Labor productivity, labor supply of the old, and economic growth

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Abstract

This study develops an overlapping generations model with human capital accumulation and endogenous labor supply of the old to examine the effects of an old agent's labor productivity on labor supply, educational investments, and economic growth. We present a unique existence of the balanced-growth-path (BGP) equilibrium and find that a rise in an old agent's labor productivity induces more labor supply of the old. Moreover, the growth rate at the BGP equilibrium is hump-shaped in an old agent's labor productivity. As an old agent's labor productivity grows, this growth rate first increases then decreases, which implies that there is no clear negative relationship between an old agent's labor productivity and economic growth.

Keywords: Human capital; OLG; Labor productivity; Labor supply of the old.

JEL Classification: J24, J26, O11.

1 Introduction

Many developed countries are facing the rapid aging of their population. Some believe that the rapid aging will slow down economic growth, because older workers' labor productivity is low relative to young workers (see, for instance, Gordon (2016)). This paper, however, proposes that this argument may not hold, even in a standard endogenous growth model.

This study develops a two-period overlapping generations (OLG) model in which both physical capital accumulation and human capital accumulation are growth engines and labor supply of the old is endogenized. Labor supply of the old in the model can be interpreted as their retirement age. The production function in the model shows constant-returns-to-scale in physical capital and human capital. A young agent is endowed with human capital, but can choose how much she wants to invest in education.

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Agents inelastically supply one unit of labor to work in order to earn a wage income. When agents become old, their wage depends on their human capital and labor productivity in old age. Agents can also decide how much of their time they wish to devote to working when they are old. Under this setting, we investigate how labor productivity of an old agent relative to a young agent affects the labor supply of the old and economic growth.

The main finding is that the economic growth rate can increase if the labor productivity of the old decreases. Why does this happen? High labor productivity of an old agent induces more educational investment when people are young because it will increase labor income in old age. At the same time, high labor productivity of an old agent raises wage income. Thus, the income effect reduces the labor supply of the old, while the substitution effect increases it. In the model, because the substitution effect dominates the income effect, an old agent supplies more labor as their labor productivity increases. This increases the aggregate effective units of labor, which decreases wage rate. Since educational investment by a young agent is a normal good, this makes a young agent to invest less in education. Specifically, when an old agent's labor productivity is too high, a young agent finds it optimal to invest less in education. The growth rate at the BGP equilibrium then decreases as an old agent's labor productivity rises. On the other hand, when an old agent's labor productivity is low, a young agent finds it optimal to invest more in education. The growth rate at the BGP equilibrium then increases as an old agent's labor productivity rises. Thus, the growth rate at the BGP equilibrium is hump-shaped in an old agent's labor productivity.

One related work to this paper is Acemoglu and Restrepo (2017). They also raise doubts about the negative relationship between aging populations and economic growth. It was shown that there is no clear negative relationship between aging and economic growth rate in the post-1990 era. The explanation given for this is the endogenous response of technology. If *aging* is defined by the ratio of old workers to young workers in the model, this paper shows that aging and economic growth rate are positively related when labor productivity of the old is low. The cause for this result is educational investment in young age, which is a different driving force from that in Acemoglu and Restrepo (2017).

Another related work to this paper is Kunze (2014), who develops an OLG model with human capital accumulation and finds a hump-shaped relationship between retirement age and economic growth. We note that the analysis by Kunze (2014) is based on a model in which retirement age is exogenous. Therefore, our paper complements the literature by analyzing the effects of retirement age on economic growth by considering an alternative setting of retirement age.

Most studies related to retirement age are interested in the effects of retirement age on pension benefits. Michel and Pestieau (2013) and Miyazaki (2014, 2016) study retirement age and optimal pay-as-you-go (PAYG) social security policy.[1] Chen and Miyazaki (2018) and Chen (2018) introduce fertility into this strand of the literature and examine the effects of retirement age and fertility on PAYG pensions.[2] Since retirement age can be either determined by law (mandatory retirement age) or optimally decided by individuals, one would like to know how these two settings of retirement age influence economic growth. While mandatory retirement age is considered in Miyazaki (2014), endogenous retirement age is examined in Miyazaki (2019). Moreover, the effects of both exogenous and endogenous retirement age are studied by Michel and Pestieau (2013), Chen and Miyazaki (2018), and Chen (2018).

[1] While Michel and Pestieau (2013) show that the first-best allocation cannot be achieved in a decentralized economy, Miyazaki (2019) argues that the first-best allocation can be achieved in such an economy.

[2] While fertility is endogenized in Chen and Miyazaki (2018), it is exogenous in Chen (2018).

The remainder of this paper is organized as follows. The next section develops a model with endogenous retirement time. Section 3 characterizes the BGP equilibrium and derives the growth rate along the BGP equilibrium. Section 4 analyzes the effects of an old agent's labor productivity on retirement age and the long-run economic growth rate. Section 5 discusses how our results will change if we consider different settings. Section 6 concludes.

2 Model

Time is discrete and continues forever; $t = 1,2,\ldots$. A population grows at rate $n > -1$, and assume that the population of the initial old is $L_0 = 1$. An agent lives for two periods: young, and old. A young agent in period t supplies h_t efficiency units of labor. The agent receives the market wage, w_t, spends it on consumption, c_t, private education, e_t, and savings, s_t. Thus, a young agent's budget constraint in period t is

$$c_t + e_t + s_t = w_t h_t. \tag{1}$$

If a young agent spends e_t for human capital accumulation in period t, her human capital in period $t+1$ will be

$$h_{t+1} = \theta e_t^{\eta} h_t^{1-\eta}, \tag{2}$$

where $\theta > 0$ and $\eta \in (0,1)$.

A young agent in period t becomes old in period $t+1$. The agent receives the return on savings, $R_{t+1}s_t$. In addition, the agent supplies $\chi l_{t+1}h_{t+1}$ efficiency units of labor, and receives the market wage, w_{t+1}, where χ is an old agent's labor productivity relative to a young agent and $l_{t+1} \in [0,1]$. The agent spends all of her income on consumption, d_{t+1}. Thus, the budget constraint in period $t+1$ is

$$d_{t+1} = w_{t+1}h_{t+1}\chi l_{t+1} + R_{t+1}s_t. \tag{3}$$

Combining Equations (1) and (3), an agent's lifetime budget constraint is

$$c_t + e_t + \frac{d_{t+1}}{R_{t+1}} = w_t h_t + \frac{w_{t+1}h_{t+1}\chi l_{t+1}}{R_{t+1}}. \tag{4}$$

Notice that the aggregate supply of capital is $K_{t+1} = N_t s_t$ and the aggregate supply of effective units of labor is $H_{t+1} = N_{t+1}h_{t+1}^{young} + N_t \chi l_{t+1} h_{t+1}^{old} = N_t(1+n+\chi l_{t+1})h_{t+1}$, where $h_{t+1} = h_{t+1}^{young} = h_{t+1}^{old}$.

An agent derives utility from consumption when she is young, and from consumption and leisure when she is old. Hence, an agent's lifetime utility is expressed by

$$U(c_t, d_{t+1}, l_{t+1}) := \ln(c_t) + \beta \ln(d_{t+1}) + \gamma \ln(1 - l_{t+1}), \tag{5}$$

where $\beta > 0$ and $\gamma > 0$ stand for the preference strength relative to the consumption when the agent is young.

Assume for simplicity that the population in this economy is constant over time and is normalized to be 1.

There is a representative firm in the economy. Its production function is expressed as

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha},$$

where $\alpha \in (0,1)$, and K_t and H_t are the aggregate capital stock and aggregate effective units of labor in the economy, respectively. Given the real wage, w_t, and the real rental rate of capital, r_t, the firm's profit in period t is

$$AK_t^{\alpha} H_t^{1-\alpha} - R_t K_t - w_t H_t.$$

Assume that capital is fully depreciated after production. Let $k_t := K_t/H_t$ denote capital per effective unit of labor in period t. Then, from the firm's problem, we derive the return of physical capital and wage rate as follows:

$$R_t = A\alpha k_t^{\alpha-1},$$
$$w_t = A(1-\alpha)k_t^{\alpha}.$$

The equilibrium concept is a standard *perfect foresight competitive equilibrium.* A *balanced-growth path (BGP)* equilibrium is an equilibrium such that for some $g = \frac{h_{t+1}}{h_t} > 0$, $\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{C_{t+1}}{C_t} = \frac{D_{t+1}}{D_t} = \frac{H_{t+1}}{H_t} = (1+n)g$ for all t, where C and D are aggregate consumption of the young and the old, respectively.

If an old agent's labor productivity is too low, then she will not supply her labor to the market. Then, no young agent invests human capital, which implies $e_t = 0$ for all t. If so, then an economy does not grow at all, and an equilibrium does not exist. To avoid non-existence of equilibrium, we impose the following assumption on χ throughout the paper.

Assumption 2.1. χ *satisfies*

$$\chi > (1+n)\frac{\alpha}{1-\alpha}\frac{\gamma}{\beta}.$$

3 Characterizing the equilibrium

The first-order conditions to an agent's problem induce

$$\begin{aligned} d_{t+1} &= \beta R_{t+1} c_t, &(6)\\ \gamma R_{t+1} c_t &= w_{t+1}\chi h_{t+1}(1-l_{t+1}), &(7)\\ R_{t+1} &= w_{t+1} l_{t+1}\chi\theta\eta e_t^{\eta-1} h_t^{1-\eta}. &(8) \end{aligned}$$

From Equation (7),

$$w_{t+1}\chi h_{t+1} l_{t+1} = w_{t+1}\chi h_{t+1} - \gamma R_{t+1} c_t \tag{9}$$

holds. Combining Equation (9) with Equation (8),

$$R_{t+1} e_t = w_{t+1} l_{t+1}\chi\eta h_{t+1} = \eta(w_{t+1}\chi h_{t+1} - \gamma R_{t+1} c_t) \Rightarrow e_t = \frac{\eta w_{t+1}\chi h_{t+1}}{R_{t+1}} - \eta\gamma c_t. \tag{10}$$

From Equations (4), (6), (9), and (10),

$$c_t = \frac{1}{1+\beta+\gamma(1-\eta)}\left[w_t h_t + \frac{(1-\eta)w_{t+1}\chi h_{t+1}}{R_{t+1}}\right]. \tag{11}$$

Since $s_t = w_t h_t - c_t - e_t$, we have

$$s_t = \frac{\beta+\gamma}{1+\beta+\gamma(1-\eta)} w_t h_t - \frac{w_{t+1}\chi h_{t+1}}{R_{t+1}} \frac{1+\eta\beta}{1+\beta+\gamma(1-\eta)}. \tag{12}$$

Using Equation (12), we obtain

$$\begin{aligned} k_{t+1} &= \frac{N_t s_t}{N_t h_{t+1}(1+n+\chi l_{t+1})} \\ &= \frac{\beta+\gamma}{1+\beta+\gamma(1-\eta)} A(1-\alpha)k_t^{\alpha} \frac{h_t}{h_{t+1}} \frac{1}{1+n+l_{t+1}\chi} - \frac{1+\eta\beta}{1+\beta+\gamma(1-\eta)} \frac{\chi}{1+n+l_{t+1}\chi} \frac{1-\alpha}{\alpha} k_{t+1}, \end{aligned}$$

where we use $w_t = A(1-\alpha)k_t^{\alpha}$ and $R_t = A\alpha k_t^{\alpha-1}$. Rearranging this equation, we have

$$k_{t+1} \frac{[1+\beta+\gamma(1-\eta)](1+n+l_{t+1}\chi)\alpha + (1+\eta\beta)\chi(1-\alpha)}{\alpha} = (\beta+\gamma)\frac{h_t}{h_{t+1}} A(1-\alpha)k_t^{\alpha} \tag{13}$$

From Equations (9) and (11),

$$1+n+l_{t+1}\chi = 1+n+\frac{(1+\beta)\chi}{1+\beta+\gamma(1-\eta)} - \frac{\gamma}{1+\beta+\gamma(1-\eta)} \frac{\alpha}{1-\alpha} \frac{1}{k_{t+1}} A(1-\alpha)k_t^{\alpha} \frac{h_t}{h_{t+1}}. \tag{14}$$

Plugging Equation (14) into Equation (13), we have

$$k_{t+1}\frac{h_{t+1}}{h_t} = \frac{[\beta(1-\alpha)+\gamma]A}{[1+\beta+\gamma(1-\eta)](1+n)+(1+\beta)\chi+\frac{(1+\eta\beta)\chi(1-\alpha)}{\alpha}} k_t^{\alpha}. \tag{15}$$

From Equations (14) and (15),

$$l_{t+1}\chi = \frac{(1-\alpha)\beta\chi - \alpha\gamma(1+n)}{\beta(1-\alpha)+\gamma}. \tag{16}$$

Note that under Assumption 1, $l_{t+1} > 0$ for all t.

Combining Equation (11) with Equation (10),

$$e_t = \frac{\eta(1+\beta)}{1+\beta+\gamma(1-\eta)} \frac{w_{t+1}\chi h_{t+1}}{R_{t+1}} - \frac{\eta\gamma}{1+\beta+\gamma(1-\eta)} w_t h_t. \tag{17}$$

Then,

$$\frac{h_{t+1}}{h_t} = \theta\left(\frac{e_t}{h_t}\right)^{\eta} = \theta\left(\frac{1-\alpha}{\alpha}\chi k_{t+1}\frac{h_{t+1}}{h_t}\frac{\eta(1+\beta)}{1+\beta+\gamma(1-\eta)} - \frac{\eta\gamma}{1+\beta+\gamma(1-\eta)}A(1-\alpha)k_t^{\alpha}\right)^{\eta}. \tag{18}$$

From Equations (18) and (15), we obtain

$$\begin{aligned} k_{t+1} = &\frac{\alpha A[\beta(1-\alpha)+\gamma]}{\theta\{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)\}^{1-\eta}} \\ &\times \frac{1}{\{(1-\alpha)\eta A[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]\}^{\eta}} \times k_t^{\alpha(1-\eta)}. \end{aligned} \tag{19}$$

From Equation (19), there is a unique stationary point, k^*,

$$k^* := \left[\begin{array}{c} \frac{\alpha A[\beta(1-\alpha)+\gamma]}{\theta\{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)\}^{1-\eta}} \\ \times \frac{1}{\{(1-\alpha)\eta A[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]\}^{\eta}} \end{array} \right]^{\frac{1}{1-\alpha(1-\eta)}}.$$

In the BGP equilibrium, the growth factor of the economy is

$$\begin{aligned} g &= \frac{h_{t+1}}{h_t} = \frac{[\beta(1-\alpha)+\gamma]A}{[1+\beta+\gamma(1-\eta)](1+n)+(1+\beta)\chi+\frac{(1+\eta\beta)\chi(1-\alpha)}{\alpha}} \frac{(k^*)^\alpha}{k^*} \\ &= \theta^{\frac{1-\alpha}{1-\alpha(1-\eta)}} \left[\frac{A\{\alpha[\beta(1-\alpha)+\gamma]\}^\alpha \{(1-\alpha)\eta[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]\}^{1-\alpha}}{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)} \right]^{\frac{\eta}{1-\alpha(1-\eta)}}. \end{aligned}$$

Lemma 3.1. *There is a unique BGP equilibrium in this model.*

4 Results

In this section, we examine how an old agent's labor productivity, χ, affects an old agent's labor supply and economic growth rate.

Proposition 4.1. *In the BGP equilibrium, an old agent supplies more labor if χ increases.*

Proof. From Equation (16),

$$l_{t+1} = \frac{(1-\alpha)\beta}{\beta(1-\alpha)+\gamma} - \frac{\alpha\gamma(1+n)}{\chi[\beta(1-\alpha)+\gamma]}.$$

If χ increases, then the second term decreases. Thus, l_{t+1} increases. *Q.E.D.*

High labor productivity of an old agent raises wage income, which decreases labor supply due to the income effect. However, the substitution effect increases labor supply of the old. This proposition shows that the substitution effect dominates the income effect.Note also that labor supply of the old is decreasing in population growth rate. Thus, recent population aging caused by low population growth rate in many developed countries will make old people to work longer, or retire later.

Proposition 4.2. *In the BGP equilibrium, the growth factor of the economy, g, satisfies*

$$\frac{\partial g}{\partial \chi} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \ \ if \ \ \chi \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \overline{\chi} := (1+n) \left\{ \frac{(1-\alpha)[1+\beta+\gamma(1-\eta)]}{\alpha(1+\beta)+(1+\eta\beta)(1-\alpha)} + \frac{\gamma}{(1-\alpha)\beta} \right\}. \tag{20}$$

Proof. See the Appendix. *Q.E.D.*

Proposition 4.2 shows that the economic growth rate in the BGP equilibrium is hump-shaped in χ. An increase in χ makes a young agent invest more in her human capital, that is, e_t increases. At the same time, an old agent supplies more labor as χ increases. This increases the aggregate effective units of labor. This decreases the wage rate, which makes a young agent to invest less in education. If the first effect dominates the second effect, then an increase in χ increases g. Otherwise, an increase in χ decreases g. Proposition 4.2 states that when χ is small, the first effect dominates the second effect, and as χ becomes larger, the second effect starts dominating the first effect. This suggests that even when an old agent's labor productivity is low, the economic growth rate can be higher than when an old agent's labor productivity is high. Note that g, decreases as n increases. Since $\overline{\chi}$ is increasing in n, as the population ages, low old people's labor productivity would be preferred to achieve higher growth of per capita consumption for the young and the old.

From Propositions 4.1 and 4.2, when χ is not so high, a positive relationship between aging and economic growth rate can be observed if aging is defined by the ratio of old workers to young workers, that is, l_{t+1} in this model. On the other hand, when χ is high, a negative relationship between aging and economic growth rate can be observed. This suggests that varying labor productivity of the old might be a factor that determines the relationship between aging and economic growth.

5 Discussion

In this section, we discuss how our results will change if we consider a different setting.

5.1 A CRRA utility function

The first case is that what happens if we consider a more general utility function such as a CRRA utility function.

Since a young agent's consumption is not important when we consider growth, consider the following modified young agent's problem:

$$\begin{aligned} \max_{e_t,s_t,d_{t+1},l_{t+1}} \quad & \beta\frac{d_{t+1}^{1-\varepsilon}}{1-\varepsilon}+\gamma\frac{(1-l_{t+1})^{1-\mu}}{1-\mu} \\ \text{s.t.} \quad & e_t+\frac{d_{t+1}}{R_{t+1}}=w_th_t+\frac{w_{t+1}h_{t+1}\chi l_{t+1}}{R_{t+1}}, \\ & h_{t+1}=\theta e_t^{\eta}h_t^{1-\eta}. \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} \gamma(1-l_{t+t1})^{-\mu} &= \beta d_{t+1}^{-\varepsilon}w_{t+1}\theta e_t^{\eta}h_t^{1-\eta}\chi \\ e_t &= \left[\frac{w_{t+1}\theta\eta\chi l_{t+1}}{R_{t+1}}\right]^{\frac{1}{1-\eta}}h_t \\ e_t+\frac{d_{t+1}}{R_{t+1}} &= w_th_t+\frac{w_{t+1}\theta e_t^{\eta}h_t^{1-\eta}\chi l_{t+1}}{R_{t+1}}. \end{aligned}$$

From these equations, we obtain

$$\frac{1}{R_{t+1}}\left[\frac{\beta w_{t+1}^{\frac{1}{1-\eta}}\theta^{\frac{1}{1-\eta}}h_t}{\gamma}\left(\frac{1}{R_{t+1}}\right)^{\frac{\eta}{1-\eta}}(1-l_{t+1})^{\mu}l_{t+1}^{\frac{\eta}{1-\eta}}\right]^{\frac{1}{\varepsilon}}=\frac{w_th_t}{\chi^{\frac{1}{(1-\eta)\varepsilon}}}+\left[\frac{w_{t+1}\theta l_{t+1}}{R_{t+1}}\right]^{\frac{1}{1-\eta}}h_t\eta^{\frac{\eta}{1-\eta}}\chi^{\frac{\varepsilon-1}{(1-\eta)\varepsilon}}. \quad (21)$$

When we use a natural-log utility function, Equation (21) holds with $\varepsilon = \mu = 1$. This implies that a change in χ does not change the value of the second term in the RHS of Equation (21). When $\varepsilon > 1$, a change in χ changes both terms in the RHS of Equation (21). Thus, a change in χ may decrease l for some parameter values, which implies Proposition 4.1 does not necessarily holds if the utility function is a CRRA function.

5.2 An elastic labor supply of a young agent

Next is to consider a case in which a young agent elastically supplies his/her labor. Consider the following modified young agent's problem:

$$\begin{aligned} \max_{e_t,s_t,d_{t+1},l_{t+1}} \quad & \sigma \ln(1-l_t^y) + \beta \ln(d_{t+1}) + \gamma \ln(1-l_{t+1}) \\ \text{s.t.} \quad & e_t + \frac{d_{t+1}}{R_{t+1}} = w_t h_t l_t^y + \frac{w_{t+1} h_{t+1} \chi l_{t+1}}{R_{t+1}}, \\ & h_{t+1} = \theta e_t^{\eta} h_t^{1-\eta}, \end{aligned}$$

where l_t^y and l_{t+1}^o are labor supply of a young agent and that of an old agent, respectively.

The first-order conditions are

$$\begin{aligned} w_t h_t (1-l_t^y) &= \frac{\sigma d_{t+1}}{\beta R_{t+1}}, \\ w_{t+1} h_{t+1} \chi (1-l_{t+1}^o) &= \frac{\gamma}{\beta} d_{t+1}, \\ w_{t+1} \theta \eta e_t^{\eta-1} h_t^{1-\eta} l_{t+1}^o \chi &= R_{t+1}. \end{aligned}$$

Combining them into the lifetime budget constraint, we obtain

$$d_{t+1} = \frac{\beta R_{t+1}}{\beta + \sigma + \gamma(1-\eta)} \left[w_t h_t + \frac{w_{t+1} h_{t+1} \chi (1-\eta)}{R_{t+1}} \right].$$

From this equation,

$$l_t^y = 1 - \frac{\sigma}{w_t h_t \left[\gamma(1-\eta) + \beta + \sigma\right]} \left[w_t h_t + \frac{w_{t+1} h_{t+1} \chi (1-\eta)}{R_{t+1}} \right],$$

and

$$l_{t+1}^o = 1 - \frac{\gamma R_{t+1}}{w_{t+1} h_{t+1} \chi [\gamma(1-\eta) + \beta + \sigma]} \left[w_t h_t + \frac{w_{t+1} h_{t+1} \chi (1-\eta)}{R_{t+1}} \right].$$

$\sigma = 0$ corresponds to the case we studied in the paper. In this setting, it seems true that a young agent's labor supply decreases as χ increases. When a young agent inelastically supplies his/her labor in the original model, an increase in χ increases an old agent's labor supply. Since an increase in χ decreases a young agent's labor supply, it is probable that an increase in χ also increases an old agent's labor supply in this case to compensate a decrease of effective labor. This may not hold if we consider a CRRA utility function.

6 Conclusion

This paper studies a two-period OLG model in which both physical capital stock and human capital stock are growth engines and an old agent endogenously chooses her labor supply. We characterize a unique BGP equilibrium and investigate the effect of an old agent's labor productivity on labor supply and on economic growth. We show that labor supply of the old will increase as an old agent's labor productivity becomes greater, and that the economic growth rate is hump-shaped in an old agent's labor productivity. Although some might expect a negative effect from an aging population on economic growth due to low labor productivity of the old, this paper's findings show that this pessimistic view is not necessarily true.

A Proof of Proposition 4.2

Proof. The sign of $\frac{\partial g}{\partial \chi}$ is determined by that of the derivative of $G := \frac{[\chi(1-\alpha)\beta - \alpha\gamma(1+n)]^{1-\alpha}}{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)}$ with respect to χ. Taking the derivative of G with respect to χ, we have

$$
\begin{aligned}
\frac{\partial G}{\partial \chi} &= \frac{(1-\alpha)[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]^{-\alpha}(1-\alpha)\beta}{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)} \\
&\quad - \frac{[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]^{1-\alpha}[\alpha(1+\beta)+(1+\eta\beta)(1-\alpha)]}{\{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)\}^2} \\
&= \frac{[\chi(1-\alpha)\beta-\alpha\gamma(1+n)]^{-\alpha}\alpha}{\{\alpha[1+\beta+\gamma(1-\eta)](1+n)+\alpha(1+\beta)\chi+(1+\eta\beta)\chi(1-\alpha)\}^2} \\
&\quad \times \left\{ \begin{array}{c} -(1-\alpha)\beta[\alpha(1+\beta)+(1+\eta\beta)(1-\alpha)]\chi \\ +(1-\alpha)^2\beta[(1+\beta)+\gamma(1-\eta)](1+n)+\gamma[\alpha(1+\beta)+(1+\eta\beta)(1-\alpha)](1+n) \end{array} \right\}.
\end{aligned}
$$

This completes the proof. *Q.E.D.*

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