Re-assessing New Keynesian paradox of flexibility

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We argue that the paradox of flexibility - that more flexible price results in worse outcomes when zero lower bound is binding - is ruled out once we consider an implicit equilibrium selection mechanism used when solving a New Keynesian model often not explicitly stated - the symmetric limit condition. Dropping the implicit mechanism leads to extraneous multiple equilibria, and breakdown of New Keynsian Phillips curve. The standard equilibrium selection in zero lower bound circumstances is questioned, given the symmetric limit condition.

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I. Introduction

When zero lower bound is binding, models in the New Keynesian tradition have the paradox of flexibility in the standard interpretation - that as price becomes flexible, deflation and lower output circumstances become worse, despite an actual flexible-price economy behaving otherwise.

In Cochrane (2017), it was argued that this is sensitive to a choice of an

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equilibrium selection mechanism and that non-standard equilibrium selection mechanisms are justified economic logic-wise.

Theoretical responses to Cochrane (2017) that argue non-standard equilibrium selection mechanisms are not justified economic logic-wise have since appeared - see Evans and McGough (2018) and García-Schmidt and Woodford (2019) as examples. They mainly involve how agents actually set expectations and the rational expectation limit of such expectation mechanisms - in other words, we should start from a model not utilizing rational expectation to select a rational expectation equilibrium.

We provide another dimension to the debate: when solving for an equilibrium, New Keynesian models have an additional equilibrium selection condition that is not explicitly stated. We call this the symmetric limit condition:

> The symmetric limit condition refers to an argument that as firms in an economy become symmetric, their equilibrium values must become symmetric as well.

We argue that rejection of the condition leads to the benchmark New Keynesian Phillips curve (PC) being unjustified and multiple equilibria prevailing in the basic New Keynesian model (see Blanchard and Kiyotaki (1987), Woodford (2003) and Galí (2015) for details of the basic New Keynesian model) that often is used to derive the three-equation model.

The argument is simple. When deriving a price-setting equation for a monopolistic competition economy, the basic New Keynesian model uses an argument that an individual firm has zero effect on aggregate price level and output because there are infinitely many firms. This enforces an equilibrium outcome of firms to be symmetric when specifications of firms are symmetric to each other.

However, we do not actually believe that there are infinitely many firms - number of firms being infinite is introduced for tractability. Furthermore, specifications of firms are generally asymmetric, which opens up for multiple equilibria even in the symmetric limit. To eliminate such extraneous equilibria, the symmetric limit condition must be imposed - as specifications of firms become symmetric, an equilibrium outcome of firms must be symmetric as well.

In New Keynesian models that follow Calvo pricing (Calvo (1983)), price rigidity introduces additional heterogeneity of firms. If specification of firms are symmetric when firms have full flexibility on pricing, heterogeneity in specifications of firms only exists due to price rigidity. The imposition of the symmetric limit condition thus requires that as price becomes flexible, an equilibrium outcome of firms becomes symmetric.

In usual circumstances, the standard equilibrium selection picks out a flexible-price limit that is equivalent to an actual flexible-price economy. When zero lower bound becomes binding, however, the paradox of flexibility arises under the standard equilibrium selection, which contradicts the symmetric limit condition.

Thus, one must choose one of the following: 1) rational expectation equilibria are useless for economic analysis, 2) reject the symmetric limit condition and accept multiple equilibria under basic New Keynesian restrictions and search for an additional specification and restriction or a different model to ensure a unique 'New Keynesian'-style equilibrium, 3) reject the standard equilibrium selection. Out of the three options, the most conservative and least disruptive option is 3).

II. Flexible-price New Keynesian model

Let us state the basic flexible-price New Keynesian model. For convenience, we assume that an economy is deterministic, but conclusions of analysis here applies to stochastic cases without loss of generality.

The representative consumer has utility function U that it maximizes:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where β is time preference discount factor, C_t is consumption, N_t is labor utilized. It is subject to the budget constraint:

$$P_t C_t + \frac{B_t}{1+i_t} \leq W_t N_t + F_t + B_{t-1} \tag{2}$$

where P_t is price level, B_t is central bank-issued bonds, i_t is nominal interest rate set on B_t, W_t is wage, and F_t is dividends paid from firms.

There is monopolistic competition in an economy - we apply the standard CES toolkit, such that:

$$C_t \equiv \left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

where C_{it} is consumption of goods at firm i. Price level P_t is defined such that $P_t C_t = \int_0^1 P_{it} C_{it} \, di$. In equilibrium, $Y_t = C_t$ and $C_{it} = Y_{it}$, and thus from now on, we will use Y and C interchangeably.

The resulting price level and demand function for Y_{it} are:

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}} \tag{4}$$

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t \tag{5}$$

Now specification of firms. Firms are assumed to utilize homogeneous labor, its only production factor, such that wage level must be same across firms. Firms have production function:

$$Y_{it} = A_t N_{it}^{1-\alpha} \tag{6}$$

with $\int_0^1 N_{it}\, di = N_t$. Firms maximize profits F_{it}, which are all given out as dividends:

$$F_{it} = P_{it} Y_{it} - W_t N_{it} \tag{7}$$

Each firm selects P_{it} to maximize profit, given its demand function for Y_{it}. Firms take W_t as given.

Since firm i is considered of negligible size given that there are infinitely many firms, we now assume that change of P_{it} does not affect P_t and Y_t. We do not need to assume that an equilibrium outcome of firms must be symmetric because firm specifications are symmetric. The profit maximization solution then says:

$$\begin{aligned} P_{it} &= \frac{\varepsilon}{\varepsilon-1} MC_{it} \\ &= \frac{\varepsilon}{\varepsilon-1} \frac{1}{1-\alpha} \frac{W_t}{A_t{}^{1/(1-\alpha)}} Y_{it}{}^{\frac{\alpha}{1-\alpha}} \\ &= \frac{\varepsilon}{\varepsilon-1} \frac{1}{1-\alpha} \frac{W_t}{A_t{}^{1/(1-\alpha)}} \left[\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t\right]^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{8}$$

where MC_{it} refers to marginal cost, when total cost is $W_t N_{it} = W_t (Y_{it}/A_t)^{1/(1-\alpha)}$.

Thus,

$$(P_{it})^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{\varepsilon-1}\frac{1}{1-\alpha}\frac{W_t}{{A_t}^{1/(1-\alpha)}}(P_t)^{\frac{\varepsilon\alpha}{1-\alpha}}(Y_t)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

Because we assumed P_t and Y_t are not changed by individual firm decisions due to firm size being negligible, Equation (9) suggests that every firm must have the same equilibrium, even when we do not impose the symmetric limit condition.

However, this result depends crucially on the number of firms being infinite. Since no one actually believe in the number of firms being infinite, and this is only introduced for tractability reasons, we have to think of the infinite-number-of-firms economy as the limit point of finite-number-of-firms economies.

In a finite-number-of-firms economy, we can no longer assume that each firm decision does not affect an aggregate outcome. Furthermore, without imposing that an equilibrium outcome of firms be symmetric because firm specifications are symmetric, heterogeneous outcomes are possible. This is especially so, given that we can slightly tweak firm specifications as to be not symmetric.

In such a case, as the infinite-number-of-firms limit is approached from finite-number-of-firms economies, heterogeneity of firm outcomes can actually increase, and we can no longer treat decisions of some firms as not affecting aggregate price level and output even in the limit. This possibility is only eliminated when we impose the symmetric limit condition.

In the appendix, we fully derive and explore multiple equilibria aspects of a flexible-price New Keynesian economy when there are finitely many firms and the symmetric limit condition is not imposed.

A. Symmetric limit condition

Required imposition of the symmetric condition has not been problematic and has been done implicitly without notice, since almost everyone accepts the symmetric limit condition.

While it is true that individual firms are different even when they are symmetric in specification - after all, New Keynesian models live in a monopolistic competition economy and each firm has some form of 'brand power' - each firm can easily copy another firm at its advantage, unless there is a restriction on that.

While our discussion of the symmetric limit condition involved the New Keynesian CES setup that dates back to Blanchard and Kiyotaki (1987), the condition is not only applicable to such a setup. Since the above outside-model economic motivation of the condition applies generally, the symmetric limit condition must be understood as an implicit equilibrium selection mechanism that is widely accepted such that it is used without much explicit notice.

B. Generalization of the multiple equilibria result

The multiple equilibria result when the symmetric limit condition is not imposed can be generalized. Thus it is not a particular specification that drives qualitative conclusions in this paper.

The spirit of New Keynesian modeling can be identifies as follows:

- Monopolistic competition. This gives us n price-setting equations when there are n firms.
- The aggregate demand effect that affects every firm. In other words, demand Y_{it} for the product of firm i depends on aggregate demand Y_t.

Since there are n firms, there are n demand equations, one for each firm producing only one variety of products.

- Consistent functional form of aggregate production and aggregate demand Y_t can be defined such that one can derive aggregate price level P_t from Y_t and budget constraints. Additionally, one can derive functional form of Y_t from functional form of P_t and demand functions as well.

There are $2n$ equations and $2n + 1$ variables (all P_{it}, all Y_{it} and Y_t) when the market uses only price signals, in fashion of traditional general equilibrium. We can eliminate variable P_{it} by setting it to be 1 and eliminate the corresponding price-setting equation by invoking Walras' law. This finally gives us $2n - 1$ equations and $2n$ variables. Thus multiple equilibria prevail if without an additional market mechanism or imposition of the symmetric limit condition.

III. Zero lower bound model

We now consider the basic deterministic sticky-price New Keynesian model. For the purpose here, we utilize the log-linearized three-equation model (around zero-inflation steady state):

$$\tilde{y}_t = \tilde{y}_{t+1} - \sigma^{-1}\left(i_t - \pi_{t+1} - r_t^n\right) \tag{10}$$

where $\tilde{y}_t$ is output gap, π_t is inflation rate, r_t^n is natural real rate of interest.

$$\pi_t = \beta\pi_{t+1} + \kappa\tilde{y}_t \tag{11}$$

where as $\kappa \to \infty$, firms become more flexible in price setting. Equation (11) is often called as the basic New Keynesian Phillips curve (NKPC).

We now introduce the zero lower bound setup in Werning (2012), which is shared by Cochrane (2017). At $0 \leq t < T$, an economy faces $r_t^n = r$, where r is negative. $i_t = 0$ at $0 \leq t < T$. At $T \leq t$, $i_t = r_t^n$ is assumed. As Cochrane (2017) emphasizes, this interest rate peg structure simplifies analysis without loss of generality - we should not think that it is this peg structure that drives qualitative conclusions.

A. *Standard equilibrium selection and paradox of flexibility*

The standard equilibrium selection picks $\pi_T = 0$ and $\pi_{T+1} = 0$, which imply $\tilde{y}_t = 0$ and $\pi_t = 0$ for $t \geq T$. Let us look at $t = T - 1$.

$$\tilde{y}_{T-1} = \sigma^{-1} r \tag{12}$$

from Equation (10),

$$\pi_{T-1} = \kappa \tilde{y}_{T-1} = \kappa \sigma^{-1} r \tag{13}$$

from Equation (11) and (12).

Now let us look at $t = T - 2$.

$$\tilde{y}_{T-2} = \tilde{y}_{T-1} - \sigma^{-1}\left(-r - \pi_{T-1}\right) = \sigma^{-1} r - \sigma^{-1}\left(-r - \kappa\sigma^{-1} r\right) \tag{14}$$

$$\pi_{T-2} = \beta\pi_{T-1} + \kappa\tilde{y}_{T-2} = \beta\kappa\sigma^{-1} r + \kappa\left[\sigma^{-1} r - \sigma^{-1}\left(-r - \kappa\sigma^{-1} r\right)\right] \tag{15}$$

The paradox of flexibility now can be identified: as $\kappa \to \infty$, $\tilde{y}_{T-2} \to -\infty$,

$\pi_{T-2} \to -\infty$. This is not a one-period event: the paradox of flexibility appears at $0 \leq t \leq T-2$.

B. *Calvo pricing and symmetric limit condition*

Under Calvo pricing, price rigidity induces heterogeneous specifications of firms - some firms are stuck at some price while other firms are free to change their price. More specifically, at each period, a firm has constant probability θ of being unable to change its price. As price rigidity is reduced, firm specifications become more symmetric. The point of the above paradox of flexibility then is that specifications becoming symmetric does not guarantee an equilibrium outcome being more symmetric as well.

But this violates the symmetric limit condition. Note also that the symmetric limit condition is implicitly invoked when solving for the New Keynesian Phillips curve in Equation (11) - symmetric firms that are freed to set price are assumed to set the same price.

To maintain the symmetric limit condition, the most conservative option is to reject the standard equilibrium selection - to look for an alternative π_T and π_{T+1} selection that respects the symmetric limit condition.

C. *Rotemberg pricing and symmetric limit condition*

In Rotemberg pricing (Rotemberg (1982)), firm specifications are symmetric even when there is price rigidity, as long as they are symmetric in the flexible-price case. Thus, it may seem that one may recast the paradox of flexibility in a Rotemberg pricing model that is free from the symmetric limit condition issue. We can no longer say that as price rigidity is reduced, firm heterogeneity is reduced, since firms are already symmetric even when price is rigid.

But reasons why we choose a particular equilibrium selection mechanism would still hold regardless of whether one uses Calvo, Rotemberg or some other means of introducing price rigidity. While we invoked the symmetric limit condition to argue that the standard equilibrium selection may not be the right one, we still have to choose a particular equilibrium selection mechanism and provide economic logic behind the selection. Such economic logic holds regardless of whether the symmetric limit condition is binding or not. Plausible selection mechanisms are explored in Cochrane (2017) - in this paper, we do not explore this matter.

Thus, unless one argues that a Calvo pricing model is mostly irrelevent to analysis of real economies, reasons for a particular equilibrium selection carry over to models with different means of introducing price rigidity.

IV. Conclusion

We argued that the symmetric limit condition is implicitly used when solving for a New Keynesian model, regardless of which explicit equilibrium selection is utilized. In zero lower bound contexts simplified as in Werning (2012), the symmetric limit condition rules out the standard equilibrium selection when there is Calvo pricing. The New Keynesian paradox of flexibility is ruled out by the symmetric limit condition. While this paper focused on Calvo pricing, unless Calvo pricing is irrelevent to economic analysis, economic logic behind an alternative equilibrium selection mechanism that is compatible with the symmetric limit condition in Calvo setups carries over to models with alternative means of introducing rigidity.

Given that empirical evidence of paradoxes given by the standard equilibrium selection, when solving a New Keynesian model, is debatable, as can be seen in Wieland (2019), analysis in this paper is not of mere theoretical curiosity. This paper provides one more compelling evidence on why the standard equilibrium selection might be wrong one to take, in light of Cochrane (2017), despite theoretical arguments supporting the standard equilibrium selection, most strongly learnability arguments as in Evans and McGough (2018). Unless one is willing to give up rational expectation analysis or find an alternative mean of carrying out economic analysis with New Keynesian emphasis, the standard equilibrium selection and the paradox of flexibility must go.

Competing Interests

Authors confirm no competing interests and no funding sources to report.

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Verification of multiple flexible-price equilibria when the symmetric limit condition is not imposed

In this appendix, we do not invoke the symmetric limit condition. Thus even if firm specifications are symmetric, an equilibrium outcome of firms is not granted to be symmetric. As before, we analyze a CES monopolistic competition economy but with finite number of firms.

The representative consumer has utility function U that it maximizes Equation (1):

$$U = \sum_{t=0}^{\infty} \beta^t \left(\frac{{C_t}^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right)$$

where β is time preference discount factor, C_t is consumption, N_t is labor utilized. It is subject to the budget constraint as in Equation (2):

$$P_t C_t + \frac{B_t}{1+i_t} \leq W_t N_t + F_t + B_{t-1}$$

where P_t is price level, B_t is central bank-issued bonds, i_t is nominal interest rate set on B_t, W_t is wage, and F_t is dividends paid from firms.

There is monopolistic competition in an economy - we apply the standard CES toolkit, such that:

$$\text{(A1)} \qquad C_t \equiv \left(\sum_i C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where C_{it} is consumption of goods at firm i.

Price level P_t, defined with $P_t C_t = \sum_i P_{it} C_{it}$, is derived as:

$$\text{(A2)} \qquad P_t = \left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

and demand function for firm i is as in Equation (5):

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$$

In equilibrium, $Y_t = C_t$ and $C_{it} = Y_{it}$, and thus from now on, we will use Y and C interchangeably.

Now specification of firms. Firms are assumed to utilize homogeneous labor, its only production factor, such that wage level must be same across firms. Firms have production function as in Equation (6):

$$Y_{it} = A_t N_{it}^{1-\alpha}$$

with $\sum_i N_{it} = N_t$. Firms maximize profits F_t defined as in Equation (7), which are all given out as dividends:

$$F_t = P_{it} Y_{it} - W_t N_{it}$$

Each firm selects P_{it} to maximize profit, given its demand function for Y_{it}. Firms take W_t as given.

Now the intuition is clear: because Y_t and P_t are now each affected by Y_{it} and P_{it}, P_{it} does depend on value of Y_{it} in the solution of the profit maximization problem. Thus, there will be multiple equilibria.

Firm i's price-setting function would be, substituting wage demand (labour supply) function coming from the consumer optimization problem and production function:

$$P_{it} = f_t(Y_{it}, \{Y_{jt}\}_{j \neq i}, \{P_j\}_{j \neq i}) \tag{A3}$$

where f_t refers to a function. Recall the demand function for firm i in Equation (5):

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$$

There are $2n - 1$ equations when there are n firms: $n - 1$ equations from Equation (A3) and n equations from Equation (5) - having set one of P_{it} to be 1 or some constant value. We invoke Walras' law to allow for elimination of an equation for P_{it}. (We know that 'excess demand' of a product must be zero when the market for other products clears.) There are $2n$ variables: $n - 1$ instances of P_{it}, n instances of Y_{it} and Y_t.

P_t is determined from $\{P_{it}\}$. Y_t can be determined from $\{Y_{it}\}$, but if we substitute Y_t with Equation (A1), then since $Y_{it} \in \{Y_{it}\}$ and given the form of Equation (A1), we would not be able to write the demand function in form of:

$$Y_{it} = g_t(\{P_{jt}\}, \{Y_{jt}\}_{j \neq i})$$

Furthermore, by construction, Equation (A2) and Equation (5) replicate Equation (A1). The proof goes as follows:

$$\begin{aligned}
C_t &= (P_t C_t) P_t^{\varepsilon-1} \left[\left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \right]^{-\varepsilon} \\
&= (P_t C_t) P_t^{\varepsilon-1} \left(\sum_i P_{it}^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i \left(P_{it}^{-\varepsilon} P_t C_t P_t^{\varepsilon-1} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i \left(\left[\frac{P_{it}}{P_t} \right]^{-\varepsilon} C_t \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\sum_i C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned} \tag{A4}$$

The first line in Equation (A4) follows from the definition of P_t in Equation (A2). The second, third and fourth line are tautological. The fifth line follows from Equation (5).

Thus there are multiple equilibria, since there are only $2n - 1$ equations for $2n$ variables.