Symmetry condition and re-assessing Blanchard-Kiyotaki decades later

Kim, Minseong

University of Illinois at Urbana-Champaign

12 October 2019

Symmetry condition and re-assessing Blanchard-Kiyotaki decades later

By Minseong Kim*

We re-assess Blanchard-Kiyotaki (1987). We conclude that there are multiple equilibria even under the flexible-price Blanchard-Kiyotaki model, and that the model only obtains equilibrium uniqueness through the symmetry condition that is partially justified only when there are infinitely many firms. Without imposition of the symmetry condition, monetary policy has a significant role in determining a flexible-price equilibrium under the Blanchard-Kiyotaki setup. While the Blanchard-Kiyotaki framework is becoming deprecated, the symmetry condition is still sometimes invoked in monopolistic competition literature, and thus logic behind it is in need of more scrutiny. We discuss implications on understanding New Keynesian paradoxes in zero lower bound circumstances.

JEL: B22, B41, E13, E30, E50, E60

Keywords: Blanchard-Kiyotaki, symmetry condition, New Keynesian economics, multiple equilibria, monetary non-neutrality, monopolistic competition, New Keynesian paradoxes, zero lower bound

* Minseong Kim: University of Illinois Urbana-Champaign, mkim146@illinois.edu

I. Introduction

Blanchard and Kiyotaki (1987), now more than three decades old, was one of the first papers in Keynesian reconstruction efforts in the DSGE framework. Even though the framework that it adopted is now becoming deprecated, there still are some traces of influence left in modern New Keynesian literature - for example, see Woodford (2003) and Galí (2015).

We argue that some of the conclusions in Blanchard and Kiyotaki (1987) that inspired later New Keynesian reconstruction efforts are not fully supported. The issue this paper raises is the symmetry condition assumed to get a unique flexible-price equilibrium in the Blanchard-Kiyotaki model. Initially, the condition seems innocuous, but more scrutiny reveals that it is not.

Thus, the re-assessment of the Blanchard-Kiyotaki model, despite becoming deprecated, has some relevance in modern macroeconomics methodology. In monopolistic competition models, the symmetry condition argument still is used to prune out some of possible equilibria. If the symmetry condition is not justified, then while this does not require a fundamental change of modern macroeconomics, a few conclusions coming from some models would be overturned. Furthermore, multiple equilibria and monetary non-neutrality are much more common than usually considered.

II. Analysis of the flexible-price Blanchard-Kiyotaki model

The point behind imposing the symmetry condition is that when firms and households are identical in their characterizations, then their equilibrium values must be the same. That is, their production Y_i must be equal such that $Y_i = Y \ \forall i$, price $P_i = P \ \forall i$.

This point seems obviously true and innocuous. But we argue that identical characterizations do not mean equilibrium outcomes are identical.

Let us state the simplified flexible-price Blanchard-Kiyotaki model. For convenience, we assume that an economy is deterministic, but conclusions of analysis here applies to stochastic cases without loss of generality.

The representative consumer has utility function U that it maximizes:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_t}^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where β is time preference discount factor, C_t is consumption, N_t is labor utilized. It is subject to the budget constraint:

$$P_t C_t + \frac{B_t}{1+i_t} \leq W_t N_t + F_t + B_{t-1} \tag{2}$$

where P_t is price level, B_t is central bank-issued bonds, i_t is nominal interest rate set on B_t, W_t is wage, and F_t is dividends paid from firms.

There is monopolistic competition in an economy - we apply the standard CES toolkit, such that:

$$C_t \equiv \left(\int_0^1 {C_{it}}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where C_{it} is consumption of goods at firm i. Price level P_t is defined such that $P_t C_t = \int_0^1 P_{it} C_{it} \, di$. In equilibrium, $Y_t = C_t$ and $C_{it} = Y_{it}$, and thus from now on, we will use Y and C interchangeably.

The resulting price level and demand function for Y_{it} are:

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}} \tag{4}$$

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t \tag{5}$$

Now specification of firms. Firms are assumed to utilize homogeneous labor, its only production factor, such that wage level must be same across firms. Firms have production function:

$$Y_{it} = A_t N_{it}^{1-\alpha} \tag{6}$$

with $\int_0^1 N_{it}\,di = N_t$. Firms maximize profits F_t, which are all given out as dividends:

$$F_t = P_{it} Y_{it} - W_t N_{it} \tag{7}$$

Each firm selects P_{it} to maximize profit, given its demand function for Y_{it}. Since firm i is considered of negligible size, we assume that change of P_{it} does not affect P_t and Y_t. Firms take W_t as given.

The profit maximization solution says that:

$$\begin{aligned} P_{it} &= \frac{\varepsilon}{\varepsilon-1} MC_t \\ &= \frac{\varepsilon}{\varepsilon-1}\frac{1}{1-\alpha}\frac{W_t}{{A_t}^{1/(1-\alpha)}} {Y_{it}}^{\frac{\alpha}{1-\alpha}} \\ &= \frac{\varepsilon}{\varepsilon-1}\frac{1}{1-\alpha}\frac{W_t}{{A_t}^{1/(1-\alpha)}} \left[\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t\right]^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{8}$$

where MC_t refers to marginal cost, when total cost is $W_t N_{it} = W_t (Y_{it}/A_t)^{1/(1-\alpha)}$. Thus,

$$(P_{it})^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{\varepsilon-1}\frac{1}{1-\alpha}\frac{W_t}{{A_t}^{1/(1-\alpha)}} (P_t)^{\frac{\varepsilon\alpha}{1-\alpha}} (Y_t)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

Because we assumed P_t and Y_t are not changed by individual firm decisions due to firm size being negligible, Equation (9) suggests that every firm must have the same equilibrium, even if we do not impose the symmetry condition externally.

But is this really correct assessment?

There are two things to note. First, in this economy, number of firms are infinite. Second, before we get an equilibrium, there is possibility that P_{it} affects P_t or Y_{it} affects Y_t if its value reaches infinity. The question is whether we can really eliminate such an equilibrium.

One may say, "why not?" After all, infinity results are nonsense. But remember that this is an infinite-number-of-firms economy, and no one really thinks that there are realistically infinite number of firms. We only do it for model tractability. Thus we need to deal with a finite-number-of-firms economy, and then consider back the infinite-number-of-firms economy.

Change Equation (3) to be:

$$C_t \equiv \left(\sum_i C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{10}$$

Price level P_t, defined with $P_t C_t = \sum_i P_{it} C_{it}$, is:

$$P_t = \left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{11}$$

and demand function for firm i is as in Equation (5):

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$$

A. Why multiple equilibria prevail

Now the intuition is clear: because Y_t and P_t are now each affected by Y_{it} and P_{it}, P_{it} does depend on value of Y_{it} in the solution of the profit maximization problem, unlike Equation (9). Thus, there will be multiple equilibria.

Firm i's price-setting function would be, substituting wage demand (labour supply) function coming from the consumer optimization problem and production function:

$$P_{it} = f_t(Y_{it}, \{Y_{jt}\}_{j\neq i}, \{P_j\}_{j\neq i}) \tag{12}$$

where f_t refers to a function. Recall the demand function for firm i in Equation (5):

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t$$

There are $2n-1$ equations when there are n firms - $n-1$ equations from Equation (12) and n equations from Equation (5)) - having set one of P_{it} to be 1 or some constant value. There are $2n$ variables - $n-1$ instances of P_{it}, n instances of Y_{it} and Y_t.

P_t is determined from $\{P_{it}\}$. Y_t can be determined from $\{Y_{it}\}$, but if we substitute Y_t with Equation (10), then since $Y_{it} \in \{Y_{it}\}$ and given the form of Equation (10), we would not be able to write the demand function in form of:

$$Y_{it} = g_t(\{P_{jt}\}, \{Y_{jt}\}_{j\neq i})$$

Furthermore, by construction, Equation (11) and Equation (5) replicate

Equation (10). The proof goes as follows:

$$
\begin{aligned}
C_t &= (P_t C_t) P_t^{\varepsilon-1} \left[\left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \right]^{-\varepsilon} \\
&= (P_t C_t) P_t^{\varepsilon-1} \left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i \left(P_{it}^{-\varepsilon} P_t C_t P_t^{\varepsilon-1} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i \left(\left[\frac{P_{it}}{P_t} \right]^{-\varepsilon} C_t \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned} \tag{13}
$$

The first line in Equation (13) follows from the definition of P_t in Equation (11). The second, third and fourth line are tautological. The fifth line follows from Equation (5).

Thus there are multiple equilibria, since there are only $2n - 1$ equations for $2n$ variables.

B. *A unique symmetric equilibrium*

We now consider the case when equilibrium variables of firms are assumed to be symmetric (identical).

Instead of writing as Equation (12), price-setting function may be written instead as, having substituted production function:

$$
\frac{W_t}{P_t} = h_t(Y_t) \tag{14}
$$

where h_t refers to a function. Equation (14) does not have the wage demand (labour supply) function substituted.

The wage demand (labour supply) function now needs to be stated, having substituted production function:

$$\frac{W_t}{P_t} = \eta_t(Y_t) \tag{15}$$

where η_t refers to a function. The interaction of wage demand and supply function interacts to allow for possibility of a unique symmetric equilibrium - given value of P_t, we can obtain W_t and Y_t (two variables) from these two equations.

C. *Summary: multiple equilibria unless the symmetry condition*

Thus, the symmetry condition has to be externally imposed in a finite-number-of-firms economy in order to secure possibility of a unique equilibrium. Otherwise, we generally have multiple equilibria. But since the symmetry condition cannot be derived, its justification is weak. We revisit the justification issue soon.

In the infinite-number-of-firms limit, it is true that asymmetry has to be extreme (infinite) for an equilibrium other than a unique symmetric equilibrium. However, it is still true that these perverse equilibria are not eliminated. We have to eliminate the perverse equilibria by imposing the no-infinity condition or the symmetry condition. Again, one must consider the point that no one really thinks an economy has infinitely many firms.

D. Justification for the symmetry condition?

One may still argue that the symmetry condition is justified. After all, firms are same in their characterizations! Would not firms then have to face the same circumstance?

But the point is that firms are not really the same. After all, the point of monopolistic competition is that each firm has something different from others - the reason why it gets limited monopoly power or price setting power. Even when production function is the same, products from different firms are not identical, despite math possibly not clearly identifying this fact.

E. Monetary (non-)neutrality

As matter of monetary neutrality goes, if we restrict to single-period analysis, there is still not much monetary policy can do, and monetary neutrality holds, despite equilibrium non-uniqueness.

However, when we move onto multiple-period analysis, where nominal interest rate i_t comes to make sense, circumstances do change. This can be seen in the canonical New Keynesian Euler equation, derived from the consumer optimization problem of our simplified Blanchard-Kiyotaki model, which is shared in both the flexible and sticky price case:

$$\left(\frac{Y_t}{Y_{t+1}}\right)^{-\sigma} = \beta(1+i_t)\frac{P_t}{P_{t+1}} \tag{16}$$

Now central bank can affect an economy by affecting growth of Y_t through i_t. Thus there is monetary non-neutrality.

III. Conclusion

This paper re-assessed the Blanchard-Kiyotaki framework - its methods and assumptions. While the framework is becoming deprecated even in macroeconomic literature, there are still some traces of the framework left. For example, one may say that the conclusion of Blanchard and Kiyotaki (1987) that frictions, such as sticky price, are needed to generate monetary non-neutrality still provides intuitive benchmark assessment in New Keynesian economics that the optimal goal of monetary policy is about reducing impacts of frictions to an otherwise frictionless economy.

What was shown is that once we carefully examine the finite-number-of-firms circumstances and justifications behind the symmetry condition, the conventional assessments of the Blanchard-Kiyotaki model are not justified. Multiple equilibria prevail, and due to the consumption Euler equation, monetary non-neutrality is there in the flexible-price Blanchard-Kiyotaki model. This supports the general assessment that multiple equilibria and monetary non-neutrality are much more common in a monopolistic competition world, even when not affected by frictions, in contrast to a perfect competition world.

In terms of understanding New Keynesian paradoxes in zero lower bound (ZLB), especially the ones about the flexible-price limit of sticky price economies not converging toward actual flexible-price economy - see Werning (2012), Kiley (2016) and Cochrane (2017) - this paper suggests that paradoxes may actually partially come from the underlying flexible-price model. Thus we have to reconsider validity of the paradoxes - more specifically, in what class of models do the paradoxes still exist?

Monetary policy more generally may not just be about correcting frictions, but also about choosing a baseline frictionless equilibrium.

REFERENCES

Blanchard, Olivier Jean, and Nobuhiro Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *The American Economic Review*, 77(4): 647–666.

Cochrane, John H. 2017. "The new-Keynesian liquidity trap." *Journal of Monetary Economics*, 92: 47 – 63.

Galí, Jordi. 2015. *Monetary Policy, Inflation, and the Business Cycle.* Princeton University Press.

Kiley, Michael T. 2016. "Policy paradoxes in the New Keynesian model." *Review of Economic Dynamics*, 21: 1 – 15.

Werning, Ivan. 2012. "Managing a Liquidity Trap: Monetary and Fiscal Policy." Manuscript.

Woodford, Michael. 2003. *Interest and Prices.* Princeton University Press.