# Modelling Asset Correlations of Revolving Loan Defaults in South Africa

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## Abstract
This paper examines the extraction of the empirical asset correlation for three datasets of monthly defaults on loans and credit cards obtained from the SARB from February 2006 to January 2017. The study makes use of the Beta and Vasicek distributions over a static period of time, as well as a rolling period of time. However two different calculation approaches (mode and percentile) are used for the Vasicek distribution assumption. We first use these three distinct calculation approaches to empirically estimate the asset correlation over a static period of time and compare them to the BCBS (Basel Committee for Bank Supervision) prescribed asset correlations. The computed empirical asset correlations are thereafter used to determine the economic capital and compare it to the economic capital determined using the BCBS prescribed asset correlations. Secondly, we use these three distinct calculation approaches to empirically estimate the asset correlation over a rolling five-year period and compare them to the BCBS' prescribed asset correlations. For both the static and five-year rolling empirical asset correlations, we show that the BCBS' prescribed asset correlations are much higher than the empirical asset correlations for the South African loans dataset. However, the opposite is found for both the credit card default and writeoff datasets which had higher empirical asset correlations. The economic capital charge calculated using the computed empirical asset correlations is lower than the economic capital calculated using the BCBS' prescribed asset correlations for the South African loans dataset, while the opposite result is found for both the credit card default and write-off datasets. This result implies that the BCBS' prescribed asset correlation is not as conservative as intended for South African bank specific credit cards and that the required capital charge stipulated by the BCBS is not sufficient to cover unexpected losses. This may have dire consequences to the South African banking system through systemic risk. Therefore, we recommend that the capital levels be raised to match the capital levels determined in this study.

**Keywords:** asset correlation, Vasicek distribution, Beta distribution, BCBS, economic capital, credit card defaults

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## Introduction

The banking sector's regulations in relation to bank capital and risks related to banking have been a massive driver of the evolution of risk management, substantially transforming banking regulation from being a completely new concept in the 70's to a globally defined process in the modern finance environment. The determination of specifically regulatory and economic capital is crucial for banks, especially after the 2008 credit crisis. Therefore, banks must ensure that they have a sufficient amount of capital reserved to maintain liquidity for their daily risk-taking activities. This is especially the case for a country such as South Africa which has had thirteen banks put under curatorship in the past thirty years, mainly due to liquidity issues (Tjiane & van Heerden, 2015; and Chummun, 2017).

The latest bank to experience such a fate was the VBS Mutual Bank which fell to its demise due to management failure to align the business with its sudden rate of growth. The VBS balance sheet grew from about R200 million to R2 billion when they started accepting deposits from municipalities, which were used to fund long term loans. They started experiencing liquidity issues when municipalities started withdrawing their deposits due to the Municipal Finance Management Act (2003) (MFMA, 2004) barring them from banking with a mutual bank (National Treasury, 2018).

For financial institutions to remain afloat, they must have sufficient liquid assets to honour their near-term (usually thirty days) obligations without requiring refunding from central banks (Martin, 2013). Thus, banks need to ensure that they keep regulatory capital that is liquid enough to be able to absorb short-term shocks that may be caused by withdrawals of deposits that are the most prone to bank runs (Nyaundi, 2015). Liquidity is a significant indication of a bank's health and an indirect indication of the economic health of a country, thus, the Basel Committee on Banking Supervision (BCBS) formulated three accords to improve the soundness and stability of the global banking system through capital regulations. The in-country banking regulators oversee the regulation process and ensure that the minimum capital requirements are maintained – in the case of South Africa, the South African Reserve Bank (SARB) is the regulator in charge. These Basel accords were designed to act as a global framework for integrating regulatory and supervisory processes of the countries which embrace their principles (Bieri, 2008).

The Basel I Accord was published and officially made available to all banks in July 1988 following a consultative paper that was published for comments in December 1987. It was approved by the Group of ten (G-10) central bank governors and was formally implemented in all internationally active banks by end of December 1992 (Gehrig & Iannino, 2017). These set of rules were aimed at regulating credit activity of banks and set out a common standard of measuring the capital adequacy and the minimum standards to be applied by all global banks (BIS, 2001). The Basel I Accord introduced a new concept for the capital requirement: the Cooke ratio that expressed minimum sum of capital as a percentage of its risk weighted assets (RWA) that a bank should allocate for its credit activity (BCBS, 1988). As the first framework to introduce global credit risk regulation, the Basel I Accord was far from perfect, and thus had its limitations as expected. As this was a living document that had to be applied to banks with an ever-changing level of risk and technology at their disposal, it makes sense that it also needed to evolve with time.

This spurred the Basel committee to formulate a revised capital framework in June 2006, called the Basel II accord which not only aimed at enhancing banks' risk management capabilities but to also improve their risk reporting using the three-pillar approach. These three pillars provide guidance on the minimum capital requirements standards, processes related to the supervisory review, and the data disclosure requirements based on market discipline (Santos, 2001; and ). The Basel II accord also gave banks the option of using either the standardised approach (based on risk weights specified by the BCBS for loan exposures) or Internal ratings-based (IRB) approach (based on their own estimates of key risk parameters and BCBS specified capital requirement formulas) (Resti, 2016). The IRB approach makes use of, amongst others, three main inputs into the capital calculation, which are the loss given default (LGD), exposure at default (EAD), and the probability of default (PD). Banks that employ the IRB approach have the freedom to estimate their own risk parameters in the calculation of capital. These methods are mainly based on researched and established credit risk management concepts which have been deeply scrutinised by the market to ensure a proper evaluation of its applicability (see Gup 2003; Lastra 2004; Chummun, and Bisschoff, 2014; and Nachane et al. 2005). In essence, the IRB approach provides us with a capital framework that is sophisticated, user-friendly, and much more meaningful, relevant, and accurate than that provided in the Basel I Accord (Stoffberg & van Vuuren 2015).

The improvements brought on by the introduction of Basel II (2006) were short-lived and could not be evaluated properly due to a collapse of the financial system and economic slowdown that occurred in the period 2007-2009 (van Dyk, 2018). The failures observed during this period were due to banks excessively growing their balance sheets by taking on a significant number of derivative products while they did not have sufficient liquidity buffers in place. In response to these shortcomings, the BCBS started a process to update the Basel II Accord to ensure protection of the financial and banking system so that it does not experience a repeat of the 2008 financial crisis. The updated Basel framework is an extension of Basel II and serves to strengthen and expand on the already established Basel II pillars. It is meant to build on Basel II and the lessons learned from the "sub-prime" crisis (BIS, 2018) through the introduction of two liquidity risk supervision standards: a standard to provide guidance for short-term liquidity (liquidity coverage ratio) and long-term liquidity (net stable funding ratio). The definition of capital was also strengthened through higher minimum capital requirements for banks (van Dyk, 2018). The BCBS also introduced a countercyclical buffer to mitigate procyclicality in the regulatory capital framework (Martin, 2013). It began to be phased in from 2016 and is projected to be completely implemented by 2019 (van Dyk, 2018).

The IRB credit risk capital methodology introduced by the Basel II framework for credit exposures is based on the asymptotic single-risk factor (ASRF) approach that provides an uncomplicated, closed form analytical solution which is relatively effortless to calculate (Vasicek 1987, 1991). The BCBS states that their usage of the ASRF in the capital calculations is not owing to any preference towards one model or the other but it was selected because of its suitability for this portfolio type. Banks are, however, encouraged to employ credit models that fit the best for their credit models (BCBS, 2005). This single systematic factor reflects the state of the global or local economy. The degree of the customer's exposure to this systematic risk factor is expressed by the asset correlation, which gives a measure of how the asset value of each obligor depends on the asset value of another obligor (Martin, 2013). Alternatively, the asset correlation may be described as a measure of the dependence between the borrowers' asset value and the general state of the economy (local or global). All borrowers are interconnected to each other through this single systematic risk factor (BCBS, 2005). The asset correlation is impossible to observe directly, and is often very difficult to estimate due to a lack of historical data. In the IRB approach, the shape of the risk-weighted formulas is determined by the asset correlations. As different obligors show different degrees of dependence to the economy, the asset correlations are also different for each asset class (see Table 1).

**Table 1: Asset correlations descriptions**

| Asset class | Correlation |
|---|---|
| Mortgages | Fixed (15 Percent) |
| Qualifying Revolving | Fixed (4 Percent) |
| Other Retail | Varies with PD $0.03.\left(\frac{1-e^{(-35PD)}}{1-e^{(-35)}}\right) - 0.16.[1 - \left(\frac{1-e^{(-35PD)}}{1-e^{(-35)}}\right)$ |
| High Volatility CRE | Varies with PD $0.12.\left(\frac{1-e^{(-50PD)}}{1-e^{(-50)}}\right) - 0.30.[1 - \left(\frac{1-e^{(-50PD)}}{1-e^{(-50)}}\right)$ |

All banks use the different levels of asset correlations in the capital calculations despite the knowledge that these asset correlation values were arbitrarily chosen and may not necessarily match the correlation levels that are suitable for their own credit portfolios (Martin, 2013). This is because banks must comply with the regulatory rules as specified by the BCBS. However, techniques that can be used to estimate empirical asset correlations from loan loss data, which can subsequently be used to measure a fair level of economic capital are of considerable interest. It is the BCBS' aim to ensure that banks closely align the regulatory capital that is required to keep them afloat with the amount of capital that banks themselves believe they should be keeping (i.e. the economic capital) (Smit & van Vuuren, 2009).

Currently, banks that are sophisticated enough can determine their own economic capital independent of the BCBS' guidance, while smaller banks that lack such sophistication still heavily rely on the BCBS for guidance, as they lack the quantitative resources and systems needed to determine their own economic capital. Economic capital is different from the regulatory as it is solely dependent on a bank's internal determination (or estimation) of the capital they need and would be defined as the capital amount that a bank would keep if there were no capital regulations. Regulatory capital is the amount of capital prescribed by a regulator that an institution needs to hold to maintain an adequate level of liquidity and solvency (Lang, 2009).

Botha and van Vuuren (2010) proposed and tested a method of extracting asset correlations using a Vasicek and Beta distribution of gross loan loss data from international markets, while Stoffberg and van Vuuren (2015) applied a Vasicek distribution of gross loan loss data from both international and South African markets. The main problem is that the technique has never been applied to a South African bank specific loan loss dataset. This technique has also never been applied from a beta distribution of loan loss data from South African market and South African bank-specific data. This investigation is particularly important for South Africa as a developing economy which has quite a history with liquidity issues and failed banks, having until today (December 2018) had thirteen banks put under curatorship in the past thirty years (Tjiane & van Heerden, 2015). This warrants an investigation into the relevance of the prescribed asset correlations in the South African environment especially in bank specific loss data.

The bank specific product in focus for this study is Credit Cards. The Experian Consumer Default Index (Experian, 2017) revealed that 66.48% of South African accounts in debt are credit facilities, of which credit card debt is the highest contributor. This could be due to easy access to credit and over-borrowing. The total contribution of credit cards to the South African GDP is 7.8 billion USD (Moody's Analytics, 2013), which is not too large when compared to the South African GDP but may collapse the banking system if all consumers could default on their credit. Therefore, it is wise for banks to consider a forward-looking approach and estimate their own empirical asset correlations using their unique internal loss data, which reflects the South African credit experience. This empirical asset correlation will be used to determine a fair level of economic capital, thus minimising the impact they may experience due to unexpected losses arising from credit card defaults.

This study seeks to assist South African banks to minimise the impact they may experience due to unexpected losses that may arise from credit card defaults. There is no published study that has reviewed bank specific asset correlations for South African credit cards, which is one of the aims of this study. Empirically estimating the asset correlations for a South African bank specific credit cards default and write-off data and comparing them to the Basel prescribed asset correlations is the first contribution made by this study. The computed empirical correlations are used in the calculation of economic capital. The second contribution is applying the Beta distribution to both South African commercial loans and bank-specific credit cards in the estimation of the empirical asset correlations. The final contribution is exploring South African bank-specific empirical asset correlations over a rolling period to assess it over different economic conditions.

The next section provides an outline of the literature related to the empirical asset correlation. The paper goes on to provide an outline of the Beta and Vasicek methodologies used in the empirical estimation of asset correlations and its usage in the calculation of economic capital. The section that follows provides the data analysis and results based on the Vasicek and Beta formulation. The final section provides the conclusion.

**Literature Review**

Research studies on the asset correlations for credit risky portfolios have been few and far between, with research largely focused on corporate loans. Examples of such studies include Bystrom (2011) and Lee et al. (2011). Research exploring asset correlations using loan loss data has however been very scarce, and thus necessitating further review. Botha and van Vuuren (2010) extracted empirical asset correlations for 100 of the largest United States of America (USA) banks and compared them to the BCBS-prescribed asset correlations. They employed both the Vasicek and beta distribution and introduced techniques to reverse-engineer asset correlations of gross loan losses from US based quarterly reports for the periods 1985 - 2009. Their findings show that the empirical asset correlations calculated using gross loan loss data are much lower than the Basel prescribed asset correlations. This implies that the asset correlations provided by the BCBS contain a certain level of conservatism. They also explored ways in which empirical asset correlations change over time. This study is beneficial to banks who are interested in calculating empirical asset correlations using their own internal data and subsequently using them to determine their own fair level of economic capital. Their research essentially introduced simple techniques that can be used to empirically estimate the asset correlations using only gross loan loss data. Such studies are of great benefit to banks that use the advanced IRB approach and want to determine their own internal measure of the asset correlation for the economic capital calculation.

Further investigation into the fairness of empirical asset correlations was done by Stoffberg and van Vuuren (2015), whose study updated the USA correlation data used in Botha and van Vuuren (2010). These authors compared these empirical asset correlations to those experienced using South African (SA) loan loss dataset for the period 1985-2014. However, they only employed the Vasicek distribution and found that both a percentile approach and mode approach are needed to fully evaluate the empirical data and BCBS' intended conservatism, as both those approaches closely fit the data when looking at their density and cumulative functions. They found similar results to Botha and van Vuuren (2010), with the main exception being the empirical asset correlation of the financing of agricultural production, which was higher than the Basel specified asset correlation for the 99.9th percentile approach, which would require a higher capital charge when compared to the capital charge produced when using the Basel specified asset correlation, which is not sufficient enough to cover all capital requirements. The other exception, was the "Qualifying revolving" asset class, mainly credit cards, which also had a higher empirical asset correlation, thus requiring a higher capital charge than the charge provided using the Basel specified asset correlations.

The result on the Qualifying revolving asset class, credit cards in particular differs from previous researchers that found empirical asset correlations which were lower than 1%. Stoffberg and van Vuuren (2015) also found that the BCBS had introduced a higher level of conservatism for South Africa than for the US. Their study also explored in detail the effect of macroeconomic events on the empirical asset correlations over a rolling period. This current study seeks to update the previous South African correlation data used in Stoffberg and van Vuuren (2015) as well as calculate empirical asset correlations embedded in a bank-specific credit card dataset. It however employs both the Beta and Vasicek distribution including the usage of the percentile and mode approach. These empirical asset correlations are subsequently used to calculate the economic capital. The macroeconomic effects on the asset correlation will also be explored by rolling the empirical asset correlations over time.

Basel prescribed asset correlations are applied in every bank that follows the IRB approach regardless of that country's development level, which implies a burden on developing countries like South Africa in that they are measured against developed countries in their effort to ensure that they have sufficient capital to carry them through various economic conditions. This provides a research opportunity to investigate whether applying these asset correlations rules universally achieves the necessary conservatism that the Basel committee intended.

Chernih, Henrad & Vanduffel (2006) analysed corporate defaults and their impact on asset correlations. They used asset value data used from the Moody's KMV Credit Monitor. They used a sample of companies' asset returns for the period 1997-2006. They divided the data into those using default data and those using asset data. They firstly estimate default correlations directly and then use assumptions regarding the joint asset value movements to back out the asset correlations. Secondly, they use the asset returns data to directly estimate asset correlations and convert them to asset correlations. Chernih, Henrad & Vanduffel (2006) results reveal that asset correlations are affected by the assumption that LGD's are independent of the PD, and that the asset correlations estimated from default data must be increased in order to avoid underestimating the dependence between the PD and LGD. They deduce that default data is the best source of default correlations as there are no intermediate processes that need to be assumed in such a case. They however highlighted the challenges of acquiring such data, which makes such estimations difficult. The scarcity of such data is a result of the unwillingness of banks to avail their internal loss data. In this study, we employ retail default data from a South African bank's credit card portfolio to determine empirical asset correlations. Our results are compared to the BCBS' prescribed asset correlations and results in previous literature. The empirical asset correlations are subsequently used to calculate a fair level of the South African bank's economic capital.

Rosch and Scheule (2004) use charge-off rates from all United States of America (USA) commercial banks to estimate empirical asset correlations for credit cards, residential mortgages and other retail loans. They employ a variant of the two-state one-factor CreditMetrics model, which is similar to the model used in the Basel II framework to calculate risk weights. They estimate asset correlations of 1% and 0.98% for for credit cards and residential mortgages respectively. The asset correlations became as low as 0.7% and 0.3% respectively when macroeconomic risk variables were included in the model. They also provide details as to how the inclusion of lagged macroeconomic data into the credit risk model may improve loss forecasts and reduce the economic capital by a considerable amount. In this study, we extract empirical asset correlations and use them to calculate the economic capital, however, we will not include macroeconomic data in our model.

Hansen et al (2008) also conducted research on the asset correlation of retail loans, as well as credit cards. They use a time-series of charge-rates which are published by the US Federal bank for all banks' exposures, aggregated loss rates from collateralised assets transactions contained within the Fitch-rated USA structured financial transactions, and loss rates for all the UK banks as published by the Bank of England (BoE) for the period 1985 - 2007. They employ the Beta distribution and find that the empirical asset correlations of retail loans, as well as those of credit cards come to around 1.3%, which is consistent with previous studies.

Bellotti and Crook (2012) conduct a study into the estimation of the empirical asset correlation of two UK datasets using a Merton one-factor model. The first dataset was a sample of credit card accounts from a UK financial institution. They used a combination of default and write-off data spanning from the 1990's to the mid-2000's. The second dataset was quarterly time-series data for all UK credit card issuers from 1990 to 2007. This dataset contains data that shows the number of accounts that were overdue on their accounts over various periods, and accounts that were written off per quarter. They find that the BCBS' prescribed asset correlations are much higher than the empirical asset correlations, which were below 1%.

Since the advent of the first Basel accord in 1988, and other amendments that were introduced in 1992 and 2008, banks have been developing complex IRB models which are meant to suit their own internal loss and performance data. A few research papers that explore the modeling of credit risk have been published. In the case of corporate loans, Fatemi and Fooladi (2006) found that one of the most important purposes that is served by the credit risk models they used was the identification of counterparty default risk. In the case of retail loan portfolio risks models, there has only been a few research papers published. This is because account level data of EAD, LGD and PD's collected by banks is often unavailable, especially for research purposes.

This study explores the extraction of empirical asset correlations for a South African bank's credit card portfolio using both default and write-off data, which has not been explored in any study. It also seeks to update the previous South African correlation data used in Stoffberg & van Vuuren (2015) and estimate the empirical asset correlations embedded in this data. It uses the Vasicek & beta distributions and empirical asset correlation extraction methodologies introduced in Botha & van Vuuren (2010). These empirical asset correlations will be compared with the Basel prescribed asset correlations and previous studies. They will thereafter be used to calculate the economic capital.

**Methodology**

This section provides a review of the Beta and Vasicek distributions, including how the different calculation approaches can be used in the extraction of the empirical asset correlations. Two different calculation approaches (mode and percentile) are used for the Vasicek distribution assumption This data is also used in the Basel capital calculation methods to calculate the economic capital.

**The Vasicek Distribution**

Botha and van Vuuren (2010) reverse-engineered the Vasicek distribution to calculate the retail asset correlation using the South African and USA empirical loss data. A Merton-type model was derived by Vasicek (1987, 1991, 2002) to extract an expression that can be used for the distribution of credit losses. Vasicek (1987) makes the assertion that the cumulative probability of the portfolio loss L given the state of the economy will be less than some arbitrary variable, x, given by:

$$P[L \leq x] = \left[N\left(\frac{\sqrt{1-\rho}.N^{-1}(x)+ N^{-1}(PD)}{\sqrt{\rho}}\right)\right] \qquad (1)$$

where ρ represents the asset correlation, N[...] represents the standard normal distribution function; $N^{-1}$ represents the inverse standard normal distribution; while PD is the average of the portfolios probability of default. This cumulative distribution provides a description of credit portfolio losses and is mainly dependent on (ρ and PD). It is defined over the interval $0 \leq x \leq 1$ and is given by:

$$F(x; PD; \rho) = \left[N\left(\frac{\sqrt{1-\rho}.N^{-1}(x) + N^{-1}(PD)}{\sqrt{\rho}}\right)\right] \qquad (2)$$

with ρ > 0 and 0 < ρ < 1. As the asset correlation ρ → 0, the distribution converges to a normal distribution, N(0,1), with probability functions PD and 1−PD. From the above function (equation 2), we can infer that $F(x;\ PD;\ \rho)\ =\ 1 - F(1 - x; 1 - PD; \rho)$ and that as $PD\ \rightarrow\ 1\ or\ PD\ \rightarrow\ 0$, then the loss distribution converges to L = 1 or L = 0 respectively. The loss distribution presented in equation 1 has been illustrated in Figure 1. This is a highly skewed and leptokurtic loan loss distribution, which is described by the following density:

$$F(x; PD; \rho) = \sqrt{\frac{1-\rho}{\rho}}.exp\left[\frac{1}{2}(N^{-1}(x))^2 - \frac{1}{2}\left(\frac{\sqrt{1-\rho}.N^{-1}(x) + N^{-1}(PD)}{\sqrt{\rho}}\right)\right] \qquad (3)$$

and has a single mode located at:

$$L_{mode} == N.\frac{\sqrt{1-\rho}}{1-2\rho}.N^{-1}(PD) \qquad (4)$$


**Source: Stoffberg & van Vuuren, 2015**

**Figure 1: The loss distribution that contains a total loss at the 99:9th percentile.**

The α-percentile value of L - which is the inverse of the loss distribution is given by:

$$L\alpha\ =\ (x\ ; PD\ ; \rho) =\ 1\ -\ F(1\ -\ x;\ 1\ -\ PD;\ \rho)\ \ . \qquad (5)$$

This is a typical, skewed loan loss distribution with all the relevant features provided in Figure 1. The EL is the average level of portfolio credit losses that a bank can reasonably expect to suffer in a particular year (BCBS, 2005), and the ‘total loss’ (L) is a Basel-defined point, which in this case is a point where 99.9% of all losses fall. The area under the curve to the left of the total loss point in Figure 1 represents 99.9% of all portfolio losses. However, the UL refers to losses above the expected loss levels up to the total loss position, which is the difference between the ’Total loss’ and EL (BCBS, 2005).

There are two approaches employed to calculate the empirical asset correlations using a Vasicek distribution, namely the mode and percentile approaches. The mode approach is mainly calculated using the portfolio PD and mode of the portfolio losses to determine

the empirical asset correlation, while the percentile approach uses the UL ($UL^{99.9\%}$) at the 99th percentile and portfolio PD to determine the empirical asset correlation.
The procedure of extracting empirical asset correlations using the mode approach has originally been outlined in Botha and Van Vuuren (2010) and proceeds as follows:

1. Source gross loss data, which can be arranged as a time series.
2. Calculate the mean loss (PD in equations 1 and 2) and the mode ($L_{mode}$ in equation 4). These values can be calculated by taking the simple average of the gross losses (PD), and by determining the most prevalent loss ($L_{mode}$) over the period.
3. Using equation 4, the asset correlation may now be determined as follows:

$$\frac{N^{-1}(L_{mode})}{N^{-1}(PD)} = \frac{\sqrt{1-\rho}}{1-2\rho} \qquad (6)$$

Therefore,

$$\left(\frac{N^{-1}(L_{mode})}{N^{-1}(PD)}\right)^2 = \frac{1-\rho}{(1-2\rho)^2} \qquad (7)$$

Let

$$\xi = \left(\frac{N^{-1}(L_{mode})}{N^{-1}(PD)}\right)^2 \qquad (8)$$

substituting into equation 7 gives:

$$\xi = \frac{1-\rho}{(1-2\rho)^2} \qquad (9)$$

$$\xi(1 - 2\rho)^2 = 1 - \rho \qquad (10)$$

$$4\xi\rho 2 + (1 - 4\xi)\rho + (\xi - 1) = 0 \qquad (11)$$

This is a quadratic equation in ρ (the asset correlation) which has solutions:

$$\rho = \frac{(4\xi-1)\pm\sqrt{8\xi+1}}{8\xi} \qquad (12)$$

The above calculation (equation 12) results in two values for the asset correlation (ρ). However, only one of the empirical asset correlations is correct. Botha and Van Vuuren (2010) calculated both ρ's to establish which one provided the most economically feasible UL. They showed that the smaller of the two ρ's should be used, as the other asset correlation value (ρ) resulted in an unrealistically high capital charge (>20%), which does not make economic sense in the Basel II framework (Guttler and Liedtke, 2007). Stoffberg and Van Vuuren (2015) also found similar results, thus only one ρ will be presented.
To empirically measure the total portfolio loss at a confidence interval of 99.9%, we may combine equations 1 and 2 - where a confidence interval of 99.9% implies that α = 0.1%):

$$F(\alpha; 1 - PD; 1 - \rho) = N.\left[\left(\frac{\sqrt{1-(1-\rho)}.N^{-1}(\alpha) + N^{-1}(1-PD)}{\sqrt{1-\rho}}\right)\right] \qquad (13)$$

and

$$Gross\ total\ loss = N.\left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(\alpha)}{\sqrt{1-\rho}}\right)\right] \qquad (14)$$

As shown in Figure 1 and equation 14, the gross total loss L, at a specified confidence level is simply the sum of the Expected and Unexpected gross loss (EL + $UL^{99.9\%}$).
Thus, the Unexpected loss at a 99.9th percentile is:

$$UL^{99.9\%} = N.\left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] - EL \qquad (15)$$

But, the Expected portfolio loss EL = PD, the portfolio's probability of default, as gross loss data is used (and the LGD =1 since we assume no recoveries). Thus, we can express the Unexpected loss as:

$$UL^{99.9\%} = N.\left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] - PD \qquad (16)$$

Equation 16 is dependent on three variables, which are the Unexpected gross loss ($UL^{99.9\%}$), the average portfolio loss (PD) and the asset correlation (ρ). If we can empirically determine the $UL^{99.9\%}$ and the PD from loan loss data, then we can manipulate equation 16 to get:

$$UL^{99.9\%} + PD = N.\left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] \qquad (17)$$

$$N^{-1}(UL^{99.9\%} + PD) = \left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] \qquad (18)$$

with ρ being the only unknown.

**The Beta Distribution**

The Vasicek distribution is one of the distributions used to describe the behaviour of credit portfolio losses of IRB banks. It was used for the calculation of the Basel II capital requirement as stipulated by the BCBS. However, there are many leptokurtic fat tailed distributions that exist, and may be employed as a best fit to credit portfolio loss data (Botha & van Vuuren, 2010). The empirical asset correlations for the datasets used in this study were compared using both the beta and vasicek distributions. The former being more popular in the Basel Committee's analysis of securitisation. Its main characteristics are the two parameters, α and β. These parameters can easily be measured using the mean (μ), and standard deviation (σ) of the empirical loan losses.
These two quantities, α and β are described using the following equations (Wolfram Research, 2009):

$$\alpha = \mu.\left(\frac{\mu.(1-\mu)}{\sigma^2} - 1\right) \qquad (19)$$

$$\beta = (1-\mu).\left(\frac{\mu.(1-\mu)}{\sigma^2} - 1\right) \qquad (20)$$

where μ is the mean of the gross loan losses and σ is the standard deviation of the gross losses. After calculating α and β (using equations 19 and 20) for the period under scrutiny. The beta distribution may be defined by the following probability density function:

$$f(x\colon \alpha,\beta) = \left(\frac{1}{B(\alpha,\beta)}.\, x^{\alpha-1}(1-x)^{\beta-1}\right) \qquad (21)$$

where B(α, β) is the beta function given by $\frac{\Gamma(\alpha).\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and where Γ(...) is the gamma function. The cumulative beta distribution is given by:

$$P(x) = \frac{\int_0^x (1-t)^{\beta-1}.t^{\alpha-1}dt}{B(\alpha,\beta)} \qquad (22)$$

$$P(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)}.\int_0^x (1-t)^{\beta-1}.t^{\alpha-1}dt,\ 1 \geq x \geq 0, \alpha,\beta > 0 \qquad (23)$$

where x is the distribution variable, and Γ is a standard Gamma function. The (EL + UL) which represent the total loss are simply the value of x when P(x) equals 99.9%, and, B(α, β) is the incomplete Beta function, which is a generalization of the Beta function. it is given by:

$$B(x,\alpha,\beta) = \int_0^x (1-t)^{\beta-1}.t^{\alpha-1}dt \qquad \alpha,\beta > 0 \qquad (24)$$

The empirical asset correlation is derived by determining the correlation value which is equal to the Basel II Total Loss at a confidence level of 99.9%.
We derive the asset correlation by empirically determining the correlation value that will be equal to the Basel Total Loss (UL + EL) at 99.9% confidence interval. The procedure of empirically extracting asset correlations proceeds as follows:

1. Source gross loan loss data, which can be arranged as a time series.
2. Calculate the mean(μ), and standard deviation(σ), of this loan loss data. These values can easily be calculated by taking the simple average(μ) of the gross loss, and the standard deviation(σ) of the gross loss, over the period under observation.
3. Calculate α and β using equation 19 and 20.
4. Determine the value of the distribution variable x when P(x) = 99.9% using equation 8. This represents the total gross loss at the 99.9th percentile ($L_{total}^{99.9\%}$), of the fitted Beta distribution.

Substitute the $L_{total}^{99.9\%}$ value obtained in step 4 into the BCBS equation for the total gross loss value (which is measured at a confidence interval of 99.9%), i.e

$$UL_{total}^{99.9\%} = N.\left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] - EL \qquad (25)$$

$$EL + UL_{total}^{99.9\%} = Total\ gross\ loss = L_{total}^{99.9\%} \qquad (26)$$

$$N^{-1}(L_{total}^{99.9\%}) = \left[\left(\frac{N^{-1}(PD)+\sqrt{\rho}.N^{-1}(99.9\%)}{\sqrt{1-\rho}}\right)\right] \qquad (27)$$

$$N^{-1}(L_{total}^{99.9\%}).\sqrt{1-\rho} = N^{-1}(PD) + \sqrt{\rho}.N^{-1}(99.9\%) \qquad (28)$$

Setting $\omega = N^{-1}$ ($L_{total}^{99.9\%}$), π = N−1(PD) and ψ = $N^{-1}$(0.999), Substituting into equation 28, and squaring both sides:

$$\omega^2.(1-\rho) = \pi^2 + 2\pi\psi + \psi^2.\rho \qquad (29)$$

$$0 = (\omega^2 + \psi^2).(\rho) + 2\pi\psi\sqrt{\rho} + (\omega^2 + \pi^2) \qquad (30)$$

This is a quadratic formula in the asset correlation ρ with solutions:

$$\rho = \frac{-(2\pi\psi)\sqrt{(2\pi\psi)^2 - 4(\omega^2+\psi^2)(\omega^2+\pi^2)}}{2(\omega^2+\psi^2)}^2 \qquad (31)$$

The asset correlation (ρ) is the only unknown in this equation, since the components π, ψ and ω are known.

**Data and Empirical Analysis**

Three datasets are used in the empirical estimation of the asset correlation. The first two datasets relate to a sample of credit card accounts from a South African bank. The third dataset contains data for all loans issued in South Africa. Due to the proprietary nature of bank-specific data within the South African banking sphere, it is not possible to reveal the name of the bank which gave us access to their data. For the purposes of this study, we will call it "Bank X". The South African loans loss data spans some ten years (i.e. June 2008 to January 2017), which covers 103 consecutive months. This data was collected from the SARB (Venter, 2017) dividing the impaired advances by the total loans on book or the period under scrutiny. Two datasets were used for the Bank X's credit card loan losses. The first dataset spans some nine years of monthly data (i.e. February 2006 to September 2015), which covers 117 consecutive months. This data was calculated by taking yearly cohorts of actual defaulted customers as a percentage of open, performing customers at the beginning of each yearly cohort. The second dataset spans ten years (i.e. January 2007 to May 2017), which covers 125 consecutive months. This data was calculated by taking the actual monthly write-off amount as a percentage of the monthly total exposure on the balance sheet.

The South African commercial loans and Bank X's loss data each represent a different Basel II asset class category for retail exposures, namely, qualifying revolving and the corporate, bank and sovereign. These asset types have been summarised in Table 2.

**Table 2: Asset classes for our data and their Basel II asset classification**

| Dataset | Basel II Classification |
|---|---|
| Bank X's Credit card loans | Qualifying Revolving exposures |
| South African commercial loans | The Corporate, Bank and Sovereign |

The losses used in this study are at a gross loss level, thus need to be converted to net losses for the economic capital calculation. These gross losses were converted to net losses using:

$$Net\ Losses = Gross\ Losses \cdot LGD \qquad (32)$$

The LGDs used are obtained from the results of the BCBS' 5th impact study (BCBS, 2006). These LGD averages have been presented in Table 3. The LGD used for the Bank X's Credit

Cards loss data is 71.6% (QRE), while the LGD used for the South African commercial loans loss data is 39.8% (Corp.).

**Table 3: LGD data for the banks which participated in the BCBS' 5th Quantitative Impact study**

| Asset Class | LGD |
|---|---|
| Residential Mortgages | 20.30% |
| Qualifying Revolving | 71.60% |
| Other Retail | 48.00% |
| Corporate, bank and sovereign | 39.80% |

First, the effect of different approaches (beta, mode and percentile approaches) of calculating the empirical asset correlations is explored for the South African commercial loans and Bank X's credit card loan loss datasets. Secondly, the empirical asset correlations from both Bank X's credit cards and
the South African commercial loans data – deducted using the Vasicek and beta distributions – are compared with the BCBS' prescribed asset correlations for the observation period. Thirdly, the economic capital calculated using the empirical asset correlations from the different calculation approaches is compared to the economic capital calculated using the Basel prescribed asset correlation for both the South African commercial loans loss data and Bank X's credit card loan loss datasets.
Finally, the South African and Bank X's credit card empirical asset correlations are explored over a rolling five-year period. These rolling empirical asset correlations are also compared to the BCBS-specified asset correlations over the period.

**Empirical Results**

The BCBS-specified asset correlations have been presented in Table 3. The "Corporate, bank and sovereign" calculation is used for the South African commercial loans loss data, as confirmed in Hill (2012). The "Retail revolving" asset correlation value is used as the Bank X's credit card asset correlation.
Three datasets have been used in this study with one representing the South African commercial loans and the other two representing Bank X's credit card loan loss data. The credit card data has been separated by the way it was calculated. The first dataset was calculated using the actual default data as the loan loss data estimate and is denoted with a PD subscript. The second dataset was determined using the write-off percentage as a loan loss estimate and is denoted by an EL subscript. The usage of the write-off and default data as loan loss estimates has never been explored directly in previous studies, but rather as a combination of the two in the definition of default (see Bellotti & Crook, 2012). Write-offs in our case represent total loss as the bank only writes off an account when they have exhausted all prospects of collecting on the outstanding debt. The default data has also been chosen as a loss estimate to match banking standards, as the long run actual default rates are used in the construction of through-the-cycle PD models.
Two distributions with three different approaches have been selected for this study, which are the beta and Vasicek (Mode and Percentile approaches) distributions. For Bank X's data, V_mode_PD, V_percentile_PD and Beta_PD represent results using the Actual Default data as the loss data estimate, while the V_mode_EL, V_percentile_EL and Beta_EL represent results using the Write-Off data as the loss data estimate.

The distributions were statistically tested to determine their goodness of fit. The Beta distribution was found to be ranked as one of the best in all three datasets using the Kolmogorov-Smirnov and Anderson-Darling test.

The CDF's were also plotted for the different datasets. For a discrete random variable, the CDF at a certain value x gives us the probability that the random variable will have at that value x. For a continuous random variable, the PDF describes the probability of finding a random variable in the area under the curve (Smit & van Vuuren, 2009). The CDF evaluated at a value x, describes the probability that a variable takes a value within a range which is less than or equal to x (Brown, 2005).These are shown in Figures 2 to 4.
The Beta (Beta_PD, Beta_EL, Beta) distribution was on average ranked 1-3 using the Kolmogorov-Smirnov Test. Even though that was the case, all the other distributions had P-values below the critical value for all distributions at significant levels 0.01 - 0.2 except for the V_Percentile_EL. Figures 2(a), 3(a) and 4(a) provide an illustration of the cumulative density function for the different approaches, plotted against their respective empirical loss data. Figure 2(b), 3(b) and 4(b) provides an illustration of the density function for the different approaches plotted against their respective empirical loss data.

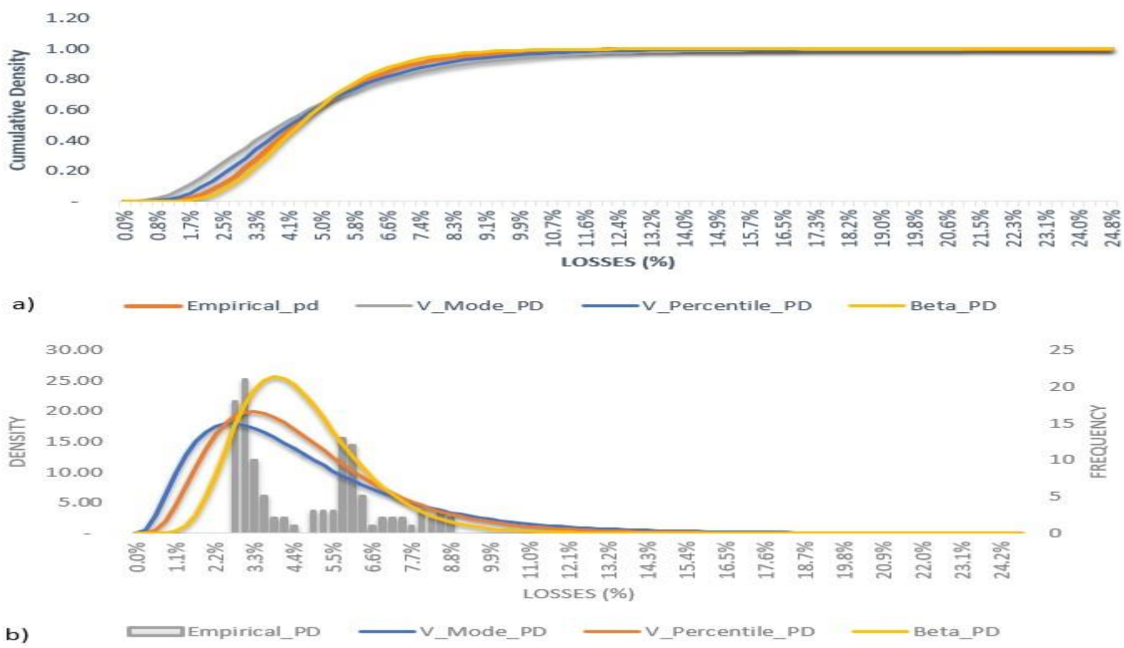


**Figure 2: (a) The cumulative, and (b) the density function for Credit Card Actual Default Rate losses from February 2006 to September 2015.**

Visually seen in Figure 2(b), both the Vasicek (V_Mode_PD and V_Percentile_PD) and Beta (Beta_PD) distributions closely fits Bank X's Credit Card Default empirical loss data, except at the point of highest frequency of losses in the case of the Beta_PD. The cumulative densities in Figure 2(a) are also closely aligned for all the distributions. This was confirmed using the Kolmogorov-Smirnov (K-S) test for statistical goodness of fit.

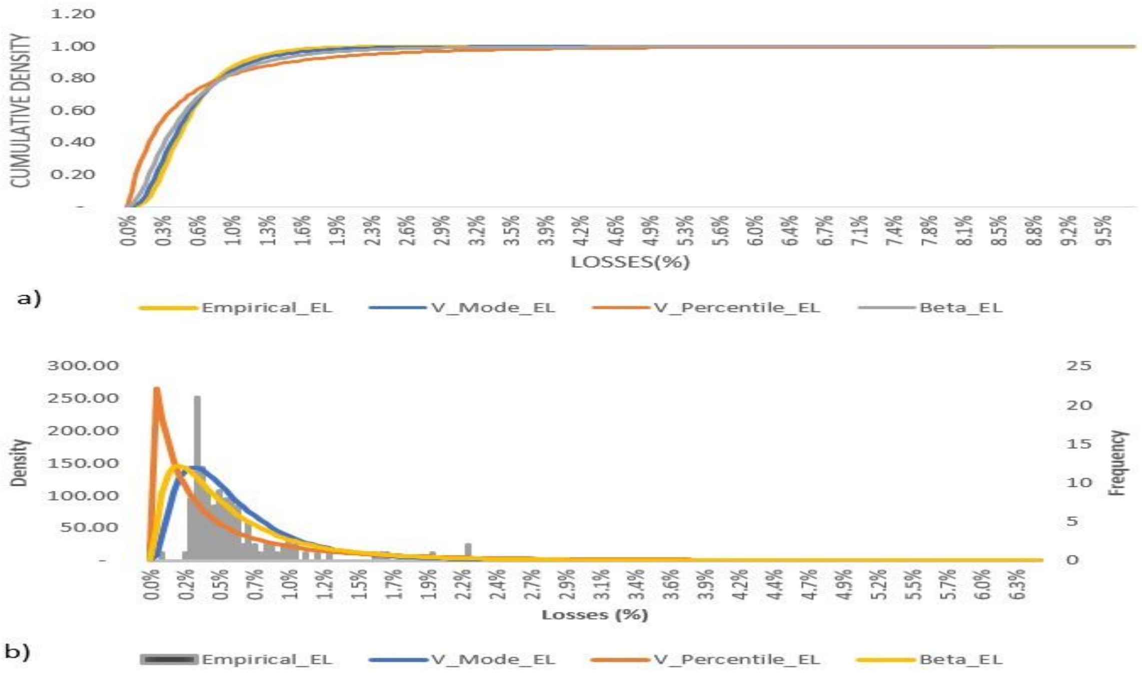


**Figure 3: (a) The cumulative, and (b) the density function for Credit Card Write-Off data from January 2007 to May 2017.**

Visually seen in Figure 3(b), both the V_Mode_EL and Beta_EL distributions closely fits Bank X's Credit Card Write-Off empirical data, while the Beta_EL shifts slightly to the left at the point of the highest frequency of losses. The V_Percentile_EL provides a sub-optimal fit to the empirical loss data. This is also supported by the statistical tests using Kolmogorov-Smirnov Test, which revealed that the V_Percentile_EL is not a good this fit to the empirical loss data. The cumulative densities in Figure 3(a) are also closely aligned for all the distributions except for the V_Percentile_EL which seems to be an outlier. This is also supported by "V_Percentile_EL - Goodness of Fit - Summary" table in the appendix I, which validates statistically using the Kolmogorov-Smirnov Test that V_Percentile_EL is not a good this fit to the empirical loss data.

Visually seen in Figure 4(b), all three of these distributions provides a sub-optimal fit to the South African commercial loans empirical loss data. The Beta and Vasicek distributions are both unimodal, thus it makes sense why they would provide a suboptimal fit to an (apparent) bimodal distribution of data. The V_Mode and beta distributions provide a good fit to the main body of the loss distribution but a poor fit to the tail region, while the V_Percentile provides a good fit to most of the loss data at low frequencies, while it aligns more to the left at the point of the highest frequency of losses. The cumulative densities illustrated in Figure 4(a) are also not as closely aligned, with the V_Percentile more closely aligned to the empirical loss data than the other cumulative densities. Even though these distributions do not visually portray a perfectly good fit to the data, the Kolmogorov-Smirnov Test results reveal that they are a good fit to the data.

The BCBS' specified asset correlations were compared with the empirical asset correlations calculated using all the gross loss data (from Equations 12, 18 and 31). Table 4, Figures 5 and 6 present a comparison the empirical asset correlations against the Basel II-specified asset correlations for Bank X's Credit Card portfolio using all three approaches, the Percentile approach (V_percentile in equation 18), and the Mode approach (V_mode in equation 12) and the Beta (Beta in equation 31) calculation. These calculation approaches were applied to different datasets, the Actual Default data (V_percentile_PD & V_mode_PD & Beta_PD) and the Write-Off (V_percentile_EL & V_mode_EL & Beta_EL) data.

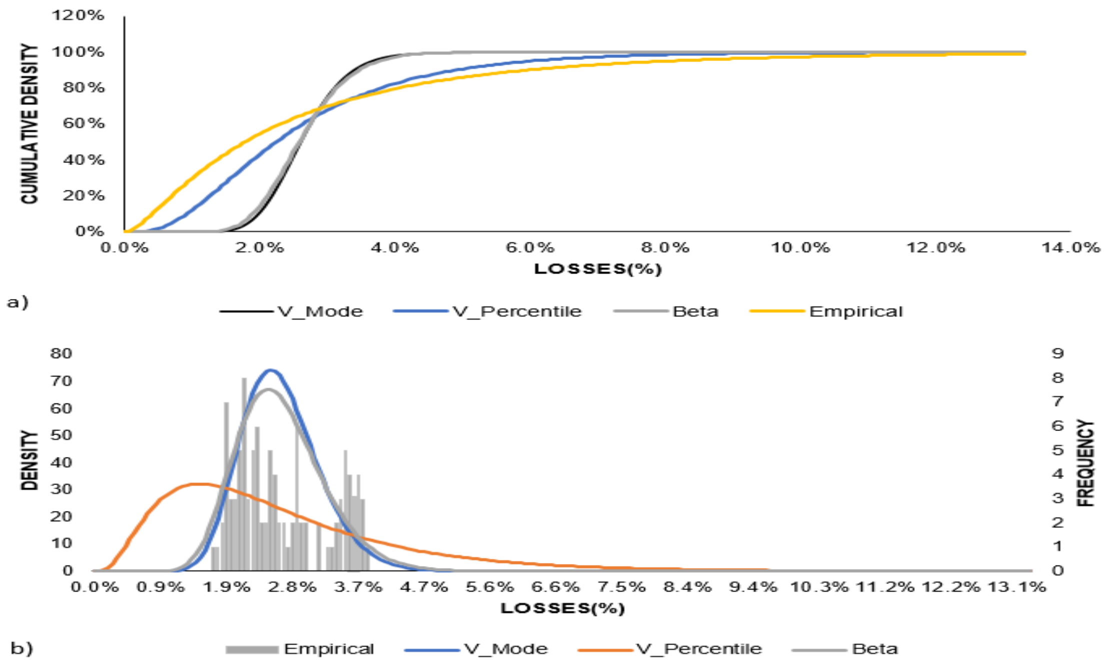


**Figure 4: (a) The cumulative, and (b) the density function for the South African Commercial Loans from June 2008 - December 2016.**

**Table 4: Empirical Correlations compared to Basel II Correlations**

| Credit Card Loans | Basel II Correlation | Empirical Correlation | Ratio a:b |
|---|---|---|---|
| V_Mode_PD | 4% | 8.47% | 0.47 |
| V_Perc_PD | 4% | 5.83% | 0.69 |
| Beta_PD | 4% | 2.99% | 1.34 |
| V_Mode_EL | 4% | 5.53% | 0.72 |
| V_Perc_EL | 4% | 17.44% | 0.23 |
| Beta_EL | 4% | 8.47% | 0.47 |

These empirical results indicate that the BCBS specified asset correlation for Credit Card is on average less conservative than the empirically calculated asset correlation as shown in Table 4. The only exception is the Beta_PD which results in an empirical asset correlation of 2.99%. This outlier result by the Beta_PD is the expected result that demonstrates the BCBS' intended conservatism and corresponds with previous studies of the Credit Card empirical asset correlation by Rosch & Scheule (2004) and Bellotti & Crook (2012). The difference though is that they found empirical asset correlation values below 1% for Credit Cards in the USA and the UK respectively.

In the case of the empirical asset correlations that are higher than the BCBS- specified asset correlations, we may take comfort in the fact that the data used by these researchers does not fully take the 2008 global credit crunch into account, as Bellotti and Crook (2012) used the time period 1990 Q2 - 2007 Q4, while Rosch and Scheule (2004) used the time period 1985 to 2001 for their research. The credit crunch dramatically increased credit losses (Newscientist, 2009; and Chummun and Bisschoff, 2014), therefore it is assumed that their results would've differed had their research included the impact of the global recession on the loss rate. Our results do however correspond to the results obtained in Stoffberg & van Vuuren (2015), which had a credit cards empirical asset correlation as high as 7%. Further research should be established to determine whether credit card portfolios from other South African banks behave in a similar manner.

One of the limitations of this study was accessing credit card and retail revolving data for the other South African banks to compare with the results acquired from Bank X's loan loss data.

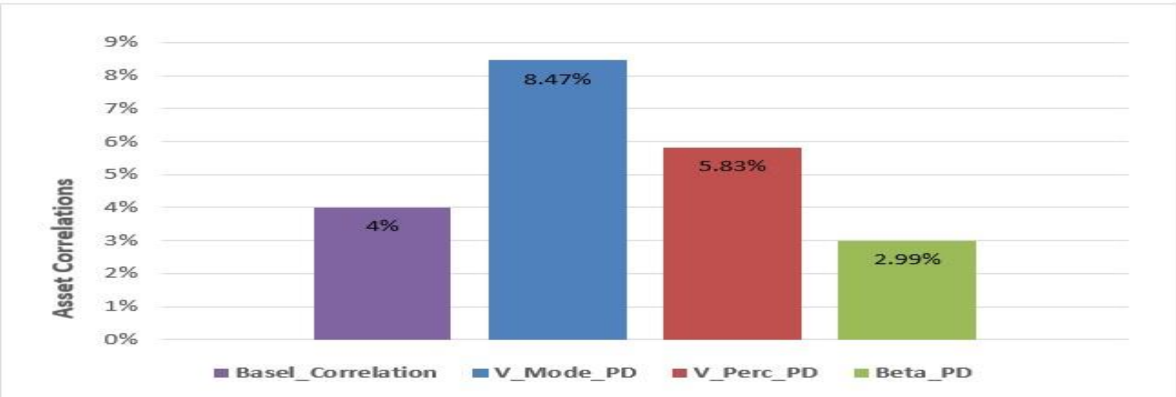


**Figure 5: Empirical Correlations compared to Basel II Correlations using the Actual Default Rate data**

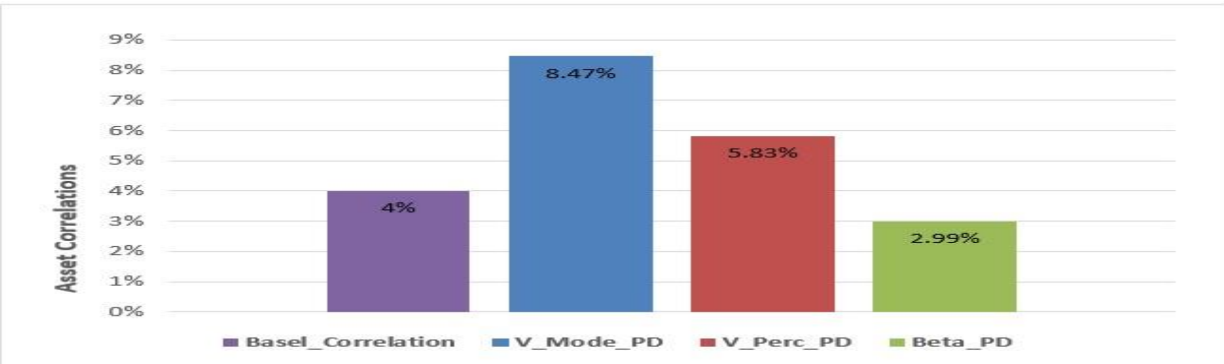

**Figure 6: Empirical Asset Correlations compared to the Basel II specified asset Correlations using the Write-Off data**

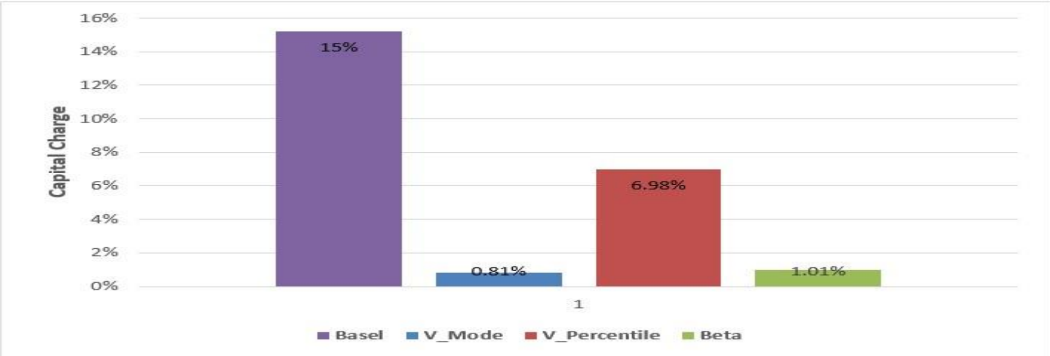


**Figure 7: South African Commercial Loans Empirical Correlations compared to Basel II Correlations**

**Table 5: Comparison of the South African Commercial Loans Empirical Asset Correlations and the BCBS' Specified Asset Correlations**

| South African Commercial Loans | Basel II Correlation | Empirical Correlation | Ratio a:b |
|---|---|---|---|
| V_Mode | 15% | 0.81% | 18.77 |
| V_Percentile | 15% | 6.98% | 2.18 |
| Beta | 15% | 1.01% | 15.11 |

Table 5 and Figure 7 compares the BCBS' specified correlations with the empirical correlations for the South African Commercial loans using all three approaches, the Percentile approach (V_percentile in equation 18), the Mode approach (V_mode in equation 12) and the Beta (Beta in equation 31) calculation. For all three approaches, it was found that the BCBS' specified asset correlation is more conservative than the empirically estimated asset correlations by a factor of 2, 15 and 19 for the Percentile, Beta and Mode approaches respectively. This is an expected result, which also agrees with the results found by Stoffberg & Van Vuuren (2015).

This study has thus far shown how to calculate the empirical asset correlation for Bank X and the South African Commercial loans loss data. However, knowledge of only the empirical asset correlation does not give us an indication of how much capital is needed to protect a bank against potential risks.

The BCBS specifies that banks that employ the IRB approach should use a downturn LGD in the capital calculation, due to the correlation between the LGD and PD not being accounted for in the capital formula (Miu & Ozdemir, 2006). Therefore, they proposed that a principles-based approach be used. This approach stipulates that banks must identify an appropriate downturn period and consider the adverse dependencies between default rates and recoveries.

The BCBS implicitly stated that using a model that possesses a systematic correlation between the LGD and PD using LGD inputs over a long term gives comparative capital when you compare it to a similar credit risk model that doesn't possess correlated LGD and PD using downturn LGD inputs. In an effort to compensate for the lack of correlation, mean LGD's need to be raised by about 35% to 41%. Therefore, downturn LGD's were obtained by increasing the LGD's in table 3 by 38%, i.e the LGD's were multiplied by 1.38. The calculated downturn LGD's are then used as inputs in the capital calculations presented in this section using the BCBS' IRB approach. For the empirical asset correlation calculations, the unaltered LGD's as presented in table 3 were used.

**Table 6. A comparison of the South Table 11: Comparison of Bank X's Credit Card Capital Charge (Relative to Basel Capital Charges)**

| Bank X's Credit Card Loans | Capital Charge using Basel II Correlations | Capital Charge using Empirical Correlations | Ratio a:b |
|---|---|---|---|
| V_Mode_PD | 9.13% | 15.84% | 0.58 |
| V_Perc_PD | 9.13% | 11.96% | 0.76 |
| Beta_PD | 9.13% | 7.47% | 1.22 |
| V_Mode_EL | 2.03% | 2.67% | 0.76 |
| V_Perc_EL | 2.03% | 8.21% | 0.25 |
| Beta_EL | 2.03% | 3.93% | 0.52 |

**Figure 8: Comparison of Bank X's Credit Cards Ratio of the Capital charge (Relative to the ratio of the Basel Capital Charge)**

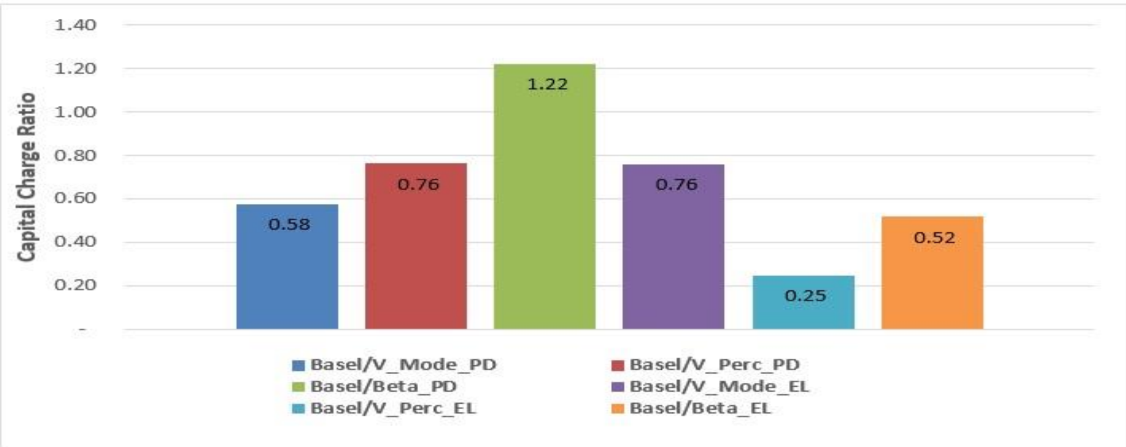

Table 6 presents a comparison between Bank X's Credit Card Capital charge relative to the Basel capital charge. Figure 8 presents a comparison of Bank X's Credit Card Capital charge ratio relative to the Basel capital charge. Bank X's capital charges have been calculated using the empirical asset correlations calculated in the previous section, while the Basel capital charge has been calculated using the BCBS specified asset correlations (Table 1). Bank X's capital charge is larger than the Basel specified capital charge for the most part (see Table 6). The only exception is the Bank X's Beta_PD Capital charge which is an advantageous result for Bank X, and provides the necessary conservatism that was intended by the BCBS.
The case where Bank X's capital charges are larger than the Basel capital charges indicate that the banks liquidity may not be enough to carry them through all their losses. The Maximum capital cover by the Basel capital charge compared to Bank X's capital charge is 0.76 (see Figure 8), again excluding the Beta_PD capital charge. This implies that the Basel capital charge is not sufficient to ensure enough capital for Bank X's Credit Card portfolio, and that the capital levels need to be reviewed and possibly increased.

**Table 7: Comparison of South African Commercial Loans Ratio of the Capital charge (Relative to the ratio of the Basel Capital Charge)**

| South African Commercial Loans | Capital Charge using Basel II Corre-lations | Capital Charge using Empirical Correlations | Ratio a:b |
|---|---|---|---|
| V_Mode | 10% | 1.18% | 8.68 |
| V_Percentile | 10% | 5.29% | 1.94 |
| Beta | 10% | 1.35% | 7.59 |

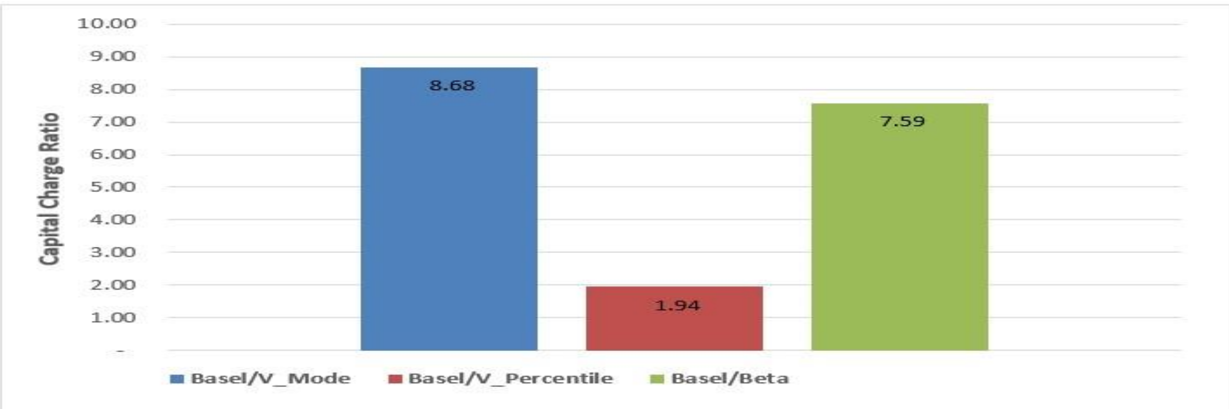


**Figure 9: Comparison of South African Commercial Loans Ratio of the Capital charge (Relative to the ratio of the Basel Capital Charge)**

A comparison of the South African commercial loans capital charge ratio relative to the Basel Capital charge has been presented in Figure 9 and Table 7. Unlike Bank X's case, the Basel asset correlations were found to be very conservative for all three approaches (See Table 7). Figure 9 shows that the BCBS capital charge was 2 times more conservative using the V_Percentile Approach, 7.6 times using the Beta approach and 8.7 times using the V_Mode approach.

The BCBS has consistently stressed that banks should always strive to have sufficient capital cover to shield them against insolvency, and placed emphasis on the need to always remain conservative (Carver, 2014), it is however clear from the estimated empirical results that this is not always the case. As shown in figure 8, the ratio of the BCBS-specified capital charge to the Bank X's Credit Cards empirical capital charge is lower than 1 for the most part. Using the BCBS' current formulation, Basel II-compliant banks that analyse their credit risk using the IRB approach, may alter a few parameters to increase the capital cushion. The LGD is one of the adjustable parameters, while the asset correlation is the other. It is clear that the manner in which the asset correlation is calculated has an impact on the regulatory capital used to protect banks against insolvency.

Banks with a large capital cushion have high costs, which are attributable to the cost of maintaining capital. Therefore, for a bank to have a competitive advantage there needs to be a balancing act between maintaining an adequate amount capital reserves and limiting the opportunity costs associated with maintaining these capital reserves. Large Basel II retail banks gain a cost and ultimately a credit pricing advantage when the required capital reserves are lower. This would result in higher Returns on Equity (ROE). Such an advantage, forces smaller, community banks to reduce their pricing and also adjust their underwriting guidelines to enable them to compete with such banks. This could have an adverse impact on their profitability and potentially their viability. It is a possibility that the BCBS wished to avoid such possibilities by adjusting the only variable they had at their disposal, which is the asset correlation. This adjustment ultimately resulted in increased capital charges for most Basel compliant banks. This is probably the reason the BCBS imposed the asset correlation for all participating banks, as a way to promote fairness and eliminate competitive advantages larger banks had compared to smaller banks.

**Exploring Empirical Asset Correlations Over A Rolling Period**

The empirical asset correlation values are highly sensitive to the parameters estimated from portfolio loss data. Figure 10, provides an illustration of how loan losses and their associated implied asset correlations vary over a five-year rolling period for South African commercial loans. Figures 11 and 12 illustrate the same information for Bank X's credit card default and credit card write-off data respectively.

The South African economy experienced the end of a benign period characterised by inflation rates within the targeted 3-6 percent, economic growth, and a manageable budget and current account deficit in mid-2007 (Padayachee, 2012). After this period, the country experienced a breach in the inflation target upper limit, increased interest rates, a drop in the country's GDP growth rate, and a fall in the mining and manufacturing output. These macroeconomic changes resulted in increase in the default rates and number of delinquencies.

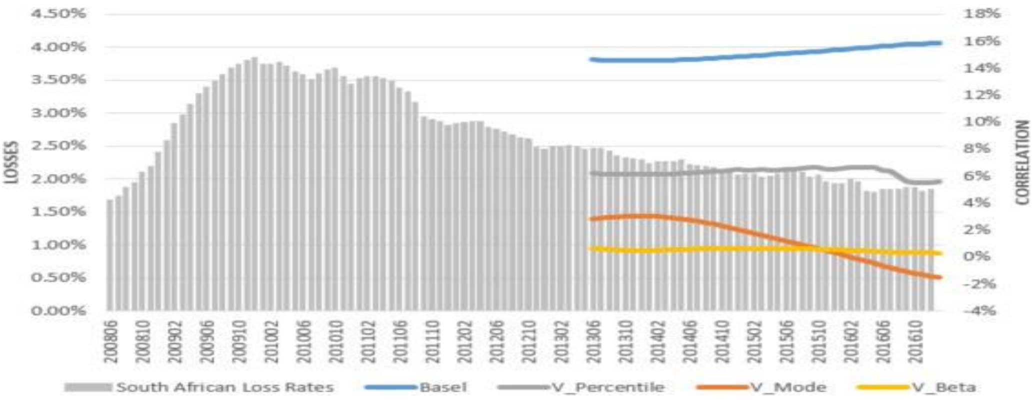


**Figure 10: A comparison of the five-year rolling empirical asset correlations to the Basel specified rolling asset correlations for the South African commercial loans data**

In Figure 10, it is clear that the worst losses were experienced around September 2009. After this point, the losses started to gradually decrease, and stabilised around 2014. Due to a lack of data going back 5 years before the global recession, it was not possible to demonstrate rolling empirical asset correlations before the global recession. The first point represents five years of data between June 2008 and June 2013. The empirical asset correlations were lower than the BCBS' prescribed asset correlations for the whole period under scrutiny. This means the BCBS' asset correlation has been extremely conservative through this whole period.

The same economic events that had an impact on the losses experienced by the South African Commercial loan data also had an impact on Bank X's loss data. Figure 11, Bank X's Credit Card Default data, shows a spike in losses much earlier than the spike seen in the South African commercial loss data (see figure 10). As Credit Cards is an unsecured product, it makes sense for it to experience

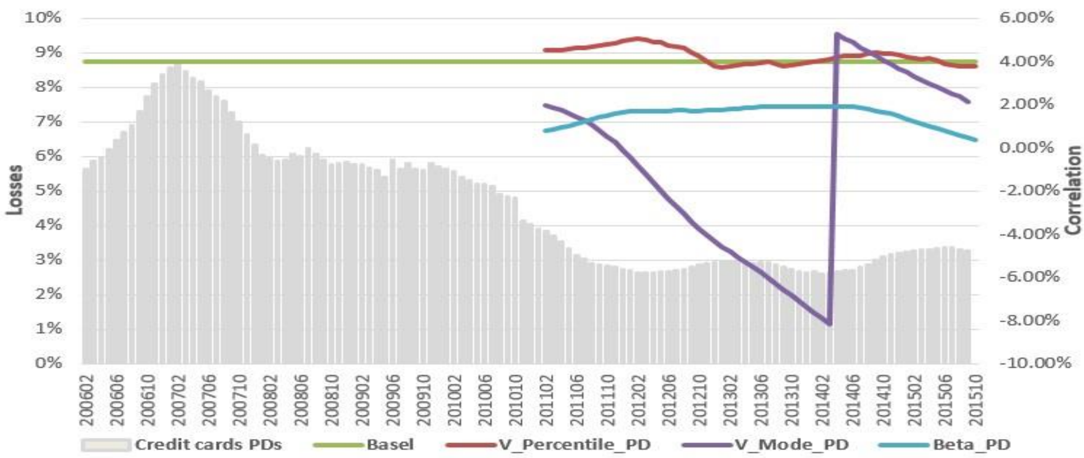


**Figure 11: 5-year rolling empirical asset correlations compared to Basel specified rolling asset correlations for Bank X's Credit Card Default Data.**

an immediate impact of the global recession, as opposed to a much secured product. Conventional wisdom has always been that consumers will likely pay their secured obligations first, Mortgages in particular, when faced with a financial crisis (Bell, 2007).

The BCBS' prescribed asset correlation is less conservative than the V_Percentile_PD empirical asset correlation from 2011 until they become equal around 2013, and continued to be quite close to each other from there on. It is assumed that the empirical asset correlation is higher due to the effects of the global recession. These changes are not evident in the BCBS' prescribed asset correlation as they prescribed a constant figure of 4%, which does not adapt to changes in the economic environment. This means the BCBS' asset correlations do not always achieve their intended con servatism.The BCBS' prescribed asset correlation is higher than the V_Mode_PD empirical asset correlation for the most part, except for a short period in 2014. The Mode approach of the Vasicek distribution produces very volatile asset correlations, as it is very sensitive to changes in the loss data. A change in the underlying data yields an exaggerated gross loss value than the previous mode. Data analysed over periods of great loss, and then periods of low losses, or vice versa, will result in the mode undergoing a "jump" to a high value and then a low value, (or otherwise) without jumping to intermediate values in-between. This will have the effect of extremely exaggerating changes in the empirical asset correlations estimated using
this approach. The mode approach is an approach that takes on the most prevalent loss experienced in the data (Sharma, 2010). In contrast, the percentile approach aims to find the 99.9th percentile of the loan loss data, which produces a smoother result over time.

The BCBS' prescribed asset correlation is more conservative than the Beta_PD for the whole period under scrutiny (see Figure 11). The Beta_PD seems to coincidentally take on an average value of the V_Mode_PD and V_Percentile_PD's empirical asset correlations. This is an advantageous result for Banks that would like to adjust their economic capital depending on the prevailing economic conditions. It also achieves the BCBS' intended level of conservatism over a long period.

Figure 12 which presents Bank X's Credit Card Write-off loss data, shows much lower loss rates compared to both the South African Commercial loan and Bank X's Credit Card Default losses. Bank X's Credit Card Write-off losses increase gradually during the crisis and experience their largest loss around October 2009, which is similar to the period that the South African Commercial loans experienced their largest losses. Even though that is the case, the losses shown in Figure 12 are not as clear, nor as large as the both the South African Commercial loans and Bank X's Credit Card Default losses. This is due to the way this loss dataset was determined as it takes the monthly write-off data as a percentage of the total loans on the balance sheet.

A write-off usually happens when a customer has not made any payments for at-least six months or when the bank considers a customer's debt as noncollectable. Santucci (2016) tracks the disposition of revolving card balances that existed in March 2009, and found that only 27.8 percent of those balances had been written-off, while 72.2 percent were paid down. Thus, we can infer that the small percentage of losses resulting from the graph in figure 12 exhibits similar behaviour.

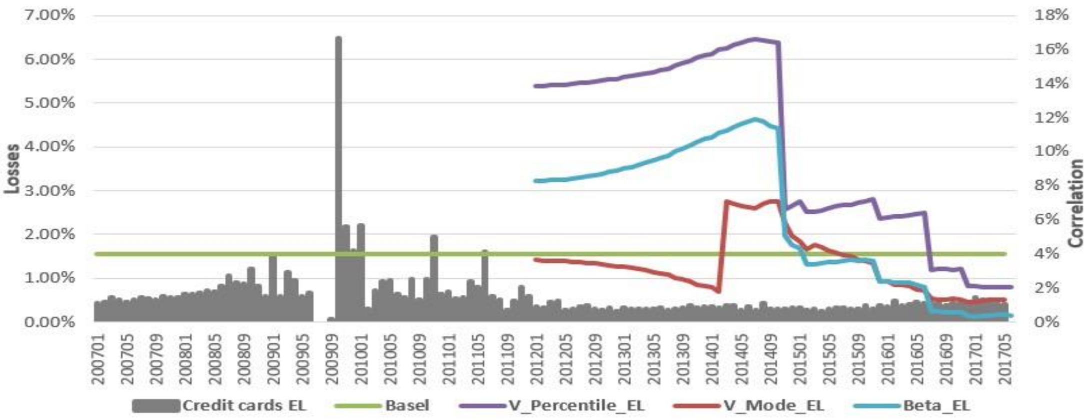


**Figure 12: A comparison of the 5-year rolling empirical asset correlations to the Basel specified rolling asset correlations for Bank X's Credit Card Write-off Data.**

Figure 12 illustrates that the BCBS' prescribed asset correlation is more conservative than the V_Mode_EL empirical asset correlation for the most part, except for a short period in 2014. The Mode approach gave very volatile results, with the empirical asset correlations being high as the effects of the recession wore off. It is therefore evident that the credit crisis resulted in an increase in the number of highly correlated assets with regards to the mode approach, and thus this should be taken into consideration when calculating in the BCBS' capital calculation. The V_Percentile_EL and Beta_EL had much higher empirical asset correlations than the BCBS' prescribed asset correlation up until the year 2015, where they continue on a downwards trend afterwards. These high correlations may be due to the effects of the recession wearing off. The V_Percentile_EL has a high correlation amount as expected because it produced a sub-optimal fit to the loss data as seen in Figure 3(b).

This section showed a trend of losses and their associated asset correlations over time. It demonstrated that the BCBS' empirical asset correlations do not always achieve their intended level of conservatism, especially in relation to a Bank X's Credit Cards population. This indicates that Bank X must review the level of capital they have reserved for Credit Cards unexpected losses and consider if it needs to be adjusted to the resulting economic capital levels. This result reveals that they would not have sufficient capital to cover all their unexpected losses in a stressed period.

The methods (Beta, Vasicek (Mode and percentile approach) that have been used in this study can be used by banks to determine when to raise or lower levels of economic capital appropriately depending on the prevailing economic conditions. This means banks using these methods have the power to determine their own unique em pirical capital requirements without blindly accepting all of the obscured parameters given in the Basel capital calculations.

## Conclusion

This study had two main objectives which will be addressed below. The first objective was to determine the empirical asset correlation values for a South African bank-specific credit cards default and write-off data, as well as for a South African commercial loan loss data and comparing these to the BCBS' prescribed asset correlations. These computed values are used to as direct inputs in the economic capital calculation and compared to the economic capital calculated using the BCBS' stated asset correlations. We used both the Beta and Vasicek distribution. To fully evaluate the empirical asset correlation using the Vasicek distribution we employed a mode approach and percentile approach, as both of them provided a good fit when looking at their cumulative and density functions. Our results reveal that the BCBS' stated asset correlations are much higher than the empirical asset correlations for the South African commercial loans dataset. However, our results on credit cards empirical asset correlations paint a different picture. It was found that the average empirical asset correlation estimated using default data was 5.76% (see table 8), while the average empirical asset correlation estimated using write-off data was 7% (see table 9).

**Table 8: The average asset correlation determined using the default data**

| Approach used | Correlation |
|---|---|
| V_Mode_PD | 8.47% |
| v_Perc_PD | 5.83% |
| Beta_PD | 2.99% |
| **Default data average** | 5.76% |

**Table 9: The average asset correlation determined using the write-off data**

| Approach used | Correlation |
|---|---|
| V_Mode_EL | 5.53% |
| Beta_EL | 8.47% |
| **Write-off data average** | 7.00% |

Table 8 presents Bank X's credit cards default data's empirical asset correlation results obtained using the three estimation approaches Beta, mode and percentile approach of the Vasicek distribution. Thus, the average empirical asset correlation determined using the default data is 5.76%. Table 9 presents Bank X's credit cards write-off data's empirical asset correlation results obtained using only two estimation approaches Beta, mode approach of the Vasicek distribution. The write-off data's empirical results estimated using the percentile approach were ignored from the final average as they provided a sub-optimal fit to the data, which resulted in extremely high asset correlations. Thus, the average empirical asset correlation determined using the write-off data is 7.00%.

**Table 10: A presentation of the correlation results based on publications**

| Researcher | Credit Cards |
|---|---|
| Mhlophe & Muteba. Mwamba (Present paper): Default data | 5.76% |
| Mhlophe & Muteba. Mwamba (Present paper): Write-off data | 7.00% |
| Stoffberg and van Vuuren (2015) | 7.00% |
| Bellotti and Crook (2012) | 0.39% |
| Hansen *et al* (2008) | 1.32% |
| Rosch and Scheule (2004) | 0.66% |

Table 10 presents correlation results based on different publications. In this table, we compare the average empirical asset correlations calculated in Table 8 and 9 for Bank X's credit cards default and write-off data respectively with the empirical asset correlations obtained from previous research studies. We observe that most of the previous studies had obtained empirical asset correlations which were considerably lower than the Basel stipulated asset correlation of 4%, with most of them obtaining empirical asset correlations less than 2%. All three of these researchers do not take the global recession into account as they used loss data for periods before 2008. The credit crunch dramatically increased credit losses (Newscientist, 2009), therefore it is assumed that their results would've differed had their research included the impact of the global recession on the loss rate. This may be evident in the results
obtained in Stoffberg & van Vuuren (2015), which had a credit cards empirical asset correlation as high as 7% for a US credit cards portfolio. Their result is consistent with the results obtained in our study. What is quite interesting is that they also use write-off data which gives exactly the same average empirical asset correlation as our credit cards empirical asset correlation, especially given that these calculations have been conducted in both a developing economy (South Africa) and a developed economy (US). This reveals that the losses experienced during the 2008 credit crisis have dramatically increased the real asset correlations, which is one of considerations that separates these two studies from the previous research that obtained lower empirical asset correlations. This result however implies that the required credit cards capital charge stipulated by the BCBS is not sufficient to cover potential unexpected losses. This implication is consistent with the economic capital calculated using the empirical asset correlations in this study, which were higher than the economic capital calculated using the BCBS' stated asset correlations. Thus, Bank X needs to increase their capital reserves to match those that have been estimated in this study.
The second objective of the study was to determine the asset correlations for a bank-specific credit cards default and write-off portfolio, as well as South African commercial loans dataset over a rolling five-year period, in order to the observe the sensitivity of the asset correlations over different economic conditions. We find that the South African commercial loans portfolio demonstrates the BCBS' intended conservatism, as it has a lower empirical asset correlation over the whole period. However, the default data had correlation which were lower until they surpassed the Basel asset correlation in 2012, while the opposite was seen in the write-off data which had a higher empirical correlation for the whole period until 2015. This demonstrates that the BCBS' intended conservatism is not always applicable especially in periods of enhanced risk.
This study has shown how to easily estimate empirical asset correlations from retail loan loss data. Using the beta, mode and percentile approaches of the vasicek distribution, this study has shown how to easily extract empirical asset correlations with only gross loss data and how these differ from the BCBS' prescribed asset correlations, and how they react to rolling them in different economic conditions. The computed empirical asset correlations are then used to calculate the economic capital.
This study is quite beneficial and the methods used are easy to apply for banks who have their own internal loss data and are interested in determining their own estimate of the correlation for both economic and regulatory capital purposes. The credit cards empirical asset correlation results imply that the BCBS' prescribed asset correlation is not as conservative as intended for South African bank specific credit cards and that the required capital charge stipulated by the BCBS (imposed by the

SARB) is not sufficient to cover unexpected losses. A higher asset correlation results in higher unexpected losses thus requiring a higher capital charge. Unexpected losses larger than Bank X's current capital buffer may have dire consequences for not only Bank X, but for the whole South African banking system through systemic risk. Therefore, we recommend that the Bank X raise their SARB imposed capital levels to match those determined in this study. Secondly, we recommend that the SARB should propose to other South African banks to use estimation approaches shown in this study to empirically estimate the asset correlations using their own internal loss data banks. These computed empirical asset correlations can thereafter be used to inform them of their real level of economic capital.
Finally, a study may consider research on empirical asset correlations and establish their impact on the Basel III capital regulations.

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