Equalized Factor Price and Integrated World Equilibrium

Guo, Baoping

Individual Researcher

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Baoping Guo[1]

Fairfax, VA, USA

Abstract – Dixit and Norman provided a remarkable result for the factor price equalization, as that world prices for commodities and equalized factor prices remain the same when the allocation of factor endowments of two countries changes within the parallelogram formed by the rays of diversification cone in Integrated World Equilibrium (IWE) diagram. What structure are for those prices? Why are the prices not touchable or visible even we knew that the entire parallelogram shares the same prices for giving two countries factor endowments? This paper explored the equalized factor prices and general trade equilibrium that embedded in the Dixit-Norman IWE diagram. The study demonstrated that the endogenous factor prices equalized are the function of world factor endowments. Moreover, the equalized price makes sure that countries participating in free trade gain from trade. This result is helpful for the studies of factor price non-equalization when countries have different productivities.

Keywords:
Factor content of trade; factor price equalization; General equilibrium of trade; Integrated World Equilibrium; IWE

1. Introduction

The Heckscher-Ohlin theorem and the factor-price equalization (FPE) theorem paved the road toward general trade equilibrium. The general equilibrium of trade and the FPE are the same issues from different angles. The Integrated World Equilibrium (Dixit and Norman, 1980) is remarkable to illustrate the FPE by trade equilibriums. It displayed the property of the FPE with mobile factors. Helpman and Krugman (1985) normalize the assumption of integrated equilibrium, which presented equilibrium analyses in a simple way. Deardorff (1994) derived the conditions of the FPE for many goods, many factors, and many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments within lenses identified.

Woodland (2013, pp39) described the importance of the general equilibrium, "General equilibrium has not only been important for a whole range of economics analyses, but especially so for the study of international trade". Deardorff (1984, pp685) said, "A trade equilibrium is somewhat more complicated". The Heckscher-Ohlin theories still do not achieve this important goal, even for the simplest $2 \times 2 \times 2$ model.

[1] Former faculty member of The College of West Virginia (renamed as Mountain State University), auther Email address: bxguo@yahoo.com.

The one focus of studies on the general equilibrium for constant returns and perfect competition is by the social utility function and direct and indirect trade utility function (offer curve). It is not easy neither for this approach to get a complete price-trade equilibrium. It did provide a framework for the solution of the equilibrium from consumption.

The factor price equalization theorem is with a fundamental influence, with long and involved discussion in the many works of literature. The FPE could imply the trade equilibrium and the Heckscher-Ohlin theorem. The FPE in the IWE does provide some hints on the price-trade equilibrium. It sets a reference for the solution equalized prices. If an analytical solution of prices do not with the mobile property of the Dixit-Norman prices, it is not a qualified solution. This study found that behind the mobile factor price equalization (PFE), there is a clear relationship of price-trade equilibrium that embedded in the IWE. It shows why the IWE is so correct and accepted widespread.

What determines factor price equalization is not an out of day topic. Trefler (1993) demonstrated that factor price equalization and the HOV theorem hold for his equivalent-productivity model. Fisher (2011) mentioned the Heckscher-Ohlin theory is valid for his model mapped from the virtual factor endowments. Theoretically, the analyses of factor price non-equalization (localization) based on the price structure of the equalized prices.

This study explored that the equalized factor prices and world commodity prices at the equilibrium are the functions of the world factor endowments. The result is consistent with the insight price inference that Dixit and Norman made four decades ago. The study also derived the autarky prices and illustrated that the equalized factor prices ensure gains from trade for countries participating in the trade.

This study is divided into five sections. Section 2 introduces the solution of price–trade equilibrium by the IWE diagram. Section 3 provides a way to estimate autarky prices. The logic is that the autarky factor endowments determine autarky prices. Section 4 presents the equilibrium for cases of two factors, two commodities, and multiple countries. Section 5 is a related discussion.

2. The Price-Trade Equilibrium by Geometric Analyses within The IWE

We take the following normal assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals, and (7) full employment of factor resources. We denote the Heckscher-Ohlin model as follows. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \qquad \text{(2-1)}$$

where A is the 2×2 technology matrix (the matrix of direct factor inputs), X^h is the 2×1 vector of commodities of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular. The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \tag{2-2}$$

where W^h is the 2×1 vector of factor prices, its elements are r rental for capital and w wage for labor, P^h is the 2×1 vector of commodity prices.

Figure 1 is a regular IWE diagram. The dimensions of the box represent world factor endowments. The origin of the home country is the lower-left corner, for the foreign country is the right-upper corner. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Suppose that an allocation of the factor endowments is at point E, where the home country is capital abundant. Point C represents the trade equilibrium point. It indicates the sizes of the consumption of the two countries.

We introduce two parameters, which are the shares of the home country's factor endowment to their world factor endowments respectively,

$$0 < \lambda_L < 1 \tag{2-3}$$

$$0 < \lambda_K < 1 \tag{2-4}$$

The factor endowments of the home country can be expressed as

$$L^H = \lambda_L L^W \tag{2-5}$$

$$K^H = \lambda_K K^W \tag{2-6}$$

where K^W is the world capital endowment, and L^W is the world labor endowment. The allocation of point E in Figure 1 is $E(\lambda_L L^W, \lambda_K K^W)$.

The factor contents of trade are

$$F_K^H = K^H - sK^W = (\lambda_K - s)K^W \tag{2-7}$$

$$F_L^H = L^H - sL^W = (\lambda_L - s)L^W \tag{2-8}$$

Using the trade balance of factor contents yields

$$\frac{r^*}{w^*} = -\frac{F_L^H}{F_K^H} = \frac{(s-\lambda_L)L^W}{(\lambda_K - s)K^W} \tag{2-9}$$

where r^* is the equalized rental, w^* is the equalized wage. Introduce a constant q as

$$q = \frac{(s-\lambda_L)}{(\lambda_K - s)} \tag{2-10}$$

Substituting it into (2-9) yields

$$\frac{r^*}{w^*} = q\frac{L^W}{K^W} \tag{2-11}$$

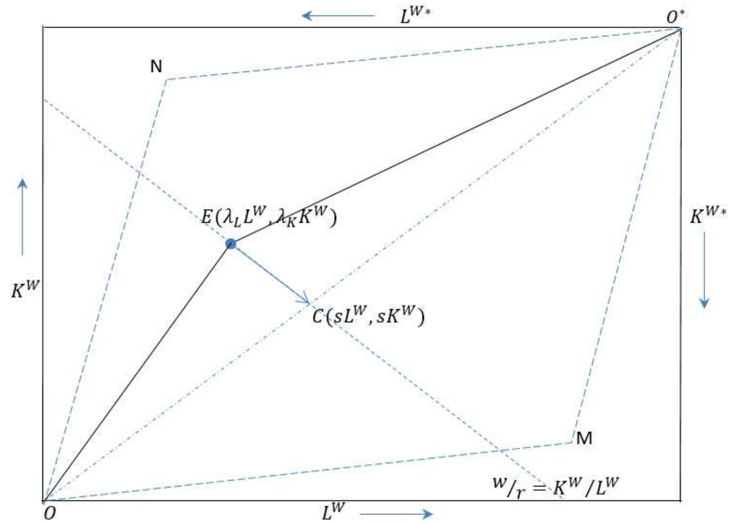

Figure 1 IWE Diagram

The factor prices and commodity prices are unchanged within the parallelogram by ONO^*M on the IWE diagram. That was proofed by Dixit and Norman (1980) and other studies. The factor price ratio (r^*/w^*) should be unchanged. Therefore, q should be constant when the allocation of factor endowments. Equation (2-11) illustrates that the rental/wage ratio is the function of the world factor endowments. This is why the FPE holds within the parallelogram formed by ONO^*M in the IWE diagram.

We have interesting to know what value q takes. At point $C(sL^W, sK^W)$, We see that $\lambda_L = s$ and $\lambda_K = s$, where s is the home country's share of GNP. There is no trade at this point. We now suppose that allocation E is nearby to C or imagine point E moves to close to its equilibrium point C. If the allocation E is above the diagonal line OO', it means that country hom is capital abundant. It also implies that $s - \lambda_L > 0$ and $\lambda_K - s > 0$. Taking $\lambda_L \to s$ and $\lambda_K \to s$ yields

$$\lim_{\substack{\lambda_L \to s \\ \lambda_k \to s}} \frac{(s-\lambda_L)}{(\lambda_K - s)} = 1 = q \tag{2-12}$$

We see that constant q equals to 1. Substituting q=1 into equation (2-10), we have the share of GNP at equilibrium as

$$s = \frac{1}{2}(\lambda_L + \lambda_K) = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-13}$$

In addition, equation (2-11) is reduced as

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{2-14}$$

This is true for any allocation of factor endowments within parallelogram ONO^*M.

With the equilibrium share of GNP (2-13) and the rental/wage ratio (2-14), we now obtain the whole equilibrium solution of the Heckscher-Ohlin model as

$$r^* = \frac{L^W}{K^W} \tag{2-15}$$

$$w^* = 1 \tag{2-16}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{2-17}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{2-18}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W}, \qquad F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W}, \quad (h = H, F) \tag{2-19}$$

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W, \qquad T_2^h = x_2^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W \;, \quad (h = H, F) \tag{2-20}$$

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^W} + \frac{L^h}{L^W}\right) \;, \qquad (h = H, F) \tag{2-21}$$

where p_i^* is world price for commodity i; T_i^h is the trade volume of commodity i in country h. Here, we assumed $w^* = 1$ by using Walras' equilibrium condition to drop one market clear condition.

We process another way to the solution of the general price-trade equilibrium. We view the equilibrium from the angle of trade competition by a trade box in the IWE diagram. Fisher (2011) proposed an insight concept of "goods price diversification cone". It is the counterpart of the diversification cone of factor endowments. The commodity prices should lie between the rays of goods price diversification cone in algebra as,

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \tag{2-22}$$

This condition will make sure that the factor prices from unit cost equation (2-2) are positive. The boundaries of the share of GNP corresponding the rays of the goods price diversification cone (2-22) can be calculated as

$$s_b^H(p) = s\left(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^F + K^H} = \lambda_K \tag{2-23}$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^F + L^H} = \lambda_L \tag{2-24}$$

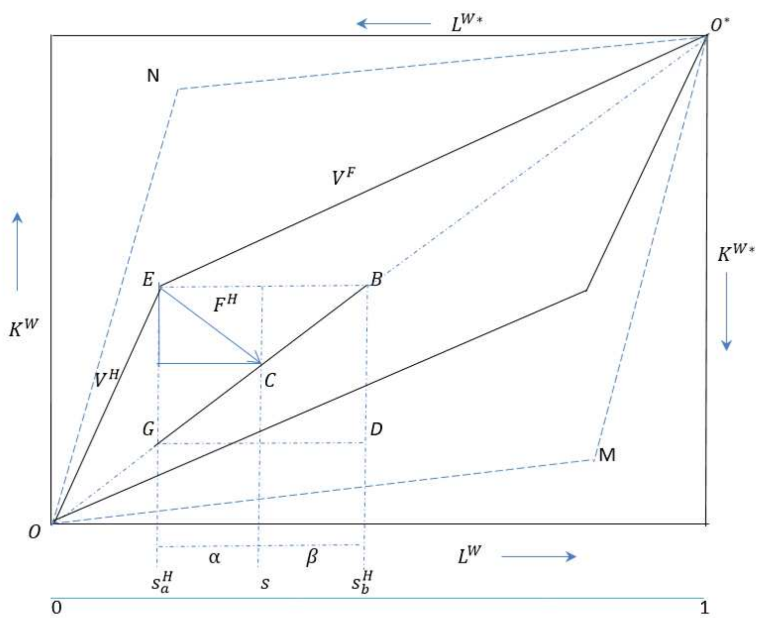

Figure 2 IWE with Trade Box

Figure 2 is an IWE diagram added with a trade box. The shares of GNP by (2-23) and (2-24) identify the trade box $EBDG$ in Figure 2. If a commodity price lies in the price diversification cone, the share of GNP lies in the trade box.

The home country's share of GNP, s, divides the trade box into two parts in Figure 2. Their lengths are α and β respectively as

$$\alpha = (\lambda_K - s), \qquad \beta = (s - \lambda_l) \tag{2-25}$$

When α increases, the home country's share of GNP increases and the foreign country's share of GNP decreases, and vice versa. In trade competitions, both countries want to reach their maximum GNP share in free trade.

Equation (2-25) can be rewritten as

$$\alpha = \left(\frac{K^H}{K^W} - s \quad\right), \qquad \beta = \left(s \quad - \frac{L^H}{L^W}\right) \tag{2-26}$$

The α and β are under constraint

$$\alpha + \beta = \left(\frac{K^H}{K^{EW}} - \frac{L^H}{L^{EW}}\right) \tag{2-27}$$

For reaching a competitive price-trade equilibrium of the model, we set a utility function as the product of redistributable shares of GNP of the two countries as

$$u = \alpha\beta \tag{2-28}$$

This simple utility function reflects the market mechanism that each country is trying to reach its larger share of GNP. The share of GNP is the function of commodity outputs and commodity prices. The utility function (2-28) reflects that one country cannot obtain gains without trade-off from another country.

Substituting (2-26) into (2-28) yields

$$u = (s - \frac{L^H}{L^W})(\frac{K^H}{K^W} - s) \tag{2-29}$$

We are interested in maximizing the utility function u, so we take differential of (2-29) with respect to s yields

$$\frac{du}{ds} = -2s + \left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-30}$$

By the first-order condition, we obtain

$$s = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-31}$$

With equation (2-31), we can get the same result of the general trade equilibrium (2-15) through (2-20).

From the factor content of trade (2-19), we see that when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$, then $F_L^H > 0$ and $F_K^H > 0$. This just states the Heckscher-Ohlin theorem. Dixit and Norman (1980) illustrated that when the allocation of the factor endowments changes, the factor prices and the commodity prices remain the same. The price solution (2-15) through (2-18) proved it analytically in a more strict condition. It explained why the same FPE holds within the parallelogram by $ONO'M$ on the IWE diagram.

3. Autarky Price and Comparative Advantage

The new logic from the last section is that world factor resource determines world prices. We now apply it to a country with an isolated market. Its "autarky" prices can be determined by its "autarky" factor endowments.

The IWE diagram itself supports the logic that autarky factor resources determine autarky prices analytically. Assuming that one country shrinks to very small, another country's autarky prices are then the world prices of the trade. Mathematically, when $V^H \to 0,$ inside the IWE box, then $V^F \to V^W$ and the world relative factor price r^* after trade will close to the relative autarky factor price of the foreign country,

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \qquad , \qquad \lim_{\substack{L^H \to 0 \\ K^H \to 0}} \frac{L^H + L^F}{K^H + K^F} = \frac{L^F}{K^F} = r^{Fa} \tag{3-1}$$

Moreover, the common commodity prices will close to the foreign country's autarky commodity prices. Therefore, we proved the autarky price formation mathematically.

Based on the above discussion, we present the autarky prices of countries that participate in free trade as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{3-2}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{3-3}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{3-4}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{3-5}$$

where superscript ha indicates the autarky price of country h.
The gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{3-6}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{3-7}$$

We add a negative sign in inequalities above since we expressed factor trade by net export, T^h . In most other works of literature, they denoted factor trade by net import. We denoted factor trade by net export. Appendix A is proof of the gain from trade by inequality (3-6). It implies that the world prices at the equilibrium will ensure the gains from trade for both countries, by the autarky prices derived.

The equilibrium prices should have some optimal properties. Guo (2019) demonstrates that the relative commodity price p_1^*/p_2^* reached its maximum value respective to world capital endowment (or to either country's capital endowments) if we assume that $K^H/K^F > L^H/L^F$ and $a_{K1}/a_{K1} > a_{L1}/a_{L2}$. In addition, it reached its minimum value respective to world labor endowment (it implies that p_2^*/p_1^* reached its maximum value respective to labor factor endowments). This result means that both countries export their products with comparative advantage at the maximum commodity price simultaneously. It implies that both countries get their maximum benefits through trade. We summarize the content of this section as a theorem as follows.

Theorem – The comparative advantage theorem

The factor price equalized when equilibrium reached. At the equilibrium, each country exports the good that has a comparative advantage. The ratio of world commodity prices at the equilibrium lies between the ratios of autarky prices of two countries. The world factor endowments, fully employed, determine world prices that assure the gains from trade for countries participating in trade. The equilibrium demonstrated the Heckscher-Ohlin theorem also.

Proof

The solution (2-15) through (2-18) shows how the world prices are determined and why it remains the same with mobile factor endowments in the IWE box. The relative factor price w/r presents an angle in Figure 1. The angle is unique for a giving IWE. Therefore, the solution is unique. The FPE is true and unchanged for an IWE with giving world factor endowments. If the solution is unique and if it satisfies the Dixit-Norman price inference, the equilibrium by the equalized factor prices is right. Appendix A proved the gains from trade as inequality (3-7).

End Proof

The equilibrium shows the unification of the Heckscher-Ohlin theorem, The FPE theorem, gains from trade, and Dixit-Norman mobile equalized prices. Each of them means others. That consolidate the existing Heckscher-Ohlin theories.

4. General equilibrium of trade for the case of two factors, two commodities, and multiple countries

We generalize the equilibrium result above to the model of two factors, two commodities, and multiple countries in this section.

In a two-country system, home and foreign, they are trade partners with each other. In a multi-country system, who is the trade partner for whom? We specify that trades are one that a country trades with the rest of the world. The trade relations are very simple now. It just likes the scenario of the two-country system from the analysis view.

Figure 3 draws an IWE diagram for two factors, two commodities, and three countries. The dimensions of the box represent world factor endowments. The vector $V^h(L^h, K^h)$ represents the factor endowments of country h, h=1, 2, and 3. The factor endowment vector V^1 of country 1 is arranged to start at origin point O. The rest of the world factor endowment is $V^2 + V^3$. It starts at the origin point O'.

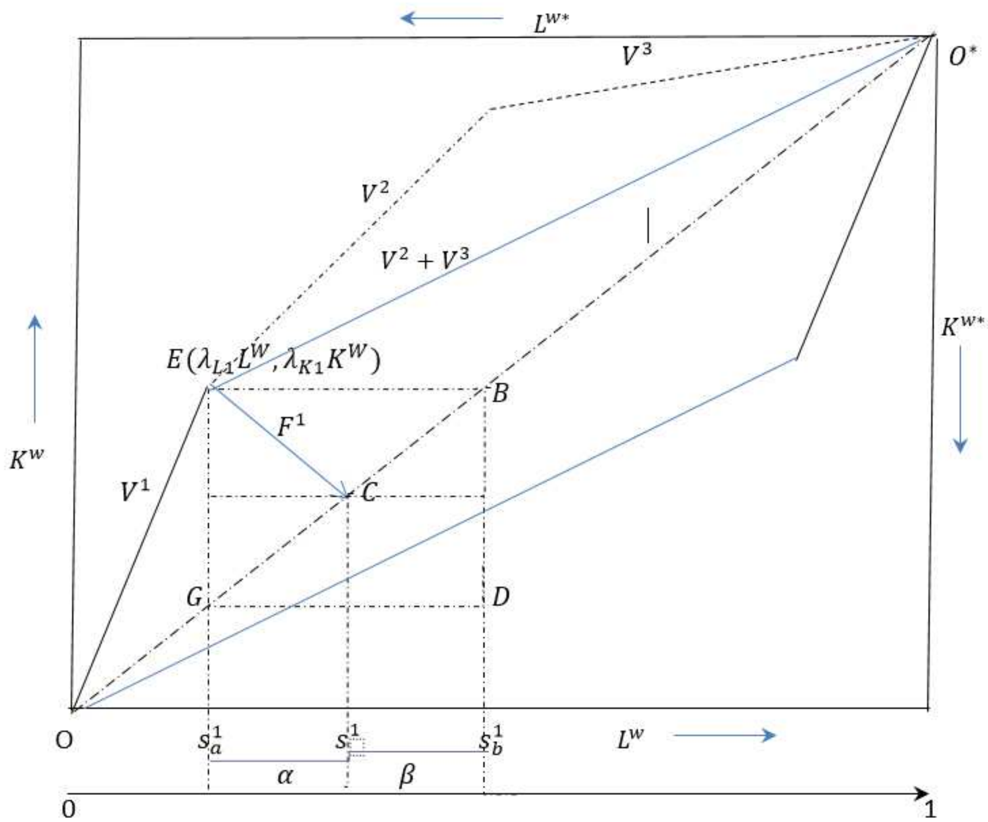

Figure 4 IWE diagram with GNP Share Box for 2 × 2 × 3 model

The algebra expression for the 2 x 2 x M model is as same as equation (2-1) and (2-2); the only difference is the country number. The country number now goes from 1 to M (In Figure 2, we only present 3 countries for illustration).

We now introduce two lists of parameters, which are the shares of factor endowments of country h to their world factor endowments respectively as

$$0 \le \lambda_{Lh} \le 1 \ , \quad 0 \le \lambda_{Kh} \le 1 \qquad (h = 1,2,\dots,M) \tag{4-1}$$

$$\sum_{h=1}^{M} \lambda_{Lh} = 1 \quad , \qquad \sum_{h=1}^{M} \lambda_{Kh} = 1 \tag{4-2}$$

The factor endowments of country h can be denoted as

$$L^h = \lambda_{Lh} L^W \qquad (h = 1,2,\dots,M) \tag{4-3}$$

$$K^h = \lambda_{Kh} K^W \qquad (h = 1,2,\dots,M) \tag{4-4}$$

The allocation of factor endowments of country 1 in Figure 3 is $E(\lambda_{L1}L^W, \lambda_{K1}K^W)$. It shows how a country trades with the rest of the world.

The factor contents of trade of country h are

$$F_K^h = K^h - s^h K^W = (\lambda_{Kh} - s^h)K^W \qquad (h = 1,2,\dots,M) \tag{4-5}$$

$$F_L^h = L^h - s^h L^W = (\lambda_{Lh} - s^h)L^W \qquad (h = 1,2,\dots,M) \tag{4-6}$$

The trade balance of factor contents for country h is

$$\frac{r^{*h}}{w^{*h}} = \frac{(s^h - \lambda_{Lh})L^W}{(\lambda_{Kh} - s^h)K^W} \qquad (h = 1,2,\dots,M) \tag{4-7}$$

where r^{*h} is the equalized rental in country h, w^{*h} is the equalized wage in country h. It displays the trade balance between country h and the rest world. Extending the result (2-12) in the last section to the equation above, we have

$$\frac{(s^h-\lambda_{Lh})}{(\lambda_{Kh}-s^h)}=1 \qquad (h=1,2,\dots,M) \tag{4-8}$$

$$\frac{r^{*h}}{w^{*h}}=\frac{L^W}{K^W} \qquad (h=1,2,\dots,M) \tag{4-9}$$

This means that the relative factor price is the same for all countries.

$$\frac{r^{*h}}{w^{*h}}=\frac{L^W}{K^W}=\frac{r^*}{w^*} \tag{4-10}$$

By assuming $w^*=1$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^h=\frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W} \qquad (h=1,2,\dots,M) \tag{4-11}$$

$$\frac{r^*}{w^*}=\frac{L^W}{K^W} \tag{4-12}$$

$$w^*=1 \tag{4-13}$$

$$p_1^*=a_{k1}\frac{L^W}{K^W}+a_{L1} \tag{4-14}$$

$$p_2^*=a_{k2}\frac{L^W}{K^W}+a_{L2} \tag{4-15}$$

$$F_K^h=\frac{1}{2}\frac{K^hL^W-K^WL^h}{L^W} \qquad (h=1,2,\dots,M) \tag{4-16}$$

$$F_L^h=-\frac{1}{2}\frac{K^hL^W-K^WL^h}{K^W} \qquad (h=1,2,\dots,M) \tag{4-17}$$

$$x_1^h=x_1^h-\frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W}x_1^W \qquad (h=1,2,\dots,M) \tag{4-18}$$

$$x_2^h=x_1^h-\frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W}x_2^W \qquad (h=1,2,\dots,M) \tag{4-19}$$

We see that

$$\sum_{h=1}^{H}s^h=\sum_{h=1}^{H}\frac{1}{2}\frac{K^hL^W+K^WL^h}{K^WL^W}=1 \tag{4-20}$$

Those are the equilibrium solution for the $2\times 2\times M$ model. We can demonstrate that all countries participating in trade gain from trade. It showed that world factor endowments determine world prices in the multi-country economy.

5. Related Discussions

The price-trade equilibrium above displayed the root of the FPE in the IWE. The trade box illustrates how the redistributable shares of GNP are distributed to each country in trade competition. It is a Pareto optimal solution since the trade box shows how social trade-off played. It is a balanced trade that the share of a country in world spending equals to its share in world income.

Dixit (2010) mentioned, "The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, 'it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.' That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor prices at the equilibrium of this study presented the Heckscher-Ohlin theorem. Guo (2019) provide a trade effect analyses based on the equilibrium of this paper, it displayed that the trade effect of changes of factor endowments is a chain effect of the Rybczynski's trade effect triggering the Stolper-Samuelson's trade effect. The equilibrium solution put all of the four-core theorems together.

The multiple-country equilibrium is more intricate in economic logic. The equation (4-21) shows that the sum of the shares of GNP of all countries equals to 1. It confirms that both the solution and the approach of this study are right.

Conclusion

The paper attained the price structure of equalized prices and the general equilibrium of trade in the 2 x 2x M Heckscher-Ohlin model. The equilibrium addresses the Heckscher-Ohlin theorem with trade volume, the factor price equalization theorem with price structure, and comparative advantage with gains from trade.

The study illustrates the economic logic that world factor resources determine world prices. Its first application is to identify autarky prices.

The solution of equalized prices is ascertained by Dixit and Norman price inference that the prices remain the same when the allocation of factor endowments changes.

The result of gains from trade is a good side effect of the trade equilibrium of this paper. It is an important property of the equilibrium and the FPE. It is what we expected. No literature described possible connections between equalized factor price and comparative advantages.

The equalized factor prices provides the theoretical background for further analyses of factor price none-equalization when countries have different productivities.

Appendix A

We express the gains from trade for the home country as

$$-W^{Ha\prime}F^H > 0 \tag{A-1}$$

Adding trade balance condition $W^{*\prime}F^H = 0$ on (A-1) yields

$$-(W^{Ha\prime} - W^{*\prime})F^H > 0 \tag{A-2}$$

We see

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix} \quad , \qquad W^* = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix} \tag{A-3}$$

Substituting them into (A-2) yields,

$$-\begin{bmatrix}\frac{L^H}{K^H}-\frac{L^W}{K^W} & 0\end{bmatrix}\begin{bmatrix}\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}\\ -\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}\end{bmatrix}>0 \tag{A-4}$$

It can be reduced to

$$-(\frac{L^H}{K^H}-\frac{L^W}{K^W})\times\frac{1}{2}\frac{K^HL^W-K^WL^H}{L^W}>0 \tag{A-5}$$

It means

$$-\left(\frac{L^H}{K^H}-\frac{L^W}{K^W}\right)\times\frac{1}{2}\frac{\frac{L^W}{K^W}-\frac{L^H}{K^H}}{L^W}K^WK^H=\left(\frac{L^H}{K^H}-\frac{L^W}{K^W}\right)^2\times\frac{1}{2L^W}K^WK^H>0 \tag{A-6}$$

It is true. So that (A-1) holds. Similarly, we can obtain

$$-W^{Fa\prime}F^F=\left(\frac{L^F}{K^F}-\frac{L^W}{K^W}\right)^2\times\frac{1}{2L^W}K^WK^F>0 \tag{A-7}$$

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