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Abstract

This paper proposes a residual-based unit root test in the presence of smooth structural changes approximated by a Fourier function. While Fourier Augmented Dickey Fuller test that introduced by Enders and Lee (2012a) allows smooth changes of the unknown form, the Residual Augmented Least Squares procedure use additional higher moment information found in non-normal errors. The test offers a simple way to accommodate an unknown number and form of structural breaks and have good size and power properties in the case of non-normal errors.

Keywords: Non-normal errors, Fourier Function, Unit root.

[Declarations of interest: none]

1. Introduction

There may be periods of sharp fluctuations in time series over time for various reasons such as natural disasters, wars over time, economic crises, and policy changes. This kind of fluctuations are called as *structural breaks* in time series literature. These structural breaks can be either in the mean of the series or in the trend or both in mean and trend. As with most econometric methods, ignoring these structural breaks in unit root tests can also lead to biased results.

Since the results of conventional unit root tests are biased in the case of structural breaks, several unit root tests have been introduced to the literature that allow structural breaks. While Perron (1989)'s study is the first to consider structural breaks in unit root testing, it was criticized for the assumption of known breakpoint. The underlying reason of these criticism is for the test strategy, which is assumed to be independent of the data, is inconsistent because the date of break is determined exogenously (Libanio, 2005). After Perron's (1989) study, several unit root tests have been introduced to the literature that determine structural breaks endogenously (Banerjee et al. (1992), Zivot and Andrews (1992), Perron (1994), Lumsdaine and Papell (1997), Clemente et al. (1998), Ohara (1999), Lanne et al. (2002), Saikkonen and Lütkepohl, (2002), and Lee and Strazicich (2003, 2013)). On the other hand, these tests have been also criticized since the predetermination of the number of structural breaks (Yilanci, 2017).

Recently, unit root tests have been introduced into the literature based on nonlinear models (see Leybourne et al. 1998 and Kapetanios et al. 2003, among others). These tests focused on the view that structural breaks in time series do not occur suddenly. The weakness of these tests is that the form of the structural breaks should be known as a priori. Therefore, in cases where the structure, number, and location of structural breaks are not known, the application of the mentioned methods could produce incorrect results. Becker et al. (2004, 2006) suggested a Fourier approach to overcome these problems. The main reason for using the Fourier function is that it can capture the behavior of unknown functions (Gallant, 1981). With the Fourier approach, the need for prior knowledge of the form, date, and number of structural breaks has been eliminated (Enders and Lee, 2012a). Fourier KPSS (Becker et al., 2006), Fourier LM (Enders and Lee, 2012b), Fourier DF (Enders and Lee, 2012a), and Fourier GLS (Rodrigues and Taylor, 2012) unit root tests were introduced to the literature with the Fourier approach.

The usage of the information about non-normal errors for unit root tests is one of the topics of interest in the literature recently. Im et al. (2014) proposed a new unit root test that uses information from non-normal errors. This test uses a simple procedure based on the method of "Residual Augmented Least Squares" (RALS) methodology proposed by Im and Schmidt (2008). This test shows significantly better power characteristics than conventional tests that dp not use the information on non-normal errors.

In the unit root test literature, information about non-normal errors is generally ignored, since limit distributions of these tests are not affected by ignoring non-normal errors. However, the notion that information represented by non-normal errors is useless or should be neglected is incorrect. On the contrary, using information about non-normal errors in unit root tests can increase the power of unit root tests. In this study, a new Fourier unit root test will be introduced to the literature that is more powerful FADF unit root test with non-normal errors.

2. Residual Augmented Least Square-Fourier Augmented Unit Root Test

The Dickey-Fuller unit root test equation can be described as follows.

$$y_t = \alpha_t + \rho y_{t-1} + \gamma_t + \varepsilon_t \tag{1}$$

Where α_t refers to a time-dependent deterministic term function. The null hypothesis of unit root is tested by examining $\rho = 1$. In cases where the form of the deterministic term is not known, an incorrectly defined deterministic term can lead to biased test results. Enders and Lee (2012a) proposed the Fourier approach for unknown deterministic term functions as

$$\alpha_{(t)} = \alpha_0 + \sum_{k=1}^{n} \alpha_k \sin\left(2\pi kt / T\right) + \sum_{k=1}^{n} \beta_k \cos\left(2\pi kt / T\right); \qquad n \le T/2 \tag{2}$$

Where n represents the approximate number of frequencies, k represents a given frequency, and T represents the number of observations. It should be remembered that if all coefficients of trigonometric terms in Equation 2 are not statistically significant, a linear process will occur and the Dickey-Fuller unit root test **arises**.

By replacing the Equation 1 into the Model 1, we obtain the FADF unit root test equation as follows:

$$\Delta y_t = \rho y_{t-1} + c_1 + c_2 t + c_3 \sin(2\pi kt / T) + c_4 \cos(2\pi kt / T); \qquad n \le T/2 \tag{3}$$

It should be noted that the critical values for the null of unit root are not dependent on the coefficients of Fourier terms or other deterministic terms. In this approach, the critical values, as in other similar tests, depend only on frequency *k* and sample size (*T*). Enders and Lee (2012a) proposed a two-step procedure in the estimation of the extended DF regression model with Fourier functions. In the first step, all models for the $1 \le k \le 5$ are estimated, and the model with the smallest residual squares is selected as the appropriate model, and in the second step, the FADF test statistics are calculated with the help of the appropriate model, and the unit root hypothesis is tested by comparing with critical values.

In economic or financial time series, the existence of non-normally distributed series is considerable. These distributions may occur for various reasons and may not be easy to distinguish from some nonlinear forms. For example, some financial variables are characterized by asymmetric distributions that may occur when there is an asymmetric relationship in the data. Moreover, some economic time series variables have a mixture of different distributions that will typically be modeled as regime transition models. If a particular nonlinear form is known, it is possible to take advantage of nonlinear tests using specific information. However, superiority of the RALS method that suggested by Im et al. (2014) is that it is not necessary to know the functional form. Instead, RALS method employs high moments of regression residuals that distributed as non-normal, and does not require nonlinear estimation techniques as it is performed in a linear frame based on the ordinary least-squares (OLS) estimation.

Im and Schmidt (2008) consider the following two-moment conditions in the RALS procedure:

$$E\left(e_t \otimes X_t\right) = 0$$

$$E\left(\left(h\left(e_t\right) - K\right) \otimes X_t\right) = 0$$

The first of these conditions specify the standard moment condition of the OLS method, while the second condition refers to the additional moment condition based on the nonlinear functions of e_t. These two conditions can be shown as.

$$\hat{w}_t = h\left(\hat{e}_t\right) - \hat{K} - \hat{e}_t \hat{D}_2 \quad t = 1,2,...T \tag{4}$$

Where $\hat{e}_t$ denotes residuals obtained from the main regression and calculated as follows.

$$\hat{K} = \frac{1}{T}\sum_{t=1}^{T} h(\hat{e}_t) \quad , \hat{D}_2 = \frac{1}{T}\sum_{t=1}^{T} h'(\hat{e}_t) \quad \text{and} \quad h(\hat{e}_t) = \left[\hat{e}_t^2, \hat{e}_t^3\right]' .$$

The equation can be shown in open form as follows.

$$\hat{w}_t = \left[e_t^2 - m_2,\ \hat{e}_t^3 - m_3 - 3m_{2t}\hat{e}_t\right]', t = 1,2,...,T \tag{5}$$

Two new series are obtained using the high moments of the errors. m_2 is the mean of the square of the residuals, and m_3 is the mean of the cube of the residuals. When these two new series are added to the main model, the non-normal information of the errors is reflected in the model.

In Equation (5), the first term is related to the constant variance condition ($E[(e_t^2 - \sigma_e^2)y_{t-1}] = 0$). This condition also increases the effectiveness of the estimators when errors are not symmetrical. On the other hand, it increases efficiency in the absence of the second term $m_4 = 3\sigma^4$ in equality. Higher moments (provided that k>3, $h(\hat{e}_t) = [\hat{e}_t^2, \hat{e}_t^3, \hat{e}_t^4, ..., \hat{e}_t^k])$) can also be used at this stage. In addition, more efficiency can be achieved when $m_{j+1} = j\sigma^2 m_{j-1}$ is not present (this equation applies only if there is a normal distribution). However, this can only be the case if higher moments are present.

Meng (2013) and Lee et al. (2015), suggested that the second and third moments were used to increase the power of the test.

Using the RALS method, we can extend the FADF test equation as follows.

$$\Delta y_t = \rho y_{t-1} + c_1 + c_2 t + c_3 \sin\left(2\pi kt / T\right) + c_4 \cos\left(2\pi kt / T\right) + c_5 \hat{w}_t + v_t \qquad n \le T/2 \tag{6}$$

The RALS-FADF test statistic (τ_{RFADF}) is obtained by estimating the model obtained as the appropriate model in the second stage by OLS and testing the null hypothesis $\rho = 0$.

THEOREM

Under the null hypothesis (τ_{RFADF}) the asymptotic distribution of the test statistic is as follows;

$$\tau_{RFADF} \rightarrow \rho.\tau_{FADF} + \sqrt{1-\rho^2.Z}$$

τ_{FADF} is the limit distribution of the t statistic of the FADF test and ρ is the long-term correlation between the FADF and the residuals of the RALS-FADF defined as follows.

$$\hat{\rho}^2 = \frac{\hat{\sigma}_{eu}^2}{\hat{\sigma}_u^2 \hat{\sigma}_e^2}$$

$\rho^2 = 1$ is valid for $FADF = RALS - FADF$. In this case, the critical value of FADF can be used instead of the critical value of RALS-FADF.

3. Monte Carlo Experiments

In this section, we examine the empirical size, and power comparison of the critical values for the proposed RALS-FADF unit root test.

3.1. Critical Values

Asymptotic critical values of RALS-FADF are reported in Table 1a and Table 1b (in Appendix) at significance levels of 1, 5 and 10%, respectively. The asymptotic critical values are based on 100000 replications for sample sizes (t= 50, 100, 250, 500, and 1000), frequency values (k=1, 2, 3, 4, and 5) and long-run correlation values between FADF and RALS-FADF residuals (ρ^2=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1).

In Table 1, it can be seen that the asymptotic distribution of test statistics depends on the frequency number (k), the number of observations (n), and the long-run correlation values. While ceteris paribus, it can be seen that increases in k and n tend to decrease critical values, while increases in ρ^2 tend to increase critical values.

3.2. Size and Power Property

In this section, we investigate the performance of the suggested RALS-FADF test using Monte Carlo simulations. All simulations are performed using 10.000 replications. We allow the error term to follow five types of non-normal distribution (χ_1^2, χ_2^2, χ_3^2 and t_2, t_3). We also allow the error term that follows the standard normal distribution for comparison purposes.

To evaluate the size and power of the test statistics, we consider the following data generating process (DGP)

$$y_t = \rho y_{t-1} + c_2 \sin(2\pi kt/T) + c_3 \cos(2\pi kt/T) + c_4 \hat{w}_t + v_t \tag{7}$$

$$v_t = v_{t-1} + \varepsilon_t \tag{8}$$

The size and power properties of the FADF and RALS-FADF are compared in Table 2 and Table 3.

Table 2. Size Property of FADF and RALS-FADF

τ	χ_1^2	χ_2^2	χ_3^2	t_2	t_3	$N(0,1)$
τ_{FADF}	0.0474	0.0476	0.0454	0.0525	0.0495	0.051
$\tau_{\text{RALS-FADF}}$	0.0527	0.0635	0.0656	0.041	0.0472	0.0922

Note: τ denotes the test statistics. τ_{FADF}, $\tau_{\text{RALS-FADF}}$ denote the test statistic for FADF test and RALS-FADF test respectively.

The results in Table 2 show that the size of the proposed test is close to 5% in non-normal distributions. When the normal distribution is used, it is seen that the size becomes distorted.

Table 3. Power Property of FADF and RALS-FADF

c_2	c_3	τ	χ_1^2	χ_2^2	χ_3^2	t_2	t_3	$N(0,1)$
0	0	τ_{FADF}	0.1611	0.167	0.1771	0.1513	0.1659	0.1786
		$\tau_{\text{RALS-FADF}}$	0.9432	0.8368	0.7307	0.5132	0.338	0.2694
0.3	0	τ_{FADF}	0.1627	0.1705	0.1802	0.1513	0.1668	0.1782
		$\tau_{\text{RALS-FADF}}$	0.9473	0.8432	0.7389	0.5131	0.3396	0.2692
0.3	0.5	τ_{FADF}	0.1748	0.178	0.184	0.1514	0.1785	0.1888
		$\tau_{\text{RALS-FADF}}$	0.945	0.8511	0.7405	0.5167	0.3559	0.2766
0	0.5	τ_{FADF}	0.185	0.1874	0.1892	0.1552	0.1886	0.2004
		$\tau_{\text{RALS-FADF}}$	0.9487	0.8632	0.7554	0.5238	0.3717	0.2886
3	5	τ_{FADF}	0.9738	0.9762	0.9798	0.3781	0.9651	0.9792
		$\tau_{\text{RALS-FADF}}$	0.9971	0.9981	0.9994	0.8527	0.9985	0.9807
0	5	τ_{FADF}	0.9992	0.9998	1	0.6022	0.9942	1
		$\tau_{\text{RALS-FADF}}$	0.9995	0.9999	1	0.9609	0.9999	1
3	0	τ_{FADF}	0.3898	0.3761	0.374	0.1789	0.377	0.3377
		$\tau_{\text{RALS-FADF}}$	0.9966	0.9824	0.9551	0.5772	0.6539	0.4281

Note: τ denotes the test statistics. τ_{FADF}, $\tau_{\text{RALS-FADF}}$ denote the test statistic for FADF test and RALS-FADF test respectively.

The overall result from Table 3 is that the RALS-FADF test is more powerful than the FADF test for all combinations of distributions. When the power properties of the RALS-FADF test are examined, it is seen that when the normal distribution of the error term is allowed, the power tends to decrease and the size skews compared to other distributions. This is expected because the RALS procedure does not provide additional information when errors are normally distributed. The RALS-FADF test shows significantly improved power compared to the FADF test when errors follow non-normal distributions with chi-square or t distribution. Table 3 also shows a decrease in power properties when the degree of freedom of non-normal distributions increases for RALS-FADF. This is also expected because when the degree of freedom of non-normal distributions increases, they approach the normal distribution with the central limit theorem.

4. Empirical Application

The validity of purchasing power parity (PPP) in BRICS (Brazil, Russia, India, China, and South Africa) countries is examined for the empirical application of the proposed test. The validity of the purchase rate parity is tested by determining whether the shocks on the real exchange rates are temporary or permanent. If the shocks on the real exchange rate are temporary, in other words, if the real exchange rate is stationary, the PPP hypothesis is valid.

Real exchange rate data from January 1995 to July 2019 were used in this study. All data used are obtained from International Monetary Fund data service. The real exchange rate series is calculated with the following formula.

$$y_{i,t} = s_{i,t} + p_{USA,t} - p_{i,t} \quad (9)$$

where $s_{i,t}$ indicates the logarithmic nominal exchange rate of *i* country. $p_{USA,t}$ and $p_{i,t}$ indicate the logarithmic price index of the USA and *i* country, respectively. Table 4 shows the unit root test results.

Table 4. RALS-FADF Unit Root Test Results

Countries	k	p	ρ^2	FADF	RALS-FADF
Brazil	2	7	0.6588	-2.8283	-3.6325**
China	1	14	0.9430	-3.7113	-3.7998**
India	1	14	0.8663	-5.7652*	-6.0469*
Russia	3	11	0.7492	-1.2439	-1.8103
South Africa	4	13	0.8503	-3.1669**	-3.1524**

Note: * and ** indicate the 1% and 5% significant values, respectively.

FADF unit root test results show that the real exchange rate series are stationary in India and South Africa. On the other hand, results of the RALS-FADF unit root show that the real exchange rate series has a unit root only for Russia. So, we can conclude that the PPP hypothesis valid in all countries except Russia.

5. Conclusion

In this study, the unit root test of Enders and Lee (2012a) is extended using the RALS estimation procedure of Im and Schmidt (2008). While unit root tests of the FADF type allow smooth transitions of the unknown form, the RALS procedure may use additional information found in non-normal errors. The simulation exercises show that while the error term follows a non-normal distribution, the power of the RALS-FADF test appears to increase dramatically. We test the validity of the purchasing power parity hypothesis for BRICS countries using the newly proposed RALS-FADF unit root test. The results show that PPP hypothesis is valid in all countries except Russia.

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Appendix

Table 1a. Critical values for Residual Augmented Fourier ADF Unit Root Test

			ρ^2									
n	k	%	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
50	1	1	-3,04406	-3,35606	-3,56832	-3,7633	-3,93475	-4,07469	-4,19873	-4,33322	-4,44891	-4,56445
		5	-2,38286	-2,66892	-2,89213	-3,09117	-3,25484	-3,40628	-3,52457	-3,65443	-3,77077	-3,87788
		10	-2,0155	-2,31156	-2,54182	-2,7408	-2,90543	-3,05334	-3,17449	-3,31444	-3,43013	-3,53788
	2	1	-2,82278	-3,06395	-3,23424	-3,37069	-3,50402	-3,62363	-3,74829	-3,83605	-3,94493	-4,04817
		5	-2,15056	-2,38258	-2,54338	-2,6832	-2,80496	-2,90897	-3,01899	-3,10594	-3,1931	-3,30247
		10	-1,802	-2,0168	-2,18016	-2,32126	-2,43495	-2,53333	-2,65012	-2,73021	-2,821	-2,91098
	3	1	-2,78799	-3,01062	-3,15164	-3,27595	-3,36709	-3,45108	-3,52213	-3,62876	-3,72343	-3,76726
		5	-2,12467	-2,31527	-2,4616	-2,57479	-2,66336	-2,76533	-2,83848	-2,91601	-2,98067	-3,05006
		10	-1,75927	-1,95579	-2,09324	-2,21367	-2,31096	-2,40851	-2,48838	-2,55193	-2,62171	-2,68219
	4	1	-2,75753	-2,95551	-3,0892	-3,19749	-3,27311	-3,3702	-3,44574	-3,52298	-3,58149	-3,64543
		5	-2,10621	-2,2809	-2,42909	-2,5176	-2,61383	-2,69176	-2,76539	-2,85087	-2,89211	-2,94951
		10	-1,74851	-1,92374	-2,07106	-2,18045	-2,27316	-2,3502	-2,41671	-2,50253	-2,55208	-2,60639
	5	1	-2,77155	-2,94958	-3,0495	-3,16102	-3,24633	-3,3314	-3,38071	-3,48175	-3,537	-3,60569
		5	-2,09586	-2,27881	-2,39115	-2,4993	-2,58998	-2,66971	-2,74156	-2,80084	-2,84857	-2,90239
		10	-1,73949	-1,91894	-2,04647	-2,15773	-2,24419	-2,32535	-2,39982	-2,46222	-2,51591	-2,57174
100	1	1	-3,05313	-3,31721	-3,5512	-3,72109	-3,85611	-4,01761	-4,12132	-4,24433	-4,33746	-4,43141
		5	-2,36457	-2,67144	-2,89821	-3,0586	-3,22237	-3,37074	-3,48206	-3,60601	-3,69762	-3,80899
		10	-2,00293	-2,30667	-2,53949	-2,71235	-2,87971	-3,03132	-3,15412	-3,27795	-3,38427	-3,49117
	2	1	-2,84523	-3,06217	-3,23348	-3,33493	-3,49413	-3,58715	-3,68756	-3,79081	-3,86351	-3,98298
		5	-2,17056	-2,38862	-2,55589	-2,68655	-2,79999	-2,91428	-3,00087	-3,10855	-3,18087	-3,27402
		10	-1,81256	-2,02611	-2,19487	-2,33211	-2,44499	-2,55425	-2,64221	-2,74734	-2,81819	-2,91066
	3	1	-2,78474	-2,99519	-3,12651	-3,2303	-3,33372	-3,42675	-3,51711	-3,59639	-3,65598	-3,75994
		5	-2,12748	-2,32539	-2,45316	-2,57682	-2,67982	-2,76063	-2,84833	-2,92037	-2,99091	-3,06284
		10	-1,76275	-1,96269	-2,10402	-2,22586	-2,33663	-2,41666	-2,5002	-2,57857	-2,6487	-2,71067
	4	1	-2,7705	-2,94464	-3,07658	-3,18467	-3,27513	-3,3641	-3,44107	-3,48828	-3,54995	-3,61013
		5	-2,10683	-2,29895	-2,43068	-2,54009	-2,64228	-2,71157	-2,78556	-2,85281	-2,91575	-2,95979
		10	-1,74755	-1,94668	-2,0741	-2,1882	-2,29903	-2,37704	-2,44724	-2,518	-2,58115	-2,63359
	5	1	-2,7949	-2,972	-3,09342	-3,16714	-3,25611	-3,32878	-3,37765	-3,44508	-3,50998	-3,57056
		5	-2,12084	-2,27673	-2,40952	-2,523	-2,61298	-2,69109	-2,76026	-2,81555	-2,87069	-2,92899
		10	-1,7598	-1,93172	-2,05838	-2,18475	-2,26989	-2,35672	-2,431	-2,4855	-2,54992	-2,59966

Table 1a. Critical values for Residual Augmented Fourier ADF Unit Root Test (Continued)

			ρ^2									
n	k	%	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
250	1	1	-3,04406	-3,35606	-3,56832	-3,7633	-3,93475	-4,07469	-4,19873	-4,33322	-4,44891	-4,56445
		5	-2,38286	-2,66892	-2,89213	-3,09117	-3,25484	-3,40628	-3,52457	-3,65443	-3,77077	-3,87788
		10	-2,0155	-2,31156	-2,54182	-2,7408	-2,90543	-3,05334	-3,17449	-3,31444	-3,43013	-3,53788
	2	1	-2,82278	-3,06395	-3,23424	-3,37069	-3,50402	-3,62363	-3,74829	-3,83605	-3,94493	-4,04817
		5	-2,15056	-2,38258	-2,54338	-2,6832	-2,80496	-2,90897	-3,01899	-3,10594	-3,1931	-3,30247
		10	-1,802	-2,0168	-2,18016	-2,32126	-2,43495	-2,53333	-2,65012	-2,73021	-2,821	-2,91098
	3	1	-2,78799	-3,01062	-3,15164	-3,27595	-3,36709	-3,45108	-3,52213	-3,62876	-3,72343	-3,76726
		5	-2,12467	-2,31527	-2,4616	-2,57479	-2,66336	-2,76533	-2,83848	-2,91601	-2,98067	-3,05006
		10	-1,75927	-1,95579	-2,09324	-2,21367	-2,31096	-2,40851	-2,48838	-2,55193	-2,62171	-2,68219
	4	1	-2,75753	-2,95551	-3,0892	-3,19749	-3,27311	-3,3702	-3,44574	-3,52298	-3,58149	-3,64543
		5	-2,10621	-2,2809	-2,42909	-2,5176	-2,61383	-2,69176	-2,76539	-2,85087	-2,89211	-2,94951
		10	-1,74851	-1,92374	-2,07106	-2,18045	-2,27316	-2,3502	-2,41671	-2,50253	-2,55208	-2,60639
	5	1	-2,77155	-2,94958	-3,0495	-3,16102	-3,24633	-3,3314	-3,38071	-3,48175	-3,537	-3,60569
		5	-2,09586	-2,27881	-2,39115	-2,4993	-2,58998	-2,66971	-2,74156	-2,80084	-2,84857	-2,90239
		10	-1,73949	-1,91894	-2,04647	-2,15773	-2,24419	-2,32535	-2,39982	-2,46222	-2,51591	-2,57174
500	1	1	-3,03991	-3,31169	-3,51289	-3,68368	-3,82865	-3,95183	-4,06609	-4,14012	-4,25814	-4,33111
		5	-2,36241	-2,64399	-2,86079	-3,04326	-3,20375	-3,3347	-3,46938	-3,56136	-3,66925	-3,75984
		10	-2,00778	-2,29109	-2,51945	-2,69511	-2,85567	-3,0107	-3,14193	-3,24978	-3,35703	-3,46625
	2	1	-2,83613	-3,07175	-3,229	-3,35168	-3,49257	-3,57074	-3,6662	-3,76786	-3,83802	-3,92884
		5	-2,17518	-2,39591	-2,54613	-2,68875	-2,80911	-2,90255	-3,00526	-3,08844	-3,1801	-3,26577
		10	-1,81493	-2,03582	-2,19232	-2,33232	-2,45076	-2,55781	-2,65685	-2,74118	-2,82803	-2,91259
	3	1	-2,80476	-2,98931	-3,14513	-3,23808	-3,33153	-3,43407	-3,51542	-3,57943	-3,65717	-3,72429
		5	-2,13533	-2,33048	-2,48065	-2,5978	-2,68927	-2,78029	-2,86112	-2,918	-2,99676	-3,05771
		10	-1,78043	-1,97019	-2,12216	-2,24895	-2,34834	-2,43776	-2,52887	-2,58949	-2,66272	-2,72569
	4	1	-2,79034	-2,98159	-3,095	-3,1951	-3,30872	-3,3659	-3,41432	-3,49176	-3,56001	-3,60018
		5	-2,12058	-2,30624	-2,43522	-2,55383	-2,64285	-2,71228	-2,78946	-2,86519	-2,91822	-2,98078
		10	-1,76123	-1,95444	-2,09664	-2,20139	-2,30952	-2,37595	-2,45608	-2,53622	-2,5944	-2,65598
	5	1	-2,79417	-2,95232	-3,08197	-3,17149	-3,23308	-3,3163	-3,38156	-3,42933	-3,49687	-3,54854
		5	-2,11353	-2,29928	-2,42238	-2,52321	-2,62928	-2,69381	-2,76267	-2,82555	-2,88514	-2,94603
		10	-1,76393	-1,93996	-2,07839	-2,17785	-2,28886	-2,36736	-2,4429	-2,50817	-2,57506	-2,62719

Table 1a. Critical values for Residual Augmented Fourier ADF Unit Root Test (Continued)

			ρ^2									
n	k	%	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	1	1	-3,03113	-3,30686	-3,51073	-3,70943	-3,83833	-3,93279	-4,0594	-4,16601	-4,22873	-4,3192
		5	-2,35855	-2,64127	-2,86801	-3,0538	-3,19764	-3,3237	-3,46075	-3,56407	-3,65628	-3,75388
		10	-1,9968	-2,28804	-2,51761	-2,71145	-2,86766	-3,00175	-3,13735	-3,25242	-3,35857	-3,46389
	2	1	-2,85185	-3,06004	-3,20241	-3,34217	-3,47851	-3,55794	-3,6745	-3,76075	-3,85709	-3,90221
		5	-2,1742	-2,38469	-2,54619	-2,6855	-2,81731	-2,90581	-3,0155	-3,09108	-3,18704	-3,2504
		10	-1,80964	-2,03428	-2,1968	-2,32969	-2,46321	-2,56409	-2,66576	-2,73867	-2,83053	-2,90972
	3	1	-2,82535	-2,98738	-3,13923	-3,24207	-3,32684	-3,42859	-3,51605	-3,56189	-3,64461	-3,70443
		5	-2,14442	-2,32625	-2,48206	-2,58011	-2,687	-2,77686	-2,85614	-2,92699	-3,00241	-3,05941
		10	-1,77766	-1,97399	-2,12472	-2,23779	-2,34432	-2,43478	-2,51247	-2,59326	-2,66592	-2,72098
	4	1	-2,81731	-2,9854	-3,09811	-3,20541	-3,27343	-3,34497	-3,43169	-3,49518	-3,55151	-3,58541
		5	-2,13043	-2,30627	-2,43531	-2,55607	-2,64158	-2,71903	-2,79971	-2,86729	-2,93189	-2,96939
		10	-1,76286	-1,95564	-2,09658	-2,20853	-2,30901	-2,39196	-2,47563	-2,53807	-2,6048	-2,65381
	5	1	-2,79026	-2,9706	-3,07517	-3,16981	-3,25081	-3,33424	-3,37829	-3,44442	-3,48445	-3,55283
		5	-2,11383	-2,30543	-2,42649	-2,54234	-2,61582	-2,70498	-2,76605	-2,82985	-2,87516	-2,94497
		10	-1,74902	-1,94927	-2,08539	-2,19468	-2,27952	-2,37041	-2,44026	-2,51947	-2,56261	-2,62288

Table 1b. Critical values for Residual Augmented Fourier ADF Unit Root Test with Constant and Trend (Continued)

			ρ^2									
n	k	%	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
50	1	1	-3.25004	-3.63443	-3.90496	-4.14221	-4.32952	-4.53168	-4.67821	-4.85183	-4.99192	-5.1367
		5	-2.59705	-2.97648	-3.25032	-3.49125	-3.68473	-3.87969	-4.02435	-4.18573	-4.31546	-4.45588
		10	-2.24465	-2.6158	-2.90545	-3.15129	-3.34988	-3.54087	-3.68971	-3.85224	-3.98329	-4.11471
	2	1	-3.09142	-3.43815	-3.68595	-3.89237	-4.07769	-4.25133	-4.42868	-4.56689	-4.69588	-4.8377
		5	-2.42263	-2.75354	-3.00609	-3.21723	-3.3933	-3.56016	-3.71436	-3.84885	-3.99013	-4.10491
		10	-2.06621	-2.3986	-2.64326	-2.85509	-3.02877	-3.19596	-3.35068	-3.48586	-3.61988	-3.73846
	3	1	-3.03881	-3.31925	-3.56027	-3.73308	-3.87395	-4.03616	-4.16609	-4.31999	-4.41985	-4.53614
		5	-2.35703	-2.63552	-2.86284	-3.03778	-3.19497	-3.33209	-3.46056	-3.59485	-3.69035	-3.8023
		10	-1.98648	-2.27667	-2.49305	-2.67442	-2.83261	-2.97366	-3.09517	-3.22201	-3.32991	-3.43186
	4	1	-2.97454	-3.25056	-3.46348	-3.61989	-3.76895	-3.91315	-4.03155	-4.14675	-4.24808	-4.3871
		5	-2.31741	-2.57579	-2.78462	-2.94343	-3.09117	-3.221	-3.3413	-3.44465	-3.5366	-3.63247
		10	-1.95671	-2.22334	-2.43386	-2.60001	-2.74031	-2.87003	-2.98035	-3.09389	-3.18168	-3.26974
	5	1	-2.96834	-3.23789	-3.40016	-3.57023	-3.71194	-3.82413	-3.93299	-4.04039	-4.16278	-4.28061
		5	-2.2987	-2.56356	-2.74862	-2.90451	-3.03975	-3.15859	-3.25831	-3.36758	-3.45759	-3.55996
		10	-1.93837	-2.20531	-2.39284	-2.55405	-2.69356	-2.81328	-2.92649	-3.01886	-3.1148	-3.20696
100	1	1	-3.26054	-3.60024	-3.88572	-4.08965	-4.27671	-4.4275	-4.56541	-4.72428	-4.83444	-4.93466
		5	-2.58598	-2.96715	-3.2381	-3.45917	-3.64647	-3.8152	-3.97178	-4.11808	-4.23302	-4.35117
		10	-2.23083	-2.62093	-2.90469	-3.11931	-3.31941	-3.49692	-3.65088	-3.80057	-3.92306	-4.04773
	2	1	-3.11031	-3.40175	-3.65182	-3.83032	-4.02956	-4.18113	-4.30695	-4.45639	-4.56833	-4.66535
		5	-2.42774	-2.74552	-2.99436	-3.18419	-3.36326	-3.53147	-3.67024	-3.80355	-3.92669	-4.03798
		10	-2.06383	-2.38934	-2.63742	-2.84568	-3.00999	-3.18323	-3.32323	-3.4693	-3.58622	-3.70541
	3	1	-3.01732	-3.29676	-3.52386	-3.68871	-3.83754	-3.98357	-4.11503	-4.23558	-4.3308	-4.44278
		5	-2.35648	-2.6438	-2.85217	-3.03097	-3.18304	-3.32027	-3.44516	-3.55782	-3.66769	-3.7818
		10	-1.99402	-2.28265	-2.49363	-2.68278	-2.84238	-2.97102	-3.09851	-3.21835	-3.32443	-3.43452
	4	1	-2.99154	-3.24324	-3.45319	-3.60634	-3.76803	-3.89202	-3.98196	-4.11767	-4.19289	-4.2996
		5	-2.31864	-2.59474	-2.78719	-2.95932	-3.09923	-3.23688	-3.34091	-3.4444	-3.5352	-3.63124
		10	-1.96553	-2.243	-2.44472	-2.61197	-2.76073	-2.88592	-3.00279	-3.10912	-3.19985	-3.29615
	5	1	-2.97758	-3.23591	-3.42487	-3.59672	-3.71242	-3.8076	-3.91623	-4.03033	-4.10551	-4.20851
		5	-2.32085	-2.55882	-2.76115	-2.91504	-3.05275	-3.17606	-3.27694	-3.37859	-3.46383	-3.54606
		10	-1.96049	-2.21322	-2.40679	-2.5726	-2.71111	-2.84052	-2.95645	-3.04825	-3.13622	-3.22467

Table 1b. Critical values for Residual Augmented Fourier ADF Unit Root Test with Constant and Trend (Continued)

			ρ^2									
n	k	%	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
250	1	1	-3.2488	-3.62114	-3.86214	-4.06989	-4.2307	-4.38247	-4.5143	-4.65584	-4.73801	-4.85953
		5	-2.58591	-2.96324	-3.22485	-3.4398	-3.62367	-3.79235	-3.93868	-4.07118	-4.18598	-4.2973
		10	-2.22812	-2.60825	-2.88588	-3.10952	-3.30365	-3.48006	-3.63197	-3.77337	-3.89588	-4.0109
	2	1	-3.0887	-3.41515	-3.64015	-3.83337	-3.99067	-4.1647	-4.28811	-4.39549	-4.5021	-4.60365
		5	-2.42693	-2.75304	-2.98347	-3.18232	-3.35059	-3.5216	-3.6526	-3.77809	-3.88878	-4.00792
		10	-2.07341	-2.3952	-2.63265	-2.83256	-3.0102	-3.17573	-3.32175	-3.45425	-3.56754	-3.68706
	3	1	-3.03931	-3.30942	-3.5107	-3.6706	-3.8227	-3.96495	-4.0685	-4.18438	-4.28366	-4.38619
		5	-2.35872	-2.64697	-2.85599	-3.02287	-3.19553	-3.32132	-3.44416	-3.55326	-3.66453	-3.75947
		10	-1.99096	-2.29669	-2.50707	-2.67616	-2.84452	-2.98314	-3.10845	-3.22438	-3.32757	-3.42913
	4	1	-3.02068	-3.26031	-3.44937	-3.60642	-3.75998	-3.8706	-3.9727	-4.07173	-4.1422	-4.26111
		5	-2.33482	-2.60235	-2.79512	-2.95327	-3.1224	-3.23365	-3.33281	-3.4417	-3.53207	-3.63109
		10	-1.96963	-2.25094	-2.45158	-2.61815	-2.77902	-2.89798	-3.00542	-3.11708	-3.20886	-3.30036
	5	1	-2.9944	-3.23556	-3.43683	-3.56116	-3.68943	-3.8214	-3.91372	-3.99555	-4.09558	-4.18573
		5	-2.32821	-2.56903	-2.77731	-2.91822	-3.0538	-3.18553	-3.28714	-3.38664	-3.47108	-3.55924
		10	-1.96364	-2.22471	-2.43169	-2.5842	-2.73047	-2.85489	-2.96194	-3.06044	-3.15193	-3.24518
500	1	1	-3.25207	-3.5837	-3.83415	-4.03992	-4.21843	-4.37381	-4.50078	-4.59222	-4.717	-4.8173
		5	-2.58407	-2.93836	-3.20343	-3.4243	-3.61692	-3.77886	-3.92622	-4.05652	-4.17181	-4.28071
		10	-2.237	-2.59849	-2.87118	-3.0982	-3.29462	-3.46995	-3.6309	-3.76107	-3.88989	-4.00288
	2	1	-3.10028	-3.42225	-3.64042	-3.84325	-4.00316	-4.13225	-4.25664	-4.36164	-4.49194	-4.59363
		5	-2.43198	-2.75066	-2.98703	-3.17889	-3.36513	-3.49435	-3.64109	-3.76414	-3.8762	-3.99358
		10	-2.07142	-2.39018	-2.63013	-2.83641	-3.01708	-3.16396	-3.30741	-3.43924	-3.55825	-3.68114
	3	1	-3.00871	-3.29276	-3.52116	-3.68947	-3.81867	-3.94979	-4.06436	-4.18565	-4.28861	-4.37955
		5	-2.36304	-2.63664	-2.86037	-3.03888	-3.16861	-3.31777	-3.4429	-3.54766	-3.65485	-3.76049
		10	-2.00831	-2.28933	-2.50935	-2.69041	-2.83741	-2.98112	-3.10822	-3.22499	-3.32096	-3.43545
	4	1	-3.00469	-3.26111	-3.45743	-3.60608	-3.76128	-3.86692	-3.94604	-4.05839	-4.16653	-4.24111
		5	-2.33238	-2.59496	-2.79945	-2.9735	-3.11097	-3.2225	-3.33309	-3.43588	-3.54444	-3.62254
		10	-1.97417	-2.24758	-2.45778	-2.62709	-2.77434	-2.89658	-3.0029	-3.12035	-3.21749	-3.30591
	5	1	-2.98685	-3.23316	-3.42661	-3.55741	-3.68509	-3.79058	-3.90019	-3.99035	-4.08989	-4.1701
		5	-2.31745	-2.57237	-2.77809	-2.92126	-3.05774	-3.18318	-3.27405	-3.37919	-3.47482	-3.56121
		10	-1.96448	-2.2257	-2.43366	-2.58093	-2.72721	-2.85165	-2.96099	-3.06389	-3.16189	-3.24555

Table 1b. Critical values for Residual Augmented Fourier ADF Unit Root Test with Constant and Trend (Continued)

n	k	%	ρ^2									
			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1000	1	1	-3.23764	-3.59173	-3.85336	-4.0583	-4.21101	-4.35501	-4.48991	-4.59941	-4.70568	-4.81266
		5	-2.58009	-2.9286	-3.20716	-3.42556	-3.61816	-3.76511	-3.93148	-4.04437	-4.16795	-4.27372
		10	-2.22325	-2.59103	-2.86825	-3.10536	-3.29575	-3.45961	-3.6288	-3.74991	-3.88434	-3.9968
	2	1	-3.10513	-3.40641	-3.63596	-3.82943	-4.00141	-4.10874	-4.24518	-4.36941	-4.47675	-4.57808
		5	-2.41927	-2.74759	-2.97582	-3.17683	-3.35067	-3.5079	-3.64192	-3.76298	-3.88879	-3.9964
		10	-2.07046	-2.39083	-2.63585	-2.83003	-3.01836	-3.17335	-3.30697	-3.43725	-3.56603	-3.67783
	3	1	-3.03771	-3.3193	-3.50522	-3.6837	-3.81936	-3.94111	-4.07457	-4.16582	-4.27033	-4.36233
		5	-2.36914	-2.64359	-2.86459	-3.03056	-3.17932	-3.31424	-3.44624	-3.54739	-3.6552	-3.75265
		10	-2.00252	-2.29649	-2.51597	-2.68473	-2.84243	-2.97736	-3.11047	-3.21784	-3.33402	-3.42847
	4	1	-3.01191	-3.27634	-3.47498	-3.61148	-3.74011	-3.86386	-3.95855	-4.06205	-4.1629	-4.2342
		5	-2.3366	-2.60277	-2.80227	-2.97468	-3.10529	-3.23339	-3.33427	-3.43595	-3.53824	-3.61803
		10	-1.97311	-2.25349	-2.46026	-2.63308	-2.77237	-2.9027	-3.01563	-3.11598	-3.21908	-3.30184
	5	1	-2.99808	-3.26303	-3.42285	-3.56706	-3.69285	-3.80993	-3.91214	-3.97711	-4.06273	-4.15139
		5	-2.31276	-2.5889	-2.78662	-2.934	-3.05274	-3.18728	-3.2884	-3.37917	-3.46858	-3.55931
		10	-1.95523	-2.23706	-2.43915	-2.59319	-2.7281	-2.8627	-2.96611	-3.06685	-3.15656	-3.24319