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University of St. Gallen

2019

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Abstract

Schumpeterian growth theory based on creative destruction was originally designed for continuous time innovation and growth models. However its recently expanding use in DSGE modelling calls for an easily usable discrete time recast. We here show how to construct a discrete time version of creative destruction fully equivalent to its continuous time counterpart.

JEL classification: O30, O40.
Keywords: R&D and Growth; Creative Destruction; Discrete Time; DSGE.

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1 Introduction

Following Benigno and Fornaro's (2018) benchmark contribution on the unemployment and growth consequences of the zero lower bound constraint of monetary policy in the presence of nominal frictions and Schumpeterian creative destruction, more and more authors[1] are currently trying to integrate creative destruction-driven growth with dynamic stochastic general equilibrium (DSGE) modelling. The source of growth used in the literature borrows much from the established research and development (R&D) and growth theory based on Schumpeterian creative destruction (Aghion and Howitt, 1992, Grossman and Helpman, 1991, etc), which has the advantage of being consistent with the microeconomic evidence that resource reallocation from less productive obsolete firms to more productive innovative firms is important for growth. However, the R&D and innovation technology used in this literature is explicitly designed for continuous time. In particular, creative destruction follows an endogenous innovation probability per unit time modelled as a Poisson process. When recast in discrete time, which is necessary for usual DSGE modelling, the simplifying properties of the Poisson process are lost, with potentially devastating complications. In particular, the discrete time models, by assuming that one innovation is possible per period, literally taken imply that if more firms are trying to innovate, it is possible that more than one happens to patent the innovation at the end of the period. With free entry of an indefinite number of R&D firms, the distribution of potential patent holders at the end of the period becomes too complex. An elegant way out of this problem is to assume that in each sector and in each period one and only one entrepreneur is randomly selected with the opportunity to try to innovate.[2] However, while insightfully introducing into creative destruction the concept of the scarcity of innovations (Scotchmer, 2004), this sacrifices free entry into R&D, at the heart of the growth driven by Schumpeterian patent races (Aghion and Howitt, 1992, Grossman and Helpman, 1991). Alternatively, in order to maintain Schumpeterian patent races, Benigno and Fornaro (2018) assume a very small time unit that approximately behaves like continuous time

[1] See, for example, Pinchetti (2016), and Cozzi et. al (2017).

[2] Aghion, Howitt, and Mayer-Foulkes (2005) pioneered this approach. Also see Aghion and Howitt (2009) for several very interesting applications (not in DSGE). See Nuno (2011) for a real business cycle application.

In this paper we will generalize Benigno and Fornaro's (2018) assumption and show how a simple to apply discrete time innovation process leads to a straightforward translation of the continuous time modelling into discrete time. This is potentially useful to microfound the generality of the Schumpeterian DSGE models. In particular, while we accept the usual discrete time models' assumption that only one innovation is found per period, we will maintain the continuous time implication that only one firm is the first to find the innovation. This is, in our opinion, very natural, because a discrete time patent race is a tractable parody of a more realistic patent race in continuous time. Hence, given the cardinality of the continuum, the probability of two firms simultaneously winning the patent race is indeed zero. We claim that this property should never be lost in the discrete time simplification of the patent race-driven growth models.

The rest of this paper is organized as follows. Section 2 shows the need of recasting innovation from continuous time to discrete time. Section 3 shows our solution to this problem. Section 4 concludes.

2 One Process fits All?

2.1 Continuous Time - a Refresh

In the standard quality ladder model of Aghion and Howitt (1992 and 2009), Grossman and Helpman (1991), Segerstrom (1998), etc. time is continuous, and there is a continuum of differentiated consumption or intermediate goods $\omega \in [0,1]$, with vertical innovation carried out by outsider R&D firms. At any time t, due to instantaneous price competition and constant returns to scale, each sector $\omega \in [0,1]$ is temporarily monopolized by the owner of the blueprint on the top quality product is $j(\omega,t) \in N$, until an outsider R&D firm manages to invent the $j(\omega,t)+1$st quality as a result of its R&D investment. Let $l(\omega,h,t)$ denote the R&D employment[3] of firm h in sector ω at date t, with $w(t)$ the corresponding real wage. It is usually assumed that resulting probability intensity of innovation per unit time by firm h is

$$I(\omega,h,t)=\frac{l(\omega,h,t)}{X(\omega,t)} \tag{1}$$

[3]Results would be identical if we assumed that also final or intermediate goods are used in R&D.

where $X(\omega, t)$ denotes a potentially time varying difficulty index of R&D in the sector.[4]

Since all innovative Poisson processes are assumed to be independent across firms and sectors, we can write the sectorial probability, $I(\omega, t)$, per unit time of a quality jump by summing (1) for the number $H(\omega, t) \in N$ of R&D firms in sector ω active at time t, that is:

$$I(\omega, t) = \sum_{h=1}^{H(\omega,t)} I(\omega, h, t). \tag{2}$$

Notice that in eq. (2) we simply summed the firm probabilities because the probability of two innovations occurring at the same time is zero. Hence the individual firm's probability of appropriating the innovation per unit time remains the same, regardless of the total flow probability.

Using (1) and (2), a generic R&D firm h's expected profit maximization at time t in an instantaneous patent race for product quality $j(\omega, t) + 1$ of value $V(\omega, j(\omega, t) + 1, t)$ can be rewritten as

$$\max_{l \geq 0} \frac{l(\omega, h, t)}{X(\omega, t)} V(\omega, j(\omega, t) + 1, t + 1) - w(t) l(\omega, h, t). \tag{3}$$

This leads to the R&D free entry (zero profit) condition

$$\frac{V(\omega, j(\omega, t) + 1, t + 1)}{X(\omega, t)} = w(t) \tag{4}$$

as in standard Schumpeterian models (Aghion and Howitt, 1992, Grossman and Helpman, 1991, Segerstrom, 1998, Howitt, 1999, etc).

2.2 Discrete Time

Let us now abandon continuous time and assume that, like the generality of DSGE models, time is discrete, $t = 0, 1, 2, ...$. As in the literature we make the following:

Assumption 1. Only one innovation can be made and patented per period.

Remark. This means that there is only one patent race per period.

[4]For example, Segerstrom (1998) and Howitt (1999) use it to elimate the strong scale effect with semi-endogenous respectively endogenous growth effects.

Consistently with the continuous time industry's innovation process in which the single firm contribution $I(\omega, h, t)$ from eq. (1) is part the total probability of success - now constrained not to exceed 1. In fact, we will also assume the following:

Assumption 2. The total probability of the new product of quality $j(\omega, t)+1$ being invented at time $t+1$ is

$$I(\omega, t) = \min \left\{ \sum_{h=1}^{H(\omega,t)} I(\omega, h, t), 1 \right\}. \tag{5}$$

So far nothing new. However, the previous two assumptions leave the door open to the possibility that more firms will win the patent race, which would contrast the very concept of realistic patent races, which do take place in continuos time. Moreover, literally relying on these two assumptions, as we try to study the R&D firm's optimizing behavior and the industry's R&D free entry condition, complications start: for example, if in a duopoly each firm taken in isolation had probability 1/3 of appropriating the quality jump, each will be the only one to make the quality jump only with probability $(1-1/3)1/3$, that is 2/9. If both firms innovate, which happens with probability 1/9, the patent has to be either shared or randomly assigned. With a generic number, $H(\omega, t)$, of firms in the industry it becomes impossible to write down an R&D free entry condition as simple as eq. (4).

3 A Simple Solution

We propose a simple and harmless solution, based on the consideration that between the beginning and the end of a discrete time runs a continuous time patent race, in which the probability of simultaneous innovation and patenting is zero. This does not require that the period is vanishingly small: if the time unit is quarterly, within three months of R&D there will be a first firm that finds the idea and patents it, thereby appropriating all the value of that period's innovation. Hence we us make the following:

Assumption 3. If firm h wins the patent race in period t no other firm $h' \neq h$ can also win it.

Remark. Our assumption means that if in reality the patent race between t and $t+1$ occurs in continuous time, the discrete time approximation shall just observe which firm has been the winner in period $[t, t+1]$, rather than allowing the completely unrealistic assumption of more firms having won that race.

We will also make the following:

Assumption 4. The probability of firm h's being the inventor of this new good, conditional on the good being invented, is

$$\frac{I(\omega, h, t)}{I(\omega, t)}. \tag{6}$$

Remark. Note that the total probability of innovation in the industry and the chances of a generic firm h succeeding in the patent rate will depend on the whole set of probability inputs $\{I(\omega, h, t)\}_{h=1}^{H(\omega,t)}$. However notice that at the aggregate economy level all single industry processes can safely be assumed independent.

Consequently, the probability of R&D success for firm h in sector ω is just the probability of the innovation happening, which depends on the aggregate R&D in sector ω, multiplied by the probability of appropriating it conditional on the innovation happening, that is:

$$I(\omega, t)\frac{I(\omega, h, t)}{I(\omega, t)} = I(\omega, h, t). \tag{7}$$

Using (1) and(7), a generic R&D firm h's expected profit maximization in a patent race in period t for product quality $j(\omega, t)+1$ of value $V(\omega, j(\omega, t)+1, t+1)$ can be rewritten as

$$\max_{l \geq 0} \frac{l(\omega, h, t)}{X(\omega, t)} E_t \left[V(\omega, j(\omega, t) + 1, t + 1)\right] - w(t)l(\omega, h, t) \tag{8}$$

where E_t is the expectation operator conditional on information up to time t. This leads to the same free entry condition

$$\frac{E_t \left[V(\omega, j(\omega, t) + 1, t + 1)\right]}{X(\omega, t)} = w(t) \tag{9}$$

as in eq. (4).

3.1 Robustness

3.1.1 Stepping on Toes

Our result can be easily generalized to R&D production functions incorporating the widely adopted Jones and Williams' (1998) "stepping-on-toes" negative externalities of industry R&D. In fact, we could rewrite (1) as

$$I(\omega, h, t) = \frac{l(\omega, h, t)}{X(\omega, t)} \left(\frac{l(\omega, t)}{X(\omega, t)} \right)^{-a} \tag{10}$$

where

$$l(\omega, t) \equiv \frac{\sum_{h'=1}^{H(\omega,t)} l(\omega, h', t)}{H(\omega, t)}$$

is the average R&D employment in industry ω in period t. The R&D firm expected profit maximization would become

$$\max_{l(\omega,h,t) \geq 0} \frac{l(\omega, h, t)}{X(\omega, t)} \left(\frac{l(\omega, t)}{X(\omega, t)} \right)^{-a} E_t \left[V(\omega, j(\omega, t) + 1, t + 1) \right] - w(t) l(\omega, h, t) \tag{11}$$

leading to the modified free entry condition

$$\frac{E_t \left[V(\omega, j(\omega, t) + 1, t + 1) \right]}{X(\omega, t)} \left(\frac{l(\omega, t)}{X(\omega, t)} \right)^{-a} = w(t). \tag{12}$$

In a symmetric equilibrium $l(\omega, h, t) = l(\omega, t)$, so that (10) simplifies to

$$I(\omega, h, t) = \left(\frac{l(\omega, t)}{X(\omega, t)} \right)^{1-a} = I(\omega, t), \tag{13}$$

which gives equilibrium first order condition

$$\frac{E_t \left[V(\omega, j(\omega, t) + 1, t + 1) \right]}{X(\omega, t)} I(\omega, t)^{\frac{-a}{1-a}} = w(t). \tag{14}$$

Given the value of the future patent, $V(\omega, t + 1)$, the R&D difficulty index, $X(\omega, t)$, and the wage rate, $w(t)$, the probability of an innovation arriving at the end of period t is

$$I(\omega, t) = \left(\frac{E_t \left[V(\omega, j(\omega, t) + 1, t + 1) \right]}{X(\omega, t) w(t)} \right)^{\frac{1-a}{a}}. \tag{15}$$

4 Final Remarks

We have provided a simple definition of the R&D investment process in the Schumpeterian innovation process that allows a direct translation of the usual continuous time R&D equations into their workable discrete time counterpart. This is potentially useful for a whole class of DSGE models with Schumpeterian growth. We claim that our simplified approach is a natural consequence of assuming that in each sector and in each period there is one underlying patent race driving innovation.

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