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Abstract

This study explores a novel channel for monetary policy to impact growth and welfare—a cash-in-advance constraint on R&D combined with R&D subsidies by seigniorage tax. In a scale-invariant Schumpeterian growth model, growth is an inverted-U function of the inflation rate. Friedman rule is suboptimal (optimal) when the elasticity of labor supply is low (high). By contrast, the inverted-U relation does not exist when R&D is subsidized by other taxes or in an AK model. Calibration confirms our prediction and finds that the growth and welfare effects of inflation are large. Using panel data for 154 countries during 1970–2014, both non-parametric cubic spline and parametric regressions show that growth is an inverted-U function of the inflation rate in samples with an annual inflation rate below 30%. The cutoff point for inflation to have a zero marginal effect on growth is around 5% in ordinary least squares estimation and 3% in instrumental variables (IV) estimation. We also find that the share of labor employed in R&D—rather than the physical capital investment rate—is an inverted-U function of inflation in IV estimation. Our empirical evidence provides support for our theory.

JEL Classification: E52 O42 O47

Keywords: cash-in-advance constraint on R&D; government subsidies of R&D with seigniorage; inflation; growth; welfare; inverted-U; panel data

Declarations of interest: none

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1 Introduction

There is substantial long-standing debate over the effect of inflation on economic growth—one fundamental issue in monetary economics (see, e.g., Tobin 1965; Sidrauski 1967; Stockman 1981; Gomme 1993; Jones and Manuelli 1995; Marquis and Reffett 1994; Dotsey and Sarte 2000; Funk and Kromen 2010; Chu and Lai 2013; Chu et al. 2015; He and Zou 2016; Chu, Ning and Zhu 2019; Chu et al. 2017; Arawatari et al. 2018; Chu et al. 2019; He 2018a,b,c; Zheng et al. 2018). In this paper, we reveal a novel channel—a cash-in-advance (CIA) constraint on R&D investment combined with government subsidies of R&D with the seigniorage revenue—through which monetary policy may impact growth and welfare. We find that growth is an inverted-U function of the inflation rate. Friedman rule is suboptimal (optimal) when the elasticity of labor supply is low (high).

We calibrate the model to estimate the growth and welfare effects of a change in the nominal interest rate. Calibration shows the following. When 7.7% (15%) of the seigniorage revenue is used in R&D subsidies, maximizing growth requires that the nominal interest rate increase from 9.6% to 11.9% (73%); the growth gain is 0.001% (0.66%), and the welfare gain is equivalent to a permanent decrease in consumption of 0.35% (1.76%). To maximize welfare, the nominal interest rate must be 0% (33.6%), and the welfare gain is equivalent to a permanent increase in consumption of 0.85% (5.07%). The growth and welfare effects are smaller when the elasticity of the labor supply is larger.

As an empirical test, we combine the most recent Penn World Table (PWT) 9.0 (explained by Feenstra, Inklaar and Timmer 2015) with the World Development Indicators (WDI) of the World Bank to build panel data for 154 countries during 1970–2014. We follow the common practice of taking five-year averages of data, which yields a balanced panel with 1,386 observations. We find the following. Both non-parametric cubic spline regression and parametric regressions indicate that growth is an inverted-U function of the inflation rate in samples with an annual inflation rate below 30%. The cutoff point for inflation to have a zero effect on growth is around 5% in ordinary least squares estimation and 3% in instrumental variables (IV) estimation.

To test the direct mechanism, we find that the share of labor employed in R&D from the UNESCO Institute for Statistics—rather than the physical capital investment rate—is an inverted-U function of the inflation rate in IV estimation. Our empirical evidence provides support for our theory.

The contribution of our study is as follows. On the theoretical side, we propose a novel channel for monetary policy to impact growth and welfare. There is a large body of literature studying government subsidies of R&D (for Europe, see Almus and Czarnitzki 2002; Czarnitzki and Hussinger 2004; for China, see Boeing 2016; Wei, Xie and Zhang 2017). The United States is no exception. We access the web page of the National Science Foundation (NSF) and find that the amount of "Domestic R&D paid for and performed by the company" is \$282,570 million for all industries, while "Domestic R&D paid for by the U.S. federal government and performed by the company" is \$26,554 million for all industries in 2014. According to the H6 release of the Federal Reserve on *Money Stock and Debt Measures*, the seasonally adjusted annual rates of growth for M1 and M2 during February 2017 to February 2018 are 6.7% and 4.0%, respectively, which would yield positive seigniorage. The amount of remittances to the Treasury "Required by the Federal Reserve Act, as amended by the FAST Act" totaled \$91,467 million in 2016 (see Section 2.2 for details). *Section 7(a) of the Federal*

Reserve Act states that the earnings transferred to the Treasury will be deposited into the general fund of the Treasury. Therefore, the seigniorage of the Federal Reserve may be used in government subsidies of R&D through many government agencies including the NSF.

Given the real-world relevance, it is important for us to investigate how this institutional feature would affect the impact of monetary policy on growth and welfare. We find that government subsidies of R&D with the seigniorage revenue would produce a *positive seigniorage effect* of a higher nominal interest rate on growth. When the CIA constraint is applied to R&D investment, a higher nominal interest rate decreases R&D and thereby growth (a *negative CIA-on-R&D effect*). At low levels of inflation, the *positive seigniorage effect* dominates, and inflation promotes growth; however, beyond a particular point, the *negative CIA-on-R&D effect* dominates, and inflation retards growth. The substantial growth and welfare effects have been presented above.

By contrast, the inverted-U relationship does not exist when R&D is subsidized by other taxes. The difference in results can be explained as follows. When the R&D subsidy comes from seigniorage, it naturally changes with the monetary policy/inflation. When the R&D subsidy comes from consumption tax, it will not change with the inflation rate. Therefore, the other taxes cannot generate the *changing* seigniorage effect, ending up being unable to produce the inverted-U result. Growth remains to be a decreasing function of inflation when R&D is subsidized by other taxes.

Moreover, the same seigniorage approach can only predict a nonlinear but monotone—negative or positive—effect of money growth on long-run growth in the AK model. In Schumpeterian growth models, R&D activities and, therefore, the levels of productivity are endogenous. In contrast, the level of technology is fixed in the AK model, which does not respond to changes in the nominal interest rate. Therefore, the growth dynamics in the AK model will be not as rich as those in Schumpeterian growth models. Our approach to the non-monotone effect of inflation on growth complements the heterogeneity approach of Chu et al. (2017), Arawatari et al. (2018) and Chu et al. (2019), who focus on heterogeneous firms, entrepreneurial ability, firms and households, respectively, and the approach of combining vertical and horizontal innovations in Zheng et al. (2018).

On the empirical side, our study contributes to our understanding of the empirical debate on the effect of inflation on growth (Kormendi and Meguire 1985; Barro 1995; Bullard and Keating 1995; Bruno and Easterly 1996; Fischer 1993; Ahmed and Rogers 2000; Kremer et al. 2011). Although many empirical studies since the 1980s have found a negative effect of inflation on growth (e.g., Kormendi and Meguire 1985), there are also studies that find a positive effect of inflation on growth (e.g., Bullard and Keating 1995; Ahmed and Rogers 2000).[1] Moreover, there are many studies that have already found a non-linear effect of inflation on growth.[2] Most studies have used a transformed variable of inflation and the transformation is also non-linear (e.g., Drukker et al., 2005; Khan and Senhadji, 2001; Kremer, Bick and Nautz, 2013), which attests to the non-linear effect of

[1] There are also critics of the findings. For instance, Barro (1995) finds that there is no relationship between pooled decade averages of growth and inflation in economies with annual inflation below 15%. Bruno and Easterly (1996) find that the results are sensitive and depend on outliers with episodes of high inflation.

[2] For instance, Khan and Senhadji (2001) have identified a threshold effect in the inflation-growth nexus. Kremer, Bick and Nautz (2013) have found the threshold—beyond which inflation would decrease growth—to be around 2% for advanced countries and 17% for developing countries. Bruno and Easterly (1999) found that 40% annual inflation seems to be the upper limit, beyond which inflation may significantly reduce growth.

inflation on growth. However, transformation makes interpretation more difficult. Although there are empirical studies testing the inverted-U relationship between inflation and growth (Bick 2010; López-Villavicencio and Mignon 2011), our study shows that the inverted-U relationship holds at low levels of inflation and we establish a causal effect.

Our study relates to the literature on the effect of R&D subsidies on growth and welfare (see Segerstrom, 1998, 2000, and references therein). According to Segerstrom (1998), R&D subsidies do not have long-run growth effects, but they have welfare effects. Segerstrom (2000) considers both horizontal and vertical R&D and finds that R&D subsidies decrease long-run growth. We model R&D subsidies as an increase in income for entrepreneurs and they are financed by seigniorage, while Segerstrom (1998, 2000) assumes that R&D expenditures are subsidized by lump-sum government taxation. However, the difference in findings is not due to how R&D is subsidized. Instead, it is because a higher R&D subsidy does not impact the long-run innovation rate in any industry in Segerstrom (1998), but it does in our model as in the seminal new growth models of Romer (1990) and Aghion and Howitt (1992) (discussed in Segerstrom, 1998).

The paper proceeds as follows. Section 2 presents the motivating evidence. Section 3 presents the model. Sections 4 and 5 provide the empirical evidence. Section 6 concludes.

2 Motivating Evidence

2.1 The Inverted-U Relationship between Growth and Inflation

As discussed (see Section 4 for details), we combine the PWT 9.0 with the WDI of the World Bank to build panel data for 154 countries during 1970–2014. Taking five-year averages of data yields a balanced panel with 1,386 observations.

The true data-generating process underlying the relationship between the outcome (growth) and the predictor (inflation) is unknown. The relationship is found to be monotone—negative or positive (see references discussed above), non-monotone (Chu et al. 2017), or monotone but non-linear (Arawatari et al. 2018). Therefore, it is useful for us to visualize the relationship between the outcome (growth) and the predictor (inflation).

Facing an unknown data-generating process, researchers have developed many flexible smoothing tools, aiming to represent the relationship between the outcome and the predictor as accurately and unbiasedly as possible. These non-parametric smoothers include kernel-based scatterplot smoothers and regression splines, among many others. We use cubic spline regression. With a small neighborhood size, the fitted function is more responsive to local disturbance and is thereby more smooth. However, it is also more prone to bias. We focus on presenting the partial regression plot between growth and inflation. That is, we first regress growth on all the other control variables (i.e., the initial output per employment, the logarithms of human capital, physical capital investment, international trade and government spending) and fixed country and effects and obtain the growth residuals. Then we regress inflation on the same set of variables to obtain the inflation residuals. We then plot the growth residuals against the inflation residuals. The data is centered at medium levels of inflation. Therefore, we choose more knots around the medium levels of inflation to ensure smoothness.

As discussed later, when the inflation rate is high enough, R&D labor will be zero. In other words, the positive growth rates in these high inflation countries may not be best explained by our R&D-based Schumpeterian model. Therefore, we drop the observations with average annual inflation above 30% (which leaves 992 observations in our sample). Figure 1a illustrates the results of cubic spline regression (with degrees of freedom being 3) for the new sample. The fitted line clearly shows an inverted-U relationship between growth and inflation in the sample with average annual inflation below 30%. To avoid the influence of the outlier, we drop the observations with average annual inflation below -5% (which drops two observations). Figure 1b illustrates the resultant cubic spline regression results. The fitted line still shows an inverted-U relationship between growth and inflation.

Figure 1. Partial Cubic Spline Regression (d.f.=3)

Therefore, using non-parametric regression splines, we find the existence of an inverted-U relationship between growth and inflation in our largest cross-country panel data (see also Bick 2010; López-Villavicencio and Mignon 2011; Kremer et al., 2013). Our non-parametric regression is just a correlation even if we use residuals from a growth regression against residuals from an inflation regression to avoid the potential bias from omitting the other important control variables listed above. We leave the establishment of a causal relationship to Section 4. In the following, we first propose a monetary Schumpeterian model to explain the inverted-U relationship between growth and inflation. Our model features important real world facts, as elaborated on in the following section.

2.2 Government Subsidies of R&D and Seigniorage

2.2.1 Government Subsidies of R&D

It is now commonly held that R&D is the engine of sustained long-run growth. Therefore, it is not surprising that in almost all countries, the government provides R&D subsidies to firms, aiming to promote business development. The provision of government subsidies to R&D has intensified in this era of globalization as each country attempts to gain an advantage over others in technology and thereby in terms of trade in exporting. Czarnitzki and Hussinger (2004) highlight:

> In recent years a growing gap in the levels of research investment between Europe and the U.S. or Japan has been observed. European governments fear the negative consequences for the long–run technological performance, growth and employment potential. For this reason, the 2002 EU member states agreed on the so–called Barcelona objectives. On this basis, the "Action Plan for Europe" has been proposed: the European R&D expenditure should be increased from currently 1.9% of GDP to 3.0% by 2010, where two thirds should be financed by the business sector, as its R&D spending is currently lagging behind the U.S. and Japan. In order to achieve this goal, national governments are requested to reinforce their national technology programs to support R&D in the business sector.

There is a large body of literature studying government subsidies of R&D (Almus and Czarnitzki 2002; Czarnitzki and Hussinger 2004; Boeing 2016; Wei, Xie and Zhang 2017). For instance, Almus and Czarnitzki (2002) note, "In 1998, the German federal government spent about 2.2 billion Euros on promoting R&D activities in the business sector." With a one-party non-democratic government, China is known for its heavy R&D subsidies to firms (reviewed by Boeing 2016), favoring large state-owned firms over private and foreign ones (see Wei, Xie and Zhang 2017).

The United States is no exception. We access https://www.nsf.gov/statistics/2018/nsf18302/ for data from "Business Research and Development and Innovation: 2014". The amount of "Domestic R&D paid for and performed by the company" is $282,570 million for all industries, while "Domestic R&D paid for by the U.S. federal government and performed by the company" is $26,554 million for all industries in 2014. The United States has many forms of government R&D subsidies, including tax credits and subsidies from local governments. For instance, we access the web page of the U.S. Department of Treasury to obtain data on federal spending. In the first quarter of 2018, the federal spending to the NSF is $539.1 million. The NSF used the funds to subsidize R&D. One can see that the amount of domestic R&D financed by the federal government [$26,554 million in 2014] is much larger than the budget of the NSF [$539.1 million in the first quarter of 2018], which means the federal government subsidies of R&D also come from other government agencies.

2.2.2 Government Subsidies of R&D by Seigniorage

In almost countries, government spending is funded by taxes, seigniorage or both. Seigniorage is important in developed countries and represents a much larger share of government spending in developing countries (see Obstfeld and Rogoff 1996, p. 527). Here we report some evidence of the importance of seigniorage in the United States. We access the https://www.federalreserve.gov/publications/2016-ar-federal-reserve-banks.htm#14894 to acquire the data on the amount of remittances to the Treasury by the Federal Reserve. The amount of remittances to the Treasury "Required by the Federal Reserve

Act, as amended by the FAST Act" totals \$91,467 million in 2016.[3]

The large amount of remittances partly comes from seigniorage. As we will show later, if the money growth rate is above zero, there will be positive seigniorage. According to the H6 release of the Federal Reserve on *Money Stock and Debt Measures*, the seasonally adjusted annual rates of growth for M1 and M2 during February 2017 to February 2018 are 6.7% and 4.0%, respectively. With regard to the "Use of Earnings Transferred To The Treasury," *Section 7(b) Division of Earnings of the Federal Reserve Act* notes, "The net earnings derived by the United States from Federal Reserve banks shall, in the discretion of the Secretary, ..., or shall be applied to the reduction of the outstanding bonded indebtedness of the United States under regulations to be prescribed by the Secretary of the Treasury."

Section 7(a) of the Federal Reserve Act states that the earnings transferred to the Treasury is deposited into the general fund of the Treasury. The general fund is used to fund the spending by all U.S. government agencies, including the NSF. Therefore, the seigniorage of the Federal Reserve is used to fund the government subsidies of R&D.

3 Monetary Schumpeterian Model

In order to explain the inverted-U relationship between growth and inflation, we introduce two pieces into the seminal contribution of Aghion and Howitt (1992). The first one is a CIA constraint on R&D. The second one is the government subsidy of R&D with the seigniorage revenue (see the stylized facts in Section 2.2). None of these two elements is new. But their combination creates a novel view of the effect of inflation on growth and welfare. It is worth discussing the following issues.

First, it makes no difference whether we use the monetary Schumpeterian quality-ladder model (Aghion and Howitt, 1992) based on Chu and Cozzi (2014), or the monetary variety-expanding model (i.e., Romer, 1990) based on He (2015). We follow the approach in Chu and Cozzi (2014) to compare the growth and welfare effects with existing monetary Schumpeterian quality-ladder models (see, e.g., Chu, Ning and Zhu, 2019; Chu et al., 2017, 2019; He, 2018, a,c). In so doing, we can appreciate further the magnitude/importance of our mechanism. Nevertheless, the results hold up if we follow our own approach in He (2015). We will briefly discuss this issue in Section 3.11.1.

Second, although government spending, including the government subsidies of R&D through the NSF, may be funded by taxes, seigniorage or both, we show that the inverted-U relationship cannot be generated when R&D subsidies are financed by other taxes in Section 3.11.2.

3.1 Households

At time t, the population size of each household is fixed at L. There is a unit continuum of identical households, which have a lifetime utility function

$$U = \int_0^\infty e^{-\rho t} \left[\ln \left(c_t \right) + \theta \ln \left(1 - l_t \right) \right] dt, \tag{1}$$

[3] The Federal Reserve states: "The FAST Act, which amended section 7(a) of the Federal Reserve Act, requires that any Reserve Bank capital surplus in excess of \$10 billion be transferred to Treasury."

where c_t is per capita real consumption of final goods and l_t is per capita supply of labor at time t. $\rho \in \left(0, \frac{1}{2}\right)$ is the rate of time preference and $\theta \geqslant 0$ captures leisure preference. For instance, Chu, Ning, and Zhu (2017) set the discount rate ρ to 0.04 (much smaller than 0.5). Each individual is endowed with one unit of labor. For the sake of simplicity, we assume that there is no population growth. Although we use elastic labor supply, our prediction on the hump-shaped response of growth to inflation does not depend on this (the magnitudes of growth and welfare effects depend on the elasticity of labor supply). In our quantitative analysis section, elastic labor supply gives a more realistic measure of growth and welfare effects.

Each household maximizes its lifetime utility subject to the following asset-accumulation equation

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t - c_t - \pi_t m_t + i_t b_t + (1 - \beta)\, \tau_t, \tag{2}$$

where a_t is the real value of equity shares in monopolistic intermediate-goods firms owned by each member of the household; r_t and w_t are the rate of real interest and the wage, respectively; m_t is the real money balance held by each person, and π_t is the inflation rate; and b_t is the real money balance borrowed by entrepreneurs to finance R&D, and its return is the nominal interest rate i_t.

The literature traditionally assumes that seigniorage revenue is rebated as a lump-sum transfer to the household, with per capita transfer as τ_t. As discussed in Section 2.2, the seigniorage revenue in the U.S. may be used to subsidize domestic R&D. Section 2.2 illustrates that the amount of remittances of the Federal Reserve to the Treasury ($96,902 million in 2014) is much larger than that of the domestic R&D paid for by the U.S. federal government. For this reason, we assume that part of (i.e., $\beta \in [0, 1]$ share of) the seigniorage revenue is used to subsidize business-promoting activities (i.e., $(1 - \beta)\, \tau_t$ is rebated to households). In so doing, we can get *a positive seigniorage effect* from a higher nominal interest rate that would promote growth (elaborated upon later).

As a side note, we will show in section 3.11.2 that using other taxes to subsidize R&D would not yield an inverted-U effect of inflation on growth. In reality, government revenue also comes from channels other than seigniorage. However, as long as seigniorage (even if part of seigniorage) is used in subsidizing R&D, there will be a hump-shaped response of growth to inflation. In other words, considering other taxes would not change the main prediction of our model. On the flip side, if we observe a hump-shaped response of growth to inflation (see our empirical sections), our model offers an original mechanism to rationalize the empirics, which also has profound policy implications. Therefore, for simplicity but without loss of generality, we abstract from considering other taxes.

The CIA constraint is given by $c_t + b_t \leq m_t$. As in Chu and Cozzi (2014), when the CIA constraint applies to R&D investment, a higher nominal interest rate will place an additional cost on the borrowing of entrepreneurs, which will generate a negative effect of an increasing nominal interest rate on growth. We refer to this effect as *a negative CIA-on-R&D effect*.

Using Hamiltonian (see the Appendix I for derivation), we can derive the no-arbitrage condition $i_t = \pi_t + r_t$ (the Fisher equation) and the optimality condition for consumption

$$\frac{1}{c_t} = \mu_t \left(1 + i_t\right), \tag{3}$$

where μ_t the Hamiltonian co-state variable on (2).

The optimal condition for labor supply is

$$\frac{\theta}{1-l_t} = w_t \mu_t, \tag{4}$$

where the left side of equation (4) is the marginal disutility of labor, and the right side of equation (4) is the marginal benefit of labor. Using (3), we rewrite the optimal condition for labor supply as

$$l_t = 1 - \frac{\theta c_t (1+i_t)}{w_t}. \tag{5}$$

According to (5), a higher nominal interest rate hurts growth by decreasing labor supply (the market size effect in Aghion, Akcigit and Fernández-Villaverde 2013). The Euler equation is

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho. \tag{6}$$

3.2 Labor Market

The fixed aggregate labor supply L has two uses. First, some labor is used in producing intermediate goods. Second, some labor is used as research input. The labor market clearing condition is

$$L_{x,t} + L_{r,t} = l_t L, \tag{7}$$

where $L_{x,t}$ and $L_{r,t}$ are the total employment in manufacturing and R&D, respectively. We define $l_{r,t} \equiv L_{r,t}/L$ as the share of employment in the R&D sector. Similarly, the share of labor in production is $l_{x,t} \equiv L_{x,t}/L$.

3.3 Final-Goods Sector

The final-goods sector is competitive. The production function of the final-goods firms is

$$y_t = \exp\left(\int_0^1 \ln x_t(j)\, dj\right), \tag{8}$$

where $x_t(j)$ denotes intermediate goods $j \in [0,1]$. The final goods firms maximize their profit, taking the price of each intermediate good j, denoted $p_t(j)$, as given. The demand function for $x_t(j)$ is

$$x_t(j) = y_t / p_t(j). \tag{9}$$

3.4 Intermediate-Goods Sector

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader. The leader in industry j has the following production function:

$$x_t(j) = \gamma^{q_t(j)} L_{x,t}(j). \tag{10}$$

The parameter $\gamma > 1$ is the step size of an improvement in productivity, and $q_t(j)$ is the number of productivity improvements that have occurred in industry j as of time t. $L_{x,t}(j)$ is the production labor in industry j. Equation (10) adopts a cost-reducing view of vertical innovation. Given $\gamma^{q_t(j)}$, the marginal cost of production for the industry leader in industry j is $mc_t(j) = w_t/\gamma^{q_t(j)}$. Standard Bertrand price competition leads to a profit-maximizing price $p_t(j)$ determined by a markup γ (the step size of innovation) over the marginal cost. The amount of monopolistic profit is

$$\Pi_t(j) = \left(\frac{\gamma - 1}{\gamma}\right) p_t(j)\, x_t(j) = \left(\frac{\gamma - 1}{\gamma}\right) y_t. \tag{11}$$

The labor income from production is

$$w_t L_{x,t}(j) = \left(\frac{1}{\gamma}\right) p_t(j)\, x_t(j) = \left(\frac{1}{\gamma}\right) y_t. \tag{12}$$

3.5 Seigniorage Revenue from Inflation

The government controls the nominal money supply, denoted M_t. It is equivalent to the case in which the nominal interest rate is chosen as the policy instrument because $i_t = \dot{M}_t/M_t + \rho$. The derivation is as follows. The per capita real money balance m_t is $m_t = M_t/(P_t L)$, where P_t is the price level of the final goods and $\dot{P}_t/P_t = \pi_t$. Therefore, $\dot{m}_t/m_t = \left(\dot{M}_t/M_t\right) - \pi_t$. On the balanced growth path, m_t and c_t grow at the same rate g_t (the balanced growth rate). Using equations (3) and (6), we have $g_t = r_t - \rho$. Therefore, $\dot{m}_t/m_t = \left(\dot{M}_t/M_t\right) - \pi_t = g_t = r_t - \rho$, which, combined with the Fisher equation, delivers $\dot{M}_t/M_t = i_t - \rho$.

The total seigniorage revenue $R_t = \tau_t L$ is $R_t = \dot{M}_t/P_t = \left(\dot{M}_t/M_t\right) m_t L$. Therefore, if the growth rate of the money supply is above zero (i.e., $\dot{M}_t/M_t > 0$, which is equivalent to $(i_t - \rho) > 0$), there will be positive seigniorage revenue; otherwise, the seigniorage revenue would be negative. Specifically,

$$R_t = \frac{\dot{M}_t}{P_t} = \left(\dot{m}_t + \pi_t m_t\right) L = \left(\frac{\dot{m}_t}{m_t} + \pi_t\right) \frac{m_t L}{y_t} y_t = (i_t - \rho)\phi_t y_t, \tag{13}$$

where we use the facts that on the balanced growth path m_t, c_t and y_t all grow at the same rate g_t and $g_t + \pi_t = i_t - \rho$; we define ϕ_t as the money-output ratio Lm_t/y_t, and ϕ_t is endogenous.

3.6 Research Arbitrage

The sole input of R&D is labor. Entrepreneurs have to borrow money from households to pay the wage bill of R&D workers (i.e., R&D is subject to the CIA constraint), raising the R&D cost by

$(1+i)$. Therefore, the zero-expected-profit condition of R&D firm $k \in [0,1]$ in each industry is

$$\lambda_t(k)\, v_t = (1+i)\, w_t L_{r,t}(k), \tag{14}$$

where $L_{r,t}(k)$ is the amount of labor hired by R&D firm k, and $\lambda_t(k)$ (the firm-level innovation rate per unit time) is $\lambda_t(k) = \frac{\varphi}{L} L_{r,t}(k)$, where φ is a constant. This assumption eliminates the scale effects (see discussions of scale effects in Laincz and Peretto, 2006; Segerstrom, 1998). The aggregate arrival rate of innovation λ_t is

$$\lambda_t = \int_0^1 \lambda_t(k)\, dk = \frac{\varphi}{L} L_{r,t} = \varphi l_{r,t}, \tag{15}$$

where $l_{r,t} = L_{r,t}/L$ is the share of employment in the R&D sector.

We denote by $v_t(j)$ the value of the monopolistic firm in industry j. In a symmetric equilibrium, $v_t(j) = v_t$ (Cozzi et al., 2007, provide a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in Schumpeterian growth models). The no-arbitrage condition for v_t is

$$r_t v_t = \widehat{\Pi}_t + \dot{v}_t - \lambda_t v_t. \tag{16}$$

Equation (16) says that the return of holding an innovation $r_t v_t$ equals the sum of the flow profit of innovation $\widehat{\Pi}_t$ and potential capital gain $(\dot{v}_t)$ less the expected capital loss $\lambda_t v_t$.

Because entrepreneurs get the seigniorage revenue, their profits will be the usual monopolistic profit from innovations (the Π_t in equation 11) plus the extra seigniorage revenue (the β share of the seigniorage revenue R_t given in equation 13). Therefore, we have

$$\widehat{\Pi}_t = \Pi_t(j) + \beta R_t = \left(\frac{\gamma - 1}{\gamma}\right) y_t + \beta(i_t - \rho)\phi_t y_t. \tag{17}$$

The last term in (17) is the government subsidies of R&D with the seigniorage revenue, which will yield the *positive seigniorage effect* (i.e., a higher nominal interest rate tends to promote R&D and thereby growth) discussed above. We model R&D subsidies as an increase in income for entrepreneurs (please see Segerstrom, 2000, for R&D subsidies through subsidizing R&D expenditures).

3.7 General Equilibrium

The general equilibrium is a time path of prices $\{p_t(j), r_t, w_t, i_t, v_t\}$ and allocations $\{c_t, l_t, b_t, m_t, y_t, x_t(j), L_{x,t}(j), L_{r,t}(k)\}$, which satisfy the following conditions at each instant of time:

- households maximize utility taking prices $\{r_t, w_t, i_t\}$ as given;
- competitive final-goods firms maximize profit taking $\{p_t(j)\}$ as given;
- monopolistic intermediate-goods firms choose $\{L_{x,t}(j), p_t(j)\}$ to maximize profit taking $\{w_t\}$ as given;
- R&D firms choose $\{L_{r,t}(k)\}$ to maximize expected profit taking $\{w_t, i_t, v_t\}$ as given;

- the labor market clears (that is, $L_{x,t} + L_{r,t} = Ll_t$);
- the final goods market clears (that is, $y_t = c_t L + \beta R_t$);
- the CIA constraint binds: $c_t + b_t = m_t$;
- the value of monopolistic firms adds up to the value of households' assets (i.e., $v_t = a_t L$);
- the amount of money borrowed by R&D is $w_t L_{r,t} = b_t L$.

3.8 Balanced Growth Path

Plugging equation (10) into (8), we have

$$y_t = \exp\left(\int_0^1 q_t(j)\, dj \ln \gamma\right) L_x = \exp\left(\int_0^t \lambda_v dv \ln \gamma\right) L_x = Z_t L_x, \tag{18}$$

where $Z_t \equiv \exp\left(\int_0^t \lambda_v dv \ln \gamma\right)$ is the level of aggregate technology. The growth rate of Z_t is

$$g_z = \lambda_t \ln \gamma = \varphi l_{r,t} \ln \gamma. \tag{19}$$

There is no transitional dynamics in our model, as proven in Proposition 1.

Proposition 1 *Given a fixed nominal interest rate i, $\beta \leqslant \frac{1}{2-\frac{1}{\gamma}}$ is sufficient (but not necessary) to ensure that the economy immediately jumps to a unique and saddle-point stable balanced growth path on which each variable grows at a constant rate.*

Proof. See the Appendix II.

According to the proof of Proposition 1, given a fixed nominal interest rate $i_t = i$, the equilibrium labor allocation is stationary on a balanced growth path. On the balanced growth path, equation (18) shows that the growth rate of total output is $g_y = g_z$. Per capita consumption is $c_t = [1 - \beta(i_t - \rho)\phi_t]\, y_t/L$, implying that c_t and Z_t must grow at the same rate: $g_c = g_z$. According to equation (19), the balanced growth rate is uniquely pinned down by the share of labor employed by R&D firms $l_{r,t}$, we solve for the equilibrium labor allocation. First, using the goods market clearing condition and the binding CIA constraint, we can solve for the money-output ratio ϕ_t as:

$$\phi_t = \frac{Lm_t}{y_t} = \frac{L(c_t + b_t)}{y_t} = [1 - \beta(i_t - \rho)\phi_t] + \frac{l_r}{\gamma l_x} \Rightarrow \phi = \frac{1}{1 + \beta(i - \rho)}\left(1 + \frac{l_r}{\gamma l_x}\right). \tag{20}$$

Using $\dot{v}_t / v_t = g$, $\lambda \widehat{\Pi}_t = (\rho + \lambda)(1 + i)\, w_t L_{r,t}$, (12), (15), and (17), we have

$$(\gamma - 1)\, l_x + \beta(i - \rho)\phi\gamma l_x = (l_r + \rho/\varphi)(1 + i). \tag{21}$$

The labor market clearing condition is

$$l_r + l_x = 1 - \theta\gamma l_x (1 + i)\, [1 - \beta(i - \rho)\phi]. \tag{22}$$

Equations (20)–(22) solve for $\{l_r, l_x, \phi\}$ as functions of the nominal interest rate i.

3.9 The Inverted-U Relationship between Growth and Inflation

As discussed, our predictions do not depend on elastic labor supply (see the results under our calibrated parameter values with elastic labor supply). Therefore, we illustrate our prediction with inelastic labor supply $\theta = 0$. Using (20) to substitute for ϕ in (21) and combining with a simplified (22): $l_r + l_x = 1$, we solve for $\{l_r, l_x\}$ as functions of the nominal interest rate i:

$$l_r = \frac{(\gamma-1)\left[1+2\beta(i-\rho)\right] - (1+i)\left[1+\beta(i-\rho)\right]\frac{\rho}{\varphi} + \beta(i-\rho)}{(\gamma-1)\left[1+2\beta(i-\rho)\right] + (1+i)\left[1+\beta(i-\rho)\right]}, \tag{23}$$

$$l_x = \frac{(1+i)\left[1+\beta(i-\rho)\right]\left(1+\frac{\rho}{\varphi}\right) - \beta(i-\rho)}{(\gamma-1)\left[1+2\beta(i-\rho)\right] + (1+i)\left[1+\beta(i-\rho)\right]}. \tag{24}$$

Proposition 2 *In the steady state, growth is an inverted-U function of the nominal interest rate when* $\beta \geqslant \frac{(\gamma-1)\left(1+\frac{\rho}{\varphi}\right)}{1+\frac{\rho}{2}+(\gamma-1)\left(2+2\rho+\frac{\rho+2\rho^2}{\varphi}\right)}$.

Proof. See the Appendix III.

It is worth discussing the logical consistency between Propositions 1 and 2. $\beta \leqslant \frac{\gamma}{2\gamma-1}$ and $\beta \geqslant \frac{\widehat{\gamma}\left(1+\frac{\rho}{\varphi}\right)}{1+\frac{\rho}{2}+\widehat{\gamma}\left(2+2\rho+\frac{\rho+2\rho^2}{\varphi}\right)} \equiv \widehat{\beta}$ (where $\widehat{\gamma} = (\gamma-1)$) are sufficient for Propositions 1 and 2, respectively, to hold. $\widehat{\gamma} = (\gamma-1)$ is very small. For instance, according to Chu, Ning and Zhu (2019), $\gamma = 1.05$, which yields $\widehat{\beta} < \widehat{\gamma} = 0.05$. Therefore, when $\widehat{\beta} \leqslant \beta \leqslant \frac{1}{2-\frac{1}{\gamma}}$, both Propositions 1 and 2 will hold. That is, the economy always immediately jumps to a unique and saddle-point stable balanced growth path, on which the balanced growth rate is an inverted-U function of the nominal interest rate.

The intuition behind Proposition 2 is as follows. First, the balanced growth rate given in equation (19) is linear in the share of labor employed by R&D firms l_r, as in standard Schumpeterian growth models (see e.g., Aghion and Howitt, 1998, ch. 2; Chu and Cozzi, 2014). Therefore, we only need to discuss the intuition why the share of labor employed by R&D firms l_r is an inverted-U function of the nominal interest rate, as elaborated on below.

Similar to the benchmark Schumpeterian model in Aghion and Howitt (1998, ch. 2), our model features a unique equilibrium that is pinned down by two conditions: the arbitrage condition between working in the manufacturing sector and working as a R&D researcher (i.e., the free labor mobility condition that equates the wage rate between the two sectors of manufacturing and R&D) and the labor market clearing condition. The labor market clearing condition under inelastic labor supply is simply $l_r + l_x = 1$. Therefore, either one of manufacturing employment or R&D employment increases, the other one must decrease. Therefore, how the share of labor employed by R&D firms l_r responds to the nominal interest rate is fully determined by the free labor mobility condition.

The free labor mobility condition is related to the return to and cost of R&D. The return to R&D can bee seen from equations (14), (16) and (17). The last term in equation (17) indicates that the government subsidies of R&D with the seigniorage revenue will yield a *positive seigniorage effect*, thereby drawing labor away from manufacturing into R&D. As a result, more R&D and thereby growth will be forthcoming. However, a higher nominal interest rate places an additional borrowing cost for entrepreneurs, as indicated by the $(1+i)$ term in (14), ending up decreasing R&D (i.e., the *negative CIA-on-R&D effect*).

Using the free labor mobility condition in equation (21), the right-hand-side (RHS) of equation (21) is the benefit of R&D, while the left-hand-side (LHS) of equation (21) is the cost of R&D. With free labor mobility, the return to R&D equals its cost. When the nominal interest rate increases, it increases the cost of R&D linearly, as indicated in the LHS of equation (21). By contrast, the nominal interest rate affects the benefit of R&D non-linearly and through two terms: the linear $(i-\rho)$ term and the non-linear ϕ (the money-output ratio) term. When the nominal interest rate increases from ρ (when the seigniorage effect is positive, which makes our explanation easier), the *positive seigniorage effect* is larger than the *negative CIA-on-R&D effect* when $\beta \geqslant \frac{(\gamma-1)\left(1+\frac{\rho}{\varphi}\right)}{1+\frac{\rho}{2}+(\gamma-1)\left(2+2\rho+\frac{\rho+2\rho^2}{\varphi}\right)} = \widehat{\beta}$ ($\widehat{\beta} < (\gamma-1)$). Why? Using equation (13), the *positive seigniorage effect* is proportional to the output (for simplicity, thinking of the money-output ratio being around 1, when $i=\rho$ in equation 20). Therefore, a one-percentage-point increase in the nominal interest rate would generate the seigniorage revenue of around 1% of output. The amount of the seigniorage revenue goes to entrepreneurs is around $1\% \times \beta$ of output. By contrast, a one-percentage-point increase in the nominal interest rate would increase the R&D cost by only 1% of the wage payment to R&D workers, which is 1% of $w_t L_r$. According to existing studies (e.g., Aghion, Akcigit and Fernández-Villaverde, 2013), the equilibrium R&D share of GDP is around 3% for the U.S. economy. Therefore, the same one-percentage-point increase in the nominal interest rate would increase the R&D cost by around $1\% \times 3\%$ of output. Obviously, when the nominal interest rate increases from ρ, the *positive seigniorage effect* significantly dominates the *negative CIA-on-R&D effect* when β is larger than 3% (which is around $\widehat{\beta}$).

The final free labor mobility condition in equation (21) equates the return to R&D with the cost of R&D, instead of the marginal return to R&D with the marginal cost of R&D. Therefore, when the nominal interest rate increases, the additional cost—due to the CIA constraint on R&D—on R&D will be $i \times w_t L_r$, not $\Delta\ i \times w_t L_r$. Therefore, as the nominal interest rate approaches infinity, the increase in R&D cost also approaches infinity. By contrast, the seigniorage approaches 100% (that is, the term $\beta(i-\rho)\phi$ in equation 21 approaches 1) of output (the upper limit, which is finite) as the nominal interest rate approaches infinity. Obviously, the increase in R&D cost dominates that in R&D subsidies with the seigniorage revenue. Therefore, the *negative CIA-on-R&D effect* dominates the *positive seigniorage effect* when the nominal interest rate is high enough.

Taken together, when β is above a threshold, when the nominal interest rate increases from zero, initially, the *positive seigniorage effect* dominates, thereby increasing R&D and growth; however, when the nominal interest rate is beyond the threshold, the *negative CIA-on-R&D effect* dominates, thereby lowering equilibrium R&D and growth. When β approaches zero, the *positive seigniorage effect* approaches zero and is always dominated by the *negative CIA-on-R&D effect*. In this case, growth is a decreasing function of the nominal interest rate. Our mechanism differs from and complements the non-linear (Arawatari et al., 2018; Zheng et al., 2018) or inverted-U (Chu et al., 2017; 2019) effect of inflation on growth.

3.9.1 Welfare and Inflation

Our focus is on the relationship between inflation and growth, but it is worth discussing optimal monetary policy (i.e., the relationship between welfare and inflation). In the following, we briefly discuss the results (we omit the proof to save space). Nevertheless, the predictions are confirmed by our calibrations results presented in Section 3.10.

We find that welfare is also an inverted-U function of inflation when β is above a threshold (and this threshold may be different from $\widehat{\beta}$) and the elasticity of labor supply is not too high. That is, the Friedman rule (i.e., the optimal nominal interest rate should be zero, see Friedman, 1969) would be suboptimal. Additionally, the nominal interest rate that maximizes growth is higher than that maximizes welfare. The intuition is as follows. We use inelastic labor supply as an example. Welfare consists of two parts: the initial level of per capita consumption that increases with manufacturing labor; long-run growth that increases with R&D labor (see Section 3.10 for details). When R&D labor increases, manufacturing labor decreases. Therefore, concerning welfare, when the nominal interest rate increases, there is an additional decreasing current consumption effect. As a result, the nominal interest rate that maximizes welfare is lower than that maximizes growth. Nevertheless, the *positive seigniorage effect* on welfare dominates (is dominated by) the *negative CIA-on-R&D effect* and the lower initial consumption effect at low (high) levels of inflation.

Under elastic labor supply, an increase in the nominal interest rate decreases labor supply through the consumption-leisure choice. As a result, total labor supply decreases and leisure increases, thereby increasing welfare. However, when the elasticity of labor supply is very high, the *seigniorage effect* decreases a lot. As a result, the gains in growth and leisure will be small when the nominal interest rate increases. Actually, the gains in growth and leisure are dominated by the losses in current consumption (see the welfare decomposition in Section 3.10). Therefore, Friedman rule is optimal.

It is worth mentioning the following. As discussed in Walsh (2010) and Chu et al. (2017), the Fisher equation gives rise to a positive long-run relationship between the inflation rate and the nominal interest rate, which is supported by empirical studies (e.g., Mishkin, 1992; Booth and Ciner, 2001). In our model, as long as $g'(i) < 1$, we have $\pi'(i) = (1 - g'(i)) > 0$. We find that $g'(i) < 1$ under our calibrated parameter values. Therefore, economic growth would also be an inverted-U function of inflation, which is tested in Section 4. Before that, we calibrate the model and simulate the quantitative effects of inflation on growth and social welfare to further increase the empirical appeals of the paper (i.e., to appreciate the importance of our mechanism).

3.10 Quantitative Analysis

Our model has the following set of structural parameters $\{\rho, \gamma, \beta, \varphi, \theta\}$. We follow Chu, Ning, and Zhu (2019) to set the discount rate ρ to a conventional value of 0.04 and the step size of innovation γ to 1.05. We need three conditions to pin down the values of $\{\beta, \varphi, \theta\}$. The first condition is the long-run GDP per capita growth of 2% in advanced countries (see Chu et al., 2017; Chu and Cozzi, 2014). The second condition is the standard moment of $l = 0.3$, following Chu and Cozzi (2014). The third condition is the share of the seigniorage revenue allocated to entrepreneurs, for which we use the numbers for the United States in Section 2: "The amount of *Domestic R&D paid for and* *performed by the company* is \$282,570 million for all industries, while *Domestic R&D paid for by the U.S. federal government and performed by the company* is \$26,554 million for all industries in 2014." Therefore, using (17), we have $\frac{\gamma-1}{\gamma}y_t/\left[\beta(i_t-\rho)\phi_t y_t\right]$ =\$282,570/\$26,554. Here we have assumed that the cost of R&D is shared according to the profit of R&D. Please note that these numbers may give the lower bound (a conservative estimation) of β (around 8% in our calibration). For instance, the amount of remittances to the Treasury totaled \$96,902 million in 2014. If the domestic R&D paid for by the U.S. federal government comes in its entirety from these remittances, then we should have the upper bound of β as $\beta = \$26,554/\$96,902 \approx 0.27$.

The upper bound of β is lower than $\frac{1}{2}$. Therefore, according to Proposition 1, the condition $\beta < \frac{1}{2}$ is sufficient for the economy to immediately jump to a unique and saddle-point stable balanced growth path. Because our model does not feature transitional dynamics, our following quantitative welfare analysis is accurate.

We have $i = 9.6\%$, the calculated sample value for advanced countries including the United States, following He (2018a): $i = \pi + r = \pi + \rho + g + n$ (where the sample mean of inflation rate is 2.71%, population growth rate is 0.89%). We first use the conservative estimate of β, and then we discuss how the change in β will affect our results. Now we pin down the values of $\{\beta, \varphi, \theta\}$ by solving the following equations:

$$g = (\varphi \ln \gamma)\, l_r = 0.02, \tag{25}$$

$$l = 0.3, \tag{26}$$

$$\frac{\gamma - 1}{\gamma}/\left[\beta(i_t - \rho)\phi_t\right] = \$282,570/\$26,554. \tag{27}$$

Solving equations (25)-(27) yields the values of $\{\beta, \varphi, \theta\}$ to be $\{0.0769, 31.42, 2.13\}$. To summarize, we pin down the parameter values $\{\rho, \gamma, \beta, \varphi, \theta\}$ as $\{0.04, 1.05, 0.0769, 31.42, 2.13\}$.

Figure 2 simulates the relationship between the nominal interest rate and economic growth, which shows an inverted-U relation between the nominal interest rate and economic growth as long as β is not too low. The case of $\beta = 0$ corresponds to that in Chu and Cozzi (2014), which shows that growth is a decreasing function of the nominal interest rate. This is expected because both the *CIA-on-R&D* and *market-size effects* are negative, while the *positive seigniorage effect* is absent.

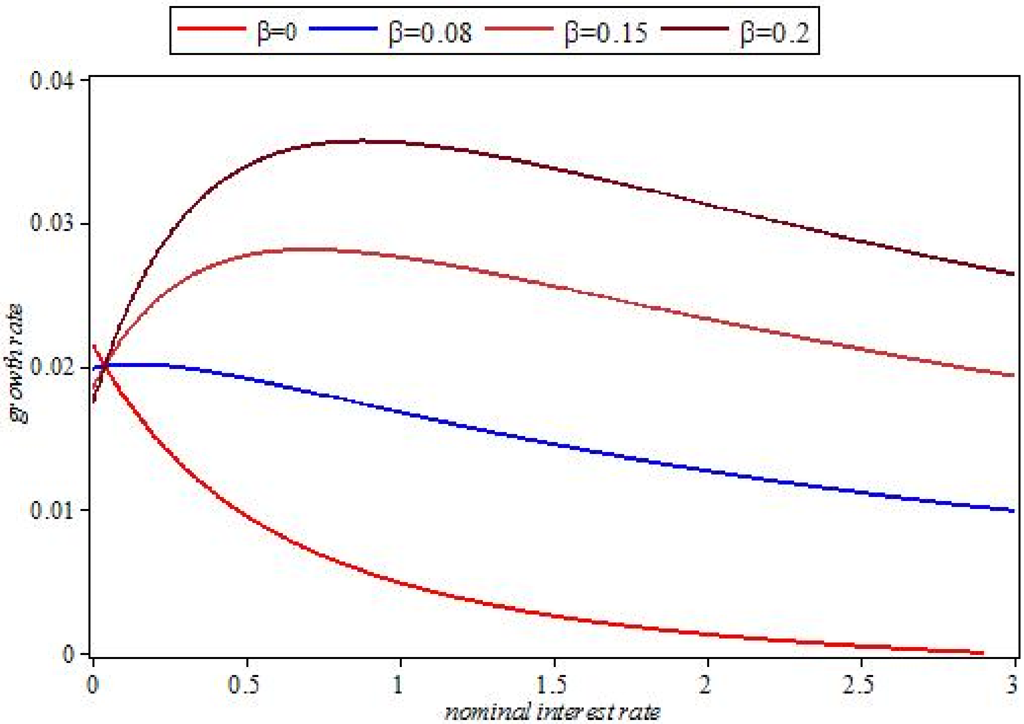

Figure 2. Growth and the Nominal Interest Rate

To calculate welfare, we impose balanced growth on (1) to have

$$U=\frac{1}{\rho}\left[\ln\left(c_0\right)+\frac{g}{\rho}+\theta\ln\left(1-l\right)\right], \tag{28}$$

where $c_0=[1-\beta(i-\rho)\phi]\,Z_0 l_x=\left[1-\frac{\beta(i-\rho)}{1+\beta(i-\rho)}\left(1+\frac{l_r}{\gamma l_x}\right)\right]l_x$, where we follow Chu and Cozzi (2014) to normalize Z_0 to unity; $g=\varphi l_r\ln\gamma$.

The welfare gain can be decomposed into three parts. First, an increase in the nominal interest rate would decrease labor supply through the consumption-leisure choice (the *market-size effect*). The decrease in labor supply means an increase in leisure, which increases welfare. The first effect is captured by the third term $\theta\ln(1-l)$ in (28). Second, an increase in the nominal interest rate changes the labor allocation between R&D and manufacturing. In our model with the *positive seigniorage effect*, R&D labor will increase and manufacturing labor will decrease. When R&D labor increases, more innovation will be forthcoming, thereby increasing the balanced growth rate. As a result, the welfare tends to increase. This second growth effect is captured by the middle term $\frac{g}{\rho}$ in (28). The first two effects are positive (negative) when the nominal interest rate increases (decreases). The third effect is reflected by the first term $\ln(c_0)$ in (28). According to the output market clearing condition, per capita consumption is a linear function of manufacturing labor. An increase in the nominal interest rate decreases total labor supply and increases R&D labor, thereby significantly decreasing manufacturing labor. Additionally, when β increases, more seigniorage and thereby more output will be used to finance entrepreneurs, which further decreases consumption. The welfare effects depend on the relative size of the three effects.

Columns 1.1 to 1.3 of Table 1 present the calibration results for our benchmark calibrated $\beta = 0.0769$. According to columns l.1 and 1.2, to maximize growth, i must increase from the benchmark value of 9.6% to 11.9%, the equilibrium rate of economic growth increases from the benchmark value of 2.000% to a maximum value of approximately 2.001% (the increase is negligible), and the welfare gain ΔU is equivalent to a permanent decrease in consumption of 0.35%. We have decomposed the welfare gain into three parts. One can see that c_0 decreases from 0.2857 to 0.2813. This negative effect on welfare dominates the two positive effects on welfare due to the increases in growth (0.001%) and leisure (from 0.7 to 0.7038). As a result, total welfare decreases.

As discussed, the nominal interest rate that maximizes growth does not necessarily maximize welfare. According to columns l.1 and 1.3, to maximize welfare, i must decrease from the benchmark value 9.6% to 0 (respecting the zero lower bound on the nominal interest rate), which means $\beta = 0.0769$ is low enough that the Friedman rule is optimal. When i decreases from 9.6% to 0, the growth rate decreases from 2.0% to 1.98%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 0.85%. Welfare decomposition shows that c_0 increases from 0.2857 to 0.3054. This positive effect on welfare dominates the two negative effects on welfare due to the decreases in growth (-0.02%) and leisure (from 0.7 to 0.6817). As a result, total welfare increases.

As discussed, our calibrated β may under-estimate the true value of β. If we assume the federal government subsidies of R&D are fully financed by the remittances to the Treasury from the Fed, then we have the upper bound of β as $\beta = \$26,554/\$96,902 \approx 0.27$. Therefore, we also report the calibration results for larger values of β. The results are presented in columns 1.4 to 1.9 of Table 1.

Columns 1.4 to 1.6 of Table 1 present the calibration results for $\beta = 0.15$. According to columns 1.4 and 1.5, to maximize growth, i must increase from the benchmark value 9.6% to 73.1%, the growth rate increases from 2.18% to a maximum value of 2.84%, and the welfare gain ΔU is equivalent to a permanent decrease in consumption of 1.76%. Similar welfare decomposition shows the effect of lower consumption on welfare dominates the two positive effects on welfare due to the increases in growth (0.66%) and leisure (from 0.6983 to 0.7622). Therefore, total welfare decreases.

According to columns l.4 and 1.6, to maximize welfare, i must increase from 9.6% to 33.6%, which means the Friedman rule is suboptimal and the benchmark value of 9.6% for the nominal interest rate is less than optimal. When i increase from 9.6% to 33.6%, the equilibrium rate of economic growth increases from 2.18% to 2.66%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 5.07%. Similar welfare decomposition shows that c_0 decreases from 0.2850 to 0.2436. This negative effect on welfare is dominated by the two positive effects on welfare due to the increases in growth (0.48%) and leisure (from 0.6983 to 0.7376). As a result, total welfare increases.

Table 1: Calibration Results for different values of β

	Column number								
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
	$\{\rho, \gamma, \varphi, \theta\} = \{0.04, 1.05, 31.42, 2.13\}$								
	$\beta = 0.0769$			$\beta = 0.15$			$\beta = 0.2$		
		max g	max U		max g	max U		max g	max U
i	9.6%	11.9%	0	9.6%	73.1%	33.6%	9.6%	90.5%	46.6%
c_0	0.2857	0.2813	0.3054	0.2850	0.1969	0.2436	0.2845	0.1790	0.2238
l_r	0.0130	0.0131	0.0129	0.0142	0.0186	0.0173	0.0151	0.0236	0.0220
l_x	0.2870	0.2831	0.3044	0.2875	0.2192	0.2551	0.2878	0.2139	0.2446
l	0.3	0.2962	0.3173	0.3017	0.2378	0.2724	0.3029	0.2375	0.2666
$1-l$	0.7	0.7038	0.6817	0.6983	0.7622	0.7376	0.6971	0.7625	0.7334
g	2.000%	2.001%	1.98%	2.18%	2.84%	2.66%	2.31%	3.61%	3.37%
ΔU	n/a	−0.35%	0.85%	n/a	−1.76%	5.07%	n/a	5.46%	14.32%

Note: i is the nominal interest rate; c_0 is initial per capita consumption; l_r and l_x are the R&D labor and manufacturing labor, respectively. l is total labor supply, and $(1-l)$ is leisure. g is the per capita growth rate. ΔU is the welfare gain (equivalent to a permanent increase in consumption). $\max g$ and $\max U$ refer to nominal interest rates that maximize growth and welfare, respectively.

Columns 1.7 to 1.9 of Table 1 present the calibration results for $\beta = 0.2$. According to columns 1.7 and 1.8, to maximize growth, i must increase from the benchmark value 9.6% to 90.5%, the growth rate increases from 2.31% to a maximum value of 3.61%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 5.46%. Similar welfare decomposition shows the effect of lower consumption on welfare is dominated by the two positive effects on welfare due to the increases in growth (1.30%) and leisure (from 0.6971 to 0.7625). As a result, total welfare increases.

According to columns 1.7 and 1.9, to maximize welfare, i must increase from 9.6% to 46.6% (i.e., both the Friedman rule and the benchmark value of 9.6% are suboptimal). When i increase from 9.6% to 46.6%, the growth rate increases from 2.31% to 3.37%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 14.32%. Similar welfare decomposition shows that c_0 decreases from 0.2845 to 0.2238. This negative effect on welfare is dominated by the two positive effects on welfare due to the increases in growth (1.06%) and leisure (from 0.6971 to 0.7334).

The substantial growth and welfare effects when β is higher can be explained as follows. According to our explanation on the mechanism of the inverted-U result following Proposition 2, when the nominal interest rate increases from ρ, the *positive seigniorage effect* significantly dominates the *negative CIA-on-R&D effect* when β is higher. Therefore, the R&D labor and thereby the balanced growth rate increases significantly when the nominal interest rate increases. This explains the large growth and welfare gains when β is higher.

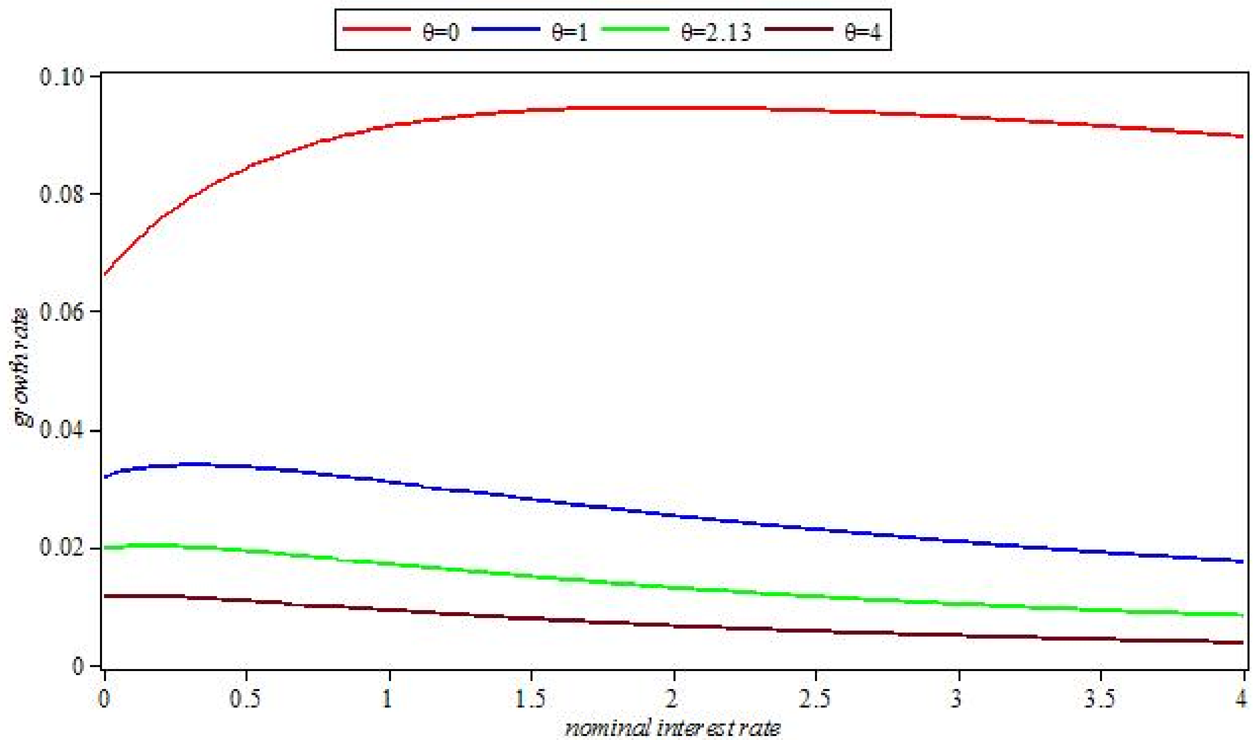

Figure 3. The effect of elastic labor supply.

Figure 3 and Table 2 illustrate that the growth and welfare effects also significantly depend on the value of θ. We use $\beta = 0.08$ as an example. The case of $\theta = 0$ corresponds to inelastic labor supply, where the above-mentioned first effect is not present. In this case, the growth and welfare effects are much larger. According to columns 2.1 and 2.2 of Table 2, to maximize growth, i must increase from 9.6% to 196%, the equilibrium rate of economic growth increases from 7.13% to a maximum value of 9.45%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 51.87%. According to columns 2.1 and 2.3, to maximize welfare, i must increase from 9.6% to 144.8%, the equilibrium rate of economic growth increases from 7.13% to a maximum value of 9.38%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 55.12%. The results when $\theta = 1$ (see columns 2.4 to 2.6) are similar to those in columns 1.1 to 1.3 of Table 1.

Columns 2.7 to 2.9 of Table 2 present the calibration results for $\theta = 4$. According to columns 2.7 and 2.8, to maximize growth, i must decrease from the benchmark value of 9.6% to 8.7%, the increase in the growth rate is negligible, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 0.15%. According to columns l.7 and 1.9, to maximize welfare, i must decrease from 9.6% to 0, which means the Friedman rule is optimal. When i decreases from 9.6% to 0, the decrease in the growth rate is around 0.004%, and the welfare gain ΔU is equivalent to a permanent increase in consumption of 1.25%. Similar welfare decomposition shows that c_0 increases from 0.1770 to 0.1910. This positive effect on welfare dominates the two negative effects on welfare due to the decreases in growth (-0.004%) and leisure (from 0.8146 to 0.8021). As a result, total welfare increases.

Table 2: Calibration Results for different values of θ

	Column number								
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
	$\{\rho, \gamma, \varphi, \beta\} = \{0.04, 1.05, 31.42, 0.08\}$								
		$\theta = 0$			$\theta = 1$			$\theta = 4$	
		max g	max U		max g	max U		max g	max U
i	9.6%	196%	144.8%	9.6%	31.8%	13.0%	9.6%	8.7%	0
c_0	0.9491	0.8056	0.8379	0.4539	0.4063	0.4459	0.1770	0.1782	0.1910
l_r	0.0465	0.0617	0.0612	0.0216	0.0221	0.0217	0.0076	0.0076	0.0076
l_x	0.9535	0.9383	0.9388	0.4561	0.4158	0.4492	0.1778	0.1789	0.1903
l	1	1	1	0.4776	0.4379	0.4709	0.1854	0.1865	0.1979
$1-l$	0	0	0	0.5224	0.5621	0.5391	0.8146	0.8135	0.8021
g	7.13%	9.45%	9.38%	3.31%	3.38%	3.33%	1.16926%	1.16933%	1.1651%
ΔU	n/a	51.87%	55.12%	n/a	-1.82%	0.08%	n/a	0.15%	1.25%

Note: i is the nominal interest rate; c_0 is initial per capita consumption; l_r and l_x are the R&D labor and manufacturing labor, respectively. l is total labor supply, and $(1-l)$ is leisure. g is the per capita growth rate. ΔU is the welfare gain (equivalent to a permanent increase in consumption). $\max g$ and $\max U$ refer to nominal interest rates that maximize growth and welfare, respectively.

3.11 Robustness Checks

3.11.1 Monetary Variety-Expanding Model

As discussed, it is intriguing to investigate whether such an inverted-U relation between inflation and growth exists in the monetary variety-expanding model based on He (2015). To save space, we omit the detailed steps. Our difference from He (2015) is that the CIA constraint now applies to R&D instead of manufacturing. Therefore, the *negative CIA-on-R&D effect* still exists. When entrepreneurs are subsidized with the seigniorage revenue, the *positive seigniorage effect* still exists.

Following He (2015), the household's problem is the same as in Section 3.1. We consider inelastic labor supply. As in He (2015), the final-goods sector is competitive. The production function of a final-good firm i is

$$y_i = \sum_{j=1}^{N} X_{ij}^{\alpha} L_i^{1-\alpha}, \tag{29}$$

where N is the number of innovations, X_{ij} is the amount of intermediate-good j used by a final-good firm i, and L_i is the labor input of a final-good firm i. $\alpha \in (0,1)$. The total labor supply is fixed at L. Working through the model, the balanced growth rate is

$$g = \frac{\widehat{\Pi}_t}{\eta\left(1+\zeta i\right)} - \rho = \frac{\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}} L + \beta\left(i-\rho\right)\phi_t \frac{y_t}{N_t}}{\eta\left(1+\zeta i\right)} - \rho, \tag{30}$$

where $\widehat{\Pi}_t$ is the profit of entrepreneurs, which consists of the profit from production $\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}L$ and the subsidies from the government $\beta\left(i-\rho\right)\phi_t\frac{y_t}{N_t}$; η is the fixed cost of each innovation (variety), and ζ captures the strength of the CIA constraint on R&D investment.

According to (30), the *negative CIA-on-R&D effect* tends to decrease growth, while the *positive seigniorage effect*, $\beta\left(i-\rho\right)\phi_t\frac{y_t}{N_t}$, tends to increase growth. The mechanism is the same as that for Proposition 2. At low levels of interest rate, the *positive seigniorage effect* is larger than the *negative CIA-on-R&D effect* when β is large. The *positive seigniorage effect* is proportional to the output. Therefore, a one-percentage-point increase in the nominal interest rate would generate the seigniorage revenue of around 1% of output. The amount of the seigniorage revenue goes to entrepreneurs is around 1% $\times$ β of output. By contrast, a one-percentage-point increase in the nominal interest rate would increase the R&D cost by only $\zeta\times$1% of η (the fixed cost of each innovation, variety). Obviously, if ζ is lower enough (it is only a sufficient condition for us to illustrate the mechanism clearly), the *positive seigniorage effect* dominates the *negative CIA-on-R&D effect*. Similarly, as the nominal interest rate approaches infinity, the increase in R&D cost also approaches infinity. By contrast, the seigniorage approaches 100% (that is, the term $\beta(i-\rho)\phi$ in equation 30 approaches 1) of output as the nominal interest rate approaches infinity. Therefore, the *negative CIA-on-R&D effect* dominates the *positive seigniorage effect* when the nominal interest rate is high enough.

In summary, our inverted-U results hold up in monetary variety-expanding models.

3.11.2 Considering Other Taxes

It is useful to show whether our inverted-U result will hold if other tax revenues instead of the seigniorage revenue are used as R&D subsidies. Answering this question would make our explanation for the hump-shaped growth-inflation relationship more compelling.

To accept our results, one needs to buy into our logic that seigniorage revenue (or inflation tax) subsidizes R&D. Our chain of reasoning is: (a) the Federal Reserve remits most of its net income to the Treasury where it goes into general funds; (b) General funds are, then, used to fund the NSF among other government agencies; and (c) the NSF then subsidizes R&D. We claim that on average 8% of seigniorage revenue goes to subsidizing R&D (see Section 3.10). For 2016, the Federal Reserve provided 2.8% of Federal Government tax revenue (the \$91.47 billion reported above divided by \$3,270 billion in total tax revenue). In other words, our 8% figure appears to be roughly three times too large. Further, just because x% of government tax revenue comes from seigniorage does not necessarily mean that x% of an increase in seigniorage revenue will go to increasing subsidies to R&D (through the NSF)—marginal effects need not correspond to average effects. Finally, is seigniorage revenue the best way to raise revenue for R&D subsidies?

Our reply to this concern is as follows. First, although the seigniorage goes to the government tax revenue pool, we cannot say all tax revenues will finance the R&D subsidies equally. The government subsidies of R&D with the seigniorage revenue can also be rationalized as follows. Considering the central bank independence (CBI) literature (e.g., Bade and Parkin 1988; Grilli, Masciandaro and Tabellini 1991; Alesina and Summers 1993), one can deem the seigniorage revenue as required by the central bank laws to subsidize growth-enhancing private activity. Official policy in most OECD countries holds that channeling credit supply to those activities generating growth is best done by a private banking system (aiming for the high return generated by growth), not by central banks. Nevertheless, it may be the case empirically that the fraction of total credit supply (including credit supply generated by money growth) used to finance growth-enhancing private activity is larger in countries with a relatively independent central bank, and that correspondingly the fraction used to finance non-productive government consumption is smaller in these countries.

Second, we can subsidize entrepreneurs with other taxes instead of seigniorage, but we will not get any inverted-U relation between inflation and growth. It is more interesting to consider other distortionary taxes (the flat-rate distortionary taxes in Aghion, Akcigit and Fernández-Villaverde 2013), but the results hold up for non-distortionary lump-sum taxes. We use the consumption tax as an example. Unless consumption tax is endogenous and responds to inflation in a similar way as seigniorage (which we think is impossible or not relevant), there is no changing positive effect and growth is always a decreasing function of inflation. We use the model with inelastic labor supply (i.e., $\theta = 0$ as in Section 3.9) for a simple illustration. The budget constraint becomes

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t - (1 - t_c)\, c_t - \pi_t m_t + i_t b_t + \tau_t, \tag{31}$$

where t_c is the distortionary consumption tax rate.

Now, we have the new final goods market clearing condition: $y_t = c_t L + G_t$, where $G_t = t_c c_t L$ is the tax revenue collected from consumption tax. We have $G_t = \frac{t_c}{1+t_c} y_t$. The government uses the consumption tax revenue to subsidize entrepreneurs. The profit of entrepreneurship $\widehat{\Pi}_t$ becomes

$$\widehat{\Pi}_t = \Pi_t (j) + G_t = \left(\frac{\gamma - 1}{\gamma} \right) y_t + \frac{t_c}{1 + t_c} y_t. \tag{32}$$

Working through the same steps, we have

$$(\gamma - 1)\, l_x + \frac{t_c}{1 + t_c} \gamma l_x = (l_r + \rho / \varphi)\, (1 + i)\, . \tag{33}$$

Equation (33) together with inelastic labor supply $l_r + l_x = 1$ would solve for $\{l_r, l_x\}$ as

$$l_r = \frac{\left[(\gamma - 1) + \frac{\gamma t_c}{1+t_c} \right] \left(1 + \frac{\rho}{\varphi} \right)}{(\gamma - 1) + \frac{\gamma t_c}{1+t_c} + 1 + i} - \frac{\rho}{\varphi}, \tag{34}$$

$$l_x = \frac{(1 + i) \left(1 + \frac{\rho}{\varphi} \right)}{(\gamma - 1) + \frac{\gamma t_c}{1+t_c} + 1 + i}. \tag{35}$$

Comparing the R&D labor l_r in (34) to that in (23), it is obvious that here the R&D labor l_r and thereby the balanced growth rate is a monotonically decreasing function of the inflation rate.

The difference in results can be seen clearly by comparing the R&D subsidies in (32) and (17). When the R&D subsides come from seigniorage, it is $\beta(i_t - \rho)\phi_t y_t$, which naturally changes with the monetary policy (i.e., i_t). By contrast, when the R&D subsides come from consumption tax, it is $\frac{t_c}{1+t_c} y_t$, which will not change with the monetary policy (i.e., i_t) or the inflation rate (it is clearer in equation (33)). Therefore, the other taxes cannot generate the *changing* seigniorage effect, ending up being unable to produce the inverted-U result. The R&D subsides with other tax revenues can increase R&D, but it is just a one-time change. Additionally, we have $\partial l_r/\partial t_c > 0$, which means long-run growth is a monotonically increasing function of the consumption tax rate (but welfare is not). Finally, whether seigniorage revenue is the best way to raise revenue for R&D subsidies is beyond the scope of this paper, we leave this issue to future studies.

3.11.3 An AK Model

Here we investigate whether such an inverted-U relation between inflation and growth exists in an AK model (we cannot exhaust all the other endogenous growth models). Doing so helps us to further understand the mechanism underlying our model. We show the details of the following case: the government uses seigniorage to subsidize household accumulation of physical capital.[4]

There is a unit continuum of identical households, which have a lifetime utility function

$$U = \int_0^\infty e^{-\rho t} \ln (c_t)\, dt, \tag{36}$$

where the variables and parameters are defined as before. Each individual is endowed with one unit of labor. Because it is an AK model, the wage rate is zero. Each household maximizes its lifetime utility given in equation (36) subject to

$$(1 - s_t)\, \dot{k}_t + \dot{m}_t = Ak_t - c_t - \pi_t m_t, \tag{37}$$

where s_t is the rate of subsidies of capital investment. The CIA constraint is $c_t + \dot{k}_t \leq m_t$. The per capita seigniorage revenue τ_t is not lump-sum transferred to households, but used to subsidize household accumulation of physical capital. That is, we have $s_t \dot{k}_t = \tau_t$.

The goods market clearing condition is $c_t + \dot{k}_t = Ak_t$. The binding CIA constraint means $c_t + \dot{k}_t = m_t$. We define $\dot{M}_t/M_t = \psi$. Therefore, $\tau_t = \psi m_t = \psi A k_t$. As a result, we have $s_t \dot{k}_t = \psi A k_t$, which gives $s_t = \psi A / \left(\dot{k}_t / k_t\right)$. Therefore, s_t is a constant on the balanced growth path.

Using Hamiltonian [see the Appendix IV for derivation], the new balanced growth rate g_2 is

$$g_2 = \frac{A}{1 + \rho + \psi - s} - \rho. \tag{38}$$

[4] Additionally, we also studied two cases. The first assumes that the seigniorage revenue is transferred back to households. In this case, the growth rate is a decreasing function of the monetary growth rate (see Dotsey and Sarte, 2000, for similar results in discrete time). The result is driven by the *negative CIA-on-investment effect* (see Stockman, 1981, for similar results in capital accumulation models). The second case introduces the assumption of government subsidies of production with the seigniorage revenue. In this case, the balanced growth rate is a decreasing (non-linear but monotone) function of the monetary growth rate, which is because the *positive seigniorage effect* is dominated by the *negative CIA-on-investment effect*. The proofs of the first two cases are available upon request.

Now plugging $s = \psi A / \left(\dot{k}_t / k_t \right) = \psi A / g_2$ into the above equation solves g_2:

$$g_2 = \frac{A}{(1 + \rho + \psi) - \frac{\psi A}{g_2}} - \rho. \tag{39}$$

There are two solutions to the quadratic function of (39): one is positive and the other is negative. Now the balanced growth rate becomes a monotone—either positive or negative—function of the monetary growth rate [see the Appendix IV for proof], which is because the *positive seigniorage effect* either dominates or is dominated by the *negative CIA-on-investment effect*.

In summary, the same approach can only predicts a nonlinear but monotone—negative or positive—effect of money growth on long-run growth in the AK model. In Schumpeterian growth models, R&D activities and, therefore, the levels of productivity are endogenous. In contrast, the level of technology is fixed in the AK model, which does not respond to changes in the nominal interest rate. Therefore, the growth dynamics in the AK model will be not as rich.

4 Empirical Evidence

Although our non-parametric regression shows an inverted-U relationship, it is just a correlation. In other words, the effect of inflation on growth may not be causal. When the effect is not causal, the inverted-U effect of inflation on growth may be spurious. Therefore, we have to establish a causal relationship between inflation and growth, which is the motivation of this section.

4.1 Empirical Specification

The vast body of empirical literature on growth regressions has established a more or less standard empirical formulation (e.g., Barro 1991; Mankiw, Romer and Weil 1992). Essentially, the rate of economic growth depends on the initial level of income per capita/worker, which captures the conditional convergence effect and the variable(s) of interest of the specific question that researchers would like to investigate. Such variables (sometimes the interaction among the variables) include physical and human capital investments and the growth rate of the labor force or population (Barro 1991; Mankiw, Romer and Weil 1992), democracy (e.g., Persson and Tabinilli 2009; Acemoglu et al. 2019 and references therein), exchange rate flexibility (e.g., Aghion et al. 2009), and debt-to-GDP ratio (e.g., Checherita-Westphal and Rother 2012), among many others. Considering the literature, our empirical specification is as follows:

$$\begin{aligned} growth_{it} = {} & \beta_0 + \beta_1 \pi_{i,t} + \beta_2 \pi_{i,t}^2 + \beta_3 \ln \left(\frac{RGDP}{emp} \right)_{i,t-1} + \beta_4 \ln(csh\_i)_{it} + \beta_5 \ln(hc)_{it} \\ & + \beta_6 \ln(Labor)_{it} + \beta_7 \ln(csh\_g)_{it} + \beta_8 \ln(Trade)_{it} + \theta_i + T_t + \varepsilon_{it}, \end{aligned} \tag{40}$$

where $growth_{it}$ is the average annual growth of real GDP per employment for country i during period t; π is the average annual rate of inflation during the same period. Our construction of the variables mainly follows Mankiw et al. (1992). The control variables include the following. The variable

$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$ is the logarithm of real GDP per employment at the beginning of each period, which is to control for conditional convergence. The variables csh_i, hc and csh_g are the physical capital investment rate, human capital indicator and the share of government spending in GDP, respectively (we use the same notation as in Feenstra et al., 2015, for easier reference by the readers). The variable $Labor$ measures labor force growth. The variable $Trade$ measures the ratio of international trade to GDP. θ_i and T_t stand for the country fixed effects and time/period fixed effects, respectively.

There are several issues worth discussing. First, since this paper uses a model of endogenous growth, it would be useful to think of the results working their way via R&D expenditure shares (or labor force shares in R&D) rather than directly by output. We test this in Section 5. Additionally, it does not make sense to control for investment rates if one of the paper's arguments is that it affects output via investment rates. The Solow model assumes that investment rates are exogenous, but in this model they are endogenous. Second, the paper controls for Solow Model variables and ends up showing conditional convergence. Clearly, the theory itself lies squarely in the Schumpeterian quality ladder framework. It seems that the empirics may not support conditional convergence. However, the real world data would involve both conditional convergence and endogenous growth (see discussions in Aghion and Howitt, 1998, ch. 12). Therefore, the investment rate is driven by both the Solow model of capital accumulation and the endogenous model of R&D innovation. Therefore, many endogenous models still use the augmented Solow model (e.g., Aghion et al., 2009).

According to equation (40), the inflation variable enters the regression in a quadratic form. The marginal effect of inflation on growth would be $\beta_1 + 2\beta_2 \times \pi_{i,t}$. Therefore, the effect of inflation on growth would depend on the level of the inflation rate if $\beta_2 \neq 0$. We are interested in testing whether $\beta_2 < 0$. However, a quadratic form with $\beta_2 < 0$ could be either a concave or an inverted-U function. The cutoff point of the inflation rate with the marginal effect of inflation on growth being zero is $\widehat{\pi}_{i,t} = -\frac{\beta_1}{2\beta_2}$. If the threshold of inflation is larger than the minimum level of inflation and smaller than the maximum level of inflation in our sample, then growth is an inverted-U function of inflation. Otherwise, growth is a concave but monotone function of inflation.

4.2 Data Sample

The recent PWT 9.0 (explained by Feenstra, Inklaar and Timmer 2015) provides the most complete and recent data for all the countries during 1950–2014. One reason for the time sample 1950–2014 may be due to the common practice in empirical growth literature of using five-year averages of the data to smooth out business cycle fluctuations. We follow the common practice by taking five-year averages of the data as well. Therefore, the time sample of 1950–2014 naturally delivers 13 non-overlapping five-year average subperiods, the first being 1950–1954 and the last being 2010–2014.

There are of course many missing data, especially during the early years and for developing countries. Moreover, we acquire the inflation data from the WDI of the World Bank, and most of this data starts from year 1960 and some even starts from the late 1990s (e.g., the share of health expenditures in GDP). Therefore, we drop the observations before 1970. That is, we focus on the time sample of 1970–2014; taking five-year non-overlapping averages of the data yields 9 subperiods, the first being 1970–1974 and the last being 2010–2014.

We then combine the PWT 9.0 data with the WDI data on the inflation rate and monetary growth rate to get our final data sample for all the countries during 1970–2014. After merging the two datasets and retaining the observations that appeared in both the PWT and the WDI, our final sample has 154 countries during 1970–2014. Taking five-year averages yields a balanced panel with 1,386 observations (there are missing data on some variables for quite a few countries).

4.3 Measuring the Inflation Rate

In existing literature (see Aghion et al., 2009, and the references on the inflation-growth nexus cited above), the inflation rate is usually measured as the percentage change in the CPI (consumer price index). As discussed, we acquired the data on the inflation rate—Inflation, consumer prices (annual %)—from the Financial Sector section of the WDI. We denote the variable as π_{CPI}.

4.4 Measuring Growth Variables

Our dependent variable is the average annual growth of real GDP per employment, following Mankiw et al. (1992), who use real GDP per working-age person. There are three measures of real GDP in the PWT 9.0 (i.e., $RGDP^e$, $RGDP^o$, and $RGDP^{NA}$). Feenstra, Inklaar and Timmer (2015, p. 3157) have provided the following guidelines:

> The variables with the R-prefix are best suited for comparisons over time, though only $RGDP^e$ and $RGDP^o$ are simultaneously suitable for over time and cross-country comparisons. The $CGDP$ and $RGDP$ series, on both the expenditure and on the output sides, are tied to multiple ICP benchmarks whenever price data for a country have been collected multiple times. If the sole object is to compare the growth performance of economies, we would recommend using the $RGDP^{NA}$ series (and this is closest to earlier versions of the PWT).

Because we focus on comparing the growth performance of economies (how inflation impacts growth performance), we use the $RGDP^{NA}$ series. We divide this series by the *emp* series in the PWT 9.0 to get real GDP per employment. Using this data, we can calculate the average annual growth of real GDP per employment for each five-year subperiod to obtain our dependent variable *growth*. For instance, the average annual growth rate (i.e., *growth*) for subperiod 2000–2004 would be [log($RGDP^{NA}/emp$ in 2004)-log($RGDP^{NA}/emp$ in 2000)]/4. Initial real GDP per employment (i.e., $(RGDP/emp)_{t-1}$) takes the value of the beginning year of each subperiod (i.e., the value of year 2000 is given for subperiod 2000–2004).

For labor force growth measure *Labor*, it is measured following Mankiw et al. (1992). That is, *Labor* is equal to $(g+\delta)$—world annual growth g plus depreciation rate δ—plus the labor force growth rate that is measured as the annual growth of the *emp* series in the PWT 9.0. We follow Mankiw et al. to use 0.05 for $(g+\delta)$. That is, we assume a 2% world annual growth and a 3% depreciation rate. The physical capital investment rate is measured by the csh_i series in the PWT 9.0. Human capital invest rate is measured by the hc series in the PWT 9.0. We add together the csh_x (the ratio of export value to GDP) and the absolute value of the csh_m (the ratio of import value to GDP; the numbers are negative) series in the PWT 9.0 to get the measure of Trade. The last control variable is the ratio of government spending to GDP, denoted csh_g, and we use the csh_g series in the PWT 9.0. We use the same notation here for easier comparison by readers. We then compute the five-year averages of the variables.

Table 3 presents the summary statistics of the final data.

[Table 3 Here]

4.5 Parametric Estimation Results

Many countries in our sample experienced very high levels of inflation. For instance, Zimbabwe's average annual inflation was 8,603% during 2004–2009. Angola's average annual inflation was 1,478% during 1995–2000. According to our model, when the inflation rate is high enough, R&D labor l_r (and thereby the balanced growth rate) will be zero. In this case, it would be meaningless to test the effect of inflation on growth when growth does not change with inflation any more. In other words, the positive growth rates in these high inflation countries may not be best explained by our R&D-based Schumpeterian model. Therefore, in the following, we focus on testing the inverted-U relationship between growth and inflation in the sample with average annual inflation below 30%.

4.5.1 Ordinary least squares regression results

We first run the LSDV (least squares dummy variable) regression (i.e., ordinary least squares regression with country and year fixed effects). It is very possible that the observations are not independent across groups (i.e., countries). That is, the inflation rates and growth rates across countries may be correlated. To deal with heteroskedasticity, we use robust standard errors. The LSDV regression results are presented in Table 4.

We first do not exclude the observations with average annual inflation above 30% to see how our empirical specification fits the recent cross-country data. According to regression 4.1 of Table 4, the estimated coefficient on the logarithm of initial real GDP per employment $ln\left(RGDP/emp\right)_{t-1}$ is negative and significant at the 1% level, showing strong evidence of conditional convergence. The estimated coefficient on the physical capital investment rate $ln\left(csh\_i\right)$ is positive and significant at the 1% level, and that on labor force growth $ln\left(Labor\right)$ is negative and insignificant at the 10% level. The estimated coefficient on human capital $\ln(hc)$ is negative and significant at the 10% level. Except for human capital, the results on other variables are consistent with the prediction of the augmented Solow model. The estimated coefficient on government spending $ln\left(csh\_g\right)$ is negative and insignificant, and that on international trade $ln\left(Trade\right)$ is positive and significant at the 1% level. Therefore, our empirical specification fits the recent cross-country data quite well.

Now we add our variables of interest. We first add the inflation rate π_{CPI}. For comparison, we first include observations with annual inflation below and above 30% (the full sample). The LSDV regression results are presented in regression 4.2 of Table 4. The estimated coefficient on the inflation rate π_{CPI} is negative, which is insignificant at the 10% level. The results on other variables remain similar to those in regression 4.1. The insignificant, negative effect of inflation on growth is consistent with previous cross-country studies (see the references cited above). In regression 4.3 of Table 4, we include both the inflation rate and its square term in the regression. The regression results indicate that growth is a U function of the inflation rate.

As discussed, it is meaningful for us to exclude the observations with such high levels of inflation. Additionally, the existing literature has detected a threshold above which inflation will hurt growth. For instance, Kremer, Bick and Nautz (2011) found this threshold to be around 2% for advanced countries and 17% for developing countries while Bruno and Easterly (1999) found that 40% annual inflation seems to be the upper limit beyond which inflation may significant reduces growth. Khan and Senhadji (2001) also focused on the non-linear relationship between inflation and growth. However, most papers used a transformed variable of inflation, and the transformation is also non-linear. For instance, Baglan and Yoldas (2014) and Kremer et al. (2011) used the logarithm of inflation when it is above 1% and 1-inflation when it is below 1% as a means of dealing with negative inflation.

Taking our discussion above and the existing literature into account, we find that when we exclude the observations with average annual inflation above 30%, growth is a significant inverted-U function of the inflation rate. The results are presented in regression 4.4 and 4.5 of Table 4. In regression 4.5 of Table 4, we include both the inflation rate and its square term in the regression. The regression results indicate that growth is a significant inverted-U function of the inflation rate. The estimated coefficient on the inflation rate π_{CPI} is negative, which is significant at the 1% level. The estimated coefficient on the square term of the inflation rate π_{CPI}^2 is also negative, which is significant at the 1% level. The F-test on the joint significance of both the inflation rate and its square term yields a p-value below 1%, meaning the inflation rate and its square term jointly have a significant effect on growth. In contrast, the estimated coefficient on the inflation rate π_{CPI} is negative, which is significant at the 5% level when we exclude the square term of the inflation rate from the regression.

According to regression 4.5, we find that the cutoff point of the inflation rate with the marginal effect of inflation on growth being zero is $\widehat{\pi}_{i,t} = 5.19\%$. The inflation rate in our sample lies on the interval [-24%, 29%] when we focus on samples with annual inflation rate below 30%. The threshold of inflation is larger than the minimum level of inflation and smaller than the maximum level of inflation, which indicates that growth is an inverted-U function of inflation in our sample.

[Table 4 Here]

4.5.2 IV regression results

It is possible that the inflation rate may be endogenous. As discussed in Bruno and Easterly (1999), it is hard to find an instrument that is relevant and time-varying. As a compromise, we follow the same strategy used in Checherita-Westphal and Rother (2012). In testing the inverted-U effect of government debt on growth for countries in the Euro area, Checherita-Westphal and Rother (2012) have used up to the fifth lag of government debt as the instruments. However, using lagged variables as instruments may be problematic when the endogenous variable is highly persistent over time. To deal with this issue, Checherita-Westphal and Rother (2012) also used the average level of government debt for the other countries in the Euro area as instruments.

Following Checherita-Westphal and Rother (2012), we also use the same Stata command ivreg2 developed by Baum et al. (2007). We also use two-step GMM (generalized method of moments) estimation. As noted in Checherita-Westphal and Rother (2012), “The two-step GMM presents some efficiency gains over the traditional IV/2-SLS estimator derived from the use of the optimal weighting matrix, the overidentifying restrictions of the model, and the relaxation of the independent and identical distribution (i.i.d.) assumption, see Baum et al (2007).”

As discussed in our theory, we have $i_t = \dot{M}_t/M_t + \rho$. Therefore, we also acquire the data on the monetary growth rate—the broad money growth (annual %)—in the Financial Sector section of the WDI. The monetary growth rate and its square term are also used as instruments when necessary. Specifically, we have used two groups of instruments. The first group uses up to fifth lags of inflation, inflation square, monetary growth rate and its square (20 excluded instruments). The second group uses up to fifth lags of the average inflation rate of all the other countries (i.e., the rest of the world), the average inflation rate of all the other countries and its square (7 excluded instruments). The number of observations is 352 (covering 100 countries), while the number of observations in regression 4.1 is 289 (covering 83 countries).[5] This difference is explained by the fact that many European countries, such as Germany and France, have joined the monetary union and use the Euro as a common currency. Therefore, these countries do not have their own monetary growth rates.

Table 5 presents the first-stage results of the IV regression results. The F-test statistics on the instruments are above 10, the rule of thumb for the existence of strong instruments in Staiger and Stock (1997). That is, the instruments jointly have significant effects on the endogenous variables (i.e., the inflation rate and its square).

[Table 5 Here]

The second-stage results are presented in Table 6. In regressions 6.1 and 6.3, we have used the lagged values of inflation and its square as the instruments. According to regression 6.1 of Table 6, where we have used heteroskedasticity-robust standard errors, the estimated coefficient on the inflation rate π_{CPI} is negative, which is significant at the 1% level. The estimated coefficient on the square term of the inflation rate π^2_{CPI} is negative, which is significant at the 1% level. The F-test on the joint significance of both the inflation rate and its square term yields a p-value below 1%, meaning the inflation rate and its square term jointly have a significant effect on growth. The Hansen-J over-identification test yields a p-value above 0.1, meaning we accept the null hypothesis that the instruments are valid.

According to regression 6.1, we find that the cutoff point of the inflation rate with the marginal effect of inflation on growth being zero is $\widehat{\pi}_{i,t} = 2.78\%$. The inflation rate in our sample lies on the interval [-24%, 29%] when we focus on samples with annual inflation rate below 30%. When

[5] Since economic growth in the model is innovation-driven and it is more likely to associate with advanced economies, it would be better to divide the whole sample into several groups based on the income level to see whether the inverted-U still holds in different groups and how income disparities among the groups alter the optimal inflation rate. First, we have considered samples with average annual inflation below 30%. That is, we have excluded countries with high levels of inflation. Therefore, the economic growth of the economies in our sample is more likely to be innovation-driven. Second, the number of observations is already small (289) in IV regressions. We have tested the inverted-U in OECD countries, but the results are not as significant, which may be due to the small sample bias.

inflation is below the cutoff value of $\widehat{\pi}_{i,t} = 2.78\%$, growth is an increasing function of the inflation rate; when inflation is beyond the cutoff point $\widehat{\pi}_{i,t} = 2.78\%$, growth becomes a decreasing function of the inflation rate. Therefore, our two-step GMM estimation results indicate that growth is an inverted-U function of inflation in our sample.

The results remain robust when we use standard errors robust to both heteroskedasticity and autocorrelation, as indicated in regression 6.3 of Table 6. The cutoff point of inflation with the marginal effect of inflation on growth being zero becomes $\widehat{\pi}_{i,t} = 9.6\%$, which still lies within our sample interval of the inflation rate. Therefore, growth remains an inverted-U function of inflation in our sample when we deal with heteroskedasticity and autocorrelation in two-step GMM estimation.

In regressions 6.2 and 6.4, we have used the average level of inflation for the rest of the world as the instrument. According to regression 6.2 of Table 6, where we have used heteroskedasticity-robust standard errors, the estimated coefficient on the inflation rate π_{CPI} is negative, which is significant at the 5% level. The estimated coefficient on the square term of the inflation rate π_{CPI}^2 is negative, which is significant at the 10% level. The F-test on the joint significance of both the inflation rate and its square term yields a p-value below 1%, meaning the inflation rate and its square term jointly have a significant effect on growth. The Hansen-J over-identification test yields a p-value above 0.1, meaning we accept the null hypothesis that the instruments are valid. The cutoff point of the inflation rate with the marginal effect of inflation on growth being zero is 2.52%, which is similar to that in regression 6.1. Therefore, our two-step GMM estimation results indicate that growth is an inverted-U function of inflation in our sample when we use the second group of instruments. The results remain robust when we use standard errors robust to both heteroskedasticity and autocorrelation, as indicated in regression 6.4 of Table 6.

[Table 6 Here]

5 Empirical Evidence on the Direct Mechanism

As discussed, Chu et al. (2017, 2019) incorporate heterogeneous firms to also predict an inverted-U relationship between inflation and growth. Therefore, it is best if we can offer more evidence on the direct mechanism. To do so, we directly test the effect of inflation on l_r (the share of R&D labor in total labor supply), denoted $R\&D\_S$. According to our discussion following Proposition 2, our model predicts that inflation has an inverted-U effect on l_r (the share of R&D labor in total labor supply). This prediction is unique to our model.

Specifically, our empirical specification is

$$R\&D\_S_{it} = \alpha_0 + \alpha_1 \pi_{i,t} + \alpha_2 \pi_{i,t}^2 + \alpha_3 (\text{Controls})_{it} + \theta_i + T_t + \varepsilon_{it}, \tag{41}$$

where $R\&D\_S$ (i.e., l_r) is measured by "Total R&D personnel per thousand employment (FTE)" from the UNESCO Institute for Statistics. Both R&D personnel and employment are full-time equivalent (FTE). The data covers the period 1996-2016 for 162 countries. (Controls) are the variables that may have an effect on R&D labor share. These variables are initial real GDP per employment (i.e., $(RGDP/emp)_{t-1}$), human capital $Human$, and trade openness $Trade$. These three variables are used in Section 4. One additional control variable is the degree of CBI. We planned to use CBI as an instrument for inflation, but the over-identifying tests show that using it as a control variable instead of an instrument is better.

5.1 Identification Strategy

In aggregate level studies, there is always a possibility of endogeneity. We use IV estimation to deal with the potential endogeneity of the inflation rate. As in Section 4, the broad money growth rate (denoted $M2g$) is a candidate instrument. Because we have the square term of inflation, we need at least two instruments. Therefore, we also use financial depth and its logarithm as additional instruments. With more instruments than endogenous variables, we can use the over-identifying tests to check the validity of the instruments.

5.2 Data Sample

We collect data on CBI from Dincer and Eichengreen (2014). We use the unweighted average of the nine aggregated variables (i.e., CBIU in Table 9 of Dincer and Eichengreen) to measure CBI, denoted CBI. The data covers 1998-2010 for more than 100 countries. For financial depth, we obtain the necessary data from the WDI. Specifically, we measure financial depth (denoted FD/GDP) as the indicator "Domestic credit to private sector (% of GDP)".

We use yearly PWT data to have many observations as possible. We then merge all the other data into the yearly PWT panel data. Doing so yields a final sample of 52 countries during 1998-2010, producing a balanced panel of 676 observations. Nevertheless, there are many missing data, especially for our dependent variable, as elaborated on below.

Table 7 presents the summary statistics of the final data. According to Table 7, our dependent variable has only 279 observations. Its mean is 6.59, which indicates that on average there are 6.59 FTE R&D personnel per thousand FTE employment. Jordan in 1998 has the largest value of R&D share (22.04 R&D personnel per thousand employment). Canada has a mean value of $R\&D\_S$ at 12.57 during 1998-2010. Canada's 0.01257 is close to our calibrated value for l_r (see columns 1.1 to 1.3 in Table 1). We also reported the summary statistics for inflation and money growth when our dependent variable has observations. The mean of inflation in our final sample used in regressions is 7.21%, with a maximum of 85.74%. The corresponding numbers for monetary growth are 16.46% and 125.03%, respectively. Therefore, there are no extreme values of inflation in our final sample, which means the results will not be driven by outliers.

[Table 7 Here]

5.3 Estimation Results

We find that our results are similar when we do not center the value of inflation. Therefore, we report the results with the original values of inflation (i.e., uncentered values).

For comparison, we first present the LSDV results in Table 8. Regression 8.1 of Table 8 reports the results with the linear form of inflation. One can see that inflation has a positive, significant effect on $R\&D\_S$ (i.e., the share of labor employed in R&D) at the 5% level. The effect becomes negative and insignificant if we use robust standard errors. Regression 8.3 of Table 8 reports the results with both inflation and its squared term in the regression. The estimated coefficient of inflation remains positive and significant at the 5% level, while its squared term is negative and insignificant. However, we find that the effect of inflation on $R\&D\_S$ is concave instead of inverted-U. The results remain similar when we use robust standard errors (see regression 8.4 of Table 8).

[Table 8 Here]

Table 9 presents the first-stage and the corresponding second-stage results of the 2SLS (two-stage least squares) estimation. Regression 9.1 of Table 9 reports the first-stage results with inflation as the dependent variable. Both monetary growth and financial depth have a positive, significant effect on inflation at the 1% level. According to regression 9.2, monetary growth has a positive, significant effect on the square of inflation, while financial depth has a positive, insignificant effect on the square of inflation. Nevertheless, both regressions 9.1 and 9.2 indicate that the F-test statistics on the instruments are above 10, the rule of thumb for the existence of strong instruments in Staiger and Stock (1997). That is, the instruments jointly have significant effects on the endogenous variables (i.e., the inflation rate and its square). Because the instruments are strong, we use 2SLS estimation.

According to the second-stage results presented in regression 9.3 of Table 9, the estimated coefficient on the inflation rate π_{CPI} is positive, which is significant at the 1% level. The estimated coefficient on the square term of the inflation rate π_{CPI}^2 is negative, which is significant at the 1% level. The over-identification tests yields a p-value much above 0.1, meaning we accept the null hypothesis that the instruments are valid. Moreover, we find that the cutoff point of the inflation rate with the marginal effect of inflation on $R\&D\_S$ (i.e., the share of labor employed in R&D) being zero is $\widehat{\pi}_{i,t} = 41.86\%$. The inflation rate in our sample lies on the interval [-10%, 86%]. Therefore, our 2SLS estimation results indicate that $R\&D\_S$ is an inverted-U function of inflation in our sample. The results remain robust when we use robust standard errors, as indicated in regression 9.4 of Table 9. When we use LIML (limited-information maximum likelihood) estimation to deal with weak instruments when we use robust standard errors, the results in regression 9.5 of Table 9 indicate that our results remain robust.

The inverted-U effect of inflation on $R\&D\_S$ (i.e., the share of labor employed in R&D) provides direct support of our mechanism. As explained, only when R&D is subsidized by seigniorage revenue is there a positive effect of inflation on R&D. Subsection 3.11.2 shows that subsidies to R&D by other taxes will not explain the data, because the other taxes cannot generate the *changing* seigniorage effect, ending up being unable to produce the inverted-U result between inflation and R&D.

[Table 9 Here]

5.4 Comparison Results with Physical Capital Investment

If we can show that the inverted-U relationship is especially strong for R&D, as opposed to other types of investment, that would help further. We also have the data on physical capital investment (see Section 4). Therefore, we conduct the same regressions with the dependent variable being physical capital investment rate $ln\,(csh\_i)$. To make the results comparable, we focus on the same sample as in the previous subsection (i.e., the sample where $R\&D\_S$ has observations). The second-stage results of IV regressions are presented in Table 10.

Regression 10.1 shows that the logarithm of physical capital investment rate (i.e., $ln\,(csh\_i)$) is a U-function of inflation. However, the estimated coefficients on inflation and its square become insignificant in LIML estimation. The results hold up when we use the level of physical capital investment (i.e., csh_i) as the dependent variable, as illustrated in regressions 10.3 and 10.4.

[Table 10 Here]

Taken together, the inverted-U result between growth and inflation is driven by the inverted-U relationship between $R\&D\_S$ and inflation. Such an inverted-U relation does not exist between physical capital investment and inflation.

6 Conclusions

This study introduces two elements into the seminal contribution of Aghion and Howitt (1992), which creates a novel view of the effect of inflation on growth and welfare by showing that growth is an inverted-U function of inflation. The results are confirmed by calibration and empirical evidence. The large growth differences across countries may be partly explained by our mechanism. Here we briefly discuss the strong policy implications of our study.

First, the positive nominal interest rate that maximizes growth is higher than that maximizes welfare. When the elasticity of labor supply is very high, the Friedman rule is optimal even if a positive nominal interest rate promotes growth. China's average annual growth of M2 was 18.3% during the period 2003–2011, generating large seigniorage revenue. With one-party dictatorship, the autocratic government of China is able to direct the seigniorage revenue to any specific sectors of the economy. China's high growth may be partially explained by our mechanism. However, China's heavy subsidies to state-owned industries may promote its growth, but it may be bad for welfare. Therefore, even without the current trade war with the U.S., China may want to re-evaluate its industrial policies that heavily subsidize state-owned industries. On the flip side, the autocratic government of China can choose growth rather than welfare as its objective. From this point of view, democratic countries do not necessarily grow faster than non-democratic ones (see Persson and Tabinilli, 2009, and Acemoglu et al., 2016, for recent democracy-growth causality studies).

Second, we treat the share of seigniorage used in R&D subsidies as given. Obviously, there are substantial growth and welfare gains if the government could endogenize the share of the seigniorage revenue used as R&D subsidies. Using Table 2, when 20% of seigniorage is used in R&D subsidies, raising the nominal interest rate to its optimal value can add 1.06% annually to growth (which is substantial), and the substantial welfare gain is equivalent to a permanent increase in consumption of 14.32%. Of course, such effects can also be generated when other taxes revenues are used as R&D subsidies. The welfare gains are also found in Segerstrom (1998), but there is no growth effect and a negative growth effect of R&D subsidies in Segerstrom (1998) and Segerstrom (2000), respectively.

It is meaningful to check whether our inverted-U results hold up in models with both horizontal and vertical R&D considered in Peretto (1998, 1999) and Segerstrom (2000) or in models with imitation and transfer of foreign technologies considered in Chu et al. (2014) in future research.

APPENDIX I: HOUSEHOLD'S DYNAMIC OPTIMIZATION

Household's Hamiltonian function is

$$H_t = \ln c_t + \theta \ln (1 - l_t) + \mu_t [r_t a_t + w_t l_t - c_t - \pi_t m_t + i_t b_t] + v_t (m_t - c_t - b_t),$$

where μ_t is the co-state variable on (2); η_t is the Lagrangian multiplier for the CIA constraint. The first-order conditions include

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \mu_t - v_t = 0, \tag{42}$$

$$\frac{\partial H_t}{\partial l_t} = \frac{\theta}{1 - l_t} - \mu_t w_t = 0, \tag{43}$$

$$\frac{\partial H_t}{\partial b_t} = \mu_t i_t - v_t = 0, \tag{44}$$

$$\frac{\partial H_t}{\partial a_t} = \mu_t r_t = \rho \mu_t - \dot{\mu}_t, \tag{45}$$

$$\frac{\partial H_t}{\partial m_t} = -\mu_t \pi_t + v_t = \rho \mu_t - \dot{\mu}_t. \tag{46}$$

Combining (44), (45) and (46) yields $v_t = \mu_t (r_t + \pi_t) = \mu_t i_t$. Plugging this condition into (42) yields

$$\frac{1}{c_t} = \mu_t (1 + i_t), \tag{47}$$

which is (3) in the main text. Rewriting (45) as

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho \tag{48}$$

yields the intertemporal optimality condition (6) in the main text.

Equation (43) is the optimal condition for labor supply in equation (4) in the main text.

APPENDIX II: PROOF OF PROPOSITION 1

We follow Chu and Cozzi (2014) to define a transformed variable $\Omega_t \equiv c_t / v_t$. We have

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_t}{v_t} = r_t - \rho - \frac{\dot{v}_t}{v_t}, \tag{49}$$

where the second equality uses the households' Euler equation by combining (3) and (6).

Using equation (16), the law of motion for v_t is

$$\frac{\dot{v}_t}{v_t} = r_t + \lambda_t - \frac{\widehat{\Pi}_t}{v_t}. \tag{50}$$

Using equation (17), we have

$$\frac{\widehat{\Pi}_t}{v_t} = \left[\frac{\gamma-1}{\gamma} + \beta\left(i-\rho\right)\phi_t\right]\frac{y_t}{v_t} = \frac{\frac{\gamma-1}{\gamma} + \beta\left(i-\rho\right)\phi_t}{1-\beta(i-\rho)\phi_t}\frac{c_t L}{v_t}, \tag{51}$$

where the last equality uses the output market clearing condition: $c_t = \left[1-\beta(i-\rho)\phi_t\right] y_t/L$. We also have $\lambda_t = \varphi l_{r,t}$. Therefore, plugging (51) and (50) into (49), we have

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\left[\frac{\gamma-1}{\gamma} + \beta\left(i-\rho\right)\phi_t\right] L}{1-\beta(i-\rho)\phi_t}\Omega_t - \left(\varphi l_{r,t} + \rho\right). \tag{52}$$

Now using (12), (14), and $c_t = \left[1-\beta(i-\rho)\phi_t\right] y_t/L$, we have

$$l_{x,t} = \frac{1+i}{\gamma\varphi}\frac{y_t}{v_t} = \frac{\left(1+i\right)L}{\gamma\varphi\left[1-\beta(i-\rho)\phi_t\right]}\Omega_t. \tag{53}$$

Now using (53) and the labor market clearing condition given in equation (22), we have

$$l_{r,t} = 1 - \frac{\left\{1+\theta\gamma\left(1+i\right)\left[1-\beta(i-\rho)\phi_t\right]\right\}\left(1+i\right)L}{\gamma\varphi\left[1-\beta(i-\rho)\phi_t\right]}\Omega_t. \tag{54}$$

Now plugging (54) into (52), we have

$$\frac{\dot{\Omega}_t}{\Omega_t} = f\left(\Omega_t\right)\Omega_t - \left(\varphi+\rho\right), \tag{55}$$

where $f\left(\Omega_t\right) = \frac{\left[\frac{\gamma-1}{\gamma}+\beta(i-\rho)\phi_t\right]L}{1-\beta(i-\rho)\phi_t} + \frac{\left\{1+\theta\gamma(1+i)\left[1-\beta(i-\rho)\phi_t\right]\right\}(1+i)L}{\gamma\left[1-\beta(i-\rho)\phi_t\right]}$. (20) shows $\phi_t = \frac{1}{1+\beta(i-\rho)}\left(1+\frac{l_{r,t}}{\gamma l_{x,t}}\right)$. Therefore, ϕ_t is a function of Ω_t, and so is $f\left(\Omega_t\right)$. Combining (53) and (54) yields

$$\frac{l_{r,t}}{l_{x,t}} = \frac{\gamma\varphi\left[1-\beta(i-\rho)\phi_t\right]\left(1+i\right)L}{\left(1+i\right)L\Omega_t} - \left\{1+\theta\gamma\left(1+i\right)\left[1-\beta(i-\rho)\phi_t\right]\right\}. \tag{56}$$

Using equation (20), we have

$$1-\beta(i-\rho)\phi_t = \frac{1}{1+\beta(i-\rho)} - \frac{\beta(i-\rho)}{1+\beta(i-\rho)}\frac{l_{r,t}}{\gamma l_{x,t}}. \tag{57}$$

Using (56) to substitute out $l_{r,t}/l_{x,t}$ in (57), we can solve for $\left[1-\beta(i-\rho)\phi_t\right]$ as

$$1-\beta(i-\rho)\phi_t = \frac{1+\frac{1}{\gamma}\beta(i-\rho)}{1+\beta(i-\rho)\left[1+\frac{\varphi}{(1+i)L\Omega_t} - \theta\left(1+i\right)\right]}. \tag{58}$$

Now using (58) to substitute out $[1-\beta(i-\rho)\phi_t]$ in $f(\Omega_t)$, we have

$$f(\Omega_t) = \frac{1+\beta(i-\rho)\left[1+\frac{\varphi}{(1+i)L\Omega_t}-\theta(1+i)\right]}{1+\frac{1}{\gamma}\beta(i-\rho)}\left(2+\frac{i}{\gamma}\right)L - L + \theta(1+i)^2 L. \tag{59}$$

Now plugging (59) in (55), we have

$$\frac{\dot{\Omega}_t}{\Omega_t} = A\Omega_t - B, \tag{60}$$

where $A = \frac{1+\beta(i-\rho)[1-\theta(1+i)]}{1+\frac{1}{\gamma}\beta(i-\rho)}\left(2+\frac{i}{\gamma}\right)L - L + \theta(1+i)^2 L$, and $B = (\varphi+\rho) - \frac{\beta(i-\rho)\varphi\left(2+\frac{i}{\gamma}\right)}{(1+i)\left[1+\frac{\beta}{\gamma}(i-\rho)\right]}$. Given a fixed i, both A and B and functions of structural parameters. We have

$$\frac{A}{L} = \frac{1+\beta(i-\rho)[1-\theta(1+i)]}{1+\frac{1}{\gamma}\beta(i-\rho)}\left(2+\frac{i}{\gamma}\right) - 1 + \theta(1+i)^2 \tag{61}$$

$$= \frac{\gamma+i+\beta(i-\rho)(2\gamma+i-1)+\theta(1+i)[(1+i)(\gamma-\beta\rho+\beta i)-\beta(i-\rho)(2\gamma+i)]}{\gamma-\beta\rho+\beta i}. \tag{62}$$

Because $\gamma-\beta\rho+\beta i = \gamma+\beta(i-\rho) > 0$, we have

$$sign(A) = sign\{\gamma+i+\beta(i-\rho)(2\gamma+i-1)+\theta(1+i)[\gamma(1+i)-\beta(i-\rho)(2\gamma-1)]\}. \tag{63}$$

Given $\rho \in \left(0,\frac{1}{2}\right)$ and $\beta \leqslant \frac{\gamma}{2\gamma-1} = \frac{1}{2-\frac{1}{\gamma}}$, we have $\beta(i-\rho)(2\gamma+i-1) > -\beta\rho(2\gamma+i-1) > \left(-\gamma-\frac{i}{2}+\frac{1}{2}\right)$ and $\beta(i-\rho)(2\gamma-1) \leqslant \gamma(i-\rho)$. Plugging these conditions into (63), we have

$$sign(A) > sign\left\{\gamma+i-\gamma-\frac{i}{2}+\frac{1}{2}+\theta(1+i)[\gamma+\gamma i-\gamma(i-\rho)]\right\} > 0. \tag{64}$$

Therefore, given $\rho \in \left(0,\frac{1}{2}\right)$ and $\beta \leqslant \frac{\gamma}{2\gamma-1} = \frac{1}{2-\frac{1}{\gamma}}$, we have $A > 0$. Equation (60) shows that the dynamics of Ω_t is characterized by saddle-point stability such that Ω_t jumps immediately to its interior steady state given by $\Omega_t = B/A$. When Ω_t is stationary and unique, l_r and l_x must be stationary and unique according to equations (53) and (54). Therefore, per capita labor supply l is stationary and unique as well. The dynamic property of the model does not depend on the sign of B. However, because $\Omega_t = c_t/v_t = B/A$, we need $B > 0$ to have a meaningful equilibrium. When $i \leqslant \rho$, we always have $B > 0$. When $i > \rho$, we have

$$sign(B) = sign\left\{(1+i)(\varphi+\rho)+\frac{(1+i)(\varphi+\rho)}{\gamma}\beta(i-\rho)-\beta(i-\rho)\varphi\left(2+\frac{i}{\gamma}\right)\right\} \tag{65}$$

$$= sign\left\{\varphi i\left(1+\frac{\beta}{\gamma}-2\beta\right)+\underbrace{\varphi+\rho+\rho i+\frac{\beta i\rho(1+i-\rho)}{\gamma}}_{+}+\underbrace{\beta\rho\left(2\varphi-\frac{\varphi+\rho}{\gamma}\right)}_{+}\right\}. \tag{66}$$

We have $sign(B) > 0$ if $1+\frac{\beta}{\gamma}-2\beta \geqslant 0$ (i.e., $\beta \leqslant \frac{1}{2-\frac{1}{\gamma}}$). Please note that $\beta \leqslant \frac{1}{2-\frac{1}{\gamma}}$ is a sufficient condition to make sure Propositions 1 and 2 are logically consistent. That is, the economy in Proposition 2 will always immediately jump to a unique and saddle-point stable balanced growth path, and the path is meaningful when $\Omega_t = c_t/v_t = B/A > 0$ (only when both $A > 0$ and $B > 0$).

APPENDIX III: PROOF OF PROPOSITION 2

As discussed, the balanced growth rate in (19) is linear in the share of labor employed in R&D l_r. Therefore, we only need to prove that l_r, given in (23), is an inverted-U function of the nominal interest rate. To make our proof easier, we define $R = \beta(i - \rho)$, $\widehat{\gamma} = (\gamma - 1)$ and rewrite l_r as

$$l_r = \frac{\Theta}{\Lambda}, \tag{67}$$

where the numerator $\Theta = \widehat{\gamma}(1 + 2R) - \left(\frac{1}{\beta}R + \rho + 1\right)(1 + R)\frac{\rho}{\varphi} + R$ and the denominator $\Lambda = \widehat{\gamma}(1 + 2R) + \left(\frac{1}{\beta}R + \rho + 1\right)(1 + R)$.

We respect the zero-lower bound on the nominal interest rate, which means $i \geqslant 0$ and $R \geqslant -\beta\rho$. Taking derivative of l_r with respect to R, we have

$$\frac{\partial l_r}{\partial R} = \frac{\Theta'\Lambda - \Lambda'\Theta}{\Lambda^2} = \frac{F(R)}{\Lambda^2}, \tag{68}$$

where we have defined

$$F(R) = \Theta'\Lambda - \Lambda'\Theta. \tag{69}$$

Plugging Λ, Θ, Λ', and Θ', into equation (69) and collecting the same items, we find out that the cubic terms of R cancel out. Therefore, $F(R)$ is a quadratic function of R:

$$F(R) = -\left(\frac{1 + 2\widehat{\gamma}}{\beta} + \frac{2\widehat{\gamma}\rho}{\beta\varphi}\right)R^2 - \frac{2\widehat{\gamma}(\rho + \varphi)}{\beta\varphi}R + \widehat{\Theta}, \tag{70}$$

where $\widehat{\Theta} = 1 + \rho + \widehat{\gamma}\left(2 + \rho - \frac{1}{\beta} - \frac{\rho}{\beta\varphi} + \frac{\rho + \rho^2}{\varphi}\right)$. According to equation (70), the graph of the quadratic function $F(R)$ is a parabola that opens downwards. Now we have

$$\begin{aligned} F(-\beta\rho) &= -2\widehat{\gamma}\beta\rho^2 - \frac{2\widehat{\gamma}\beta\rho^3}{\varphi} - \beta\rho^2 + \frac{2\widehat{\gamma}\rho^2}{\varphi} + 2\rho\widehat{\gamma} + \rho + 1 + 2\widehat{\gamma} + \rho\widehat{\gamma} - \frac{\widehat{\gamma}}{\beta} - \frac{\rho\widehat{\gamma}}{\beta\varphi} \\ &+ \frac{\widehat{\gamma}\rho^2 + \widehat{\gamma}\rho}{\varphi} = \widehat{\gamma}\left[2 + 3\rho + \frac{\rho}{\varphi} + \frac{3\rho^2}{\varphi} - 2\beta\rho^2 - \frac{\rho}{\beta\varphi} - \frac{1}{\beta} - \frac{2\beta\rho^3}{\varphi}\right] + 1 + \rho - \beta\rho^2. \end{aligned} \tag{71}$$

Using $\beta \leqslant 1$ and $\rho \in \left(0, \frac{1}{2}\right)$, we rewrite (71) as

$$\begin{aligned} F(-\beta\rho) &= \widehat{\gamma}\left[2 + 2\rho + \frac{\rho}{\varphi} + \frac{2\rho^2}{\varphi} + \underbrace{\rho(1 - 2\beta\rho)}_{+} + \underbrace{\frac{\rho^2}{\varphi}(1 - 2\beta\rho)}_{+} - \frac{1}{\beta}\left(1 + \frac{\rho}{\varphi}\right)\right] \\ &+ 1 + \frac{\rho}{2} + \underbrace{\frac{\rho}{2}(1 - 2\beta\rho)}_{+} > \widehat{\gamma}\left[2 + 2\rho + \frac{\rho}{\varphi} + \frac{2\rho^2}{\varphi} - \frac{1}{\beta}\left(1 + \frac{\rho}{\varphi}\right)\right] + 1 + \frac{\rho}{2}. \end{aligned} \tag{72}$$

Therefore, according to (72), we have

$$F(-\beta\rho) > 0 \text{ if } \beta \geqslant \frac{\widehat{\gamma}\left(1+\frac{\rho}{\varphi}\right)}{1+\frac{\rho}{2}+\widehat{\gamma}\left(2+2\rho+\frac{\rho+2\rho^2}{\varphi}\right)}. \tag{73}$$

Taken together, if $\beta \geqslant \frac{\widehat{\gamma}\left(1+\frac{\rho}{\varphi}\right)}{1+\frac{\rho}{2}+\widehat{\gamma}\left(2+2\rho+\frac{\rho+2\rho^2}{\varphi}\right)}$, $F(R) = 0$ has a unique solution (a unique global maximum) $R = R^* > -\beta\rho$, as illustrated in Figure A.1. The maximum point of $F(R)$ may be on the left of $(-\beta\rho)$ because $F'(-\beta\rho) < 0$ if $\beta < \frac{\widehat{\gamma}\left(1+\frac{\rho}{\varphi}\right)}{\rho+2\rho\widehat{\gamma}\left(1+\frac{\rho}{\varphi}\right)} \approx \frac{\widehat{\gamma}}{\rho}$. Nevertheless, $F'(-\beta\rho) < 0$ is not necessary for our inverted-U result.

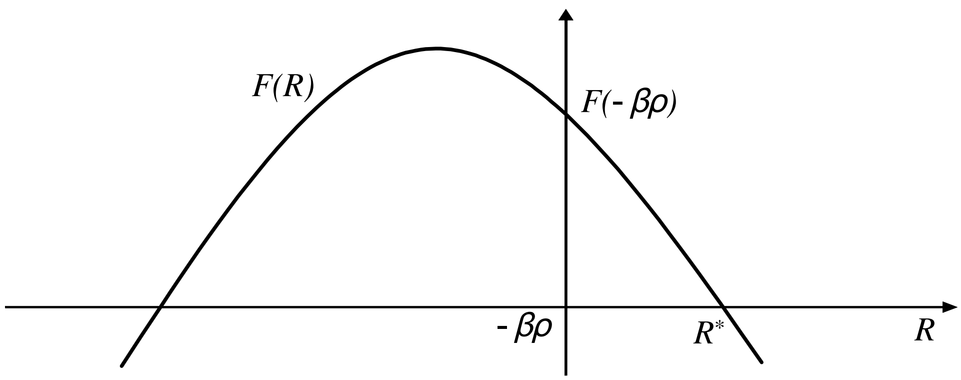

Figure A.1. The Graph of *F(R)*

Taken together, when $R \in [-\beta\rho, R^*)$ or $i \in \left[0, \frac{R^*}{\beta}+\rho\right)$, $sign\left\{\frac{\partial l_r}{\partial R}\right\} = sign\{F(R)\} > 0$; when $R \in (R^*, +\infty)$ or $i \in \left(\frac{R^*}{\beta}+\rho, +\infty\right)$, $sign\left\{\frac{\partial l_r}{\partial R}\right\} = sign\{F(R)\} < 0$. Therefore, l_r is an inverted-U function of the nominal interest rate. More strictly, we have $F'(R^*) < 0$, using $\frac{\partial l_r}{\partial R} = \frac{F(R)}{\Lambda^2}$ yields

$$\frac{\partial^2 l_r}{\partial R^2}|_{R=R^*} = \frac{F'(R^*)}{\Lambda^2} - \frac{F(R^*)}{\Lambda^3}\Lambda' = \frac{F'(R^*)}{\Lambda^2} < 0. \tag{74}$$

Equation (74) shows that l_r is a concave function of R at least around R^*. Therefore, l_r is an inverted-U function of the nominal interest rate around $i^* = \frac{R^*}{\beta} + \rho > 0$.

APPENDIX IV: GOVERNMENT SUBSIDIES OF CAPITAL ACCUMULATION BY HOUSEHOLDS IN THE AK MODEL

We denote investment by $\dot{k}_t = x_t$. Household's Hamiltonian function is

$$H_t = \ln c_t + \mu_t x_t + v_t\left[Ak - c_t - \pi_t m_t - (1-s_t)\, x_t\right] + \eta_t\left(m_t - c_t - x_t\right),$$

where μ_t and v_t are the co-state variables on $\dot{k}_t$ and $\dot{m}_t$, respectively; η_t is the Lagrangian multiplier for the CIA constraint.

The first-order conditions include

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - v_t - \eta_t = 0, \tag{75}$$

$$\frac{\partial H_t}{\partial x_t} = \mu_t - v_t (1 - s_t) - \eta_t = 0, \tag{76}$$

$$\frac{\partial H_t}{\partial k_t} = A v_t = \rho \mu_t - \dot{\mu}_t, \tag{77}$$

$$\frac{\partial H_t}{\partial m_t} = -v_t \pi_t + \eta_t = \rho v_t - \dot{v}_t. \tag{78}$$

Following Dotsey and Sarte (2000), on the balanced growth, $c_t = \chi A k$ and $\dot{k}_t = (1 - \chi) A k$, where χ is a constant. On the balanced growth path, s_t is a constant (shown in the text), and $g_2 = \dot{c}_t / c_t = -\dot{\lambda}_t / \lambda_t$, where we define $\lambda_t = (v_t + \eta_t)$. We conjecture $\dot{v}_t / v_t = \dot{\mu}_t / \mu_t$. Using (76) yields $\dot{\lambda}_t / \lambda_t = \dot{\mu}_t / \mu_t$. Now using (77), we have $v_t / \mu_t = (\rho + g_2) / A$, confirming our conjecture. Then from (76), we have $\eta_t / \mu_t = [A - (1 - s)(\rho + g_2)] / A$. Plugging these conditions into (78) yields

$$-\frac{\dot{v}_t}{v_t} = g_2 = \frac{\eta_t}{v_t} - \pi_t - \rho = \frac{A - (1 - s)(\rho + g_2)}{(\rho + g_2)} - \pi_t - \rho. \tag{79}$$

We have $m_t = M_t / P_t$. Therefore, $\dot{m}_t / m_t = \left(\dot{M}_t / M_t \right) - \pi_t = \psi - \pi_t$. Combining the binding CIA constraint and the goods market clearing condition yields $\dot{m}_t / m_t = \dot{k}_t / k_t = g_2$. Therefore, $\pi_t + g_2 = \psi$. We now have

$$g_2 = \frac{A}{1 + \rho + \psi - s} - \rho, \tag{80}$$

which is (38) in the main text. Now we rewrite (39) as $(1 + \rho + \psi)(g_2 + \rho) - \frac{\psi A}{g_2}(g_2 + \rho) = A$. Total differentiating this equation with respect to the nominal interest rate, we have

$$g_2' = \frac{A (g_2 + \rho) - g_2 (g_2 + \rho)}{(1 + \rho + \psi) g_2 + \frac{\rho \psi A}{g_2}}. \tag{81}$$

Using (39), we have $g_2 (g_2 + \rho) = \frac{A g_2 + \psi A g_2 + \rho \psi A}{1 + \rho + \psi}$. Now we have $A (g_2 + \rho) - g_2 (g_2 + \rho) = \frac{\rho A + \rho^2 A + \rho A g_2}{1 + \rho + \psi}$. Plugging this condition into (81) yields

$$g_2' = \frac{\rho A + \rho^2 A + \rho A g_2}{(1 + \rho + \psi)^2 g_2 + (1 + \rho + \psi) \frac{\rho \psi A}{g_2}} \begin{cases} > 0 \text{ if } g_2 > 0, \\ < 0 \text{ if } -(1 + \rho) < g_2 < 0. \end{cases} \tag{82}$$

APPENDIX V: STEPS OF QUANTITATIVE ANALYSIS (NOT FOR PUBLICATION)

We use Maple 16 to conduct the calibration for an elastic labor supply case.

Step 1. Calibrating the parameters

The Maple program is as follows:

```
eq1:=(gamma-1)*lx+beta*(i-rho)*xi*gamma*lx-(lr+rho/phi)*(1+i);
eq2:=lr+lx-1+theta*(1+i)*gamma*lx*(1-beta*(i-rho)*xi);
eq3:=xi*(1+beta*(i-rho))-1-lr/gamma/lx;
sols1:= solve({eq1,eq2,eq3}, {lx,lr,xi});
lx1:=subs(sols1,lx);
p1:=subs(sols1,xi);
lr:=subs(sols1,lr);
f1:=(gamma-1)/gamma/beta/(i-rho)/(1+lr/gamma/lx1)*(1+beta*(i-rho))-282570/26554;
g := phi*ln(gamma)*lr;
l := 1-theta*(1+i)*gamma*lx1*(1-beta*(i-rho)*p1);
g1 := eval(g, [gamma = 1.05, rho = 0.4e-1, i = 0.96e-1]);
l1:=eval(l,[gamma = 1.05, rho = 0.4e-1,i = 0.96e-1]);
f11 := eval(f1, [gamma = 1.05, rho = 0.4e-1, i = 0.96e-1]);
sols2 := solve({f11, g1 = 0.2e-1, l1 = .3}, {phi, beta, theta})
```

Step 2. Welfare analysis

```
u1 := (ln((1-beta*(i-rho)*p1)*lx1)+phi*lr*ln(gamma)/rho+theta*ln(1-lr-lx1))/rho;
equ1 := eval(u, [gamma = 1.05, rho = 0.4e-1, beta = 0.8e-1, phi = 31.41654624, theta = 4, i = 0.8691102130e-1]);
u := (ln((1-beta*(i-rho)*p1)*lx1*(1+x))+phi*lr*ln(gamma)/rho+theta*ln(1-lr-lx1))/rho;
equ := eval(u1, [gamma = 1.05, rho = 0.4e-1, beta = 0.8e-1, phi = 31.41654624, theta = 4, i = 0.96e-1]);
solve(equ = -56.49537768, {x});
```

Step 3. Drawing the figures

We use Figure 3 as an example. The Maple program is as follows:

```
g100 := eval(g, [gamma = 1.05, rho = 0.4e-1, beta = 0., phi = 31.41654624, theta = 2.129298312]);
g101 := eval(g, [gamma = 1.05, rho = 0.4e-1, beta = 0.8e-1, phi = 31.41654624, theta = 2.129298312]);
g102 := eval(g, [gamma = 1.05, rho = 0.4e-1, beta = 0.15, phi = 31.41654624, theta = 2.129298312]);
g103 := eval(g, [gamma = 1.05, rho = 0.4e-1, beta = .2, phi = 31.41654624, theta = 2.129298312]);
plot1 := plot(g100, i = 0 .. 3, color = red, axes = boxed, labels = [nominal*interest*rate, growth*rate], labeldirections = [horizontal, vertical], tickmarks = [10, 6], view = [0 .. 3, 0 .. 0.4e-1]);
plot2 := plot(g101, i = 0 .. 3, color = blue, axes = boxed, labels = [nominal*interest*rate, growth*rate], labeldirections = [horizontal, vertical], tickmarks = [10, 6], view = [0 .. 3, 0 .. 0.4e-1]);
plot3 := plot(g102, i = 0 .. 3, color = orange, axes = boxed, labels = [nominal*interest*rate, growth*rate], labeldirections = [horizontal, vertical], tickmarks = [10, 6], view = [0 .. 3, 0 .. 0.4e-1]);
plot4 := plot(g103, i = 0 .. 3, axes = boxed, labels = [nominal*interest*rate, growth*rate], labeldirections = [horizontal, vertical], tickmarks = [10, 6], view = [0 .. 3, 0 .. 0.4e-1]);
plots[display]([plot1, plot2, plot3, plot4]);
```

APPENDIX VI: STEPS FOR GENERATING THE CROSS-COUNTRY PANEL DATA (NOT FOR PUBLICATION)

Step 1. Generating the yearly panel data for 182 countries during 1950-2014 using PWT 9.0

We first access www.ggdc.net/pwt (redirect to https://www.rug.nl/ggdc/productivity/pwt/). Doing so gives us the most complete cross-country yearly panel data by Penn World Table (PWT) 9.0 (explained by Feenstra, Inklaar and Timmer 2015). The webpage states: "PWT version 9.0 is a database with information on relative levels of income, output, input and productivity, covering 182 countries between 1950 and 2014."

We first download the PWT 9.0 data in excel format. We then download the "Program package"—"The Stata data files and do-files necessary to replicate and customize PWT", which delivers programs90.zip. We unzip the file to find the gen_final_pwt.do file. From the Stata13 "file" menu, choose "import" "Excel spreadsheet" and open the downloaded excel data (remember to choose "import first row as variable names"). Now run the gen_final_pwt.do file, and we will get pwt90_output.dta (the yearly panel data for 182 countries during 1950-2014).

Step 2. Generating the five-year average panel data for 182 countries during 1950-2014

We then run the following do file to get the five-year average panel data:

gen rgdpl=rgdpna/emp // using the $RGDP^{NA}$ series, as discussed in the main text
gen growth=(log(rgdpl[_n])-log(rgdpl[_n-4]))/4 // calculating annual growth for each five-year interval
gen inirgdpl=rgdpl[_n-4] // initial real GDP per employment
gen lninirgdpl=log(inirgdpl)
gen emp_g=(log(emp[_n])-log(emp[_n-4]))/4 // calculating annual employment growth
gen labor=emp_g+0.05
gen lnlabor=log(labor*100)
gen v=year-1950
gen v1=int(v/5) // to get five year intervals
egen group=group(country v1) // to get five year intervals for each country
by group, sort: egen pop_mean= mean(pop) // calculating five-year averages
by group, sort: egen hc_mean= mean(hc)
gen lnhc=log(hc_mean)
by group, sort: egen csh_i_mean= mean(csh_i)
gen lncsh_i=log(csh_i_mean*100)
by group, sort: egen csh_g_mean= mean(csh_g)
gen lncsh_g=log(csh_g_mean*100)
by group, sort: egen csh_x_mean= mean(csh_x)
gen lncsh_x=log(csh_x_mean*100)
by group, sort: egen csh_m_mean= mean(csh_m)

```
gen lncsh_m=log((-csh_m_mean)*100)
gen lntrade=log((csh_x_mean-csh_m_mean)*100)
sort country year
save "E:\PWT_2018\my_pwt90_001.dta", replace //generating my own PWT 9.0 dta
preserve
gen vv=year-1949
keep if mod(vv,5)==0 //keeping only the necessary data points
save "E:\PWT_2018\my_pwt90_002.dta", replace
```

Step 3. Merging the other variables into my PWT data

Step 3.1. Generating any interested variable (e.g., CPI) using the WDI

For the CPI (consumer price index) inflation data, we first access the World Development Indicators (WDI) of the World Bank to download the excel files. The CPI data covers the period 1960-2017 (it is in the country by year format). We first use the following long.do program to turn it into a variable with the same format as in my PWT 9.0 data (the stata dta file). To do so, we need to search in Stata and install “dm88_1” in order to use the “renvars” command. The stata do file is as follows:

```
*deleting the first three rows of the data spreadsheet in the downloaded excel file of the WDI. Importing the data with the first row as variable names
renvars E-BJ \ var1960-var2017
reshape long var, i(CountryName) j(year)
rename var cpi_pi
drop if year>2014
gen v=year-1960
gen v1=int(v/5)
egen group=group(CountryName v1)
by group, sort: egen cpi_pi_m= mean(cpi_pi)
rename CountryName country
rename CountryCode countrycode
sort country year
gen vv=year-1959
keep if mod(vv,5)==0
save "E:\PWT_2018\cpi_pi.dta", replace
```

Step 3.2. Merging the interested variable (e.g., CPI) into my PWT data

Now we merge the interested variable into my PWT dta data. The stata do file is as follows:

```
use "E:\PWT_2018\my_pwt90_002.dta", clear
sort country year
```

```
merge country year using "E:\PWT_2018\cpi_pi.dta"
keep if _merge==3
save "E:\PWT_2018\my_pwt90_final.dta", replace
```

Repeating steps 1-3 can build any cross-country panel data using the PWT 9.0 and the WDI. If there is no data needed from the PWT 9.0, one can start from and repeat step 3 to generate cross-country panel data. Yearly cross-country panel data can be generated similarly.

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Table 3: Descriptive statistics

	Observations	Mean	Standard deviation	Minimum	Maximum
growth (%)	1168	1.23	4.33	-30.08	30.59
π_{CPI} *(%)*	1094	44.71	339.73	-23.82	8603.28
$M2g$ *(%)*	1022	57.77	696.37	-71.22	21671.69
$\ln(RGDP/emp)_{t-1}$	1174	9.88	1.22	6.22	13.32
$\ln(hc)$	1068	0.72	0.35	0.01	1.31
$\ln(csh\_i)$	1303	2.96	0.58	0.48	5.82
$\ln(Labor)$	1156	1.92	0.37	-1.29	3.25
$\ln(csh\_g)$	1303	2.90	0.50	0.86	5.20
$\ln(Trade)$	1303	3.63	0.95	-1.75	6.98

Note: The growth data are from the PWT 9.0, covering 154 countries during
1970-2014. We take five-year averages to avoid the influence from business cycles.
growth is annual growth of real GDP per employment (in percentage term).
π_{CPI} is the CPI inflation rate of the WDI from the World Bank (WB) (in percentage term).
$M2g$ is the monetary growth rate of the WDI from the WB (in percentage term).
RGDP/emp is real GDP per employment (in 2011 us$). *hc* measures human capital.
csh_ i is the investment rate. *Labor* is the employment growth plus 0.05.
csh_ g is the ratio of government spending to GDP.
Trade is the sum of exports and imports as a share of GDP.
csh_ i, csh_ g, Labor and Trade are multiplied by 100 before taking logarithms.

Table 4. LSDV Regressions Results (five-year non-overlapping average during 1970-2014)
Dependent variable: average annual growth of real GDP per employment

	Regression number				
	4.1	4.2	4.3	4.4	4.5
Indep. Variable		Full Sample			$\pi_{CPI} < 30$
π_{CPI}		-0.001	-0.0046^{***}	-0.05^{**}	-0.34^{***}
		(0.0008)	(0.0013)	(0.03)	(0.11)
π^2_{CPI}			$5.5 \times 10^{-7***}$		-0.0043^{***}
			(1.6×10^{-7})		(0.0016)
$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$	-6.25^{***}	-5.77^{***}	-5.77^{***}	-5.00^{***}	-5.04^{***}
	(0.72)	(0.65)	(0.65)	(0.61)	(0.61)
$\ln(csh\_i)$	2.06^{***}	1.59^{***}	1.59^{***}	2.42^{***}	2.06^{***}
	(0.43)	(0.42)	(0.42)	(0.43)	(0.43)
$\ln(hc)$	-3.18^{*}	-1.85	-1.85	-3.06^{*}	-2.90^{*}
	(1.79)	(1.81)	(1.81)	(1.77)	(1.75)
$\ln(Labor)$	-0.59	-1.36^{**}	-1.36^{**}	-1.83^{***}	-1.82^{***}
	(0.65)	(0.61)	(0.61)	(0.69)	(0.69)
$\ln(csh\_g)$	-0.20	-0.54	-0.54	-0.52	-0.55
	(0.52)	(0.48)	(0.48)	(0.46)	(0.45)
$\ln(Trade)$	2.02^{***}	3.03^{***}	3.03^{***}	1.68^{***}	1.68^{***}
	(0.45)	(0.58)	(0.58)	(0.44)	(0.44)
Country fixed effects	YES	YES	YES	YES	YES
Time fixed effects	YES	YES	YES	YES	YES
F-test on π, π^2					F(2, 711)=4.83
(p-value)					(0.0083)
π turning point					-39.5 [5.19]
sample range					(-69, -15) [(-24, 29)]
R^2	0.40	0.45	0.47	0.51	0.52
Observations	1013	935	935	851	851

Note: *growth* is annual growth of real GDP per employment (in percentage term).
π_{CPI} is the CPI inflation rate of the WDI from the WB (in percentage term). For the turning points, the first number is the centered inflation, while the numbers in brackets are the original values.
RGDP/emp is real GDP per employment (in 2011 us$). *hc* measures human capital.
csh_i is the investment rate. *Labor* is the employment growth plus *0.05*. *csh_g* is the ratio of government spending to GDP. *Trade* is the sum of exports and imports as a share of GDP.
***Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level
(robust standard errors in parentheses)

Table 5. IV Regressions Results (First-stage results)

Sample: five-year non-overlapping average during 1970-2014 with annual inflation below 30%

	Regression number			
	5.1	5.2	5.3	5.4
	First-stage dependent variable as			
	π_{CPI}	π^2_{CPI}	π_{CPI}	π^2_{CPI}
F test of excluded instruments (p-value)	F(20,176)=7.83 (0.00)	F(20,176)=10.46 (0.00)	F(7,35)=7977 (0.00)	F(7, 35)=507 (0.00)
Observations	289	289	352	352

Note: Instruments: 5.1-5.2: up to fifth lags of inflation, inflation square, monetary growth rate and its square (20 excluded instruments); 5.3-5.4: the average inflation rate of the other countries, its square, up to fifth lags of the average inflation rate of the other countries (7 excluded instruments).

Table 6. IV Regressions Results (two-step GMM)

Second-stage results. Dependent variable: average annual growth of real GDP per employment

Sample: five-year non-overlapping average during 1970-2014 with annual inflation below 30%

	Regression number			
	6.1	6.2	6.3	6.4
Indep. Variable	Correct for heteroskedasticity		Correct for heteroskedasticity and autocorrelation	
π_{CPI}	-1.51^{***}	-2.88^{*}	-1.35^{***}	-3.04^{**}
	(0.49)	(1.52)	(0.49)	(1.34)
π_{CPI}^2	-0.018^{***}	-0.041^{*}	-0.016^{**}	-0.044^{**}
	(0.007)	(0.023)	(0.007)	(0.020)
$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$	-7.22^{***}	-8.13^{***}	-7.10^{***}	-8.30^{***}
	(1.29)	(1.30)	(1.28)	(1.36)
$\ln(csh\_i)$	1.67^{**}	1.34	1.89^{***}	1.09
	(0.69)	(1.08)	(0.66)	(1.00)
$\ln(hc)$	2.52	3.71	3.31	4.18
	(3.13)	(4.26)	(3.09)	(4.02)
$\ln(Labor)$	-3.30^{***}	-2.75^{***}	-3.37^{***}	-2.95^{***}
	(0.90)	(0.64)	(0.82)	(0.69)
$\ln(csh\_g)$	-2.48^{***}	-1.12^{*}	-2.24^{***}	-0.92
	(0.62)	(0.63)	(0.56)	(0.58)
$\ln(Trade)$	3.22^{***}	2.03^{**}	2.74^{***}	1.94^{**}
	(0.75)	(0.83)	(0.72)	(0.78)
Country fixed effects	YES	YES	YES	YES
Time fixed effects	YES	YES	YES	YES
Hansen J test	17.23	9.10	12.63	9.44
(p-value)	(0.51)	(0.11)	(0.81)	(0.09)
Test on π, π^2	chi2(2)=18.38	chi2(2)=15.10	chi2(2)=17.45	chi2(2)=13.58
(p-value)	(0.0001)	(0.0005)	(0.0002)	(0.0011)
π turning point	-41.9 [2.78]	-35.1 [9.6]	-42.2 [2.52]	-34.5 [10.2]
sample range	(-69,-15) [(-24,29)]	(-69,-15) [(-24,29)]	(-69,-15) [(-24,29)]	(-69,-15) [(-24,29)]
R^2	0.33	0.25	0.34	0.23
Observations	289	352	289	352

Note: Endogenous variables: inflation and its square. Instruments: 6.1, 6.3: up to fifth lags of inflation and its square, monetary growth rate and its square; 6.2, 6.4: the average inflation rate of the other countries, its square, up to fifth lags of the average inflation rate of the other countries.

***Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level

(robust standard errors in parentheses)

Table 7: Descriptive statistics

	Observations	Mean	Standard deviation	Minimum	Maximum
$R\&D\_S$	279	6.59	5.57	0.04	22.14
π_{CPI} (%)	657	8.77	20.12	-10.07	325.00
	279	7.21	11.60	-10.07	85.74
$M2g$ (%)	667	181.09	4204.92	-99.88	108613.3
	277	16.46	16.61	-25.55	125.03
FD/GDP	675	51.84	47.18	1.27	312.12
ln(*FD/GDP*)	675	3.50	1.04	0.24	5.74
CBI	668	0.45	0.20	0.1	0.79
$\ln(RGDP/emp)_{t-1}$	676	10.07	1.07	6.73	12.31
$\ln(hc)$	676	0.90	0.25	0.11	1.30
$\ln(Trade)$	676	3.85	0.65	2.06	6.13

Note: The growth data are from the PWT 9.0, covering 52 countries during 1998-2010 (yearly panel data).
$R\&D\_S$ measures l_r (i.e., the share of R&D labor in total labor supply), which is measured by "Total R&D personnel per thousand employment (FTE)" from the UNESCO Institute for Statistics (http://data.uis.unesco.org/index.aspx?queryid=63#).
π_{CPI} is the CPI inflation rate of the WDI from the WB (in percentage term).
$M2g$ is the monetary growth rate of the WDI from the WB (in percentage term).
FD/GDP is financial depth from the WDI (in percentage term). *CBI* is degree of central bank independence (ranges from 0 to 1) from Dincer and Eichengreen (2014).
RGDP/emp is real GDP per employment (in 2011 us$). *hc* measures human capital.
$Trade$ is the sum of exports and imports as a share of GDP. We also reported the summary statistics for π_{CPI} and $M2g$ when $R\&D\_S$ has observations.

Table 8. LSDV Regressions Results (yearly panel data during 1998-2010)
Dependent variable: R&D employment share $R\&D\_S$

	Regression number			
Indep. Variable	8.1	8.2	8.3	8.4
π_{CPI}	0.030^{**}	-0.001	0.030^{**}	0.030^{*}
	(0.014)	(0.0008)	(0.014)	(0.016)
π^2_{CPI}			-0.0002	-0.0002
			(0.0002)	(0.0003)
CBI			3.77^{***}	3.77^{***}
			(0.76)	(0.65)
$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$	-6.25^{***}	-5.77^{***}	0.29	0.29
	(0.72)	(0.65)	(0.71)	(0.70)
$\ln(hc)$	-3.18^{*}	-1.85	4.85	4.85
	(1.79)	(1.81)	(2.96)	(2.90)
$\ln(Trade)$	2.02^{***}	3.03^{***}	-2.20^{***}	-2.20^{***}
	(0.45)	(0.58)	(0.41)	(0.44)
Country fixed effects	YES	YES	YES	YES
Time fixed effects	YES	YES	YES	YES
F-test on π, π^2			F(2, 222)=4.99	F(2, 222)=3.87
(p-value)			(0.0076)	(0.0222)
π turning point			85.96	85.96
sample range			[-10.07, 85.74]	[-10.07, 85.74]
R^2	0.99	0.99	0.99	0.99
Observations	278	278	278	278

Note: $R\&D\_S$ is measured by "Total R&D personnel per thousand employment (FTE)".
π_{CPI} is the CPI inflation rate of the WDI from the WB (in percentage term).
CBI is the degree of central bank independence.
RGDP/emp is real GDP per employment (in 2011 us$). hc measures human capital.
$Trade$ is the sum of exports and imports as a share of GDP.
***Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level
(8.1, 8.3 standard errors in parentheses) (8.2, 8.4 robust standard errors in parentheses)

Table 9. 2SLS Regressions Results (yearly panel data during 1998-2010)

	Regression number				
	9.1	9.2	9.3	9.4	9.5
	First-stage results		Second-stage results		
	Dependent variable as				
Indep. Variable	π_{CPI}	π_{CPI}^2	$R\&D\_S$	$R\&D\_S$	$R\&D\_S$
π_{CPI}			0.222^{***}	0.222^{***}	0.232^{***}
			(0.065)	(0.077)	(0.085)
π_{CPI}^2			-0.0026^{***}	-0.0026^{**}	-0.0028^{**}
			(0.00096)	(0.0011)	(0.0012)
$M2g$	0.23^{***}	18.71^{***}			
	(0.04)	(3.17)			
FD/GDP	0.10^{***}	3.87			
	(0.03)	(2.77)			
$\ln(FD/GDP)$	-10.06^{***}	-384.41^{*}			
	(2.60)	(210.95)			
CBI	-14.36^{*}	-744.40	4.58^{***}	4.58^{***}	4.61^{***}
	(8.21)	(664.82)	(0.97)	(0.97)	(1.01)
$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$	8.55	554.05	0.80	0.80	0.83
	(7.65)	(620.07)	(0.89)	(1.04)	(1.08)
$\ln(hc)$	-45.16	-4819.85^{*}	0.57	0.57	0.23
	(31.06)	(2516.24)	(4.29)	(3.74)	(3.93)
$\ln(Trade)$	-6.83	-550.07	-1.75^{***}	-1.75^{***}	-1.74^{***}
	(4.51)	(365.62)	(0.54)	(0.60)	(0.61)
Country fixed effects	YES	YES	YES	YES	YES
Time fixed effects	YES	YES	YES	YES	YES
F test of excluded	F(3,219)=21.79	F(3,219)=15.44			
instruments (p-value)	(0.0000)	(0.0000)			
Sargan test (p-value)			0.3434	0.3597	
π turning point			41.86	41.86	41.41
sample range			[-10.1, 85.7]	[-10.1, 85.7]	[-10.1, 85.7]
R^2	0.66	0.49	0.97	0.97	0.97
Observations	276	276	276	276	276

Note: Endogenous variables: inflation and its square. Instruments: M2g, FD/GDP, and $\ln(FD/GDP)$.

9.5 used LIML estimation to deal with weak instruments.

***Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level

(9.1-9.3 standard errors in parentheses) (9.4, 9.5 robust standard errors in parentheses)

Table 10. IV Regressions Results between Inflation and Physical Capital Investment
Second-stage results (yearly panel data during 1998-2010)

	Regression number			
	10.1	10.2	10.3	10.4
	Estimation methods			
	2SLS	LIML	2SLS	LIML
	Dependent variable as			
Indep. Variable	$\ln(csh\_i)$	$\ln(csh\_i)$	csh_i	csh_i
π_{CPI}	-0.023^{**}	-0.065	-0.001	-0.013
	(0.011)	(0.076)	(0.003)	(4.03)
π^2_{CPI}	0.00033^{**}	0.001	0.00003	0.002
	(0.00016)	(0.0012)	(0.00004)	(0.062)
CBI	0.12	0.05	0.02	-0.18
	(0.18)	(0.38)	(0.05)	(6.65)
$\ln\left(\frac{RGDP}{emp}\right)_{t-1}$	0.13	0.03	0.14^{***}	-0.16
	(0.15)	(0.35)	(0.03)	(9.60)
$\ln(hc)$	0.23	1.71	0.17	4.77
	(0.71)	(2.60)	(0.13)	(144.58)
$\ln(Trade)$	0.58^{***}	0.58^{***}	0.10^{***}	0.12
	(0.12)	(0.18)	(0.03)	(0.67)
Country fixed effects	YES	YES	YES	YES
Time fixed effects	YES	YES	YES	YES
Over-ID test (p-value)	0.0001		0.0039	
R^2	0.66	0.19	0.77	.
Observations	276	276	276	276

Note: Endogenous variables: inflation and its square. Instruments: M2g, FD/GDP, and $\ln(FD/GDP)$. Over-ID test means over-identification test.
***Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level
(robust standard errors in parentheses)