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# **A Search Theoretic Model of Part-Time Employment and Multiple Job Holdings**

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28 August 2019

Online at <https://mpra.ub.uni-muenchen.de/97003/>

MPRA Paper No. 97003, posted 07 Jan 2020 13:50 UTC

ESSAYS ON MACROECONOMICS  
AND LABOR ECONOMICS

A Dissertation

Submitted to the Faculty

of

Purdue University

by

Andrew D. Compton

In Partial Fulfillment of the

Requirements for the Degree

of

Doctor of Philosophy

May 2019

Purdue University

West Lafayette, Indiana

## ABSTRACT

Compton, Andrew D. PhD, Purdue University, May 2019. Essays on Macroeconomics and Labor Economics . Major Professor: Trevor S. Gallen, Victoria Prowse, John M. Barron, and Seunghoon Na.

This paper develops a search-matching model of the labor market with part-time employment and multiple job holdings. The model is calibrated to data from the CPS between 2001 and 2004. Workers are able to choose their search intensity and are allowed to hold two jobs while firms can choose what type of worker to recruit. When compared to the canonical Diamond-Mortensen-Pissarides model, this model performs quite well while capturing some empirical regularities. First, the model generates recruiting and vacancy posting rates that move in opposite directions. Second, part-time employment is up to 10 times more responsive than full-time employment. Third, the model suggests that multiple job holding rates are more flexible than observed in the data with the rate changing by as much as 4 percentage points compared to 0.1 percentage points in the data. Finally, the full model is able to capture compositional changes during recessions with the full-time rate declining and the part-time rate increasing. It also produces an empirically consistent increase in the unemployment rate as well as a decrease in output. The DMP model is more muted than in the data for both.

## 0.1 Introduction

Between 1996 and 2014, roughly 20% of the labor force were part-time workers or worked multiple jobs.<sup>1</sup> Faberman et al. (2017) suggest that a worker's job

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<sup>1</sup>See Appendix ??.

prospects, search behavior, and firm recruiting differ based on the workers employment status. Further, they suggest that roughly 55% of workers would be willing to take an additional job under the right circumstances.<sup>2</sup> Together, these facts suggest that modelling all jobs as full-time (FT) jobs overlooks some fundamental features of the labor market. Overlooking part-time (PT) employment and multiple job holdings (MJH) takes on greater importance given the persistently high rate of part-time employment following the 2007-2009 recession, which has become an area of growing interest. Valletta and van der List suggest the part-time employment rate is higher now than in the past, even with business cycles and industry accounted for (Economic Letters - FRBSF, 2015-19) In particular, they show that part-time employment for economic reasons is around one percentage point higher than in the past under similar circumstances, suggesting there may have been a structural change in the labor market. Further, the persistently high rate of part-time employment has prompted the Federal Open Market Committee to consider the part-time employment rate in addition to the unemployment rate when gauging the status of the labor market (FOMC minutes from July 2016).

The goal of this paper is to explore the relationship between search frictions, part-time employment, and multiple job holdings. Part-time employment is closely related to multiple job holdings as search frictions can make taking a part-time job and looking for a secondary full-time job more attractive to workers. On the other hand, firms are also considering who they will recruit and hire and may look more or less favorably on multiple job holders and part-time workers depending on how difficult it is to match with a given type of worker. For instance, if firms observe that it is difficult to match with an unemployed worker, they may be more willing

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<sup>2</sup>This rate is comparable to the rate for employed workers looking for a new job and to findings from Paxson and Sicherman (1996) who suggest that 50% of all workers will have multiple jobs at some point in their life.

to accept a multiple job holder. In both cases, workers and firms can use part-time employment and multiple job holdings to smooth their expected outcomes.

To this end, I develop a model building on the Diamond-Mortensen-Pissarides (DMP) framework, extended to allow for PT employment and MJH. In the canonical DMP model, all jobs are assumed to be full-time and require the same skill set. In addition, workers can only hold one job. Extensions of the DMP model have allowed for variable productivity and skills, but there has been limited research allowing for both variable hours and multiple job holdings. In my model, workers search for full-time and part-time jobs simultaneously. In addition, they can hold multiple jobs. Firms can recruit part-time and full-time workers simultaneously as well as multiple job holders.

Separately calibrating my model to U.S. labor market data from December 2001 to December 2004 and January 2015 to December 2016, the model compares favorably to the DMP model and a model with only multiple job holdings along shared dimensions. Using data on job loss probabilities and the consumer Price Index for the 2007-2009 recession, the full model performs well at matching the unemployment rate response of 1.70 pp in the data with a difference of 2.64 pp compared to a difference of 0.2 pp in the DMP model. The results also suggest that multiple job holdings could be more responsive than what is observed with an decrease of 4 pp in the model compared to a decrease of 0.03 pp in the data. As suggested by Abraham et al. (2013), multiple job holdings can be difficult to observe in the data as workers and firms each have different incentives to report. This could be why it is difficult to establish the relationship between economic conditions and multiple job holding rates as the CPS may under-report or over-report depending on conditions.

Additionally, the standard DMP model is qualitatively consistent with the relationship between the unemployment rate and the full-time employment rate (a

negative correlation of  $-0.66$ ); however, the base model cannot say anything about the relationship between the unemployment rate and the part-time rate (A positive correlation of  $0.51$ ). By considering all jobs to be full-time jobs, the implications of policy and structural changes in the standard DMP model may not be consistent with a model that explicitly separates full-time and part-time employment. Full-time and part-time employment can respond differently, but because the full-time employment rate is over four times as large as the part-time employment rate, the response in FT employment dominates. In the full model, the part-time rate is typically 10 times more responsive than the full time rate in percentage terms, and up to twice as responsive in percentage point terms. The part-time employment rate does not always move in the same direction as the full-time employment rate as in the case of a change in recruiting cost. Multiple job holdings can also affect the observed part-time rate as in the case of recessionary conditions that lead to a higher than expected part-time rate as a result of a much lower multiple job holding rate. Altogether, this suggests that part-time employment and multiple job holdings can have implications for policy depending on who the policymaker cares about.

The next subsection discusses related literature followed by relevant stylized facts from the data. In Section ??, a simple environment and equilibrium with multiple job holdings, but only one type of job, is described. Section ?? presents the environment with both PT employment and multiple job holdings as well as the corresponding steady-state equilibrium. In Section ??, the model is calibrated to match U.S. data from December 2001 to December 2005. Finally, results are presented in Section ?? and discussed in section ??.

### 0.1.1 Related Literature

This paper builds on three branches the literature. First, I consider the literature on joint-search and multiple job holdings. Guler, Guvenen, and Violante (2012) construct a one-sided joint-search model of a household in which two individuals with pooled utility and consumption search for and hold jobs simultaneously. I extend their model to allow for firm choice regarding whether to hire a secondary worker and household and firm choice regarding FT versus PT employment.

Zhao (2016) extends the GGV model to multiple job holders and suggests that the opportunity to hold multiple jobs makes holding part-time work more valuable as it provides workers with more opportunities to find full-time work and smooth their income over states. I extend this model by introducing firms to the problem, and discuss the role of search and recruiting behavior in this context. Since previous work is only one sided, nothing can be said about the impact of firm-side labor market policy on employment status, worker flows, and search behavior. I am able to evaluate not only how policies affect workers, but also how firm choices affect workers. Firms prove to be especially important for determining outcomes.

The second branch of the literature is on search intensity. Pissarides (2000) provides a simple model for thinking about search intensity, but the model implies that workers should search less when the labor market is slack. To reconcile this with the fact that workers search harder under slack labor market conditions, Shimer (2004) uses an urn-ball matching function to induce higher search. I extend this framework by allowing workers to take two jobs simultaneously in a simplified framework. I also introduce multiple matching functions depending on a worker's current employment status and the job they are searching for.

Finally, I consider the literature on on-the-job search. Building off of Burdett and Mortensen's (1998) seminal work on on-the-job search, Christensen et al. (2005),

Hornstein, Krusell, and Violante (2011), and Faberman et al. (2017) have introduced variable search intensity into models of on-the-job search. As in these papers, I allow for on-the-job search as well as variable search effort and success depending on a worker's employment status. Faberman et al. is especially pertinent as they find that employed workers differ from unemployed workers in their search behavior and firms recruiting to them differently as well. This is in line with Davis, Faberman, and Haltiwanger (2013) who find that firms often use informal recruiting methods when hiring workers. I include these elements by allowing for variable search intensity and recruiting intensity depending on a worker's current state. I extend these models by introducing firms which allows for endogenous wages. This generates differential wages across employment status which supports the fact that employed workers face a different offer distribution than unemployed workers. Finally, Gavazza, Mongey, and Violante (2018) find that firm recruiting intensity and vacancy rate move in opposition such that during recessions, vacancies decline while recruiting intensity increases. Similar behavior occurs in the presence of multiple job holding and part-time employment. While aggregate vacancies may decline, changes in recruiting intensity can increase the effective vacancy rate for certain types of jobs.

### **0.1.2 Stylized Facts**

My model targets four facts from the Survey of Consumer Expenditure (SCE) used by Faberman et al. (2017). First, 55.5% of workers would be willing to take an additional job and 68.4% who would be willing to take a new job. Second, full-time workers on average have 1.18 jobs compared to 1.41 jobs for part-time workers ( $t = 5.1647$ ). These two facts suggest that multiple job holdings is an option that workers consider when making employment decisions, and that it is particularly relevant for part-time workers. Third, workers who would be willing to take an additional job



send out 1.13 applications per month which yield 0.5 contacts per month in addition to 1.55 unsolicited contacts per month. This is compared to unemployed workers who send out 6.97 applications per month and yield 0.72 contacts per month in addition to 0.57 unsolicited contacts per month. Those seeking an additional job not only display different job search behavior, but their success appears to be different from unemployed workers. Firms also seek out multiple job holders more actively than they do unemployed workers. While some of these differences are likely due to signaling and skill, some are likely due to matching frictions. These facts reinforce the notion that search by employed individuals is different than for unemployed individuals.

Finally, part-time workers send out twice as many job applications per month as do full-time workers, however, the success rate for full-time workers is higher with both receiving roughly 0.57 contacts per month. In addition, full-time workers receive 1.76 unsolicited contacts per month compared to 1.16 per month for part-time workers. This suggests that part-time worker search behavior is different from full-time worker search behavior and that the search frictions they face differ to a degree. In the canonical DMP model, all of these workers are treated the same despite having different search behavior and outcomes. If firms and workers are interacting differently depending on the worker's status, then the implications of the DMP model may miss compositional changes among the employed. To address this, my model treats unemployed workers, full-time workers, part-time workers, and multiple job holders differently to account for these differences.

Next, consider two stylized facts from the Current Population Survey (IPUMS-CPS). First, I calculate monthly worker flows<sup>3</sup> for potential workers aged 25-54 using the CPS. The structure of the CPS shows the employment status of workers for two four month periods, allowing for two sets of three monthly transitions. Focusing

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<sup>3</sup>The method used to derive worker flows is provided in Appendix ??

primarily on the time period from 2006 to 2012<sup>4</sup>, the number of workers moving from unemployment to part-time employment increases despite a drop in the flow rate. Since the stock of unemployed workers is getting larger and firms and workers are shifting their search and recruiting to part-time work, the increased stock outweighs the lower job finding rate for workers.<sup>5</sup> While the 2001 recession seems to generate similar trends, the limited time frame makes it more difficult to parse from the overall trend.

Second, there is persistence in flows to part-time for economic reasons during the entire sample period. Flows between unemployment and part-time employment for economic reasons increased throughout the 2000s, and peaked following the 2007-2009 recession. While the monthly flow counts have tapered off, they are still at an elevated level compared to the beginning of the recession. Many workers would like full-time employment, but are unable to find anything other than part-time employment suggesting there may have been a structural shift during the 2000s as suggested by Valletta and van der List (2015). Part-time work can be used by workers to smooth income while they search for a full-time job. This could lead to multiple job holdings as workers are not necessarily working the hours they desire.

Altonji and Paxson (1988) theorize workers switch jobs when faced with hourly constraints and are more willing to accept a pay cut. An alternative theory by Perlman (1966) and Shishko and Rostker (1976) suggests workers respond by holding

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<sup>4</sup>While the NBER limits the most recent recession to 2007-2009, a wider time period captures the entrance and exit from the trough.

<sup>5</sup>The flows between full-time and part-time employment as seen in Figure ??, are substantial month-to-month with an average of 3.7 million workers becoming part-time and 3.8 million becoming full-time. These transitions dwarf the other values combined. Warren (2016) provides a nice explanation for what is happening in this case by showing that firms facing search frictions and recruiting costs can find it optimal to switch workers between part-time and full-time rather than firing them in response to productivity shocks. My model will not be able to account for this effect as I perform my analysis using steady-states which do not allow for such a transition to exist endogenously in equilibrium. While I have considered the case of variable worker-firm productivity, the problem space becomes very intractable making it difficult to interpret.

second jobs. Paxson and Sicherman (1996) find workers faced with hourly constraints will respond by holding multiple jobs; However, the multiple job holding rate is acyclical. This suggests that the while there may be more workers willing to work multiple jobs, the contraction in the number of vacancies can cancel out any effect. It also suggests that it is caused by some structural elements of the economy that should not necessarily be overlooked, especially when looking at part-time employment.

Overall, it appears that unemployed, part-time, and full-time workers behave differently and have different labor market outcomes. They not only search in different ways, but they receive job offers in different ways as well suggesting firms view each type of worker differently. To get at some of these facts, I model the search frictions and trade-offs that workers and firms face when choosing search and recruiting intensity. I allow for these choices to be made based on current status and the desired job type. Once the equilibrium is described and the model is calibrated, I perturb some of the structural components of the model to see how they affect part-time employment and other labor market outcomes.

## **0.2 Multiple Job Holdings**

In order to get a better understanding of the choices that workers are making, I consider a simplified framework with workers who can hold two jobs simultaneously, but there is no distinction between part-time and full-time employment. Multiple job holdings is particularly relevant when considering part-time employment as part-time workers are more likely to have multiple jobs and the option of additional employment increases the flow value of being a part-time worker. An unemployed worker's goal is to get at least one job offer and then search for an additional job once employed.

## Environment

Consider a discrete time job search model where time goes on forever. There is a continuum of infinitely lived, risk-neutral workers with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t y_t$$

where  $y_t$  is the worker's instantaneous income at time  $t$  and  $\beta \in (0, 1)$  is the discount factor.

Workers can be in one of three states: unemployed, employed with one job (primary), or employed with two jobs (primary and secondary). While unemployed or employed in one job, workers are assumed to always be searching for a job, but can choose the intensity with which they search. Unemployed workers can choose search intensity  $s_1$ , but they must pay weakly convex search cost  $\sigma_1(s_1)$ , while workers with one job choose their search intensity  $s_2$ , but pay weakly convex search cost  $\sigma_2(s_2)$ . While unemployed, workers receive some value of leisure  $z_1$ . If a worker is employed in one job, they receive some residual value of leisure  $z_2$  as well as primary wage  $w_1$ . If a worker has two jobs, then they receive both the primary wage  $w_1$  and the secondary wage  $w_2$ , but they have no residual value of leisure. There is one type of firm that can be in one of three states: vacant, employing one primary worker, or employing one secondary worker. Firms can post vacancies  $v$  and choose whether to recruit to unemployed workers ( $a_1$ ) or employed workers ( $a_2 = 1 - a_1$ ) while paying weakly convex recruiting cost  $C(a_1, a_2)$ . Firms that hire a worker of type  $i \in \{1, 2\}$  receive output  $p_i$  and pay wage  $w_i$ .

Workers and firms are matched pairwise according to two constant returns to scale (CRTS) matching functions, one for the primary jobs and one for secondary jobs. As before the matching function depends on the effective mass of firms ( $a_i v$ )

and the effective mass of workers ( $s_i l$ ) where  $l \in \{u, l_1\}$ . The rate at which matches of type  $i \in \{1, 2\}$  are formed between a firm and worker is given by

$$m_i(\bar{s}_i l, a_i v)$$

where the effective mass of workers depends on average search intensity over all workers  $\bar{s}_i$  and  $l_i \in \{u, l_1\}$ . As a simplification, denote average market tightness such that  $m_i(\bar{\theta}_i) = m_i(\bar{s}_i l, a_i v)$  and denote individual market tightness as  $m_i(\theta_i) = m_i(s_i l, \bar{s}_i l, a_i v)$ . An unemployed worker who chooses to search with intensity  $s_1$  matches with at least one firm with probability

$$q_1(\theta_1, u) = \frac{m_1(\theta_1)}{u}$$

upon which they can only accept one job offer. Notice that the probability of a match depends on individual search intensity in addition to average search intensity. Similar to unemployed workers, an employed worker who chooses to search with intensity  $s_2$  matches with at least one firm with probability

$$q_2(\theta_2, l_1) = \frac{m_2(\theta_2)}{l_1}$$

which also depends on individual and average search intensity.<sup>6</sup> The probability that a firm fills their vacancy with a primary worker is

$$p_1(\bar{\theta}_1, v) = \frac{m_1(\bar{\theta}_1)}{v}$$

which does not depend on the individual level of search intensity. Finally, the probability that a firm fills their vacancy with a secondary worker is given by

$$p_2(\bar{\theta}_2, v) = \frac{m_2(\bar{\theta}_2)}{v}$$

which does not depend on individual search intensity as before.

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<sup>6</sup>In equilibrium,  $\bar{s}_i = s_i$  and  $\bar{\theta}_i = \theta_i$  since all workers are ex-ante homogeneous.

After a worker and firm match for a primary job, they face some risk that the job is destroyed with probability  $\lambda_1$ , in which case the worker transitions to unemployment, and the firm decides whether to post a vacancy. Should they match for a secondary job, then workers and firms face risk that the job is destroyed with probability  $\lambda_2$ , in which case the worker transitions to holding one job and searching for a second, and the firm decides whether to post a vacancy. In addition, secondary firms also have some risk of becoming a primary employer if their employees primary job is destroyed with probability  $\lambda_1$ . While the job destruction probability is denoted differently for each job type,  $\lambda_2$  could theoretically equal  $\lambda_1$ .

## **Equilibrium**

At the beginning of each period, workers and firms find out if they matched. If a an unemployed worker matches with one firm, they agree with the firm on the primary wage. If an employed worker matches with a firm, they agree with the firm on the secondary wage. A worker with one job receives the primary wage as well as the residual value of leisure, and they choose their search intensity while paying some search cost. If a worker is employed in two jobs, they receive both the primary and secondary wage. If a worker remains unemployed, they receive the instantaneous value of leisure and choose their search intensity while paying some search cost. If a firm matches with a worker of type  $i$ , they receive the corresponding value of output and pay the corresponding wage. If a firm remains vacant, they choose their recruiting intensity and pay some recruiting cost.

## Firm's Problem

Firms start by posting a vacancy and choosing recruiting intensity  $a_1$  for unemployed workers and recruiting intensity  $a_2 = 1 - a_1$  for employed workers. Firms choose their recruiting intensity to maximize flow value

$$V = \max_{a_1} \left\{ -C(a_1, a_2) + \beta V + \beta p_1(\bar{\theta}_1, v)[J_1 - V] + \beta p_2(\bar{\theta}_2, v)[J_2 - V] \right\} \quad (1)$$

where they pay recruiting cost  $C(a_1, a_2)$  and match with a primary worker with probability  $p_1(\bar{\theta}_1, v)$  and with a secondary worker with probability  $p_2(\bar{\theta}_2, v)$ . If they match with a primary worker, they receive flow value

$$J_1 = x_1 - w_1 + \beta J_1 + \beta \lambda_1 [V - J_1] \quad (2)$$

where instantaneous income is the value of output  $x_1$  and instantaneous cost is wage  $w_1$ . They also face some risk that the job is destroyed with probability  $\lambda_1$  in which case they choose whether to open a vacancy. If they match with a secondary worker, they receive flow value

$$J_2 = x_2 - w_2 + \beta J_2 + \beta \lambda_2 [V - J_2] + \beta \lambda_1 [J_1 - J_2] \quad (3)$$

where instantaneous income is the value of output  $x_2$  and instantaneous cost is wage  $w_2$ . They also face some risk that the job is destroyed with probability  $\lambda_2$  in which case they choose whether to open a vacancy. They also face some risk that their employees primary job is destroyed with probability  $\lambda_1$  in which case they become the primary employer. Because the goods market is perfectly competitive, firms will post vacancies until the flow value of posting an additional vacancy  $V = 0$  in equilibrium.

The choice of recruiting intensity for full-time workers  $a_f$  is given by equation (??)

$$\frac{\partial C}{\partial a_1} = \beta \frac{\partial p_1(\bar{\theta}_1, v)}{\partial a_1} J_1 + \beta \frac{\partial p_2(\bar{\theta}_2, v)}{\partial a_1} J_2$$

where firms do not account for the effect that recruiting has on wages. As the firm increases their recruiting intensity for unemployed workers they pay some direct cost of recruiting denoted on the LHS. On the RHS, there are both costs and benefits. First, there are gains to the probability that the firm matches with an unemployed worker, but costs due to a fall in the probability of matching with an employed worker.

### Worker's Problem

Unemployed workers receive some value of leisure  $z_1$  and choose their search intensity in order to maximize the flow value

$$U = \max_{s_1} \left\{ z_1(1 - h_1(s_1)^\nu) + \beta(U + q_1(\theta_1, u)[E_1 - U]) \right\} \quad (4)$$

where they pay search cost  $z_1 h_1(s_1)^\nu$ . Their choice of search intensity affects the probability that they match with one firm for a primary job with probability  $q_1(\theta_1, u)$ . If a worker has only one job, they choose their search intensity to maximize their flow value

$$E_1 = \max_{s_1} \left\{ w_1 + z_2(1 - h_2(s_2)^\nu) + \beta E_1 + \beta \lambda_1 [U - E_1] + \beta q_2(\theta_2, l_1) [E_2 - E_1] \right\} \quad (5)$$

where wage  $w_1$  and residual value of leisure  $z_2$  is their instantaneous income. They face some risk that the job is destroyed with probability  $\lambda_1$  in which case they become unemployed, and some probability  $q_2(\theta_2, s_2)$  that they match with a secondary firm and become a multiple job holder. If a worker has two jobs, they receive flow value

$$E_2 = w_1 + w_2 + \beta E_2 + \beta(\lambda_1 + \lambda_2)[E_1 - E_2] \quad (6)$$



where the wages  $w_1$  and  $w_2$  are their instantaneous income and they face some risk of losing one job with probability  $(\lambda_1 + \lambda_2)$  in which case they transition to single job holding.

The choice of search intensity  $s_1$  for an unemployed worker is given by equation (??)

$$z_1 h_1(s_1)^{\nu-1} = \beta \left( \frac{\partial q_1(\theta_1, u)}{\partial s_1} [E_1 - U] + q_1(\theta_1, u) \frac{\partial [E_1 - U]}{\partial s_1} \right) \quad (7)$$

where  $\alpha_1 = 1 - \beta(1 - \lambda_1)$ ,  $\alpha_2 = 1 - \beta(1 - \lambda_1 - \lambda_2)$ ,  $\alpha_{1u} = \alpha_1 + \beta(q_1(1) + q_1(2))$ , and  $\alpha_{21} = \alpha_2 + \beta q_2(1)$ . On the LHS, workers pay some direct cost from increasing search intensity  $s_1$  while the RHS denotes the indirect costs and benefits of increasing search intensity. If an unemployed worker increases their search intensity, they increase the probability of matching with any number of firms. The choice of search intensity  $s_2$  for an employed worker is given by equation (??).

$$z_2 h_2(s_2)^{\nu-1} = \beta \left( \frac{\partial q_2(\theta_2, l_1)}{\partial s_2} [E_2 - E_1] + q_2(\theta_2, l_1) \frac{\partial [E_2 - E_1]}{\partial s_2} - \beta \lambda_1 \frac{\partial [E_1 - U]}{\partial s_2} \right) \quad (8)$$

The LHS contains the direct cost of increasing search intensity  $s_2$  while the RHS contains both costs and benefits. Focusing on the RHS, the worker gains through an increase in the probability that they match with a secondary firm.

## Wage Determination

When a worker-firm match is formed, they bargain over the wage which reduces to the axiomatic Nash Bargaining solution.<sup>7</sup> Workers and firms have full information

<sup>7</sup>The Nash bargaining solution used here is introduced in Diamond (1982) and Pissarides (1984a). Justification for this solution is provided in Binmore, Rubinstein, & Wolinsky.

about each other and the worker has bargaining power  $\gamma$  while the firm has bargaining power  $1 - \gamma$ . Thus the wage for a job of type  $i \in \{1, 2\}$  is determined by

$$w_1 = \operatorname{argmax}(E_1 - U)^\gamma (J_1 - V)^{1-\gamma} \quad (9)$$

$$w_2 = \operatorname{argmax}(E_2 - E_1)^\gamma (J_2 - V)^{1-\gamma} \quad (10)$$

which yields a system of two equations for wages  $w_1$  and  $w_2$  as in equations (??) and (??).

$$w_1 = \frac{\gamma \alpha_{21} x_1 - (1 - \gamma) [\alpha_{21} (z_2 - z_1 + \sigma_1 - \sigma_2) + \beta q_2(\theta_2, l_1) (w_2 - z_2 + \sigma_2)]}{\alpha_{21}} \quad (11)$$

$$w_2 = \frac{\gamma \alpha_{1u} [\alpha_1 x_2 + \beta \lambda_1 (x_1 - w_1)] - (1 - \gamma) \alpha_1 [\alpha_{1u} (\sigma_2 - z_2) + \beta \lambda_1 (w_1 + z_2 - z_1 + \sigma_1 - \sigma_1)]}{\alpha_{1u} \alpha_1} \quad (12)$$

The wage for a job of type  $i$  depends not only on the surplus generated from creating a job of type  $i$ , but also on the surplus generated from creating a job of type  $-i$ . If  $x_2$  were to increase without a corresponding increase in  $x_1$ , then  $w_2$  would increase while  $w_1$  decreases. Similarly, if  $x_1$  were to increase,  $w_1$  would decrease while  $w_2$  decreases.

## Steady-State

**Definition 0.2.1** *The steady-state equilibrium consists of a list  $(u, l_1, v, a_1, w_1, w_2, s_1, s_2)$  that solves the unemployment flow equation*

$$q_1(\theta_1, u)u = \lambda_1(l_1), \quad (13)$$

*the single job holder employment flow equation*

$$[q_2(\theta_2, l_1) + \lambda_1]l_1 = q_1(\theta_1, u)u + (\lambda_1 + \lambda_2)l_2, \quad (14)$$

the job creation condition for vacancies

$$C(a_1, a_2) = \beta p_1(\theta_1, v) \left( \frac{x_1 - w_1}{\alpha_1} \right) + \beta p_2(\theta_2, v) \left( \frac{\alpha_1(x_2 - w_2) + \beta \lambda_1(x_1 - w_1)}{\alpha_1 \alpha_2} \right), \quad (15)$$

the firm's recruiting intensity maximization equation (??), two wage setting conditions (??) and (??), and the worker's two search intensity maximization equations (??) and (??).

When workers and firms have the option to hold multiple jobs, their choices depend on not only on their current employment status, but also their future employment status which could include a second job. Search intensity and recruiting intensity depend not only on the direct and indirect costs and benefits of searching/recruiting for a given type of job, but also the indirect costs and benefits for the other type of job. Even if the two types of jobs are identical in every way, the problem does not reduce to the standard DMP model unless multiple job holdings is turned off entirely. All together, this suggests that multiple job holdings should be considered alongside part-time and full-time employment.

### 0.3 Part-Time Employment and Multiple Job Holdings

Now that I have described the multiple job holding choice, I consider the full model with both multiple job holdings and a full-time/part-time choice. Because it is extremely rare to go from unemployed to multiple job holdings and vice versa, I do not allow a worker to accept more than one offer per period. This simplifies the model and the analysis without causing major problems since the probability is so small that excluding it will not have much effect on behavior.

## Environment

As before, consider a discrete time job search model where time goes on forever. There is a continuum of infinitely lived, risk-neutral workers with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t y_t$$

where  $y_t$  is the worker's instantaneous income at time  $t$  and  $\beta \in (0, 1)$  is the discount factor.

Workers can be in one of 6 states: unemployed, employed in a part-time job, employed in a full-time job, employed in a primary part-time job and secondary full-time job, employed in two part-time jobs, and employed in a primary full-time job and secondary part-time job. Each worker is endowed with 240 hours of time per month with a full-time job taking 160 hours and a part-time job taking 80 hours.<sup>8</sup> Workers can search for a job until they have at most two jobs. There is one type of firm that can be in one of 6 states: vacant, employing a primary, secondary, or dual part-time worker, and employing a primary or secondary full-time worker. Firms can only employ one worker regardless of how much time the job takes. With endogenous wages and endogenous recruiting intensity, firms are indifferent between hiring a worker for a full-time job or a part-time job.

Workers and firms are matched pairwise according to five CRTS matching functions depending on the worker's state. Vacant firms choose their recruiting intensity for full-time and part-time jobs as well as for primary and secondary job holders which effects the rate at which they match with a given type of worker. In addition, workers can choose search intensity which effects the rate at which they match with a firm for a given job type. Thus, the matching function depends on the

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<sup>8</sup>A histogram of primary working hours shows a mass of workers at 20 hours and another at 40 hours of work per week which is equivalent 80 and 160 hours per month assuming 4 weeks in a month. In addition, a histogram of secondary work hours shows a big mass of workers at 10 hours and another at 20 hours. For simplicity, I ignore the 10 hour mass and focus on 20.

effective mass of firms and the effective mass of workers. The rate at which matches are formed between a firm and an unemployed worker in state is given by

$$m_i u(s_{iu}, \bar{s}_{iu}, u, a_s, a_i, v) = \frac{s_{iu} u (1 - a_s) a_i v}{\bar{s}_{iu} u + (1 - a_s) a_i v}$$

where  $u$  is the mass of unemployed workers searching for a job of type  $i \in \{\text{full-time } (f), \text{ part-time } (p)\}$ , and  $v$  is the mass of vacancies. Unemployed workers search for full-time jobs and part-time jobs with respective search intensities  $s_{fu}$  and  $s_{pu}$  while firms recruit to full-time and part-time workers with respective recruiting intensities  $a_f$  where  $a_p = 1 - a_f$ . In addition, firms choose to recruit to unemployed workers with intensity  $(1 - a_s)$  with  $a_s$  being the recruiting intensity for secondary job holders. One way to think of recruiting intensity is as the effective fraction of vacancies that are directed toward each type of worker. The rate of matching also depends on the average search intensity for a job of type  $i$  denoted by  $\bar{s}_{iu}$ .<sup>9</sup> The rate at which vacancies are filled with an unemployed worker and a job of type  $i$  is given by

$$p_{iu} = \frac{s_{iu} u (1 - a_s) a_i}{\bar{s}_{iu} u + (1 - a_s) a_i v}$$

while the job finding rate for unemployed workers for a job of type  $i$  is given by

$$q_{iu} = \frac{s_{iu} (1 - a_s) a_i v}{\bar{s}_{iu} u + (1 - a_s) a_i v}.$$

The rate at which secondary matches for a job of type  $i$  are formed between a firm and worker who is in state  $j \in \{\text{full-time } (f), \text{ part-time } (p)\}$  is given by

$$m_{ij}(s_{ij}, \bar{s}_{ij}, l_j, a_s, a_i, v) = \frac{s_{ij} l_j a_s a_i v}{\bar{s}_{ij} l_j + a_s a_i v}$$

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<sup>9</sup>I depart from the urn-ball matching function in Section ?? as it tends to break down for extreme differences in recruiting and search intensities. Stevens (2007) shows that while the urn-ball matching function desirable properties in a discrete time framework, it does not satisfy the property that  $m(0, v) = m(u, 0)$ . As an alternative Stevens proposes the ‘‘telephone-line’’ matching technology that I implement here. This technology has the added benefit of working in continuous-time.

where  $l_j$  is the mass of workers currently working a job of type  $j$ . Full-time workers search for part-time jobs with intensity  $s_{pf}$  while part-time workers searchers for full-time and part-time jobs with respective search intensities  $s_{fp}$  and  $s_{pp}$ . As before, firms recruit for full-time and part-time workers and choose secondary recruiting intensity  $a_s$ . The rate at which vacancies are filled with a worker of type  $j$  and a job of type  $i$  is given by

$$p_{ij}(s_{ij}, \bar{s}_{ij}, l_j, a_s, a_i, v) = \frac{s_{ij}l_j a_s a_i}{\bar{s}_{ij}l_j + a_s a_i v}$$

while the job finding rate for a worker of type  $j$  for a job of type  $i$  is given by

$$q_{ij}(s_{ij}, \bar{s}_{ij}, l_j, a_s, a_i, v) = \frac{s_{ij}a_s a_i v}{\bar{s}_{ij}l_j + a_s a_i v}.$$

Workers who choose to search with intensity  $s_{ij} > 0$  must pay some cost defined by the weakly convex cost function  $\sigma_j(s_{ij})$ . Firms recruiting with intensity  $a_f \in [0, 1]$  and  $a_s \in [0, 1]$  must pay weakly convex recruiting cost  $C(a_f, a_s)$ .

The matching functions exhibit two important traits for workers as they capture frictions due to congestion as well as exhibiting increasing returns to personal search intensity. As workers increase their search intensity on average ( $\bar{s}_{ij}$ ), the effective mass of workers searching for a job increases which results in a lower job finding rate. On the other hand, as an individual worker increases their search intensity ( $s_{ij}$ ), they increase their personal job finding rate. In a steady-state equilibrium, individual search intensity and average search intensity for all workers are the same.

For both workers and firms, jobs are destroyed when job specific shocks arrive to occupied jobs at an exogenous Poisson rate depending on the type of job. Thus, shocks arrive to primary part-time jobs at rate  $\lambda_{p \rightarrow u}$ , secondary part-time jobs at rate  $\lambda_{pf \rightarrow f}$ , secondary dual part-time jobs at rate  $\lambda_{pp \rightarrow p}$ , primary full-time jobs at rate  $\lambda_{f \rightarrow u}$ , and finally secondary full-time jobs at rate  $\lambda_{pf \rightarrow p}$ . In this model, these shocks move worker productivity from being high enough to make production profitable to

being low enough to lead to worker-firm separation. Because the surplus generated for each job depends on the previous state, the necessary shock required to make a given job unproductive differs depending on the prior and current job status. In addition to job loss, full-time workers transition to part-time work at rate  $\lambda_{f \rightarrow p}$  while part-time workers transition to full-time work at rate  $\lambda_{p \rightarrow f}$ .

While unemployed, workers receive some flow value from leisure  $\chi_u b$  where  $b$  is the base value of leisure and  $\chi_u$  transforms  $b$  based on the current state, which in this case is unemployment. Part-time workers receive value of leisure  $\chi_p b$  and full-time workers receive some value of leisure  $\chi_f b$ , both being transformed based on the respective state. This reflects the fact that workers have 240 hours of time per month, but a full-time job only uses 160 and a part-time job only uses 80.

Upon being matched, each full-time firm-worker pair produces final output  $x_f$  expressed in units of utility, and each part-time firm-worker pair produces final output  $x_p = x_f(0.5^{2/3})$ .<sup>10</sup> Firm-worker pairs bargain over the wage that workers receive and firms pay depending on the state that the worker is in and moving into. Firms must also pay a full-time employment tax  $T$  when they employ a full-time worker.

## Equilibrium

At the beginning of each period, workers receive their remaining value of leisure and wages if they are employed. Firms receive the final value of output and pay

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<sup>10</sup>The evidence regarding worker productivity suggest that while part-time workers tend to be paid less than their full-time counterparts, most of this difference is due to difference in job requirements and worker heterogeneity. There does not appear to be any sizable difference between productivity for a part-time worker and a full-time worker who are otherwise identical. Since my model is assuming homogeneous worker types for now, I will assume that the final value of output for a part-time worker only differs based on the number of hours worked compared to a full-time worker. I assume that the production function is Cobb-Douglas in time spent working, so a worker who work half as much as a full-time worker produces  $(0.5^{2/3})$  what a full-time worker produces. The two thirds comes from the labor share in the standard C-D function. For reference, see Aaronson and French (2004), Hirsch (2005), Manning and Petrongolo (2008), and Künn-Nelen, de Grip, and Fouarge (2013)

wages. Workers then choose how intensely to search for various jobs depending on their current state while vacant firms choose how many vacancies to post and how intensely to recruit for part-time and full-time jobs as well as how intensely to recruit for unemployed and employed workers. At the end of the period, some workers and firms match at a Poisson rate and then bargain over the wage.

### Firms Problem

All firms start out posting one vacancy and choosing recruiting intensity for FT ( $a_f$ ) and PT ( $a_p = 1 - a_f$ ) jobs as well as primary ( $1 - a_s$ ) and secondary jobs ( $a_s$ ) while paying recruiting cost  $C(a_f, a_s)$ . They receive flow value

$$(1 - \beta)V = \max_{a_f, a_s} \left\{ -C(a_f, a_s) + \beta[p_{fu}(J_f - V) + p_{fp}(J_{f \leftarrow p} - V)] \right. \\ \left. + \beta[p_{pu}(J_p - V) + p_{pp}(J_{p \leftarrow p} - V) + p_{pf}(J_{p \leftarrow f} - V)] \right\} \quad (16)$$

and face some possibility that their FT vacancy is filled by unemployed or PT workers at rates  $p_{fu}$  and  $p_{fp}$  respectively. They also face some possibility that their PT vacancy is filled by unemployed, PT, or FT workers at rates  $p_{pu}$ ,  $p_{pp}$ , and  $p_{pf}$ .

If a firm matches with an unemployed FT job seeker, they receive flow value

$$(1 - \beta)J_f = x_f - w_{f \leftarrow u} - T + \beta\lambda_{f \rightarrow u}[V - J_f] + \beta\lambda_{f \rightarrow p}[J_p - J_f] \quad (17)$$

in which case the firm receives the final value of output  $p$ , but they must pay wage  $w_{f \leftarrow u}$ . If a firm matches with a PT worker, they receive flow value

$$(1 - \beta)J_{f \leftarrow p} = x_f - w_{f \leftarrow p} - T + \beta\lambda_{fp \rightarrow p}[V - J_{f \leftarrow p}] + \beta\lambda_{p \rightarrow u}[J_f - J_{f \leftarrow p}] \quad (18)$$

in which case they receive the final value of output  $x_f$ , but they must pay wage  $w_{f \leftarrow p}$ . At this point, neither firm has any open vacancies, but they face some risk of their FT job being destroyed at rates  $\lambda_{f \rightarrow u}$  or  $\lambda_{fp \rightarrow p}$  respectively. In addition, a FT employer



that matches with an unemployed worker faces some risk of the FT job becoming a PT job at rate  $\lambda_{f \rightarrow p}$ .

If a firm matches with an unemployed PT job seeker, they receive flow value

$$(1 - \beta)J_p = x_p - w_{p \leftarrow u} + \beta\lambda_{p \rightarrow u}[V - J_p] + \beta\lambda_{p \rightarrow f}[J_f - J_p] \quad (19)$$

in which case the firm receives the final value of output  $0.5p$ , but they must pay wage  $w_{p \leftarrow u}$ . If a firm matches with a FT worker, they receive flow value

$$(1 - \beta)J_{p \leftarrow p} = x_p - w_{p \leftarrow p} + \beta\lambda_{pp \rightarrow p}[V - J_{p \leftarrow p}] + \beta\lambda_{p \rightarrow u}[J_p - J_{p \leftarrow p}] \quad (20)$$

in which case they receive the final value of output  $0.5p$ , and they pay wage  $w_{p \leftarrow f}$ . If a firm matches with a PT worker, they receive flow value

$$(1 - \beta)J_{p \leftarrow f} = x_p - w_{p \leftarrow f} + \beta\lambda_{fp \rightarrow f}[V - J_{p \leftarrow f}] + \beta\lambda_{f \rightarrow u}[J_p - J_{p \leftarrow f}] \quad (21)$$

in which case they receive the final value of output  $x_p$ , and they pay wage  $w_{p \leftarrow p}$ . Depending on if the firm matches with an unemployed, FT, or PT worker, the firm faces some risk that their PT job will be destroyed at rate  $\lambda_{p \rightarrow u}$ ,  $\lambda_{fp \rightarrow f}$ , or  $\lambda_{pp \rightarrow p}$  respectively. In addition, a PT employer that matches with an unemployed worker faces some risk of the PT job becoming a FT job at rate  $\lambda_{p \rightarrow f}$ . While it is possible for the firm to have more than one PT worker, the model is restricted to one PT worker as allowing for additional PT workers causes the number of states to grow exponentially leading to the model becoming intractable. In addition, the trade-off between FT and PT workers is my primary concern, so having additional states is outside the realm of my analysis.

## Worker's Problem

All unemployed workers receive some value of leisure  $\chi_u b$  where  $\chi_i$  is the fraction of the unemployed value of leisure that a worker receives when they are in state  $i$ .<sup>11</sup> They choose their search intensities for FT and PT jobs simultaneously while paying cost  $\sigma_u(s_{fu}, s_{pu})$ . While unemployed, workers receive flow value

$$(1 - \beta)U = \max_{s_{fu}, s_{pu}} \left\{ \chi_u b - \sigma_u(s_{fu}, s_{pu}) + \beta q_{fu}[E_f - U] + \beta q_{pu}[E_p - U] \right\} \quad (22)$$

and continue search until they receive a PT or FT job offer.

If an unemployed worker accepts a FT job, they receive flow value

$$(1 - \beta)E_f = \max_{s_{pf}} \left\{ w_{f \leftarrow u} + \chi_f b - \sigma_f(s_{pf}) + \beta \theta_p q_p [E_{fp} - E_f] + \beta \lambda_{f \rightarrow p} [E_p - E_f] + \beta \lambda_{f \rightarrow u} [U - E_f] \right\} \quad (23)$$

and start searching for a PT job while receiving primary wage  $w_{f \leftarrow u}$  and some leftover value of leisure  $\chi_f b$ . In addition, they pay search cost  $\sigma_f(s_{pf})$ . While in this state, the worker faces some risk of losing their FT job at Poisson rate  $\lambda_{f \rightarrow u}$  or having their FT job become a PT job at rate  $\lambda_{f \rightarrow p}$ . Upon accepting a secondary PT job in addition to their FT job, the worker receives flow value

$$(1 - \beta)E_{fp} = w_{f \leftarrow u} + w_{p \leftarrow f} + \beta \lambda_{fp \rightarrow f} [E_f - E_{fp}] + \beta \lambda_{f \rightarrow u} [E_p - E_{fp}] \quad (24)$$

wherein they receive primary FT wage  $w_{f \leftarrow u}$  and secondary PT wage  $w_{p \leftarrow f}$ . Since they no longer have any unused time, they receive no value of leisure and do not search for any jobs. They also face risk of losing their FT job at rate  $\lambda_{f \rightarrow u}$  and their PT job at rate  $\lambda_{fp \rightarrow f}$ .

Should an unemployed worker accept a PT job, they receive flow value

$$(1 - \beta)E_p = \max_{s_{fp}, s_{pp}} \left\{ w_{p \leftarrow u} + \chi_p b - \sigma_p(s_{fp}, s_{pp}) + \beta q_{fp} [E_{pf} - E_p] + \beta q_{pp} [E_{pp} - E_p] \right\}$$

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<sup>11</sup>I assume that  $\chi_{pp} = \chi_f$  since a workers with two part-time jobs and full-time workers both work 40 hours per week.

$$\left. + \beta\lambda_{p \rightarrow f}[E_f - E_p] + \beta\lambda_{p \rightarrow u}[U - E_p] \right\} \quad (25)$$

while searching for both a PT and a FT job simultaneously. They receive primary wage  $w_{p \leftarrow u}$  and some leftover value of leisure  $\chi_p(b)$  while paying search cost  $\sigma_p(s_{fp}, s_{pp})$ . While in this state, they face some risk that they lose their primary PT job at rate  $\lambda_{p \rightarrow u}$  and some risk that their PT job becomes a FT job at rate  $\lambda_{p \rightarrow f}$ . Upon accepting a secondary FT job in addition to their PT job, the worker receives flow value

$$(1 - \beta)E_{pf} = w_{f \leftarrow p} + w_{p \leftarrow u} + \beta\lambda_{p \rightarrow u}[E_f - E_{pf}] + \beta\lambda_{fp \rightarrow p}[E_p - E_{pf}] \quad (26)$$

wherein they receive primary PT wage  $w_{p \leftarrow u}$  and secondary FT wage  $w_{f \leftarrow p}$ . Since they no longer have any unused time, they receive no value of leisure and do not search for any jobs. They also face risk of losing their FT job at rate  $\lambda_{fp \rightarrow p}$  and their PT job at rate  $\lambda_{p \rightarrow u}$ . Upon accepting a dual PT job in addition to their primary PT job, the worker receives flow value

$$(1 - \beta)E_{pp} = w_{p \leftarrow u} + w_{p \leftarrow p} + \chi_{pp}b + \beta(\lambda_{p \rightarrow u} + \lambda_{pp \rightarrow p})[E_p - E_{pp}] \quad (27)$$

wherein they receive primary PT wage  $w_{p \leftarrow u}$  and dual PT wage  $w_{f \leftarrow p}$ . They also receive leftover value of leisure  $\chi_{pp}b$ . Finally, they face some risk of losing their primary PT job at rate  $\lambda_{p \rightarrow u}$  and their dual PT job at rate  $\lambda_{pp \rightarrow p}$ . At this point, they do not search for any additional jobs. While it is possible for workers in the real world to hold more than two jobs, this restriction matches well with the data.<sup>12</sup>

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<sup>12</sup>Averett (2001) finds that moonlighting men tend to hold one full-time job and one part-time job while women tend to hold two part-time jobs. Similarly, Hipple (2010) finds that 92% of multiple job holders only hold two jobs.

## Job Creation Condition

The number of vacancies in equilibrium is determined endogenously by the job creation condition. First, set the flow value of posting a vacancy  $V = 0$ . The number of vacancies in the market is endogenous and depends on each firm's profit maximization. As such, profit maximization implies that the value of one more vacancy is zero as a positive value would result in an additional vacancy. This zero profit condition arises from the goods market being perfectly competitive. Thus, the flow value of posting a vacancy is  $V = 0$ . This allows for solving equations (??)-(??) for the flow values themselves. Plugging these equations into equation (??) gives the job creation condition

$$C(a_f, a_s) = \beta \left( p_{fu} J_f + p_{fp} J_{f \leftarrow p} + p_{pu} J_p + p_{pp} J_{p \leftarrow p} + p_{pf} J_{p \leftarrow f} \right) \quad (28)$$

which defines the firm's choice to post a vacancy.

## Wage Determination

When a worker-firm match is formed, they engage in an alternative offers bargaining game, which reduces to the axiomatic Nash Bargaining solution, to determine each wage. The worker of type  $j \in \{u, f, p\}$  and the firm have full information about each other such that they bargain over the total surplus generated by the match for a job of type  $i \in \{f, p\}$ . Thus, each wage  $w_{i \leftarrow j}$  is determined by equations (??) if the worker is unemployed and (??) if the worker is employed.

$$w_{i \leftarrow u} = \operatorname{argmax} (E_i - U)^{\gamma_i} (J_i - V)^{1-\gamma_i} \quad (29)$$

$$w_{i \leftarrow j} = \operatorname{argmax} (E_{ji} - E_j)^{\gamma_i} (J_{i \leftarrow j} - V)^{1-\gamma_i} \quad (30)$$

## Optimal Search and Recruiting Intensity

Workers choose their search intensity optimally to maximize the flow value of their current state. Workers do not internalize the impact their search intensity will have on wages. Each worker chooses their search intensity until the marginal cost of search is equivalent to the marginal benefit which comes from changes in matching rates and the value of future states.

$$0 = \frac{\partial \sigma_u}{\partial s_{ui}} + \beta \frac{\partial q_{iu}}{\partial s_{iu}} [E_i - U] + \beta q_{iu} \frac{\partial [E_i - U]}{\partial s_{iu}} + \beta q_{-iu} \frac{\partial [E_{-i} - U]}{\partial s_{iu}} \quad \forall i \in \{f, p\} \quad (31)$$

$$0 = \frac{\partial \sigma_j}{\partial s_{ij}} + \beta \frac{\partial q_{ij}}{\partial s_{ij}} [E_{ji} - E_j] + \beta \sum_i q_{ij} \frac{\partial [E_{ji} - E_j]}{\partial s_{ij}} - \beta \lambda_{ju} \frac{\partial [E_j - U]}{\partial s_{ij}} - \beta \lambda_{j,-j} \frac{\partial [E_{-j} - E_j]}{\partial s_{ij}} \quad \forall (i, j) \in \{(f, p), (p, f), (p, p)\} \quad (32)$$

Equations (??) and (??) respectively define the optimal search intensity  $s_{fu}$  for an unemployed worker searching for a FT job, the optimal search intensity  $s_{pu}$  for an unemployed worker searching for a PT job, the optimal search intensity  $s_{pf}$  for a single FT job holder searching for a PT job, the optimal search intensity  $s_{fp}$  for a single PT job holder searching for a FT job, and the optimal search intensity  $s_{pp}$  for a single PT job holder searching for a secondary PT job.

$$\frac{\partial C_v(a_f, a_s)}{\partial a_f} = \beta \sum_i \sum_j \frac{\partial p_{ij}}{a_f} J_{ij} \quad \forall (i, j) \in \{(f, u), (p, u), (p, f), (f, p), (p, p)\} \quad (33)$$

$$\frac{\partial C_v(a_f, a_s)}{\partial a_s} = \beta \sum_i \sum_j \frac{\partial p_{ij}}{a_s} J_{ij} \quad \forall (i, j) \in \{(f, u), (p, u), (p, f), (f, p), (p, p)\} \quad (34)$$

Similar to workers, firms choose their recruiting intensities  $a_f$  and  $a_s$  optimally to maximize the flow value of their current state. As with workers, firms do not

internalize the impact that their decisions will have on market conditions such that they take wages as a given. Equation (??) defines the optimal recruiting intensity ( $a_f$ ) for a firm recruiting for a full-time worker. This implicitly defines the recruiting intensity ( $a_p = 1 - a_f$ ) for a firm searching for a full-time worker. Similarly, equation (??) defines the optimal recruiting intensity ( $a_s$ ) for firms that want to hire a worker who already has a primary job. Again, this implicitly defines the recruiting intensity ( $1 - a_s$ ) for a firm looking to hire an unemployed worker. In both cases, firms adjust their recruiting intensity until the marginal cost of recruiting intensity is equivalent to the marginal benefit which come from changes in the vacancy filling rates and the cost of recruiting.

### Worker Flows

In the steady-state, the mean rate of unemployment, single FT, single PT, primary FT and secondary PT, primary PT and secondary FT, and dual PT job holdings should be constant. In a given time interval without growth or turnover in the labor force, the mean number of workers who enter into unemployment is  $[\lambda_{p \rightarrow u} l_p + \lambda_{f \rightarrow u} l_f] Ldt$  where  $l_f$  is the rate of single FT job holdings, and  $l_p$  is the rate of single PT job holdings. During the same time interval, the mean number of workers moving out of unemployment is  $[q_{fu} + q_{pu}] u Ldt$  where  $u$  is the unemployment rate. In the steady state, the evolution of the mean rate of unemployment as

$$\dot{u} = \lambda_{p \rightarrow u} l_p + \lambda_{f \rightarrow u} l_f - [q_{fu} + q_{pu}] u$$

which can be rewritten to define the unemployment rate as in equation (??).

$$\lambda_{p \rightarrow u} l_p + \lambda_{f \rightarrow u} l_f = [q_{fu} + q_{pu}] u \quad (35)$$

Over the same time interval, flows into single PT job holdings ( $l_p$ ) is

$$[q_{pu} u + (\lambda_{p \rightarrow u} + \lambda_{pp \rightarrow p}) l_{pp}] Ldt + \lambda_{f \rightarrow u} l_{fp} Ldt + \lambda_{fp \rightarrow p} l_{pf} Ldt + \lambda_{f \rightarrow p} l_f Ldt$$

where  $l_{pp}$  is the rate of dual PT job holdings,  $l_{fp}$  is the rate of primary FT and secondary PT job holdings, and  $l_{pf}$  is the rate of primary PT and secondary FT job holdings. By assuming that all employment rates must sum to one,  $l_{pf} = 1 - u - l_f - l_p - l_{pp} - l_{fp}$ . Outflows are  $[q_{fp} + q_{pp} + \lambda_{p \rightarrow u} + \lambda_{p \rightarrow f}]l_p Ldt$ . In the steady state, we can write the evolution of the mean rate of single PT job holdings as

$$\begin{aligned} \dot{l}_p = & q_{pu}u + (\lambda_{p \rightarrow u} + \lambda_{pp \rightarrow p})l_{pp} + \lambda_{f \rightarrow u}l_{fp} + \lambda_{fp \rightarrow p}l_{pf} + \lambda_{f \rightarrow p}l_f \\ & - [q_{fp} + q_{pp} + \lambda_{p \rightarrow u} + \lambda_{p \rightarrow f}]l_p \end{aligned}$$

which can be rewritten to define the rate of single PT job holdings as in equation (??).

$$q_{pu}u + (\lambda_{p \rightarrow u} + \lambda_{pp \rightarrow p})l_{pp} + \lambda_{f \rightarrow u}l_{fp} + \lambda_{fp \rightarrow p}l_{pf} + \lambda_{f \rightarrow p}l_f = [q_{fp} + q_{pp} + \lambda_{p \rightarrow u} + \lambda_{p \rightarrow f}]l_p \quad (36)$$

Similarly, the for single FT job holdings, dual PT job holdings, and primary FT and secondary PT job holdings are

$$q_{fu}u - q_{pf}l_f + \lambda_{fp \rightarrow f}l_{fp} + \lambda_{p \rightarrow u}l_{pf} + \lambda_{p \rightarrow f}l_p = \lambda_{f \rightarrow u}l_f + \lambda_{f \rightarrow p}l_f, \quad (37)$$

$$q_{pp}l_p = (\lambda_{p \rightarrow u} + \lambda_{pp \rightarrow p})l_{pp}, \quad (38)$$

and

$$q_{pf}l_f = (\lambda_{f \rightarrow u} + \lambda_{fp \rightarrow f})l_{fp}, \quad (39)$$

respectively.

## Steady-State

**Definition 0.3.1** *The steady-state equilibrium consists of a vector  $(u, l_p, l_f, l_{fp}, l_{pp}, v, a_f, a_s, w_{f \leftarrow u}, w_{f \leftarrow p}, w_{p \leftarrow u}, w_{p \leftarrow f}, w_{p \leftarrow p}, s_{fu}, s_{pu}, s_{pf}, s_{fp},$  and  $s_{pp})$  that solves the unemployment flow equation (??), part-time single job holdings flow equation (??), full-time single job holdings flow equation (??), the part-time dual job holdings flow equation (??), the primary full and secondary part-time job holdings flow equation (??), the job creation condition for vacancies (??), the firm's recruiting intensity*

maximization equations (??) and (??), five wage setting conditions (??) and (??), and the worker's five search intensity maximization equations (??) and (??).

#### 0.4 Calibration

The model is calibrated to match monthly data from the U.S. from December 2001 to December 2004. This time frame serves as a baseline which can be perturbed to analyze the effects of recessions. All fixed parameters are summarized in Table ??.

First, the job destruction rates  $\lambda_{f \rightarrow u} - \lambda_{fp \rightarrow f}$  are set according to HP filtered monthly data on workers employment status from the Current Population Survey (CPS). The design of the CPS allows one to distinguish how many jobs an individual has and whether they are full-time or part-time jobs. Since workers can be observed for two sets of four consecutive months, there are 6 observations per worker for flows between different states. Aggregating up, the average primary full-time rate  $\lambda_{f \rightarrow u} = 0.023$ , secondary full-time rate  $\lambda_{fp \rightarrow p} = 0.037$ , primary part-time rate  $\lambda_{p \rightarrow u} = 0.046$ , secondary dual part-time rate  $\lambda_{pp \rightarrow p} = 0.156$ , and the secondary part-time rate  $\lambda_{fp \rightarrow f} = 0.191$ .<sup>13</sup>

The value of leisure  $\chi_i b$  is  $\chi_u b = b$  which serves as the based value of leisure. Using annual data from the 2003-2015 American Time Use Survey (ATUS), I calculate the average amount of time an individual has for leisure based on whether they are unemployed, part-time, full-time, or working multiple jobs. The data suggests that part-time workers have 29.36% as much time for leisure as unemployed workers while full-time workers have 5.4% as much leisure as unemployed workers. Thus, the value of leisure for part-time workers  $\chi_p b$  and  $\chi_f b$  are set equal to 29.36% and 5.4% of the value of leisure  $b$  for unemployed workers respectively. The final value of output for full-time workers  $x_f = 1$  is chosen as the numeraire. The final value of output for part-time workers is calculated based on a Cobb-Douglas production function that depends

<sup>13</sup>The derivation method is outlined in Appendix ??.



on hours worked and a labor share of  $2/3$ . The discount rate  $\beta = (1 - r) = 0.9959$  is set according to a real annual interest rate of 5 percent. The full-time employment tax  $T = 0$  is set as a baseline which can be perturbed. The worker's share of the surplus generated from a match  $\gamma_f = \gamma_p = 0.5$  are set in order to satisfy the Hosios condition.

All jointly calibrated parameters are summarized in Table ?? following a monthly frequency. Workers receive some value of leisure  $b = 0.9668$  which is calibrated so the ratio of effective vacancies to the aggregate unemployment level is 0.44 to match the CPS and JOLTS data. The total cost of recruiting intensities  $a_f$  and  $a_s$  is

$$C(a_f, a_s) = c(1 - a_s)(a_f + a_p) + Ca_s(a_f + a_p)$$

where  $c$  is the marginal cost of posting an additional vacancy to unemployed workers and  $C$  is the marginal cost of posting to employed workers. Since there is no concrete evidence to suggest that recruiting for part-time workers is more costly than recruiting for full-time workers, the recruiting cost is the same for both types, but I assume that recruiting costs for employed and unemployed workers differ as the methods of contact can differ. The cost parameter  $c = 0.3418$  is calibrated based on data summarized in Silva & Toledo (2009) which suggests that the average cost of recruiting for and hiring a new employee is roughly 30.23% of the worker's output. The cost parameter  $C = 0.2929$  is calibrated with the search cost parameters in order to match matching probabilities for workers.

The total cost of search intensity while unemployed, employed full-time, and employed part-time are

$$\sigma_u(s_{fu}, s_{pu}) = \chi_u b(1 - h_f s_{fu} - h_p s_{pu})$$

$$\sigma_f(s_{pf}) = \chi_f b(1 - H_f h_p s_{pf})$$

$$\sigma_p(s_{fp}, s_{pp}) = \chi_p b(1 - H_p h_f s_{fp} - H_p h_p s_{pp})$$

where  $h_f$  is the cost of search for a full-time job and  $h_p$  is the cost of searching for a part-time job. While employed in a full-time or part-time job, these cost parameters are transformed by  $H_f$  and  $H_p$  to reflect additional constraints on an individual's ability to find a job while working as well as reflecting an aversion to holding a second job. Combined, with the cost parameter  $C$ , these values are calibrated to match the average job finding probabilities for a worker of type  $i \in \{u, f, p\}$  finding a job of type  $j \in \{f, p\}$  which are estimated using monthly CPS data from December 2001 to December 2004.<sup>14</sup>

## 0.5 Results

Calibrating the baseline model to the data as outlined in the previous section generates the results displayed in Table ???. The rate of unemployment and each employment rate are all within a reasonable distance of the actual observed values. The first thing that stands out in Table ??? is that workers search more intensely for part-time jobs than full-time jobs with search intensity  $s_{pu} > s_{fu}$  and  $s_{pp} > s_{fp}$ . This is primarily because firms recruit more intensely for full-time workers with  $a_f = 0.6761$  being twice as large as recruiting intensity for part-time workers. Because  $a_f$  is so high, there is a lower incentive for workers to search for a full-time job compared to a part-time job.

The wage for primary part-time jobs is greater than part-time worker productivity. This result likely stems from the probability that the job could become a full-time job at some point. In addition, for firms, they are willing to take on the loss because they can avoid paying the cost of posting and maintaining a vacancy in addition to perhaps ending up with a full-time worker. In an attempt to mitigate

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<sup>14</sup>Using the same method used for estimating job loss probabilities. The derivation method is outlined in Appendix ???.

this result, the leisure values of being employed  $\chi_f$  and  $\chi_p$  were calibrated in lieu of the cost parameters  $H_f$  and  $H_p$ , but this did not affect the results as they function in similar ways in equilibrium. Since  $H_f$  and  $H_p$  function as disutility parameters for search, I opted to stick with this calibration strategy.

## Comparing Models

One of the purposes of this paper is to examine how well a model of part-time employment and multiple job holdings compares to the standard Diamond-Mortensen-Pissarides model. Given shocks to the final value of output, the cost of search, and the cost of recruiting, the full model compares favorably to the DMP model while also showing changes in employment composition. In addition, turning off part-time employment and focusing on MJH compares favorably as well. In the tables below, each exogenous parameter is perturbed for the DMP model, the model with MJH, and the model with both PT employment and MJH. Finally, I compare how the models perform when parameters are set according to their levels during the 2007-2009 recession.

First, the final value of output is perturbed as in Table ???. For the models with MJH and the full model, all final values of output are perturbed by the same percentage. In all cases, the unemployment rate decreases as the final value of output increases by 1%. Focusing on the models with the baseline value of leisure, the DMP model and full model compare very favorably with the decreases in the unemployment rate being almost identical at 59.81% and 58.73% respectively. Of interest is the changing composition of part-time employment and full-time employment. In the DMP model, employment increases by 5.27%, but FT employment rate increases by 2.64% in the full model. Making up the rest of the gap between the two models, the PT employment rate increases by 18.65%. Given how well the two models compare

in employment, they do not compare well for the vacancy rate. As the final value of output increases, the vacancy rate drops in the full model compared to an increase in the DMP model. This is because firms are shifting their recruiting behavior from secondary workers to primary workers in the full model.

The same set of comparative statics are performed with the baseline value of leisure reduced by 1% as in the second set of columns in Table ???. Given a lower value of leisure, the full model becomes more responsive to a shock to the final value of output while the DMP and MJH models are less responsive. For instance, the unemployment rate response drops from 59.81% to 36.43% for the DMP model while it increases from 58.73% to 99.01% in the full model. In addition, the PT rate increases by 22.3% compared to a 18.65% increase in the baseline full model. On the other hand, the FT rate and vacancy rate responses becomes smaller for all the models. This suggest that part-time employment plays an important role as a means of smoothing income and employment for workers and firms when they have the ability to change their search and recruiting behaviors.

Next, I examine an increase in the cost of search in Table ??. For the DMP model, there is only one search cost while the MJH and full models have more than one search cost which are increased by the same percentage. The results are quite different for the three models. In the full model, an increase in the cost of search leads to a large increase in search for part-time employment which results in an increase in employment and a decrease in the unemployment rate. These stark differences harken back to the comparative statics for the cost of full-time and part-time search in the previous section. For an increase in the cost of full-time search, the comparative statics look similar to the results for the DMP and MJH models, but a proportional increase in the cost of part-time search dominates this effect and results in lower unemployment and higher employment rates, especially for part-time employment.

Looking again at the case of a lower value of leisure, the unemployment rate and part-time rate are more responsive in the full model compared to the DMP and MJH models while the full-time rate and vacancy rate are less responsive. This result is qualitatively consistent with the results from the case of a shock to the final value of output.

Now, consider an increase in the cost of recruiting for all types of workers in Table ?? . As in the case of an increase in the cost of search, the DMP model differs from the full model, but, unlike previously, the MJH model compares favorably. Increasing recruiting cost leads to lower vacancy rates across the board as it becomes more costly for the firm to post a vacancy. This leads to an increase in the unemployment rate in the DMP and MJH model. On the other hand, an increase in the cost of recruiting leads to firms shifting towards posting secondary vacancies and part-time vacancies which results in a higher part-time rate. In turn, this leads to a decrease in the unemployment rate. In the absence of part-time employment, the DMP and MJH models behave similarly.

Finally, I compare how the models perform when there is a recessionary shock. To accomplish this, I set parameters according to their levels during the 2007-2009 recession as in Table ?? . According to the Bureau of Labor Statistics (BLS), the largest drop in the Consumer Price Index (CPI) was from September 2008 to December 2008 when prices fell by 3.34%. Thus, I decrease the final values of output  $x_f$  and  $x_p$  by 3.34%. In addition, each job destruction and transition rate is set according to the average value from the CPS. Comparing the steady-state values for each set of parameters should be sufficient as there are few dynamic elements in these models. Nash Bargaining will result in the model moving to the new steady-state very quickly if dynamics were introduced.

The results in Table ?? suggest that the full model has some advantages over the DMP model. The full model does well at matching the responses in the unemployment rate, full-time rate, and part-time rate, especially their values at the trough of the recession. Unfortunately, the response in the MJH rate was too strong with the MJH dropping to 0.12% as opposed to the lowest rate observed in the data at 5.29%. The DMP model produces relatively muted responses with the unemployment rate jumping from 8.1% to 8.3% which is below both the average and the trough values in the data. In addition, the decline in real GDP is only 0.22% which is far below the average and trough values from the Bureau of Economic Analysis at -1.69 and -3.98 respectively. The full model does provide a stronger response in real GDP with a decline of 6.8%. This is largely a result of the strong decline in the multiple job holding rate. Overall, the full model captures compositional changes among employed workers which are not captured by the DMP model.

## 0.6 Discussion

The primary objective of this paper was to understand how workers and firms interact in a market with multiple job holdings. The second objective was to see how they might behave differently than in the standard Diamond-Mortensen-Pissarides model with only one job. First, consider how firms respond to changes in productivity. As the final value of output increases, firms create fewer vacancies, but due to shifting recruiting behavior, the effective vacancy rate for unemployed workers is actually higher while the effective vacancy rate is lower for workers who already have a job. Compare this with the conclusion by Gavazza, Mongey and Violante (2018) who suggest that vacancies and recruiting behavior are often working in different directions, especially during recessions. One major difference is that in the model presented here, recruiting behavior also takes into account multiple job hold-

ers whereas GMV does not. In addition, the ability of firms to hire part time workers becomes important as well when considering shocks to the cost of recruiting. When recruiting becomes more expensive, firms shift towards less costly recruitment of already employed workers as well as workers searching for part-time jobs. This can actually lead to a 16% lower unemployment rate, but also a 17% higher part-time rate such that workers may be worse off on average. These results suggest that the ability to shift recruitment from primary to secondary and full-time to part-time play an important role in firm responses. In their absence, firms become less responsive and act in opposite ways.

Common in the empirical literature is the notion that some workers take part-time jobs as a means of smoothing their income, especially during recessions when the rate of workers who are part-time for economic reasons increases. This general idea seems to hold. For instance, decreasing the value of leisure results in a lower unemployment rate and higher full-time and part-time rates, however, the part-time rate increases by 20% compared to a 2.5% increase in the full-time rate. This corresponds to the part-time rate increasing by roughly 3 percentage points compared to full-time's 2 pp increase. This corroborates the notion that part-time employment is an important margin for adjustment as unemployment becomes less valuable. In addition, as the value of leisure falls, unemployment and part-time employment become much more responsive to shocks as seen with increasing search cost, recruiting cost, and final value of output. In particular, the part-time employment response increases while the full-time rate decreases further supporting the use of part-time employment as a means of smoothing ones utility.

Overall, the model presented here performs relatively well compared to the DMP model when introducing recessionary shocks. In this case, plugging in data from the 2007-2009 recession generates a large increase in the unemployment rate

from 7.6% to 10.24% compared with the DMP models increase from 8.1 - 8.3%. The average and trough value for the unemployment rate are 8.84% and 9.3% respectively. The full model also produces a sharp decline in the full-time employment rate from 75.8% to 70.5% compared to the data suggesting a rate of 69.0% on average. In addition, the part-time employment rate jumps from 16.6% to 19.2% which is in line with the observed increase to 17.0% in the data. Most of the overshooting in the part-time rate is due to the larger than expected decline in the multiple job holding rate.

One possible reason why multiple job holdings may not be responsive in the data is due to reporting problems. Discussing the stark difference between household and establishment level employment data, Abraham et al. (2013) conclude that multiple job holdings is likely under-reported during recessions, but it is possible that the effect could be in the opposite direction as there are competing incentives for firms to not report workers and workers to not report income. These incentives change depending on market conditions and are mostly related to shielding wages from taxes. In this light, my model produces an unlikely decline in the multiple job holdings rate, but it suggests that the rate may decline contrary to the conclusions of Hirsh, Husain, & Winters (2016). Given the shortcomings of the data on multiple job holdings, it may be hard to distinguish which is the correct conclusion.

## **0.7 Conclusion**

In this paper, I start by documenting several facts regarding part-time employment and multiple job holdings. Data from the Survey of Consumer Expenditure suggests that a worker's search behavior and job offers depend on their current employment status. In addition, multiple job holdings appears to be more important for part-time workers as they are more willing to work multiple jobs than are full-



time workers. To this end, I construct a search theoretic model that includes both part-time employment and multiple job holdings in addition to full-time employment. I allow for firms to recruit to different types of workers in different ways and I allow workers to choose their search intensity depending on their current employment status.

Allowing for part-time work and multiple job holdings in the Diamond-Mortensen-Pissarides model generates some novel results. First, variable recruiting results in the vacancy rate responding to shocks in the opposite direction of the DMP model. Despite this, the effective vacancy rate for a given job can still move in the same direction. Second, when comparing the full model to the DMP model, a recessionary shock in the full model produces results that are more in line with the data. This includes a larger jump in the unemployment rate and output as well as capturing the compositional changes within the employment rate. Finally, multiple job holdings are important when part-time employment is included. The model suggest that multiple job holding rates can vary quite dramatically and this drives some of the response in the part-time employment rate. The model presented here provides a way of looking behind the veil to see how firms and workers are responding. Given the importance of multiple job holdings in this model, future work should consider how firm size effects part-time employment and multiple job holdings.

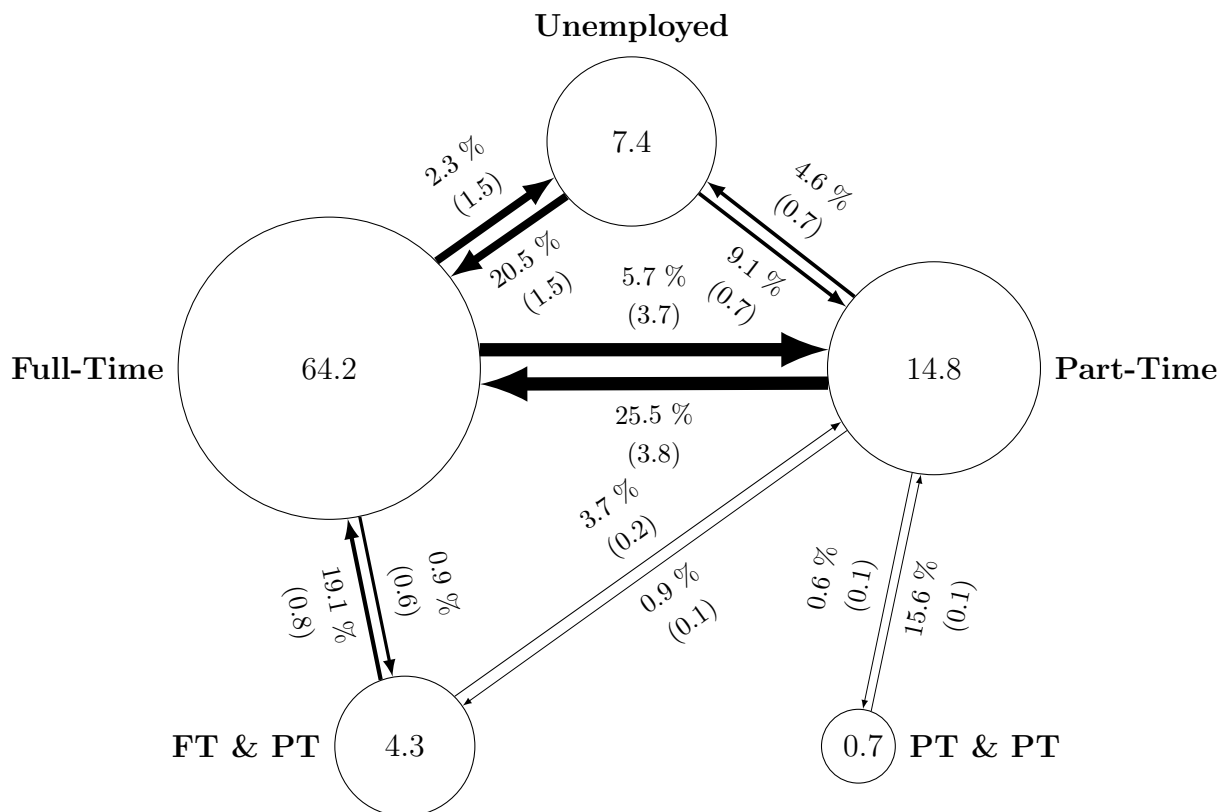


Figure 1.: Non-Recession Worker Flows (25-54 Age Group)

Average calculated using HP-filtered monthly CPS data from December 2001 to December 2004. HP filtered data is used to remain consistent with Figure ???. Numbers inside circle represent average monthly count in millions. Percentages above/below arrows represent average monthly probability of moving from one employment status to another. Finally, number in parentheses above/below arrows represent average monthly count of workers transitioning from one employment status to another.

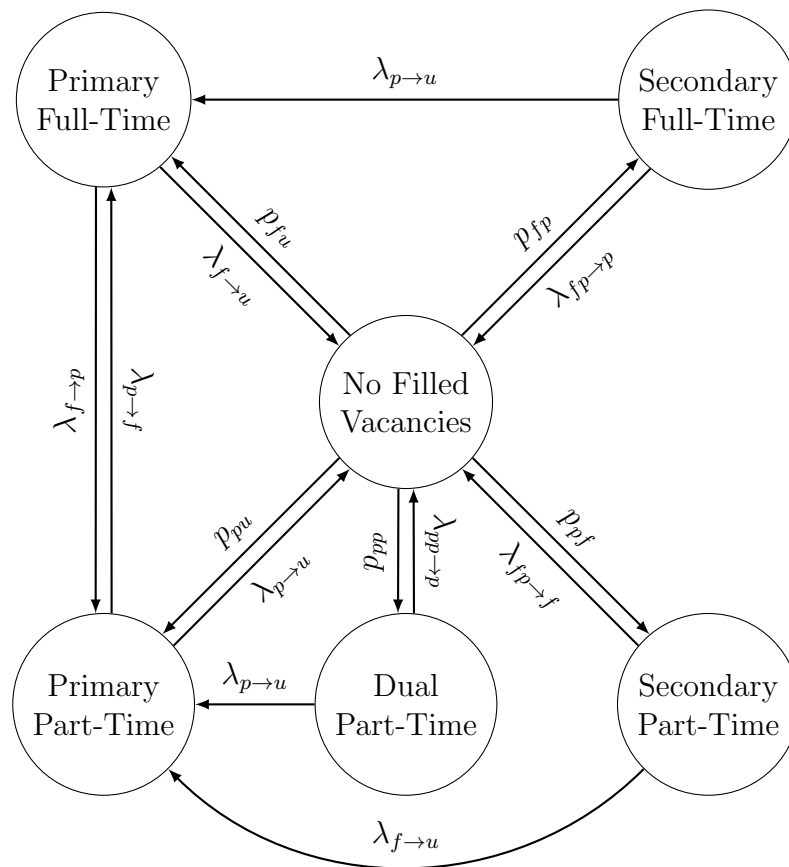


Figure 2.: Firm Flows

Figure ?? shows the different types of workers a firm could have as well as how their relationship with their worker could evolve. From the firms perspective, their pairing could be destroyed, or their worker's other pairing could be destroyed.

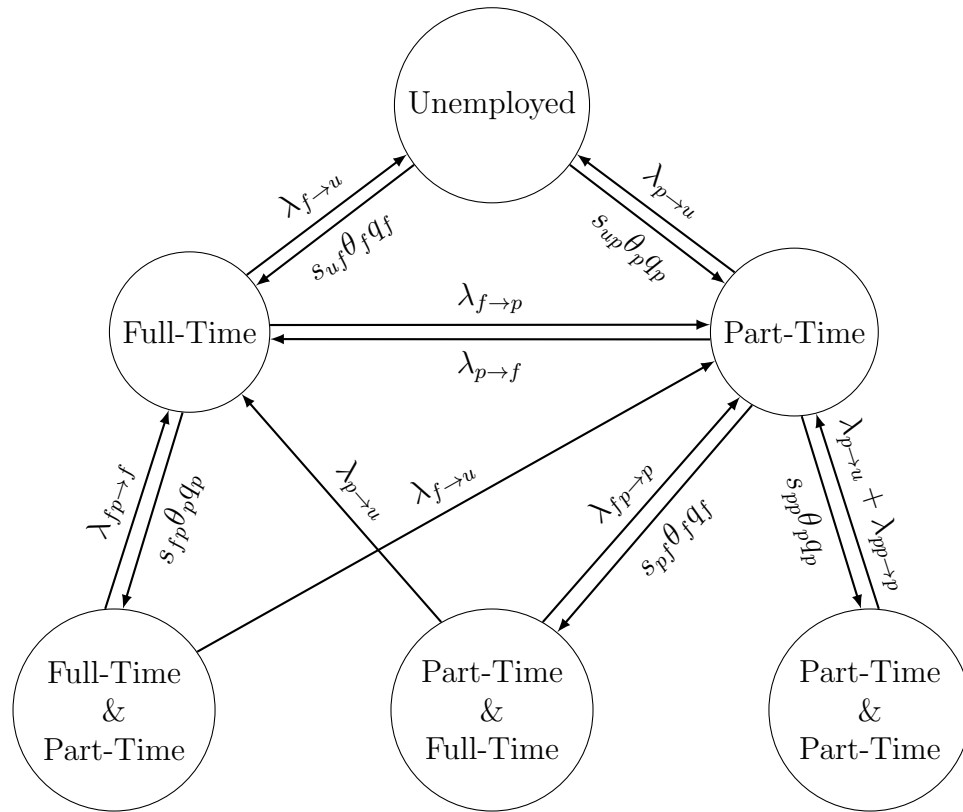


Figure 3.: Worker Flows

Figure ?? shows the different states a worker could be in as well as how their current state could evolve through getting a new job or having one of their jobs destroyed.

Table 1.: Independently Calibrated Parameters

Parameter	Value	Target/Source
$\lambda_{f \rightarrow u}$	0.023	CPS probability of losing primary FT job = 0.023
$\lambda_{fp \rightarrow p}$	0.037	CPS probability of losing secondary FT job = 0.037
$\lambda_{p \rightarrow u}$	0.046	CPS probability of losing primary PT job = 0.046
$\lambda_{pp \rightarrow p}$	0.156	CPS probability of losing secondary dual PT job = 0.156
$\lambda_{fp \rightarrow f}$	0.191	CPS probability of losing secondary PT job = 0.191
$\chi_u$	1	Base value of leisure
$\chi_p$	0.2936	% of unemployed leisure time for PT worker from ATUS
$\chi_f$	0.0540	% of unemployed leisure time for FT worker from ATUS
$x_f$	1	$x_f = (1^{2/3})$
$x_p$	0.63	$x_p = (0.5^{2/3})$
$r = (1 - \beta)$	0.00407	Annual interest rate 5%
$T$	0	Baseline
$\gamma_f$	0.5	Hosios Condition
$\gamma_p$	0.5	Hosios Condition

Table 2.: Jointly Calibrated Parameters

Parameter	Value	Targets
$h_f$	0.0432	Probability of finding primary FT job $q_{fu} = 0.205$
$h_p$	0.0040	Probability of finding primary PT job $q_{pu} = 0.091$
$H_f$	7784	Probability of finding secondary PT job $q_{fp} = 0.009$
$H_p$	1347	Probability of finding secondary dual PT job $q_{pp} = 0.006$
$C$	0.2929	Probability of finding secondary FT job $q_{pf} = 0.009$
$c$	0.3418	Total cost of recruiting = 0.3203 (Silva & Toledo (2009))
$b$	0.9668	Ratio of effective vacancies to unemployed $a_s v/u = 0.44$

All Targets matched with sum of squared errors  $< 10^{-10}$

Table 3.: Baseline Results

Variable		Baseline	Data	Variable		Baseline	$w_{j \leftarrow i} / w_{f \leftarrow u}$
Unemployment Rate	$u$	0.0804	0.0764	Primary FT wage	$w_{f \leftarrow u}$	0.9434	1
FT Rate	$l_f$	0.7151	0.7074	Primary PT wage	$w_{p \leftarrow u}$	0.7796	0.8264
PT Rate	$l_p$	0.1594	0.1597	Secondary PT wage	$w_{p \leftarrow f}$	0.3243	0.3438
FT/PT Rate	$l_{fp}$	0.0289	0.0473	Secondary FT wage	$w_{f \leftarrow p}$	0.5984	0.6343
PT/PT Rate	$l_{pp}$	0.0126	0.0079	Dual PT wage	$w_{p \leftarrow p}$	0.3923	0.4158
Variable		Baseline	$s_{ji} / s_{fu}$	Variable		Baseline	
U search for FT	$s_{fu}$	0.4454	1	Vacancy Rate	$v$	0.0805	
U search for FT	$s_{pu}$	0.1817	0.4079	FT Recruiting	$a_f$	0.6761	
FT search for PT	$s_{pf}$	0.0099	0.0222	Primary Recruiting	$a_s$	0.4393	
PT search for FT	$s_{fp}$	0.0065	0.0146				
PT search for PT	$s_{pp}$	0.0205	0.0460				

Table ?? shows the steady-state results for the calibrated model. To put the wage in perspective, I divide each by the primary FT wage to get  $w_{j \leftarrow i} / w_{f \leftarrow u}$ . Similarly, I divide each search intensity by the search intensity for a primary FT job to get  $s_{ji} / s_{fu}$ .

Table 4.: 1% increase in final value of output

	Baseline Value of Leisure			99% Baseline Value of Leisure		
	DMP	MJH	Full	DMP	MJH	Full
%Δ Unemp. Rate	-59.81	-45.34	-58.73	-36.43	-35.61	-99.01
%Δ FT Rate	5.27	4.00	2.64	1.23	1.66	-0.39
%Δ PT Rate	-	-	18.65	-	-	22.30
%Δ MJH Rate	-	167.46	-99.83	-	61.37	-37.85e3
%Δ Vacancy Rate	63.86	97.08	-41.35	35.75	50.52	-37.09

Table 5.: 1% increase in cost of search

	Baseline Value of Leisure			99% Baseline Value of Leisure		
	DMP	MJH	Full	DMP	MJH	Full
%Δ Unemp. Rate	1.39	-0.45	-16.18	1.09	1.20	-29.12
%Δ FT Rate	-0.12	0.04	0.23	0.04	-0.06	-0.10
%Δ PT Rate	-	-	17.07	-	-	19.11
%Δ MJH Rate	-	-15.86	-14.77	-	-10.28	-95.99
%Δ Vacancy Rate	-0.25	-6.12	-12.23	-0.08	-5.79	-10.66

Table 6.: 1% increase in cost of recruiting

	Baseline Value of Leisure			99% Baseline Value of Leisure		
	DMP	MJH	Full	DMP	MJH	Full
% $\Delta$ Unemp. Rate	2.27	0.56	-2.45	-2.00	-2.26	32.11
% $\Delta$ FT Rate	-0.20	-0.05	-0.03	0.07	0.11	-0.47
% $\Delta$ PT Rate	-	-	16.16	-	-	16.80
% $\Delta$ MJH Rate	-	-6.12	-1.83	-	9.17	-0.41
% $\Delta$ Vacancy Rate	-1.40	-3.42	-3.09	0.07	4.62	6.39

Table 7.: Parameters

Parameter	12/2001-12/2004	12/2007-6/2009
$x_f$	1	0.966
$x_p$	0.63	0.603
$\lambda_{f \rightarrow p}$	0.057	0.059
$\lambda_{p \rightarrow f}$	0.255	0.244
$\lambda_{f \rightarrow u}$	0.023	0.022
$\lambda_{pf \rightarrow p}$	0.037	0.035
$\lambda_{p \rightarrow u}$	0.046	0.045
$\lambda_{pp \rightarrow p}$	0.156	0.156
$\lambda_{fp \rightarrow f}$	0.191	0.177

Table 8.: Results for Recessionary Shock

	Baseline*			2007-2009)			
	DMP	Full	Data	DMP	Full	Data	Trough
Unemployment Rate	0.0810	0.0760	0.0764	0.0830	0.1024	0.0884	0.0930
FT Rate	0.9190	0.7579	0.7639	0.9170	0.7053	0.6902	0.6839
PT Rate	-	0.1661	0.1597	-	0.1923	0.1676	0.1702
MJH Rate	-	0.0407	0.0565	-	0.0012	0.0538	0.0529
% $\Delta$ RGDP	-	-	-	-0.220	-6.776	-1.689	-3.983

\* Baseline corresponds to Dec. 2001 - Dec. 2004

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## A. Worker Flow Derivation

Worker flows were calculated using the Current Population Survey (IPUMS-CPS). Starting in 1994, the CPS started asking questions regarding multiple job holdings. In particular, they started asking individuals if they worked more than one job in the prior week, and if so, how many. The survey also asks about an individuals work history. Particularly useful for me, they ask whether the individual is unemployed, part-time or full-time for economic or non-economic reasons, or if they are not part of the labor force. Combining these questions gives me a reasonable way to measure the fraction of workers in each employment state and the transitions between.

The data is not without shortcomings though. Data from 1995 showed much more fluctuation in the number of individuals in each sample month, so I have elected to drop the years 1994 and 1995.<sup>1</sup> In addition, I have elected to keep only those individuals between the ages of 25 and 54.<sup>2</sup> This way I can minimize transitions from being in school to working and from working into early retirement. Thus, my sample consists of all individuals between the ages of 25 and 54 who were sampled between January 1996 and December 2014

Once I have calculated the weighted sums of individuals in each state and moving between each state, I can calculate the probability of a worker moving from one state to another. This probability is denoted by

$$f_{t,i \rightarrow j} = \frac{M_{t,i \rightarrow j}}{M_{t-1,i}}$$

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<sup>1</sup>Individuals are surveyed for 4 months, then ignored for 8 months, and re-surveyed for 4 months. Sample months are labeled 1-8 to represent the 8 total months that individuals get sampled

<sup>2</sup>Similar to Shimer (2012)

where  $t$  is the current time period,  $t - 1$  is the previous time period,  $i$  is the previous state of employment,  $j$  is the current state of employment, and  $M$  is the mass of workers in a given state. Once I have this probability, I can back out the Poisson rate using the equation  $Pr(X < x) = f_{t,i \rightarrow j} = 1 - e^{-\lambda_{t,i \rightarrow j}x}$ . Setting  $x = 1$ , I can solve for the Poisson arrival rate

$$\lambda_{t,i \rightarrow j} = -\ln(1 - f_{t,i \rightarrow j})$$

Once these rates are calculated, I apply a HP filter to the data in order to extract the underlying trend.

## B. Part-Time and Full-Time Employment

In order to get a better understanding of the choices that workers are making, I consider a simplified framework with workers and firms restricted to either one part-time job or one full-time job. The worker's goal is to receive a job offer for either a part-time or full-time job, but receiving more than one job offer is no more valuable than receiving only one.

### B.1 Environment

Consider a discrete time job search model where time goes on forever. There is a continuum of infinitely lived, risk-neutral workers with lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t y_t$$

where  $y_t$  is the worker's instantaneous income at time  $t$  and  $\beta \in (0, 1)$  is the discount factor.

Workers can be in one of 3 states: unemployed, employed part-time, or employed full-time. Each worker is endowed with 40 hours of time with a part-time job requiring 20 hours and a full-time job requiring 40 hours. While unemployed, workers are assumed to always be searching for a job, but can choose the intensity with which they search for part-time work ( $s_p$ ) or full-time work ( $s_f$ ), however, they must pay weakly convex search cost  $\sigma(s_p, s_f)$ . Search intensity can be thought of as some measure of the number of applications a worker sends out as well as the quality of each application such that search intensity  $\{s_f, s_p\} \in \mathbb{R}$ . Searching with intensity  $s_i = 0$  is



equivalent to not searching at all. Unemployed workers receive value of leisure  $z$  while employed workers receive wage  $w_i$  with  $i \in \{f, p\}$ . There is one type of firm that can be in one of 3 states: vacant, employing a part-time worker, or employing a full-time worker. Firms can post vacancies  $v$  and choose whether to recruit to full-time workers ( $a_f$ ) or part-time workers ( $a_p = 1 - a_f$ ) while paying weakly convex recruiting cost  $C(a_p, a_f)$ . Firms that hire a worker of type  $i$  receive output  $p_i$  and pay wage  $w_i$ .

Having variable search intensity and recruiting intensity is an important component in this model for two primary reasons. First, empirical results show that workers do not search in the same manner for every type of job nor do they search in the same manner when employed and unemployed. Workers use different methods of search as well as choosing different search hours depending on the type of job they are searching for and their current job status (Holzer (1988), Pissarides & Wadsworth (1994), Aguiar, Hurst, & Karabarbounis (2013), Faberman et al. (2016)). Thus, it is important that workers in this model be able to change their search intensity according to their current employment status and desired job rather than being stuck searching with the same intensity across all jobs. Likewise, firms do not recruit for full-time and part-time positions in the same manner or the same quantity (Russo, Gorter, & Schettkat (2001)). Likewise, they recruit to workers differently depending on their current employment status (Faberman et al. (2016)). In Section ??, I show the importance of this margin. Without the intensive margin of search and recruiting, the results look dramatically different.

Workers and firms are matched pairwise according to two CRTS matching functions, one for full-time jobs and one for part-time jobs. The matching function depends on the effective mass of firms ( $a_i v$ ) and the effective mass of workers ( $s_i u$ ).

The rate at which matches of type  $i \in \{f, p\}$  are formed between a firm and worker is given by

$$m_i(\bar{s}_i u, a_i v)$$

where the effective mass of workers depends on average search intensity over all workers  $\bar{s}_i$ . The probability that any one unit of search intensity results in a job offer is:

$$\mu_i(\theta_i) = m_i(1, \theta_i) = \frac{m_i(\bar{s}_i u, a_i v)}{\bar{s}_i u}$$

where  $\theta_i$  represents market tightness. Given market tightness, an individual worker who chooses to search with intensity  $s_i$  has individual matching probability

$$q_i(\theta_i, s_i) = 1 - (1 - \mu_i(\theta_i))^{s_i}$$

which is the probability that a worker receives at least one job offer resulting from their search effort. At this point it is worth noting that in equilibrium,  $\bar{s}_i = s_i$  since all workers are homogeneous. This matching probability has some important properties. First, as an individual worker increases their search intensity  $s_i$ , they increase the probability that they match with a firm, but all workers are homogeneous, aggregate search intensity  $\bar{s}_i$  will increase which results in a lower probability that any one unit of search intensity produces a job offer such that the individuals matching probability falls as well resulting in an ambiguous response. Because the measure of workers who match in a given period is  $uq_i(\theta_i)$ , the probability that a firm's vacancy is filled is

$$p_i(\theta_i, \bar{s}_i) = \frac{q_i(\theta_i, \bar{s}_i)}{\theta_i \bar{s}_i}$$

which does not depend on the individual level of search intensity. After a worker and firm match, they both face some exogenous probability  $\lambda_i$  that the job is destroyed which depends on the type of job.

## B.2 Equilibrium

At the beginning of each period, workers and firms find out if they matched. If a worker matches with a part-time firm, they agree with the firm on the part-time wage. If they match with a full-time firm, they agree with the firm on the full-time wage. The worker receives the corresponding wage and the firm receives the corresponding final value of output and pays the corresponding wage. If a worker remains unemployed, they receive the instantaneous value of leisure and choose their search intensity and pay some search cost. If a firm remains vacant, they choose their recruiting intensity and pay some recruiting cost.

## B.3 Firm's Problem

Firms start by posting a vacancy and choosing recruiting intensity  $a_p = 1 - a_f$  for part-time jobs and recruiting intensity  $a_f$  for full-time jobs. Firms choose their recruiting intensity to maximize flow value

$$V = \max_{a_f} \left\{ -C(a_p, a_f) + \beta V + \beta p_f(\theta_f, \bar{s}_f)[J_f - V] + \beta p_p(\theta_p, \bar{s}_p)[J_p - V] \right\} \quad (\text{B.1})$$

where they pay recruiting cost  $C(a_p, a_f)$  and match with a part-time worker with probability  $p_p(\theta_p, \bar{s}_p)$  and with a full-time worker with probability  $\beta p_f(\theta_f, \bar{s}_f)$ . If they match with a worker of type  $i \in \{f, p\}$ , they receive flow value

$$J_i = x_i - w_i + \beta J_i + \beta \lambda_i [V - J_i] \quad (\text{B.2})$$

where instantaneous income is the value of output  $x_i$  and instantaneous cost is wage  $w_i$ . They also face some risk that the job is destroyed with probability  $\lambda_i$  in which case they choose whether to open a vacancy. Because the goods market is perfectly competitive, firms will post vacancies until the flow value of posting an additional vacancy  $V = 0$  in equilibrium.

The choice of recruiting intensity for full-time workers  $a_f$  is given by equation (??).

$$\begin{aligned} \frac{\partial C}{\partial a_f} = & \beta \left[ \frac{\partial p_f(\theta_f, \bar{s}_f)}{\partial a_f} \left( \frac{x_f - w_f(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\alpha_f} \right) + \frac{\partial p_p(\theta_p, \bar{s}_p)}{\partial a_f} \left( \frac{x_p - w_p(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\alpha_p} \right) \right] \\ & - \left[ \frac{\partial w_f(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\partial a_f} \left( \frac{p_f(\theta_f, \bar{s}_f)}{\alpha_f} \right) + \frac{\partial w_p(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\partial a_f} \left( \frac{p_p(\theta_p, \bar{s}_p)}{\alpha_p} \right) \right] \end{aligned} \quad (\text{B.3})$$

The LHS denotes the direct cost of increasing recruiting intensity  $a_f$  while the RHS denotes the indirect gains and costs that increased recruiting has on matching probabilities and wages. First, increased recruiting intensity  $a_f$  increases the full-time matching probability for a firm while decreasing the part-time matching probability since  $a_p = 1 - a_f$ . In addition, increased recruiting intensity  $a_f$  increases the wage  $w_f$  that the firm pays to a full-time worker while decreasing the wage  $w_p$  that they pay to a part-time workers.

#### B.4 Worker's Problem

Unemployed workers receive some value of leisure  $z$  and choose their search intensity for part-time and full-time employment in order to maximize the flow value

$$U = \max_{s_f, s_p} \left\{ z(1 - h(s_p + s_f)^\nu) + \beta(U + q_f(\theta_f, s_f)[E_f - U] + q_p(\theta_p, s_p)[E_p - U]) \right\} \quad (\text{B.4})$$

where they pay search cost  $zh(s_p + s_f)^\nu$ . Their choice of search intensity affects the probability that they match with a firm for a part-time job with probability  $q_p(\theta_p, s_p)$  or for a full-time job with probability  $q_f(\theta_f, s_f)$ . If a worker matches with a firm of type  $i \in \{f, p\}$ , they receive flow value

$$E_i = w_i + \beta E_i + \beta \lambda_i [U - E_i] \quad (\text{B.5})$$

where wage  $w_i$  is their instantaneous income and they face some risk that the job is destroyed with probability  $\lambda_i$  in which case they become unemployed.

The choice of search intensity for employment of type  $i \in \{f, p\}$  is given by

$$(\Omega_{si} + \Omega_{wi})\Delta_1 + \Omega_{qi}\Delta_2 = \beta(B_{qi} + B_{wi})\Delta_1 \quad (\text{B.6})$$

where the left-hand side of this equation corresponds to the cost of increasing search intensity  $s_i$  while the right-hand side corresponds to the benefits from increasing search intensity  $s_i$ . Workers must pay both direct and indirect cost that result from their choice. First, they pay cost

$$\Omega_{si} = \alpha_f \alpha_p z \nu h(s_p + s_f)^{\nu-1}$$

which corresponds to the direct marginal cost of changing their search intensity. The term  $\alpha_i = 1 - \beta(1 - \lambda_i)$  is the discount term for a firm of type  $i$  and  $\sigma = z(1 - h(s_p + s_f)^\nu)$  is the cost of search intensity. Second, any increase in search intensity  $s_i$  will decrease the wage  $w_i$ , so the worker pays indirect cost

$$\Omega_{wi} = -\beta \alpha_{-i} q_i(\theta_i, \bar{s}_i) \frac{\partial w_i(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\partial s_i}$$

which results from a decrease in market tightness. This is similar to the result from the canonical DMP model wherein falling market tightness reduces the wage. Both of these costs are discounted by the discount term  $\Delta_1$ .

$$\Delta_1 = \alpha_f \alpha_p + \beta \sum_i \alpha_{-i} q_i(\theta_i, \bar{s}_i)$$

Finally, The worker pays discounted indirect cost

$$\Omega_{q_i} \Delta_2 = \left[ \beta \alpha_{-i} \frac{\partial q_i(\theta_i, \bar{s}_i)}{\partial s_i} \right] \left[ \alpha_f \alpha_p \sigma(\mathbf{s}) + \beta \sum_i \alpha_{-i} q_i(\theta_i, \bar{s}_i) w_i(\boldsymbol{\theta}, \bar{\mathbf{s}}) \right]$$

which results from an increase in the discount term for the value of being unemployed. This means that an increase in search intensity  $s_i$  increases  $q_i$  which increases the denominator of the flow value of being unemployed  $U$  resulting in a decrease in  $U$ . In addition to these costs, an unemployed worker can benefit from increasing search intensity  $s_i$  as seen on the RHS of equation (??). First, workers get direct benefit

$$B_{q_i} = \alpha_{-i} \frac{\partial q_i(\theta_i, \bar{s}_i)}{\partial s_i} w_i(\boldsymbol{\theta}, \bar{\mathbf{s}})$$

which comes from an increase in the probability of matching with a firm resulting from an increase in search intensity  $s_i$ . Second, workers receive indirect benefit

$$B_{w_i} = \alpha_i q_{-i}(\theta_{-i}, \bar{s}_{-i}) \frac{\partial w_{-i}(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\partial s_i}$$

from an increase in the wage for the other type of job  $-i$ . As a worker increases search intensity  $s_i$  holding  $s_{-i}$  constant, a higher wage must be paid in order for workers to be willing to work a job of type  $-i$ . Both of these benefits are discounted at rate  $\beta \Delta_1$ .

## B.5 Wage Determination

When a worker-firm match is formed, they bargain over the wage which reduces to the axiomatic Nash Bargaining solution. Workers and firms have full information about each other and the worker has bargaining power  $\gamma$  while the firm has bargaining power  $1 - \gamma$ . Thus the wage for a job of type  $i \in \{f, p\}$  is determined by

$$w_i = \operatorname{argmax}(E_i - U)^\gamma (J_i - V)^{1-\gamma} \quad (\text{B.7})$$

which yields a system of two equations which can be solved for wages  $w_p$  and  $w_f$  as in equation (??). It is important to note that neither wage can be independently determined. They depend on each other.

$$w_i = \frac{\left(\gamma x_i + \frac{(1-\gamma)\alpha_i z(1-h(s_p+s_f)^\nu)}{\alpha_i + \beta q_i}\right) + \left(\gamma x_{-i} + \frac{(1-\gamma)\alpha_{-i} z(1-h(s_p+s_f)^\nu)}{\alpha_{-i} + \beta q_{-i}}\right) \left(\frac{(1-\gamma)\beta q_{-i}}{\alpha_{-i} + \beta q_{-i}}\right)}{1 - (\alpha_i + \beta q_i)(\alpha_{-i} + \beta q_{-i})} \quad (\text{B.8})$$

The wage for a job of type  $i$  depends not only on the surplus generated from creating a job of type  $i$ , but also on the surplus generated from creating a job of type  $-i$ . Assuming  $x_f > x_p$  implies that as  $x_p$  increases, both wages  $w_p$  and  $w_f$  will increase.

## B.6 Steady-State

**Definition B.6.1** *The steady-state equilibrium consists of a list  $(u, l_f, v, a_f, w_f, w_p, s_f, s_p)$  that solves the unemployment flow equation*

$$m_p(\bar{s}_p u, a_p v) + m_f(\bar{s}_f u, a_f v) = \lambda_f l_f + \lambda_p (1 - u - l_f), \quad (\text{B.9})$$

*the full-time employment flow equation*

$$m_f(\bar{s}_f u, a_f v) = \lambda_f l_f, \quad (\text{B.10})$$

the job creation condition for vacancies

$$c = \beta p_f(\theta_f, \bar{s}_f) \left( \frac{x_f - w_f(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\alpha_f} \right) + \beta p_p(\theta_p, \bar{s}_p) \left( \frac{x_p - w_p(\boldsymbol{\theta}, \bar{\mathbf{s}})}{\alpha_p} \right), \quad (\text{B.11})$$

the firm's recruiting intensity maximization equation (??), two wage setting conditions (??), and the worker's two search intensity maximization equations (??).

## B.7 Numerical Examples

To illustrate how the model works in this simplified environment, I parameterize the model and perform some comparative statics assuming a time period of one month. First, the elasticity of the search cost function is  $\nu = 1$ , the value of leisure is  $z = 1$ , the cost of recruiting is fixed at  $c = 0.1x_f$ , and the discount factor is  $\beta = 0.995$ . For worker's and firms, the full-time and part-time job destruction rates are  $\lambda_f = 0.04$  and  $\lambda_p = 0.08$  respectively, the probability of matching is  $\mu_i = \theta_i(1 - e^{-1/\theta_i})$ , and worker's bargaining power is  $\gamma = 0.5$ . The final value of output for full-time workers is  $x_f = 1.6$  while the final value of output for part-time workers follows a Cobb-Douglas production function with a labor-income share of  $2/3$  so that  $x_p = x_f(0.5)^{2/3} = 1.01$ . Finally, the marginal cost of search  $h = 0.001$ .

The results shown in Table ?? line up well with the observed data with an unemployment rate of 7%, a full-time employment rate of 70%, and a part-time employment rate of 23%. There are a few things that are unusual. First, search effort for part-time employment is 12 times as large as search effort for full-time employment which is not empirically consistent with observations that suggest part-time search effort is lower than full-time search effort. One of the properties of the matching function is that as vacancies fall, search effort rises. Since firms are only recruiting for part-time workers with intensity  $a_p = 0.174$ , workers are induced to search harder



Table B.1.: Numerical Elasticities

	$u$	$l_f$	$l_p$	$s_f$	$s_p$	$v$	$a_f$	$a_p$	$w_f$	$w_p$
Baseline	0.071	0.698	0.231	0.183	2.227	0.115	0.826	0.174	1.279	1.186
$h$	0.002	-0.000	-0.000	-0.008	-0.006	0.007	0.002	-0.008	-0.001	-0.001
$z$	-0.995	0.232	-0.396	5.282	7.433	-2.354	-0.635	3.022	0.369	0.425
$\lambda_f$	0.746	-0.222	0.442	0.562	0.540	-0.015	-0.097	0.462	0.011	-0.004
$\lambda_p$	0.167	0.055	-0.219	0.033	-0.979	-0.125	-0.194	0.922	0.107	0.136
$\gamma$	1.628	-0.667	1.517	-2.098	-2.646	-1.260	-0.607	2.888	0.347	-0.065
$x_f$	1.649	0.001	-0.512	-3.274	-2.153	2.461	0.597	-2.838	0.272	0.041
$x_p$	-0.421	-0.167	0.636	0.371	-1.737	-0.015	-0.132	0.626	0.377	0.599

for part-time employment than full-time employment. This suggests that separable cost of search may be needed. Second, the wage for part-time workers is higher than the final value of output for part-time workers. This is likely due to the high value of leisure and firms willingness to take a smaller loss from employing a part-time worker as opposed to continuing to post a vacancy.

Perturbing the model generates some interesting results as well. First, increased marginal cost of search  $h$  has a limited effect, but unemployment increases while both part-time and full-time employment decrease as expected. In addition, workers search with less intensity since the cost of search is higher. Second, increasing the full-time job destruction rate  $\lambda_f$  results in increased unemployment, increased part-time employment, and decreased full-time employment. It also results in increased search effort for both part-time and full-time employment. This result aligns with Aguiar, Hurst, and Karabarbounis (2013) who find that the value of leisure is lower in bad economic conditions and search intensity is higher. Third, increasing the part-time job destruction rate  $\lambda_p$  leads to similar results with increased unemployment, increased full-time employment and decreased part-time employment. Interestingly, search intensity increases for full-time employment, but decreases for part-time

employment. This is likely due to the dominating effect that full-time employment exerts in the model. Finally, if the value of leisure  $z$  is increased, the unemployment rate decreases due to a large increase in search effort. Interestingly, part-time employment decreases while full-time employment increases even with increased firm recruiting for part-time workers. I posit that this stems from the higher job destruction rate for part-time employment compared to full-time employment.

There are some very inconsistent results. First, increasing the full-time final value of output  $x_f$  results in higher unemployment even if the full-time employment rate increases. This is inconsistent with the prior literature and largely stems from a large decrease in search effort by workers; however, increasing the part-time final value of output  $x_p$  results in decreased unemployment and full-time employment, but higher part-time employment as one would expect. If both are increased proportionally, then the full-time effect dominates the part-time effect resulting in higher unemployment. Even though both wages increase, the response from search effort induces a decrease in unemployment. Scheduling costs, which are analogous to an increase in the final value of output for a firm, are one interesting theory as to why part-time employment has risen in recent years. With more efficient scheduling, part-time employment can become more valuable as it allows firms greater scheduling flexibility. As an example consider the case of  $n$  160 hour blocks that need to be filled. You could fill each block with one full-time worker who works 160 hours per month or two part-time workers who work 80 hours each. This means that an employer has  $n!$  ways to fill these blocks with full-time workers or  $(2n)!$  ways with part-time workers. It is fairly obvious to see that the number of ways a block can be filled with part-time workers is growing at a much faster rate than for full-time workers which implies that there may be lower scheduling costs associated with part-time workers and any decrease in part-time costs could have a disproportionate impact. In the context of the model,

a decrease in scheduling costs is associated with a higher final value of output and higher part-time employment which would be consistent with the scheduling cost theory.

Without multiple job holdings, the model is fairly consistent with the data even without calibrating the model. When workers and firms have a choice between full-time and part-time employment, their search intensity and recruiting intensity depend not only on the direct and indirect costs and benefits of searching/recruiting for a given type of job, but also the indirect costs and benefits for the other type of job. If the two types of jobs are identical in every way, then the problem reduces to the standard DMP model, so it is relatively tractable. When workers and firms have the option to hold multiple jobs, their search intensity and recruiting intensity depend not only on the direct and indirect costs and benefits of searching/recruiting for a given type of job, but also the indirect costs and benefits for the other type of job. Even if the two types of jobs are identical in every way, the problem does not reduce to the standard DMP model unless multiple job holdings is turned off entirely.

One final observation regards the relationship between the unemployment rate and the full-time rate. As can be seen in Table ??, the unemployment rate  $u$  and the full-time rate  $l_f$  usually move in opposite directions. A natural question to ask is under what conditions, the relationship switches as it does when perturbing  $\lambda_p$  and  $x_p$ .

**Proposition B.7.1** *So long as the following condition holds, the full-time rate will always move in the opposite direction of the unemployment rate:*

$$\frac{\lambda_f - \lambda_p - u \left[ \left( \frac{\partial q_f}{\partial v} + \frac{\partial q_p}{\partial v} \right) \frac{dv}{dl_f} + \left( \frac{\partial q_f}{\partial a_f} + \frac{\partial q_p}{\partial a_f} \right) \frac{da_f}{dl_f} + \left( \frac{\partial q_f}{\partial s_f} \right) \frac{ds_f}{dl_f} + \left( \frac{\partial q_p}{\partial s_p} \right) \frac{ds_p}{dl_f} \right]}{\lambda_p + q_p + q_f + u \left( \frac{\partial q_f}{\partial u} + \frac{\partial q_p}{\partial u} \right)} < 0$$

The derivation for Proposition ?? is provided in Appendix ?. As soon as the gains to matching are positive for an increased unemployment rate, the full-time employment rate and unemployment rate will begin to move in the same direction. This switching is most obvious for changes in  $\lambda_p$  for which  $u$  and  $l_f$  begin to move in the same direction. This becomes apparent again when both search intensities drop and vacancies rise as it does for a change in  $x_p$ . This causes the gains from higher unemployment to increase.

To remedy some of the inconsistencies described above, I consider the case of multiple job holdings in addition to part-time employment. Multiple job holdings is particularly relevant when considering part-time employment as part-time workers are more likely to have multiple jobs and the option of additional employment increases the flow value of being a part-time worker. For instance, if being a full-time worker becomes more valuable, then fewer workers will be employed part-time, but with multiple job holdings, the value of being a part-time worker also increases as they have the option of working a secondary full-time job. This mutates dampens the response of part-time employment. Finally, I consider the case of separable search cost with the cost of search being allowed to vary depending on the type of job a worker is searching for.

## B.8 Uniqueness

Given that workers can hold more than one job and firms can recruit more than one type of worker, it is not clear that there will be a unique solution. Consider the job creation curve and wage setting curve that relate how the vacancy rate and wage are related. Since there are two wages, the wage setting curve could be non-monotonic or upward sloping which would result in more than one equilibrium.

**Proposition B.8.1** *If the following conditions hold, then there exist a unique triplet  $(v, w_1, w_2)$  for the vacancy rate and both wages such that there exist a unique steady-state equilibrium for the above problem.*

$$q_2 < \frac{1}{\beta} \quad (\text{B.12})$$

$$\begin{aligned} \alpha_1 \alpha_2 C(a_1) \left[ \alpha_2 \frac{\partial p_1}{\partial v} + \beta \lambda_1 \frac{\partial p_2}{\partial v} \right] &> x \left[ \beta \lambda_1 (\lambda_1 p_2 - \beta \lambda_1 p_2) - p_1 \alpha_1 \alpha_2 \right] \frac{\partial p_2}{\partial v} \\ &+ x \left[ p_2 \alpha_1 \alpha_2 \right] \frac{\partial p_1}{\partial v} - w_2 \left[ p_2 \beta \lambda_1 \alpha_1 \right] \frac{\partial p_2}{\partial v} \\ &+ w_2 \alpha_1 \left[ p_1 \alpha_2 + \beta \lambda_1 p_2 + p_2 \alpha_2 \right] \frac{\partial p_1}{\partial v} \end{aligned} \quad (\text{B.13})$$

$$\alpha_1 C(a_1) \frac{\partial p_2}{\partial v} > \beta (x - w_1) \left( p_2 \frac{\partial p_1}{\partial v} - p_1 \frac{\partial p_2}{\partial v} \right) \quad (\text{B.14})$$

$$0 < \beta \left[ \frac{(1 - \gamma) \beta \lambda_1 (w_1 + \sigma_2 - \sigma_1)}{(1 - \beta + \beta \lambda_1 + \beta q_1)^2} \right] \frac{\partial q_1}{\partial v} \quad (\text{B.15})$$

The proof for this proposition is given below. Essentially, it must be shown that the wage setting curves cross the job creation curve at only one point regardless of whether the two curves are both upward sloping. It is not enough to show that the two curves are monotonically moving in opposite directions since the two curves are allowed to do so given the other wage and recruiting intensity. As such, a continuum of solutions can also be ruled out.

**Proof** Given  $w_2$ , (??) can be plugged into (??) to form a single equation in terms of  $v$ . Similarly, given  $w_1$ , (??) can be plugged into (??) to form another equation in terms of  $v$ . Since the job creation condition is set equal to zero, it follows that

$$f(v) = h(v) + g(v) = 0$$

is an implicit function defined in terms of  $v$ . In order for a unique solution to exist, there must be a unique optimum that satisfies:

$$f'(v) = h'(v) + g'(v) = 0$$

As long as  $h'(v) = -g'(v)$ , a unique optimum exist. ■

### C. Derivation of Proposition ??

**Proof** Fully differentiating the unemployment flow equation, we get:

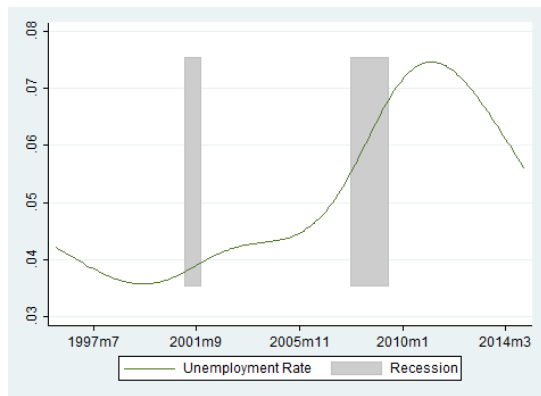
$$\begin{aligned} du(q_f + q_p + \lambda_p) &= (\lambda_f - \lambda - p)dl_f - u\left(\frac{\partial q_f}{\partial u} du + \frac{\partial q_f}{\partial v} dv + \frac{\partial q_f}{\partial a_f} da_f + \frac{\partial q_f}{\partial s_f} ds_f\right) \\ &\quad - u\left(\frac{\partial q_p}{\partial u} du + \frac{\partial q_p}{\partial v} dv + \frac{\partial q_p}{\partial a_f} da_f + \frac{\partial q_p}{\partial s_p} ds_p\right) \end{aligned}$$

which can be rearranged as

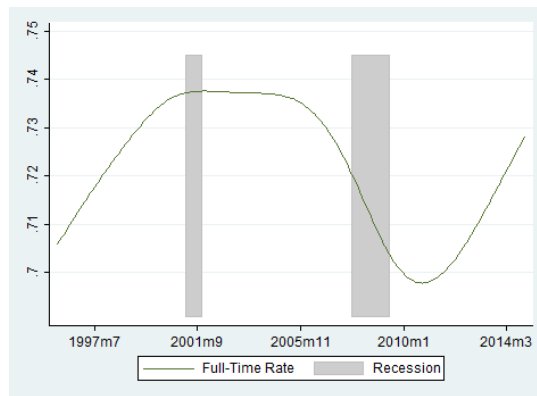
$$\frac{du}{dl_f} = \frac{\lambda_f - \lambda_p - u\left[\left(\frac{\partial q_f}{\partial v} + \frac{\partial q_p}{\partial v}\right)\frac{dv}{dl_f} + \left(\frac{\partial q_f}{\partial a_f} + \frac{\partial q_p}{\partial a_f}\right)\frac{da_f}{dl_f} + \left(\frac{\partial q_f}{\partial s_f}\right)\frac{ds_f}{dl_f} + \left(\frac{\partial q_p}{\partial s_p}\right)\frac{ds_p}{dl_f}\right]}{\lambda_p + q_p + q_f + u\left(\frac{\partial q_f}{\partial u} + \frac{\partial q_p}{\partial u}\right)}$$

■

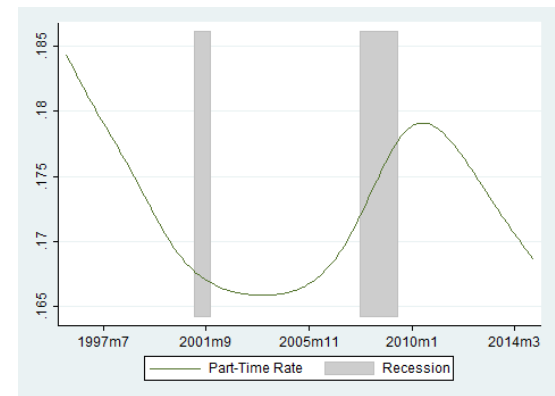
## D. Tables and Figures



(a) U Rate



(b) FT Rate



(c) PT Rate

Figure D.1.: Employment Rates



Table D.1.: Descriptive Statistics – Non-Recession\*

Employment Status	Percent of Population	Millions of Persons	Flow Into	Percent	Millions of Persons	Hazard Rate	
U	7.6	7.4	{	FT	20.5	1.5	1.6
				PT	9.1	0.7	2.4
				U	70.4	5.2	0.4
FT	70.7	64.2	{	PT	5.7	3.7	2.9
				FT / PT	0.9	0.6	4.7
				U	2.3	1.5	3.8
				FT	91.1	58.4	0.1
PT	16.0	14.8	{	FT	25.5	3.8	1.4
				FT / PT	0.9	0.1	4.7
				Dual PT	0.6	0.1	5.1
				U	4.6	0.7	3.1
				PT	68.4	10.1	0.4
FT / PT	4.7	4.3	{	FT	19.1	0.8	1.7
				PT	3.7	0.2	3.3
				FT / PT	77.2	3.3	0.3
Dual PT	0.8	0.7	{	PT	15.6	0.1	1.9
				Dual PT	84.4	0.6	0.2

\*Average calculated using HP-filtered monthly CPS data from December 2001 to December 2004

Table D.2.: Jointly Calibrated Parameters

Parameter	2001-2004*	2015-2016*	New Target	
$b$	0.9668	0.9568	$a_s v / u = 0.88$	
$c$	0.3418	0.6130	Total cost of recruiting = 0.3203	
$h_f$	0.0432	0.0464	} $q_{uf} = 0.1773$	
$h_p$	0.0040	0.0053		$q_{up} = 0.0873$
$H_f$	7784.1	8067.5		$q_{fp} = 0.0073$
$H_p$	1346.5	1348.9		$q_{pf} = 0.0066$
$C$	0.2929	0.2022		$q_{pp} = 0.0054$

\* Time periods correspond to Dec. 2001 - Dec. 2004 and Jan. 2015 - Dec. 2016.