above, the model lends itself well to further adaptations and changes in order to study aspects that have not been dealt with in depth in this article. On the supply side, for example, the assumption about the homogeneity of the product can be removed and a certain level of differentiation can be introduced, with an element of heterogeneity both amongst the firms in one locality and amongst different localities. A possible formulation of the model in this sense can include transport costs and the hypothesis of spatially positioning the tourist population in a kind of Hotelling (1929) logic, in which the tourist decides on the basis of the distance that separates him from the tourist localities. This formulation, in our opinion, would add a further element of instability, because the changes that can affect transport costs are dependent on factors beyond the control of local authorities, especially the cost of energy resources.

The reformulations of the model that can be made on the demand side are much richer and more articulated. One possible hypothesis could be to provide for different types of tourists (for example ecotourism v mass tourism) and therefore add the dynamic aspect of forming individual preferences for a particular type of tourism to the evolutionary mechanism of information transmission already considered, thus including aspects that go from environmental ethics to education[23].

A second hypothesis could be to remove the assumption of a constant tourist population, thus allowing it to grow or decrease as the specific parameters vary, as for example, available income. This hypothesis can also be applied to the population of firms, which was considered to be constant in the model: an evolutionary mechanism could be assumed that favours the proliferation of firms in those localities where profits are highest.

The increased complexity of analysis, together with the size of this paper, have advised us against including these effects here.

[23]Interesting ideas in this sense can be found in the research of Bisin-Verdier (2001) and Young (1998) on the mechanisms of the intergenerational transmission of culture and behaviour.

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