Market Structure and Organizational Form

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Abstract

This paper studies the determinants of a firm's organizational form in the context of an imperfectly competitive industry. There are two kinds of organizational forms: the multi-divisional form (M-form) and the unitary form (U-form). An M-form firm suffers from ignorance of demand externalities among different products and double marginalization is eliminated. In contrast, in a U-form firm, demand externalities are taken into consideration and double marginalization exists. A firm's optimal choice of organizational form depends on the market structure.

Keywords: Organizational form, market structure, oligopoly, multi-divisional form, unitary form

JEL Classification: D43, L13, L23

1. Introduction

Two kinds of organizational forms have been studied extensively in the literature: the unitary organizational form (U-form) and the multi-divisional organizational form (M-form).[1] A U-form firm is organized by functions, such as production and marketing. The Ford Motor Company before World War II was an example of the U-form firm. An M-form firm is organized by products and each product has its own production and sales divisions. An example of an M-form firm is the General Motors Company. As firms frequently engage in costly reshuffling of their structure, one interesting thing to know is what determines a firm's choice of organizational form.

This article studies how market structure affects a firm's choice between the U-form and the M-form organizational form. Market structure refers specifically to the number of firms in the industry, and how market demand for different products may be related. Each firm produces two products. To produce each product, both production and marketing activities are needed. In a U-form organization, there is a middle manager of production and a middle manager for marketing.

[1] Chandler (1962) records that du Pont, General Motors, and some other firms' changed their organizational modes from the U-form to the M-form in the 1920s. At that period, many firms began to produce multiple products. As a result, top managers could not supervise all decisions. Top managers had two options. The first option was to keep organizing firms by functional departments and give managers of functional departments more authority. The second one was to adopt the M-form organizational mode. By giving product managers more authority, top managers can concentrate on more strategic issues. See Williamson (1985) for some additional study on this issue.

Each functional manager maximizes his or her division's profit. In an M-form organization, there is a middle manager for each product. Each product manager maximizes his or her product division's profit. Firms are assumed to engage in Cournot competition.

Each organizational form has its own advantages and disadvantages. In a U-form firm, demand externalities between different products are taken into consideration. However, a U-form firm may suffer from double marginalization because the middle manager of production chooses transfer prices to maximize the production division's profit. An M-form firm eliminates double marginalization. On the other hand, a product manager in an M-form firm may be only concerned with the profit of the product she supervises, ignoring the fact that market demand for different goods are interdependent.

In this article, we show that a firm's optimal choice of organizational form depends on the number of firms in the industry, and whether goods are substitutes or complements. As firms face the same price, a firm has a higher profit, if and only if, it has a higher output. Whether a U-form or an M-form firm has a higher output depends on whether the two products are substitutes or complements. When the two products are substitutes, it is shown that adopting the M-form organizational mode gives a firm higher profit than adopting the U-form organizational mode. When there are multiple firms in the industry, the over-expansion of output of an M-form firm is beneficial to this firm as this expansion makes other firms in the industry less aggressive. In contrast, the U-form firm's output is too low, but it is a positive externality to other firms. As a result, an M-form firm makes a higher profit than a U-form firm when there are multiple firms in the industry. In this sense, the economics is similar to that of Fershtman and Judd (1987), except that the choice of organizational form is probably a far more credible commitment than the contractual commitments considered in their paper.[2] When products are strong complements, the U-form firm has a higher output and profit because the demand externalities are taken into consideration. As the number of firms in the industry increases, whether the difference of output increases or not, depends on whether the slope of the U-form firm's reaction curve is larger or smaller than that of an M-form firm.

Two issues merit some explanation. The first issue is that neither the U-form nor the M-form organizational mode may be optimal. Even if this is the case, a study of the choice between

[2] Related studies include Baye, Crocker, and Ju (1996) and Tan and Yuan (2003). Both papers study a two-stage game. In the first stage, firms choose their divisionalization or divestiture strategies. In the second stage, divisions engage in quantity or price competition.

the U-form and the M-form organizational mode can still be justified. One reason is that the optimal organizational form may be too complex to be implemented in real world situations. As the U-form and the M-form organizations are commonly observed in real world situations, there is some merit in understanding the advantages and disadvantages of adopting these organizational forms. The second issue is that one may wonder whether a U-form firm suffers from double marginalization or not. In real world situations, double marginalization is frequently observed. How to decide transfer prices between different functional departments is not a trivial issue in a large organization.[3] Based on survey data, Eccles and White (1988) find that mandated market-based transfer pricing is one of the three kinds of transfer pricing policies commonly used by firms.[4] When a firm adopts this kind of pricing policy, internal transactions are valued at market prices. "Buying profit centers that pay market price commonly complain that the intermediate good is being 'marked up twice', once by the selling profit center and again by the buying profit center" (Eccles and White 1988, p. S31). Eccles and White (1988) record a case in which the transfer price of an intermediate input is twice the production cost of that input.

For the literature on organizational forms, see Holmstrom and Tirole (1991), Aghion and Tirole (1995), Maskin, Qian, and Xu (2000), and Qian, Roland, and Xu (2002). The approach of modeling organizational forms used in this article is similar to that of Aghion and Tirole (1995). In their paper, a firm produces two products and each product needs production and marketing activities. A U-form organizational form is organized into the production department and the marketing department. An M-form organizational firm is organized by products. None of the above papers studies how a firm's choice of organizational form is affected by the market structure. However, market structure, such as the number of firms in the industry and how market demand for different products is related, provides the basic environment in which firms operate. As a firm's marginal benefit and marginal cost are affected by market structure, a firm's optimal choice of organizational form is affected by market structure. In fact, the influence of market structure on a firm's behavior is well recognized in the literature. For example, Fershtman and Judd (1987) show that a firm's choice of incentive scheme depends on market structure. When a firm is a monopolist in an industry, owners of this firm will try to get the managers to maximize profit. When a firm is

[3] See Eccles and White (1988) for the conflict between middle level managers about transfer prices.
[4] The other two methods are mandated full cost transfers and exchange autonomy transfers.

one of multiple firms in the industry, owners will provide incentives to managers to expand output to take advantage of the strategic interaction among firms.

The rest of the paper is organized as follows. Section 2 sets up the model and compares the equilibrium output and profit of the two organizational forms. In Section 3, equilibrium organizational forms are studied. Section 4 studies the special case of linear market demand. Section 5 discusses some assumptions of this paper and concludes.

2. The Model

There are n firms in an industry, $n \geq 1$. For all these n firms, suppose m of them adopt the U-form and $n - m$ of them adopt the M-form organizational mode. Each firm produces two products: i and j. Producing each product requires two activities: production and marketing. How activities are organized depends on the decision of the top manager of a firm. There are three levels of managers in each firm: top, middle, and low-level managers. The top manager of a firm chooses which organizational form to adopt. If a top manager chooses the M-form, for each of the two products, the production stage and the marketing stage will be supervised by the same middle manager. The two middle managers in an M-form firm will be called the middle manager for product i and the middle manager for product j. If the top manager chooses the U-form, one middle manager will supervise the two production departments and the other middle manager will supervise the two marketing departments. The two middle managers in a U-form firm will be called the middle manager for production and the middle manager for marketing. Middle managers choose quantities of production. A low-level manager follows the instruction of a middle manager on how much to produce.

All firms have the same fixed and marginal costs of production. The fixed cost of production is f. Let the constant marginal cost of production be denoted by c. The marginal cost of marketing is also assumed to be constant and is normalized to zero.

When multiple firms produce the same product, they engage in Cournot competition. For $x, y = i, j$ and $x \neq y$, let p^x denote the price of product x. Let Q^x denote total industry output of product x. Market demand for product x is given by an inverse demand function

$$p^x = p^x(Q^x, Q^y),\ x, y = i, j, \text{ and } x \neq y. \tag{1}$$

Let a and b be positive constants. A special case of (1) is given by

$$p^x = a - bQ^x - dQ^y,\ x, y = i, j, \text{ and } x \neq y. \tag{2}$$

It is assumed that the inverse demand functions are symmetric with respect to the two products. As a result, $\frac{\partial p^x(Q^x,Q^y)}{\partial Q^y} = \frac{\partial p^y(Q^y,Q^x)}{\partial Q^x}$ for all Q^x and Q^y.

Whether the two products are complements or substitutes depends on the sign of $\partial p^x(Q^x, Q^y)/\partial Q^y$. If $\partial p^x(Q^x, Q^y)/\partial Q^y < 0$, the two products are substitutes; if $\partial p^x(Q^x, Q^y)/\partial Q^y > 0$, the two products are complements; if $\partial p^x(Q^x, Q^y)/\partial Q^y = 0$, the market demand of the two goods is independent. If $\partial p^x(Q^x, Q^y)/\partial Q^y = \partial p^x(Q^x, Q^y)/\partial Q^x$, the two products are perfect substitutes.

The following assumptions about the inverse demand functions are made.

ASSUMPTION 1: $\frac{\partial p^x(Q^x,Q^y)}{\partial Q^x} < 0$.

ASSUMPTION 2: $\left|\frac{\partial p^x(Q^x,Q^y)}{\partial Q^x}\right| \geq \left|\frac{\partial p^x(Q^x,Q^y)}{\partial Q^y}\right|$.

Assumption 1 implies that inverse demand functions have a negative slope. Assumption 2 implies that the demand of a product is influenced more by its own price change than by the price change of the other product. Assumptions 1 and 2 together lead to $\frac{\partial p^x(Q^x,Q^y)}{\partial Q^x} + \frac{\partial p^x(Q^x,Q^y)}{\partial Q^y} < 0$. This inequality rules out the possibility that a product's price may increase when total quantity supplied increases.

Decisions in a U-form Firm

In this subsection, the optimal decisions in a U-form firm are studied. In a U-form firm, the middle manager of production decides how much to produce first. The marketing department cannot sell more than what the production department produces. It is assumed that the manager of production chooses transfer prices to maximize her division's profit, rather than the whole firm's profit. However, when choosing transfer prices, she takes it into account that her decisions will affect final demand for her firm's products and thus her division's profit.

The assumption that the middle manager of production sets transfer prices to maximize her division's profit rather than the firm's profit merits some discussion. Why doesn't the top manger decide the amount of production and the transfer prices? For a small firm, a top manger may be able to make all the important decisions. However, for a large firm, the top manager may not have as much information about cost and market demand as a middle manager has. Whether a middle manager is evaluated by her absolute performance or by a rank-order tournament, a middle manager's payoff increases with her own department's revenue. Therefore, a middle level manager has incentives to maximize her division's profit.

The middle manager of marketing chooses quantities to sell to maximize the marketing division's profit. Let t^x denote product x's transfer price between the production department and the marketing department, $x = i, j$. Let q_u^x denote a representative U-form firm's product x output. Since the same middle manager supervises the sale of both products, this manager will take into account the fact that market demands are interdependent. Given the transfer prices charged by the production departments, this middle manager of marketing maximizes the sum of the profits of the two products

$$(p^i - t^i)q_u^i + (p^j - t^j)q_u^j. \tag{3}$$

Taking first order condition with respect to q_u^x leads to

$$p^x - t^x + q_u^x \frac{\partial p^x}{\partial Q^x} + q_u^y \frac{\partial p^y}{\partial Q^x} = 0. \tag{4}$$

The middle manager of production chooses transfer prices to maximize the production division's profit. Through backward induction, a U-form firm's middle manager of production can figure out the final demand as a function of transfer prices. She chooses t^i and t^j to maximize her division's profit

$$(t^i - c)q_u^i + (t^j - c)q_u^j. \tag{5}$$

Taking first order condition with respect to t^x yields

$$q_u^x + (t^x - c)\frac{dq_u^x}{dt^x} + (t^y - c)\frac{dq_u^y}{dt^y} = 0. \tag{6}$$

Differentiation of (4) with respect to q_u^x, q_u^y, and t^x leads to

$$\left(2\frac{\partial p^x}{\partial Q^x} + q_u^x \frac{\partial^2 p^x}{\partial^2 Q^x} + q_u^y \frac{\partial^2 p^y}{\partial^2 Q^x}\right) dq_u^x$$
$$+ \left(\frac{\partial p^x}{\partial Q^y} + \frac{\partial p^y}{\partial Q^x} + q_u^x \frac{\partial^2 p^x}{\partial Q^x \partial Q^y} + q_u^y \frac{\partial^2 p^y}{\partial Q^x \partial Q^y}\right) dq_u^y = dt^x, x, y = i, j. \tag{7}$$

The above system can be expressed as

$$\begin{vmatrix} 2\frac{\partial p^i}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial^2 Q^i} + q_u^j \frac{\partial^2 p^j}{\partial^2 Q^i} & \frac{\partial p^i}{\partial Q^i} + \frac{\partial p^j}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + q_u^j \frac{\partial^2 p^j}{\partial Q^i \partial Q^j} \\ \frac{\partial p^i}{\partial Q^j} + \frac{\partial p^j}{\partial Q^i} + q_u^j \frac{\partial^2 p^j}{\partial Q^i \partial Q^j} + q_u^i \frac{\partial^2 p^i}{\partial Q^i \partial Q^j} & 2\frac{\partial p^j}{\partial Q^j} + q_u^j \frac{\partial^2 p^j}{\partial^2 Q^j} + q_u^i \frac{\partial^2 p^i}{\partial^2 Q^j} \end{vmatrix}$$
$$\times \begin{vmatrix} dq_u^i \\ dq_u^j \end{vmatrix} = \begin{vmatrix} dt^i \\ 0 \end{vmatrix}.$$

The determinant of the coefficient matrix is

$$\Delta \equiv \left(2\frac{\partial p^i}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial^2 Q^i} + q_u^j \frac{\partial^2 p^j}{\partial^2 Q^i}\right)^2 - \left(\frac{\partial p^i}{\partial Q^j} + \frac{\partial p^j}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + q_u^j \frac{\partial^2 p^j}{\partial Q^i \partial Q^j}\right)^2. \quad (8)$$

Application of the Cramer's rule leads to

$$\frac{dq_u^i}{dt^i} = \left(2\frac{\partial p^j}{\partial Q^j} + q_u^i \frac{\partial^2 p^i}{\partial^2 Q^j} + q_u^j \frac{\partial^2 p^j}{\partial^2 Q^j}\right)/\Delta, \quad (9a)$$

$$\frac{dq_u^j}{dt^i} = -\left(\frac{\partial p^i}{\partial Q^j} + \frac{\partial p^j}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + q_u^j \frac{\partial^2 p^j}{\partial Q^i \partial Q^j}\right)/\Delta. \quad (9b)$$

Define

$$\Delta_1 \equiv \left(2\frac{\partial p^j}{\partial Q^j} + q_u^i \frac{\partial^2 p^i}{\partial^2 Q^j} + q_u^j \frac{\partial^2 p^j}{\partial^2 Q^j}\right) - \left(\frac{\partial p^i}{\partial Q^j} + \frac{\partial p^j}{\partial Q^i} + q_u^i \frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + q_u^j \frac{\partial^2 p^j}{\partial Q^i \partial Q^j}\right). \quad (10)$$

Since demand is symmetric, equations 8 and 10 lead to

$$\frac{\Delta}{\Delta_1} = 2\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j}\right) + q_u\left(2\frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + \frac{\partial^2 p^i}{\partial^2 Q^i} + \frac{\partial^2 p^j}{\partial^2 Q^i}\right). \quad (11)$$

From equations 9 and 10, it is clear that

$$\frac{dq_u^i}{dt^i} + \frac{dq_u^j}{dt^i} = \frac{\Delta_1}{\Delta}. \quad (12)$$

In a symmetric equilibrium, $t^i = t^j = t$. Plugging equation 12 into 6 yields

$$t = c - q_u^x \frac{\Delta}{\Delta_1}. \quad (13)$$

From 13, the transfer price will be larger than the production cost if and only if $-\Delta/\Delta_1$ is positive. The following assumption is sufficient for $-\Delta/\Delta_1$ to be positive.

ASSUMPTION 3: $T_u \equiv \left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j}\right) + q_u\left(2\frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + \frac{\partial^2 p^i}{\partial^2 Q^i} + \frac{\partial^2 p^j}{\partial^2 Q^i}\right) < 0.$

Assumption 3 states that a U-form firm's marginal revenue must not rise with its rivals' outputs. It is a weak assumption. Similar assumptions are standard in Cournot analysis.[5] For example, for X denoting the output of a product and p denoting the price, Novshek (1985) shows that $p'(X) + Xp''(X) \leq 0$ guarantees the existence of a Cournot equilibrium. Assumption 3 is the

[5] Dixit (1986) and Shapiro (1989) provide more motivation for these assumptions. Dixit (1986) also illustrates the relationship between these kinds of assumptions and stability conditions.

analogy of the above inequality for the case when there are two products. If Assumption 3 is satisfied, the reaction curve of a U-form firm has a negative slope.

The following assumptions are similar to Assumption 3.

ASSUMPTION 4: $T_m \equiv \left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j}\right) + q_m \left(\frac{\partial^2 p^i}{\partial^2 Q^i} + \frac{\partial^2 p^j}{\partial Q^i \partial Q^j}\right) < 0.$

ASSUMPTION 5: $2T_u + \left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j}\right)$

$$+2mq_u \left(2\frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + \frac{\partial^2 p^i}{\partial^2 Q^i} + \frac{\partial^2 p^j}{\partial^2 Q^i} + q_u \left(\frac{\partial^3 p^i}{\partial^3 Q^i} + 2\frac{\partial^3 p^i}{\partial^2 Q^i \partial Q^j} + \frac{\partial^3 p^i}{\partial Q^i \partial^2 Q^j}\right)\right) < 0.$$

Assumption 4 states that an M-form firm's marginal revenue must not rise with its rivals' outputs. If Assumption 4 is satisfied, the reaction curve of an M-form firm has a negative slope. Assumption 5 states that a U-form firm's marketing department's marginal revenue must not rise with other firms' outputs. If Assumption 5 is satisfied, the transfer price does not decrease with the U-form firm's output.[6] Novshek (1985) shows that if a firm's marginal revenue is everywhere a declining function of the aggregate output of other firms, a Cournot equilibrium exists. In his proof, firms are not required to be identical. From Novshek (1985), Assumptions 3-5 guarantee the existence of a Cournot equilibrium.

In a symmetric equilibrium, $q_u^i = q_u^j = q_u$. Plugging equation 13 into 4 leads to one of the two equations defining optimal output choices

$$p^x - c + q_u \frac{\partial p^x}{\partial Q^x} + q_u \left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) = 0. \tag{14}$$

Decisions in an M-form Firm

In this subsection, the output decisions of an M-form firm are studied. It is assumed that the middle manager of product x's payoff is not affected by the profit of product y. In other words, a tournament incentive scheme or a profit-sharing incentive scheme is not used here. One reason why these incentive schemes are not used is that it is hard to prevent collusion of middle managers or collusion between the top manager and a middle manager, see Holmstrom and Tirole (1991) for a related argument. In addition, when there are more than one firm in the industry, a profit-sharing scheme may decrease this firm's output and benefit its competitors, see Freshtman and Judd (1987) for a study of strategic manipulation of incentives in oligopolistic competition.

[6] For the linear demand defined in equation 2, Assumptions 3-5 are satisfied if Assumptions 1 and 2 are satisfied.

Let q_m^x denote this firm's output of product x, $x = i, j$. The middle manager of product x is concerned only with product x's profit and he ignores demand externality imposed on product y. Thus, the middle manager of product x maximizes the divisional profit

$$(p^x - c)q_m^x. \tag{15}$$

Taking first order condition with respect to q_m^x yields the second equation defining optimal output choices

$$p^x - c + q_m^x \frac{\partial p^x}{\partial Q^x} = 0. \tag{16}$$

Comparison of the Two Organizational Forms

In this subsection, the output and profit of the two organizational forms are compared. In a symmetric equilibrium, $q_m^i = q_m^j = q_m$. Equations 13 and 14 lead to

$$V_1 \equiv p^x - t + q_u \frac{\partial p^x}{\partial Q^x} + q_u \frac{\partial p^y}{\partial Q^x} = 0. \tag{17}$$

Equation 16 leads to

$$V_2 \equiv p^x - c + q_m \frac{\partial p^x}{\partial Q^x} = 0. \tag{18}$$

In a symmetric equilibrium, all U-form firms produce the same amount of output q_u. All M-form firms produce the same amount of output q_m. In a symmetric equilibrium, output of the two products are the same and $Q^i = Q^j = Q$. Thus, total industry output is given by

$$Q = mq_u + (n - m)q_m. \tag{19}$$

Define a constant

$$\delta \equiv \frac{m}{n}.$$

Then

$$Q \equiv n(\delta q_u + (1 - \delta)q_m). \tag{19'}$$

The profit from adopting the U-form organizational mode is

$$\pi_u = 2(p(Q) - c)q_u - f. \tag{20a}$$

The profit from adopting the M-form organizational mode is

$$\pi_m = 2(p(Q) - c)q_m - f. \tag{20b}$$

As firms receive the same price for their product, a firm has a higher profit, if and only if, it has a higher output. The following proposition compares the output and the profit of an M-form firm with that of a U-form firm.

PROPOSITION 1. When there are multiple firms in the industry, an M-form firm produces more (less) than a U-form firm and has a higher (lower) profit if and only if $\left(\frac{\partial p^y}{\partial Q^x}+\frac{\Delta}{\Delta_1}\right)<0$ ($\left(\frac{\partial p^y}{\partial Q^x}+\frac{\Delta}{\Delta_1}\right)>0$).

PROOF. From equations 14 and 18, $q_m > q_u$ if and only if $\left(\frac{\partial p^y}{\partial Q^x}+\frac{\Delta}{\Delta_1}\right)<0$. ■

The intuition behind Proposition 1 is the following: There are two factors affecting the ranking of a U-form firm's output with that of an M-form firm. The first term in the bracket ($\partial p^y/\partial Q^x$) measures demand interdependence of the two products. The second term in the bracket (Δ/Δ_1) measures the degree of double marginalization.[7] If the two products are substitutes, the term in the bracket is negative and an M-form firm produces more of both products than a U-form firm. The reason is that both the elimination of double marginalization and the ignorance of negative demand externalities in an M-form firm increase an M-form firm's output. If $\partial p^y/\partial Q^x$ is sufficiently positive such that $\left(\frac{\partial p^y}{\partial Q^x}+\frac{\Delta}{\Delta_1}\right)>0$, an M-form firm produces less than a U-form firm because of the ignorance of positive demand externalities in an M-form firm.

Differentiation of 17 with respect to q_u, q_m, n, and m yields

$$\frac{\partial V_1}{\partial q_u}dq_u+\frac{\partial V_1}{\partial q_m}dq_m=-\frac{\partial V_1}{\partial n}dn-\frac{\partial V_1}{\partial m}dm. \tag{21}$$

Differentiation of 18 with respect to q_u, q_m, n, and m yields

$$\frac{\partial V_2}{\partial q_u}dq_u+\frac{\partial V_2}{\partial q_m}dq_m=-\frac{\partial V_2}{\partial n}dn-\frac{\partial V_2}{\partial m}dm. \tag{22}$$

Equations 21 and 22 can be expressed as

$$\begin{vmatrix}\frac{\partial V_1}{\partial q_u} & \frac{\partial V_1}{\partial q_m}\\ \frac{\partial V_2}{\partial q_u} & \frac{\partial V_2}{\partial q_m}\end{vmatrix}\begin{vmatrix}dq_u\\ dq_m\end{vmatrix}=\begin{vmatrix}-\frac{\partial V_1}{\partial n}\\ -\frac{\partial V_2}{\partial n}\end{vmatrix}dn+\begin{vmatrix}-\frac{\partial V_1}{\partial m}\\ -\frac{\partial V_2}{\partial m}\end{vmatrix}dm. \tag{23}$$

From equations 17, 18, and 19', it can be shown that

$$\frac{\partial V_1}{\partial q_u}=n\delta T_u+\frac{\partial p^i}{\partial Q^i}+\frac{\partial p^i}{\partial Q^j}-\frac{\partial t}{\partial q_u}, \tag{24}$$

$$\frac{\partial V_1}{\partial q_m}=n(1-\delta)T_u<0, \tag{25}$$

[7] For a detailed study on double marginalization, see Spengler (1950).

$$\frac{\partial V_1}{\partial n} = (\delta q_u + (1-\delta) q_m) T_u < 0, \tag{26}$$

$$\frac{\partial V_1}{\partial m} = (q_m - q_u) T_u, \tag{27}$$

$$\frac{\partial V_2}{\partial q_u} = n\delta T_m < 0, \tag{28}$$

$$\frac{\partial V_2}{\partial q_m} = n(1-\delta) T_m + \frac{\partial p^i}{\partial Q^i} < 0, \tag{29}$$

$$\frac{\partial V_2}{\partial n} = (\delta q_u + (1-\delta) q_m) T_m < 0, \tag{30}$$

$$\frac{\partial V_2}{\partial m} = (q_m - q_u) T_m. \tag{31}$$

The determinant of the coefficient matrix for equation 23 is

$$T \equiv \frac{\partial V_1}{\partial q_u}\frac{\partial V_2}{\partial q_m} - \frac{\partial V_1}{\partial q_m}\frac{\partial V_2}{\partial q_u}$$

$$= \frac{\partial p^i}{\partial Q^i} n\delta T_u + \left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right) n(1-\delta) T_m + \frac{\partial p^i}{\partial Q^i}\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right). \tag{32}$$

From equations 11 and 13, it can be shown that

$$\frac{\partial t}{\partial q_u} =$$

$$-2T_u - 2mq_u\left(2\frac{\partial^2 p^i}{\partial Q^i \partial Q^j} + \frac{\partial^2 p^i}{\partial^2 Q^i} + \frac{\partial^2 p^j}{\partial^2 Q^i} + q_u\left(\frac{\partial^3 p^i}{\partial^3 Q^i} + 2\frac{\partial^3 p^i}{\partial^2 Q^i \partial Q^j} + \frac{\partial^3 p^i}{\partial Q^i \partial^2 Q^j}\right)\right).$$

Under Assumption 5,

$$\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u} < 0. \tag{33}$$

From 33 and Assumptions 1-5, $T > 0$.

The following lemma studies how each firm's output and total output change with the number of firms in the industry while the proportion of M-form firms does not change.

LEMMA 1. Under Assumptions 1-5, (i) each firm's output decreases with the total number of firms in the industry; (ii) total output increases with the total number of firms in the industry.

PROOF. (i) Applying the Cramer's rule, from equation 23, it can be shown that

$$\frac{dq_u}{dn} = -(\delta q_u + (1-\delta) q_m)\frac{\partial p^i}{\partial Q^i}\frac{T_u}{T} < 0, \tag{34}$$

$$\frac{dq_m}{dn} = -(\delta q_u + (1-\delta) q_m)\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right)\frac{T_m}{T} < 0. \tag{35}$$

(ii) Differentiation of equation 19 with respect to n yields

$$\frac{dQ}{dn} = q_m + m\frac{dq_u}{dn} + (n-m)\frac{dq_m}{dn}. \tag{36}$$

Plugging equations 34 and 35 into 36 yields

$$\frac{dQ}{dn} = \delta q_u + (1-\delta)q_m + n\delta\frac{dq_u}{dn} + n(1-\delta)\frac{dq_m}{dn}$$
$$= \frac{\partial p^i}{\partial Q^i}\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right)\frac{(\delta q_u + (1-\delta)q_m)}{T} > 0. \blacksquare$$

Lemma 1 is consistent with results from conventional Cournot competition. When one additional firm enters the industry, for a given level of market demand, existing firms' marginal revenue decreases. As a result, existing firms' quantities of production decreases. As the increase of output from the new firm dominates the decrease of output from existing firms, total industry output increases when the total number of firms increases.

From equations 20a and 20b, firms have different profits, if and only if, they have different output levels. Thus, $q_m - q_u$ may be viewed as the advantage of being an M-form firm. Suppose currently an M-form has a higher level of profit. Thus, a new firm entering the industry will adopt the M-form firm. Does the advantage of choosing the M-form persist with the entry of a new M-form firm? This question motivates the following proposition which studies how the number of firms in an industry affects the advantage of choosing the M-form.

PROPOSITION 2. If the proportion of M-form firms does not change, the advantage of being an M-form firm increases with the number of firms in the industry if and only if

$$\frac{T_u}{\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}} > \frac{T_m}{\frac{\partial p^i}{\partial Q^i}}. \tag{37}$$

PROOF. From Lemma 1, both types of firms' output decreases as n increases. Thus, what matters is the magnitude of the decrease in output.

From equations 34 and 35, it is clear that

$$\frac{dq_u/dn}{dq_m/dn} = \frac{\frac{\partial p^i}{\partial Q^i}T_u}{\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right)T_m}. \tag{38}$$

Thus, $dq_u/dn > dq_m/dn$ if and only if 37 holds. ■

In inequality 37, T_u (T_m) measures how a U-form (an M-form) firm's marginal revenue changes with other firms' output. The expression $\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}$ measures how average revenue changes with output from the viewpoint of the marketing department of a U-form firm, and $\frac{\partial p^i}{\partial Q^i}$ measures how average revenue changes with output for an M-form firm manager. Thus, the left-hand side (right-hand side) measures the slope of the U-form (M-form) firm's reaction curve. If the U-form firm's slope is larger (smaller) than that of an M-form firm, its output contracts more (less) than the M-form firm.[8]

3. Equilibrium Organization Mode

In this section, the firms' equilibrium choice of organizational forms is studied. In equilibrium, no firm can make a profit by switching to a different organizational form. In addition, each firm should make a nonnegative profit in equilibrium.

The following lemma studies the impact of a firm's switching from the M-form to the U-form when the number of firms in the industry is fixed.

LEMMA 2. Under Assumptions 1-5, if an M-form firm switches to U-form, (i) except for the firm that switches from the M-form to the U-form, any other firm's output increases if and only if $\left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) < 0$; (ii) total output decreases if and only if $\left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) < 0$; (iii) every other firm's profit increases if and only if $\left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) < 0$.

PROOF. (i) Applying Cramer's rule, from equations 27 and 31, it can be shown that

$$\frac{dq_u}{dm} = -(q_m - q_u)\frac{\partial p^i}{\partial Q^i}\frac{T_u}{T}, \tag{39}$$

$$\frac{dq_m}{dm} = -(q_m - q_u)\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right)\frac{T_m}{T}. \tag{40}$$

From equation 39, $dq_u/dm > 0$ if and only if $q_m > q_u$. From equation 40, $dq_m/dm > 0$ if and only if $q_m > q_u$.

(ii) Differentiation of 19 with respect to m yields

$$\frac{dQ}{dm} = (q_u - q_m) + m\frac{dq_u}{dm} + (n - m)\frac{dq_m}{dm}. \tag{41}$$

Plugging equations 39 and 40 into 41 yields

$$\frac{dQ}{dm} = -\frac{\partial p^i}{\partial Q^i}\left(\frac{\partial p^i}{\partial Q^i} + \frac{\partial p^i}{\partial Q^j} - \frac{\partial t}{\partial q_u}\right)\frac{(q_u - q_m)}{T}. \tag{42}$$

[8] If the demand is linear, equation 37 degenerates to $2b + 3d > 0$.

From 42, $dQ/dm < 0$ if and only if $q_m > q_u$.

(iii) Differentiation of 20a with respect to m yields

$$\frac{d\pi_u}{dm} = 2(p-c)\frac{dq_u}{dm} + 2\left(\frac{\partial p}{\partial Q^i} + \frac{\partial p}{\partial Q^j}\right)\frac{\partial Q}{\partial m} q_u > 0.$$

Differentiation of 20b with respect to m yields

$$\frac{d\pi_m}{dm} = 2(p-c)\frac{dq_m}{dm} + 2\left(\frac{\partial p}{\partial Q^i} + \frac{\partial p}{\partial Q^j}\right)\frac{\partial Q}{\partial m} q_m > 0. \blacksquare$$

The intuition behind Lemma 2 is the following: part (i) of Lemma 2 comes from the fact that reaction functions have a negative slope. As a U-form firm produces less than an M-form under the condition that $\left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) < 0$, each firm will produce more when the number of U-form firms increases. For part (ii), there are two effects on industry output when an M-form firm switches to the U-form. First, every other firm produces more. Second, the number of M-form firms decreases. An M-form firm produces more than a U-form firm and the two effects work on opposition directions. Part (ii) of Lemma 2 says that the second effect dominates the first one. For part (iii), since total industry output decreases as the number of U-form firms increases, market price increases as m increases. When an M-form firm switches to the U-form, every other firm's output increases. Every other firm's profit increases as price also increases. If $\left(\frac{\partial p^y}{\partial Q^x} + \frac{\Delta}{\Delta_1}\right) > 0$, $q_u > q_m$. In this case, if a firm switches from the M-form to the U-form, the opposite results obtain.

4. Linear Demand

In this section, the case of linear demand is studied. This additional structure leads to stronger results. Also, the impact of a change in the degree of substitution can be parameterized and examined. The inverse demand function is given by equation 2. In this linear case, $\partial p^y/\partial Q^x + \Delta/\Delta_1 < 0$ requires that $2b + 3d > 0$. If $d > 0$, the two products are substitutes; if $d = 0$, the demand for the two products are independent; if $d < 0$, the two products are complements.

Equations 2, 14, and 17 lead to

$$q_u = \frac{b(a-c)}{(b+d)[b(m+3)+3(b+d)(n-m)]}, \tag{43a}$$

$$q_m = \frac{3(a-c)}{b(m+3)+3(b+d)(n-m)}.^{9} \tag{43b}$$

From equations 43a and 43b, when d increases, q_u and q_m decrease. Thus, each firm's output decreases with the degree of substitutability between the two products. A U-form firm's output decreases at a higher rate than that of an M-form firm.

Plugging equations 47a and 47b into equation 19 yields

$$Q = \frac{mb+3(n-m)(b+d)}{(b+d)[b(m+3)+3(b+d)(n-m)]}(a-c). \tag{44}$$

In a symmetric equilibrium, market prices of the two products are the same, $p^i = p^j = p$. Plugging 44 into 2 leads to

$$p - c = \frac{3b(a-c)}{b(m+3)+3(b+d)(n-m)}. \tag{45}$$

From equations 43a and 45, the profit of a U-form firm is

$$\pi_u = \frac{6b^2(a-c)^2}{(b+d)[b(m+3)+3(b+d)(n-m)]^2} - f. \tag{46a}$$

From equations 43b and 45, the profit of an M-form firm is

$$\pi_m = \frac{18b(a-c)^2}{[b(m+3)+3(b+d)(n-m)]^2} - f. \tag{46b}$$

The following proposition provides a necessary and sufficient condition for all firms to adopt the M-form (the U-form) organizational mode.

PROPOSITION 3. If demand is linear, all firms adopting the M-form mode is the unique equilibrium if and only if $2b + 3d > 0$. If $2b + 3d = 0$, a firm is indifferent between the two organizational forms. All firms adopting the U-form mode is the unique equilibrium if and only if $2b + 3d < 0$.

PROOF. From equations 46a and 46b, $\pi_m > \pi_u$ if and only if $2b + 3d > 0$. ■

As b is always positive, $d < 0$ is necessary for $2b + 3d < 0$. Thus, Proposition 3 highlights the role played by the coefficient d. There are some remarks for Proposition 3. First, for $d = 0$, $2b + 3d \geq 0$ is satisfied. Thus, all firms adopting the M-form mode is the unique equilibrium if the market demand for the two products is independent. Second, for $2b + 3d = 0$,

[9] When $m = 0$ and $d = 0$, from 43a, $q_m = (a-c)/(b(n+1))$. This is the usual Cournot output level when there are n firms in the industry. When $m = n$ and $d = 0$, from equation 43b, $q_u = (a-c)/(b(n+3))$. With the existence of double marginalization, this output is less than the usual Cournot output.

as firms are indifferent between the two organizational forms, the M-form mode coexists with the U-form mode. Finally, a U-form monopolist's profit is always less than an M-form monopolist's profit if products are substitutes. So a monopolist's profit is higher if it adopts the M-form even when goods are perfect substitutes ($b = d$). This is surprising, as we may have thought that either organization form has some disadvantages and no one form will dominate the other.

In the literature, there are some empirical and experimental studies comparing the performance of the U-form and the M-form firms. Studying a sample of petroleum firms during the period 1955-1973, Armour and Teece (1978) conclude that empirical results are broadly consistent with the M-form hypothesis which states that the M-form leads to a higher profit level than the U-form organizational mode. In Burton and Obel's (1988) experimental study, they conclude that the M-form hypothesis is supported. In a laboratory experiment in which students assumed the divisional and departmental manager roles, the M-form organization leads to greater total profits than the U-form under which format opportunistic behavior is possible.

5. Conclusion

If product quantities can be contracted on *ex ante*, organizational forms will not matter. In real world situations, many contingencies cannot be foreseen and contracted upon. As a result, organizational forms play important roles in affecting managers' incentives. A natural question is which factors determine a firm's optimal choice of organizational form. Both the U-form and the M-form organizational modes have advantages and disadvantages. In this paper, it is shown that a firm's optimal choice of organizational form is affected by market structure.

One important assumption in this paper is that firms provide homogenous products and engage in quantity competition. If firms produce differentiated products and engage in price competition, the result will be more complicated. As the type of reaction curve is only one of the two factors affecting a firm's organizational choice (the other one being the tradeoff between double marginalization and the ignorance of demand externalities), changing the type of competition is not likely to reverse the results established in this article. In the case of homogenous product, as firms receive the same price, a firm has a higher profit if and only if it has a higher level of output. With differentiated products, firms have different quantities of production and also different prices. It is difficult to compare the profit levels of different types of firms. For example, consider the case when the two products are substitutes. With the existence of double marginalization and the consideration that the two products are substitutes, the price of a U-form firm will be higher than that of an M-form firm. But it is unclear whether the profit of a U-form firm is higher or lower than that of an M-form firm.

In this paper, both organizational forms are assumed to have the same level of fixed cost of production. An alternative approach is that for a U-form firm, the fixed cost is f, while in an M-form firm, the fixed cost is $2f$. Thus a U-form firm has the benefits of economies of scale. One source of economies of scale may come from the large volume purchase of raw materials and joint marketing in a U-form firm. The existence of significant economies of scale may justify the existence of large U-form firms.

The model may be extended in various directions. First, in this article, each firm produces only two products. The case where a firm produces more than two products may be studied. Second, there are no demand or cost uncertainties. These kinds of uncertainties may play important roles in affecting firms' optimal choice of organizational forms. Finally, it has been assumed that marginal production cost and marginal marketing cost are constant. Generalization to a general cost function may be an interesting topic for future research.

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