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Abstract

We show the existence of involuntary unemployment without assuming wage rigidity. Key points of our analysis are indivisibility of labor supply and increasing returns to scale. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition with indivisibility of labor supply and increasing returns to scale technology.

Keywords: involuntary unemployment, monopolistic competition, indivisible labor supply, increasing returns to scale

JEL Classification No.: E12, E24.

1 Introduction

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firms' behavior is ad-hoc. In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition according to Otaki (2007), Otaki (2009), Otaki (2011) and Otaki (2015) with increasing returns to scale technology, and show the existence of involuntary

[1] Lavoie (2001) presented a similar analysis.

unemployment without assuming wage rigidity. Key points of our analysis are indivisibility of labor supply and increasing returns to scale. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is divisible and it can be small, there exists no unemployment. In the next section we analyze the relation between indivisibility of labor supply with increasing returns to scale technology and the existence of involuntary unemployment. We show that the real wage rate is increasing with respect to the employment, on the other hand the reservation real wage rate for individuals is constant given the expected inflation rate. Thus, when the real wage rate is larger than the reservation real wage rate, there exists no mechanism to reduce the difference between them without increasing unemployment. In Section 3 we present a general analysis of divisibility and indivisibility of labor supply. In Appendix 4 we present details of calculations.

2 Indivisible labor supply and involuntary unemployment

We consider a two-period (young and old) overlapping generations model under monopolistic competition according to Otaki (2007, 2009, 2011 and 2015). There is one factor of production, labor, and there is a continuum of goods indexed by $z \in [0, 1]$. Each good is monopolistically produced by Firm z. Consumers are born at continuous density $[0, 1] \times [0, 1]$ in each period. They can supply only one unit of labor when they are young (the first period).

2.1 Consumers

We use the following notations.

$c^i(z)$: consumption of good z at period i, $i = 1, 2$.
$p^i(z)$: the price of good z at period i, $i = 1, 2$.
$X^i = \left\{\int_0^1 c^i(z)^{1-\frac{1}{\eta}} dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}$, $i = 1, 2$, $\eta > 1$.
β: disutility of labor, $\beta > 0$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
L: employment of each firm and the total employment.
L_f: population of labor or employment at the full-employment state.
$y(L)$: labor productivity, which is increasing with respect to the employment, $y(L) \geq 1$.

δ is the definition function. If a consumer is employed, $\delta = 1$; if he is not employed, $\delta = 0$. The labor productivity is $y(L)$. It is increasing with respect to the employment of a firm. We define the employment elasticity of the labor productivity as follows.

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

We assume $0 < \zeta < 1$. Increasing returns to scale means $\zeta > 0$.

The utility of consumers of one generation over two periods is

$$U(X^1, X^2, \delta, \beta) = u(X^1, X^2) - \delta\beta.$$

We assume that $u(X^1, X^2)$ is homogeneous of degree one (linearly homogeneous). The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = \delta W + \Pi.$$

$p^2(z)$ is the expectation of the price of good z at period 2. The Lagrange function is

$$\mathcal{L} = u(X^1, X^2) - \delta\beta - \lambda \left(\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz - \delta W - \Pi \right).$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial X^1} \left\{ \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \tag{1}$$

and

$$\frac{\partial u}{\partial X^2} \left\{ \int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \tag{2}$$

They are rewritten as

$$\frac{\partial u}{\partial X^1} X^1 \left\{ \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right\}^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z)c^1(z), \tag{3}$$

$$\frac{\partial u}{\partial X^2} X^2 \left\{ \int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right\}^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z)c^2(z). \tag{4}$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}, \; P^2 = \left(\int_0^1 p^2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}.$$

They are price indices. By some calculations we obtain (please see Appendix)

$$u(X^1, X^2) = \lambda \left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz \right] = \lambda(\delta W + \Pi), \tag{5}$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

$$P^1X^1 + P^2X^2 = \delta W + \Pi. \tag{7}$$

The indirect utility of consumers is written as follows

$$V = \frac{1}{\varphi(P^1, P^2)}(\delta W + \Pi) - \delta\beta. \tag{8}$$

$\varphi(P^1, P^2)$ is a function which is homogeneous of degree one. The reservation nominal wage W^R is a solution of the following equation.

$$\frac{1}{\varphi(P^1, P^2)}(W^R + \Pi) - \beta = \frac{1}{\varphi(P^1, P^2)}\Pi.$$

From this

$$W^R = \varphi(P^1, P^2)\beta.$$

The labor supply is indivisible. If $W > W^R$, the total labor supply is L_f. If $W < W^R$, it is zero. If $W = W^R$, employment and unemployment are indifferent for consumers, and there exists no involuntary unemployment even if $L < L_f$. Indivisibility of labor supply may be due to the fact that there exists minimum standard of living even in the advanced economy (please see Otaki (2015)).

Let $\rho = \frac{P^2}{P^1}$. This is the expected inflation rate (plus one). Since $\varphi(P^1, P^2)$ is homogeneous of degree one, the reservation real wage is

$$\omega^R = \frac{W^R}{P^1} = \varphi(1, \rho)\beta.$$

If the value of ρ is given, ω^R is constant.

2.2 Firms

Let

$$\alpha = \frac{P^1X^1}{P^1X^1 + P^2X^2} = \frac{X^1}{X^1 + \rho X^2},\ 0 < \alpha < 1.$$

From (3) $\sim$ (7),

$$\alpha(\delta W + \Pi)\left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{-1} c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Since

$$X^1 = \frac{\alpha(\delta W + \Pi)}{P^1},$$

we have

$$\left(X^1\right)^{\frac{1}{\eta}-1} = \left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{-1} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(\delta W + \Pi)\left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}-1} c^1(z)^{-\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}} P^1c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1}\right)^{\frac{1}{\eta}} P^1\left(p^1(z)\right)^{-1}.$$

Hence,

$$c^1(z) = \frac{\alpha(\delta W + \Pi)}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. The total demand for good z is written as

$$c(z) = \frac{Y}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

Y is the effective demand defined by

$$Y = \alpha(WL + \Pi) + G + M.$$

G is the government expenditure and M is consumption by the old generation of good z (about this demand function please see Otaki (2007), Otaki (2009)). The total employment, the total profits, the total government expenditure and the total consumption by the old generation are

$$\int_0^1 L dz = L,\ \int_0^1 \Pi dz = \Pi,\ \int_0^1 G dz = G,\ \int_0^1 M dz = M.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)} = -\eta\frac{Y}{P^1}\frac{p^1(z)^{-1-\eta}}{(P^1)^{-\eta}} = -\eta\frac{c(z)}{p^1(z)}.$$

Since $c(z) = Ly(L)$, the profit of Firm z is

$$\pi(z) = p^1(z)Ly(L) - WL.$$

P^1 is given for Firm z. $y(L)$ is the productivity of labor, which is increasing with respect to the employment L.

The employment elasticity of the labor productivity is

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$Ly(L) + \left[p^1(z)(y(L) + Ly') - W\right]\frac{\partial L}{\partial p^1(z)} = 0. \tag{9}$$

From $c(z) = Ly(L)$,

$$\frac{\partial c(z)}{\partial p^1(z)} = (y(L) + Ly')\frac{\partial L}{\partial p^1(z)}.$$

Thus, (9) is rewritten as

$$c(z) + \left[p^1(z) - \frac{W}{y(L) + Ly'}\right]\frac{\partial c(z)}{\partial p^1(z)} = 0.$$

From this

$$p^1(z) = \frac{W}{y(L) + Ly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1+\zeta)y(L)} + \frac{1}{\eta}p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is smaller than that in a case without increasing returns to scale given the value of W.

2.3 Involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L)}.$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L). \tag{10}$$

If ζ is constant, this is increasing with respect to L.

The aggregate supply of the goods is equal to

$$WL + \Pi = P^1 L y(L).$$

The aggregate demand is

$$\alpha(WL + \Pi) + G + M = \alpha P^1 L y(L) + G + M.$$

Since they are equal,

$$P^1 L y(L) = \alpha P^1 L y(L) + G + M,$$

or

$$P^1 L y(L) = \frac{G + M}{1 - \alpha}.$$

In real terms[2]

$$Ly(L) = \frac{1}{1-\alpha}(g + m), \tag{11}$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

[2] $\frac{1}{1-\alpha}$ is a multiplier.

(11) means that the employment L is determined by $g + m$. It can not be larger than L_f. However, it may be strictly smaller than L_f ($L < L_f$). Then, there exists *involuntary umemployment*. Since the real wage rate $\omega = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)$ is increasing with respect to L, and the reservation real wage rate ω^R are constant, if $\omega > \omega^R$ there exists no mechanism to reduce the difference between them without increasing unemployment.

Summary of discussions The real aggregate demand and the employment are determined by the real value of $g + m$. The employment may be smaller than the population of labor, then there exists involuntary unemployment.

The real wage rate is increasing with respect to the employment and the reservation real wage rate are constant. Then, if the real wage rate is larger than the reservation real wage rate, there exists no mechanism to reduce the difference between them reducing unemployment.

Comment on the nominal wage rate In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G + M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G + M$, the real aggregate supply and the employment increases. Partition of the effects by an increase in $G + M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[3].

3 Divisibility and indivisibility of labor supply

The utility of the representative consumer is

$$U(X^1, X^2, l) = u(X^1, X^2) - G(l),$$

with the budget constraint

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = Wl + \Pi.$$

l is labor supply of an individual ($0 < l \leq 1$), and $G(l)$ is a function of disutility of labor which is strictly increasing, differentiable and strictly convex. Similarly to (8), we obtain the following indirect utility given l,

$$V = \frac{1}{\varphi(P^1, P^2)}(Wl + \Pi) - G(l). \tag{12}$$

Maximization of V with respect to l implies

$$W = \varphi(P^1, P^2)G'(l). \tag{13}$$

[3] Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining.

Let $\rho = \frac{P^2}{P^1}$. (13) is rewritten as

$$\omega = \varphi(1, \rho)G'(l), \tag{14}$$

where $\omega = \frac{W}{P^1}$ is the real wage rate. If the value of ρ is given, l is obtained from (14) as a function of ω. l is increasing in ω because $G'' > 0$. In our model, however, from (10) $\omega = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L_f l)$. Thus, we have

$$\left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L_f l) = \varphi(1, \rho)G'(l). \tag{15}$$

The value of l is obtained from (15). It does not depend on W given ρ. The total labor supply is $L_f l$. It is constant. L_f is the population of labor. If $L_f l$ is not larger than the labor demand, there exists no unemployment, that is, full-employment is realized. Then, the aggregate supply of the goods is

$$P^1 L_f l y(L_f l).$$

The aggregate demand is

$$\alpha\left(W L_f l y(L_f l) + \Pi\right) + G + M = \alpha P^1 L_f l y(L_f l) + G + M.$$

Since they are equal,

$$P^1 L_f l y(L_f l) = \alpha P^1 L_f l y(L_f l) + G + M.$$

This means

$$P^1 = \frac{G + M}{(1 - \alpha)L_f l y(L_f l)}.$$

Because $L_f l$ is constant, the price P^1 is determined by $G + M$. Then, the nominal wage is set by $W = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L_f l)P^1$. In real terms

$$L_f l y(L_f l) = \frac{g + m}{1 - \alpha}, \tag{16}$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

(16) is an identity not an equation. Thus, we should write it as follows.

$$L_f l y(L_f l) \equiv \frac{g + m}{1 - \alpha}.$$

On the other hand, (11) in the previous section is an equation not an identity.

If

$$\left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L_f l) \geq \varphi(1, \rho)G'(l) \text{ for any } 0 < l \leq 1,$$

consumers choose $l = 1$, and then the labor supply is indivisible.

4 Concluding Remark

In this paper we have examined the existence of involuntary umemployment using a monopolistic competition model with increasing returns to scale. We have derived involuntary unemployment from indivisibility of labor supply. We think that although the labor supply must not be infinitely divisible, it need not be infinitely indivisible. In the future research we want to consider the effects of fiscal policies in a state with involuntary unemployment.

Appendix: Derivations of (5), (6), (7) and (8)

From (3) and (4)

$$\frac{\partial u}{\partial X^1}X^1\left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^1}X^1=\lambda\int_0^1 p^1(z)c^1(z)dz,$$

$$\frac{\partial u}{\partial X^2}\left\{\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^2}X^2=\lambda\int_0^1 p^2(z)c^2(z)dz.$$

Since $u(X^1,X^2)$ is homogeneous of degree one,

$$u(X^1,X^2)=\frac{\partial u}{\partial X^1}X^1+\frac{\partial u}{\partial X^2}X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z)c^1(z)dz}{\int_0^1 p^2(z)c^2(z)dz}=\frac{\frac{\partial u}{\partial X^1}X^1}{\frac{\partial u}{\partial X^2}X^2},$$

and

$$u(X^1,X^2)=\lambda\left[\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz\right]=\lambda(\delta W+\Pi). \tag{5}$$

From (1) and (2), we have

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}c^1(z)^{1-\frac{1}{\eta}}=\lambda^{1-\eta}p^1(z)^{1-\eta},$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left\{\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}c^2(z)^{1-\frac{1}{\eta}}=\lambda^{1-\eta}p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\lambda^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz,$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left\{\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right\}^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz = \lambda^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1} = \lambda\left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = \lambda P^1,$$

and

$$\frac{\partial u}{\partial X^2} = \lambda\left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = \lambda P^2.$$

From them we get

$$u(X^1, X^2) = \lambda(P^1X^1 + P^2X^2),$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

and

$$P^1X^1 + P^2X^2 = \delta W + \Pi. \tag{7}$$

Since $u(X^1, X^2)$ is homogeneous of degree one, λ is a function of P^1 and P^2, and $\frac{1}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $\delta W + \Pi$. We obtain the following indirect utility function.

$$V = \frac{1}{\varphi(P^1, P^2)}(\delta W + \Pi). \tag{8}$$

$\varphi(P^1, P^2)$ is a function which is homogenous of degree one.

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