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Abstract

This paper develops a quality-ladder growth model with elastic labor supply and distortionary taxes to analyze the effects of different subsidy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to research and development (R&D). The model is calibrated to the US data to compare the growth and welfare implications of these subsidies. The main results are as follows. First, a coordination of all instruments attains the social optimum. Second, as for the use of a single instrument, the R&D subsidy is less growth-enhancing and welfare-improving than the other subsidies. Finally, as for the use of a mix of any two instruments, subsidizing the production of final goods and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare.

JEL classification: D61; E62; O31; O38
Keywords: Economic Growth; R&D; Quality Ladder; Subsidies

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1 Introduction

In this study, we explore the effects of subsidization on economic growth and social welfare in a Schumpeterian economy with elastic labor supply and distortionary taxes. In many industrialized economies where research activities for innovations are the major engine of growth, it is observed that research and development (R&D) activities are highly intervened by government policies. The necessity of government intervention on R&D activities has been justified by a spate of endogenous growth theory literature that highlight the presence of a positive R&D externality, since inventors of new products face knowledge spillovers and it is difficult for them to fully appropriate the benefits of innovations (e.g., Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Jones and Williams (2000)).[1] In subsequent R&D-based growth models, the granting of monopoly rights to innovators and subsidies to R&D are the two major forms of government policy instruments exploited to deal with such an R&D externality.[2] On the one hand, granting monopoly power to successful innovators in the form of patent protection allows for pricing their goods above marginal costs, resulting in monopolistic profits that would create sufficient incentives for entrepreneurs to perform R&D activities.[3] Such a monopoly right, however, reduces the demand for production inputs to the level below the first-best allocation, and hence, unavoidably leads to a distortion. On the other hand, given that innovations are usually costly, subsidizing R&D investment seems able to effectively promote research activities to stimulate growth,[4] thereby generating sizable effects on welfare. Nevertheless, implementing such an R&D policy still cannot remove the distortion of monopoly pricing, since this policy instrument mainly affects the competitive R&D sector instead of the monopolistic production sector.

To internalize R&D externalities and remove the monopoly-pricing distortion, the existing studies exploiting the R&D-based growth framework have used a subsidization-policy regime, which includes subsidies to manufacturing and subsidies to R&D, in the framework of R&D-based growth. In the presence of inelastic labor supply and lump-sum taxes through which subsidies are financed, the model of Barro and Sala-I-Martin (2003) with expanding varieties of new products shows that subsidies to manufacturing (through either final goods produced by competitive firms or the purchase of intermediate goods produced by monopolistic firms) are able to effectively restore the social optimum, which, however, cannot be achieved by subsidizing R&D activities. In contrast, the model of Acemoglu (2009) with improving quality of existing products implies that subsidizing both manufacturing and R&D is able to remove the distortions

[1] In addition to this positive R&D externality, there can also be negative R&D externalities due to duplicative R&D (i.e., congestion externalities in Jones and Williams (1998, 2000)) and business stealing (Bloom *et al.* (2013)). However, the positive R&D externality tends to substantially outweigh the negative externalities (Grossmann *et al.* (2013)), and this is consistent with the existing empirical evidence suggesting that the social return to R&D exceeds the private return by a wide margin. See Hall *et al.* (2010) for a complete review of the econometric literature on measuring the private and social returns to R&D.

[2] See Park (2008) for a strengthening of patent protection across countries since the signing of the agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) in 1994. In addition, see Impullitti (2010) for evidence on the widespread use of R&D subsidization in OECD countries since 1980s.

[3] The recent empirical evidence in Brown *et al.* (2017) finds that protection for intellectual property has positive effects on R&D in OECD economies.

[4] See, for example, Minniti and Venturini (2017) who find that R&D tax credits have positive effects on productivity growth of the US economy.

and replicate the socially optimal allocation.[5] Moreover, considering elastic labor supply with the financing option altered to distortionary taxes, the analysis of Zeng and Zhang (2007) reveals that in a variety-expansion model, using a single subsidy alone or even their combination cannot reach the social optimum. Nevertheless, these related studies have not examined the growth and welfare effects of the subsidization regime in a quality-ladder model with elastic labor supply and distortionary taxes.[6] Therefore, the novel contribution of our study is to fill this gap.

To meet the above objective, in this paper, we examine the growth and welfare effects of three types of subsidy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to R&D.[7] Specifically, this study revisits the implications of these subsidies and their combinations by means of growth maximization and welfare maximization. In addition, as for the model setting, we extend the R&D-based growth framework of Acemoglu (2009) with quality improvement by incorporating elastic labor supply and introducing distortionary taxes to finance subsidies. Additionally, this model is calibrated to the US data to conduct a quantitative exercise for the above analysis.

The findings of this study are summarized as follows. First, the inefficiencies present in this model originate from the *static* distortions in monopoly pricing and labor supply, along with the *dynamic* distortion in R&D externalities. It is found that subsidies to manufacturing are more effective in eliminating the former distortions, whereas subsidies to research (in fact, it is an R&D tax in the first-best outcome) are more effective in eliminating the latter one. Thus, an optimal mix of all policy instruments restores the social optimum. Second, in the calibrated economy, when only a single subsidy tool is used, subsidizing R&D investment is less effective than subsidizing manufacturing in terms of promoting growth and raising welfare. This is because the benefits of innovations and the removal of inefficiencies are both less sensitive to the decrease in research expenditures resulting from subsidies to R&D than to the increase in production volume induced by subsidies to manufacturing. Third, as for the use of a combination of any two instruments, subsidizing the production of final goods and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare. The reason is that using the two forms of subsidies to manufacturing together expands the dimension that enlarges the production size and hence is most growth-stimulating. Nevertheless, as mentioned above, these two subsidy instruments remove the same type of distortions, so their combination generates less welfare as compared to the combinations of policy instruments involving R&D subsidies, which remove different types of distortions. These results highlight the importance of the coordinated use of subsidies to R&D and subsidies to manufacturing in raising social welfare.

This study relates to the vast literature that explore the effects of R&D subsidies in the R&D-based growth models; see for example, Segerstrom (1998), Lin (2002), Dinopoulos and Syropoulos (2007), Şener (2008), Impullitti (2010), Chu and Cozzi (2018), and Yang (2018), in which either variety expansion or quality improvement is considered as the process of innovation, in addition

[5] Yang (2018) shows that mixing the subsidies to manufacturing and R&D will also achieve the social optimum even if labor supply is elastic.

[6] The advantage of considering a Schumpeterian-type quality-ladder model is that both positive and negative R&D externalities are present, so the problem of underinvestment or overinvestment in R&D could occur, depending on the relative strength of the different types of R&D externalities.

[7] Throughout this study, subsidies to the production of final goods and those to the purchase of intermediate goods are collectively called subsides to manufacturing.

to Peretto (1998), Segerstrom (2000), Chu *et al.* (2016), and Chu and Wang (2019), in which the two dimensions of innovation are combined. While inspiring, the aforementioned studies mainly focus on financing subsidy costs with non-distortionary taxes, ruling out the distortionary effects of taxes on aggregate equilibrium allocations. Our study differs from theirs by considering the impacts of R&D subsidies when subsidization is financed by distortionary labor income taxes that distort the consumption-labor decision.

This study also relates to the literature on R&D-based growth models that consider the mixed use of subsidies to research and intermediate goods. Grossmann *et al.* (2013) show that in a semi-endogenous growth model put forward by Jones (1995), a combination of a time-varying subsidy to R&D and a constant subsidy to intermediate-goods production can achieve the socially optimal growth path. Furthermore, Li and Zhang (2014) show that in the Matsuyama (1999) model of growth through cycles, using subsides to R&D and the purchase of intermediate goods, either individually or jointly, yields significant welfare gains. However, the analysis of these interesting studies focuses on dynamic general equilibrium frameworks with inelastic labor supply.[8] Therefore, the present paper complements their studies by investigating the welfare implications when labor is supplied elastically. It turns out that elastic labor supply plays a crucial role in our model in attaining the social optimum.

The rest of this paper is organized as follows. Section 2 presents the model setup. Section 3 characterizes the decentralized equilibrium and explores the growth effect of subsidies. Section 4 derives the first-best optimal outcome and analyzes the subsidy policy that restores the social optimum. Section 5 performs a numerical analysis in a calibrated economy to evaluate the growth-maximizing and welfare-maximizing subsidy instrument(s). Section 6 concludes the study.

2 The model

In this study, we extend the version of the quality-ladder growth model in Acemoglu (2009) (Chapter 14), which originates from Grossman and Helpman (1991), by incorporating (a) subsidies to the production of final goods, the purchase of intermediate goods, and the expenditures on R&D, and (b) elastic labor supply. Moreover, this model introduces distortionary labor income taxes to finance the subsidies. This study analyzes the growth and welfare implications of subsidization by controlling one or a mix of these policy instruments.

2.1 Households

Suppose that the economy admits a unit continuum of identical households, and the lifetime utility function of each household is given by

$$U = \int_0^\infty e^{-\rho t}[\ln C_t + \theta \ln(1 - L_t)]dt, \tag{1}$$

[8] One exception is Nuño (2011), who finds that the optimal mix of subsidies to research and intermediate-goods production can replicate the first-best allocations in a Schumpeterian growth model with business cycles. Nevertheless, the financing of subsidies in his analysis relies on a lump-sum tax on households.

where ρ is the discount rate, C_t is the household's consumption of final goods, and L_t is the labor supplied by a household. The parameter $\theta > 0$ indicates the intensity of leisure preference relative to consumption.

There is no population growth in this economy.[9] Each household chooses consumption C_t and labor supply L_t to maximize its lifetime utility (1) subject to the instantaneous budget constraint such that

$$\dot{A}_t = r_t A_t + W_t(1-\tau_t)L_t - C_t, \tag{2}$$

where A_t is the real value of financial assets owned by each household, W_t is the real wage rate, r_t is the real interest rate, and τ_t is the tax rate on labor wage. The standard dynamic optimization implies the consumption-labor decision given by

$$(1-\tau_t)W_t(1-L_t) = \theta C_t, \tag{3}$$

and the usual Euler equation given by

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{4}$$

Moreover, the households own a balanced portfolio of all firms in the economy. Finally, the transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}\mu_t A_t = 0$, where μ_t is the Lagrange multiplier associated with the constraint (2) in the current-value Hamiltonian of the household. This condition implies that neither asset or debt will remain at the end of the planning horizon.

2.2 Final goods

Final goods Y_t are produced competitively by using labor and a continuum of intermediate goods according to the following production function:

$$Y_t = \frac{L_t^{1-\beta}}{\beta}\int_0^1 q_t(\nu)X_t(\nu)^\beta d\nu, \qquad \beta \in (0,1) \tag{5}$$

where L_t is the level of labor, $X_t(\nu)$ is the quantity of intermediate good in line $\nu \in [0,1]$ whose quality is $q_t(\nu)$, and β measures the importance of intermediate good ν relative to labor in final goods production. In addition, the quality $q_t(\nu)$ evolves as follows:

$$q_t(\nu) = \lambda^{n_t(\nu)} q_0(\nu), \tag{6}$$

where $\lambda > 1$ represents the step size of each quality improvement, $n_t(\nu)$ is the number of innovations in line ν that have occurred between time 0 and time t. Then, the profit function of the

[9] Our model results are robust to the consideration of population growth in that the counterfactual scale effect is sterilized in a fully-endogenous approach. See the detailed discussions in Cozzi (2017a).

competitive final-goods producers is given by

$$\hat{\pi}_t = (1+s_{y,t})\frac{L_t^{1-\beta}}{\beta}\int_0^1 q_t(\nu)X_t(\nu)^{\beta}d\nu - W_tL_t - (1-s_{x,t})\int_0^1 P_t(\nu)X_t(\nu)d\nu, \tag{7}$$

where $s_{y,t} \in (0,1)$ ($s_{y,t} \in (-1,0)$) is the subsidy rate (tax rate) to the production of final goods and $s_{x,t} \in (0,1)$ ($s_{x,t} \in (-1,0)$) is the subsidy rate (tax rate) to the purchase of intermediate goods.

With free entry and profit maximization, (5) yields the conditional demand functions for the inputs, namely, the demand of labor:

$$L_t = (1+s_{y,t})(1-\beta)\frac{Y_t}{W_t}, \tag{8}$$

and the demand for the intermediate good ν:

$$X_t(\nu) = \left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}}\left(\frac{q_t(\nu)}{P_t(\nu)}\right)^{\frac{1}{1-\beta}}L_t, \tag{9}$$

where $P_t(\nu)$ is the price of the ν-th intermediate good relative to the final goods.

2.3 Intermediate goods

In each industry line $\nu \in [0,1]$, intermediate goods are produced by a monopolistic leader who holds a patent on the latest innovation and replaced by the products of an entrant who has a new innovation due to the *Arrow replacement effect*. The marginal cost of producing a unit of intermediate good is $\psi q_t(\nu)$ units of final goods, where $\psi \in (0,1)$. Thus, the ν-th intermediate goods producer maximizes her profits $\pi_t(\nu) = [P_t(\nu) - \psi q(\nu)]x_t(\nu)$ subject to the intermediate goods demand in (9), which yields the profit-maximizing price such that

$$P_t(\nu) = q_t(\nu), \tag{10}$$

where, without loss of generality, we have normalized $\psi = \beta$. Substituting (10) into (9) generates the quantity of intermediate good ν:

$$X_t(\nu) = \left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}}L_t. \tag{11}$$

Additionally, according to (11), the profit function of the monopolistic firm is given by

$$\pi_t(\nu) = (1-\beta)\left(\frac{1+s_{y,t}}{1-s_{x,t}}\right)^{\frac{1}{1-\beta}}q_t(\nu)L_t, \tag{12}$$

which shows that the monopolistic profit is increasing in the quality of the products.

Technological progress in this model stems from the realizations of quality improvements in

$q_t(v)$ across all industry lines. Define the aggregate quality index Q_t by a combination of the total quality of intermediate goods:

$$Q_t = \int_0^1 q_t(v) dv. \tag{13}$$

Substituting (11) and (13) into the final goods production function in (5) yields the total output in the final goods sector such that

$$Y_t = \frac{1}{\beta} \left(\frac{1 + s_{y,t}}{1 - s_{x,t}} \right)^{\frac{\beta}{1-\beta}} Q_t L_t, \tag{14}$$

which shows that the aggregate output is linearly increasing the aggregate quality of intermediate goods. Next, using (11), the aggregate spending on intermediate goods is obtained by

$$X_t \equiv \int_0^1 P_t(v) X_t(v) dv = \beta \left(\frac{1 + s_{y,t}}{1 - s_{x,t}} \right)^{\frac{1}{1-\beta}} Q_t L_t. \tag{15}$$

Accordingly, the labor wage rate is given by

$$W_t = (1 + s_{y,t}) \left(\frac{1-\beta}{\beta} \right) \left(\frac{1 + s_{y,t}}{1 - s_{x,t}} \right)^{\frac{\beta}{1-\beta}} Q_t. \tag{16}$$

Finally, using (12) and (13) yields the aggregate profit of the intermediate-goods sector, which is given by

$$\Pi_t \equiv \int_0^1 \pi_t(v) dv = (1-\beta) \left(\frac{1 + s_{y,t}}{1 - s_{x,t}} \right)^{\frac{1}{1-\beta}} Q_t L_t. \tag{17}$$

Observing (17) reveals that the total monopolistic profits Π_t created by all inventions is increasing in the policy instrument $s_{y,t}$ to the production of final goods and $s_{x,t}$ to the purchase of intermediate goods, respectively.

2.4 Innovations and R&D

Denote the real value of a firm who holds the most recent innovation in line v by $V_t(v)$. Accordingly, the Hamilton-Jacobi-Bellman (HJB) equation for $V_t(v)$ is given by

$$r_t V_t(v) = \pi_t(v) + \dot{V}_t(v) - p_t(v) V_t(v), \tag{18}$$

which is the no-arbitrage condition for the value of the asset (in the form of a patented innovation). Equation (18) implies that the return on this asset $r_t V_t(v)$ equals the sum of the profit flow $\pi_t(v)$, the capital gain $\dot{V}_t(v)$, and the potential losses $p_t(v) V_t(v)$ that occur due to creative destruction, where $p_t(v)$ denotes the Poisson arrival rate of the next successful innovation in each instant of time. Specifically, following the lab-equipment assumption, the formulation of $p_t(v)$ is given by

$$p_t(\nu) = \frac{\zeta z_t(\nu)}{L_t q_t(\nu)}, \tag{19}$$

where $\zeta > 0$ is R&D productivity and $z_t(\nu)$ are the units of final goods spent in R&D. Equation (19) means that the probability of the next successful innovation is increasing in R&D expenditures $z_t(\nu)$ whereas decreasing in quality $q_t(\nu)$; research on more advanced products becomes more difficult, so one unit of R&D spending is proportionately less effective when applied to a more sophisticated product. Moreover, to eliminate the scale effect in this model, we use the *fully endogenous* solution by assuming that the arrival rate of innovations depends on the R&D expenditures per unit of labor.[10]

New innovations in each line are invented by R&D firms, who have free entry into the research market and incur positive expenditures on R&D subject to policy interventions in the form of subsidization (taxation) at the rate of $s_{r,t} \in (0,1)$ ($s_{r,t} \in (-1,0)$). Hence, the expected profit of an R&D firm who spends $z_t(\nu)$ in R&D in line ν that has quality q at time t must be zero such that $p_t(\nu)V_t(\nu) - (1 - s_{r,t})z_t(\nu) = 0$, and it, together with (19), implies the zero-expected-profit condition as follows:

$$V_t(\nu) = \frac{(1 - s_{r,t})q_t(\nu)L_t}{\zeta\lambda}. \tag{20}$$

Since A_t is the market aggregate value of firms in the intermediate-goods sector, using (13) yields

$$A_t = \int_0^1 V_t(\nu)d\nu = \frac{(1 - s_{r,t})Q_t L_t}{\zeta\lambda}, \tag{21}$$

implying that A_t is increasing in the aggregate quality of goods.

2.5 Government budget

Suppose that the policymaker can intervene the production of final goods, the purchase of intermediate goods, and the expenditures on R&D by choosing the policy tools $s_{y,t}$, $s_{x,t}$, and $s_{r,t}$, respectively. These government interventions are financed by the distortionary tax levied on the household's labor income, such that

$$\tau_t W_t L_t = s_{y,t} Y_t + s_{x,t} X_t + s_{r,t} Z_t, \tag{22}$$

where $Z_t \equiv \int_0^1 z_t(\nu)d\nu$ is the total spending on R&D. In (22), the left-hand side is the tax revenues collected from the household and the right-hand side is the expenditures for subsidization. Hence, in this model, the government can implement the subsidy (or tax) instruments to affect the input allocations and steer the market economy.

[10] In the current literature, the fully endogenous solution proposed by Peretto (1998), Young (1998), and Howitt (1999) and the semi-endogenous solution proposed by Jones (1995), Kortum (1997), and Segerstrom (1998) are the two main approaches to remove the scale effect. See Cozzi (2017a,b) for detailed discussions.

3 Decentralized equilibrium

An equilibrium consists of a sequence of allocations $[C_t, Y_t, L_t, X_t(\nu), z_t(\nu)]_{t=0,\nu\in[0,1]}^{\infty}$ and a sequence of prices $[r_t, W_t, P_t(\nu), q_t(\nu), V_t(\nu)]_{t=0,\nu\in[0,1]}^{\infty}$. In each instant of time,

• households choose $[C_t, L_t]$ to maximize their utility taking $[r_t, W_t]$ as given;

• competitive final-goods firms produce $[Y_t]$ and choose $[W_t, X_t(\nu)]$ to maximize profits taking $[P_t(\nu), q_t(\nu)]$ as given;

• monopolistic leaders for intermediate goods produce $[X_t(\nu)]$ and choose $[P_t(\nu)]$ to maximize profits;

• R&D firms choose $[z_t(\nu)]$ to maximize profits taking $[q_t(\nu), V_t(\nu)]$ as given;

• the goods market clears such that $Y_t = C_t + X_t + Z_t$;

• the financial market clears such that $A_t = \int_0^1 V_t(\nu)d\nu$.

3.1 Balanced growth path

In this subsection, we define the decentralized equilibrium and prove that the economy jumps to a unique and stable balanced growth path (BGP). Hence, for an arbitrary path of subsidy rates $[s_{y,t}, s_{x,t}, s_{r,t}]_{t=0}^{\infty}$, we obtain the following result.

Proposition 1. *Holding constant s_y, s_x, and s_r, the economy jumps to a unique and stable balanced growth path along which $\{W_t, Q_t, X_t, Z_t, Y_t, C_t\}$ grow at the same and constant rate (i.e., g), and $L_t = L$ is stationary.*

Proof. See Appendix A. □

From Proposition 1, given a stationary time path of the policy levers, we can derive the steady-state levels of some variables along the BGP as follows. First, for a given level of quality $q(\nu)$ (which is constant over time until there is a new innovation in this line), the value of a firm in line ν (i.e., $V(\nu)$) does not change between time t and time $t + \Delta t$ (where Δt is an interval of time), namely $\dot{V}_t(\nu) = 0$. Thus, using (18) implies that $V(\nu)$ will be constant such that

$$V(\nu) = \frac{\pi(\nu)}{r + p(\nu)}, \tag{23}$$

where $\pi(\nu)$, r, and $p(\nu)$ are the steady-state levels of monopolistic profits, the interest rate, and the arrival rate of successful innovations in line ν, respectively. Then, combining (12), (20), and (23) yields

$$r + p(\nu) = \zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{\beta}}\left(\frac{1}{1-s_r}\right), \tag{24}$$

which implies that along the BGP, the arrival rate of next successful innovations is independent of the line index ν, denoted by p. In addition, the aggregate expenditures on R&D can be expressed by

$$Z_t = \int_0^1 z(\nu)d\nu = \frac{p}{\zeta}Q_tL. \tag{25}$$

By Proposition 1, substituting (24) into (4) yields

$$g = r - \rho = \left[\zeta\lambda(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}\left(\frac{1}{1-s_r}\right) - p\right] - \rho. \tag{26}$$

Then, to pin down the growth rate of the aggregate quality index Q_t, we know that in an interval of time Δt, there are $p_t\Delta t$ sectors that experience one innovation, and this increases their productivity by λ. Hence, we have the dynamics of Q_t given by

$$Q_{t+\Delta t} = p_t\Delta t\int_0^1 \lambda q_t(\nu)d\nu + (1-p_t\Delta t)\int_0^1 q_t(\nu)d\nu = Q_t[1+p_t\Delta t(\lambda-1)] \tag{27}$$

Now subtracting Q_t from both sides in (27), dividing it by Δt, and taking the limit as $\Delta t \to 0$ yields

$$g = \frac{\dot{Q}_t}{Q_t} = p(\lambda-1), \tag{28}$$

where $\dot{Q}_t = \lim_{\Delta t\to 0}(Q_{t+\Delta t} - Q_t)/\Delta t$. Then, combining (26) and (28) yields the steady-state arrival rate of innovations such that

$$p = \zeta(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}\left(\frac{1}{1-s_r}\right) - \frac{\rho}{\lambda}, \tag{29}$$

and the steady-state growth rate of aggregate quality is obtained by substituting (29) into (28), such that

$$g = \zeta(1-\beta)(\lambda-1)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{1-\beta}}\left(\frac{1}{1-s_r}\right) - \frac{(\lambda-1)\rho}{\lambda}. \tag{30}$$

It can be seen that the growth rate of aggregate quality in (30) is independent upon the size of labor supply. The scale effect is therefore removed. Additionally, we have the following result.

Lemma 1. *The steady-state growth rate of aggregate quality g is increasing in the subsidy rate s_y to final-goods production, the subsidy rate s_x to the purchase of intermediate goods, and the subsidy rate s_r to R&D.*

Proof. Equation (30) shows that g is increasing in s_y, s_x, and s_r. □

Intuitively, on the one hand, either a higher subsidy rate s_y to the production of final goods or a higher subsidy rate s_x to the purchase of intermediate goods can increase the demand for intermediate goods $X_t(\nu)$ in (9), which raises the profits of monopolistic firms in the intermediate-goods sector brought by innovations. On the other hand, a higher subsidy rate s_r to R&D decreases the cost of research. The above policy changes increase the benefits of innovations and raise the incentives for R&D, so more resources are reallocated toward conducting research activities; namely R&D expenditures tend to rise. Hence, (19) implies that the economy exhibits a higher arrival rate of the next successful innovation, leading to a higher rate of economic growth

g.[11] These comparative statics for the subsidy rates are consistent with those in Barro and Sala-I-Martin (2003), Zeng and Zhang (2007), and Yang (2018).

Moreover, using (22), we can compute the steady-state rate of labor income tax such that

$$\tau = \frac{1}{1-\beta}\left(\frac{s_y}{1+s_y}\right) + \frac{\beta^2}{1-\beta}\left(\frac{s_x}{1-s_x}\right) + \frac{p}{\zeta}\left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{1+s_y}\right)^{\frac{1}{1-\beta}}(1-s_x)^{\frac{\beta}{1-\beta}}s_r, \tag{31}$$

which is a composite function of the subsidy rates s_y, s_x, and s_r, where the steady-state arrival rate of innovations p is given by (29). In addition, using (31), we can derive the steady-state level of labor supply such that

$$L(s_y, s_x, s_r) = \frac{1}{1 + \frac{\theta}{(1-\beta)(1-\tau)(1+s_y)}\left[1 - \beta^2\left(\frac{1+s_y}{1-s_x}\right) - \frac{\beta p}{\zeta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{-\beta}{1-\beta}}\right]}. \tag{32}$$

From the above analysis, it can be seen that in the case of a higher s_y, s_x, and s_r, τ would increase because heavier taxation is required to balance the government budget, and such a higher rate of labor income tax would decrease the supply of labor, which is captured by the negative relationship between τ and L in (32). Nevertheless, a higher s_y, s_x, and s_r raises the growth rate g as shown in Lemma 1, and the enlarged production volume of output Y_t induces higher demand for labor L in equilibrium. Therefore, the overall effects of s_y, s_x, and s_r on L depend on the relative strength of these two opposing forces, and thus, seem analytically difficult to assess. We leave this discussion for the numerical analysis later on.

4 Optimal policy analysis

In this section, we study the socially optimal solution that maximizes the welfare of the model economy, followed by an analysis of the optimal policy regime showing how an appropriate joint choice on subsidy rates can be made so as to replicate the first-best allocation.

4.1 Socially optimal solution

As for the first-best outcome, the social planner chooses a time path of consumption C_t and labor supply L_t to maximize the households' lifetime utility given by (1), subject to the resource constraint $Y_t = C_t + X_t + Z_t$ and the technology constraint given by

$$\dot{Q}_t = \frac{\zeta(\lambda-1)Z_t}{L_t}, \tag{33}$$

which is obtained by combining (19) and (28). Moreover, in the intermediate-goods sector, the socially optimal price $p_t(\nu)$ in line ν equals the marginal cost of production $\beta q_t(\nu)$. Therefore,

[11] Recall that in Proposition 1, the steady-state growth rate of aggregate quality equals the counterparts of output and consumption.

the demand for intermediate goods in line ν is given by

$$X_t(\nu) = \beta^{\frac{1}{\beta-1}} L_t. \tag{34}$$

Aggregating these demand functions across all industry lines yields the total expenditures on the purchase of intermediate goods such that

$$X_t = \int_0^1 P_t(\nu) X_t(\nu) d\nu = \beta^{\frac{\beta}{\beta-1}} Q_t L_t. \tag{35}$$

Next, the total output in social optimum can be found by substituting (34) into (5), which is given by

$$Y_t = \frac{(L_t)^{1-\beta}}{\beta} \int_0^1 q_t(\nu) X_t(\nu)^{\beta} d\nu = \beta^{\frac{1}{\beta-1}} Q_t L_t. \tag{36}$$

Therefore, using (34), (36), and the resource constraint in (33), we obtain the dynamics of aggregate quality as follows:

$$\dot{Q}_t = \frac{\zeta(\lambda-1)}{L_t} \left[\beta^{\frac{1}{\beta-1}} Q_t L_t - \beta^{\frac{\beta}{\beta-1}} Q_t L_t - C_t \right]. \tag{37}$$

Then, the social planner's solution can be derived by setting up the following current-value Hamiltonian:

$$\hat{H}_t(C_t, L_t, Q_t, \hat{\mu}_t) = \ln C_t + \theta \ln(1 - L_t) + \hat{\mu}_t \left\{ \frac{\zeta(\lambda-1)}{L_t} \left[\beta^{\frac{1}{\beta-1}} Q_t L_t - \beta^{\frac{\beta}{\beta-1}} Q_t L_t - C_t \right] \right\}, \tag{38}$$

where $\hat{\mu}_t$ is the costate variable associated with the constraint (37). Thus, the first-order conditions are respectively given by

$$\frac{\partial \hat{H}_t}{\partial C_t} = 0 \Rightarrow \frac{1}{C_t} = \frac{\hat{\mu}_t \zeta(\lambda-1)}{L_t}; \tag{39}$$

$$\frac{\partial \hat{H}_t}{\partial L_t} = 0 \Rightarrow \frac{\hat{\mu}_t \zeta(\lambda-1) C_t}{(L_t)^2} = \frac{\theta}{1 - L_t}; \tag{40}$$

$$\frac{\partial \hat{H}_t}{\partial Q_t} = \hat{\mu}_t \zeta \beta^{\frac{1}{\beta-1}} (1-\beta)(\lambda-1) = \rho \hat{\mu}_t - \dot{\hat{\mu}}_t, \tag{41}$$

with the trasversality condition $\lim_{t \to 0} e^{-\rho t} \hat{\mu}_t Q_t = 0$. Multiplying (37) by $\hat{\mu}_t$ and multiplying (41) by Q_t, respectively, we can use (39) to obtain a differential equation such that $\dot{\hat{\mu}}_t Q_t + \hat{\mu}_t \dot{Q}_t = \rho \hat{\mu}_t Q_t - 1$, implying that $\hat{\mu}_t Q_t$ must jump to its steady-state value given by $1/\rho$. This implies that the dynamical behavior of the model in the social optimum is also characterized by saddle-point stability.

Moreover, by inserting (39) into (40), we can see that the first-best level of labor supply L_t^* is stationary such that

$$L^* = \frac{1}{1+\theta}. \tag{42}$$

Accordingly, combining the saddle-point stability condition with (37), (39), (41) implies that in the first-best outcome, $-\hat{\mu}_t$, Q_t, and C_t all grow at the same rate given by

$$g^* = \zeta\beta^{\frac{1}{\beta-1}}(1-\beta)(\lambda-1)-\rho. \tag{43}$$

Comparing the (steady-state) equilibrium rate of economic growth g in (30) and first-best rate of economic growth g^* in (43) reveals that g can be higher or lower than g^*, depending on the values of the parameters $\{\zeta, \lambda, \beta, \rho\}$ in g^*. This implication, which is well known in the Schumpeterian growth model, is associated with various sources of R&D externalities. Specifically, a higher ζ or λ implies a worsening of the *surplus-appropriability problem* and a higher β implies a worsening of the *business-stealing effect*; both of these effects are a positive externality, making g^* exceed g. In contrast, a higher ρ implies a strengthening of the *intertemporal-spillover effect*, which is a negative externality, making g^* lower than g.

4.2 First-best policy instruments

In this subsection, we consider a combination of policy instruments (including the form(s) of subsidies and/or taxes) that the policymaker can use to replicate the first-best optimal outcome. A comparison between the steady-state equilibrium as shown in Subsection 3.1 and the socially optimal outcome as shown in Subsection 4.1 reveals that the inefficiencies in the decentralized setting arise from three layers of distortions as follows.

Monopoly pricing. The first distortion is present in the ratio of intermediate-goods expenditure and total outputs X_t/Y_t. This ratio equals the relative importance of intermediate goods in final production β in the social optimum where no policy interventions are involved, whereas it equals $\beta^2[(1+s_y)/(1-s_x)]$ in equilibrium where both the subsidy rates for final-goods production and the purchase of intermediate goods are involved. It can be seen that without these policy instruments (i.e., $s_y = s_x = 0$), the ratio X_t/Y_t in equilibrium is always lower than in the first-best outcome, producing the allocation inefficiencies. As shown in Acemoglu (2009), this distortion stems from the monopoly rights protected by patents to preserve incentives for inventors to create higher quality products. Thus, if these policy tools are set to satisfy the following condition:

$$\frac{1+s_y}{1-s_x} = \frac{1}{\beta}, \tag{44}$$

then this layer of monopolistic distortion will be eliminated.

R&D externalities. The second distortion is present in the allocation of aggregate R&D expenditures Z_t, which determines the arrival rate of innovations p (i.e., (29)) and the growth rate of aggregate quality (i.e., (30)) in the steady state. Given that the setting $(1+s_y)/(1-s_x) = 1/\beta$ holds the optimal ratio of X_t and Y_t, the R&D subsidy rate s_r is the feasible policy lever that can adjust the equilibrium level of aggregate spending on R&D. Specifically, when the value of s_r induces $g(s_r)|_{(1+s_y)/(1-s_x)=1/\beta} > (<) g^*$, a too high (low) level of R&D expenditures is made in the decentralized equilibrium, again producing allocation inefficiencies. As shown in Subsection 4.1, this distortion stems from the presence of different types of R&D externalities in the canonical Schumpeterian growth model (i.e., the inclusion of the surplus-appropriability problem, the business-stealing effect, and the intertemporal-spillover effect), and the wedge between g and g^* is determined by parameter values that represent the overall impact of these R&D externalities. Therefore, if the choice of the R&D subsidy policy is designed to satisfy $g(s_r)|_{(1+s_y)/(1-s_x)=1/\beta} = g^*$, then this layer of distortion also will be eliminated. This yields the first-best design of the R&D subsidy rate given by

$$s_r^* = \frac{1}{1 - \frac{\zeta\lambda}{\rho}\beta^{\frac{1}{\beta-1}}(1-\beta)(\lambda-1)}. \tag{45}$$

Notice that since g^* in (43) is nonnegative, s_r^* becomes negative, implying that the first-best R&D policy is to tax the aggregate research spending. This is because when $s_r = 0$, given that the optimal ratio of X_t and Y_t holds (i.e., $(1+s_y)/(1-s_x) = 1/\beta$), the steady-state growth rate g becomes always higher than the socially optimal counterpart g^*, meaning that the negative R&D externalities dominates the positive R&D externalities in the model. To remove this inefficiency, imposing an R&D tax (i.e., $s_r^* < 0$) helps increase the cost of research, thus reducing the research incentives and the resulting R&D level to equate the steady-state growth rate and the first-best growth rate.[12]

Consumption-leisure tradeoff. The third distortion is present in the supply of labor L_t, which determines the level of leisure in the steady state (i.e., $1 - L$). The first-best level of labor supply L^* equals $1/(1+\theta)$, depending only on the leisure preference θ, whereas the steady-state level of labor supply L is given by (32), depending on the subsidy rates s_y, s_x, s_r and the labor income tax rate τ, in addition to other parameters. Given that the setting $(1+s_y)/(1-s_x) = 1/\beta$ holds the optimal ratio of X_t and Y_t and that $s_r = s_r^*$ holds the optimal spending Z_t on R&D, there is one degree of freedom in the set of policy tools $\{s_y, s_x, s_r\}$ that can adjust the equilibrium level of labor supply L. Specifically, when the choice of $\{s_y, s_x, s_r\}$ induces $L(s_y, s_x, s_r) > (<) L^*$, too much (little) labor is supplied in the decentralized equilibrium, which also produces an allocation inefficiency. This distortion comes from the use of labor income taxes in the presence of elastic labor supply, and it is determined by the consumption-leisure decision in (3). Thus, if the policy mix $\{s_y, s_x, s_r\}$ is chosen to satisfy the condition such that

$$L(s_y, s_x, s_r) = L^* = \frac{1}{1+\theta}, \tag{46}$$

then the last layer of distortion will be removed and the social optimum can be attained accordingly. Notice that if labor is supplied inelastically instead (i.e., $\theta = 0$), then this distortion from the consumption-leisure tradeoff no longer exists. In this case, using only one lever of the policy mix $\{s_y, s_x\}$ together along s_r by taking either s_y or s_x as given is sufficient for remedying the distortions from monopoly pricing and R&D externalities.[13]

[12] As for the implications for optimal R&D policy in scale-invariant growth frameworks, in the model of Segerstrom (1998) with diminishing technological opportunities (DTO), either R&D taxes or subsidies are optimal for small-sized innovations, where R&D taxes are optimal for sufficiently large-sized innovations. However, in the model of Dinopoulos and Syropoulos (2007) with rent protection activities (RPA), R&D taxes are optimal for small- and large-seized innovations, and R&D subsidies are optimal only for medium-sized innovations. This result still applies to the model with both DTO and RPA, as shown in Şener (2008).

[13] For example, under $s_x = 0$, the first-best outcome is attained by setting $s_y = (1-\beta)/\beta$ and s_r to (45), respectively. Alternatively, under $s_y = 0$, the first-best outcome is attained by setting $s_x = 1 - \beta$ and s_r to (45), respectively.

Summarizing the above results yields the following proposition.

Proposition 2. *The economy can achieve the first-best outcome in equilibrium with an optimal mix of policy instruments $\{s_y, s_x, s_r\}$ determined by (44), (45), and (46).*

Proof. Proven in the text. □

This result is in a sharp contrast to that in Zeng and Zhang (2007). In their variety-expansion model with distortionary taxes and elastic labor supply, the subsidies to the production of outputs and the purchase of intermediate goods are equivalent in terms of their growth and welfare effects. Therefore, the two subsidy rates are combined to become an effective subsidy rate to production (i.e., s_f in their context), which reduces one degree of freedom in policy implementation. In their model, only two subsidy tools (i.e., the production subsidy and R&D subsidy) can be used to optimize the equilibrium allocations. In the presence of three distortions as described above, no first-best combination of policy tools $\{s_f, s_x\}$ exists to attain the social optimum.[14] Nevertheless, in our quality-ladder model, the subsidy rate to output production s_y and that to the user cost of intermediate goods s_x operates separately. Hence, a combination of subsidy tools $\{s_y, s_x, s_r\}$ suffices to eliminate all distortions in our model.[15]

5 Quantitative analysis

In this section, we calibrate the model to the US data to perform a quantitative analysis. First, we numerically evaluate the effects of three subsidy instruments in terms of growth maximization. Then, we quantify the effects of the subsidy instruments in terms of welfare maximization by considering (a) the case where a mix of all instruments is implemented, (b) the case where a single instrument is implemented, and (c) the case where a combination of any two instruments is implemented, respectively.

5.1 Calibration

To perform this numerical analysis, the strategy is to assign steady-state values to the following structural parameters $\{\rho, \beta, \zeta, \lambda, \theta, s_y, s_x, s_r\}$. We choose a standard value of 0.05 for the discount rate ρ. As for the production parameter, we calibrate the value of β by setting the markup ratio $P_t(\nu)/[\psi q_t(\nu)] = 1/\beta$ to 1.5, which is consistent with the markup values of the US economy considered in Hornstein (1993) and Devereux *et al.* (1996). As for the R&D productivity, we calibrate the value of ζ by following Zeng and Zhang (2007) to choose the average growth rate of GDP (i.e., g in (30)) in the US for the last 30 years, which has been roughly 3%, according to

[14] However, the social optimum might be attained in the model of Zeng and Zhang (2007) if the mix of policy instruments is expanded to include a consumption tax τ_c in addition to the manufacturing subsidy s_f and the R&D subsidy s_r.

[15] In the blocking-patents model of Yang (2018) with subsidization, given elastic labor supply and a lump-sum tax, the distortions from monopoly pricing and the consumption-leisure tradeoff are consolidated to one layer of distortion from the (inverse) supply of labor in manufacturing terms. In addition to the distortion from R&D externalities, by fixing the patent-policy regime, an optimal mix of production subsidy and R&D subsidy will suffice to help recover the first-best outcome.

the Conference Board Total Economy Database. As for the step size of quality improvement, we calibrate the value of λ by setting the time between arrivals of innovation $1/p$ to about 3 years, as in Acemoglu and Akcigit (2012). As for the leisure preference parameter, we calibrate the value of θ by matching the standard moment of labor supply L to $1/3$. Finally, given that the US has used an R&D subsidy but not the manufacturing subsidy (including subsides to the production of final goods and to the purchase of intermediate goods) in the past three decades, we choose the market values of $s_y = s_x = 0$ and follow Grossmann *et al.* (2013) to calibrate the value of s_r by targeting the current R&D subsidy rate in the US, which is approximately 6.6% (OECD (2009, 2013)). Table 1 summarizes the values of the parameters and variables in this quantitative exercise.

Table 1: Calibration

Targeted Moments						g 0.030	p 0.333	L 0.333
Parameters	ρ 0.050	β 0.667	ζ 1.063	λ 1.090	θ 1.845	s_y 0.000	s_x 0.000	s_r 0.066

5.2 Numerical results

Before proceeding to the cases in which the policy instruments are employed, this analysis starts from the comparisons in the growth rate and welfare level between the decentralized equilibrium in which realistic values are calibrated (i.e., the benchmark case) and an extreme scenario in which no policy tools are introduced (i.e., the no-policy case). The purpose of this exercise is to quantify the differences in growth and welfare of the equilibrium level in our model as compared to in the original quality-ladder model. The growth rates and welfare levels of these two cases are shown in Table 2. It can be seen that as compared to our benchmark case, the growth rate declines by 0.230% (percentage point) and the welfare level declines by 0.0825% (percent change) when all policy interventions are dismantled.[16] Notice that in this comparison, the equilibrium subsidy rates to final-goods production and the purchase of intermediate goods are identical in the benchmark case and in the no-policy case (i.e., $s_y = s_x = 0$). Therefore, the growth and welfare differences between the two cases are only driven by the presence of subsidies to R&D, which effectively stimulates the arrival rate of innovations in equilibrium, as shown in (29). This result tends to justify the use of R&D subsidies in promoting growth and raising welfare with the current US policy in the absence of the use of any manufacturing subsidies.[17]

Moreover, from the no-policy case to the benchmark case, the labor income tax rate τ rises from 0 to 4.14% to finance the use of R&D subsidies s_r, and the labor supply L rises slightly from

[16] See Appendix A.2 for the derivation of the steady-state welfare function. The welfare difference is expressed as the usual equivalent variation in consumption flow such that $\exp(\rho\Delta U) - 1$, where ΔU denotes the difference in the steady-state welfare.

[17] Note that the main focus of this exercise is not on the sizes of the growth-maximizing and welfare-maximizing subsidy/tax rates, but rather on the comparisons in the policy effectiveness for growth and welfare among the decentralized equilibrium and the outcomes with the optimal policy instrument(s).

0.3329 to 0.3333 in response. Corresponding to the discussion in Subsection 3.1, this result implies that the effect of s_r through the growth channel that stimulates the demand for production labor dominates the counterpart through the taxation channel that stifles the supply of labor.

Table 2: Growth and Welfare under the benchmark case and the no-policy case

Benchmark	g	U	L	τ
$(s_y = s_x = 0, s_r = 0.066)$	0.0300	-10.2996	0.3333	0.0414
No-policy	g	U	L	τ
$(s_y = s_x = s_r = 0)$	0.0277	-10.3161	0.3329	0.0000

5.2.1 Growth-maximizing subsidization

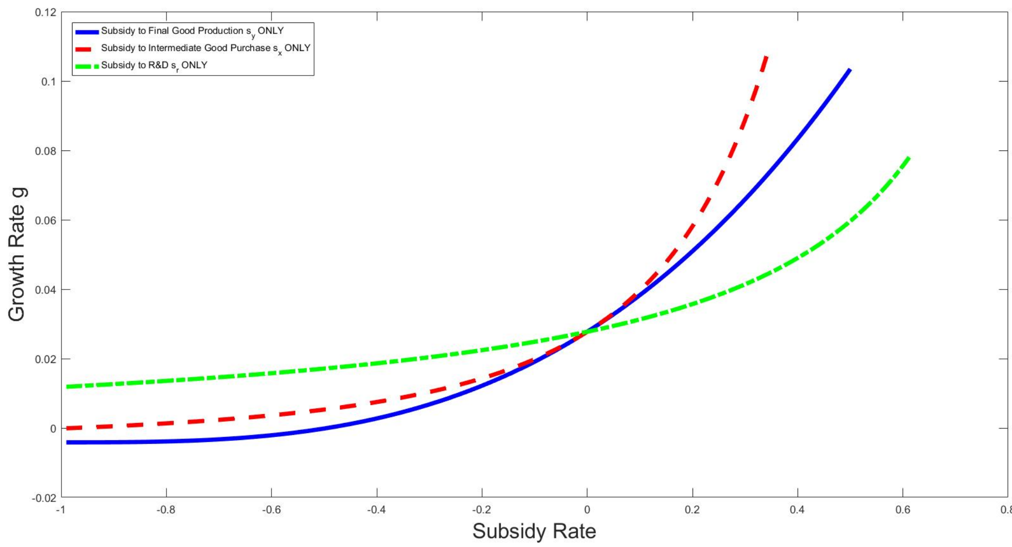
According to Lemma 1, a higher rate of subsidies to the production of final goods (i.e., s_y), the purchase of intermediate goods (i.e., s_x), and R&D (i.e., s_r) leads to a quantitatively identical effect on the steady-state rate of economic growth (namely, a higher g). Therefore, this subsection quantifies and compares the size of the effect of each subsidy instrument in terms of growth maximization.

First, we consider the case in which the three policy instruments are used. As shown in Table 3, the maximized rate of economic growth g is 10.76% and the growth-maximizing rates of subsidy are given by $s_y = 0.09$, $s_x = 0.27$, and $s_r = 0.05$, respectively. Since the level of R&D subsidy rate in the current case (i.e., $s_r = 0.05$) does not differ considerably from the counterpart in the benchmark case (i.e., $s_r = 0.066$), the significant increase by 7.76% (percentage point) in g is mainly driven by the use of subsidies in manufacturing s_y and s_x. It can be seen that among these growth-maximizing rates of subsidy, s_x is the largest whereas s_r is the smallest, implying that s_r tends to be less effective in enhancing economic growth than s_y and s_x. In other words, in this model, the benefits of innovations are much more sensitive to the increase in monopolistic profits (due to more production sales in final goods) rather than the reduction in research costs. In addition, as compared to the benchmark case, the labor income tax rate τ rises to 78.71% to finance the higher level of subsidy expenditure. However, the supply of labor increases dramatically to 0.9997; this may be due to the large growth effect that stimulates the demand for labor in manufacturing. Table 3 also reveals that using all subsidy tool will generate excess growth compared to the growth rate of 8.99% in the social planner's solution (which will be derived in Subsection 5.2.2).

Table 3: Growth maximization under a combination of all instruments

All subsidies	g	L	τ
$(s_y = 0.09, s_x = 0.27, s_r = 0.05)$	0.1076	0.9997	0.7871

Second, we consider the case in which only a single subsidy instrument is used. Figure 1 depicts the relationships between the growth rate g and each of the policy levers $\{s_y, s_x, s_r\}$, and it verifies the implication of Lemma 1 such that the growth rate g is monotonically increasing in each of the subsidy rates.[18] Notice that the pattern in Figure 1 demonstrates that in the presence of subsidization (i.e., $s_y > 0, s_x > 0, s_r > 0$), the growth effect of subsidies to the purchase of intermediate goods s_x (i.e., the red dotted line) is more significant than the growth effects of other two subsidies s_y and s_r (i.e., the blue solid line and the green dotted line, respectively). This



Notes: The range in consideration for s_y is $[-1, 0.5]$, for s_x is $[-1, 0.341]$, and for s_r is $[-1, 0.612]$, respectively.

Fig. 1. The relationship between the growth rate and a single subsidy rate

comparison in growth effectiveness is clearly shown in Table 4, which presents the situations of growth maximization for each subsidy tool. Specifically, under the growth-maximizing rate of 49.9% for subsidies to final goods production s_y (with a tax rate τ of 99.87%), the growth rate is 10.32% and the labor supply is 0.0261, whereas under the growth-maximizing rate of 34.1% for subsidies to the purchase of intermediate goods s_x (with a tax rate τ of 68.99%), the growth rate is 10.73% and the labor supply is 0.9848. Nevertheless, under the growth-maximizing rate of 61.2% for R&D subsidies s_r (with a tax rate τ of 99.87%), the growth rate is 7.8% and the labor supply is 0.078. This result implies that when only a single subsidy instrument is implemented to stimulate growth, subsidizing R&D (the purchase of intermediate goods) is least (most) effective, at the expense of the smallest (largest) taxation per wage rate (i.e., τL); this policy implication differs from the comparison in the growth effectiveness of subsidy instruments in Zeng and Zhang (2007) such that the R&D subsidy is more growth-enhancing than the other subsidies. Additionally, the above analysis justifies the fact in Table 3 that the growth-maximizing rate of s_x (s_r) is the largest (smallest) among the three subsidy tools if the choice of all tools becomes available.

Finally, it is interesting to see how the growth effect changes when a mix of any two subsidy instruments is used. Table 5 displays the growth-maximization solutions for three different combinations of the subsidy rates accordingly. It can be seen that the three strategies of pol-

[18] In this numerical analysis, to ensure that the consumption level is positive and that the labor supply is bounded between 0 and 1, we restrict the range of s_y to $[-1, 0.5]$, of s_x to $[-1, 0.341]$, and of s_r to $[-1, 0.612]$, respectively.

Table 4: Growth maximization under a single instrument

	g	L	τ
Subsidies to production of final goods ($s_y = 0.4990$, $s_x = 0$, $s_r = 0$)	0.1032	0.0261	0.9987
Subsidies to purchase of intermediate goods ($s_y = 0$, $s_x = 0.3410$, $s_r = 0$)	0.1073	0.9848	0.6899
R&D subsidies ($s_y = 0$, $s_x = 0$, $s_r = 0.6120$)	0.0780	0.0198	0.9987

icy combinations produce similar rates of growth; they are all higher than the growth rates by using a single subsidy tool but lower than the growth rate by using the three tools together. Notice that the policy combinations with subsidization to R&D generate lower growth rates than the one without it, which confirms the previous finding that s_r is the least effective to enhance growth. In particular, subsidizing the mix of final-goods production and R&D with $s_y = 47.5\%$ and $s_r = 5\%$ yields the lowest growth-maximizing rate of 10.35% (with a tax rate of 99.98%), whereas subsidizing the manufacturing factors with $s_y = 42\%$ and $s_x = 6.5\%$ yields the highest growth-maximizing rate of 10.75% (with a tax rate τ of 98%).

Table 5: Growth maximization under a combination of two instruments

	g	L	τ
Subsidies to manufacturing ($s_y = 0.4200$, $s_x = 0.0650$, $s_r = 0$)	0.1075	0.9999	0.9800
Subsidies to production of final goods and R&D ($s_y = 0.4750$, $s_x = 0$, $s_r = 0.0500$)	0.1035	0.0033	0.9998
Subsidies to purchase of intermediate goods and R&D ($s_y = 0$, $s_x = 0.3300$, $s_r = 0.0500$)	0.1074	0.9918	0.7071

5.2.2 Welfare-maximizing subsidization

Optimizing a mix of all instruments. The analysis now quantifies the implications of welfare-maximizing subsidies when the three policy instruments $\{s_y, s_x, s_r\}$ are implemented, as displayed in Table 6. Using our benchmark calibration, the optimal mix of all subsidy instruments is given by $s_y^* = 0.395$, $s_x^* = 0.07$, and $s_r^* = -0.745$. Recalling Proposition 2, the use of this optimal mix of subsidy rates induces the decentralized equilibrium to achieve the first-best solution. Intuitively, the first-best outcome in this model is restored by adjusting three policy levers to remedy the three distortions occurring in the decentralized equilibrium. First, given $s_y = s_x = 0$ in equilibrium, the fraction $(1 + s_y)/(1 - s_x) = 1$ implies that the subsidy rate to the production of final goods is less compatible with the subsidy rate to the purchase of intermediate goods in the sense that $(1 + s_y)/(1 - s_x)$ is smaller than its optimal value $1/\beta$. This is the first inefficiency stemming from the distorted ratio of intermediate-goods expenditure and total outputs X_t/Y_t in equilibrium in which subsidies to manufacturing are absent. Second, there is a layer of inefficiency stemming from the allocation on the aggregate R&D spending, because with the suboptimal subsidy rate to R&D (i.e., $s_r = 0.066$), the equilibrium growth rate of $g = 0.030$ is substantially lower than the socially optimal one $g^* = 0.0575$. Third, there is another layer of inefficiency stemming from the supply of labor, since in the presence of distortionary labor income tax τ, the suboptimal subsidy rates $\{s_y, s_x, s_r\}$ yield a level of labor supply at $L = 0.333$ in equilibrium, which is smaller than the socially optimal level at $L^* = 0.3530$. Thus, when the subsidy rates to manufacturing are raised and the subsidy rate to R&D is lowered to their first-best levels $\{s_y^*, s_x^*, s_r^*\}$, respectively, the above three layers of distortions are eliminated by reallocating the resources in the use of final goods and in the supply of labor. As a result of correcting inefficiencies, the increment in welfare from the decentralized equilibrium to the social optimum is very considerable (i.e., approximately 75.055% of consumption).

Table 6: Welfare maximization under a combination of all instruments

All subsidies	g^*	U^*	L^*	τ^*
($s_y^* = 0.395$, $s_x^* = 0.07$, $s_r^* = -0.745$)	0.0575	0.8990	0.3530	0.7391

Two interesting points around the above results are worthwhile discussing. First, notice that the first-best rate of R&D subsidy s_r^* becomes negative (i.e., an R&D tax) under the benchmark calibration. This is because when the subsidy rates to manufacturing are fixed at their first-best levels (i.e., $s_y^* = 0.3950$ and $s_x^* = 0.0700$), the equilibrium outcome with no R&D subsidies (i.e., $s_r = 0$) will generate a higher growth rate to the economy as compared to the social optimum (i.e., $g = 0.1035 > g^* = 0.0575$). Hence, to depress the equilibrium R&D, the level of s_r has to be lowered to be smaller than 0. The mechanism for the first-best rate of R&D subsidy to be negative in this model is similar to the mechanism for the first-best rule of profit division to be positive in Yang (2018), who considers the effects of blocking patents in a quality-ladder model; a negative R&D subsidy rate and a positive profit-division rule play the same role in mitigating the R&D level and the growth rate. In other words, achieving the social optimum in quality-ladder models requires a policy instrument that can be growth-depressing, since the class of quality ladder models features R&D externalities (e.g., the business-stealing effect) that could lead to suboptimally excess growth. This is in a sharp contrast to the policy analysis of subsidization in variety-expansion models. For example, in Barro and Sala-I-Martin (2003) with inelastic labor and lump-sum taxes, using a subsidy to the production of final goods or to the purchase of intermediate goods alone, either of which is growth-enhancing, can induce the decentralized equilibrium to achieve the social optimum, given that the equilibrium growth rate in their setting is lower than the socially optimal growth rate. In Zeng and Zhang (2007) who consider the model of Barro and Sala-I-Martin (2003) with elastic labor and distortionary taxes, a combination of analogous subsidy tools cannot achieve the first-best outcome.

Additionally, the majority of welfare improvements moving from the decentralized equilibrium to the social optimum stems from remedying the distortion in monopoly pricing. This can be seen in the following policy experiment that decomposes the distortions present in the current framework. Consider an intermediate case where the subsidy rates are given by $s_y = 0$, $s_x = 0.0669$, and $s_r = 0.3642$, and the resulting level of welfare is $U = -10.2063$. In this case, the equilibrium growth rate g and the equilibrium labor supply L attain their first-best levels of

g^* and L^*, respectively. Therefore, given that the ratio $(1+s_y)/(1-s_x)$ in the intermediate case (which is 1.0717) does not drastically differ from its counterpart in the benchmark case (which is unity), the welfare difference between the benchmark case and the intermediate case mainly stems from the distortions in R&D externalities and the consumption-labor tradeoff, denoted by $\xi_1 = 0.468\%$. In addition, it is straightforward to see that the welfare difference between the intermediate case and the first-best case stems from the distortion in monopoly pricing, denoted by $\xi_2 = 74.240\%$. Accordingly, it is obvious that the magnitude of ξ_2 is much more significant than that of ξ_1. This finding is consistent with Nuño (2011) and Yang (2018), both of whom argue that most of the welfare losses in the decentralized equilibrium of R&D-based growth models are attributed to the presence of suboptimal choices of policy tools that affect the resource allocation in the monopolistic intermediate-goods sector.

Sensitivity. To examine the sensitivity of the above numerical analysis, we consider the following exercises with respect to the structural parameters $\{\beta, \rho, \lambda, \zeta, \theta\}$. First, we vary the value of β to 0.6849 such that the implied markup ratio $1/\beta = 1.46$ is consistent with the average level of the empirical estimates in De Loecker *et al.* (2018) for US firms during 1996–2005. Second, the values of $\{\rho, \lambda, \zeta\}$ are changed so that the resulting optimal growth rate g^* is maintained at the rate of 0.0501, which is the value generated by setting β alone to 0.6849 in the first sensitivity exercise.[19] Additionally, we recalibrate the model by using the leisure preference parameter $\theta = 2$. Table 7 presents the welfare level U^*, the growth rate g^*, the labor supply L^*, the consumption level C_0^*, and the tax rate τ^*, respectively, under the alternative sets of structural parameters.

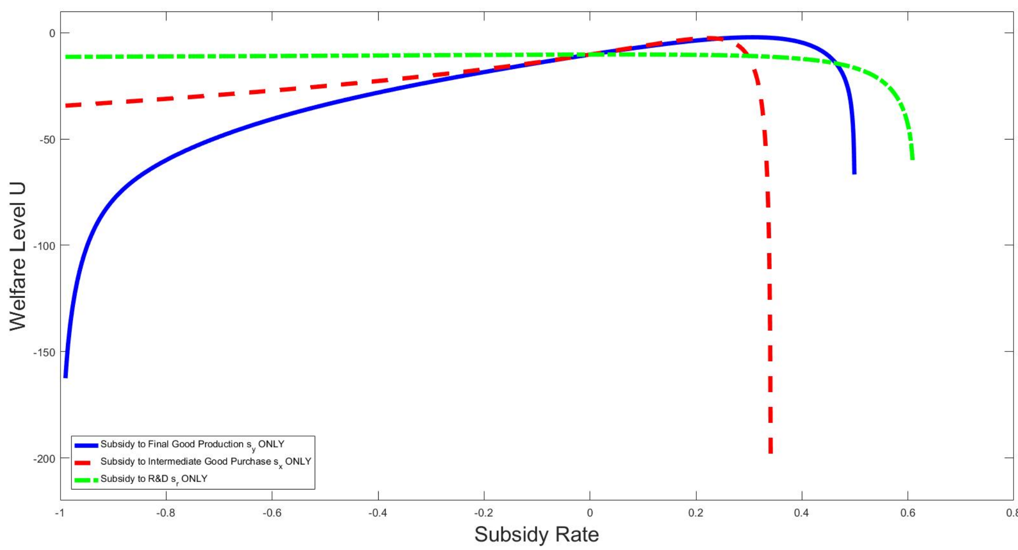
It can be seen that, the qualitative pattern and the quantitative magnitude of the main results are quite robust. First, the optimal subsidy rate to output production s_y^* continues to be the largest among the policy instruments, whereas the optimal subsidy rate to R&D s_r^* is still negative. This result continues to imply that s_y^* and s_x^* are the two subsidy rates that eliminate most of the inefficiencies. Second, under the new parameter settings of ζ, λ, ρ and θ, the welfare level U^* declines, as compared to the counterpart in the benchmark first-best case as shown in Table 6. The reason is as follows. A lower ζ or λ (and also a higher ρ or θ) raises C_0^* and reduces L^*, yielding a positive effect on U^*. Furthermore, this parameter change decreases g^*, yielding a negative effect on U^*. The latter effect outweighs the former effect, leading the overall level of U^* to decrease. Nevertheless, under a higher value of β, the considerable rise in C_0^* generates a sufficiently positive welfare effect, in addition to the positive welfare effect brought by the reduction in L^*, to dominate the negative welfare effect from the decrease in g^*. Hence, the resulting level of welfare U^* in this case increases as compared to the benchmark first-best case.

Optimizing a single instrument. Then, we consider the cases in which only one single instrument is used. Figure 2 depicts the relationships between the welfare level U and each of policy levers $\{s_y, s_x, s_r\}$. It can be seen that the welfare level U exhibits an inverted-U shape for each of the subsidy rates. In addition, Table 8 presents the details on welfare maximization by optimizing each subsidy tool. Specifically, the welfare-maximizing rate of 30.70% for subsidies to final goods production s_y (with a tax rate τ of 48.32%) yields a growth rate of 6.70%, a welfare level of -2.1760,

[19]Notice that from (43), the parameter values of $\{\beta, \rho, \lambda, \zeta\}$ determine the socially optimal growth g^*. Given that g^* is nonlinear in β, it is technically convenient to firstly set the alternative value of β, yielding a new value of g^*. Then the alternative values of other values are set to match this implied first-best growth rate in order to compare the welfare effects of these parameters in removing distortions.

Table 7: Sensitivity checks

$\beta = 0.6849$	U^*	g^*	L^*	C_0^*	τ^*
$(s_y^* = 0.4038,\ s_x^* = 0.0385,\ s_r^* = -0.8452)$	1.2208	0.0501	0.3339	0.8251	0.6711
$\zeta = 0.9891$	U^*	g^*	L^*	C_0^*	τ^*
$(s_y^* = 0.4422,\ s_x^* = 0.0385,\ s_r^* = -0.8452)$	-0.6820	0.0501	0.3339	0.7502	0.6799
$\lambda = 1.0838$	U^*	g^*	L^*	C_0^*	τ^*
$(s_y^* = 0.4459,\ s_x^* = 0.0360,\ s_r^* = -0.8542)$	-0.6776	0.0501	0.3349	0.7524	0.6793
$\rho = 0.0574$	U^*	g^*	L^*	C_0^*	τ^*
$(s_y^* = 0.4382,\ s_x^* = 0.0412,\ s_r^* = -0.9600)$	-1.6970	0.0501	0.3316	0.7967	0.6602
$\theta = 2$	U^*	g^*	L^*	C_0^*	τ^*
$(s_y^* = 0.4462,\ s_x^* = 0.0358,\ s_r^* = -0.7433)$	-0.4325	0.0576	0.3181	0.6654	0.7001



Notes: The range in consideration for s_y is $[-1, 0.5]$, for s_x is $[-1, 0.341]$, and for s_r is $[-1, 0.612]$, respectively.

Fig. 2. The relationship between the welfare level and a single subsidy rate

and a labor supply of 0.3240, whereas the welfare-maximizing rate of 22.30% for subsidies to the purchase of intermediate goods s_x (with a tax rate τ of 38.27%) yields a growth rate of 6.38%, a welfare level of -2.5863, and a labor supply of 0.4117. However, the welfare-maximizing rate of 6.00% for R&D subsidies s_r (with a tax rate τ of 3.74%) yields a growth rate of 2.98%, a welfare level of -10.2995, and a labor supply of 0.3333. This result of welfare maximization implies that when only a single subsidy instrument is adopted to promote welfare, subsidizing R&D (the production of final goods) is least (most) effective, at the expense of the smallest (largest) taxation per wage rate τL. The reason is as follows. Most of inefficiencies in this model are realized by correcting the distortions in monopoly pricing and the consumption-leisure tradeoff, mainly through the use of s_y and s_x. Therefore, when the subsidy rate is implemented alone, s_y and s_x tend to be more welfare-effective than s_r, given that the latter is used to mainly correct the distortion brought by R&D externalities.[20] [21] This policy implication again differs from the comparison in the welfare effectiveness of subsidy instruments in Zeng and Zhang (2007) such that the R&D subsidy can be more welfare-improving than the other subsidies.

Table 8: Welfare maximization under a single instrument

	U	L	τ
Subsidies to production of final goods ($s_y = 0.3070$, $s_x = 0$, $s_r = 0$)	-2.1760	0.3240	0.4832
Subsidies to purchase of intermediate goods ($s_y = 0$, $s_x = 0.2230$, $s_r = 0$)	-2.5963	0.4117	0.3827
R&D subsidies ($s_y = 0$, $s_x = 0$, $s_r = 0.0600$)	-10.2995	0.3333	0.0374

Optimizing a mix of two instruments. We also consider a policy experiment in which a combination of any two instruments is optimized. Table 9 reports the quantitative implications of the optimal mix of any two subsidy tools under the benchmark parameterization. The relationships between a combination of two subsidy tools and the welfare level are depicted in Figures 3, 4, 5, respectively, for an easier view from *different* angles. Specifically, subsidies to manufacturing (namely the combination of s_y and s_x) yield a welfare level of -2.0785, which is lower than the welfare level under the combination of s_x and s_r (i.e., -0.1520) and under the combination of s_y and s_r (i.e., 0.7471).

It is obvious that an optimal mix of two subsidy rates is substantially welfare-improving than the decentralized equilibrium and the outcomes optimizing a single subsidy. For example, the welfare level under the combination of s_y and s_x, which is the lowest one among the three combinations of any two subsidies, is higher than the welfare level under the decentralized equilibrium by 50.841% of consumption. Moreover, the welfare level under the combination of s_y and s_x is higher than the welfare level under optimizing s_y alone, which is the largest one among the outcomes using a single subsidy, by 0.489% of consumption. Nevertheless, the welfare level under an optimal mix of two subsidy rates is much lower than the counterpart under the optimal combination of three subsidy rates (i.e., the social optimum).[22]

Notice that among the three combinations of any two policy instruments, the combinations with subsidies to R&D (namely either subsidizing the production of final goods s_y and R&D s_r or subsidizing the purchase of intermediate goods s_x and R&D s_r) are more welfare-enhancing than subsidies to manufacturing only (i.e., the mix of subsidies to output production s_y and the purchase of intermediate goods s_x). This finding is in line with the argument in Zeng and Zhang

[20] The welfare improvements from the decentralized equilibrium to the outcomes by optimizing the subsidy to output production and to the purchase of intermediate goods are significantly large (which are 50.1073% and 46.9857% of consumption, respectively), whereas the counterpart by optimizing R&D subsidy alone is marginally small (which is roughly 0.001% of consumption).

[21] Similar to the analysis of growth maximization, this welfare-maximization analysis justifies our findings in Table 6 such that the welfare-maximizing rate of s_y (s_r) is the largest (smallest) among the three subsidy tools if the choice of all tools becomes available.

[22] The welfare level under the combination of s_y and s_x, the combination of s_x and s_r, and the combination of s_y and s_r, is apparently lower than the welfare level under the socially optimal outcome by 160.53%, 5.3955%, and 0.7624% of consumption, respectively.

Table 9: Welfare maximization under a combination of two instruments

Subsidies to manufacturing	U	L	τ
($s_y = 0.2300$, $s_x = 0.0600$, $s_r = 0$)	-2.0785	0.3525	0.6461
Subsidies to production of final goods and R&D	U	L	τ
($s_y = 0.4950$, $s_x = 0$, $s_r = -0.7300$)	0.7471	0.3173	0.7309
Subsidies to purchase of intermediate goods and R&D	U	L	τ
($s_y = 0$, $s_x = 0.3100$, $s_r = -0.7100$)	-0.1520	0.4352	0.2271

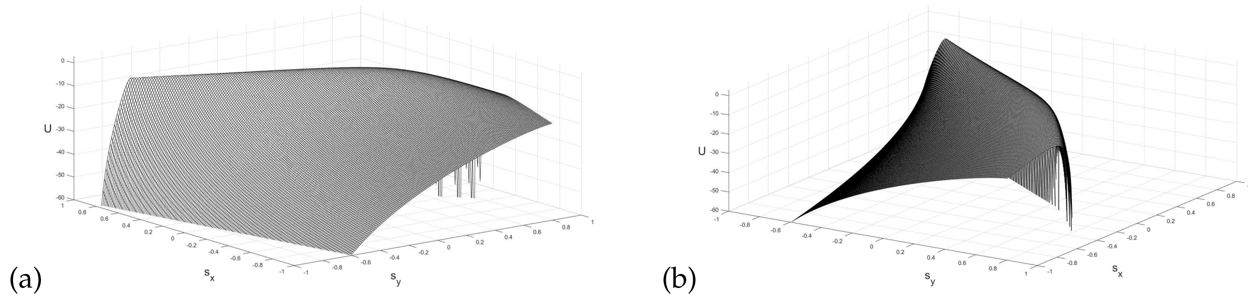


Fig. 3. Welfare effect of subsidies to manufacturing

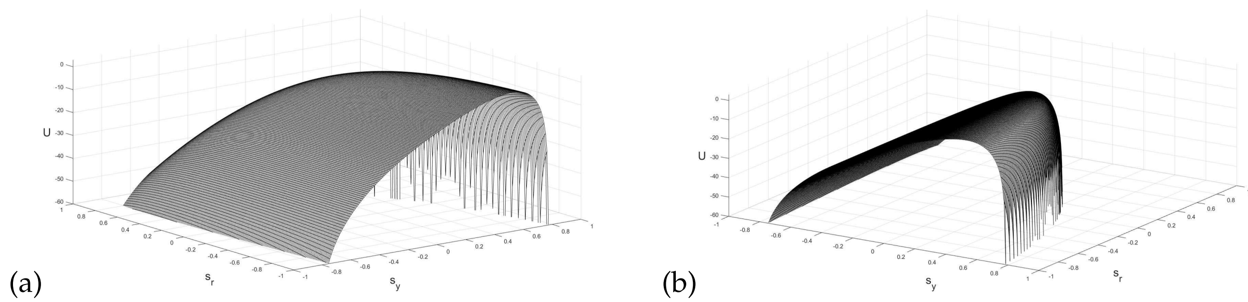


Fig. 4. Welfare effect of subsidies to production of final goods and R&D

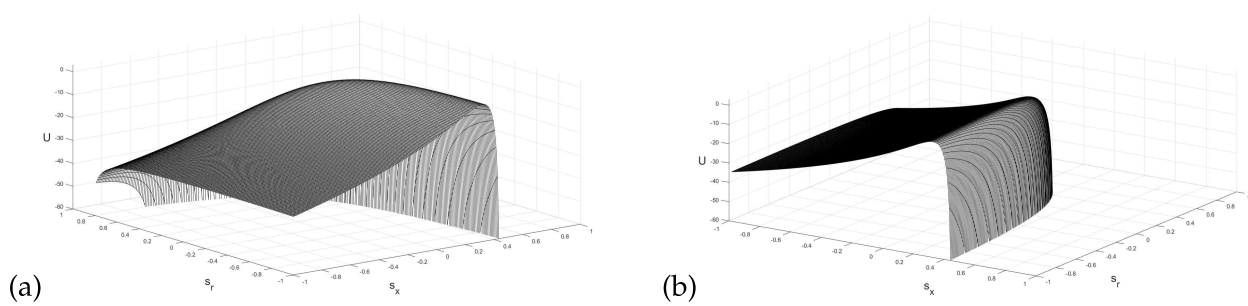


Fig. 5. Welfare effect of subsidies to purchase of intermediate goods and R&D

(2007) such that the joint use of subsidies is to take advantage of their relative strength in correcting different types of distortions. Specifically, subsidies to manufacturing tend to be more effective in eliminating the distortions from monopoly pricing and the consumption-leisure tradeoff, both of which are considered as the *static* efficiency losses, whereas subsidies to R&D tend to be more effective in eliminating the distortion from R&D externalities, which is considered as the *dynamic* efficiency losses. As a result, mixing the subsidies that remedy different types of inefficiencies (i.e., both static and dynamic inefficiencies) does better than mixing the subsidies that remedy the same type of inefficiencies (i.e., only the static inefficiency).

6 Conclusion

In this study, we explore the growth and welfare implications of a subsidization-policy regime in a quality-ladder endogenous growth model with elastic labor supply, where subsidies are financed by distortionary labor income taxes. This subsidization regime includes three policy instruments: subsidies to the production of final goods, subsidies to the purchase of intermediate goods, and subsidies to R&D. In this model, the equilibrium allocations are subject to three layers of distortions, namely, the distortion on the monopoly pricing, the consumption-leisure tradeoff, and the R&D externalities. Therefore, the policymaker can adjust the equilibrium allocations to mitigate these distortions by properly implementing the subsidy tools.

The result in the current study differs substantially from those in the literature. In the presence of lump-sum taxes, Barro and Sala-I-Martin (2003) show that in a variety-expansion model with inelastic labor supply, the social optimum can be attained by subsidizing manufacturing (through either the production of final output or the purchase of intermediate products), whereas the analysis of Acemoglu (2009) (and Yang (2018)) implies that in a quality-ladder model with inelastic (elastic) labor supply, the social optimum can be achieved by subsidizing manufacturing and research together. With elastic labor supply and distorionary taxes, Zeng and Zhang (2007) show that in a variety-expansion model, the social optimum cannot be restored by using a single type of subsidies or their combination. Nevertheless, under a similar setting of labor and taxes as in Zeng and Zhang (2007), our result shows that in a quality-ladder model, the mix of subsidy instruments to the production of final goods, the purchase of intermediate goods, and research is able to replicate the first-best optimal outcome by correcting all distortions occurring in the decentralized equilibrium. Specifically, subsidies to manufacturing tend to remove the static distortions on monopoly pricing and the consumption-leisure tradeoff, whereas subsidies to R&D tend to remove the distortion on R&D externalities. Therefore, the process of innovation is crucial in determining the possibility of which the social optimum is attained in a decentralized equilibrium with the aid of subsidies.

To quantify the effectiveness of the subsidy tools in promoting economic growth and raising social welfare, this model is calibrated to the US data to perform a numerical analysis on growth maximization and welfare maximization. First, the use of more types of subsidies ameliorates the effects on maximizing growth and welfare. Second, as for the use of a single instrument, we find that the R&D subsidy is less growth-enhancing and welfare-improving than the other subsidies. Finally, as for the use of a mix of any two instruments, subsidizing final-goods production and the purchase of intermediate goods is most effective in promoting growth but least effective in raising welfare. These quantitative results in the comparisons of the growth and welfare effectiveness among subsidy tools differ significantly from some existing literature (e.g., Zeng and Zhang (2007)) showing that R&D subsidy is more growth-enhancing and welfare-improving than the other types of subsidies. Although subsidizing R&D investment is the common practice as observed in many industrial countries (such as the US), the present study provides an important policy implication on extending the dimensionality of the fiscal-policy (or industrial-policy) system by increasing the number of subsidy/tax tools, given that the mechanisms of these dimensions work differently in allocating resources and eliminating inefficiencies.

Appendix A

A.1 Proof of Proposition 1

In this proof, we examine the stability of this model given a stationary path of $s_{y,t}$, $s_{x,t}$, and $s_{r,t}$. First, define the transformed variable $\Omega_t \equiv C_t/Y_t$. Then, taking the log of Ω_t and differentiating it with respect to time yields

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} \tag{A.1}$$

From (21), the asset-market clearing condition is given by

$$A_t = \int_0^1 V_t(\nu)d\nu = \frac{(1-s_r)}{\zeta\lambda} Q_t L_t, \tag{A.2}$$

and recalling from (14), the aggregate final-goods production function is given by

$$Y_t = \frac{1}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} Q_t L_t. \tag{A.3}$$

Using (A.2) and (A.3), we obtain

$$Y_t = \frac{1}{\beta}\left(\frac{1+s_y}{1-s_x}\right)^{\frac{\beta}{1-\beta}} \left(\frac{\zeta\lambda}{1-s_r}\right) A_t = \chi_1 A_t, \tag{A.4}$$

where $\chi_1 \equiv [(\zeta\lambda/\beta)/(1-s_r)][(1+s_y)/(1-s_x)]^{\beta/(1-\beta)} > 0$ is a composite parameter. Hence, (A.4) implies

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t}. \tag{A.5}$$

In addition, using the household's budget constraint (2) and (A.5) yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} = r_t + (1-\tau_t)\frac{W_t L_t}{A_t} - \frac{C_t}{A_t} = r_t + (1-\tau_t)\frac{\chi_1 W_t L_t}{Y_t} - \frac{\chi_1 C_t}{Y_t}. \tag{A.6}$$

Applying the definition of Ω_t and the Euler equation (4), we have

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = -\rho - (1-\tau_t)\frac{\chi_1 W_t L_t}{Y_t} + \chi_1\Omega_t = \chi_1\Omega_t - (1-\tau_t)\chi_1(1-\beta) - \rho, \tag{A.7}$$

where the last equality uses the labor share of output such that $W_t L_t / Y_t = (1 + s_y)(1 - \beta)$ in (8).

Next, to derive the relationship between τ_t and Ω_t, we combine (5) and (9) to obtain the ratio of aggregate expenditures on intermediate goods to final goods:

$$X_t = \beta^2 \left(\frac{1 + s_y}{1 - s_x} \right) Y_t. \tag{A.8}$$

Moreover, substituting (8) and (A.8) into the government budget constraint $\tau_t W_t L_t = s_y Y_t + s_x X_t + s_r Z_t$ in (22) yields the ratio of aggregate expenditures on research to final goods:

$$Z_t = \frac{Y_t}{s_r} \left[\tau_t (1 + s_y)(1 - \beta) - s_x \beta^2 \left(\frac{1 + s_y}{1 - s_x} \right) - s_y \right]. \tag{A.9}$$

Then, using (A.8) and (A.9) in the final-goods resource constraint $Y_t = C_t + X_t + Z_t$, a few steps of manipulation yield

$$\tau_t = \underbrace{\frac{s_r}{(1 + s_y)(1 - \beta)} \left[1 + \frac{\beta^2 \left(\frac{1 + s_y}{1 - s_x} \right) (s_x - s_r) + s_y}{s_r} \right]}_{\equiv \chi_2} - \frac{s_r \Omega_t}{(1 + s_y)(1 - \beta)}, \tag{A.10}$$

where χ_2 is a composite parameter.

Finally, substituting (A.10) into (A.7) yields a one-dimensional differential equation in Ω_t:

$$\frac{\dot{\Omega}_t}{\Omega_t} = (1 - s_r)\chi_1 \Omega_t - \chi_1 (1 - \chi_2)(1 + s_y)(1 - \beta) - \rho. \tag{A.11}$$

Therefore, given that Ω_t is a control variable and that its coefficient in (A.11) is positive (i.e., $(1 - s_r)\chi_1 = (\zeta \lambda / \beta)[(1 + s_y)/(1 - s_x)]^{\beta/(1-\beta)} > 0$), the dynamics of Ω_t is characterized by saddle-point stability such that Ω_t jumps immediately to its steady-state value given by

$$\Omega = \frac{\chi_1 (1 - \chi_2)(1 + s_y)(1 - s_x) + \rho}{(1 - s_r)\chi_1}, \tag{A.12}$$

where the parameter space is restricted to ensure $\Omega > 0$. Given the stationary Ω, (A.10) immediately follows that $\tau_t = \tau$ is also time invariant, and then X_t / Y_t from (A.8) and Z_t / Y_t from (A.9) are both stationary. It then implies that variables $\{Y_t, C_t, X_t, Z_t\}$ share the identical growth rate.

We now prove that L_t must be stationary as well. Dividing the both sides of (3) by Y_t, we have

$$(1 - \tau) \left(\frac{W_t}{Y_t} - \frac{W_t L_t}{Y_t} \right) = \theta \Omega. \tag{A.13}$$

Inserting (8) and (A.10) into (A.13) yields

$$\frac{W_t}{Y_t} = \frac{\theta \Omega + (1 + s_y)(1 - \beta)(1 - \chi_2) + s_r \Omega}{1 - \chi_2 + s_r \Omega / [(1 + s_y)(1 - \beta)]}, \tag{A.14}$$

which implies

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{Y}_t}{Y_t}. \tag{A.15}$$

Taking the log of (8) and differentiating the resulting equation with respect to time, together with (A.15), yields

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{W}_t}{W_t} = 0. \tag{A.16}$$

Therefore, $L_t = L$ must be stationary holding constant s_y, s_x and s_r. Moreover, from (16), we eventually have

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{X}_t}{X_t} = \frac{\dot{Z}_t}{Z_t}. \tag{A.17}$$

A.2 Derivation of the steady-state welfare function

The steady-state welfare function is obtained by imposing the BGP in the utility function (1). Integrating it yields

$$U_0 = \frac{1}{\rho}\left[\ln C_0 + \frac{g}{\rho} + \theta \ln(1-L)\right], \tag{A.18}$$

where C_0 is the initial level of consumption. Using (3) and (16), C_0 can be reexpressed as follows:

$$C_0 = \left(\frac{1-\tau}{\theta}\right)(1+s_y)^{\frac{1}{1-\beta}}\left(\frac{1}{1-s_x}\right)^{\frac{\beta}{1-\beta}}\left(\frac{\beta}{1-\beta}\right)Q_0(1-L), \tag{A.19}$$

where Q_0 is the initial level of aggregate quality. Then, substituting (A.19) into (A.18) yields

$$U = \frac{1}{\rho}\left\{\begin{array}{l} \ln(1-\tau) - \ln\theta + \left(\frac{1}{1-\beta}\right)\ln(1+s_y) + \left(\frac{\beta}{1-\beta}\right)\ln\left(\frac{1}{1-s_x}\right) + \ln\frac{\beta}{1-\beta} \\ + \left[\frac{\zeta}{\rho}(\lambda-1)(1-\beta)\left(\frac{1+s_y}{1-s_x}\right)^{\frac{1}{\beta}} - \frac{\lambda-1}{\lambda}\right] + (1+\theta)\ln(1-L), \end{array}\right\}, \tag{A.20}$$

where the exogenous terms have dropped and τ and L are given by (31) and (32), respectively. Hence, given that τ and L are functions of the policy instruments $\{s_y, s_x, s_r\}$, U is also a function of the policy instruments $\{s_y, s_x, s_r\}$.

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